Echoes of Entanglement
from Quantum Metrology to Chaos

Monika Schleier-Smith

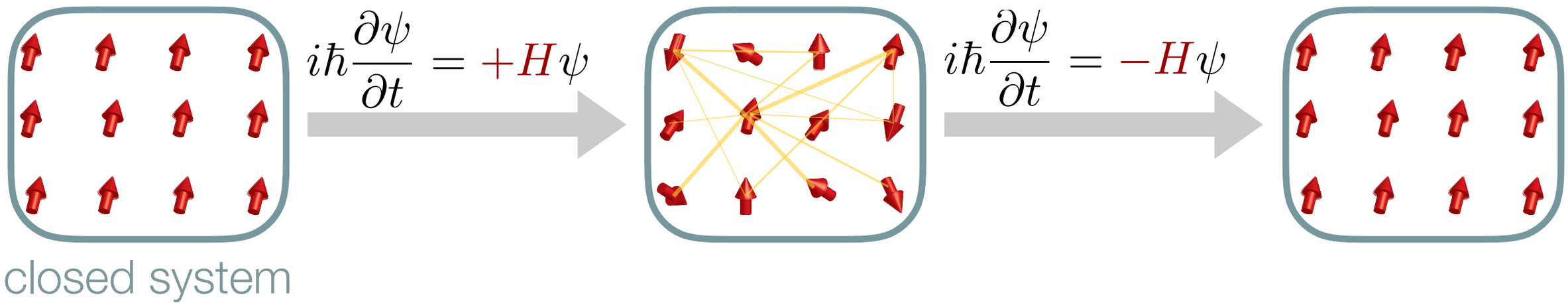
July 25, 2017

Emily Davis
Gregory Bentsen
Tracy Li
Brian Swingle
Patrick Hayden

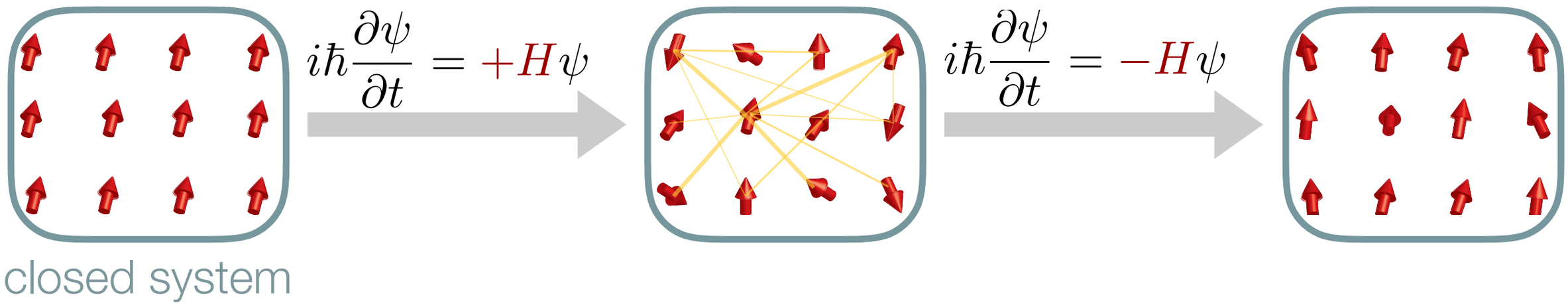
Stanford University

What if we could rewind time?

If we could go back in time…



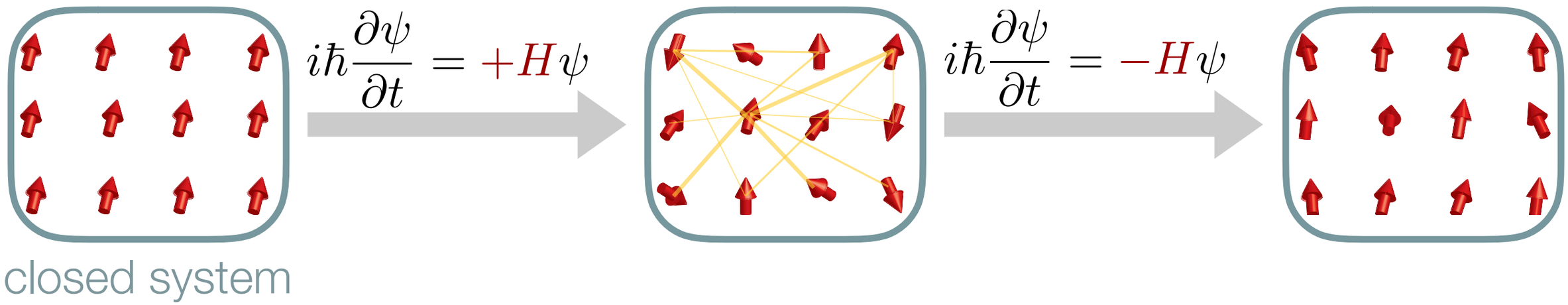
If we could go back in time…



Motivations

- Probing chaos via sensitivity to perturbations (“Loschmidt echo”)

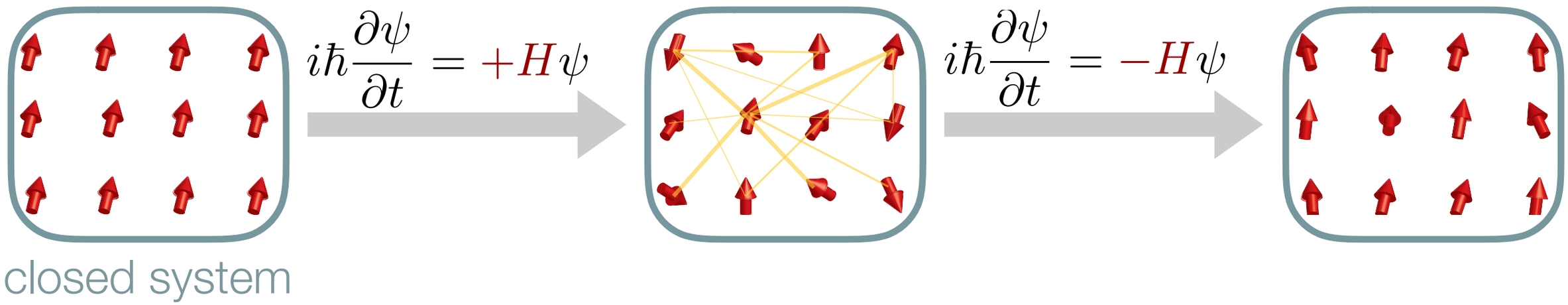
If we could go back in time…



Motivations

- Probing chaos via sensitivity to perturbations (“Loschmidt echo”)
- Exploiting sensitivity to perturbations for precision measurements

If we could go back in time…



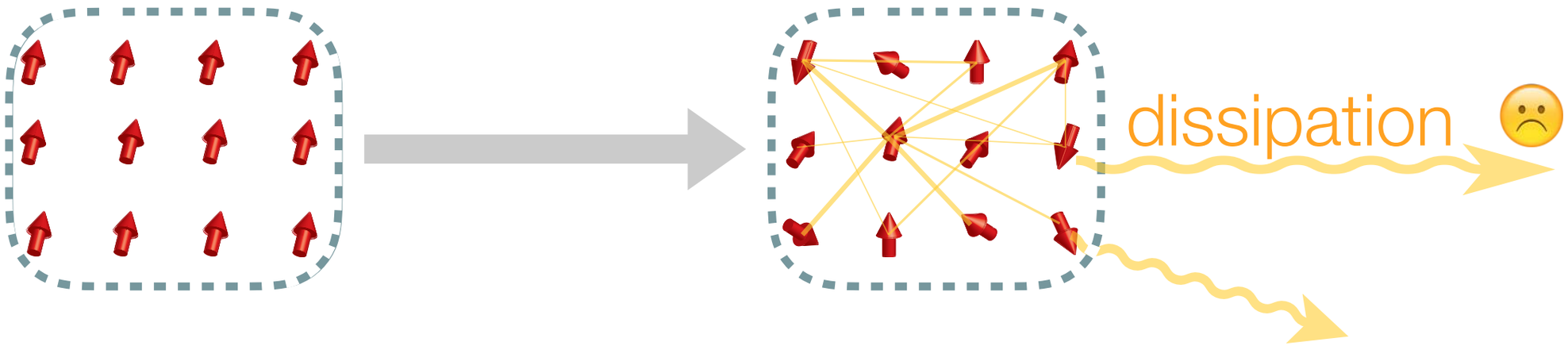
Motivations

- Probing chaos via sensitivity to perturbations (“Loschmidt echo”)
- Exploiting sensitivity to perturbations for precision measurements

Questions

- Role of many-body entanglement?

If we could go back in time…



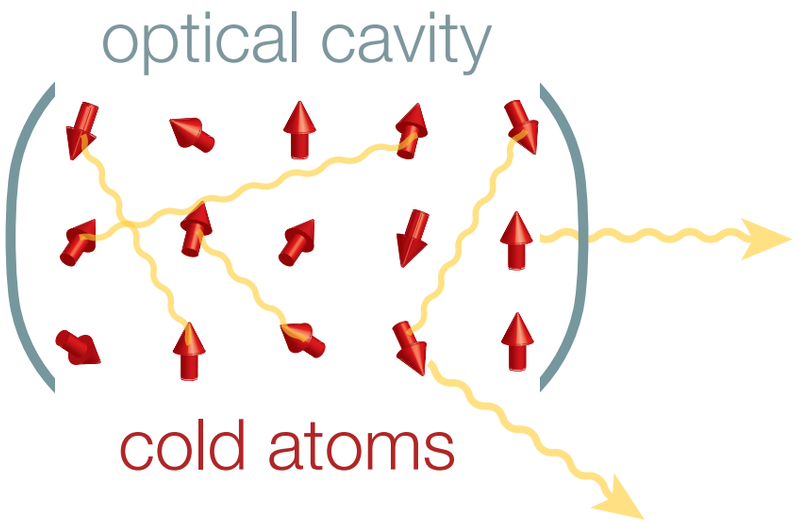
Motivations

- Probing chaos via sensitivity to perturbations (“Loschmidt echo”)
- Exploiting sensitivity to perturbations for precision measurements

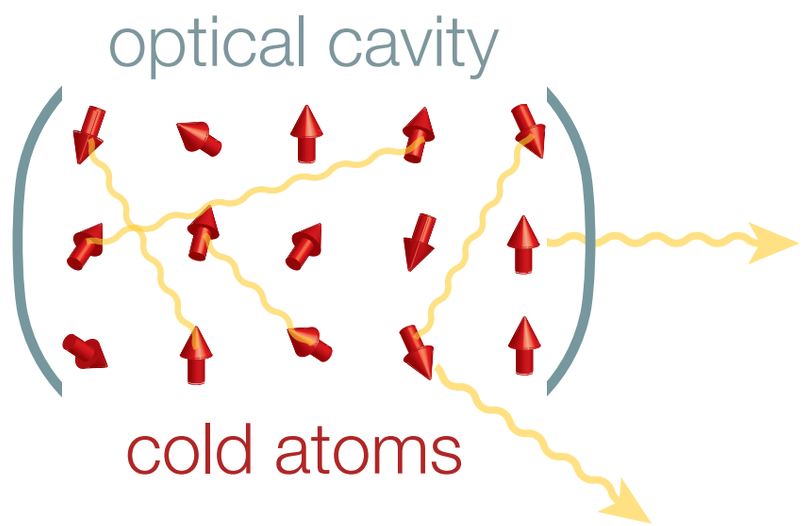
Questions

- Role of many-body entanglement?
- Usefulness of *approximate* time reversal in an open quantum system?

Model System



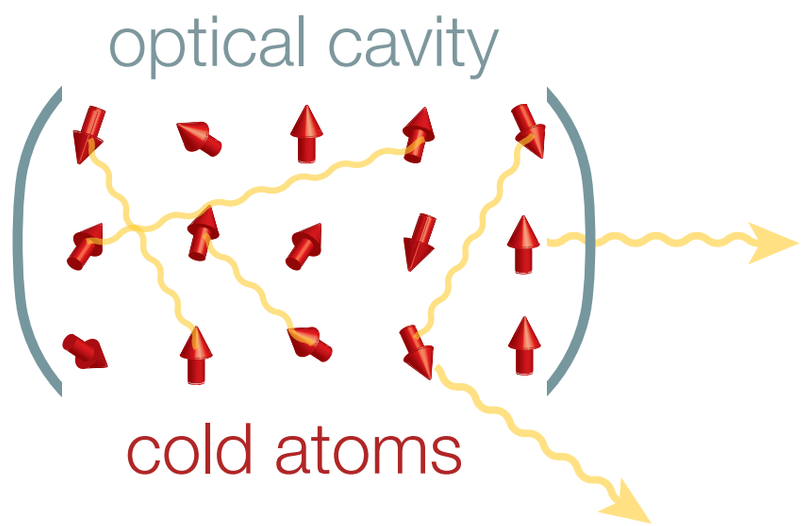
Model System



Photon-mediated interactions

- Easy to switch on/off and control sign
- Quantitative understanding of interaction-to-dissipation ratio

Model System



Photon-mediated interactions

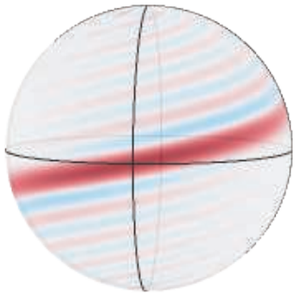
- Easy to switch on/off and control sign
- Quantitative understanding of interaction-to-dissipation ratio
- **Non-local** → entangling atoms *en masse* for quantum metrology
  → topological encoding of quantum information?
  → novel quantum simulations: spin glasses; black holes?

