Dirac Physics and Topology in Photonics

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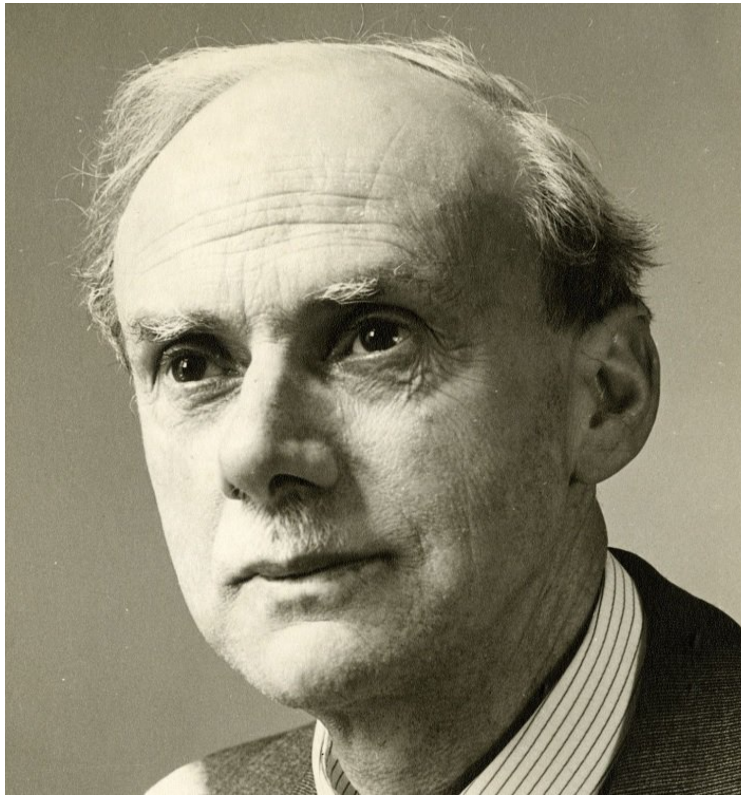
Outline

- Dirac equation and band topology
- Simulation of Dirac physics in photonics
- Summary

Dirac equation



Dirac equation (1928)

$$\left(\sum_{n=1}^{3} \alpha_n p_n + \beta m\right)\psi = i\frac{\partial}{\partial t}\psi$$

$$\alpha_n^2 = \beta^2 = 1_{4\times 4}$$

$$\alpha_n \alpha_j + \alpha_j \alpha_n = 0,\ \forall\, n \neq j$$

$$\alpha_n \beta + \beta \alpha_n = 0$$

Niels Bohr: *"What are you working on, Mr. Dirac?"*
Paul Dirac: *"I'm trying to take the square root of something."*

- A consequence of solving the "negative probability" issue in the Klein-Gordon equation ("square root" of the K-G eqn.)

K-G eqn. (1926):

$$\left(\sum_{n=1}^{3} p_n^2 + m^2\right)\psi = -\frac{\partial^2}{\partial t^2}\psi$$

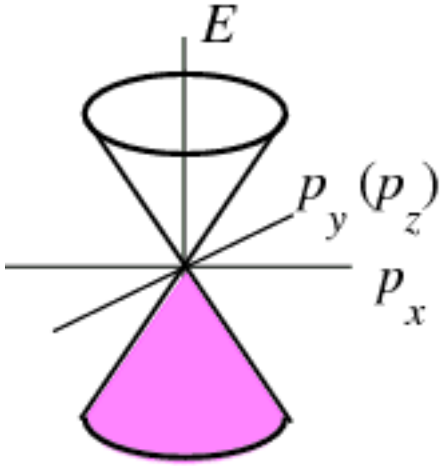
$$H_{KG} := H_D^2$$

Massless Dirac equation & Weyl equations

Weyl equations:

$$\left(\sum_{n=1}^{3} \alpha_n p_n\right)\psi = i\frac{\partial}{\partial t}\psi \quad \Rightarrow \quad H \equiv \sum_{n=1}^{3} \alpha_n p_n = \begin{pmatrix} H_L & 0 \\ 0 & H_R \end{pmatrix}$$

$$H_L = -\sum_n \sigma_n p_n \quad (n=x,y,z) \qquad H_R = \sum_n \sigma_n p_n \quad (n=x,y,z)$$

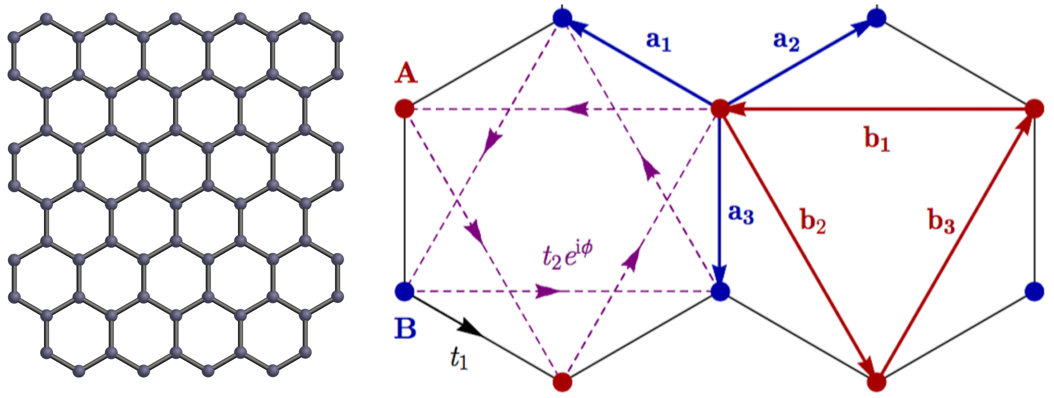


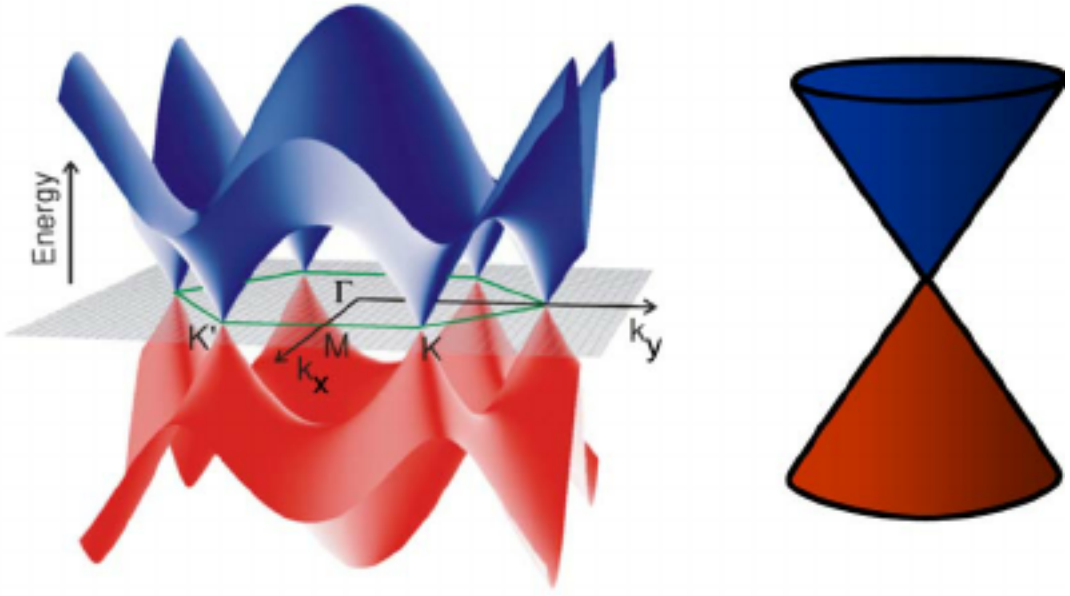
- Linear crossing points: Dirac (*4*fold) and Weyl (*2*fold) points

- A Dirac point consists of two Weyl points with opposite chirality.
  More generally, $H_{WP} = \sum_{nj} \sigma_n v_{nj} p_j$, *its chirality is* $N_w = sign(det(\hat{v}))$

- Dirac/Weyl points were realized recently in novel electronic materials

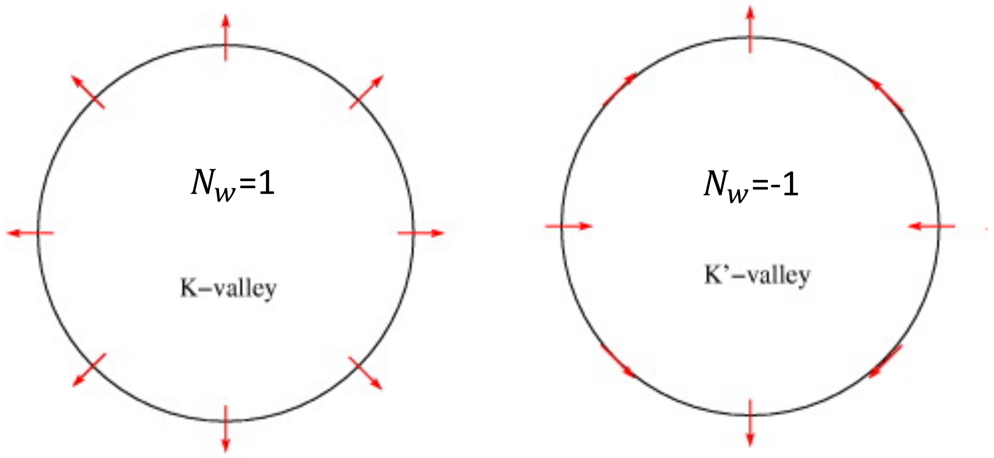
Massless 2D Dirac equation (graphene)





Two chiral fermions of *opposite* chirality $H = v_F(\tau_z\sigma_x p_x + \sigma_y p_y)$, τ_z=$\pm$ for K and K' valleys

$\sigma_z = \pm 1$ represents A & B lattice sites



Introducing mass in Dirac equation:

- Breaking inversion: $H_{m1} = m_1\sigma_z, \quad m_1 = V_A - V_B$
- Breaking time-reversal: $H_{m2} = m_2\tau_z\sigma_z, \quad \tau_z=\pm,$ $m_2 = -3\sqrt{3}t_2\sin\phi$

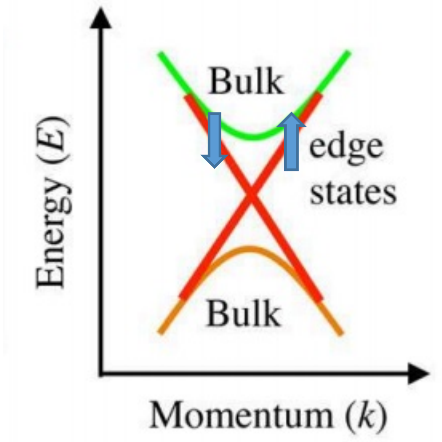
Haldane, Phys. Rev. Lett. **61**, 2015 (1988).

2D Dirac electron & topology

- Topological Chern number is the integration of the Berry curvature over the Brillouin zone
  $C = \frac{1}{2\pi} \iint dk_x dk_y \, \Omega$ (for more, see Xiao et al. Rev. Mod. Phys. 82, 1959 (2010).)

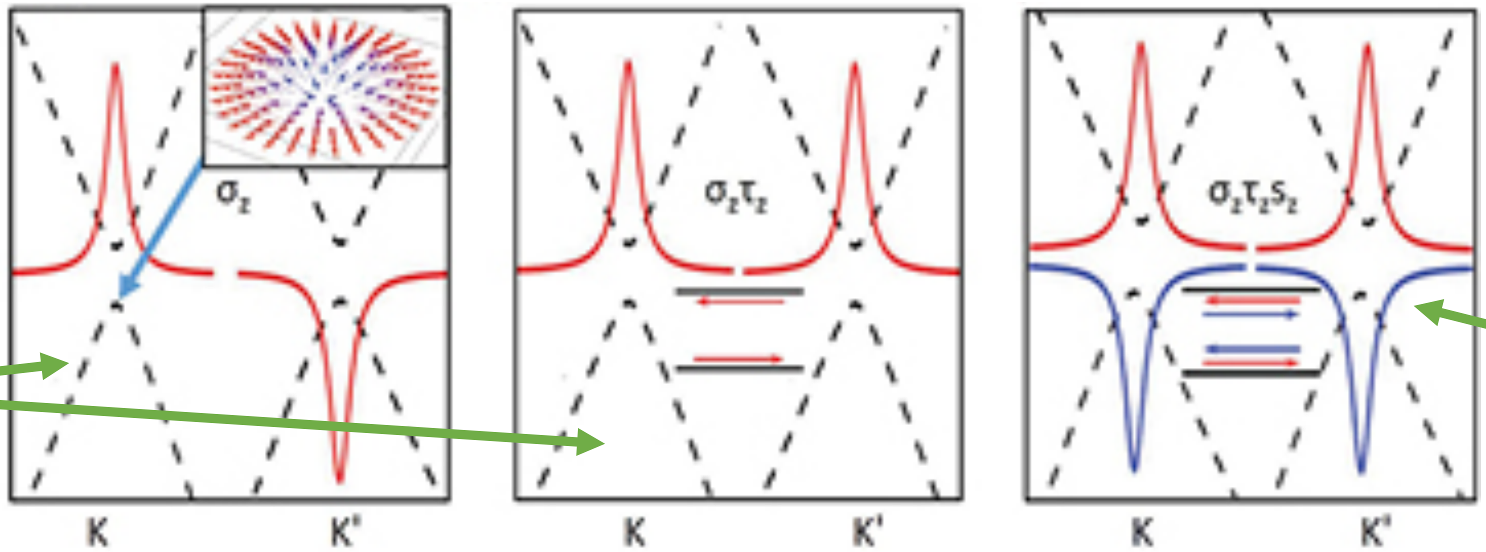
- Chern number of a 2D Dirac electron is simply: $C = \frac{1}{2} sign(m * N_w)$
  "a short-cut to topological material/number"

- Z_2 topological insulators: $H_{m3} = m_3 \tau_z \sigma_z s_z$



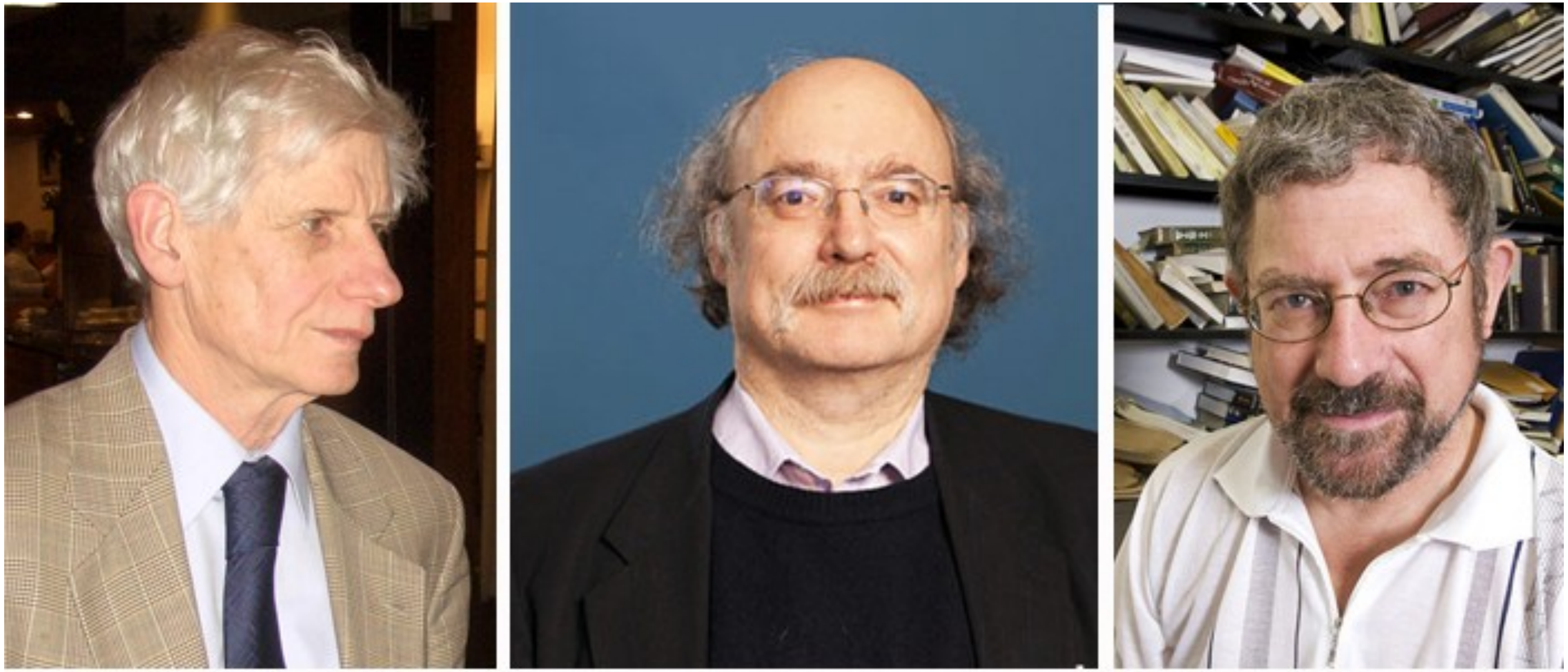
Red curves:
Berry curvature
Black: band-structure

Haldane model



Kane-mele Model of time-reversal symmetry (Z_2) topological insulators

Nobel Prize for Physics 2016:
“for theoretical discoveries of topological phase transitions and topological phases of matter”



Thouless Haldane Kosterlitz

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Why topology in photonics?

