Open Quantum Systems
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thp Institute for Theoretical Physics University of Cologne

Probing Topology in Finite Temperature and Non-Equilibrium Quantum States

+ Non-equilibrium Phase Transition to Chaos

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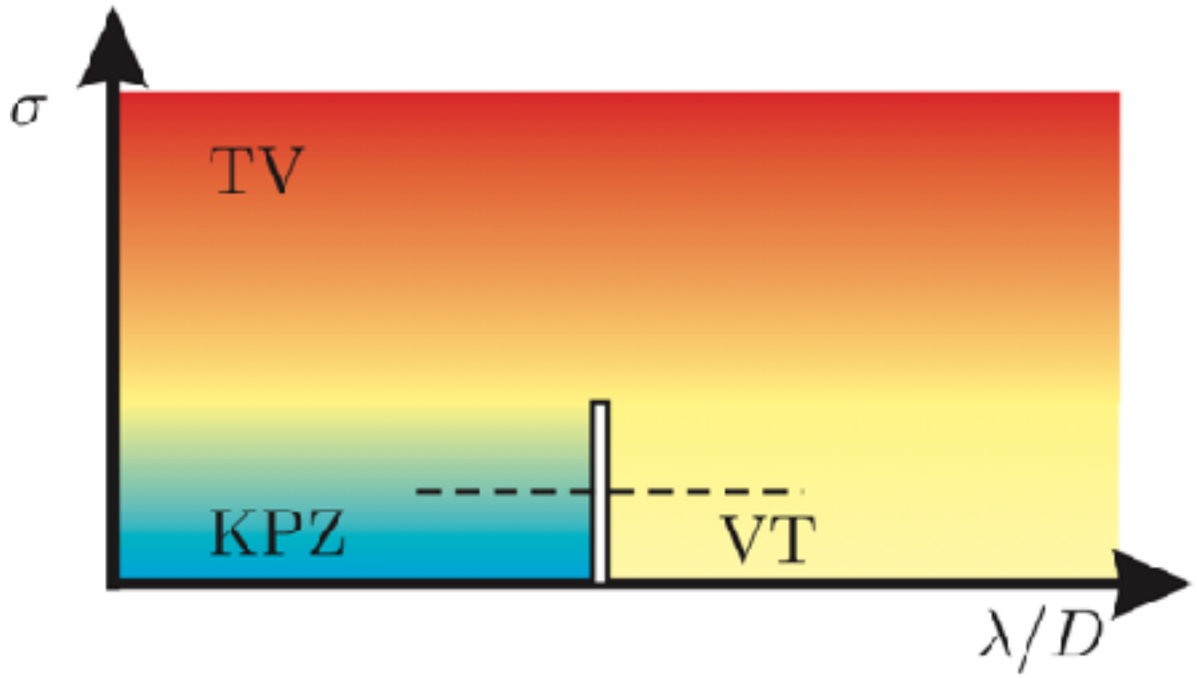
Based on

C.-E. Bardyn, L. Wawer, A. Altland, M. Fleischhauer, SD, arxiv:1706:02741

€ € € : erc European Research Council

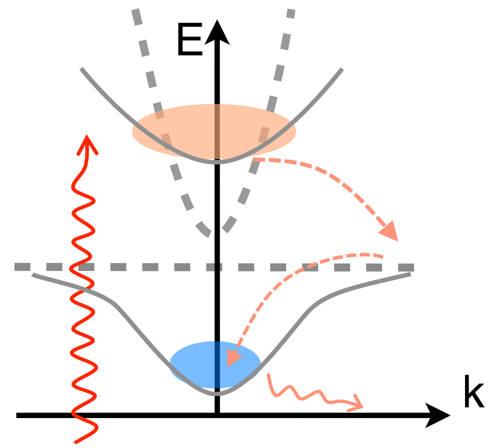
DFG Deutsche Forschungsgemeinschaft

I. Non-equilibrium Phase Transition to Chaos



Exciton-polariton dynamics at small frequency

- starting point:



- driven-dissipative stochastic Gross-Pitaevski equation

$$i\partial_t \phi = \left[-\frac{\nabla^2}{2m} - \mu + i(\gamma_p - \gamma_l) + (g - i\kappa)\,|\phi|^2 \right] \phi + \zeta$$

$$\phi = \rho e^{i\theta}$$

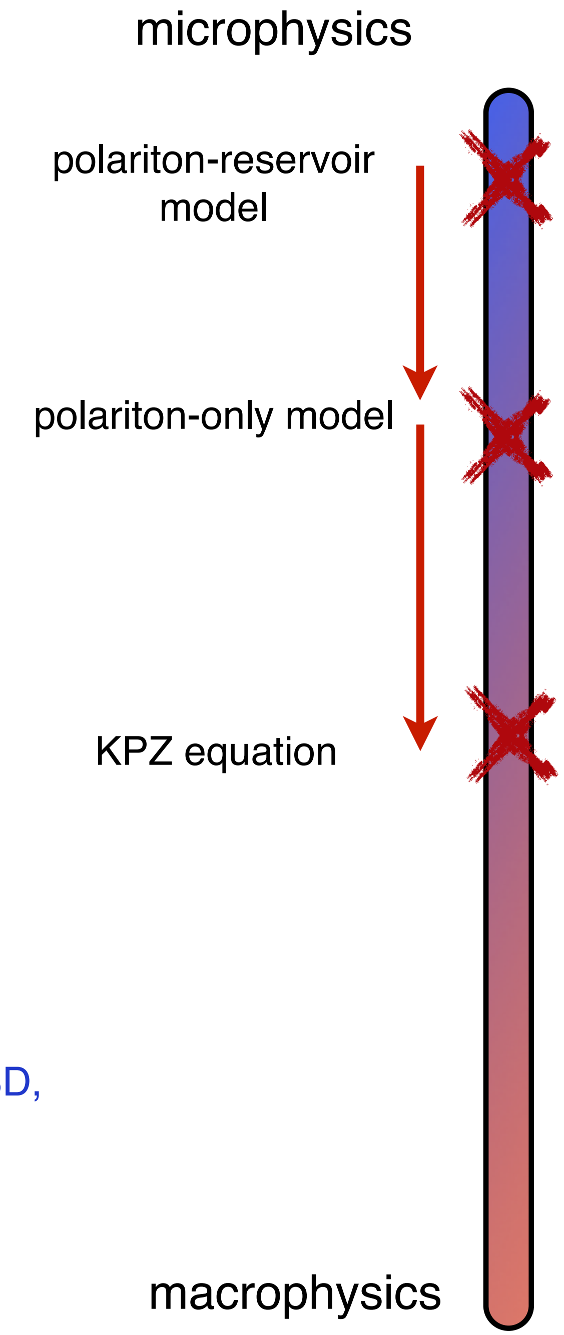
- effective low frequency dynamics

E. Altman, L. Sieberer, L, Chen, SD,
J. Toner, PRX (2015)

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

phase diffusion   phase nonlinearity   Markov noise

form of the KPZ equation   Kardar, Parisi, Zhang, PRL (1986)



- nonlinearity: single-parameter measure of non-equilibrium strength
(ruled out in equilibrium by detailed balance (symmetry))

Compact KPZ

L. Sieberer, G. Wachtel, E. Altman, SD, PRB (2016)
G. Wachtel, L. Sieberer, SD, E. Altman, PRB (2016)

- wait a second — we ignored a fundamental symmetry of polaritons so far: local discrete gauge invariance

$$\phi(t,\mathbf{x}) = \rho(t,\mathbf{x})e^{i\theta(t,\mathbf{x})} \qquad \theta_{t,\mathbf{x}} \mapsto \theta_{t,\mathbf{x}} + 2\pi n_{t,\mathbf{x}}$$

- Teaching symmetry to KPZ equation:

$$\theta_{t+\epsilon,\mathbf{x}} = \theta_{t,\mathbf{x}} + \epsilon\left(\mathcal{L}[\theta]_{t,\mathbf{x}} + \eta_{t,\mathbf{x}}\right) + 2\pi n_{t,\mathbf{x}}$$

lattice regularized deterministic term

stochastic difference equation

$$Z = \sum_{\{\tilde{n}_{t,\mathbf{x}}\}} \int \mathcal{D}[\theta] e^{iS[\theta,\tilde{n}]}$$

discrete noise MSRJD functional integral

non-eq. lattice gauge theory

$$Z \propto \sum_{\substack{\{n_{vX},\tilde{n}_{vX},\\ \mathbf{J}_{vX},\tilde{\mathbf{J}}_{vX}\}}} \int \mathcal{D}[\phi,\tilde{\phi},\mathbf{A},\tilde{\mathbf{A}}] e^{iS[\phi,\tilde{\phi},\mathbf{A},\tilde{\mathbf{A}},n_v,\tilde{n}_v,\mathbf{J}_v,\tilde{\mathbf{J}}_v]}$$

non-equilibrium electrodynamic theory

Non-equilibrium electrodynamic duality:

Electric field <=> smooth phase fluct. (KPZ)

Charges <=> vortices

Dynamical & non-equilibrium analog of Kosterlitz-Thouless construction

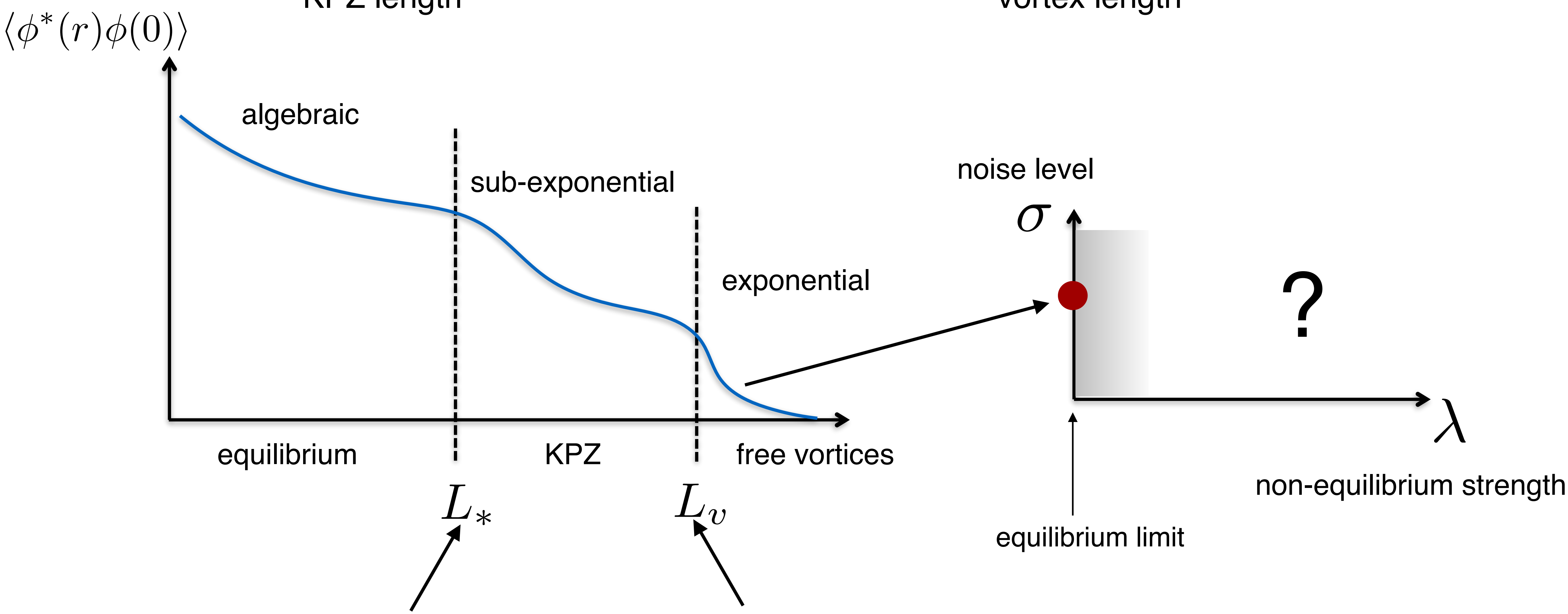
L. Sieberer, G. Wachtel, E. Altman, SD, PRB (2016)
G. Wachtel, L. Sieberer, SD, E. Altman, PRB (2016)