Outline

Background: photon-mediated interactions

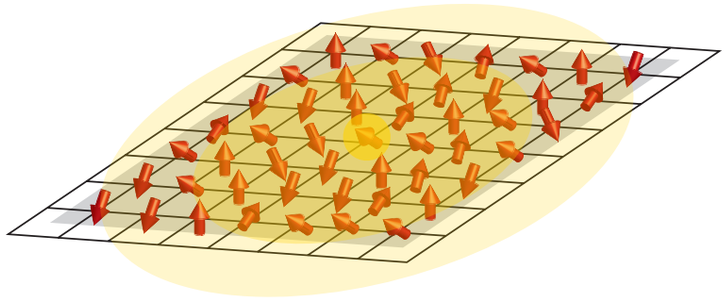
Experiment design and status

Quantum Metrology with robustness to noise



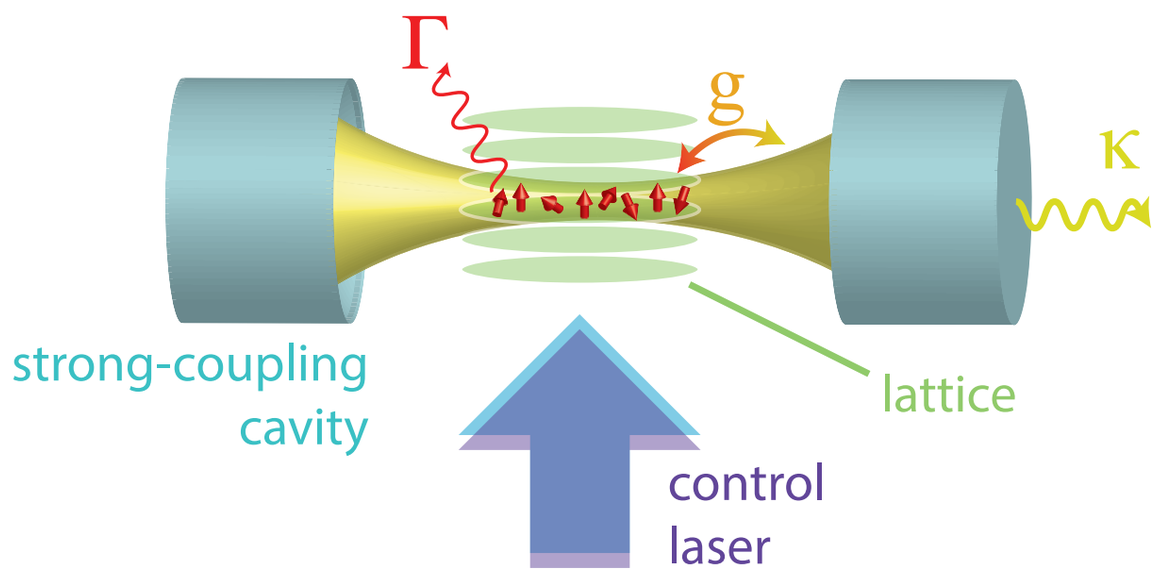
Echo spectroscopy for approaching the Heisenberg limit

Quantum Chaos: information scrambling

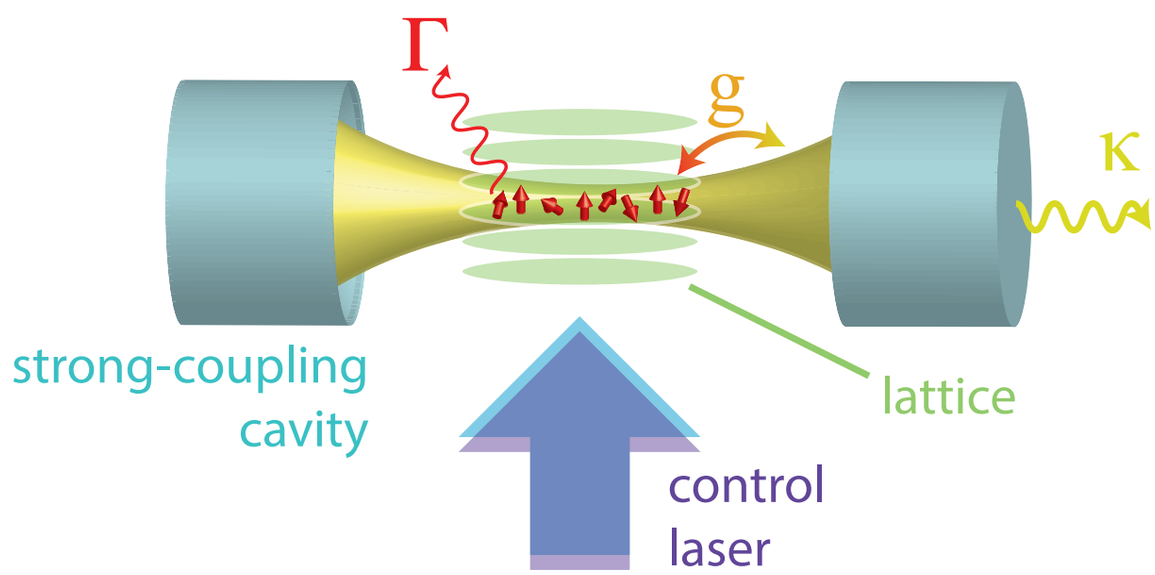


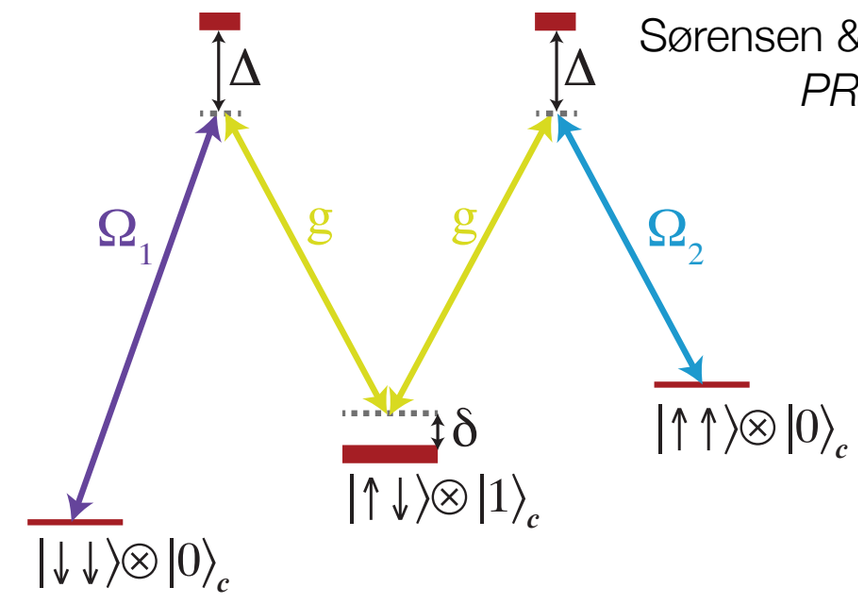
Towards quantum simulations of black holes?

Photon-Mediated Spin Interactions



Photon-Mediated Spin Interactions



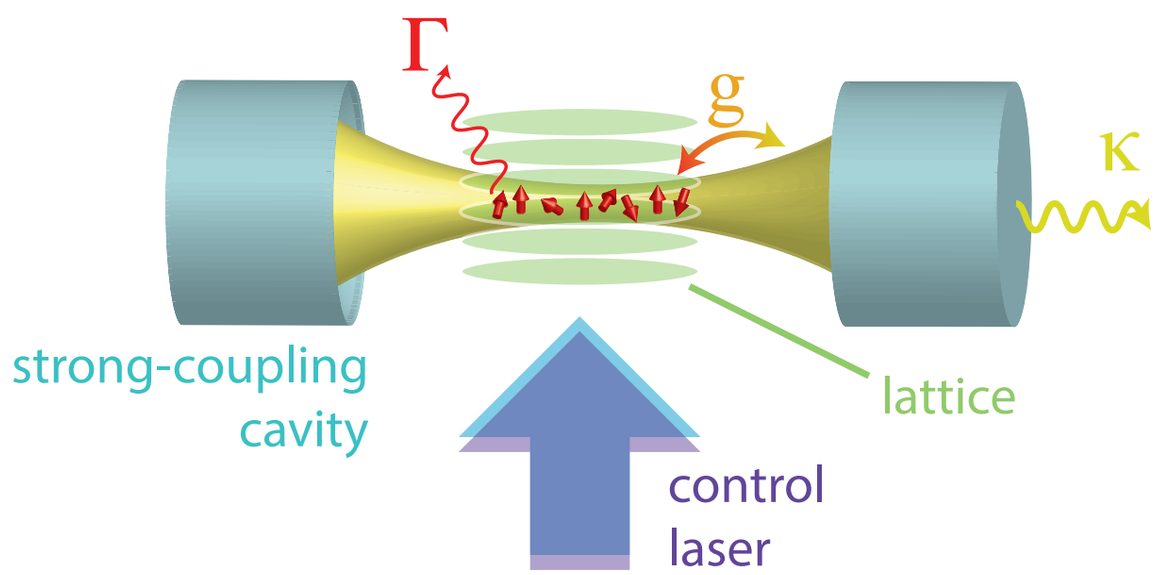


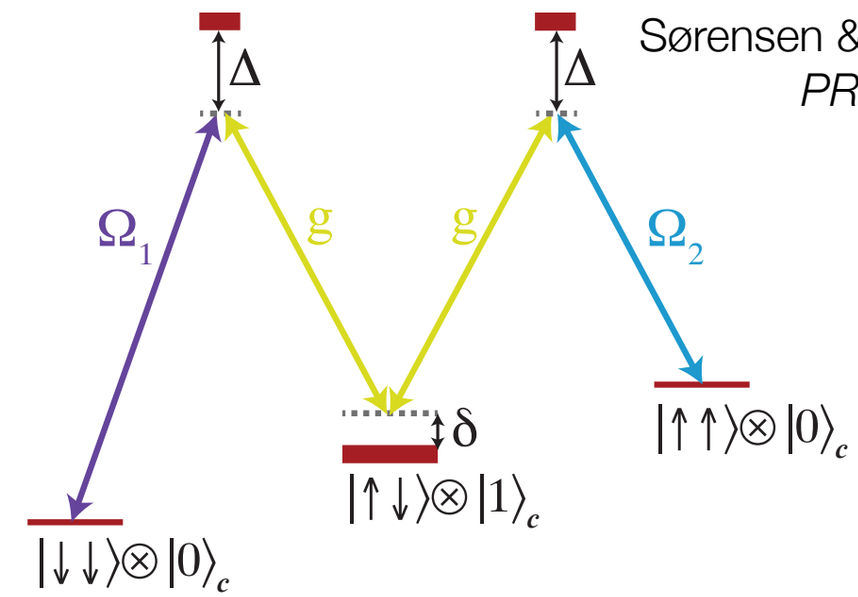
Sørensen & Mølmer, *PRL* (2002).

Pairwise correlated spin flips: $H \propto \sum_{i,j} (s_+^i + s_-^i)(s_+^j + s_-^j) \propto \sum_{i,j} s_x^i s_x^j$

- Spatial addressing enables controlled interactions between arbitrary pairs

Photon-Mediated Spin Interactions



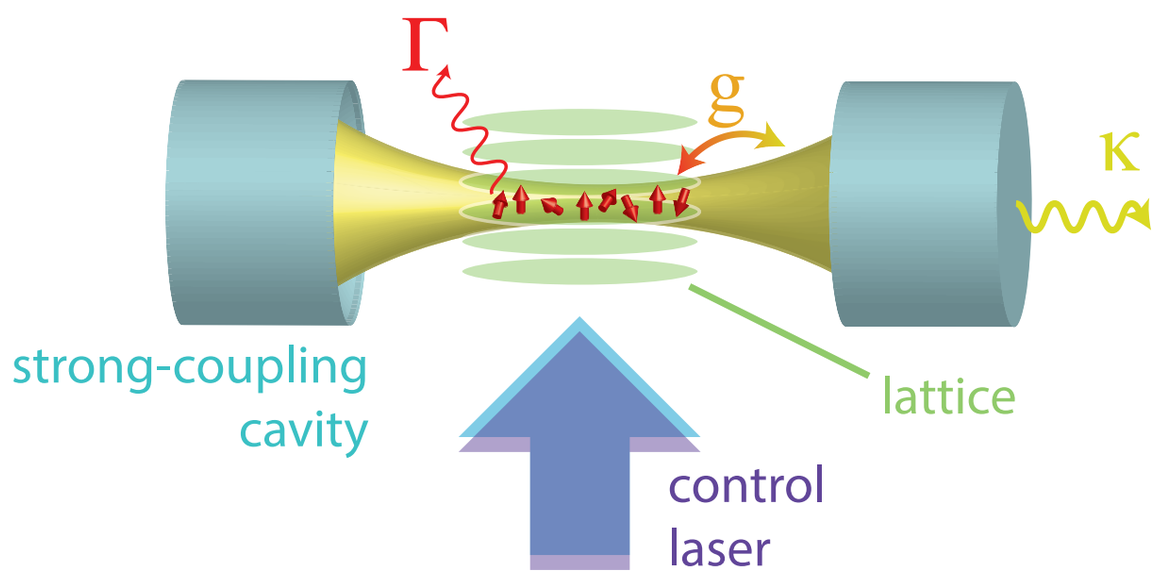


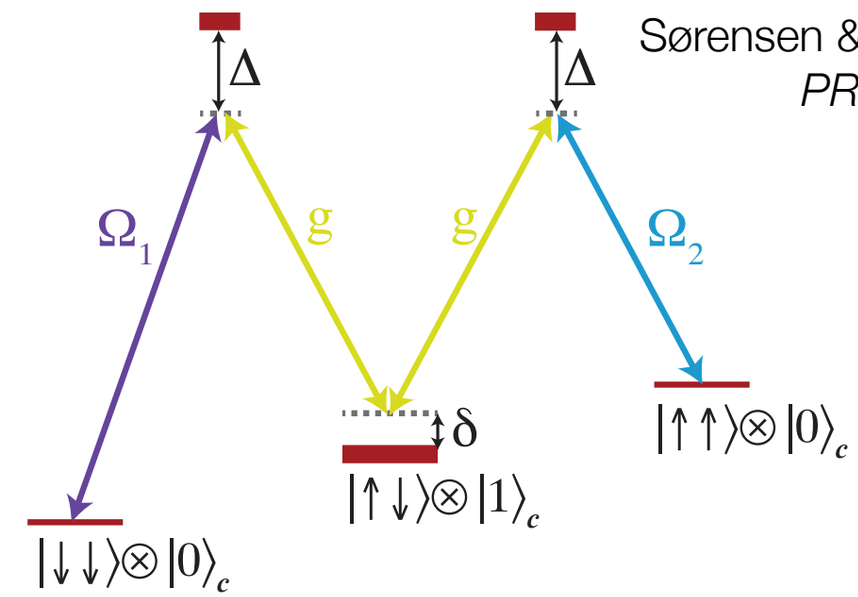
Sørensen & Mølmer, *PRL* (2002).

Pairwise correlated spin flips: $H \propto \sum_{i,j} (s_+^i + s_-^i)(s_+^j + s_-^j) \propto \sum_{i,j} s_x^i s_x^j$

- Spatial addressing enables controlled interactions between arbitrary pairs
- Sign of interaction controlled by sign of detuning **δ**

Photon-Mediated Spin Interactions



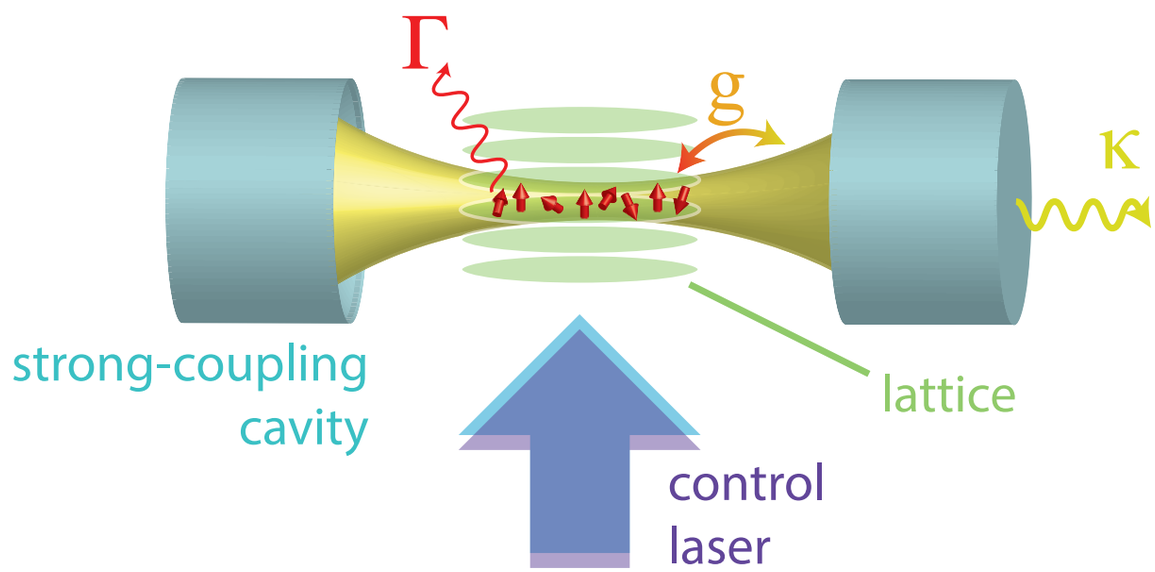


Sørensen & Mølmer, *PRL* (2002).