1. **Topological light-trapping at interfaces**
   edge/surface states

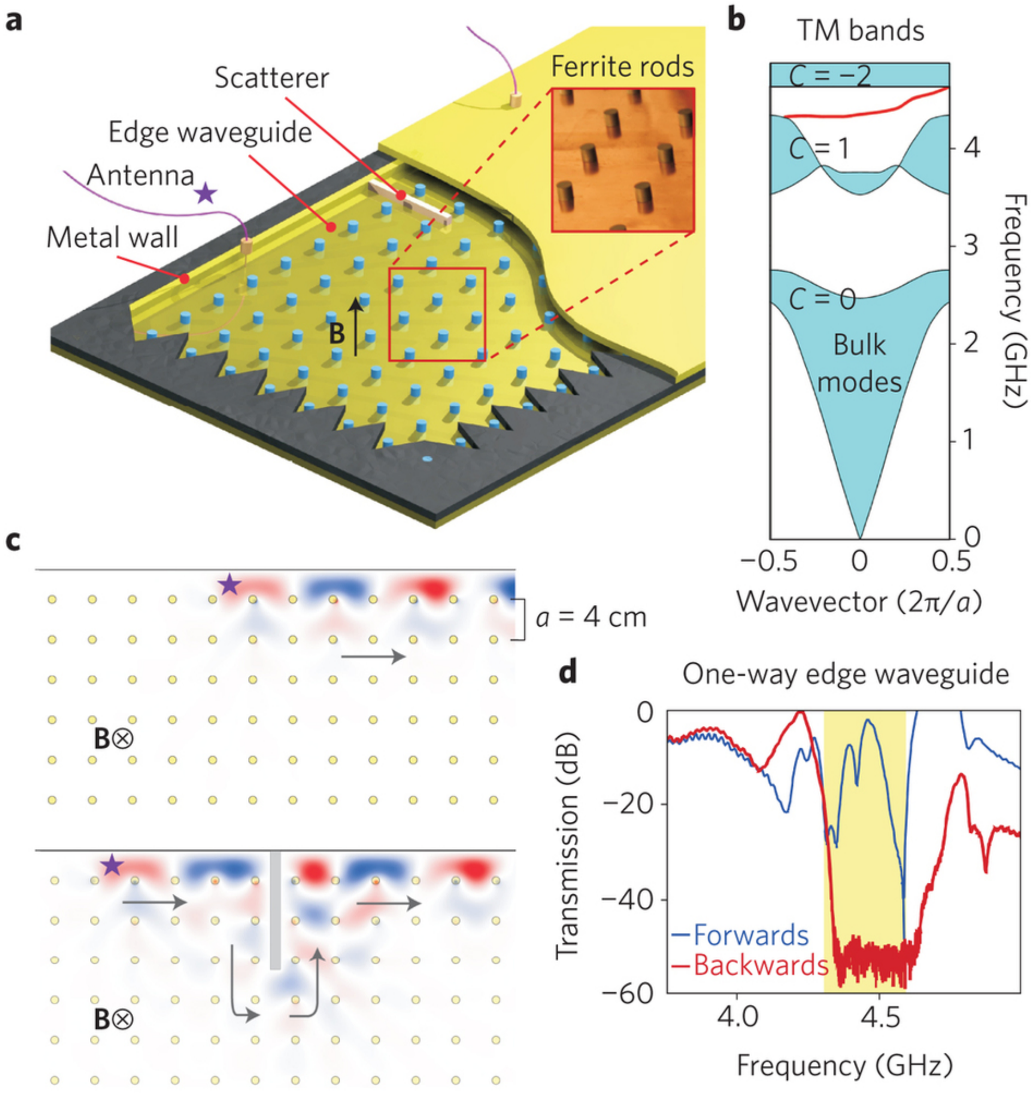
2. **Robust waveguiding**
   e.g., one-way edge waveguide

3. **Zero refractive-index behaviors**
   effectively n=0 for light propagating at $\omega = \omega_{Dirac}$

4. ***Dirac cones, synthetic gauge fields…***

5. **Anomalous refraction**, etc.

Topological light-trapping & one-way waveguiding

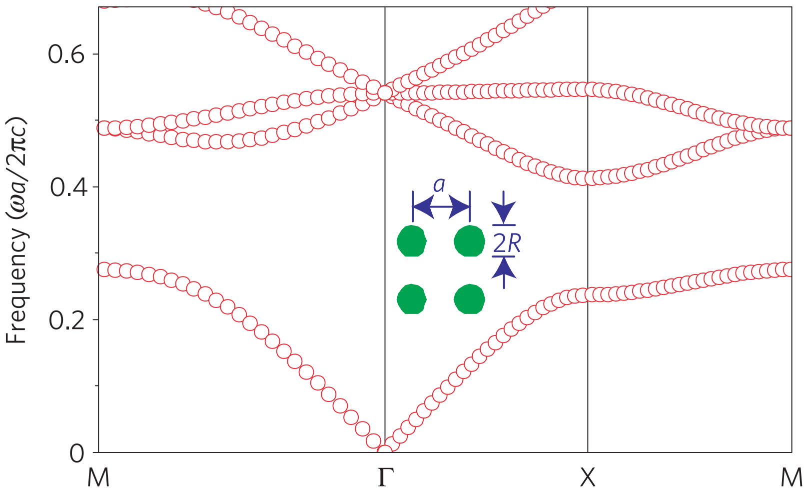


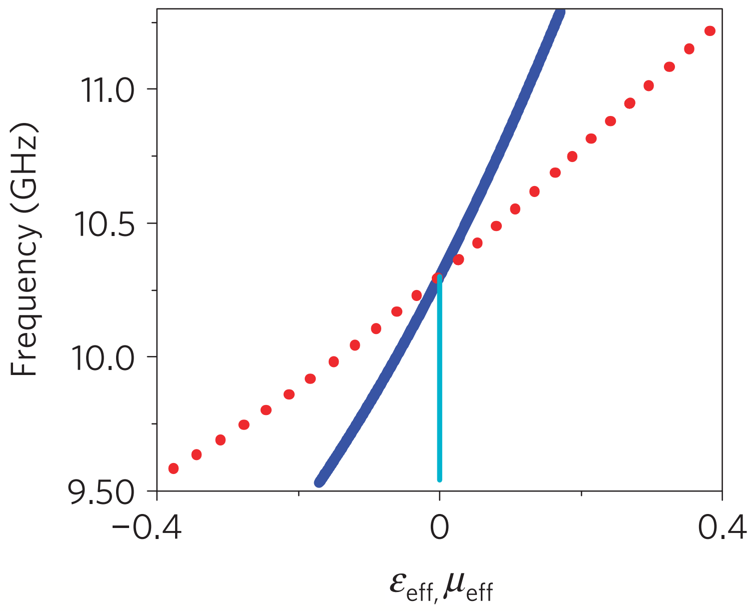
Theory:
Haldane and Raghu
Phys. Rev. Lett. **100**, 013904 (2008)
Wang et al.
Phys. Rev. Lett. **100**, 013905 (2008)

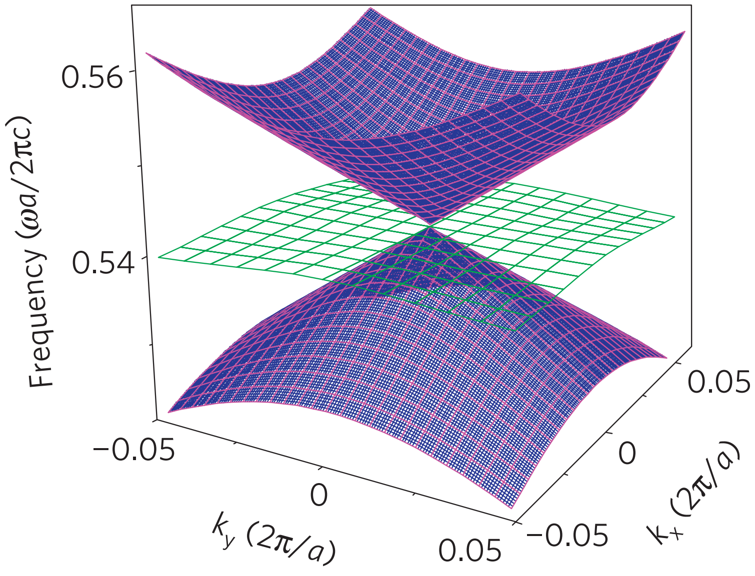
Experiment:
Wang et al. Nature **461**, 772-775 (2009)

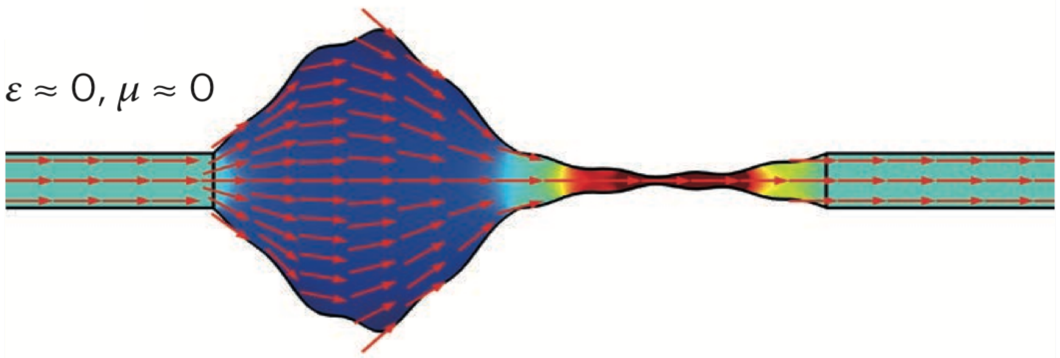
- ***Breaking time-reversal symmetry via magneto-optical effects***
- ***Topological light trapping on the edge***
- ***Robust unidirectional light-flow***

Effective Zero-Index Media



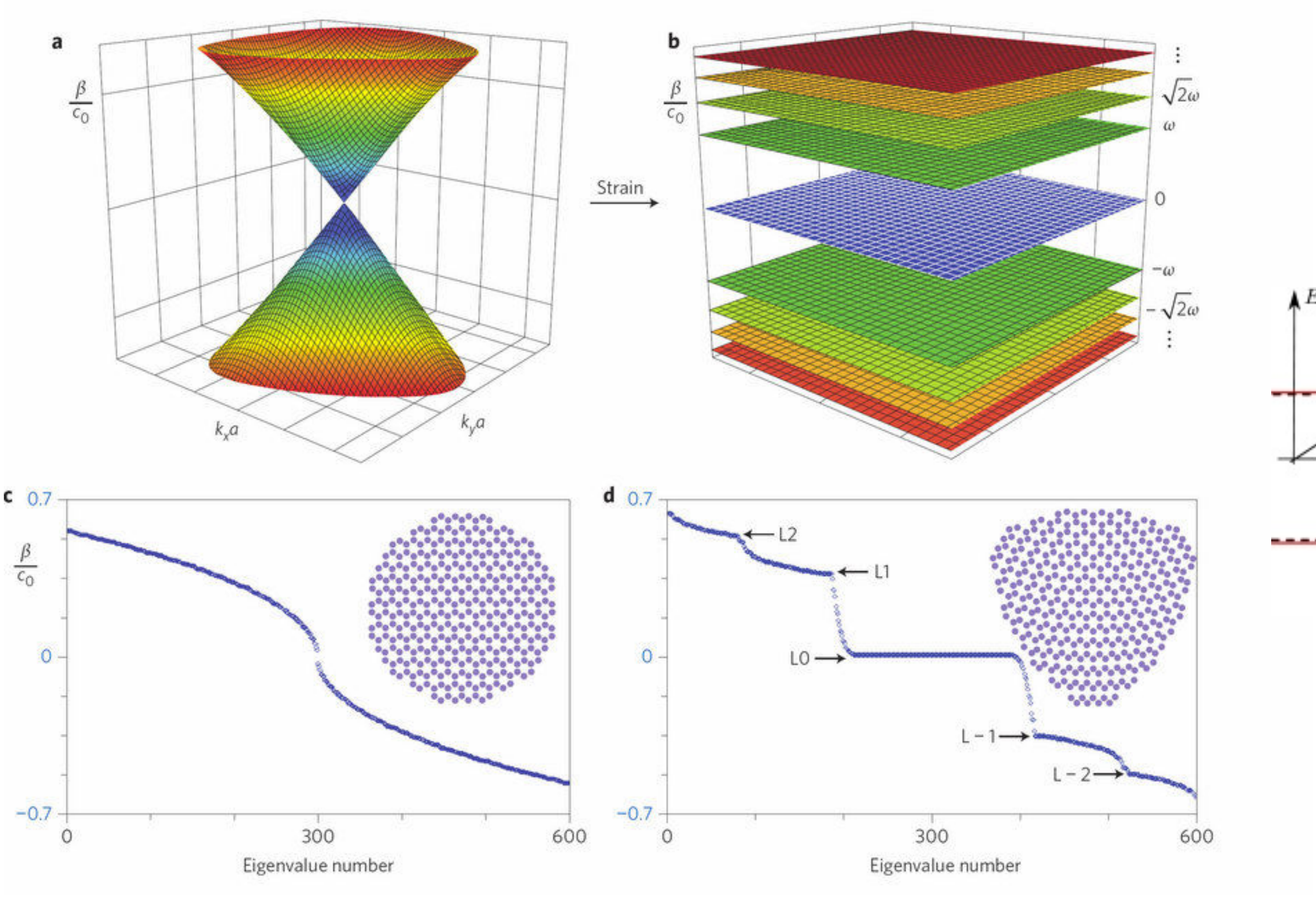


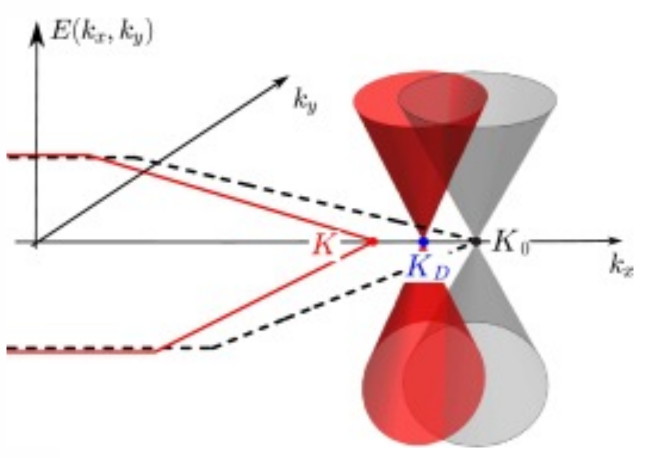




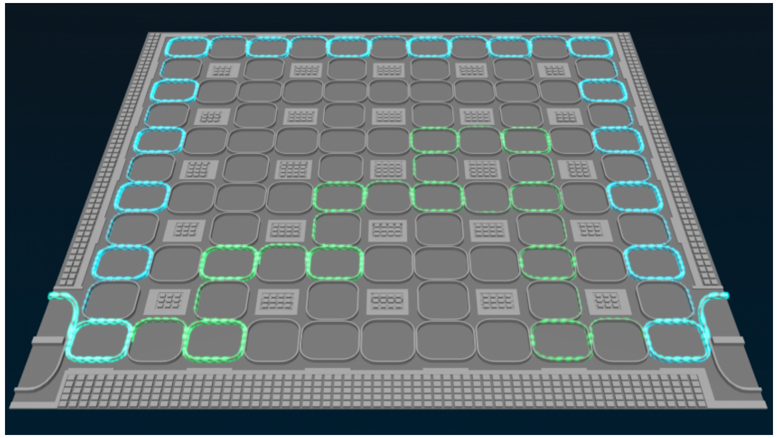
"***Liquid like propagation of light***", *Huang et al. Nat. Mater.* **10**, 582 (2011); *Liberal & Engheta Nat. Photon.* **11**, 149 (2017).