2D: Competing Length Scales and Suppression of KT

- two emergent length scales:

$$L_* = a_0 e^{\frac{16\pi D^3}{\lambda^2 \Delta}} \qquad L_v = a_0 e^{\frac{2D}{\lambda}}$$

KPZ length | vortex length



Mechanism: smooth non-linear phase diffusion | noise-activated vortex unbinding

Kosterlitz-Thouless physics fragile to non-equilibrium perturbation

- Exponentially large in deviation from equilibrium sets challenge to experiment

Strong non-equilibrium: Compact KPZ vortex turbulence

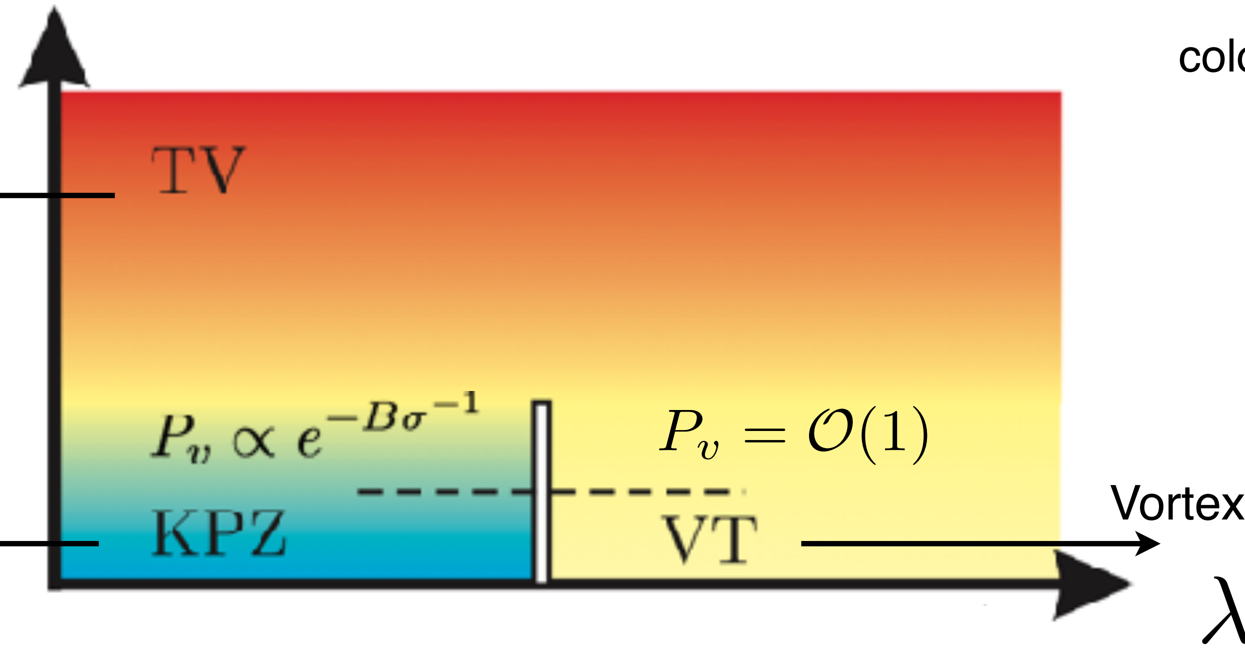
L. He, L. Sieberer, SD PRL (2017)

- In search of the phase diagram for XP condensates

noise level σ

Noise activated vortices (TV)

KPZ / equilibrium dominated physics



color code: vortex density on space-time plane

P_v

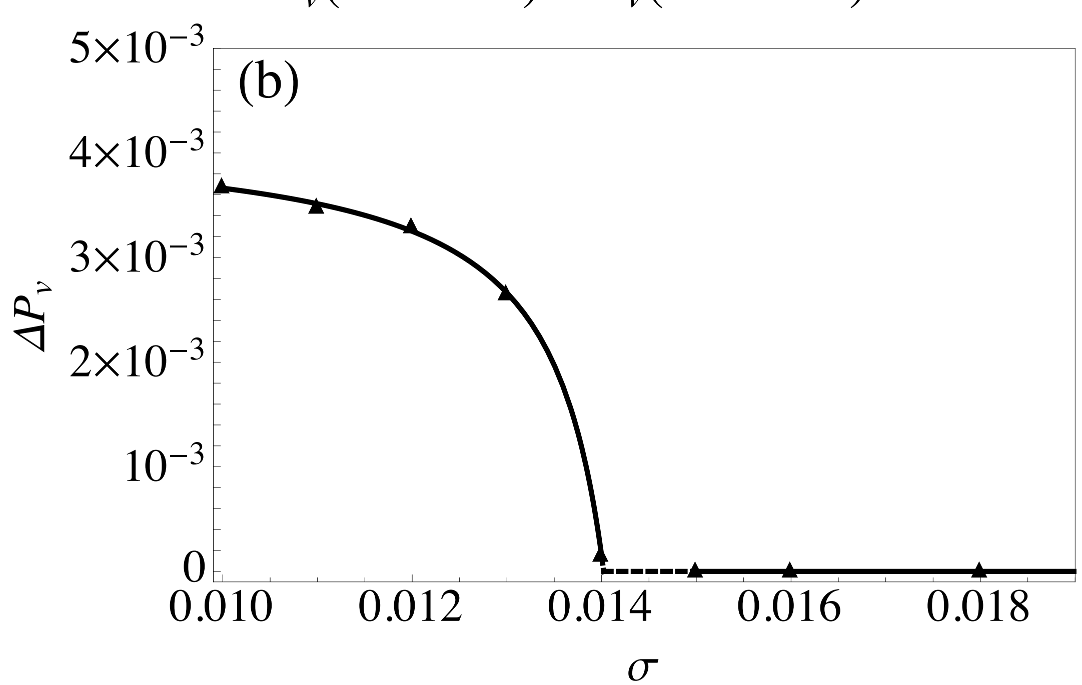
Vortex turbulence (VT)

first order non-equilibrium phase transition

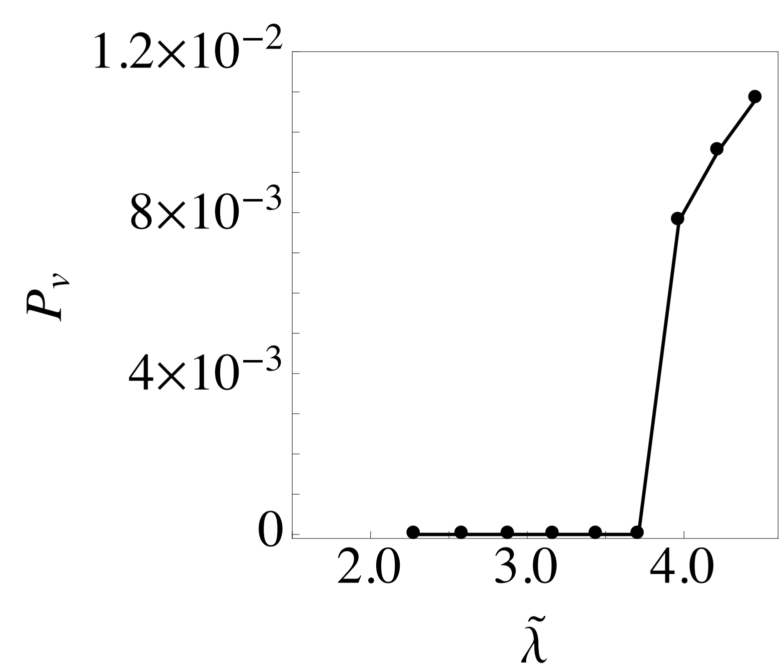
non-equilibrium strength

one dimension (vortex = space-time defect)

$P_v(\tilde{\lambda} \to \tilde{\lambda}^{*+})$ - $P_v(\tilde{\lambda} \to \tilde{\lambda}^{*-})$



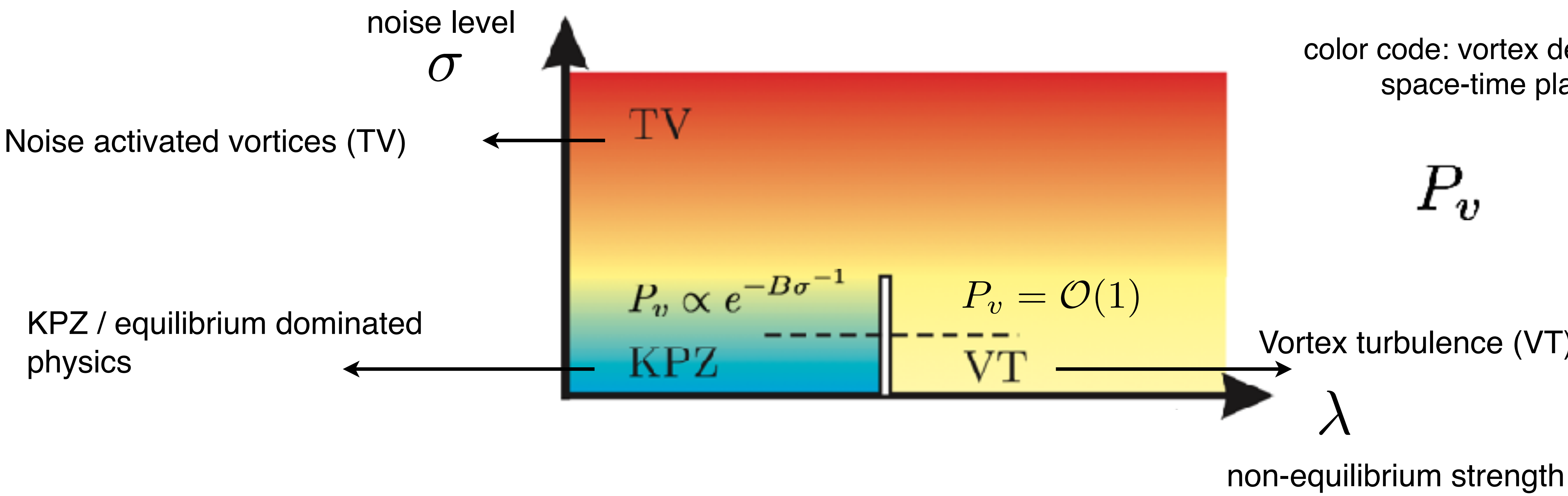
two dimensions (preliminary data)



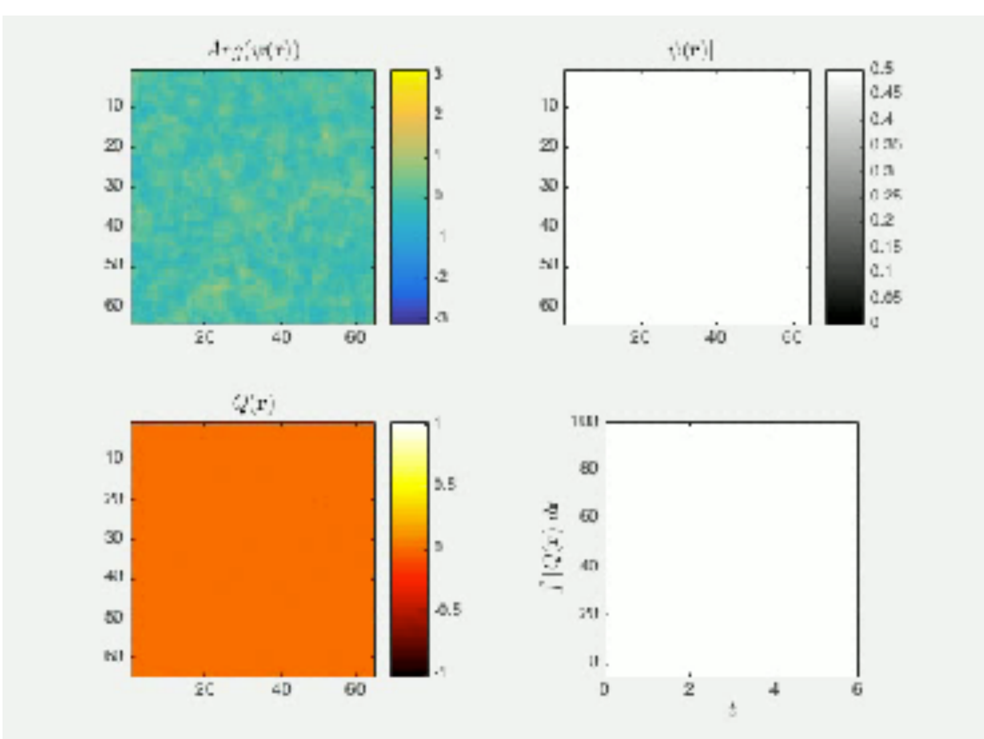
Strong non-equilibrium: Compact KPZ vortex turbulence