Pairwise correlated spin flips: $H \propto \sum_{i,j} (s_+^i + s_-^i)(s_+^j + s_-^j) \propto \sum_{i,j} s_x^i s_x^j$

- Spatial addressing enables controlled interactions between arbitrary pairs
- Sign of interaction controlled by sign of detuning **δ**
- Coherent interactions for **δ≫κ** and strong coupling $\eta \equiv 4g^2/(\kappa\Gamma) \gg 1$

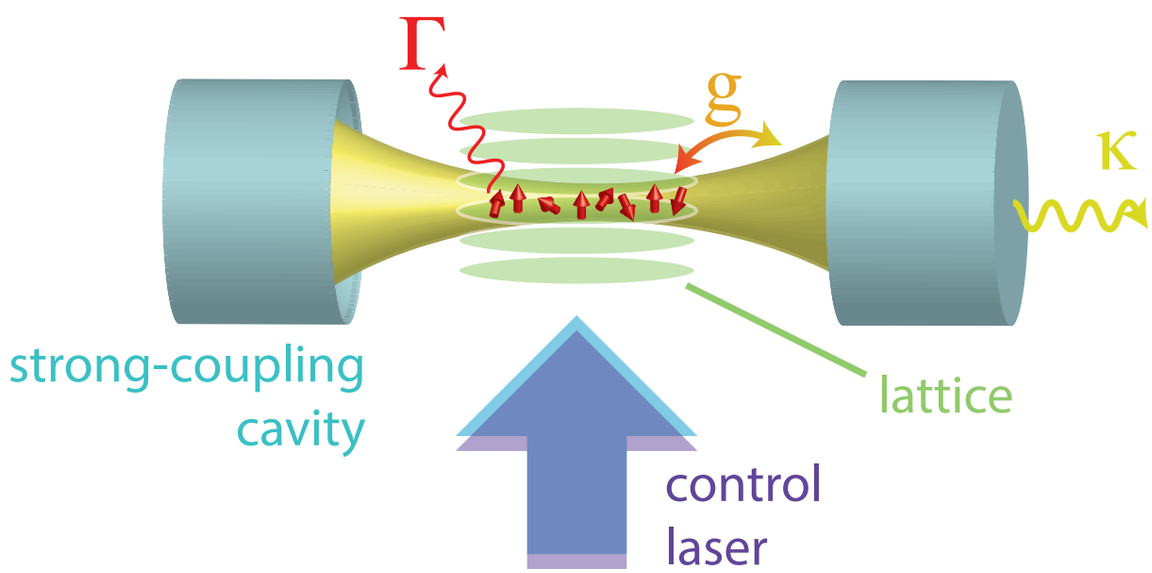
Experiment Design



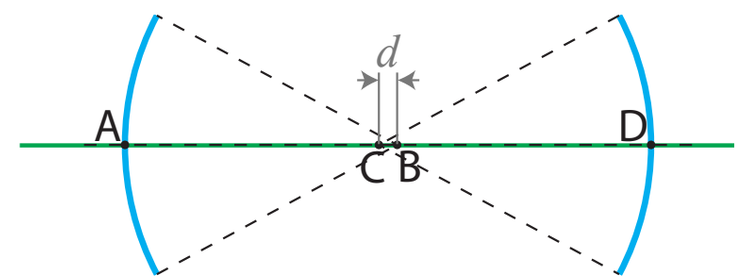
~ 10^1 - 10^3 atoms

- Strong coupling: $\eta \equiv \frac{4g^2}{\kappa\Gamma} \sim \frac{F\lambda^2}{w^2} \gg 1$
- Optical access for addressing from side
- Confinement in 3D lattice

Experiment Design



~ 10^1 - 10^3 atoms



- Strong coupling: $\eta \equiv \frac{4g^2}{\kappa\Gamma} \sim \frac{F\lambda^2}{w^2} \gg 1$
- Optical access for addressing from side
- Confinement in 3D lattice