Synthetic Gauge Fields & Photonic Landau Levels

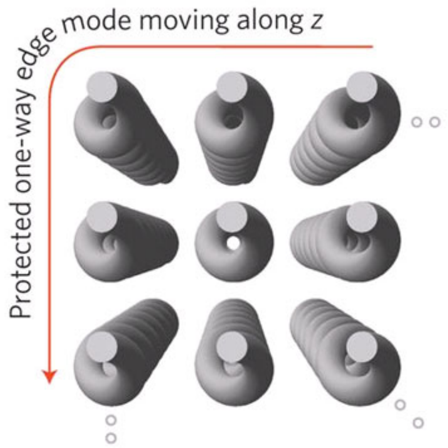




Creating optical topology using only dielectrics



Hafezi et al., Nat. Photon. 7, 1001 (2013)



Rechtsman et al, Nature 496, 196 (2013)

- No topological light-trapping (light-trapping achieved by each lattice site)
- No subwavelength physics, no strong light-matter interaction, etc.

Simulating Dirac physics for optics: challenges

$$\left(\sum_{n=1}^{3} \alpha_n p_n\right)\psi = i\frac{\partial}{\partial t}\psi \quad \Longrightarrow \quad H \equiv \sum_{n=1}^{3} \alpha_n p_n = \begin{pmatrix} H_L & 0 \\ 0 & H_R \end{pmatrix}$$

$$H_L = -\sum_n \sigma_n p_n \quad (n=x,y,z) \qquad H_R = \sum_n \sigma_n p_n \quad (n=x,y,z)$$

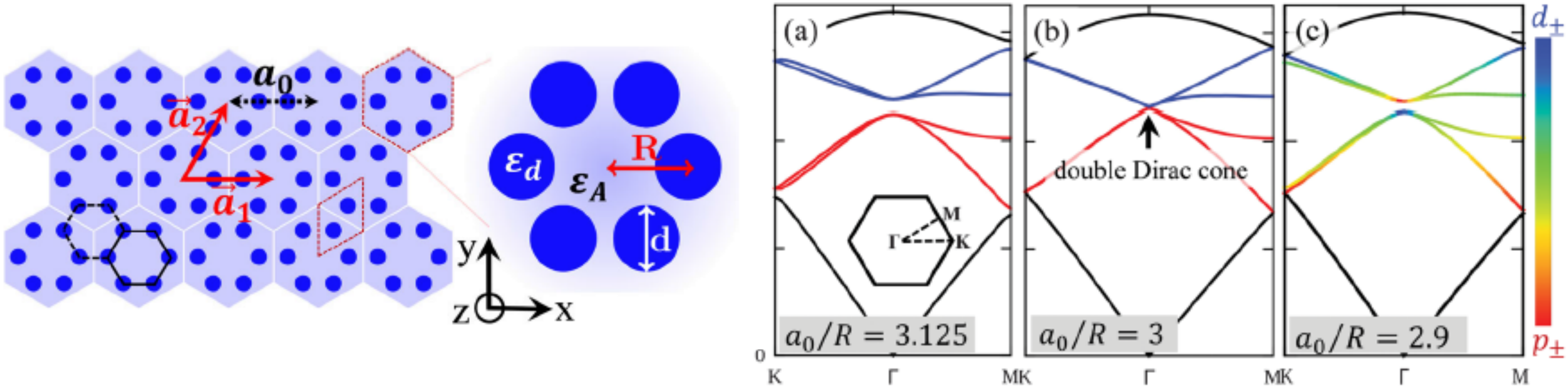
- "Spin": Kramers double degeneracy is ***absent*** for photons (polarization, EM duality symmetry, …. not working)
- "Orbital": Parity inversion to ensure *k-linear* Hamiltonian
- Need to realize both simultaneously (mapping from Maxwell equations to Dirac equation!)

$$H_{Maxwell} := H_D^2$$

- **Using only dielectric materials**

Subwavelength photonic crystals

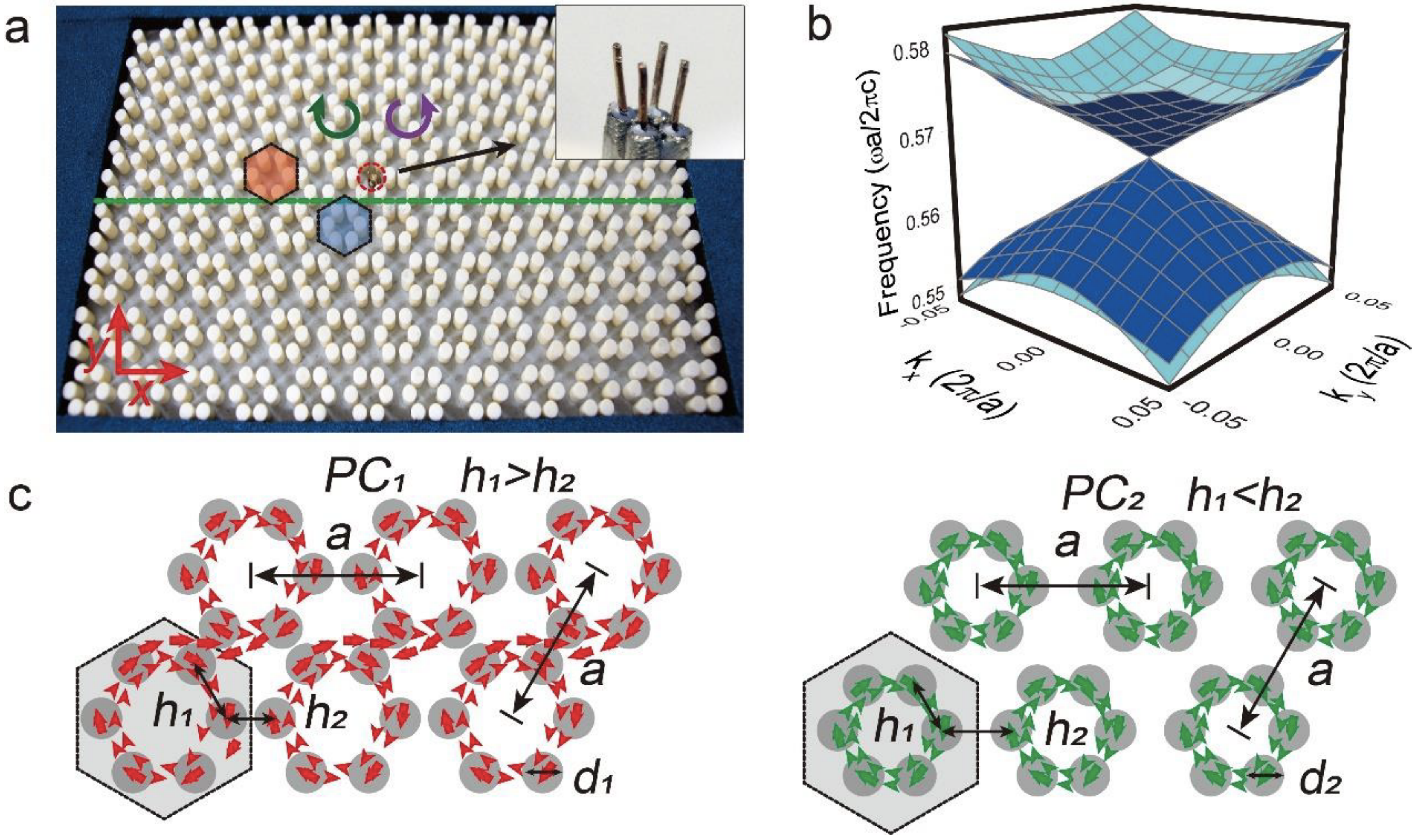
Wu & Hu, Phys. Rev. Lett. 114, 223901 (2015)



- Dielectric photonic crystal, but 2D (still assuming metallic cladding in z direction)

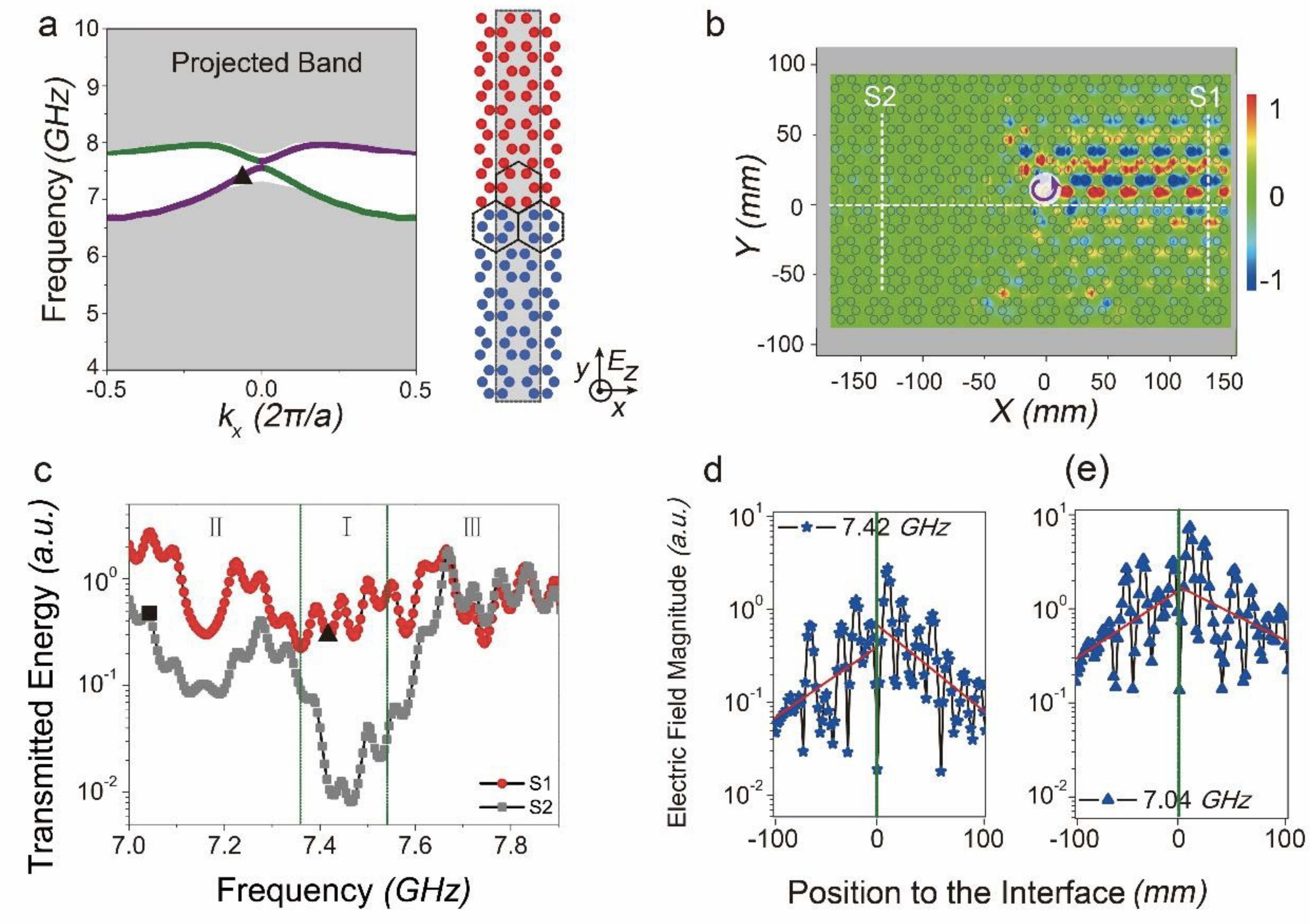
- Solution: A slab photonic crystal design with only dielectrics
  Barik et al. New J. Phys. ***18****, 113013 (2016). Zhu et al., in preparation*

Experimental realization



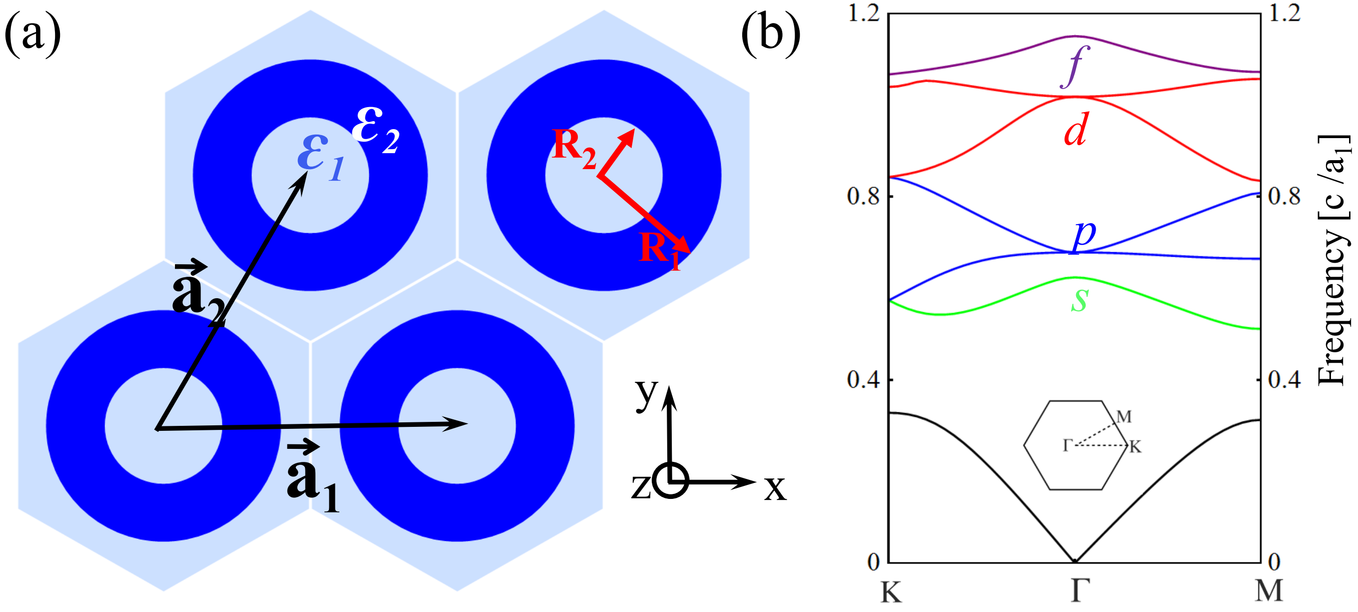
Yang, JHJ, Hu, Hang, arXiv:1610.07780

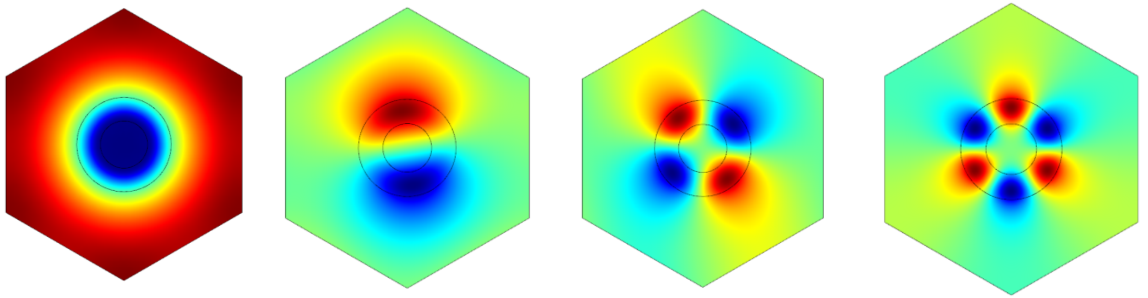
Pseudo-spin of edge states



Yang, JHJ, Hu, Hang, arXiv:1610.07780

A simple core-shell dielectric photonic crystal





C$_6$ symmetry ensures:

p and d states

Doubly degenerate

@Brillouin zone center

(“*pseudo-Kramers degeneracy*”)

Xu, Wang, Xu, Chen, & JHJ, Opt. Express 24, 18059 (2016)

Physics and Observations

$$\nabla \times \frac{1}{\varepsilon(\vec{r})} \nabla \times \vec{h}_{n,\vec{k}}(\vec{r}) = \frac{\omega^2}{c^2} \vec{h}_{n,\vec{k}}(\vec{r})$$

$$\mathcal{H} = \nabla \times \frac{1}{\varepsilon(\vec{r})} \nabla \times$$

$$\int_{u.c.} d\vec{r} \vec{h}^*_{n',\vec{k}}(\vec{r}) \cdot \vec{h}_{n,\vec{k}}(\vec{r}) = \delta_{nn'}$$