L. He, L. Sieberer, SD PRL (2017)

- numerical solution of stochastic GPE and c-KPZ:

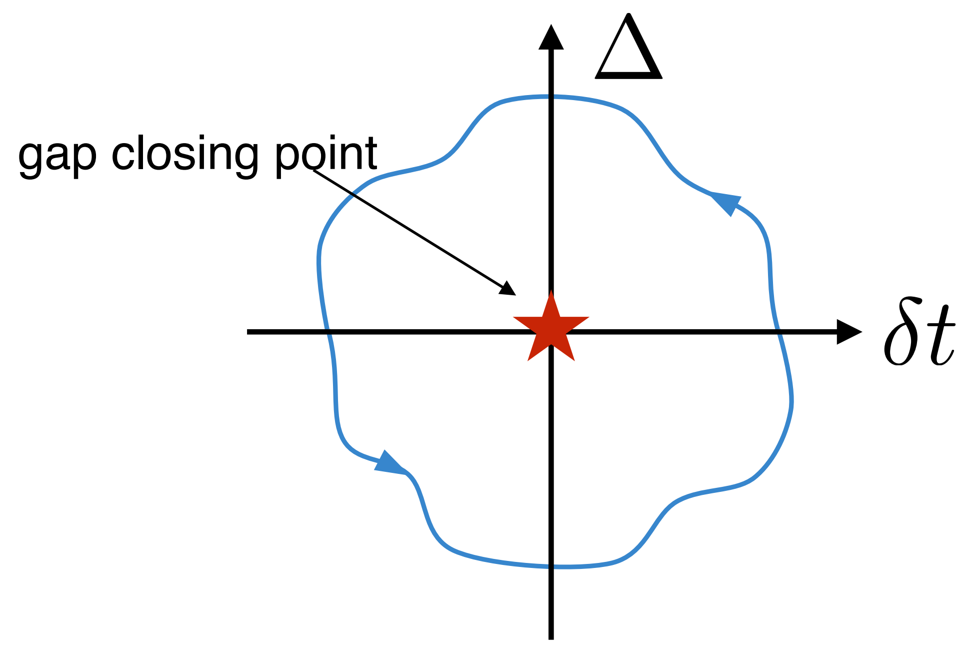


- mechanism of the phase transition



➡ deterministic limit: how does the system generate its own noise?

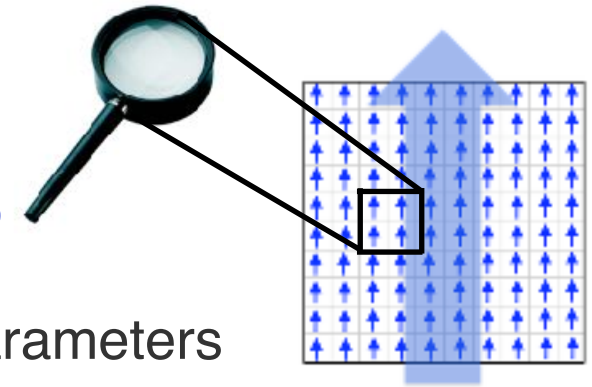
II. Topological quantization of observables in mixed quantum states



$$P = \frac{1}{2\pi}\mathrm{Im}\ln\langle\psi_0|\hat{T}|\psi_0\rangle = \frac{1}{2\pi}\varphi_{\mathrm{Z}} \qquad \varphi_{\mathrm{Z}} = \oint dk A_0(k)$$

Motivation: Topological States of Matter

Nayak et al., RMP (2008)
Hasan and Kane, RMP (2010)
Qi and Zhang, RMP (2011)



magnetization visible locally

- Characteristics of topological states of matter
  - New paradigm for ordered states of matter: beyond Landau's local order parameters
    - ➡ Nonlocal "order parameters": Topological invariants
    - ➡ Topo

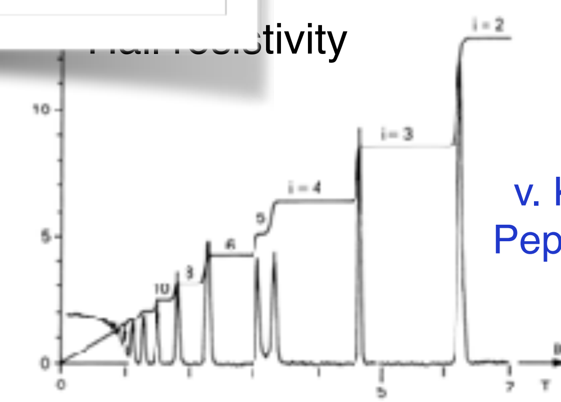
er et al. PRB (2008); Kitaev (2009)
Zirnbauer, PRB (1997)

All statements for pure quantum states at zero temperature!

➡ What about finite temperatures?

➡ What about non-equilibrium states?

  - Observable consequences: e.g. Quantum Hall effect
    - ➡ quantized bulk responses (e.g. Hall response)
    - ➡ robust edge states (e.g. chiral edge modes)
    - ➡ may carry non-abelian exchange statistics (e.g. Majorana edge states in topological superconductors)



v. Klitzing, Dorda, Pepper, PRL (1980)

Majorana edge mode

- Applications: topological quantum memories and computing

Motivation: Topological States of Matter for Mixed States?

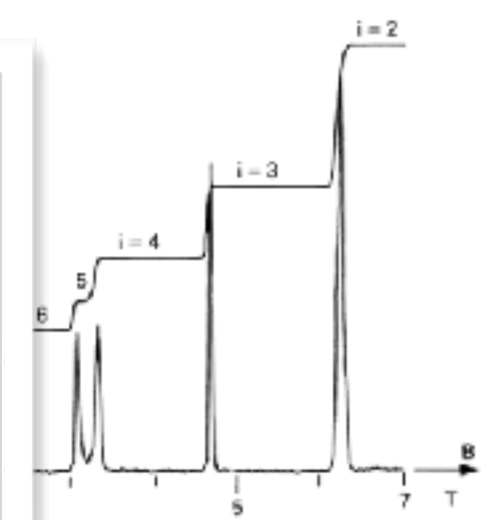
- Discouraging common wisdom: topological quantization corrupted at finite T
- e.g. Quantum Hall effect

  - Hall respon

    σ_x

  - Finite temp

    σ_x



Is topological quantization of observables a privilege of equilibrium ground states? Of pure states?

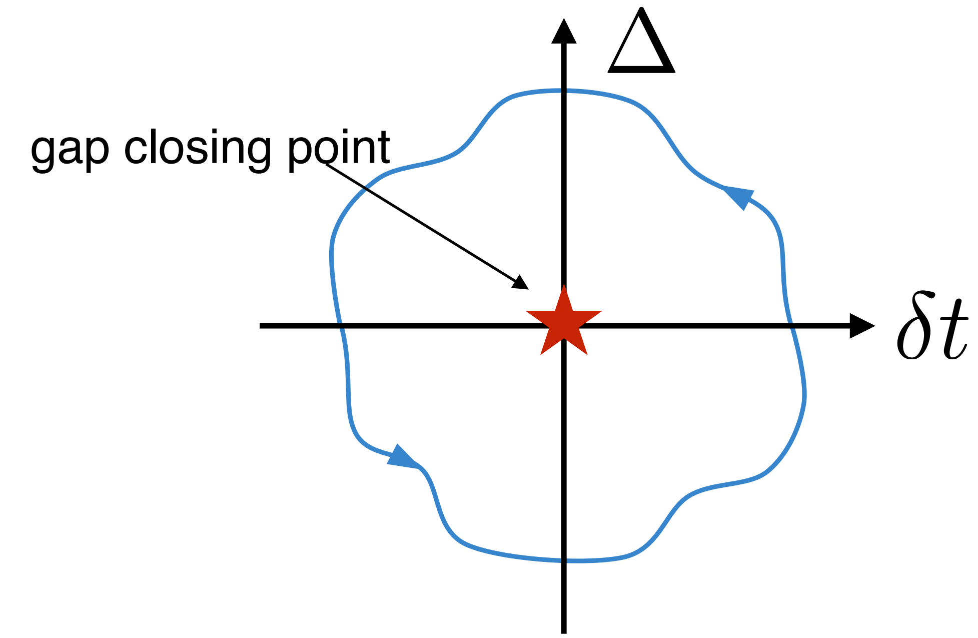
A way out: the focus is on single-particle observables

- Formal constructions of topological invariants possible for density matrices
- But not unique and related to system observables

Viyuela et al. PRL 2014, PRL 2014, Huang, Arovas PRL 2014, Budich, SD PRB (2015)

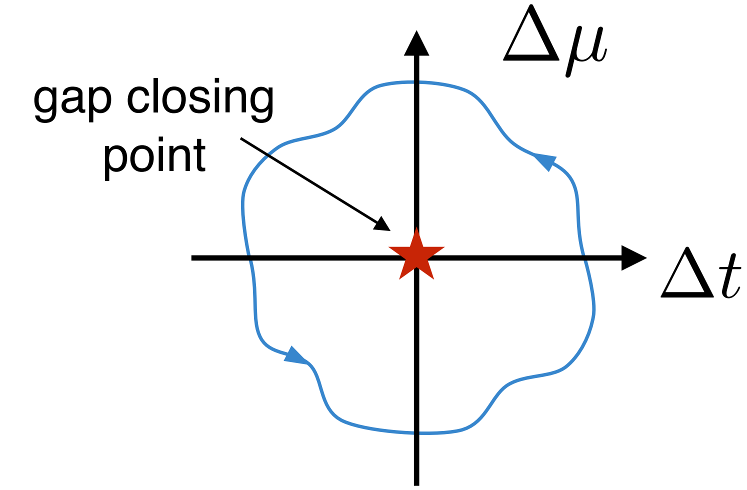
Viyuela et al., arxiv (2016)

Topological quantization of observables at zero temperature



$$P = \frac{1}{2\pi}\mathrm{Im}\ln\langle\psi_0|\hat{T}|\psi_0\rangle = \frac{1}{2\pi}\varphi_{\mathrm{Z}} \qquad \varphi_{\mathrm{Z}} = \oint dk A_0(k)$$

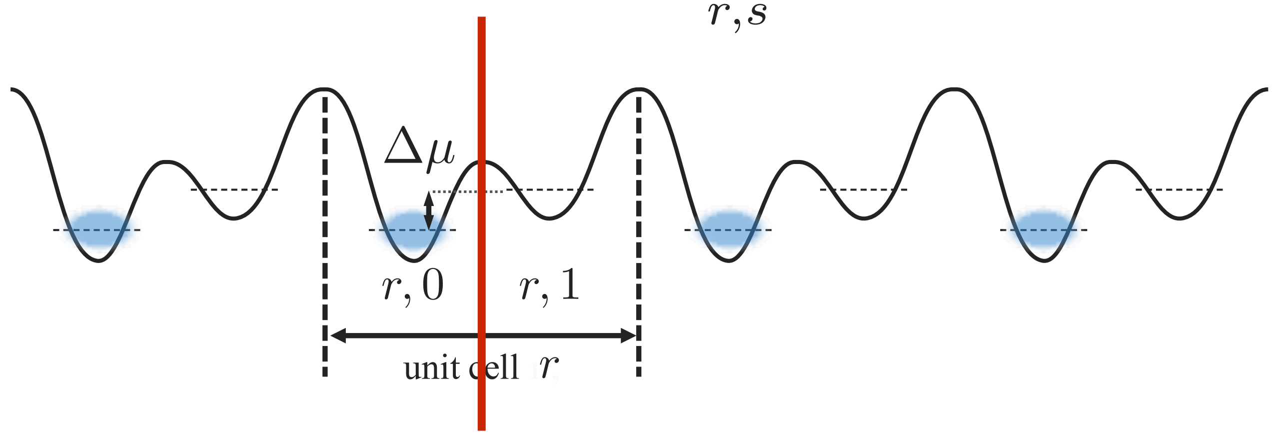
Topological quantization of observables: zero temperature