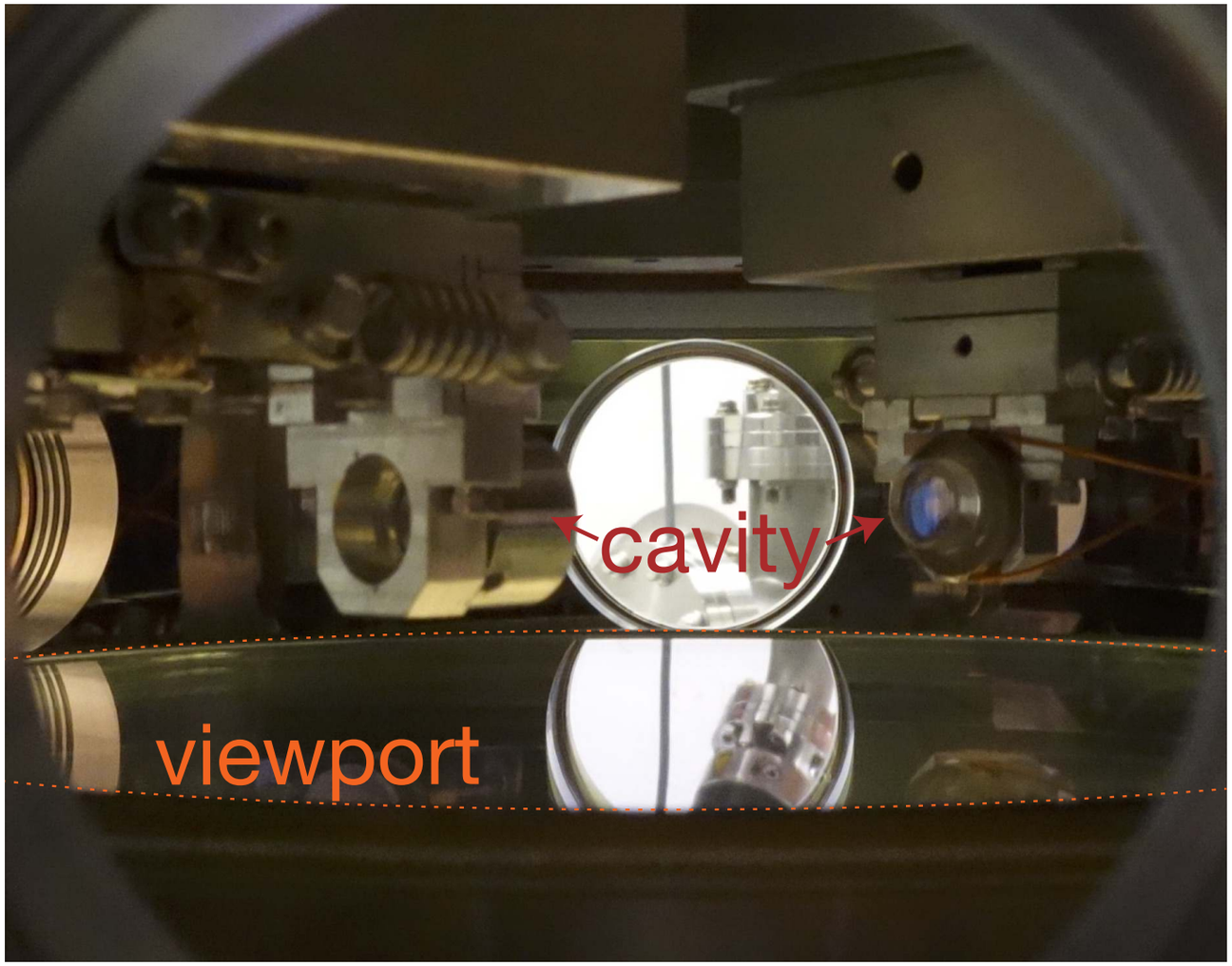
$\Rightarrow$ Near-concentric resonator

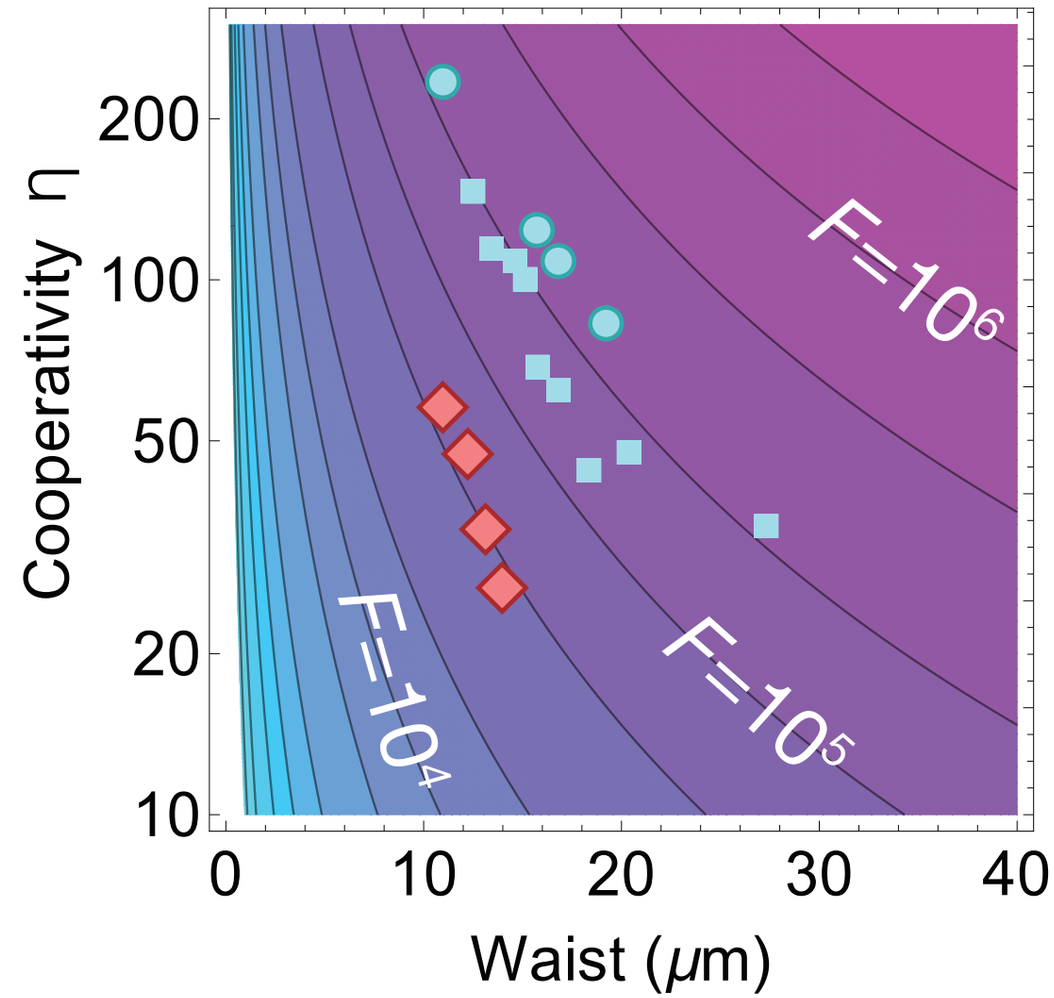
Length L ~ 5 cm

Waist w ~ 12 μm

Non-degenerate modes

Strong Coupling with Optical Access

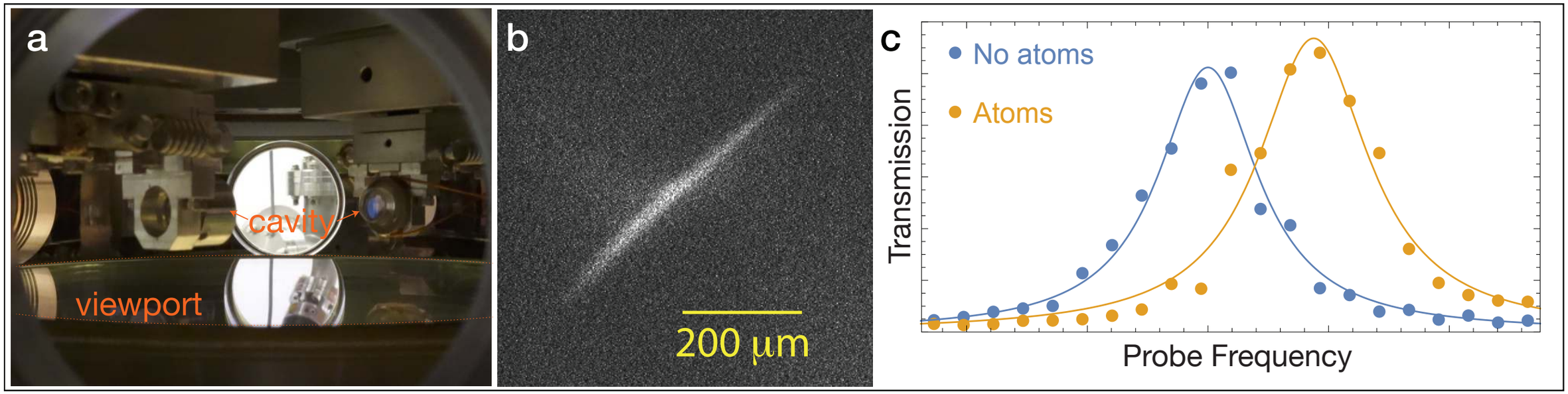




Excellent optical access + single-atom cooperativity η ~ 50

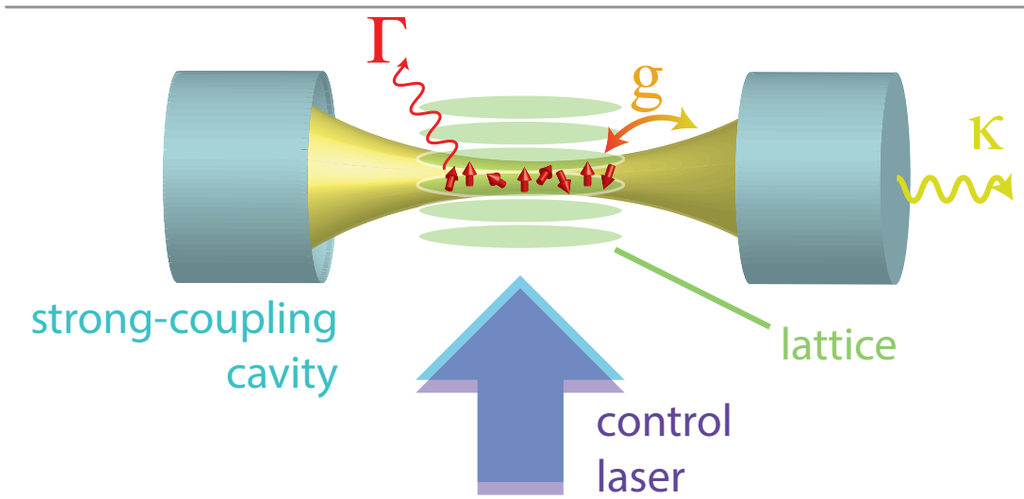
Atoms in the Cavity

Image of atoms trapped in cavity



Dispersive measurement of ~100 atoms with far-detuned probe

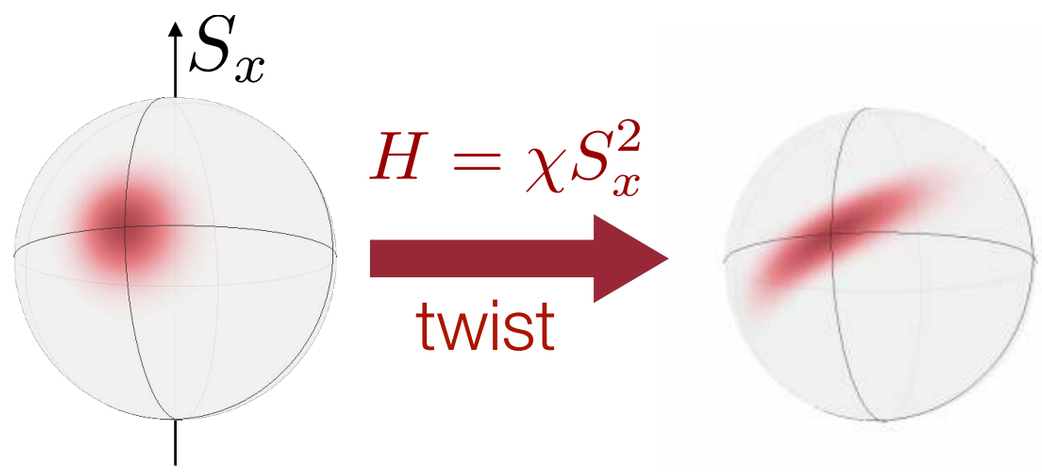
Photon-Mediated Spin Interactions



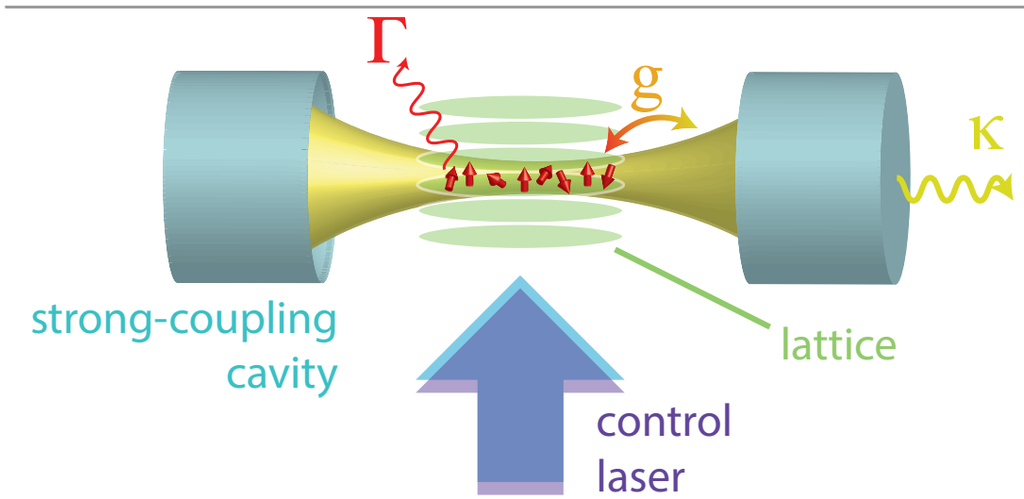
$$H \propto \sum_{i,j} (s_+^i + s_-^i)(s_+^j + s_-^j) \propto \sum_{i,j} s_x^i s_x^j$$

Simple limit:
all-to-all interaction

collective spin $\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$



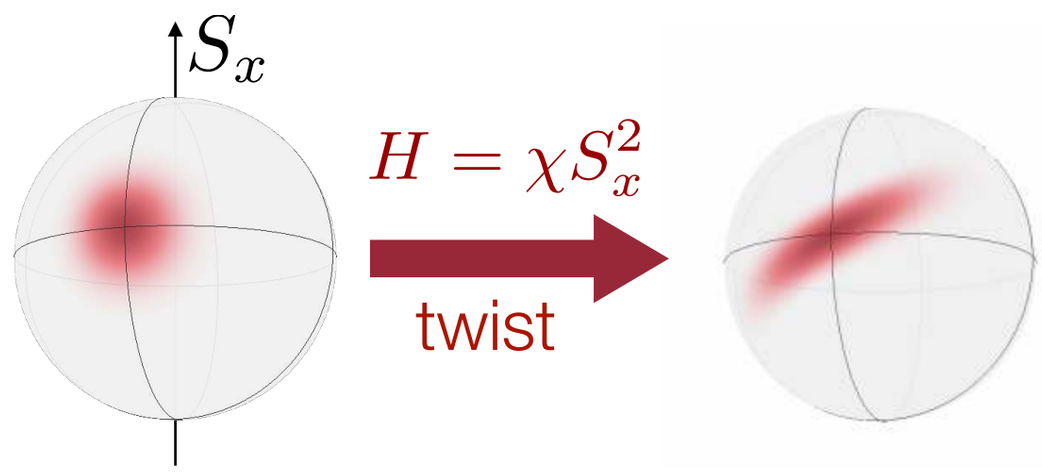
Photon-Mediated Spin Interactions



$$H \propto \sum_{i,j} (s_+^i + s_-^i)(s_+^j + s_-^j) \propto \sum_{i,j} s_x^i s_x^j$$

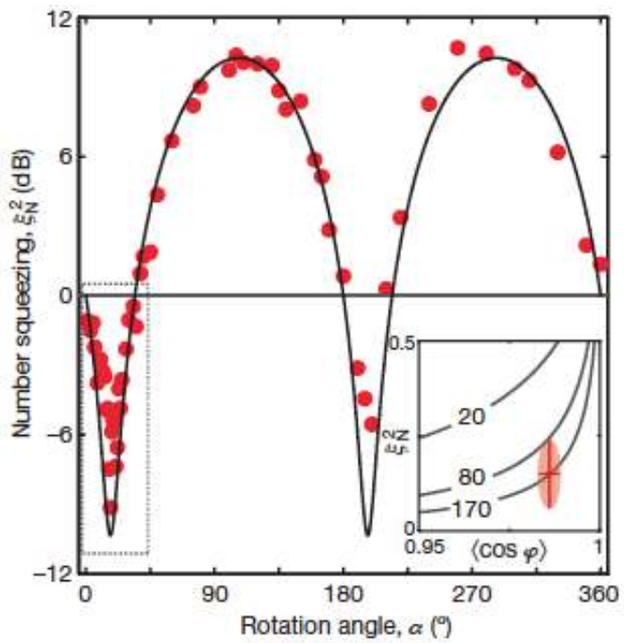
Simple limit:
all-to-all interaction

collective spin $\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$

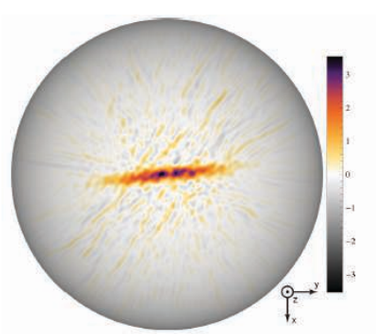


One-Axis Twisting Experiments

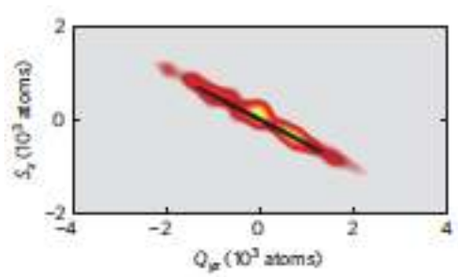
BECs



Gross, . . . & Oberthaler, *Nature* (2010).

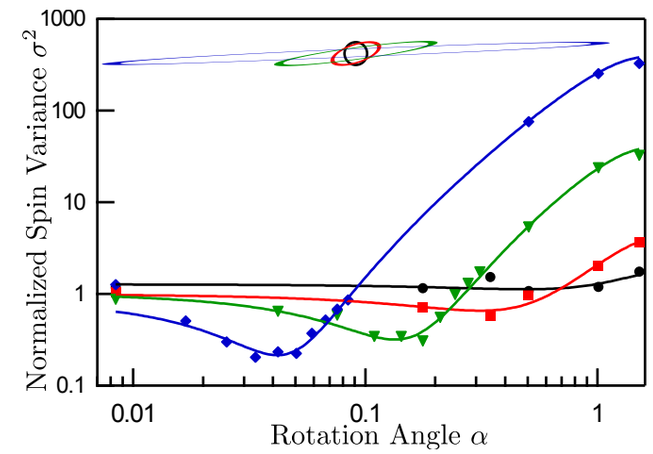


Riedl, . . . & Treutlein, *Nature* (2010).



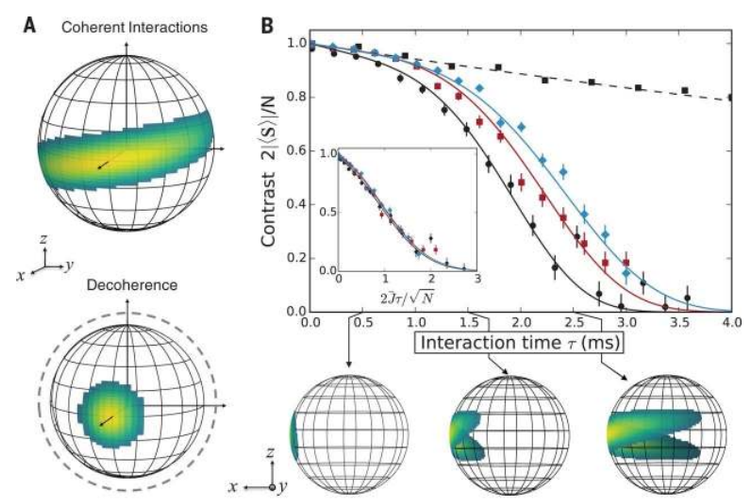
Hamley, . . . & Chapman, *Nature Physics* (2011).

Cavity QED



Leroux, MS-S & Vuletic, *PRL* (2010).
Hosten, … & Kasevich, *Science* (2016).

Ion traps

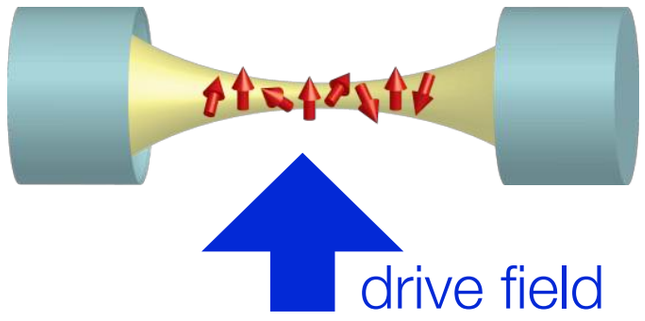


Bohnet, … & Bollinger, *Science* (2016).
Also: Monz, … & Blatt *PRL* (2011).

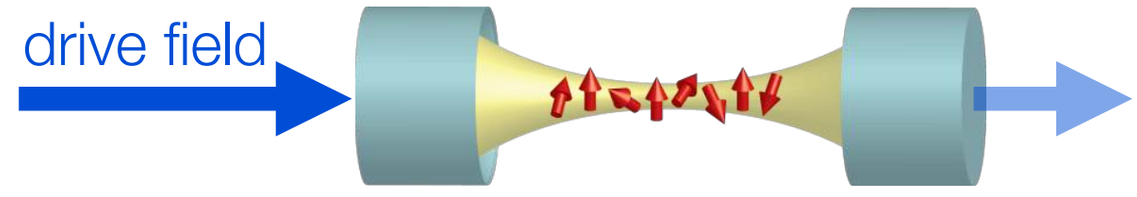
Two Approaches

…to generating spin-spin interactions

$$\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$$



Interaction : $H = \chi S_x^2$

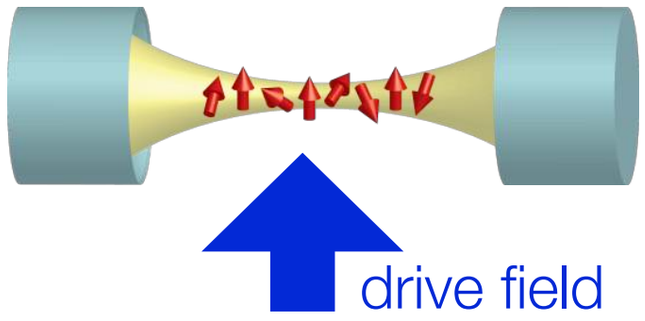


Interaction : $H = \chi S_z^2$

Two Approaches

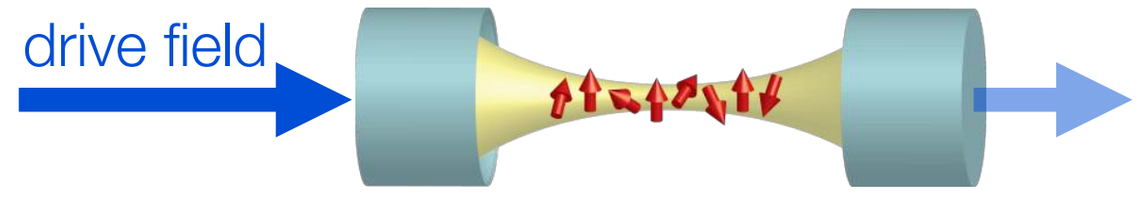
…to generating spin-spin interactions

$$\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$$



$$\text{Interaction}: \; H = \chi S_x^2$$

- *Spatial control of interactions*
- *Technically demanding*

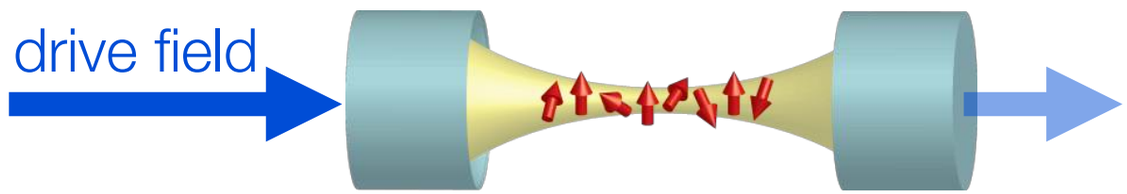


$$\text{Interaction}: \; H = \chi S_z^2$$

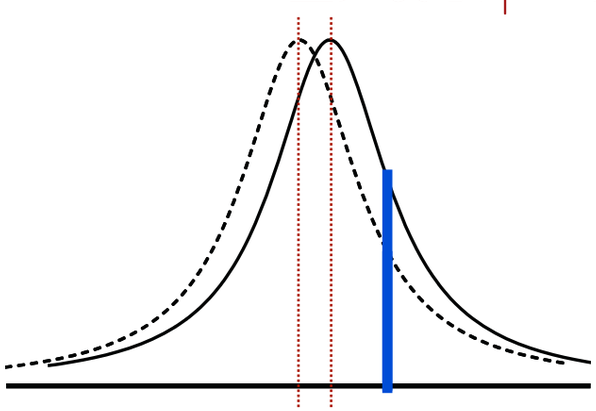
- *Global interactions only*
- *Simpler: already demonstrated!*

Cavity Feedback Squeezing

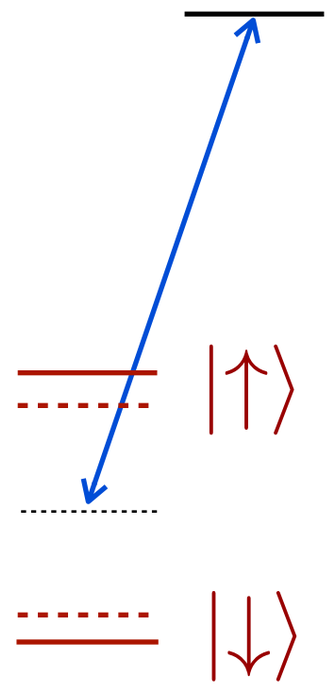
MS-S, ID Leroux & V Vuletic,
PRA **81**, 021804*(R)* (2010).