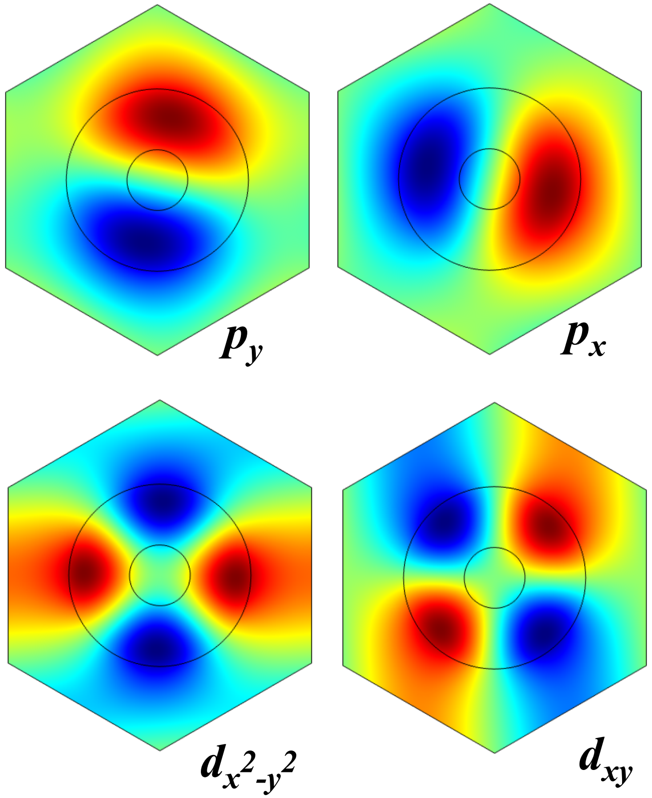
Four bands $k \cdot P$ expansion of the above "Hamiltonian" near the Brillouin zone center :

Spin up

$$H_{\mathrm{eff}}(k_x, k_y) = \begin{pmatrix} H(k) & 0 \\ 0 & H^*(-k) \end{pmatrix}$$

Spin down

$$H(k) = \begin{pmatrix} \frac{\omega_p^2}{c^2} & Ak_+ \\ A^*k_- & \frac{\omega_d^2}{c^2} \end{pmatrix} \qquad k_\pm = k_x \pm ik_y$$



Spin up

$$p_+ = p_x + ip_y$$
$$d_+ = d_{x^2-y^2} + id_{xy}$$

Spin down

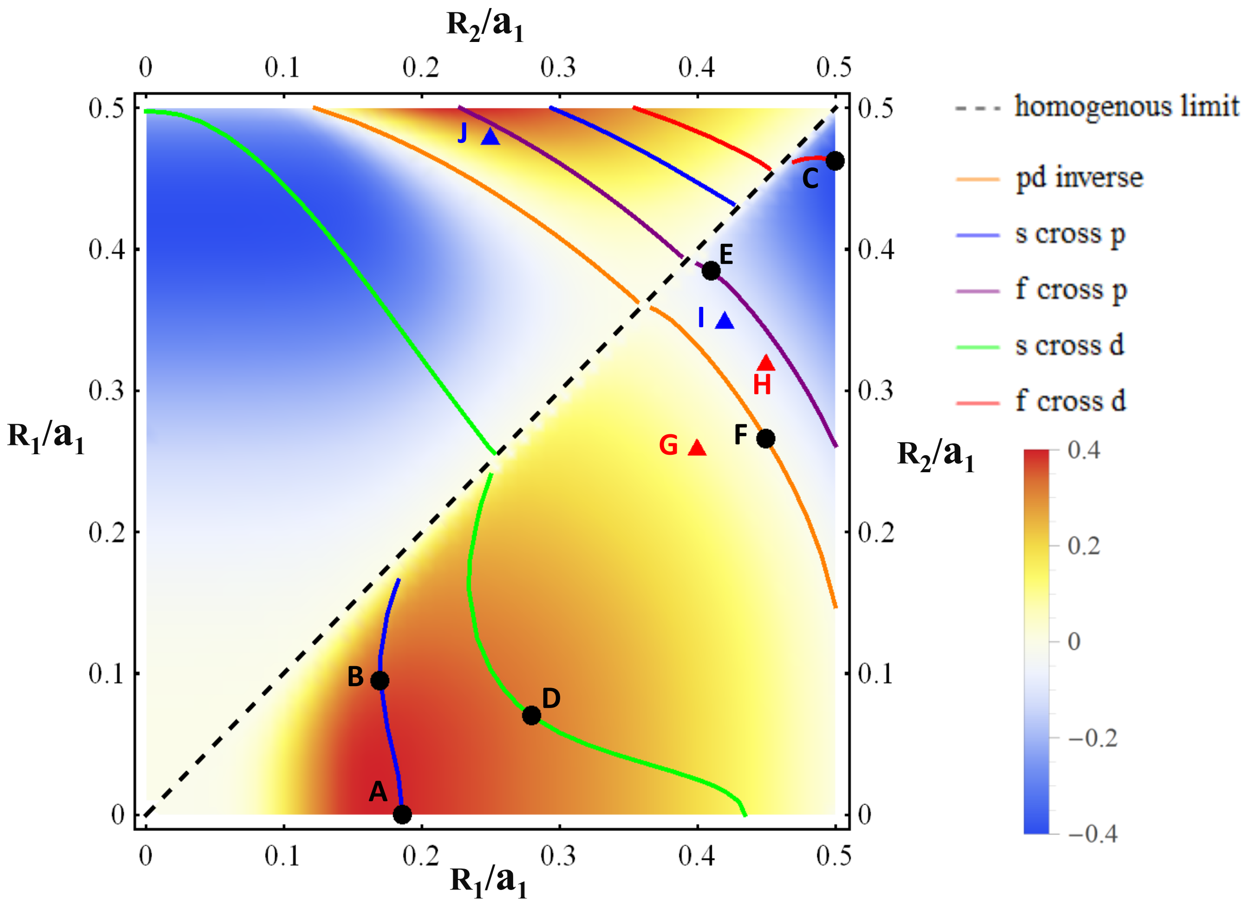
$$p_- = p_x - ip_y$$
$$d_- = d_{x^2-y^2} - id_{xy}$$

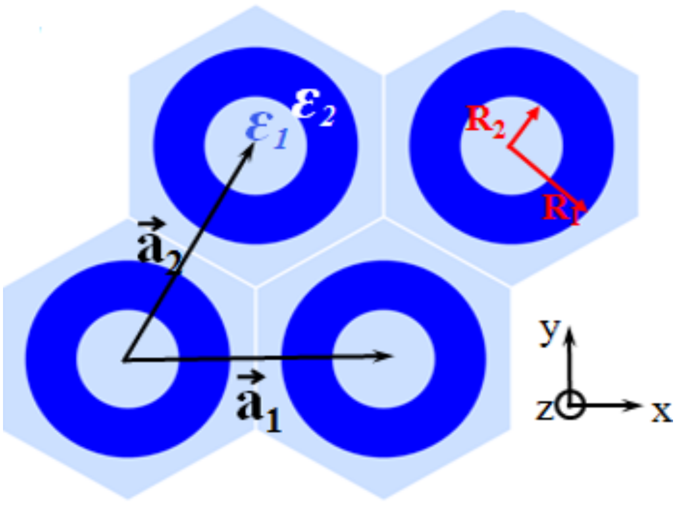
- $\omega_p < \omega_d$ Normal photonic band gap
- $\omega_p > \omega_d$ "p-d inversed" bands: Z_2 topological insulator of light

BHZ, Science 314, 1757 (2006)

Xu, Wang, Xu, Chen, & JHJ, Opt. Express 24, 18059 (2016)

2D core-shell photonic crystal: phase diagram



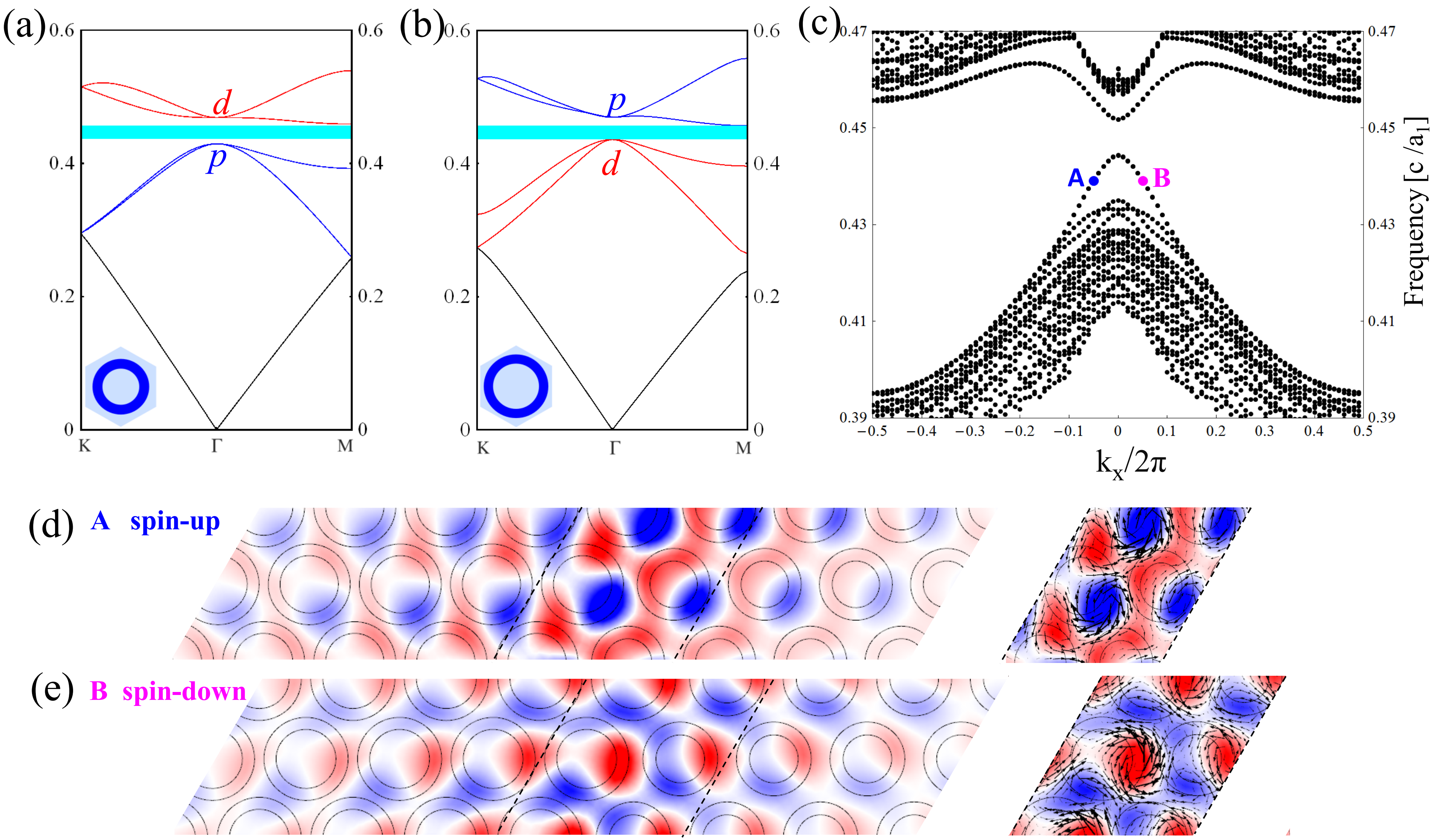


ε_2=12 (silicon)

ε_1=1

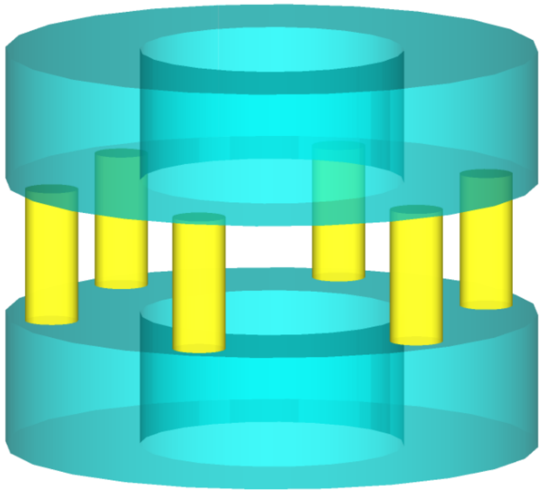
Xu, Wang, Xu, Chen, & JHJ, Opt. Express 24, 18059 (2016)

Edge states: unpaired 1D massive Dirac photon



Xu, Wang, Xu, Chen, & JHJ, Opt. Express 24, 18059 (2016)

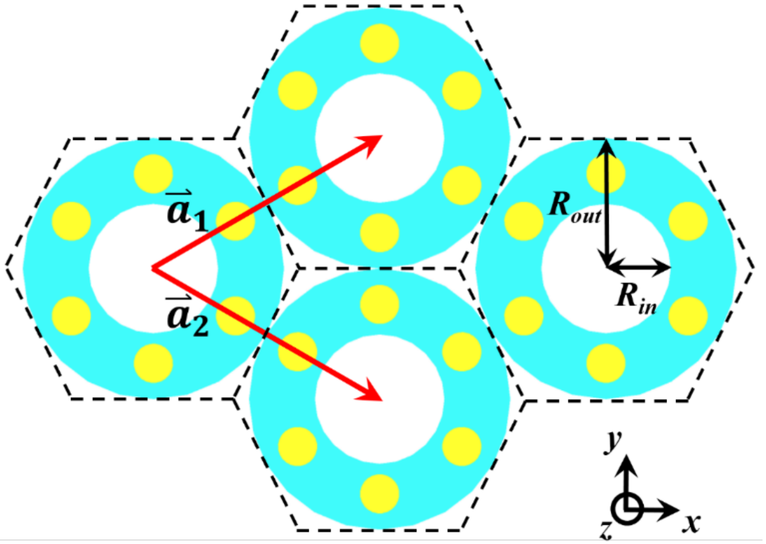
3D Z_2 Dirac points in dielectric photonic crystal