- Prime example: adiabatic Thouless pump in one dimension
- Pumping of one unit of charge if and only if a critical point is encircled

- Starting point: Rice-Mele model

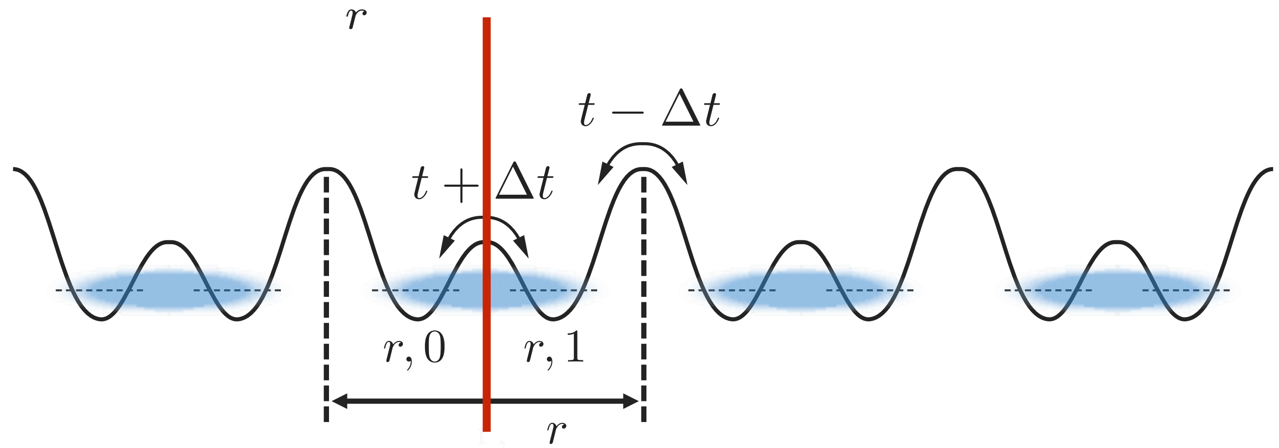
$$H_{\mathrm{RM}} = H_{\mathrm{SSH}} - \frac{\Delta\mu}{2}\sum_{r,s}(-1)^s a^\dagger_{r,s} a_{r,s}$$



Rice, Mele, PRL (1982)

- Special case: SSH model

$$H_{\mathrm{SSH}} = -\sum_r \left[(t+\Delta t)a^\dagger_{r,1}a_{r,0} + (t-\Delta t)a^\dagger_{r+1,0}a_{r,1}\right] + \mathrm{h.c.}$$

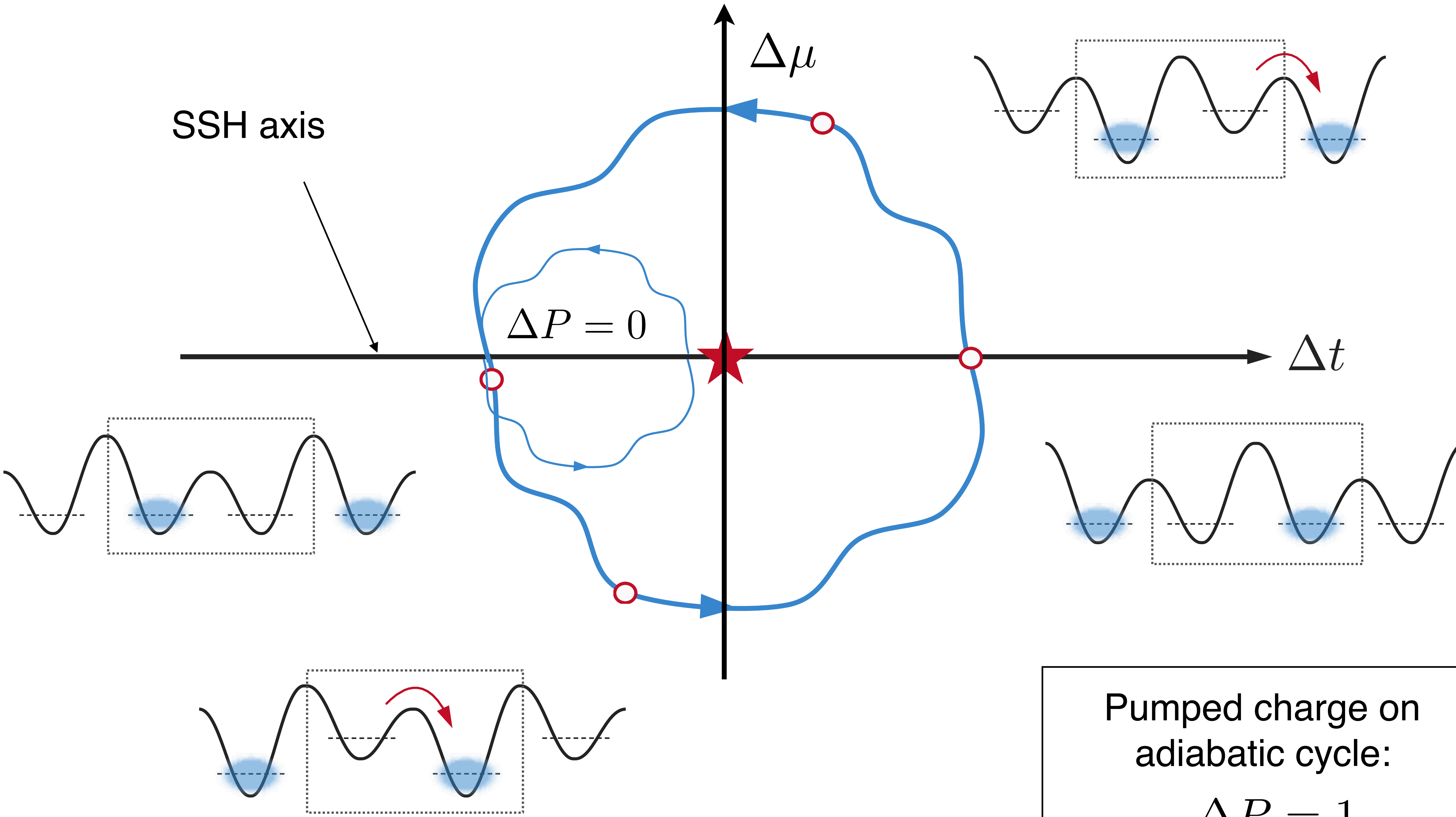


Su, Schrieffer, Heeger, PRL (1979)

Gap closing point at $\Delta t = 0$

Thouless charge pump: intuition

$$H_{\mathrm{RM}} = H_{\mathrm{SSH}} - \frac{\Delta\mu}{2}\sum_{r,s}(-1)^s a^\dagger_{r,s} a_{r,s}$$



Pumped charge on adiabatic cycle:

$$\Delta P = 1$$

What is topological about it?

Thouless, PRB (1983)
Thouless, Niu, J. Phys. A (1984)
Resta, PRL (1998)

$(\hbar = e = 1)$

- More precisely: demonstrate topological quantization of accumulated charge for ground states:

$$\Delta P = \oint dt\, I(t) \qquad I = \langle \psi_0 | \tfrac{1}{M} \hat{P} | \psi_0 \rangle$$

Accumulated charge — Current integrated over adiabatic cycle

- Resta: current can be written as total derivative — Resta, PRL (1998)

$$I(t) = \partial_t P(t)$$

- with "Resta polarization"

$$P = \frac{1}{2\pi} \mathrm{Im} \ln \langle \psi_0 | \hat{T} | \psi_0 \rangle, \quad \hat{T} \equiv e^{i\delta k \hat{X}}$$

$$\hat{X} \equiv \sum_j \hat{x}_j$$

Position operator

$$\delta k = 2\pi / L$$

- quantization is obvious:

$$\langle \psi_0 | \hat{T} | \psi_0 \rangle (t) = e^{i\varphi(t)}$$

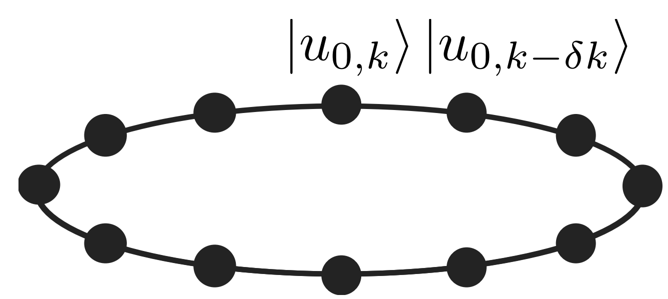
➡ Is it a topological quantization? What is the value of ΔP ?

What is topological about it?

- Topological origin:
  - T generates translations of all single-particle momenta $k \rightarrow k - \delta k$

$$P = \frac{1}{2\pi} \operatorname{Im} \ln \langle \psi_0 | \hat{T} | \psi_0 \rangle, \qquad \hat{T} \equiv e^{i \delta k \hat{X}}$$

  - focus on Rice-Mele: non-interacting, translation invariant [real space] 2 band model:
  - $|\psi_0\rangle$ as Slater determinant of ground state Bloch functions:



$$P = \frac{1}{2\pi} \operatorname{Im} \ln \underbrace{\prod_k \langle u_{0,k} | u_{0,k-\delta k} \rangle}_{\text{Wilson loop}}$$

  - Introduce a gauge connection

$$\langle u_{0,k} | u_{0,k-\delta k} \rangle = e^{i \delta k A_0(k)} \qquad A_0(k) = i \langle u_{0,k} | \partial_k u_{0,k} \rangle$$

  - Identifies Resta polarization as geometric phase (Zak phase)

$$P = \frac{1}{2\pi} \varphi_{\mathrm{Z}} \qquad \varphi_{\mathrm{Z}} = \oint dk A_0(k)$$

What is topological about it?

- Topological origin of accumulated charge:

Chern number

$$\Delta P = \oint dt \partial_t P(t) = \frac{1}{2\pi} \oint dt dk\, \partial_t \underbrace{A_0(k,t)} = \frac{1}{2\pi} \oint dt dk\, \underbrace{F_0(k,t)} = C \in \mathbb{Z}$$

Berry connection

Berry curvature

$$A_{\alpha,0} \equiv i \langle u_{k,t,0} | \partial_\alpha u_{k,t,0} \rangle$$

$$F_0 \equiv \partial_t A_{k,0}(k,t) - \partial_k A_{t,0}(k,t)$$

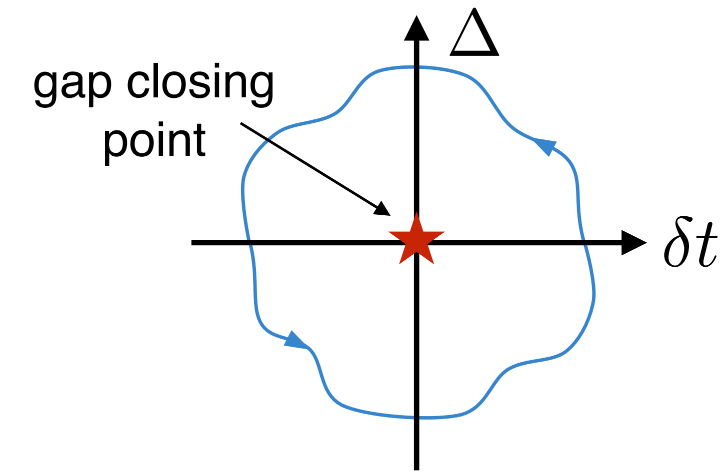
- Summary (T = 0)