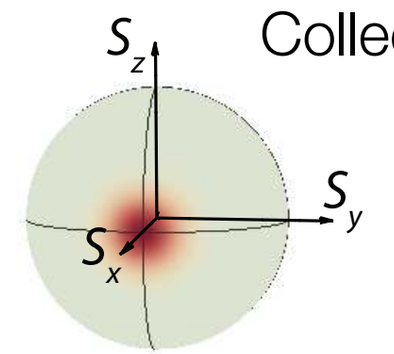
$$\Delta\nu \propto N_\uparrow - N_\downarrow \propto S_z$$



cavity resonance
is shifted by atoms

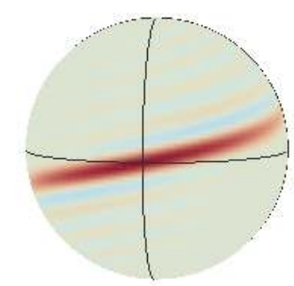


atomic levels
are shifted by light



Collective spin

$$H = \chi S_z^2$$



effective interaction
mediated by light

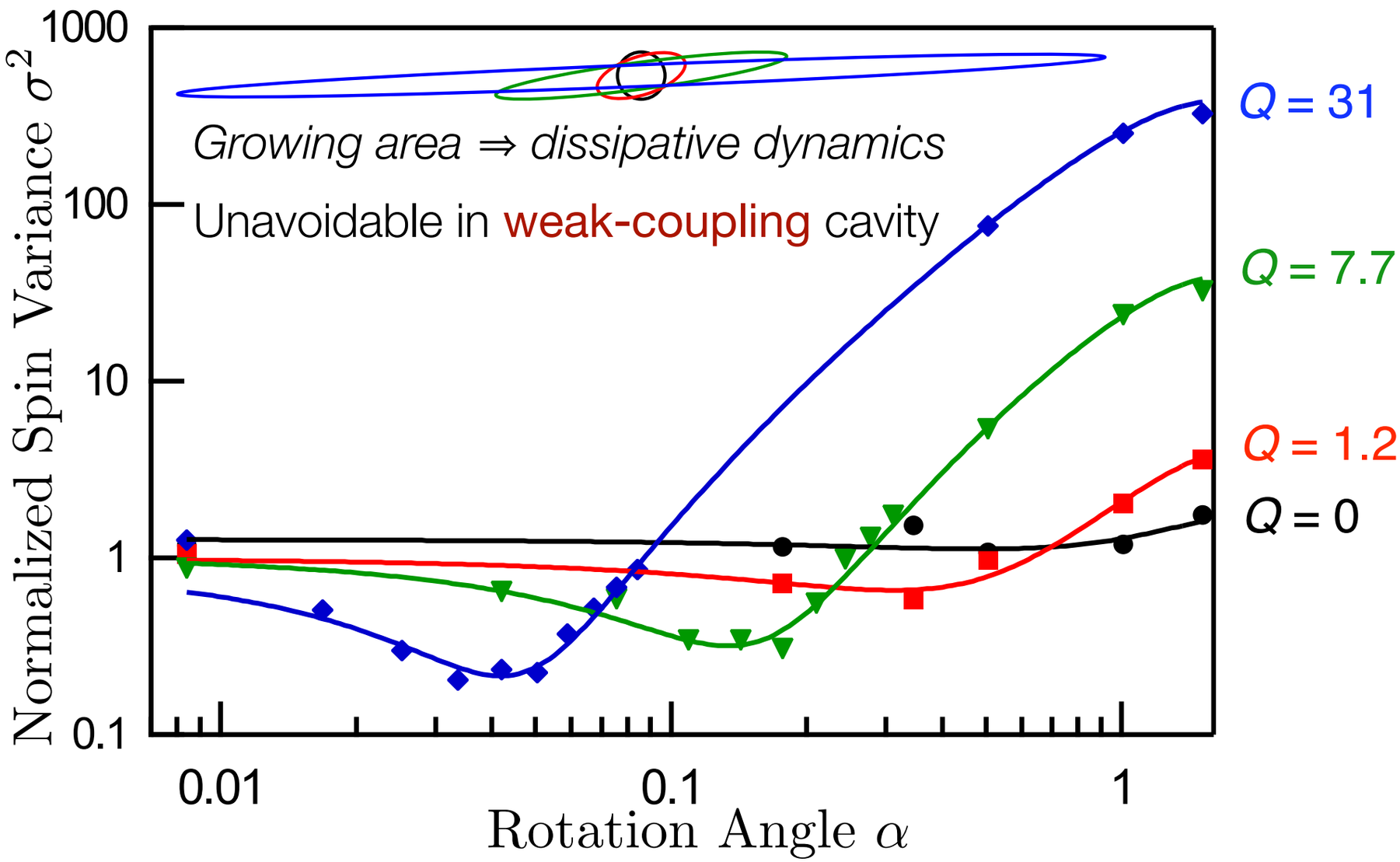
"one-axis twisting"
[Kitagawa & Ueda, PRA, 1993.]

Spin Squeezing

ID Leroux, MS-S & V Vuletic,
PRL **104**, 073602 (2010).

Twisting strength $Q = N\chi t = \begin{pmatrix}\text{\# of photons scattered} \\ \text{into cavity per atom}\end{pmatrix}$

$N = 4\times10^4$ atoms
$\eta = 0.1$, $\delta=\kappa/2$

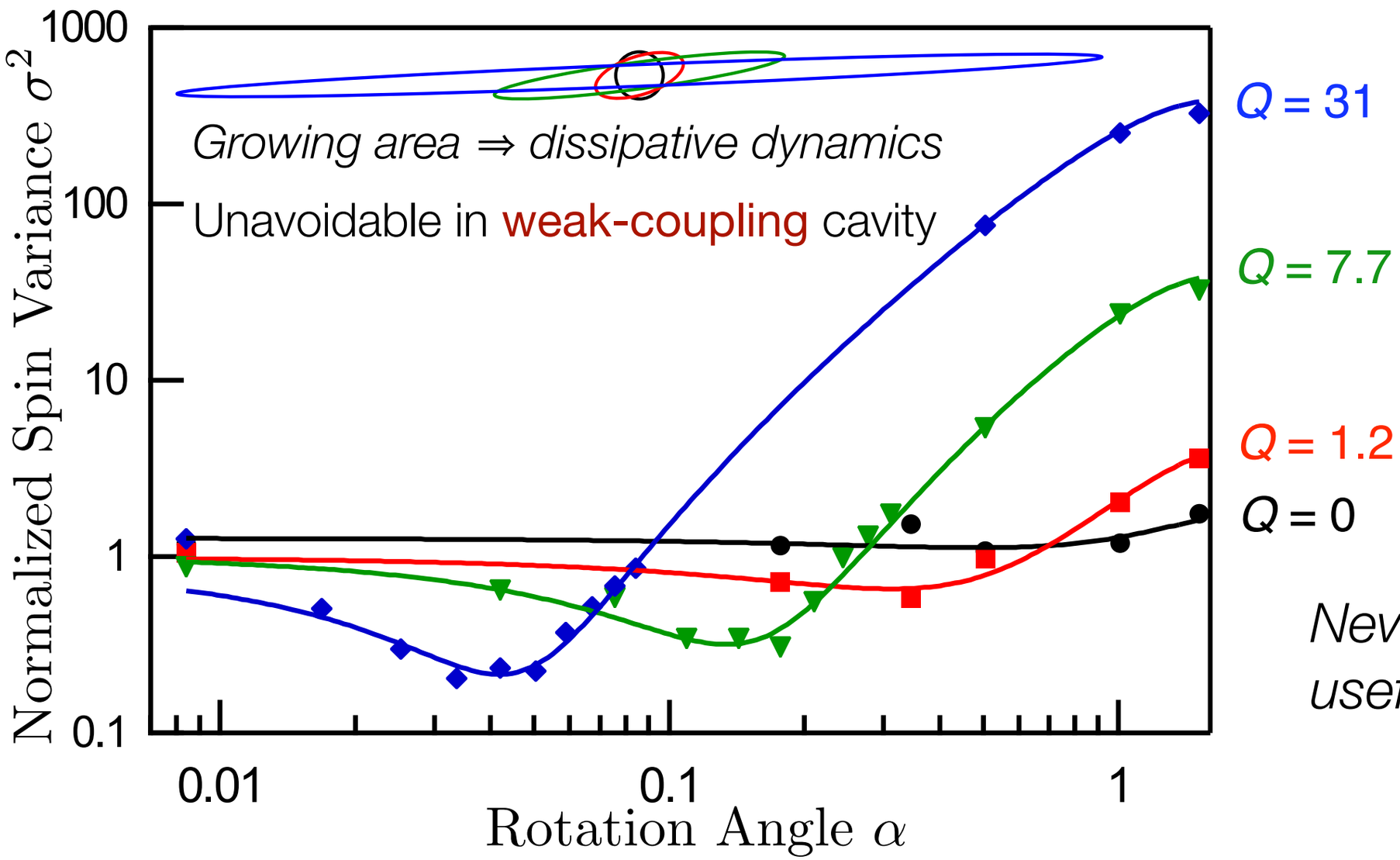


Spin Squeezing

ID Leroux, MS-S & V Vuletic,
PRL **104**, 073602 (2010).

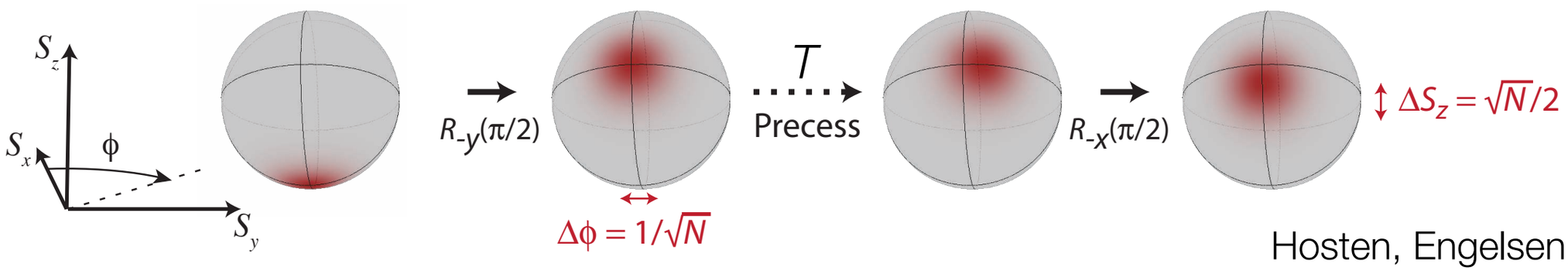
Twisting strength $Q = N\chi t = \left(\begin{array}{c}\text{\# of photons scattered}\\ \text{into cavity per atom}\end{array}\right)$

$N = 4\times10^4$ atoms
$\eta = 0.1$, $\delta=\kappa/2$

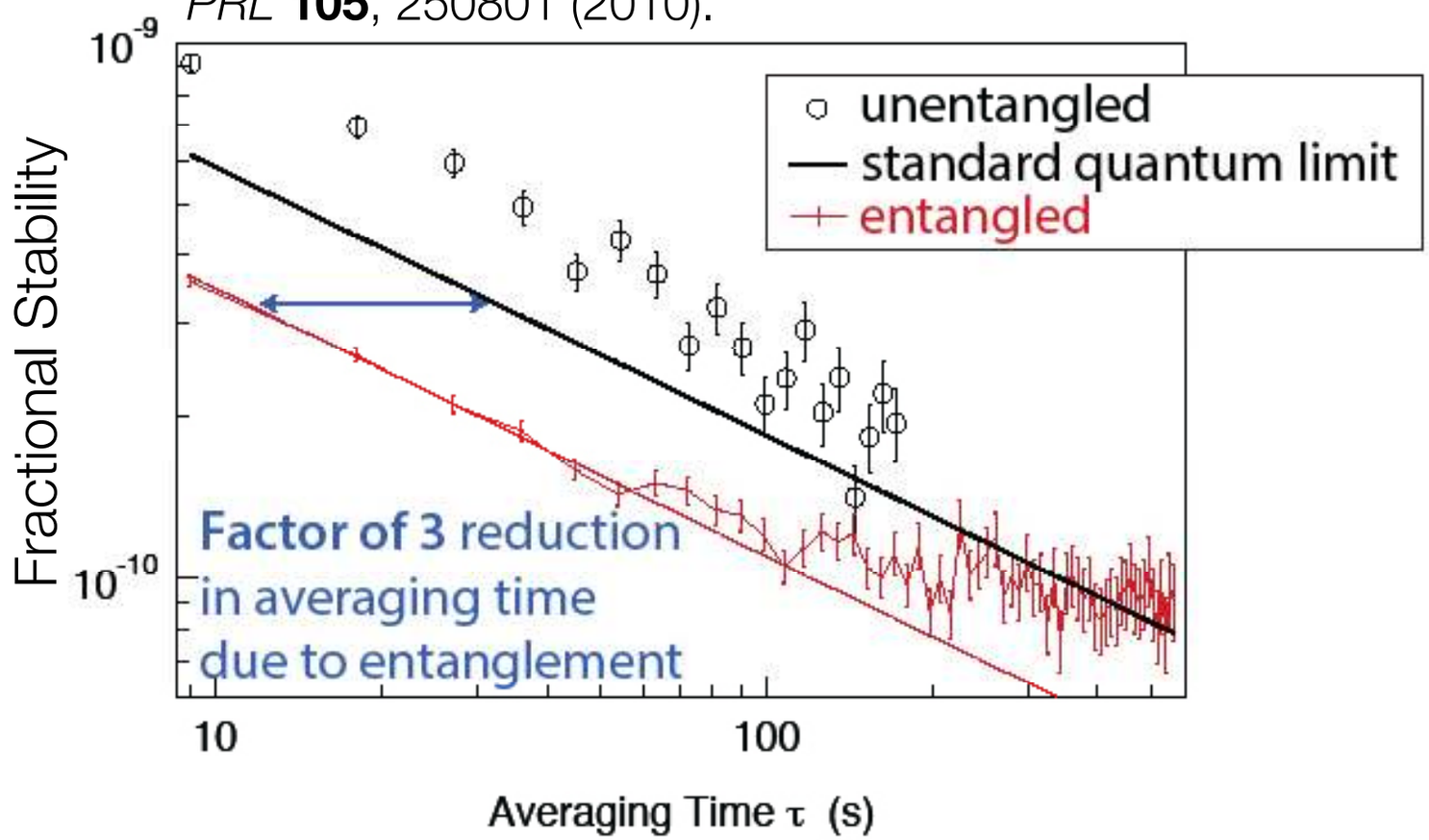


Nevertheless:
useful entanglement!