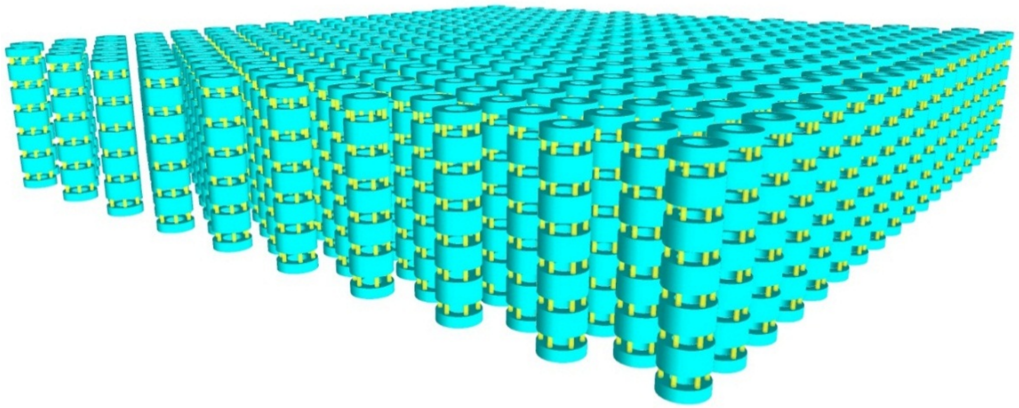


$\varepsilon=12$

$R_{out}=0.5a$

$R_{in}=0.4a$

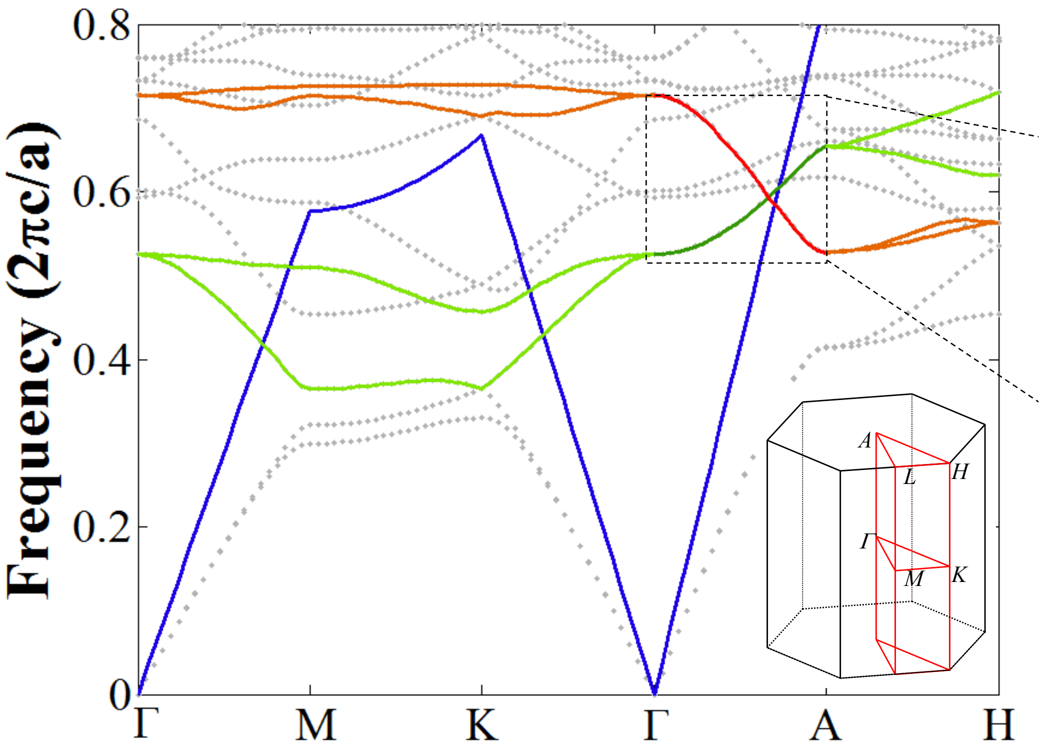


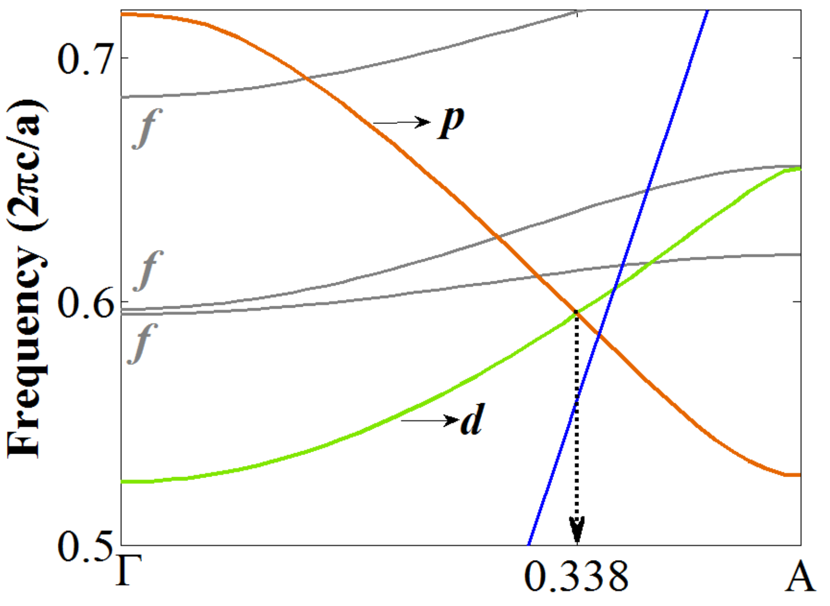


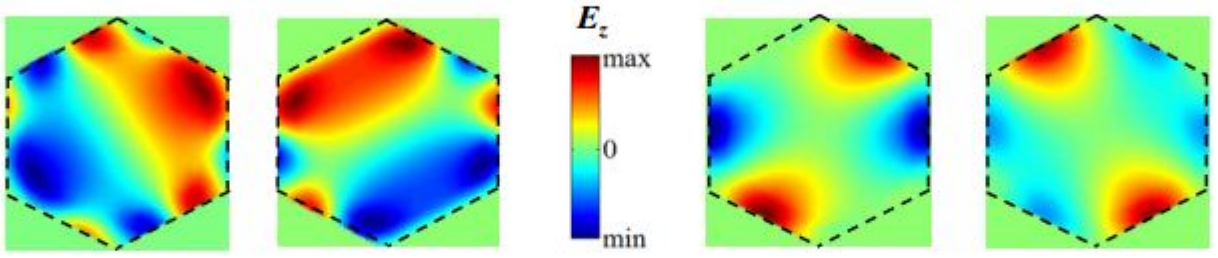
- Hexagonal photonic crystal
- C_{6v} symmetry
- Inversion symmetry

Wang, JHJ *et al.* Phys. Rev. B 93, 235155 (2016)

3D Z_2 Dirac points in dielectric photonic crystal





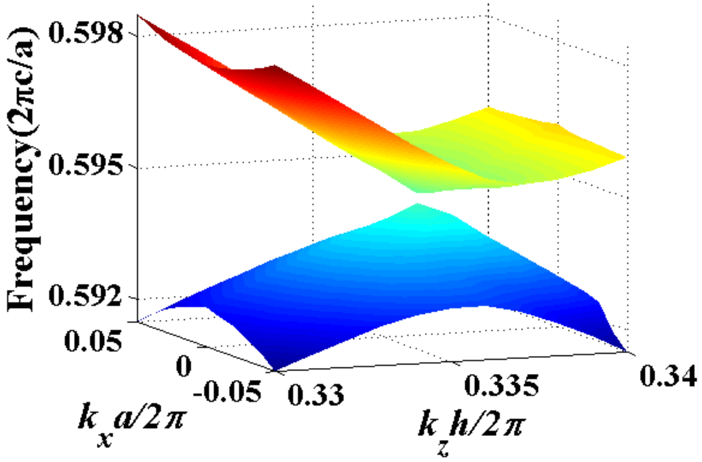


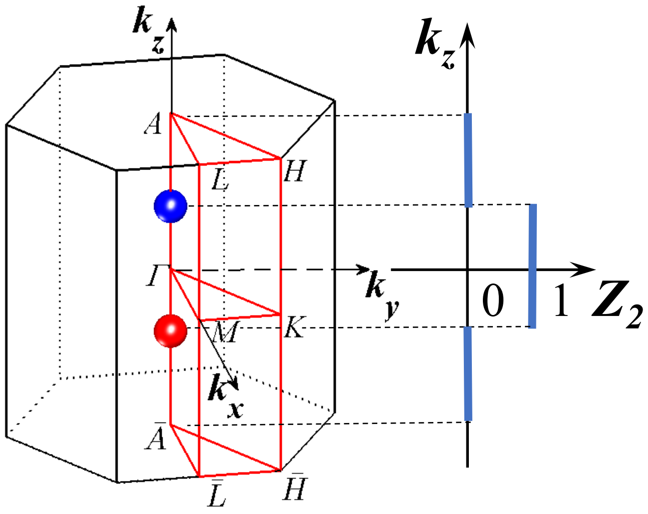
- C_{6v} symmetry
- M_z mirror symmetry

$$(p_+, k_z) \xrightarrow{C_6, TR} (p_-, -k_z) \xrightarrow{M_z} (p_-, k_z)$$

Wang, JHJ *et al.* Phys. Rev. B 93, 235155 (2016)

3D Z_2 Dirac points in dielectric photonic crystal





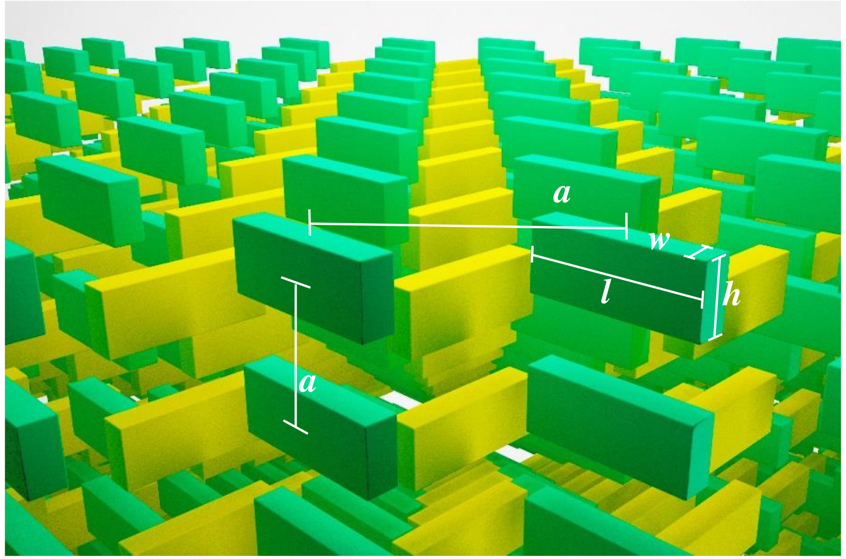
3D (paired) topological Dirac points

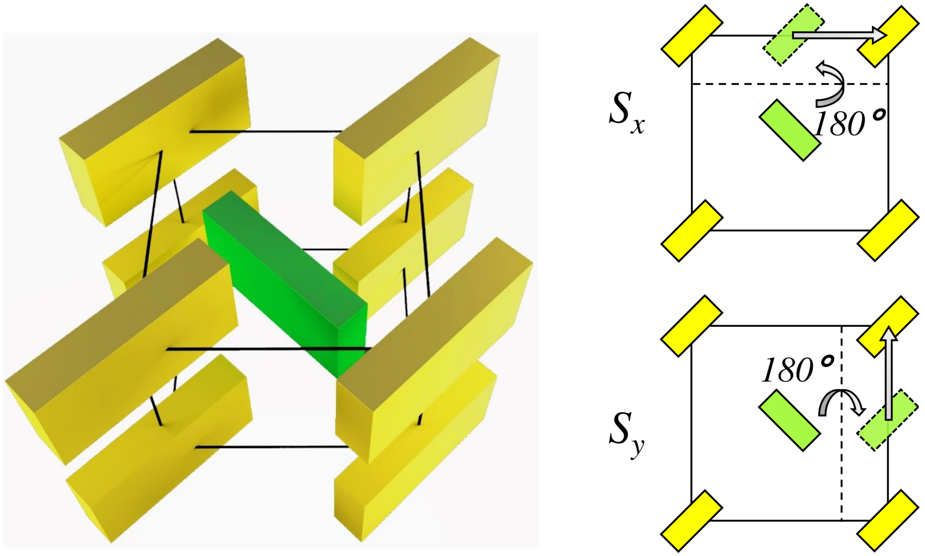
- **Differing from**
  Unpaired Dirac point from fine-tuning (critical, unstable)
- **Parent state of photonic weak topological insulator**
  (Slobozhanyuk *et al.*, Nat. Photon. 2017)

$$\mathcal{H} = \frac{2\omega_0}{c^2}\begin{pmatrix} \frac{\omega_d^2(\vec{k})}{2\omega_0} & v_\parallel k_\parallel e^{-i\theta_k} & 0 & 0 \\ v_\parallel^* k_\parallel e^{i\theta_k} & \frac{\omega_p^2(\vec{k})}{2\omega_0} & 0 & 0 \\ 0 & 0 & \frac{\omega_d^2(\vec{k})}{2\omega_0} & v_\parallel^* k_\parallel e^{i\theta_k} \\ 0 & 0 & v_\parallel k_\parallel e^{-i\theta_k} & \frac{\omega_p^2(\vec{k})}{2\omega_0} \end{pmatrix}$$

Wang, JHJ *et al.* Phys. Rev. B 93, 235155 (2016)

All-dielectric Non-symmorphic Photonic crystals

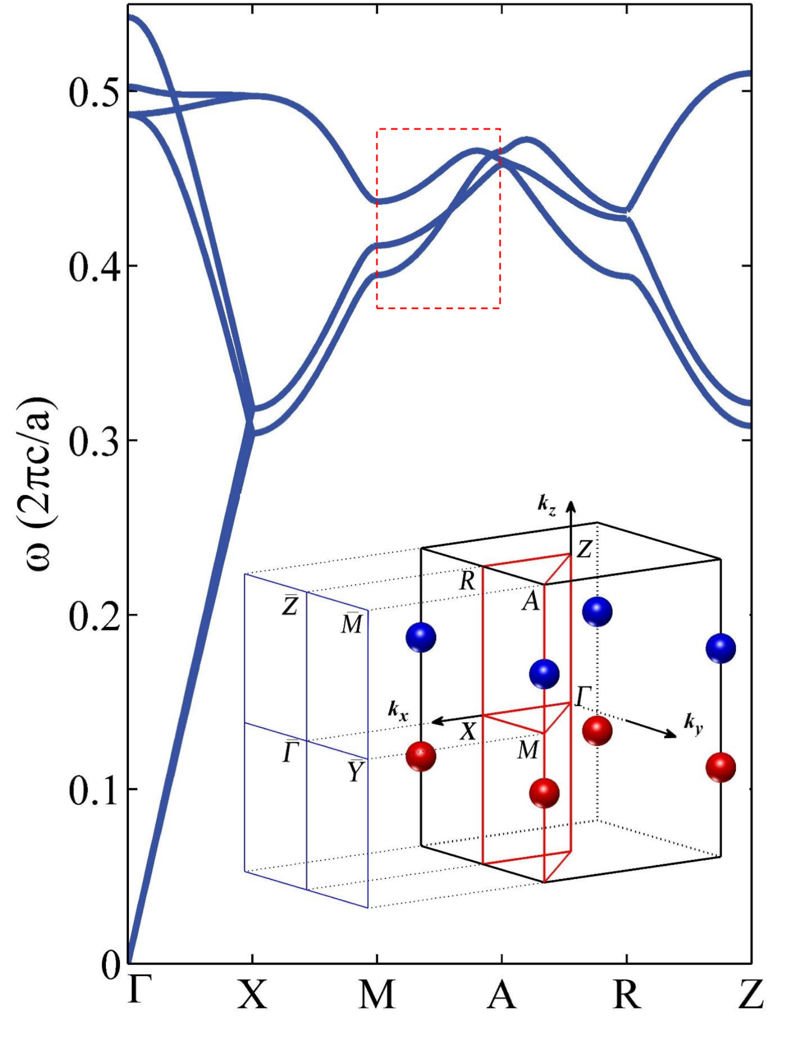


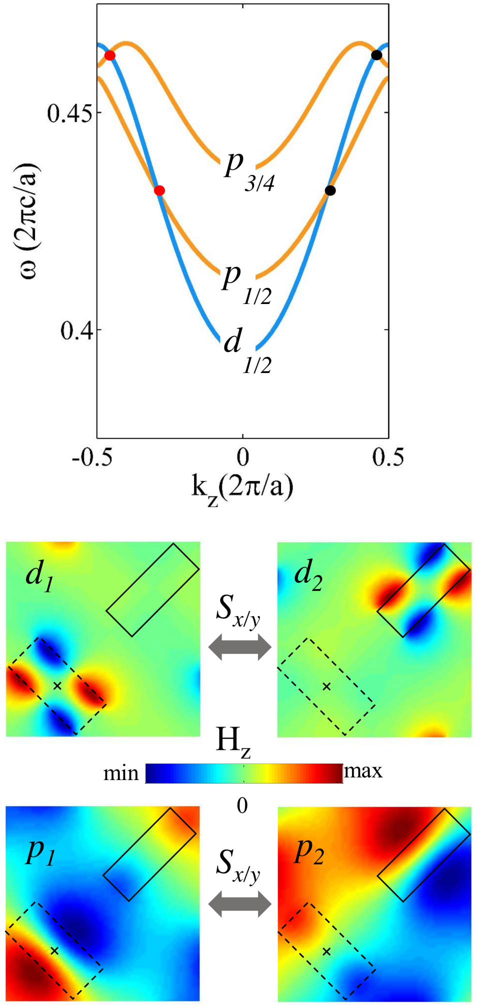


- **Tetragonal photonic crystal**
- **Screw symmetries**

Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

Screw Symmetry induced double degenerate planes





$$\Theta_x \equiv S_x\mathcal{T} : (x, y, z, t) \rightarrow (0.5a + x, 0.5a - y, 0.5a - z, -t),$$

$$\Theta_y \equiv S_y\mathcal{T} : (x, y, z, t) \rightarrow (0.5a - x, 0.5a + y, 0.5a - z, -t),$$