Accumulated charge (observable) $\Leftrightarrow$ Accumulated Resta polarization $\Leftrightarrow$ Topological invariant

$$\oint dt I = \Delta P = C$$

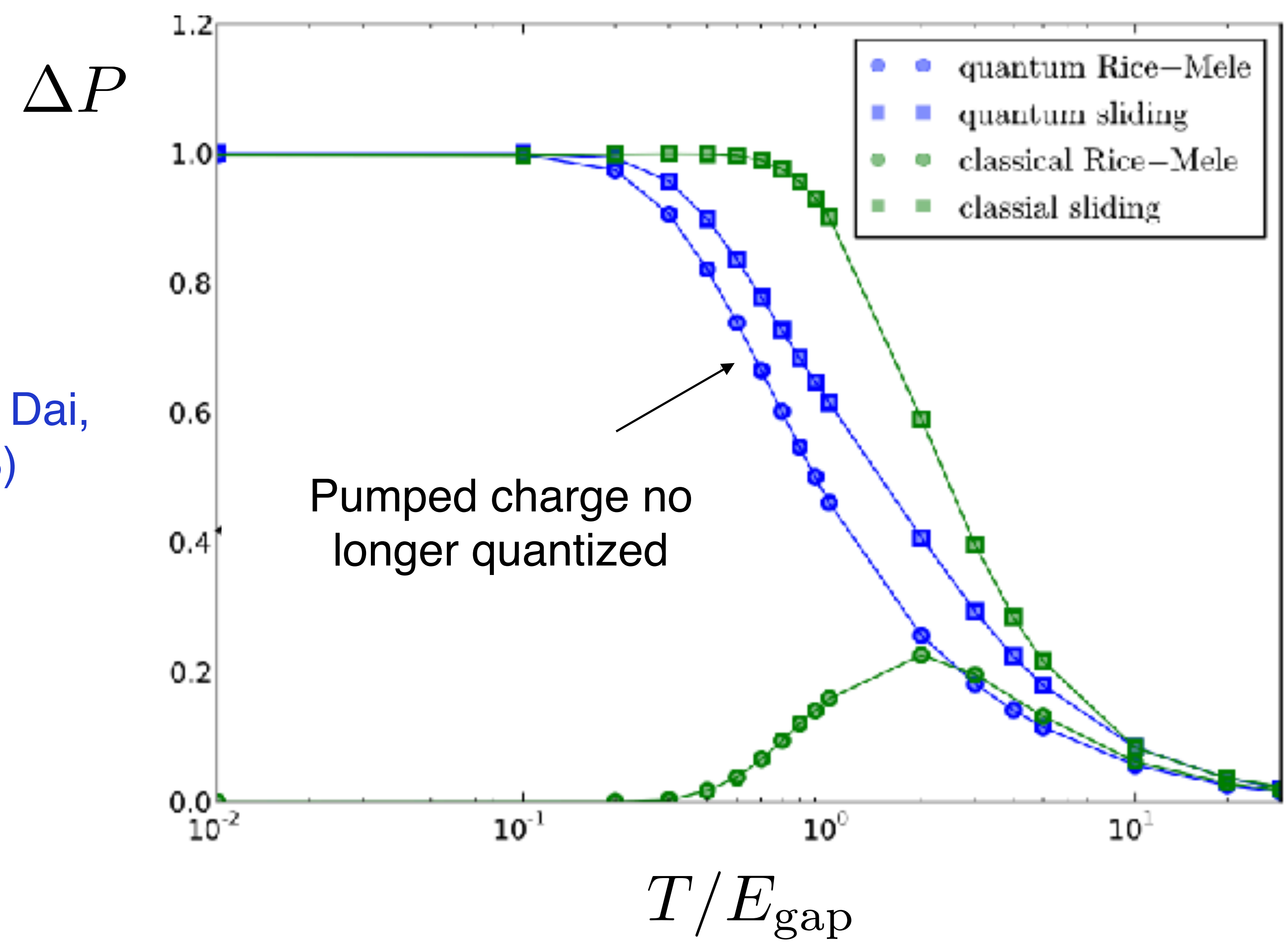
➡ We identified a topologically quantized observable

Failure of topological quantization at finite temperature



- Common wisdom: topological quantization corrupted at finite T (e.g. conductance in quantum Hall effect)
- Concrete example: Accumulated charge through Thouless pump

Wang, Troyer, Dai, PRL (2013)

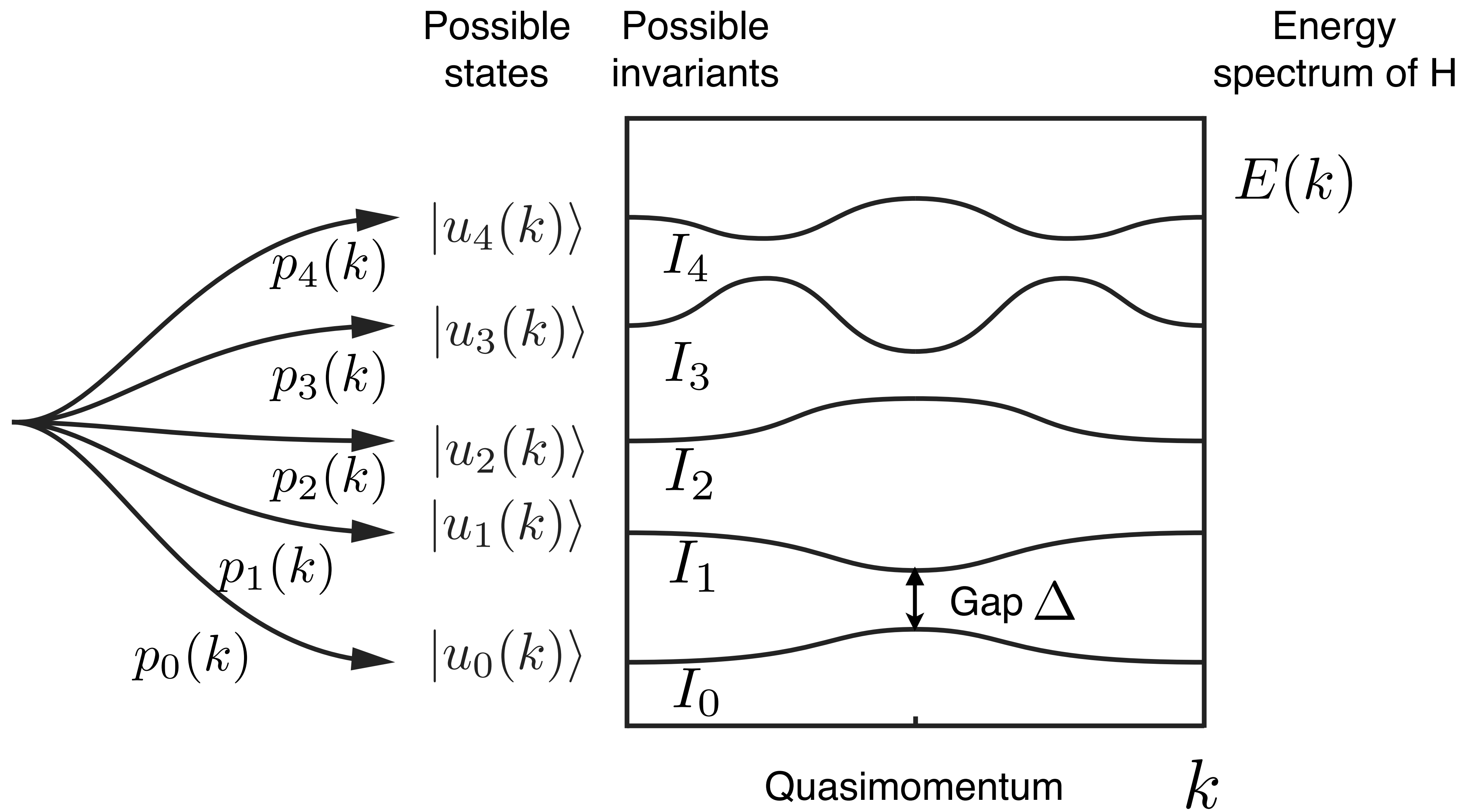


Failure of topological quantization at finite temperature

- Physical reason

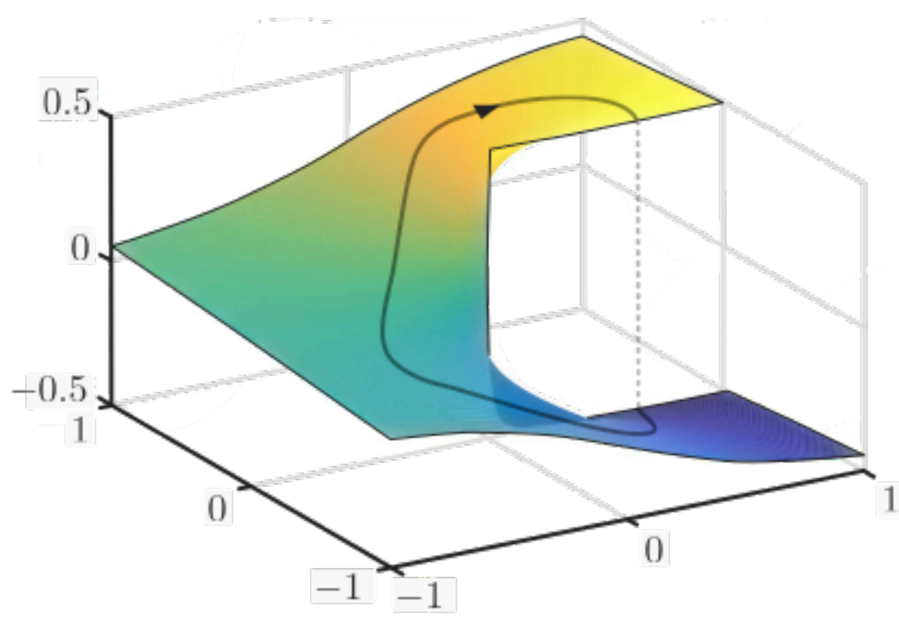
Density matrix

$$\rho = e^{-\beta H}$$



- Every band has its own Berry connection and Chern number
- Probability of finding invariant n: $p_n(k) = e^{-\beta E_n(k)}$
- Intensive corrections quantization only possible up to intensive corrections $\sim e^{-\beta\Delta}$?

Topological quantization of observables in mixed quantum states



$$\varphi_{\mathrm{E}} = \mathrm{Im}\,\mathrm{Tr}\ln(\mathbf{1} + \prod_k e^{-\beta\epsilon_k\sigma_z} e^{i\delta k\mathcal{A}_k\cdot\sigma})$$

Key Results

- Ensemble geometric phase: Natural extension of the Resta polarization / Zak phase to mixed states

$$\varphi_{\mathrm{E}} = \operatorname{Im}\ln\operatorname{Tr}\left(\rho e^{i\delta k\hat{X}}\right) \qquad \delta k = \frac{2\pi}{L}$$

equilibrium or non-equilibrium density matrix

- Reduction to Zak phase in thermodynamic limit

$$\varphi_{\mathrm{E}} = \varphi_{\mathrm{Z}} + \Delta(N) \qquad \Delta(N) \xrightarrow{N\to\infty} 0 \qquad \varphi_{\mathrm{Z}} = \oint dk\, A_{k,0}, \quad A_{k,0} = i\langle u_{k,0}|\partial_k|u_{k,0}\rangle$$

➡ Emergent geometric phase, different from formal (Uhlmann) approaches

- Exact topological quantization for cyclic adiabatic parameter changes (correction terms do not contribute to winding of Zak phase)

$$\Delta\varphi_{\mathrm{E}} = \oint d\phi\, \partial_\phi \varphi_{\mathrm{E}} = C$$

Chern number of lowest band

- Observability via many-body interferometry

➡Topological quantization persists in suitable many-body correlators

Projection mechanism

- Gaussian states (e.g. real space)

$$\rho = \frac{1}{\mathcal{Z}} \exp\Big(- \sum_{i,j} \hat{a}_i^\dagger G_{ij} \hat{a}_j \Big) \qquad \Leftrightarrow \langle \hat{a}_i^\dagger \hat{a}_j \rangle = [f(G)]_{ij} \qquad f(G) = (e^G + \mathbf{1})^{-1}$$

- G hermitean matrix, equilibrium or non-equilibrium, in equilibrium $G = \beta H$