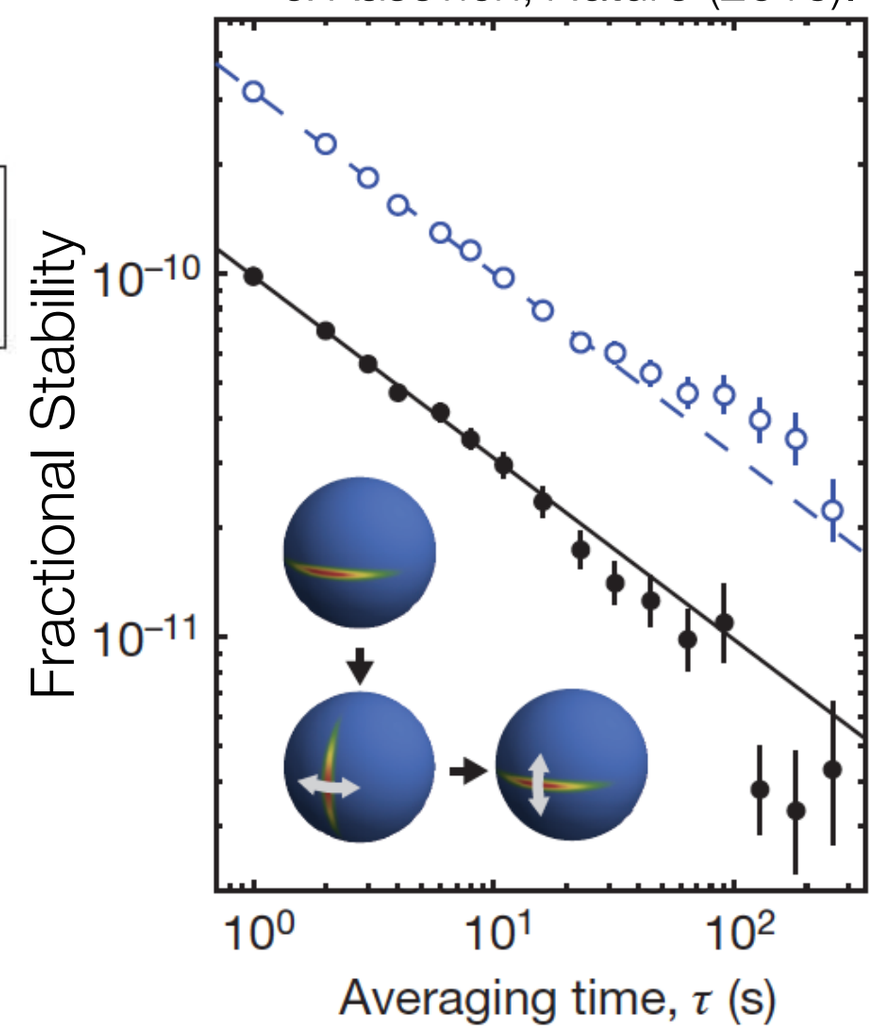
Enhancing Atomic Clocks



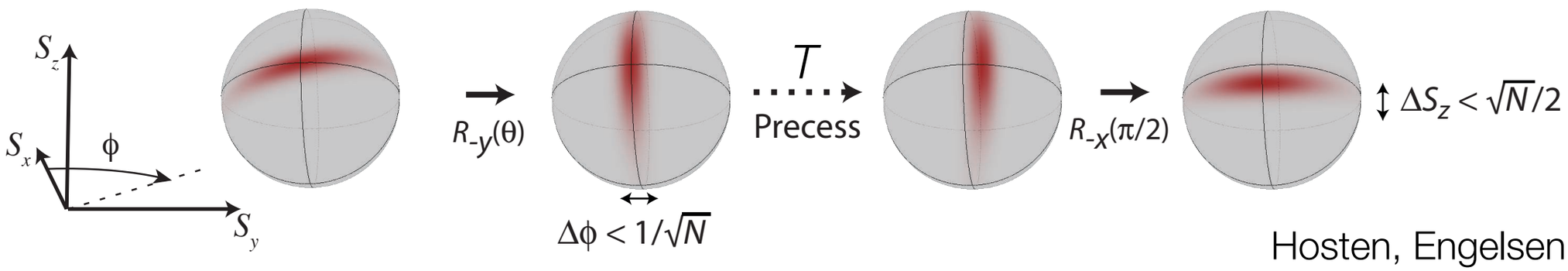
ID Leroux, MS-S & V Vuletic, *PRL* **105**, 250801 (2010).



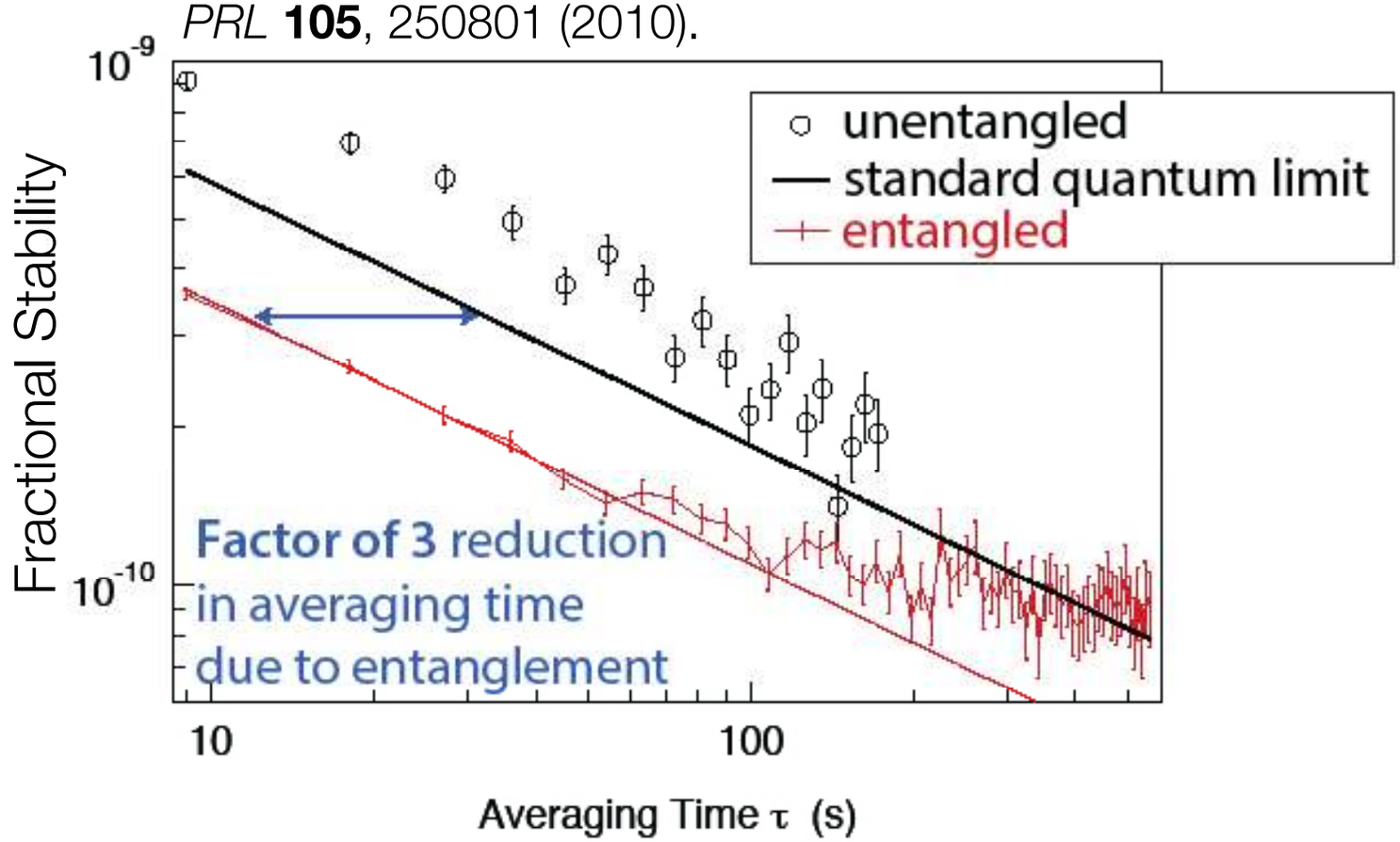
Hosten, Engelsen, Krishnakumar & Kasevich, *Nature* (2016).



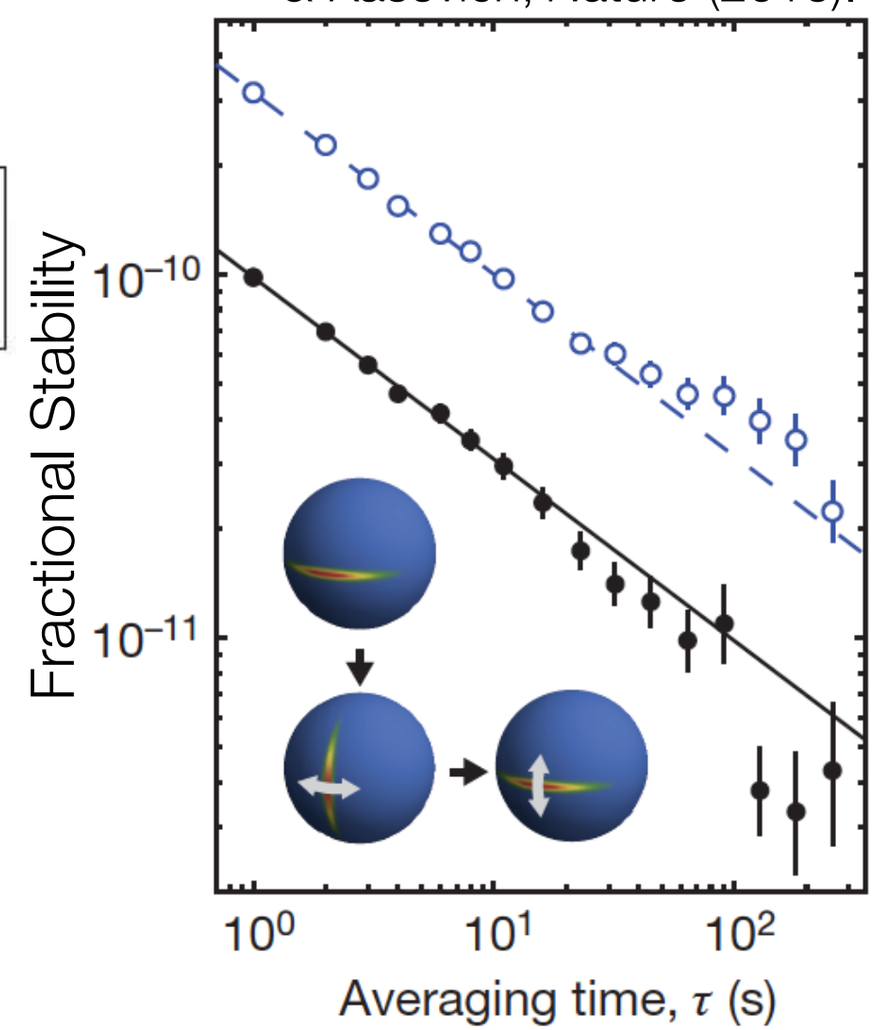
Enhancing Atomic Clocks



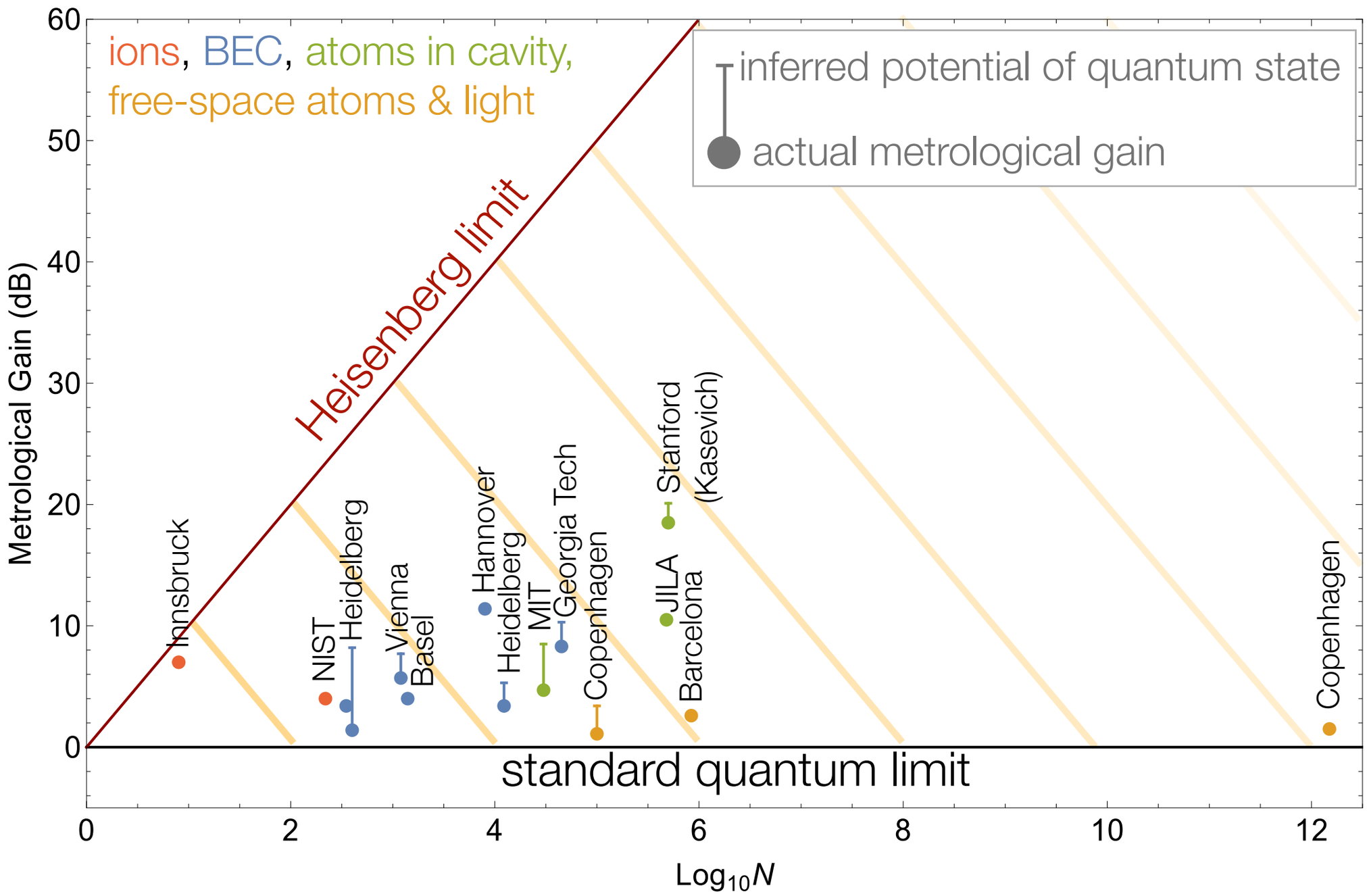
ID Leroux, MS-S & V Vuletic, *PRL* **105**, 250801 (2010).