$\mathcal{T}$: time-reversal (complex conjugation)

$$\Theta_x^2 \Psi_{n\vec{k}}(\vec{r}) = e^{-ik_x a}\Psi_{n\vec{k}}(\vec{r}).$$

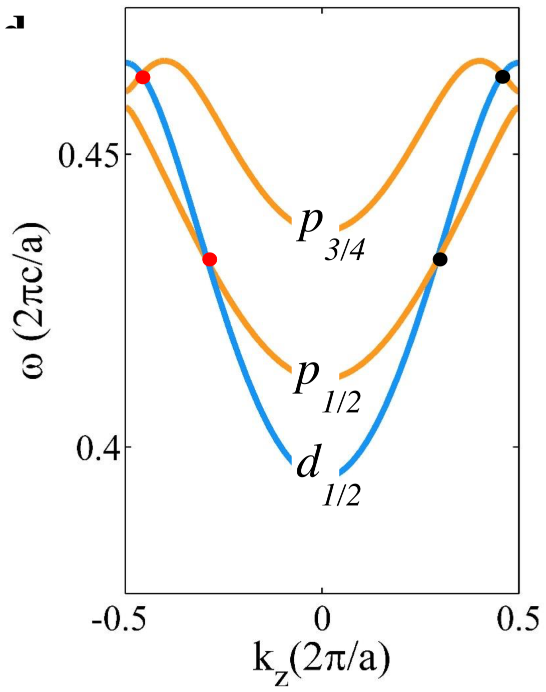
On the $k_x = \frac{\pi}{a}$ plane

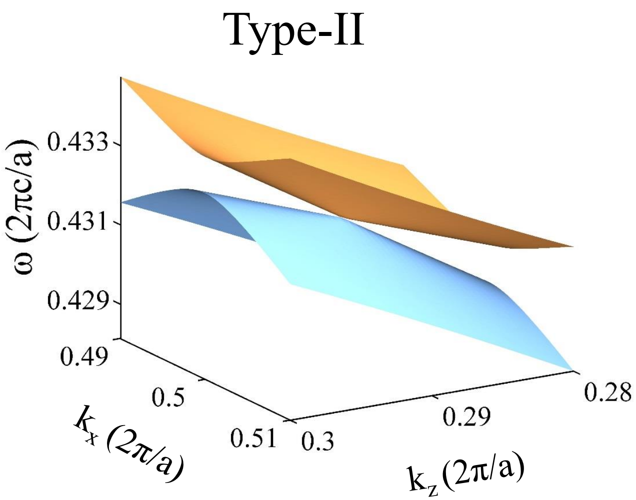
$$\Theta_x^2 = -1.$$

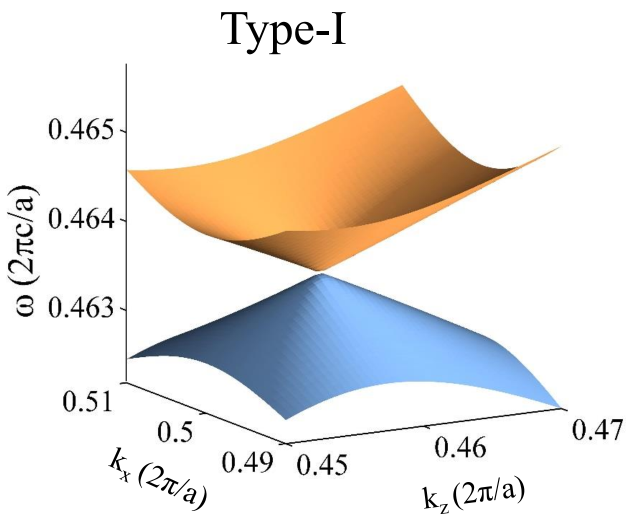
- Double degeneracy for all bands!
- Same for the $k_y = \pi/a$ plane!

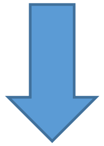
Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

Type-II and type-I Dirac points

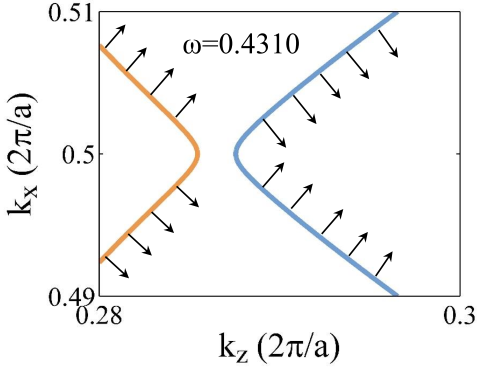


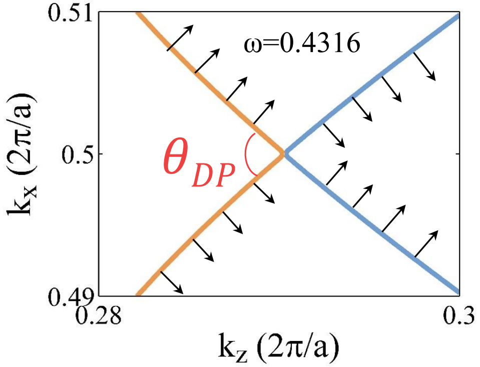






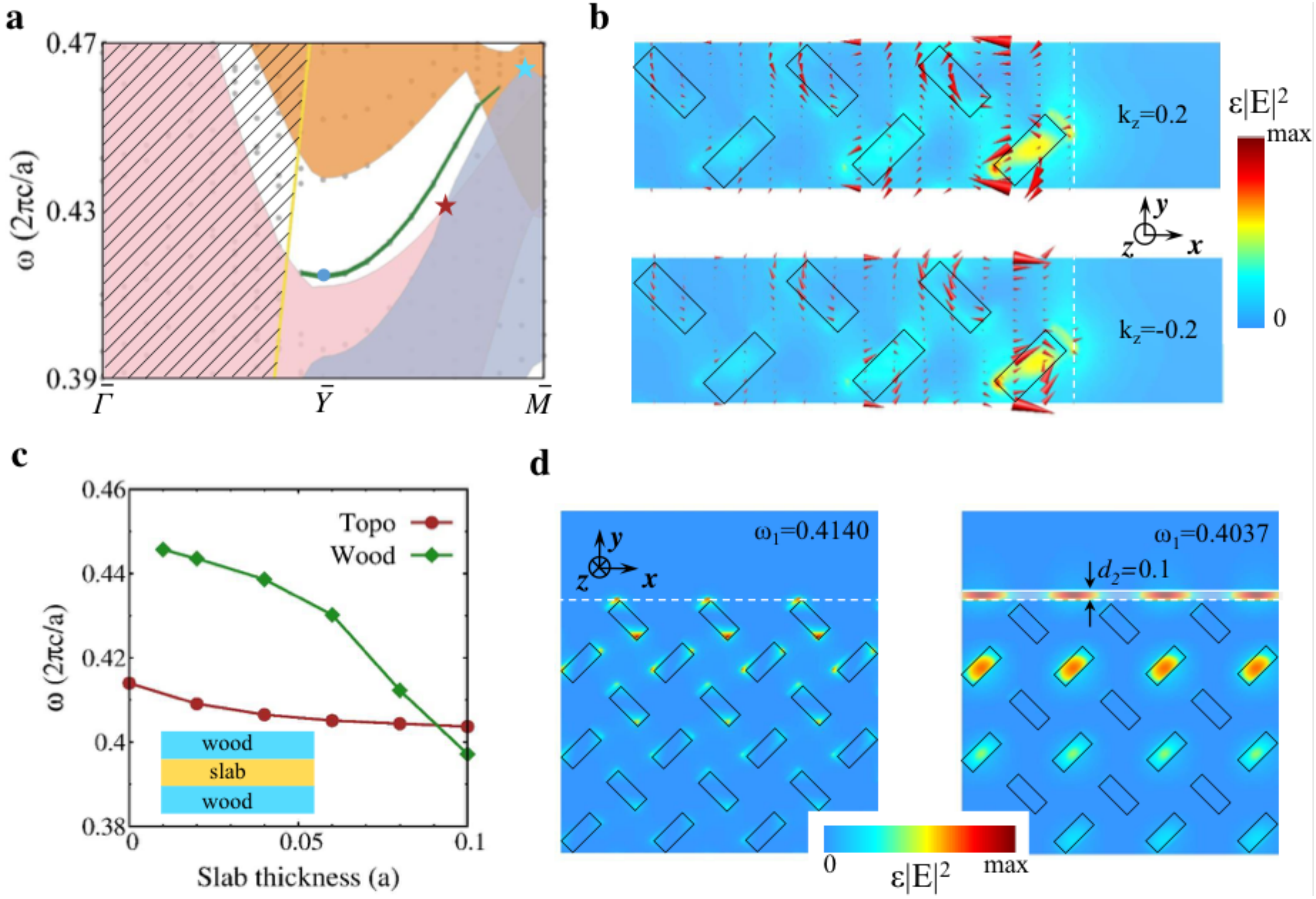
Hyperbolic isofrequency contours →





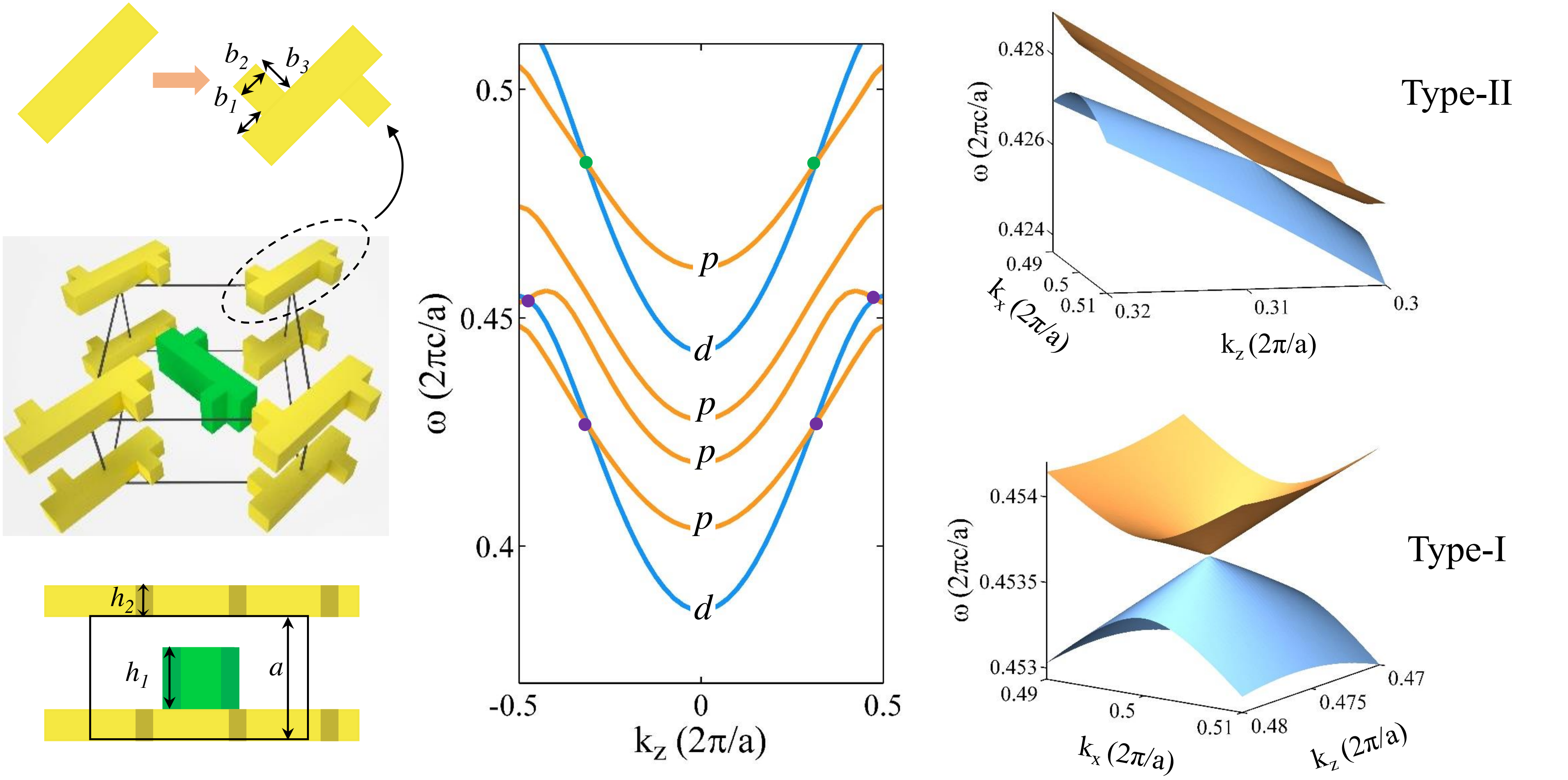
Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

Robust light-trapping on photonic-crystal---air interfaces



Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

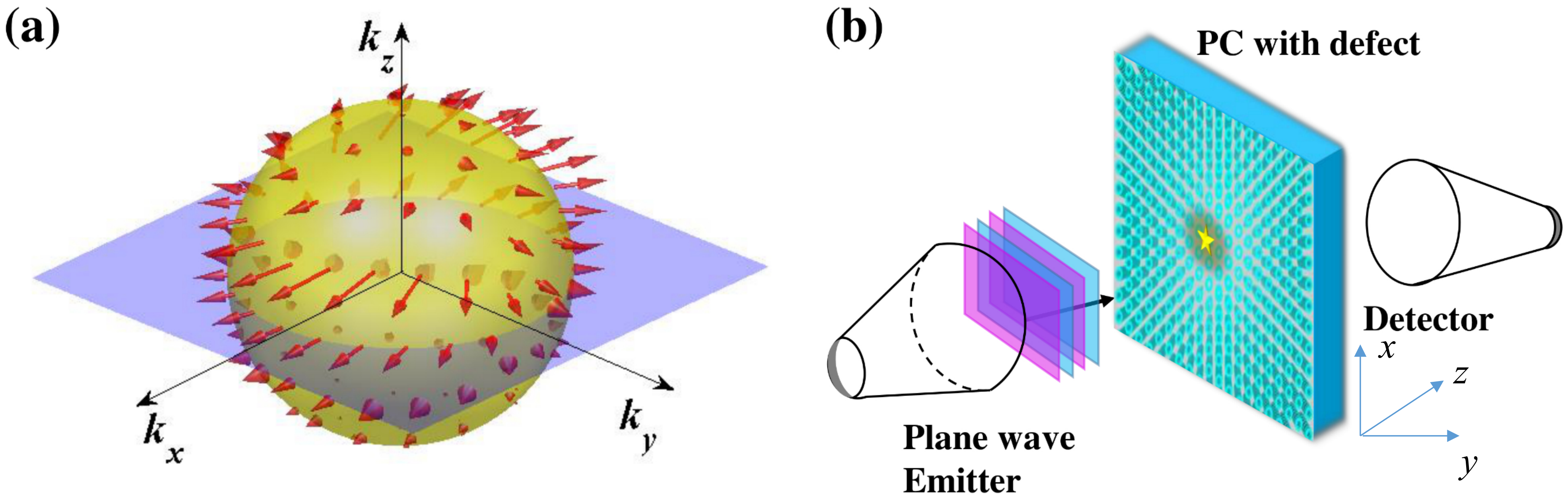
Type-II and type-I Weyl points



Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

Angular-momentum---wavevector correlation

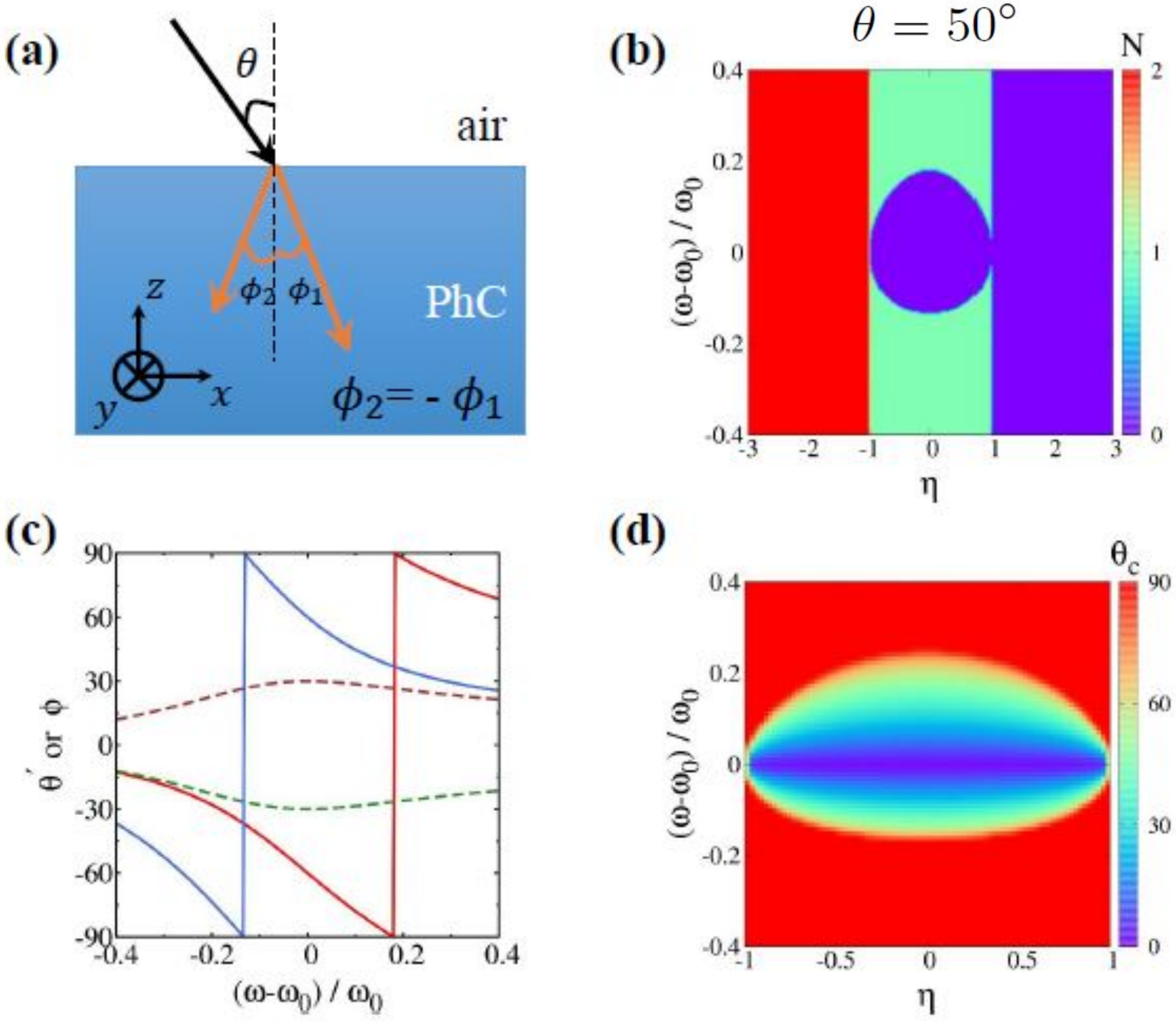
Wang, JHJ *et al.* Phys. Rev. B 93, 235155 (2016)