- many-body correlator $\varphi_{\mathrm{E}} = \mathrm{Im}\ln\mathrm{Tr}(\rho\hat{T}) = \mathrm{Im}\mathrm{Tr}\ln[\mathbf{1} + (\mathbf{1} - f(G))^{-1} f(G) T]$

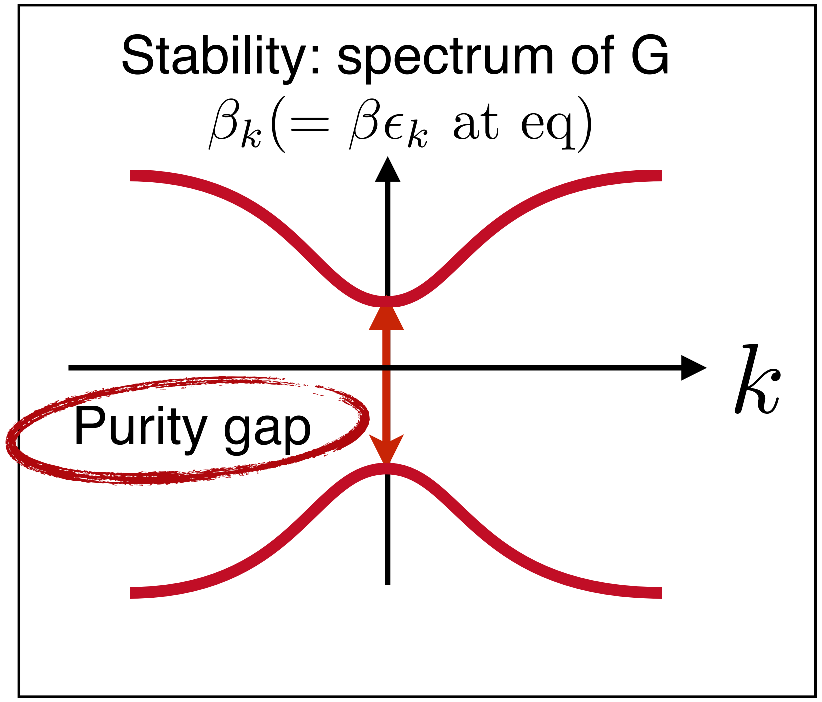
- interpretation (momentum space): $G = \sum_k G_k |k\rangle\langle k|, \quad T = \sum_k \mathbf{1}|k+1\rangle\langle k|$

- treat G as ordinary Hamiltonian: “Bloch basis”

$$G_k = U_k B_k U_k^\dagger, \quad B_k \equiv \mathrm{diag}_s(\beta_{k,s})$$

  - B_k collects “purity eigenvalues” Probability
  - U_k collects eigenvectors of G “Bloch functions” Topology

$$G_k |u_{k,s}\rangle = \beta_{k,s} |u_{k,s}\rangle$$



➡ Full analogy to pure state discussion, but no mention of equilibrium

Projection mechanism

- Ensemble geometric phase

$$\varphi_{\mathrm{E}} = \mathrm{Im}\ln\mathrm{Tr}(\hat{T}) = \mathrm{Im}\mathrm{Tr}\ln[\mathbf{1} - f(G) + f(G)T] = \mathrm{Im}\mathrm{Tr}\ln[\mathbf{1} + (\mathbf{1} - f(G))^{-1} f(G)T]$$

- Evaluate in momentum space $G = \sum_k G_k |k\rangle\langle k|, \quad T = \sum_k \mathbf{1}|k+1\rangle\langle k|$

- many-body correlator

$$\begin{aligned}\varphi_{\mathrm{E}} &= \mathrm{Im}\ln\mathrm{Tr}\left(\rho e^{i\delta k \hat{X}}\right) \\ &= \mathrm{Im}\mathrm{Tr}\ln(\mathbf{1} + \prod_k \frac{f(B_{k+1})}{\mathbf{1} - f(B_{k+1})} U_{k+1}^{\dagger} U_k)\end{aligned}$$

Momentum and band space

eg. two bands, particle hole symmetry

$e^{-\beta_k \sigma_z}$ occupation probability

$e^{i\delta k \mathcal{A}_k \cdot \sigma}$ translation operator

$$\mathcal{A}_k \cdot \sigma = \sum_{i=0}^{3} \mathcal{A}_k^i \sigma_i$$

➡ Path ordered matrix product, reminiscent of Wilson loop

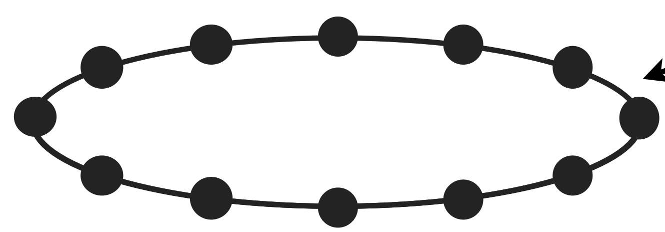
Projection mechanism (two bands, equilibrium)

- many-body correlator, e.g. two bands, particle hole symmetry

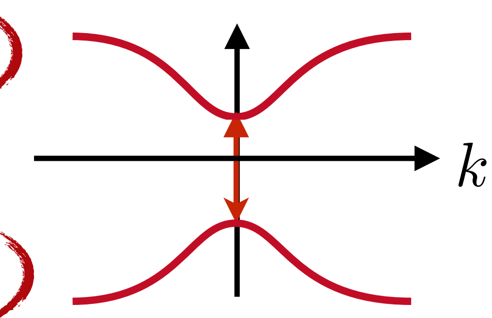
$$\varphi_{\mathrm{E}} = \mathrm{Im}\mathrm{Tr}\ln\Big(\mathbf{1} + \prod_k \underbrace{e^{-\beta_k \sigma_z}}_{\text{probability}} \underbrace{e^{i\delta k \mathcal{A}_k\cdot\sigma}}_{\text{geometry}}\Big) \qquad \mathcal{A}_k\cdot\sigma = \sum_{i=0}^{3}\mathcal{A}_k^i\sigma_i$$

- Assume all matrices commute

Brillouin zone



- Key point:
  - Upper band: exponentially (in system size) suppressed $\quad (e^{-\beta_k})^N = e^{-N\beta_k}$
  - Lower band: exponentially (in system size) enhanced $\quad (e^{+\beta_k})^N = e^{+N\beta_k}$



➡ Effective projection onto lowest purity band = ground state

$$\varphi_{\mathrm{E}} \to \mathrm{Im}\mathrm{Tr}\ln e^{i\oint dk[P_{0,k}\mathcal{A}_k P_{0,k}]} = i\oint dk\langle u_{0,k}|\partial_k|u_{0,k}\rangle = \oint dk A_{0,k} = \varphi_{\mathrm{Z}}$$

Projection mechanism (two bands, equilibrium)

- many-body correlator, e.g. two bands, thermal state, particle hole symmetry

$$\varphi_{\mathrm{E}} = \mathrm{Im}\,\mathrm{Tr}\ln\Big(\mathbf{1} + \prod_k \underbrace{e^{-\beta_k \sigma_z} e^{i\delta k \mathcal{A}_k \cdot \sigma}}\Big)$$

probability geometry

$$\mathcal{A}_k \cdot \sigma = \sum_{i=0}^{3} \mathcal{A}_k^i \sigma_i$$

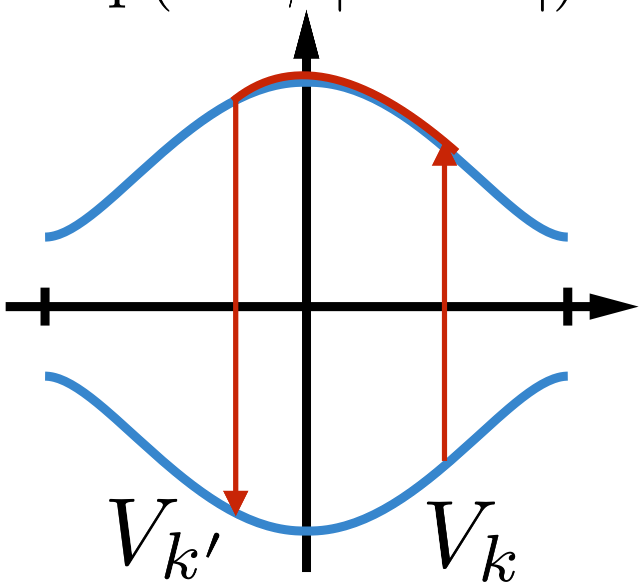
- non-commuting parts: perturbative corrections

$$e^{i\delta k \sum_{i=0}^{3} \mathcal{A}_k^i \sigma_i} = \underbrace{e^{i\delta k \sum_{i=0,3} \mathcal{A}_k^i \sigma_i}} + \underbrace{\delta k \sum_{i=1,2} \mathcal{A}_k^i \sigma_i} + \mathcal{O}(\delta k^2)$$

Diagonal, commutes $\quad V_k$ transition between bands

Excursion penalty

$\sim \exp(-\Delta\beta|k-k'|)$



- Expand and sum all possible second order processes (analogous second order time dependent perturbation theory)

$$\varphi_{\mathrm{E}} = \varphi_{\mathrm{Z}} + \Delta(N) \qquad \Delta(N) \sim [N\beta\Delta\epsilon]^{-2}$$

Ensemble geometric phase (EGP)

➡ Emergent U(1) geometric phase in thermodynamic limit

Topological nature of accumulated phase for adiabatic cycle

- EGP difference integer quantized by construction (phase variable defined mod(2pi))

$$\frac{1}{2\pi}\Delta\varphi_{\mathrm{E}} = \oint d\phi\, \partial_\phi \varphi_{\mathrm{E}}(\phi) = \frac{1}{2\pi}(\varphi_{\mathrm{E}}(\phi_f) - \varphi_{\mathrm{E}}(\phi_i)) \in \mathbb{Z}$$

final/initial parameter of adiabatic cycle $\phi_f = \phi_i$

- This quantization is of topological origin: N dependent correction cannot contribute to integer value

$$\frac{1}{2\pi}\Delta\varphi_{\mathrm{E}} = \frac{1}{2\pi}\oint d\phi \oint_{\mathrm{BZ}} dk \partial_\phi A_0(k,\phi) = \frac{1}{2\pi}\iint d\phi\, dk\, F_0(k,\phi) = C \in \mathbb{Z},$$