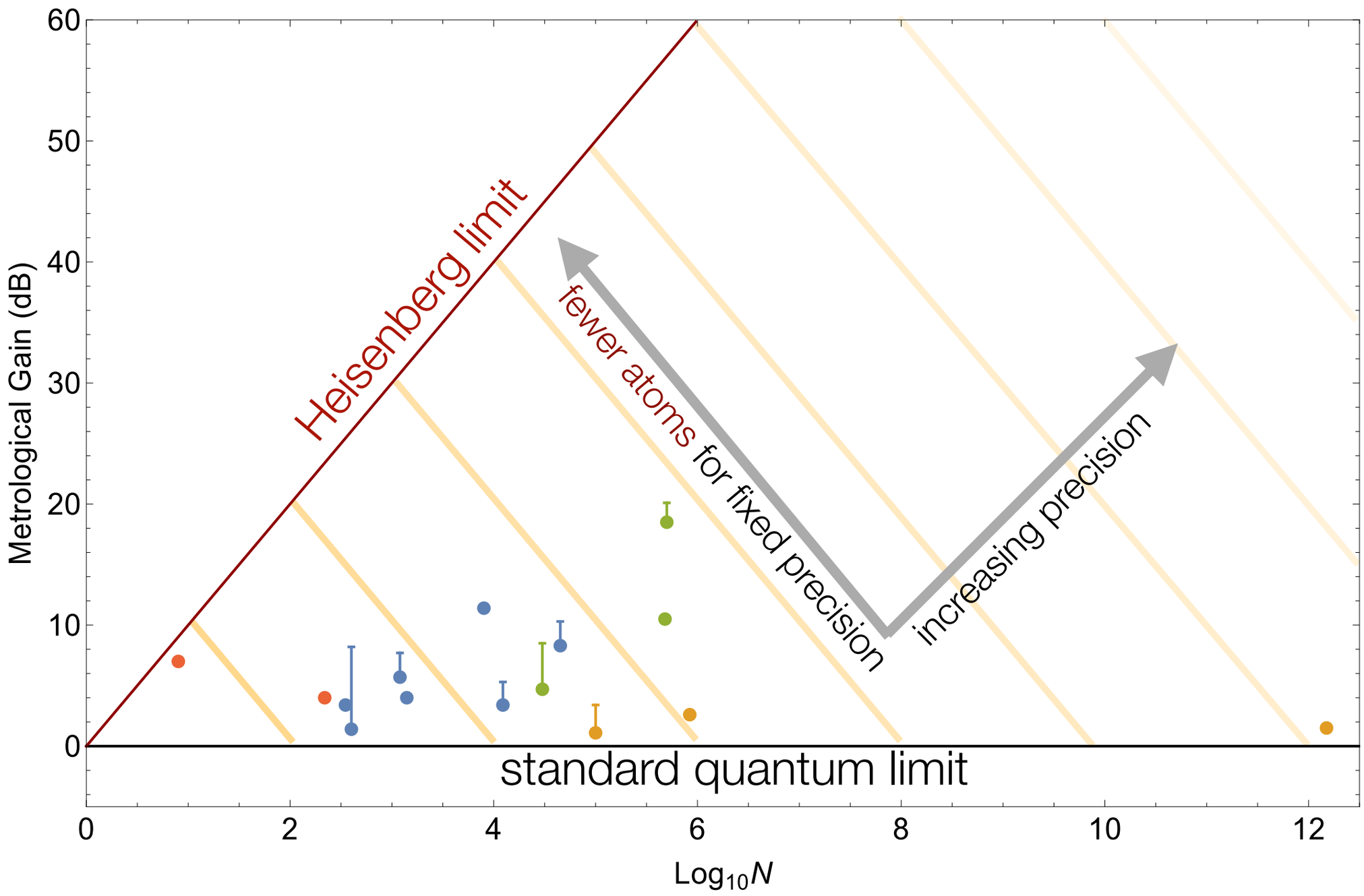
Hosten, Engelsen, Krishnakumar & Kasevich, *Nature* (2016).



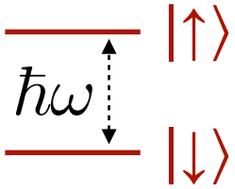
Entanglement-Enhanced Measurements



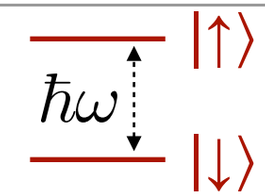
Approaching the Heisenberg Limit?



Towards the Heisenberg Limit?



Towards the Heisenberg Limit?



Fundamental limit set by Heisenberg Uncertainty Principle $\Delta E \Delta T \geq \hbar/2$

$$\Delta E \leq \frac{N}{2}\hbar\omega \;\Rightarrow\; \Delta(\omega T) \geq \frac{1}{N}$$

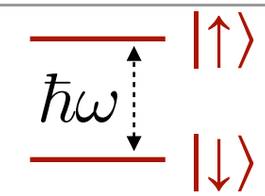
Heisenberg Limit

Reaching the Heisenberg Limit requires a state with *maximum uncertainty* in energy

$$|\psi\rangle_{\text{cat}} = \frac{|\uparrow\uparrow\uparrow\ldots\uparrow\rangle + |\downarrow\downarrow\downarrow\ldots\downarrow\rangle}{\sqrt{2}}$$

Towards the Heisenberg Limit?

Fundamental limit set by Heisenberg Uncertainty Principle $\Delta E \Delta T \geq \hbar/2$



$$\Delta E \leq \frac{N}{2}\hbar\omega \;\Rightarrow\; \Delta(\omega T) \geq \frac{1}{N}$$

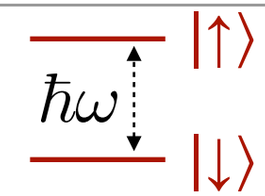
Heisenberg Limit

Reaching the Heisenberg Limit requires a state with *maximum uncertainty* in energy

$$|\psi\rangle_{\text{cat}} = \frac{\overbrace{|\uparrow\uparrow\uparrow\ldots\uparrow\rangle}^{\text{Alive}} + \overbrace{|\downarrow\downarrow\downarrow\ldots\downarrow\rangle}^{\text{Dead}}}{\sqrt{2}}$$

GHZ ("cat") state

Towards the Heisenberg Limit?



Fundamental limit set by Heisenberg Uncertainty Principle $\Delta E \Delta T \geq \hbar/2$

$$\Delta E \leq \frac{N}{2}\hbar\omega \;\Rightarrow\; \Delta(\omega T) \geq \frac{1}{N}$$

Heisenberg Limit

Reaching the Heisenberg Limit requires a state with *maximum uncertainty* in energy

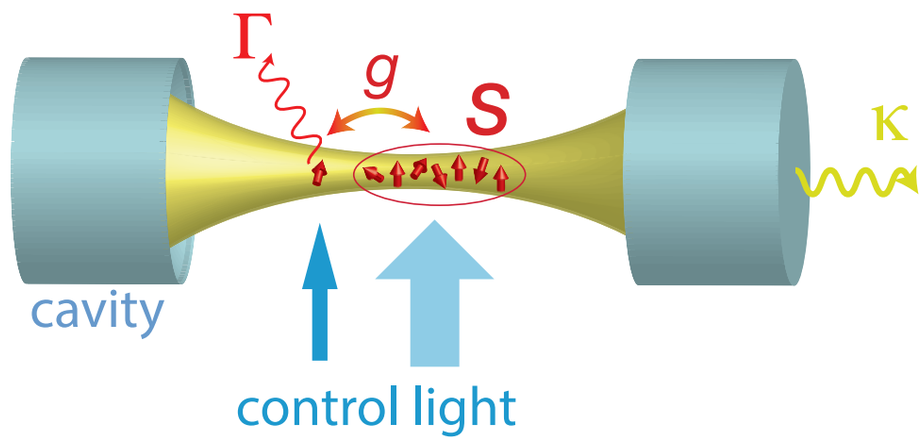
$$|\psi\rangle_{\text{cat}} = \frac{\overbrace{|\uparrow\uparrow\uparrow\ldots\uparrow\rangle}^{\text{Alive}} + \overbrace{|\downarrow\downarrow\downarrow\ldots\downarrow\rangle}^{\text{Dead}}}{\sqrt{2}}$$

GHZ ("cat") state

- ***Making and detecting these states at large N?***
- ***Easier alternatives?***

Routes to Cat States?

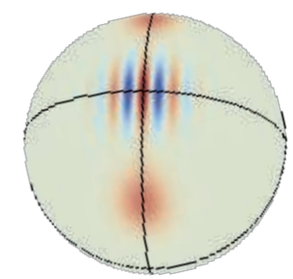
Qubit-Ensemble Interface



single-atom state → photon polarization → many-atom kitten/cat

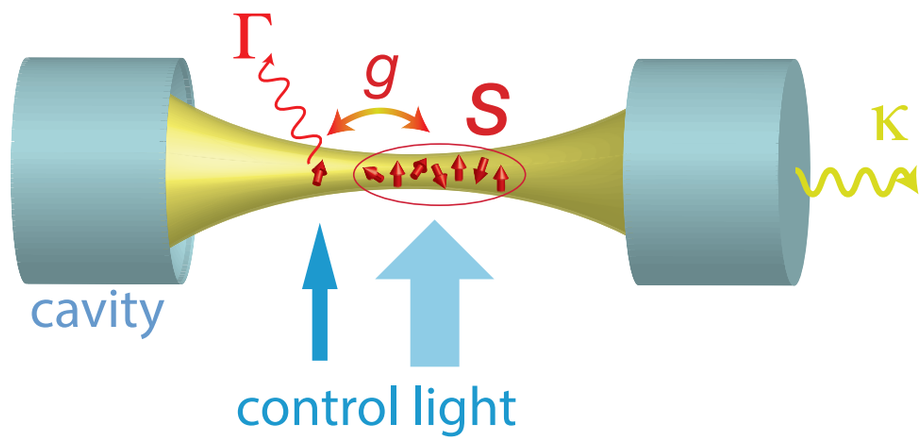
$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



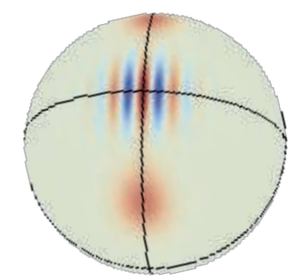
Routes to Cat States?

Qubit-Ensemble Interface



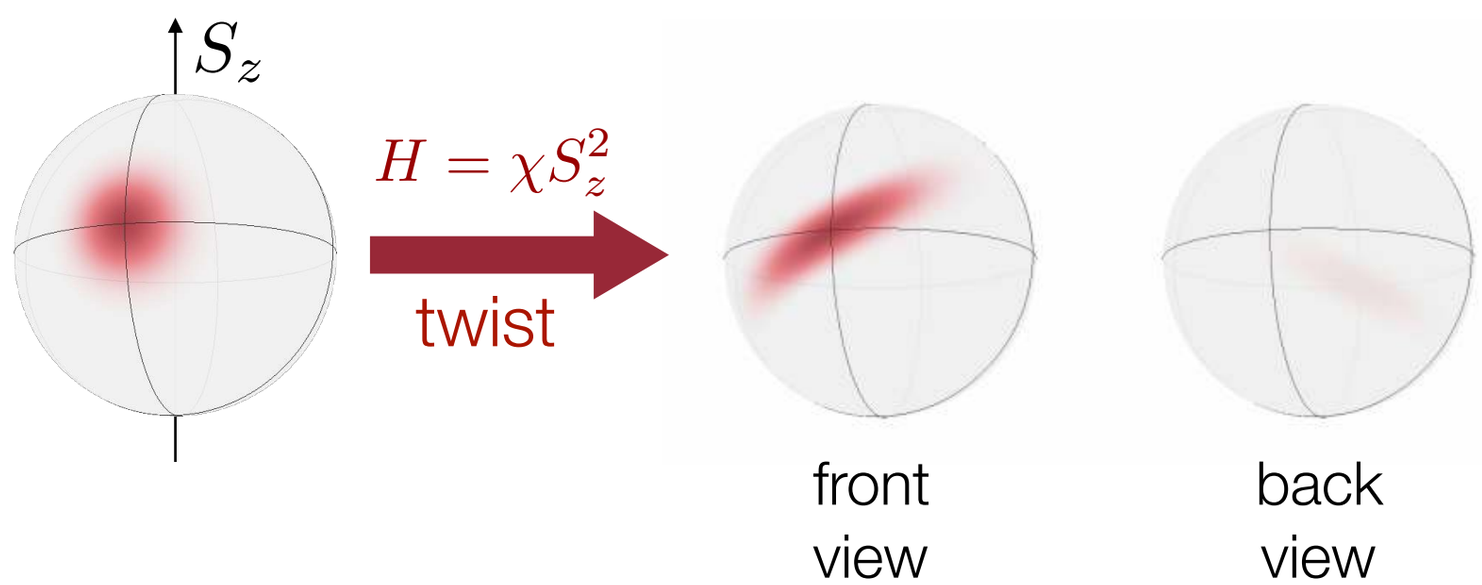
single-atom state → photon polarization → many-atom kitten/cat

$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \qquad \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



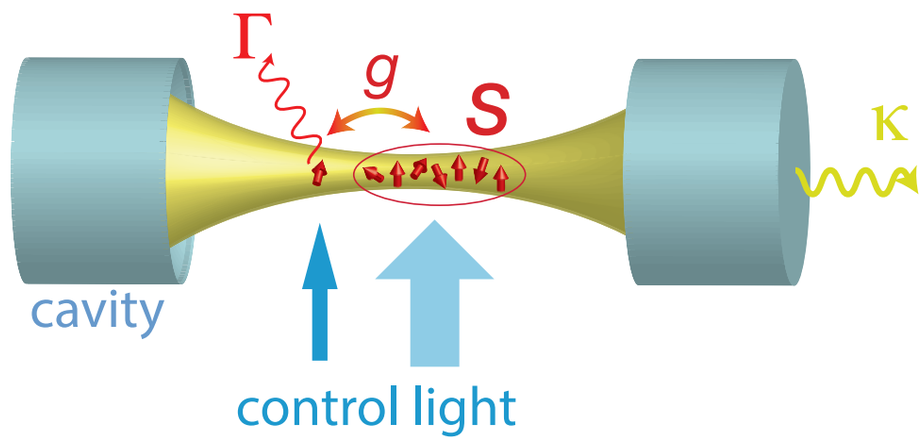
Collective Interactions

collective spin $\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$



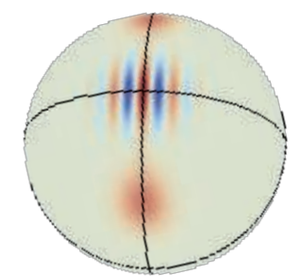
Routes to Cat States?