- **Angular-momentum—wavevector locking of Weyl/Dirac points leads to suppressed back-scattering in the bulk photonic crystal**

Condensed-matter experiment: Liang, et al., Nat. Mater. **14**, 280 (2015)

Refraction: Type-II and type-I Dirac points



$\theta = 50^\circ \quad \eta = -2$

$$\omega = \omega_0 + v(\eta q_z \pm q)$$

Type II: $|\eta| > 1$

Type I: $|\eta| < 1$

A pair of Dirac points ("two valleys") with opposite η

Type II: anomalous refraction with valley contrast

Type I: normal conical refraction no valley contrast

Wang, Chen, Hang, Kee & JHJ, arXiv:1703.09899

Summary

➢ Our photonic architectures

- Point group symmetry: core-shell photonic crystals
- Non-symmorphic screw symmetry: block photonic crystals

➢ Our discoveries:

- 2D topological insulators & 3D topological Dirac/Weyl points
- Special edge states: 1D unpaired massive Dirac photons
- Photonic topological angular-momentum---wavevector correlation
- Topologically robust light-trapping on photonic-crystal---air interfaces
- Anomalous refraction for type-II Dirac (Weyl) points

Further questions

➢ Topological optics

- **Topological cavities (suppressing inhomoegeneous broadening)**
- **Nonlinear topological optics : polaritons**
- **Quantum effects? (nonreciprocal photon emission)……**
- **…..**

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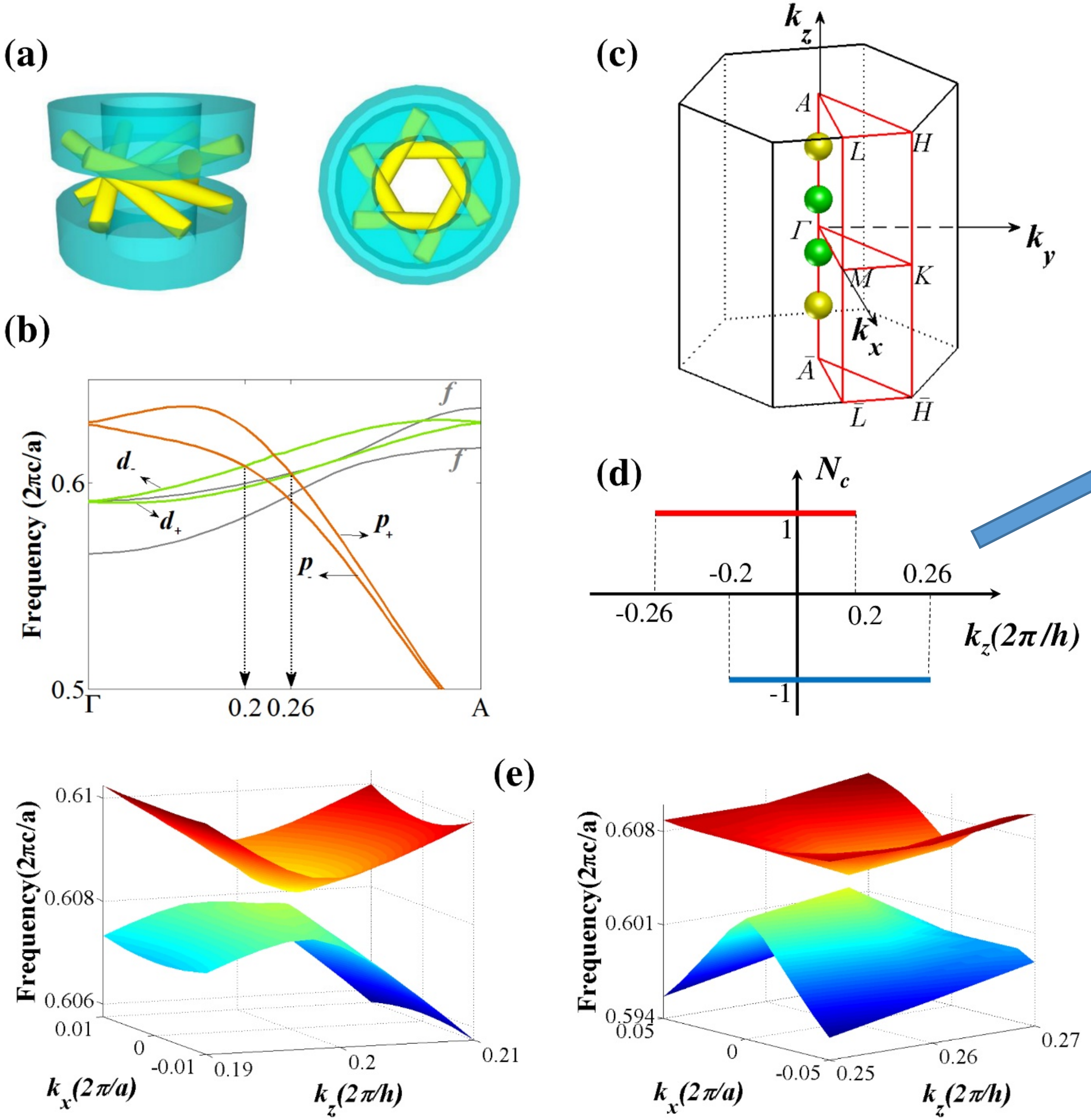
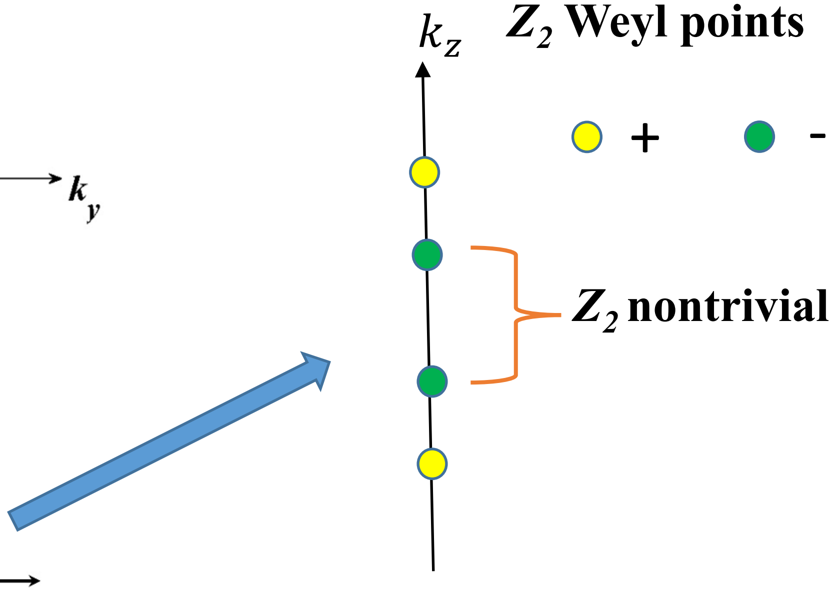
Breaking inversion symmetry: Z_2 Weyl Points

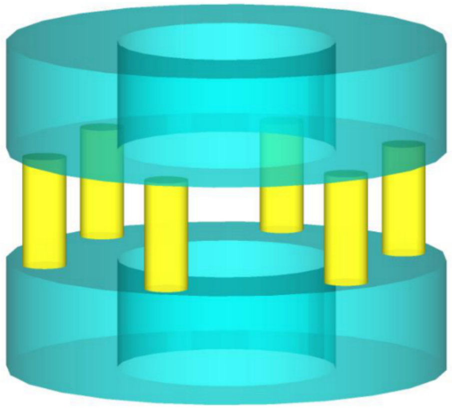


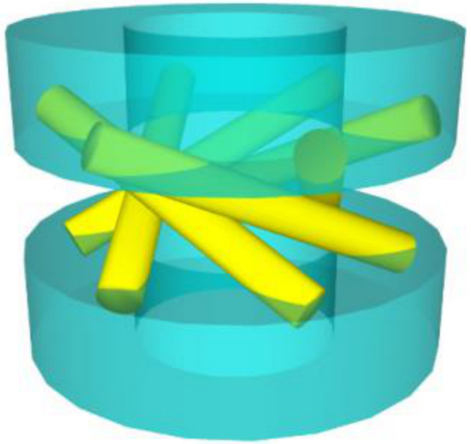


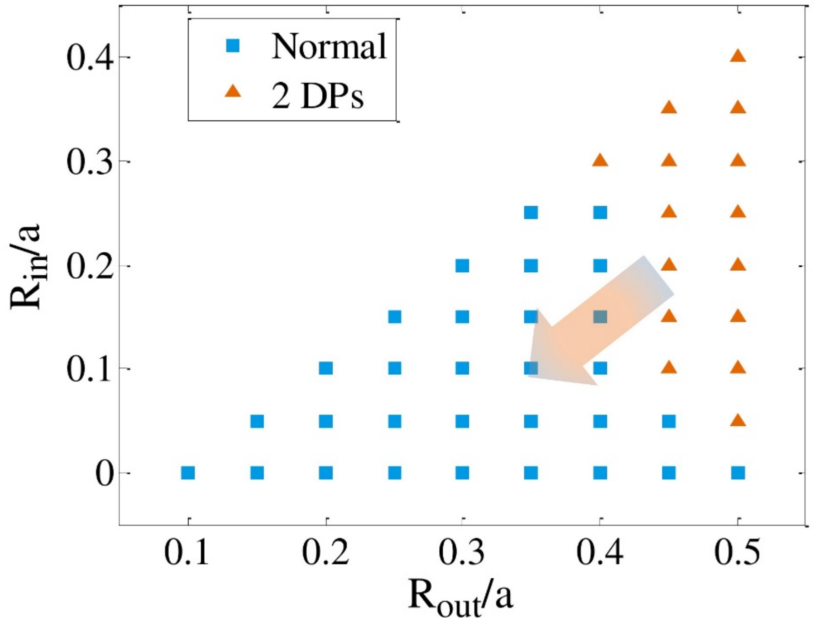
A pair of Dirac points --->
Two pairs of Weyl points

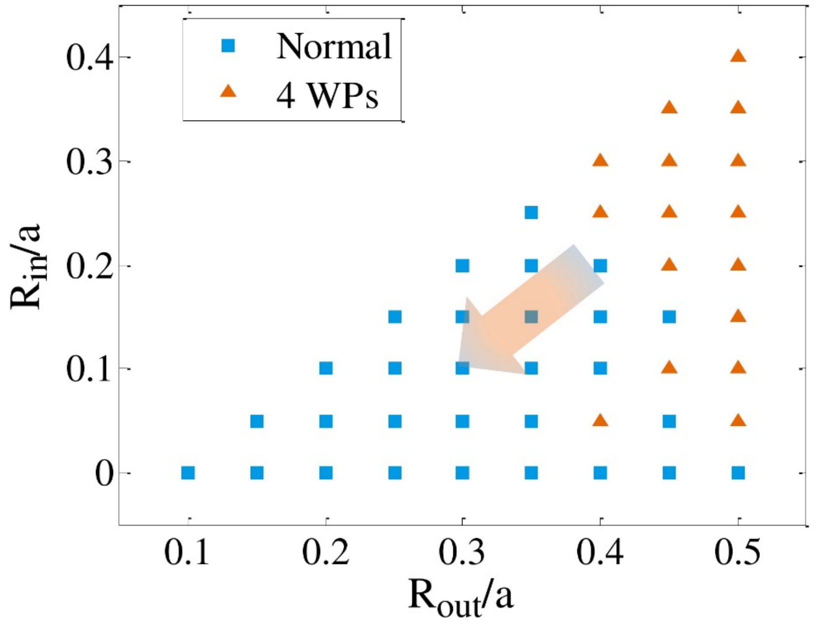
Wang, Jiang *et al.* Phys. Rev. B **93**, 235155 (2016)