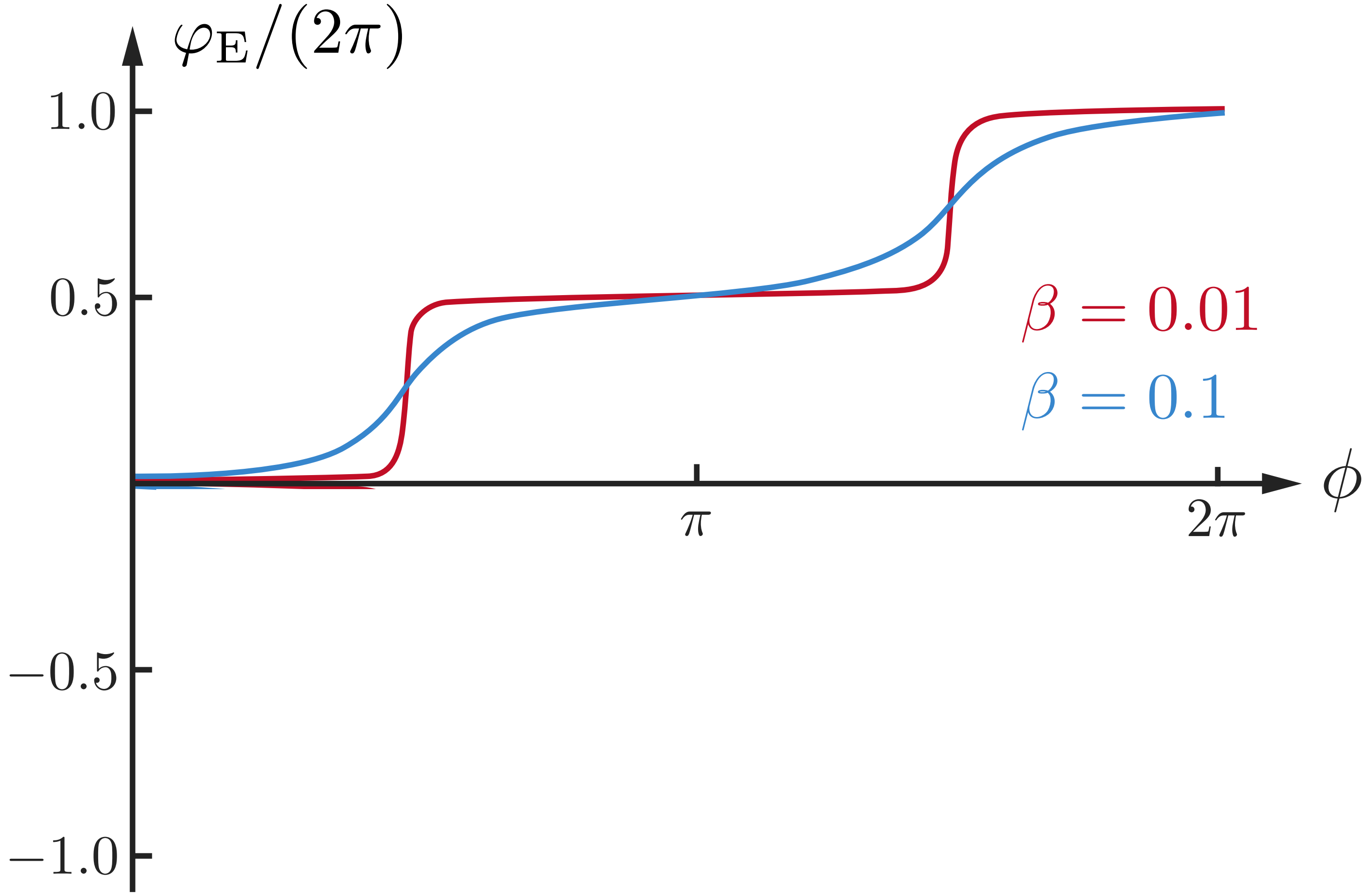
Berry connection of lowest purity band

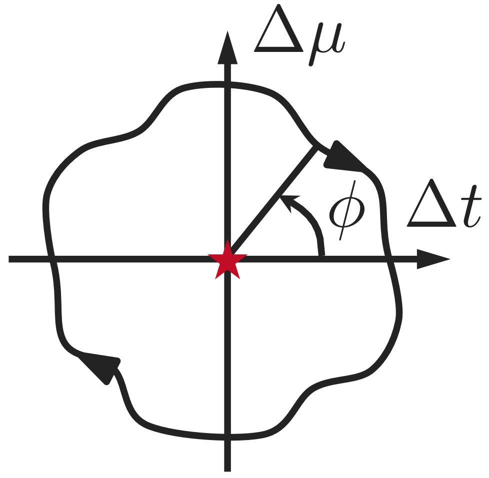
Berry curvature of lowest purity band

- *Value* of EGP difference connected to Chern number of lowest purity band, i.e. the topology of the density matrix

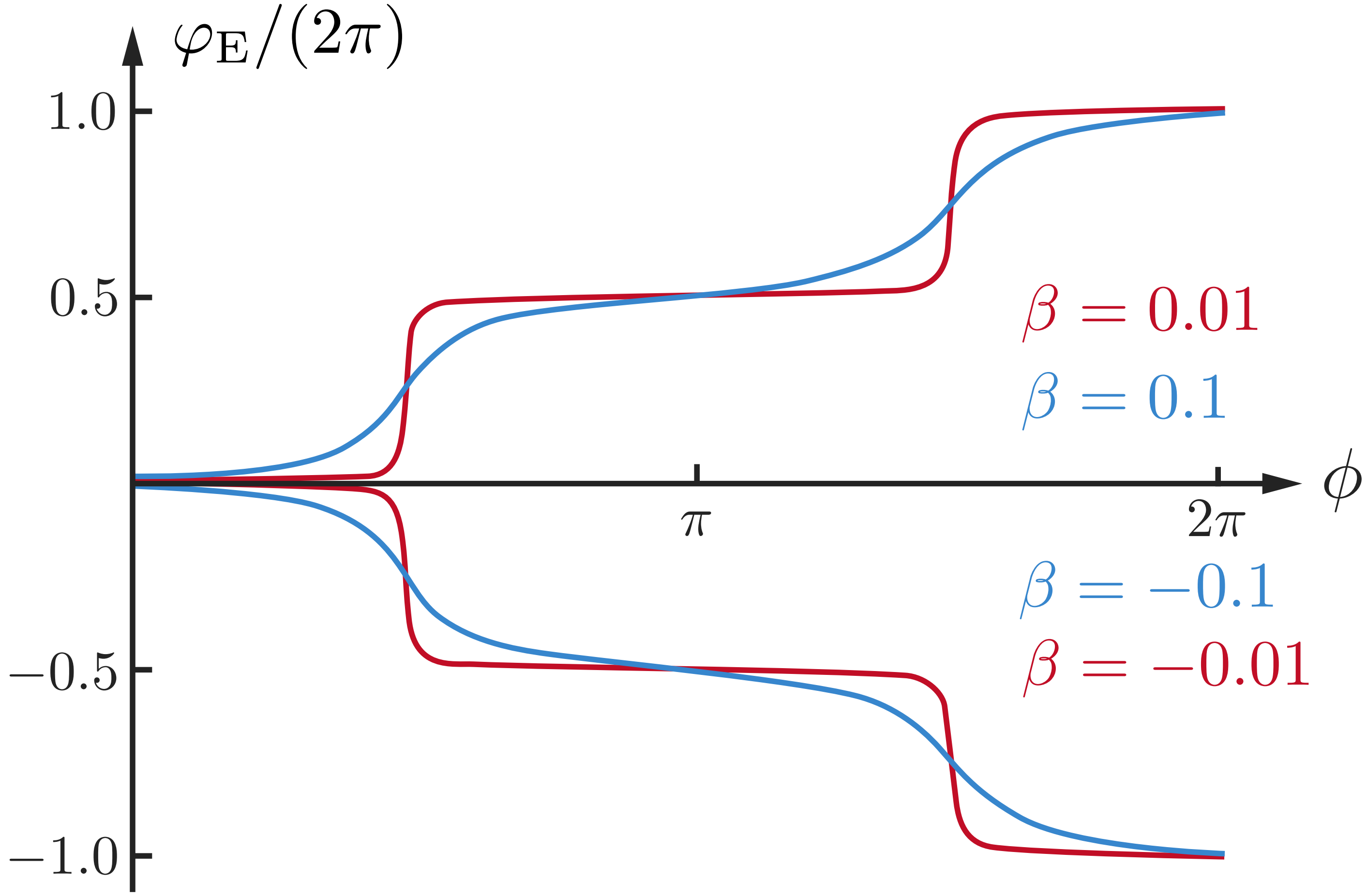
➡ toplogical quantization of EGP *difference* in irrespective to system size N

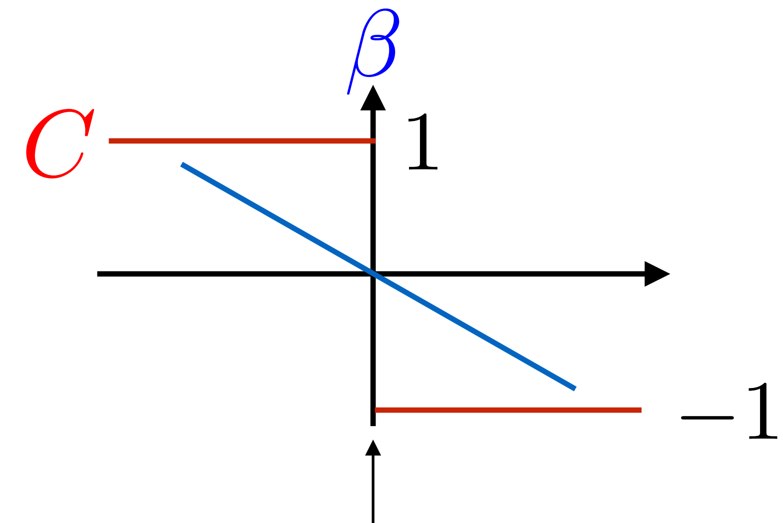
Topological quantization at finite T





Infinite temperature topological phase transition

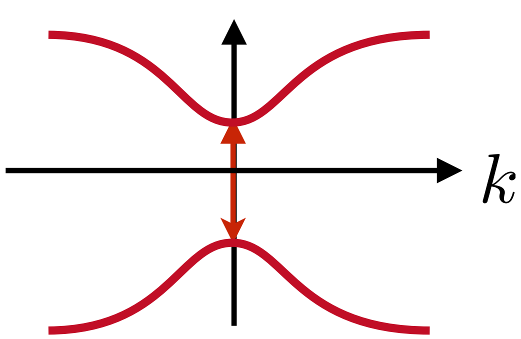




Infinite temperature

Completely structureless states

band population inversion



ultracold atoms: Bloch group, Science (2013)

- Coordinate singularities for $\beta_k = \beta \epsilon_k = 0$ → $\epsilon_k = 0$ for some k; → $T = \infty$

➡ Topological phase transition at infinite temperature detected by EGP

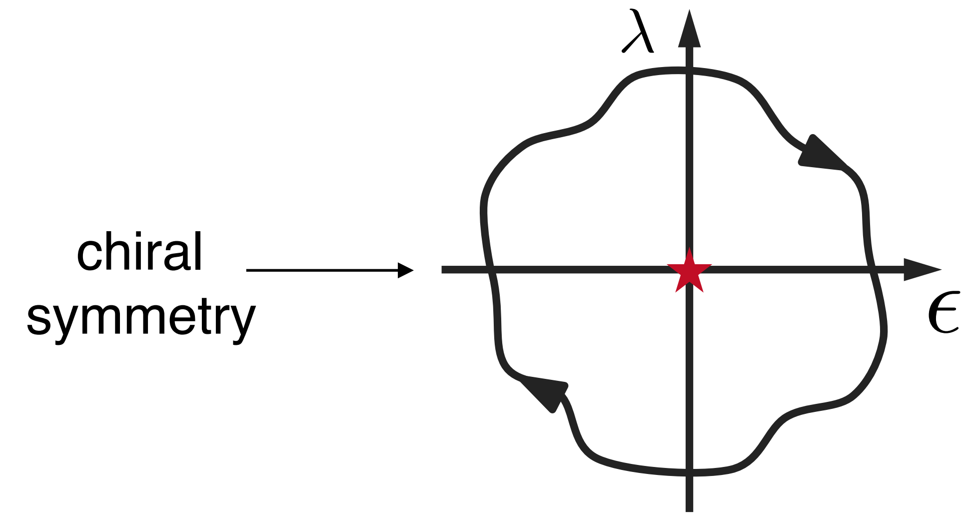
“Purity gap closings” and non-equilibrium phase transition

- Topological Lindblad dynamics: Driven open analog Rice-Mele model (Linzner et al. PRA (2016))

$$\partial_t \rho = \sum_{r,s} \left(2L_{r,s}\rho L^\dagger_{r,s} - \{L^\dagger_{r,s}L_{r,s}, \rho\}\right)$$

$$L_{r,0} = \sqrt{1+\epsilon}\left[(1-\lambda)\left(\hat{a}^\dagger_{r,0} + \hat{a}_{r,1}\right) + (1+\lambda)\left(\hat{a}_{r,0} - \hat{a}^\dagger_{r,1}\right)\right]$$

$$L_{r,1} = \sqrt{1-\epsilon}\left[(1-\lambda)\left(\hat{a}^\dagger_{r+1,0} + \hat{a}_{r,1}\right) + (1+\lambda)\left(\hat{a}_{r+1,0} - \hat{a}^\dagger_{r,1}\right)\right]$$



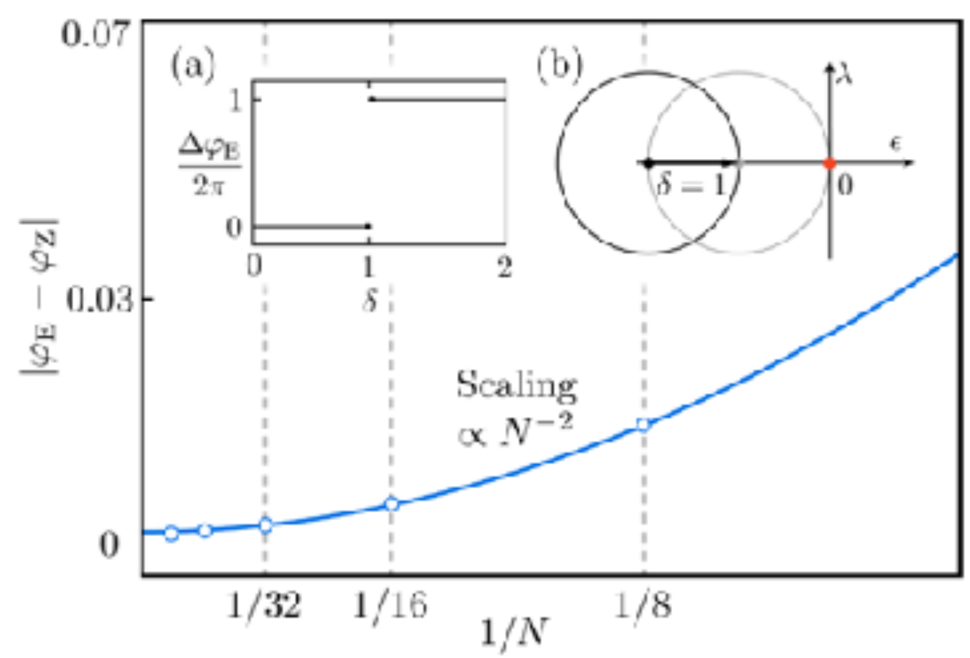
- Steady state density matrix shares symmetries of Rice-Mele model
- Spectral gap closing point is replaced by a purity gap closing point

$$\beta_{k_*}(\epsilon = \lambda = 0) = 0$$

- Spectral (damping) gap is open

$$\Delta_d > 0$$

- no divergent length/time scales at such critical point!



➡ EGP detects non-equilibrium topological phase transitions without thermodynamic signatures

Observability

- Measuring the expectation value of a unitary matrix
- Measuring a genuine many-body operator (not: single particle current)

Smallest possible lattice momentum

$$\varphi_{\mathrm{E}} = \mathrm{Im} \ln \langle \hat{T} \rangle \qquad \hat{T} = \exp(i\delta k \sum_i x_i \hat{a}_i^\dagger \hat{a}_i) \qquad \delta k = 2\pi / L$$

➡ Interferometric detection for mesoscopic setup of ultracold atoms

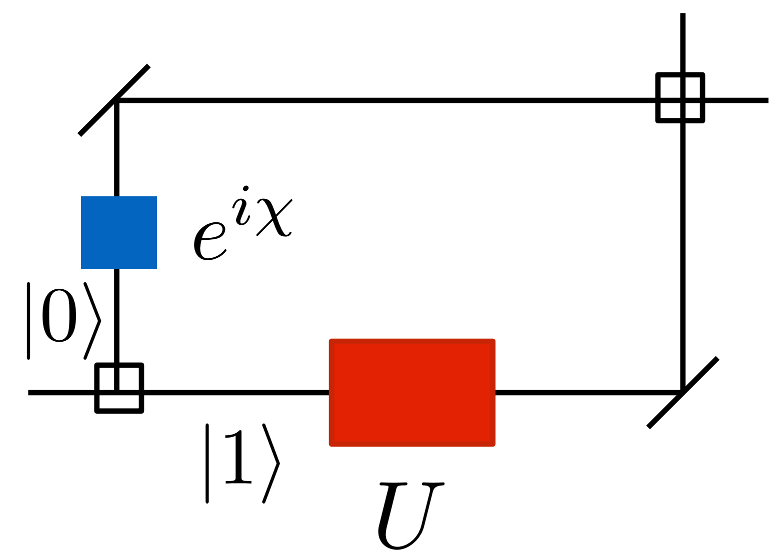
---

- Measuring the unitary: Partial trace of an interferometer-system unitary   Sjoeqvist et al., PRL (2000)

  - Interferometer Hilbert space $|0\rangle \,, |1\rangle$
  - Desired total unitary

$$\mathcal{U} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes U + \begin{pmatrix} e^{i\chi} & 0 \\ 0 & 0 \end{pmatrix} \otimes \mathbf{1}$$

interferometer   fermion system

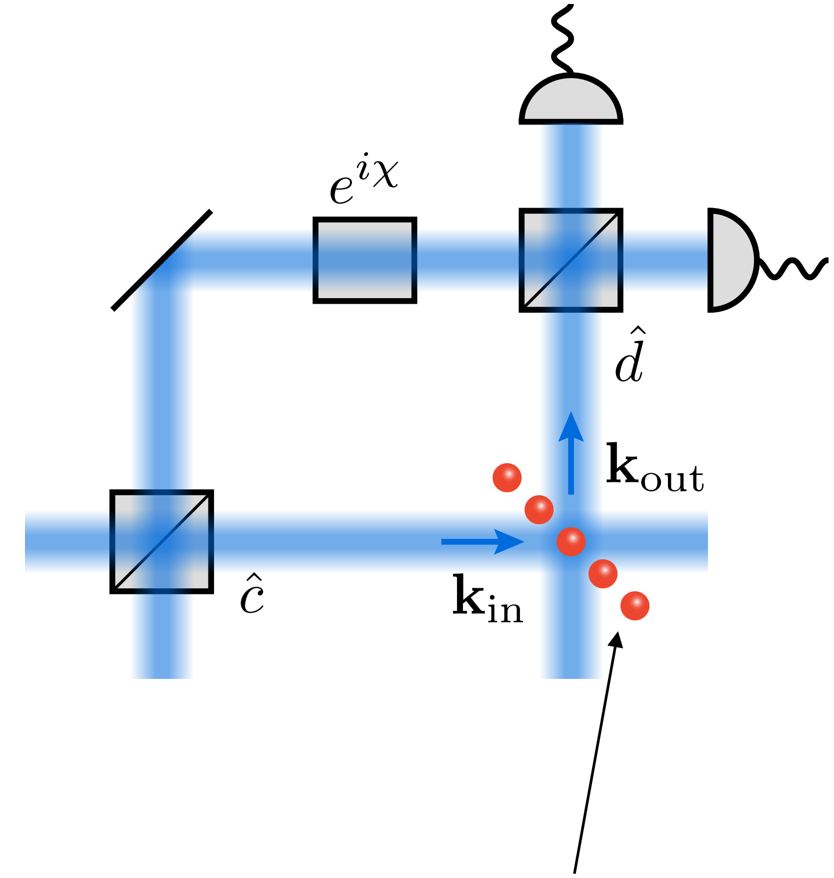


  - Including beam splitters, mirrors, output intensity along $|0\rangle$

$$I \sim \mathrm{tr}[U\rho U^\dagger + \rho + (e^{i\chi}\rho U^\dagger + h.c.)] \sim 1 + |\langle U \rangle| \cos[\chi - \arg\langle U \rangle]$$

Observability

- Here: Mach-Zehnder interferometer with a mirror replaced by fermion system

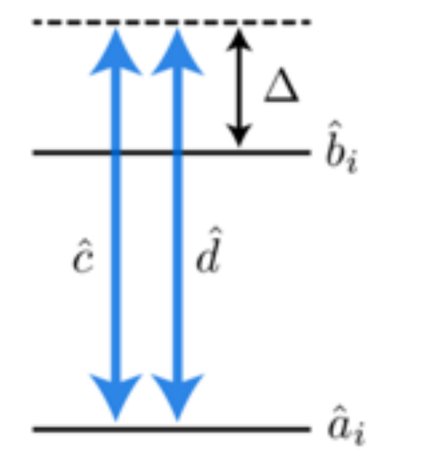


- Interferometer modes c, d coupling off-resonantly internal fermion states a, b, linear mode function profile

$$H_{\text{eff}} = \hat{X}\left[\eta_\gamma \hat{c}^\dagger \hat{c} + \eta_\delta \hat{d}^\dagger \hat{d} + \left(\eta_{\gamma\delta}\hat{c}^\dagger \hat{d} + \text{H.c.}\right)\right]$$

- input-output relation

$$\begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}_{\text{out}} = \langle \hat{T} \rangle \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{Independent of probed state}} \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}_{\text{in}}$$



- EGP picked up by interferometer, measured via scanning reference phase χ

$$\langle \hat{T} \rangle = |\langle \hat{T} \rangle| e^{i\varphi_{\text{E}}}$$

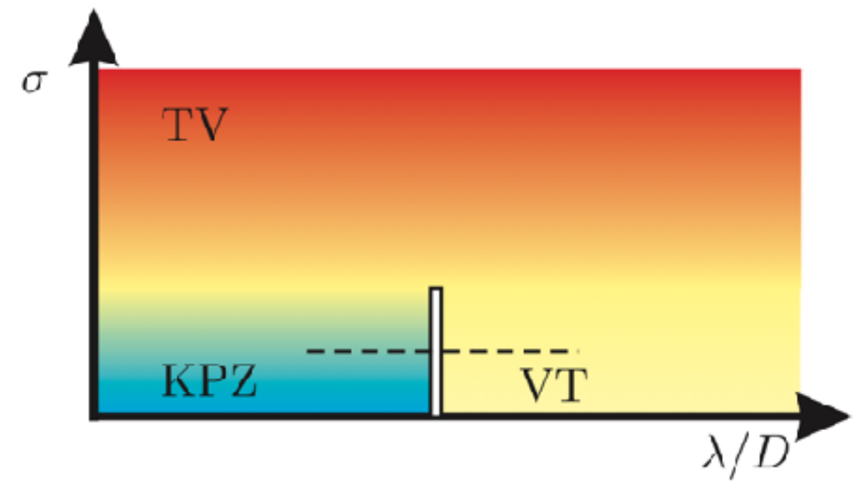
- Visibility limited by amplitude of signal $|\langle \hat{T} \rangle| \approx \exp(-\frac{N}{2\pi} e^{-\Delta\beta/2})$ (Purity gap: Δ)

➡ Interferometric measurement of finite density mesoscopic fermion systems

$$N \approx 40, \quad T \approx 0.2 T_{\text{F}}$$

Conclusions & Outlook

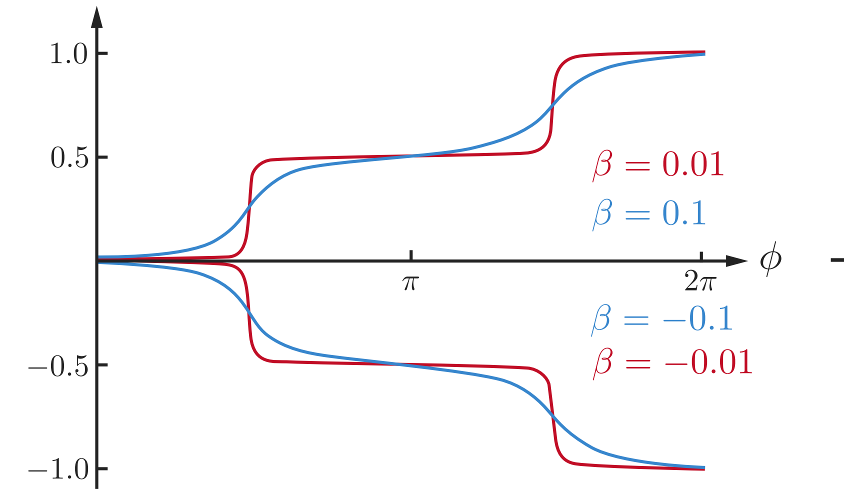
I. Phase diagram of Exciton-Polaritons



- Emergent Hamilton dynamics at strong non-equilibrium drive
- Explains phase transition as transition to chaos

II. Topology in mixed quantum states

- Exact toplogical quantization persists in certain many-body correlators
    - Many-body purification mechanism and emergent Zak phase
    - Detects non-equilibrium topological phase transitions
    - Observable via interferometry in mesoscopic samples



➡ Validity of construction for interacting or disordered systems?

➡ Topological classification of higher dimensional mixed states via many-body correlators?

➡ Topology of bosonic non-equilibrium mixed states with equipartitioned occupations of selected bands (ultracold atoms, polaritons..)?