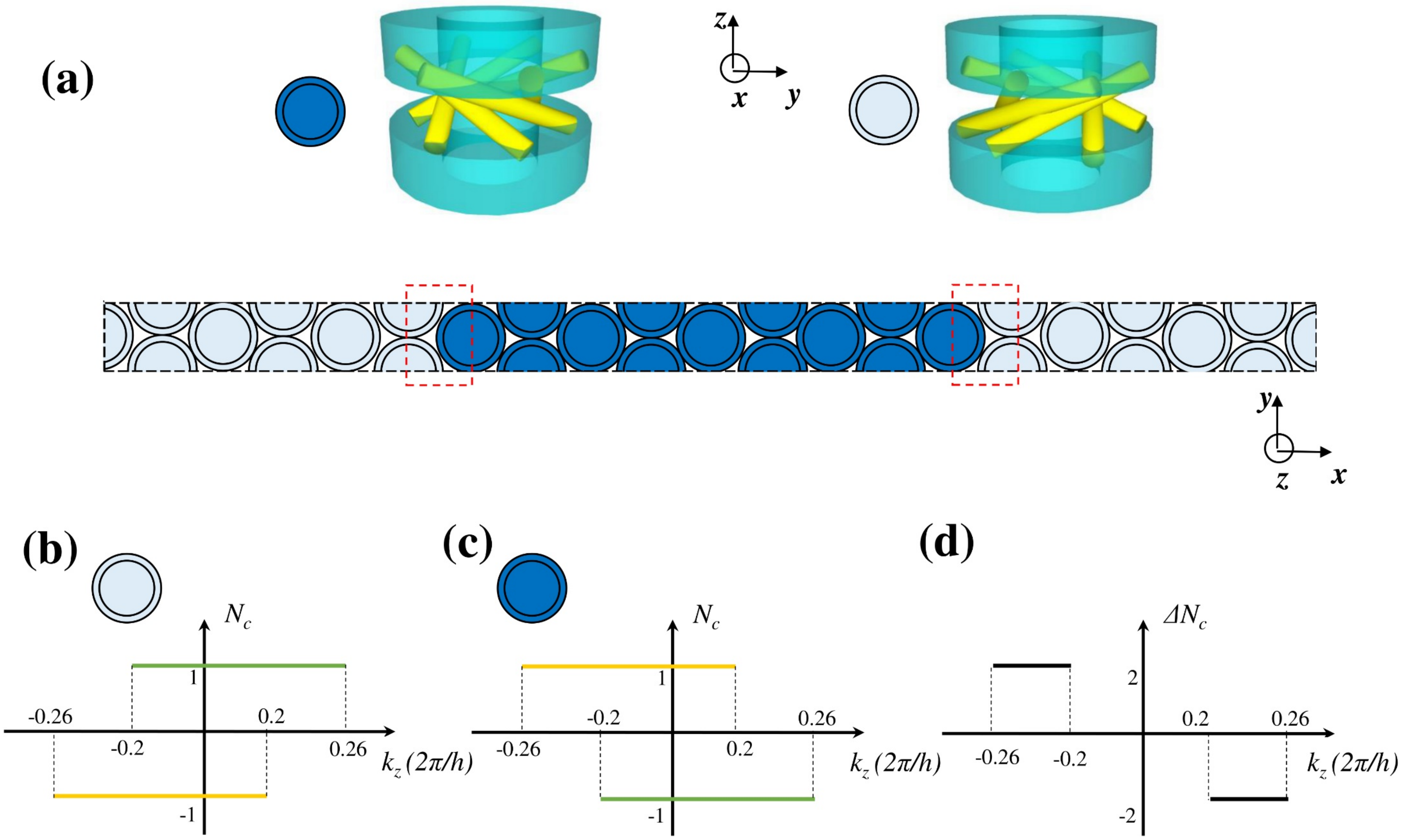
Phase diagram & robustness



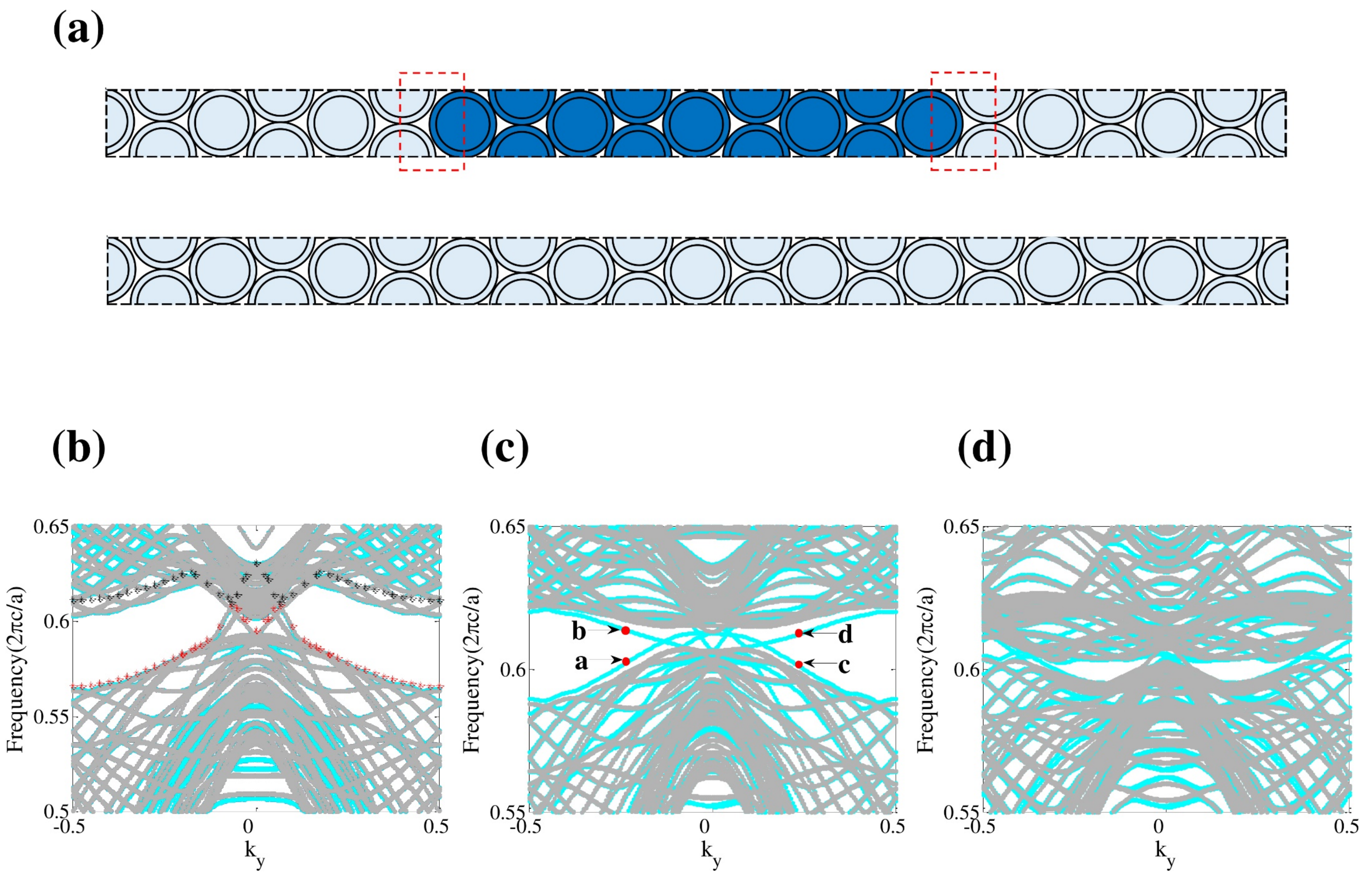




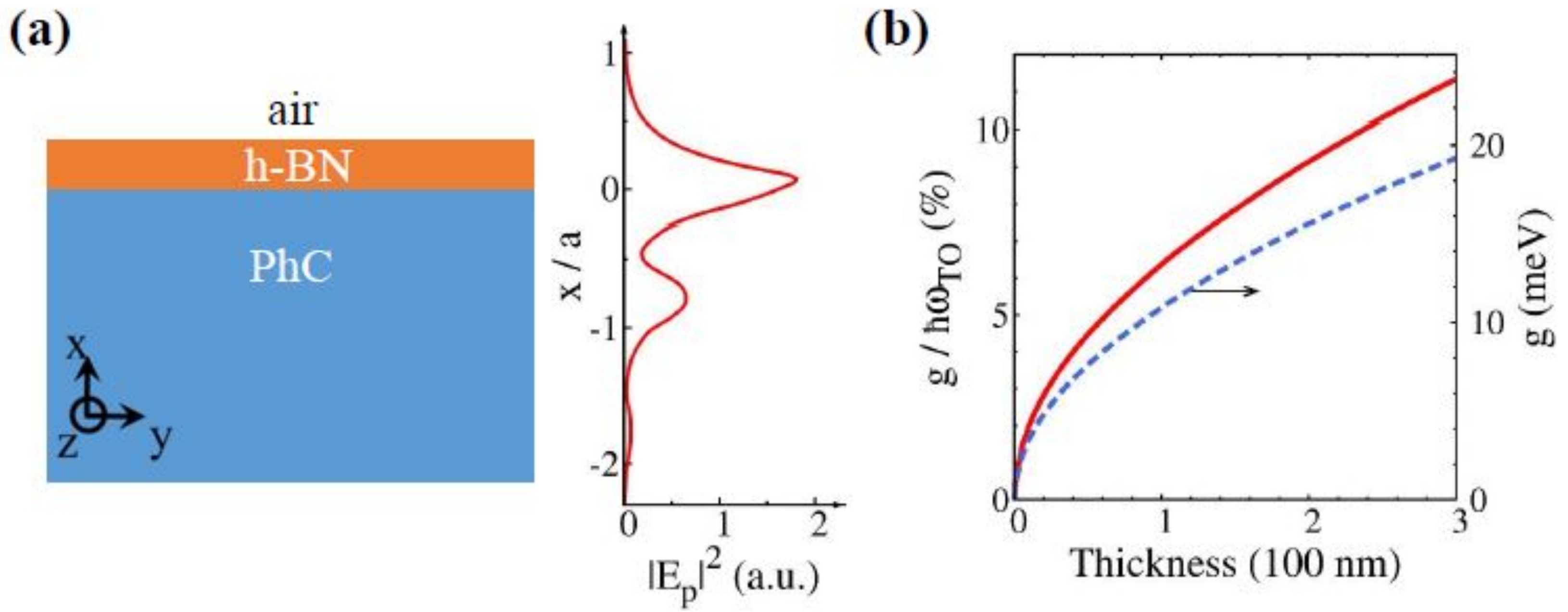




(a)
z
x
y
y
z
x
(b)
(c)
(d)
N_c
N_c
ΔN_c
1
-1
-0.26
-0.2
0.2
0.26
$k_z\,(2\pi/h)$
1
-1
-0.26
-0.2
0.2
0.26
$k_z\,(2\pi/h)$
2
-2
-0.26
-0.2
0.2
0.26
$k_z\,(2\pi/h)$


(a)
(b)
(c)
(d)
0.65
0.6
0.55
0.5
Frequency(2πc/a)
-0.5
0
0.5
k
y
b
a
d
c

Strong coupling from topological surface states



Strong light-matter interaction due to topological light-trapping @ photonic-crystal---air interfaces

Wang, Chen, Hang, Kee & Jiang, arXiv:1608.02437 (2016)

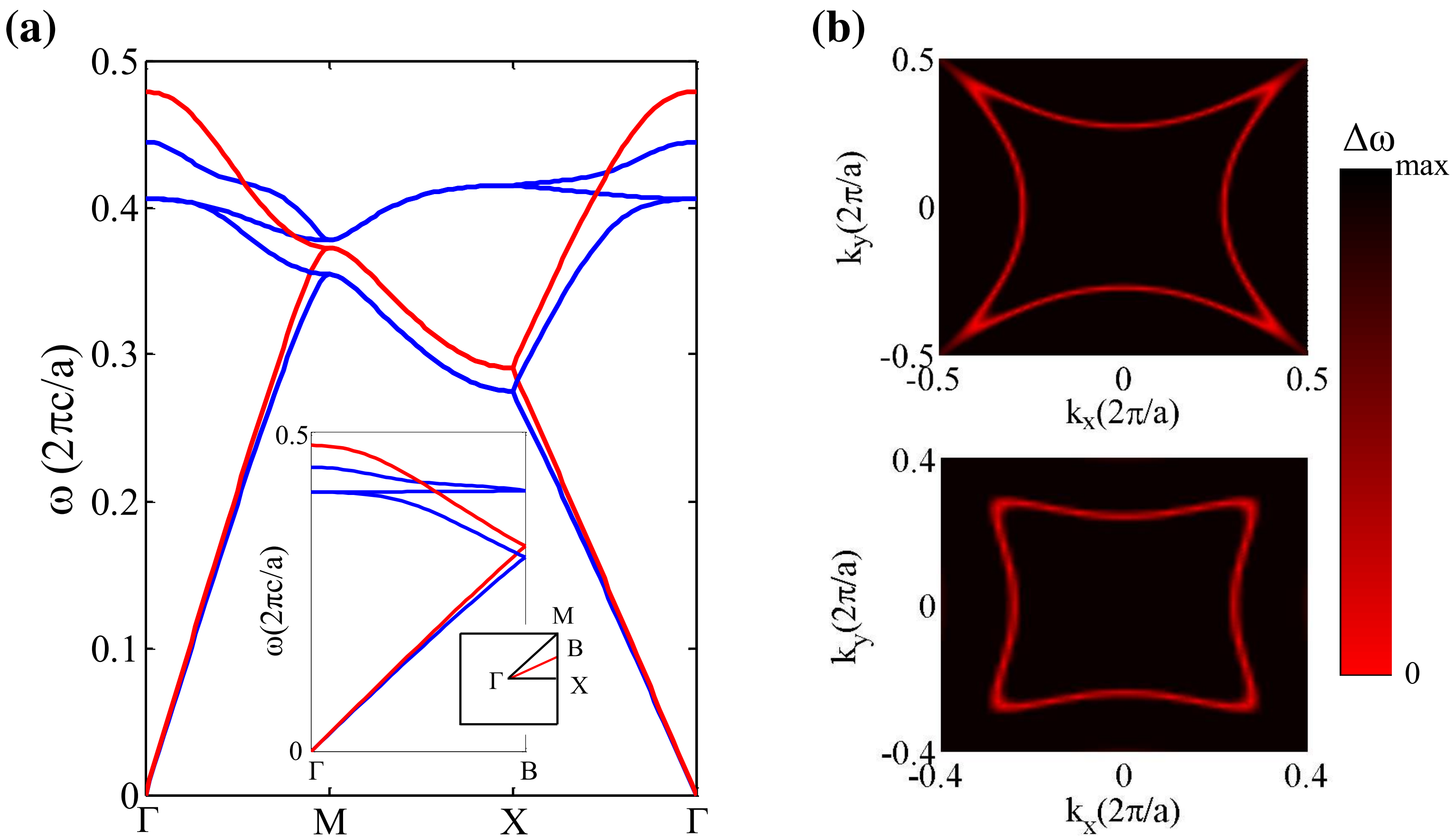
Deterministic photonic Line-node

Red: M_z= -1

Blue: M_z=1

"Partner switching mechanism"

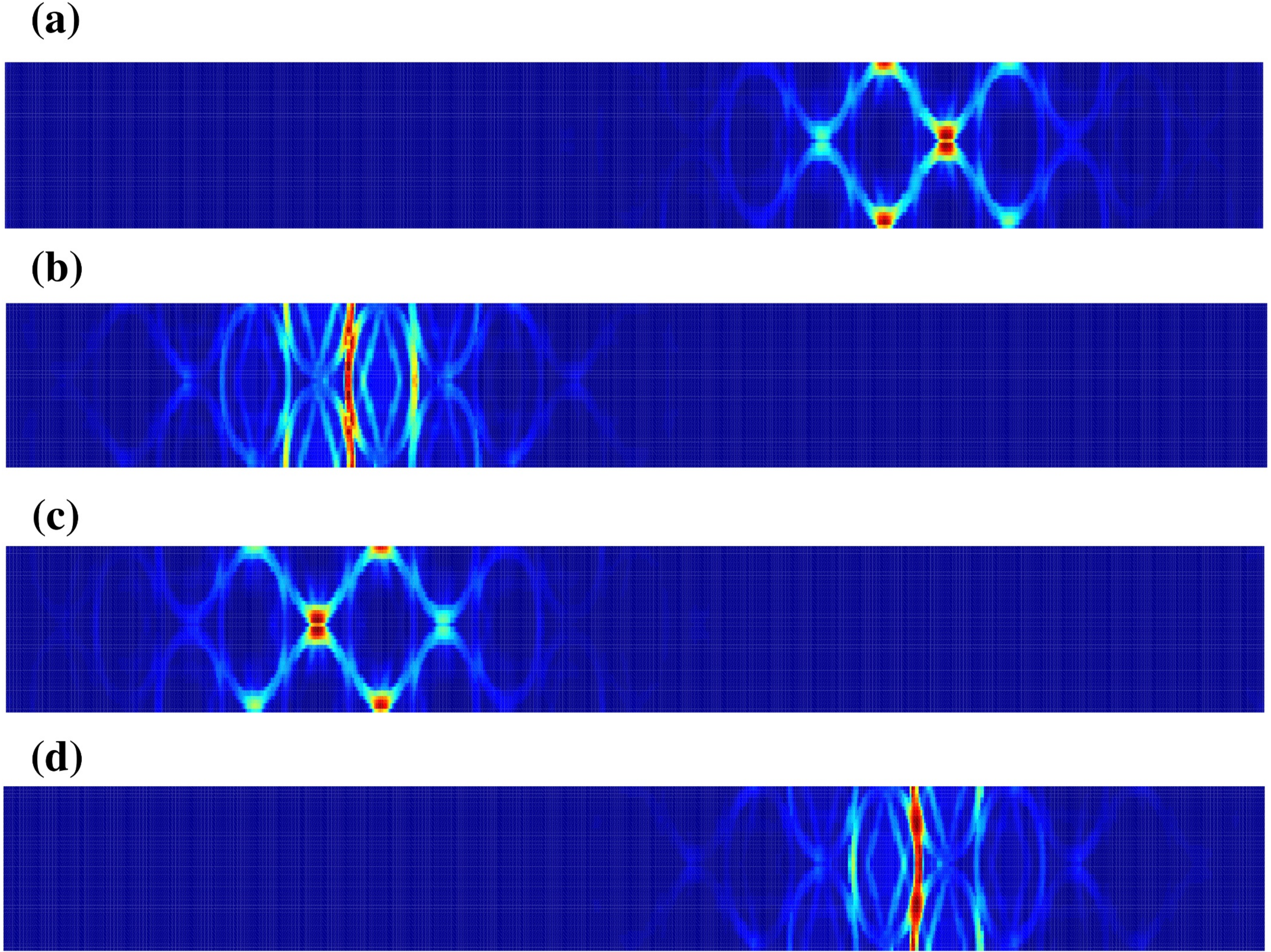
Degeneracy at zero frequency is due to gauge symmetry



$$M_z\Theta_i|m_z\rangle = \Theta_i M_z|m_z\rangle = m_z\Theta_i|m_z\rangle$$

Lowest linenode is deterministic!

Wang, Chen, Hang, Kee & Jiang, arXiv:1608.02437 (2016)


(a)
(b)
(c)
(d)