Qubit-Ensemble Interface



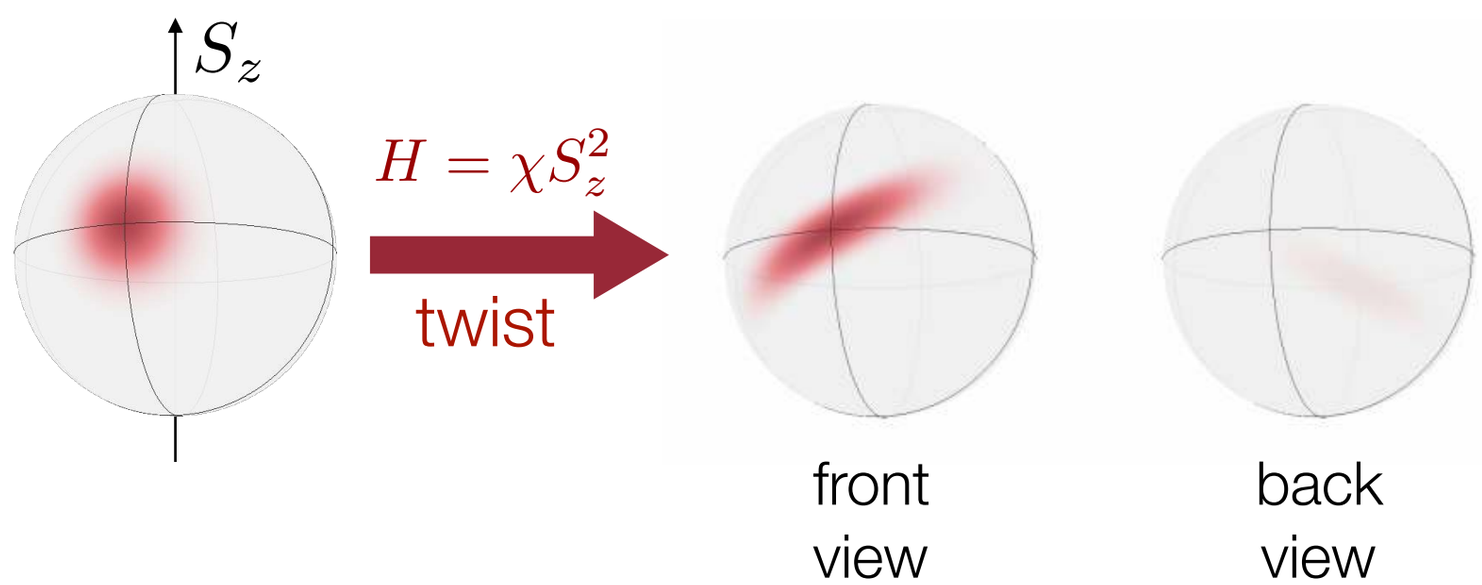
single-atom state → photon polarization → many-atom kitten/cat

$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \qquad \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

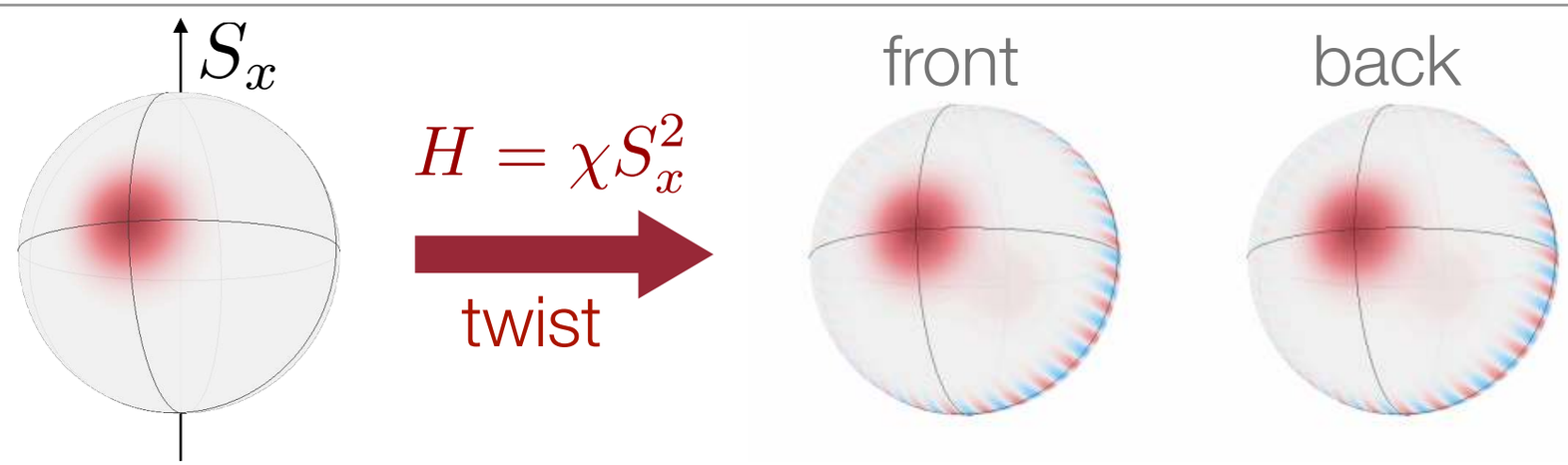


Collective Interactions

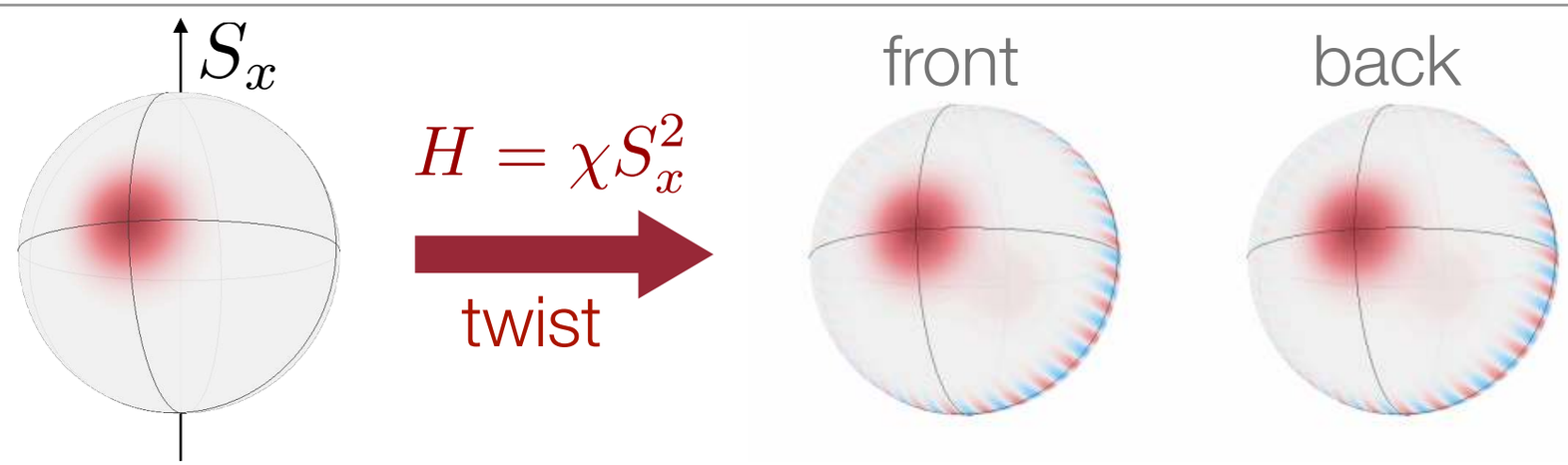
collective spin $\mathbf{S} = \sum_{i=1}^{N} \mathbf{s}_i$



Twisting to the Heisenberg Limit?



Twisting to the Heisenberg Limit?

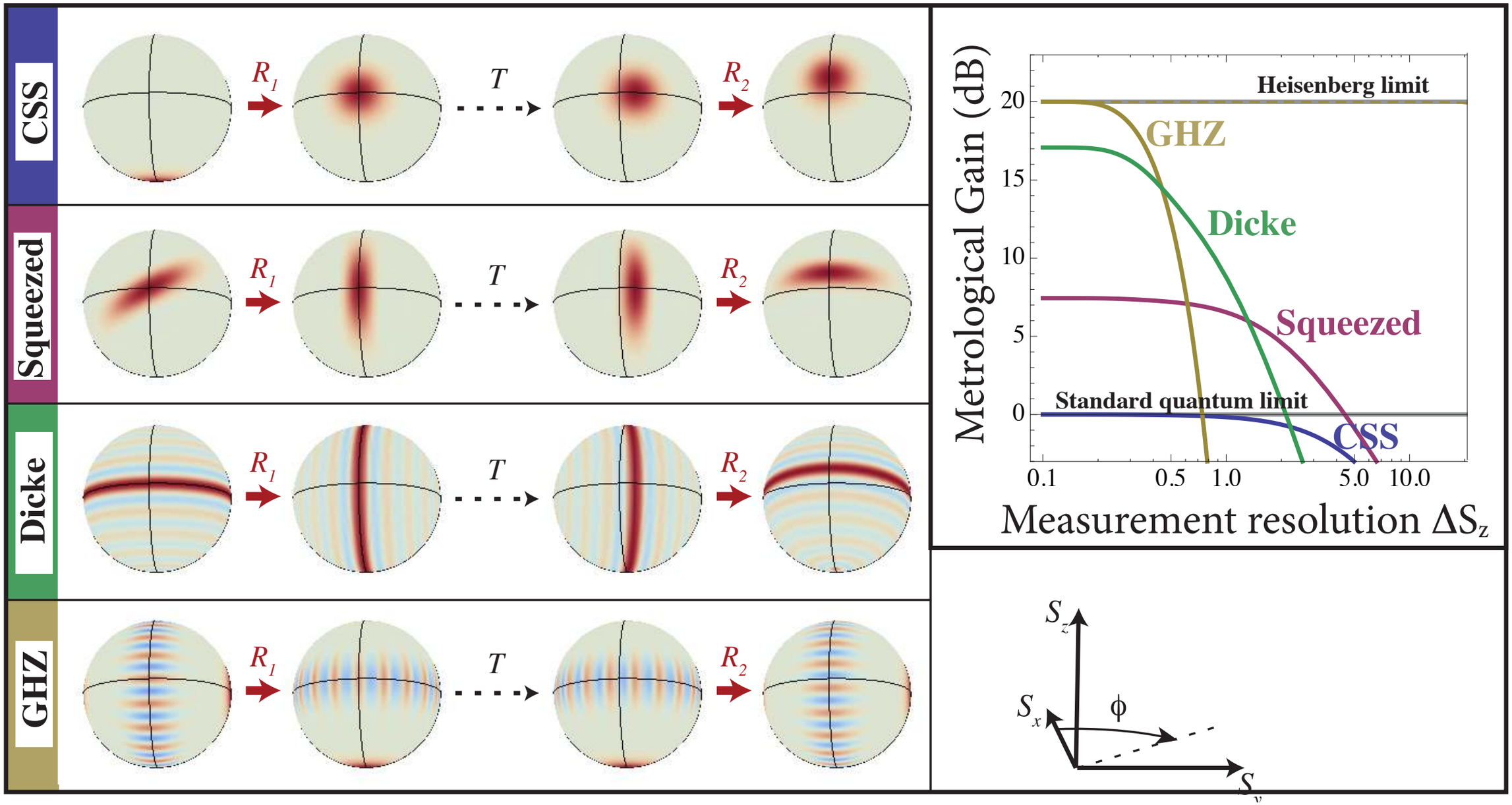


Demonstrated with up to 14 ions [Monz…& Blatt, *PRL* 2011]

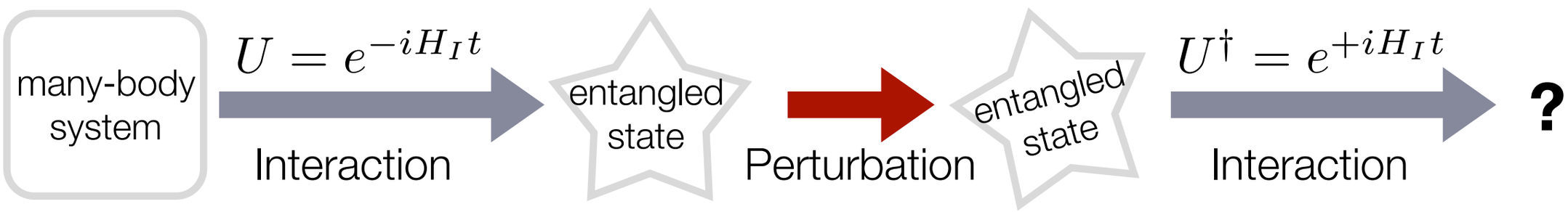
Scaling to larger ensembles of neutral atoms?

- Preparation via $H = \chi S_x^2$ requires *extreme twisting* ($\chi t = \pi/2$)
- Useless after loss of a *single* atom
- Detection requires *single-atom resolution* $\Delta S_z \ll 1$

Metrological Gain *vs.* Detection Noise



Echo Spectroscopy

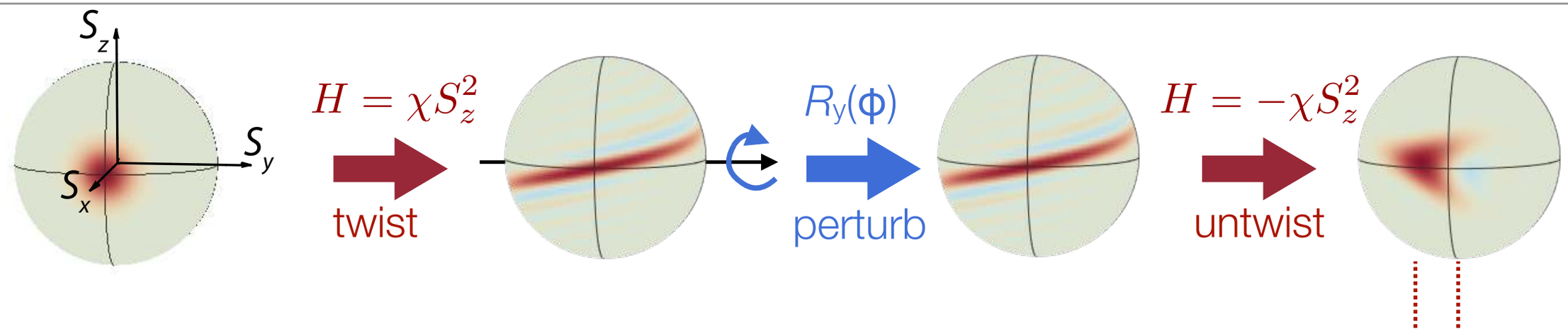


Detect perturbation by measuring whether system returns to *initial state*, rather than directly detecting the entangled state

Inspiration: Loschmidt echo (diagnostic of chaos)

One-Axis Twisting Echo

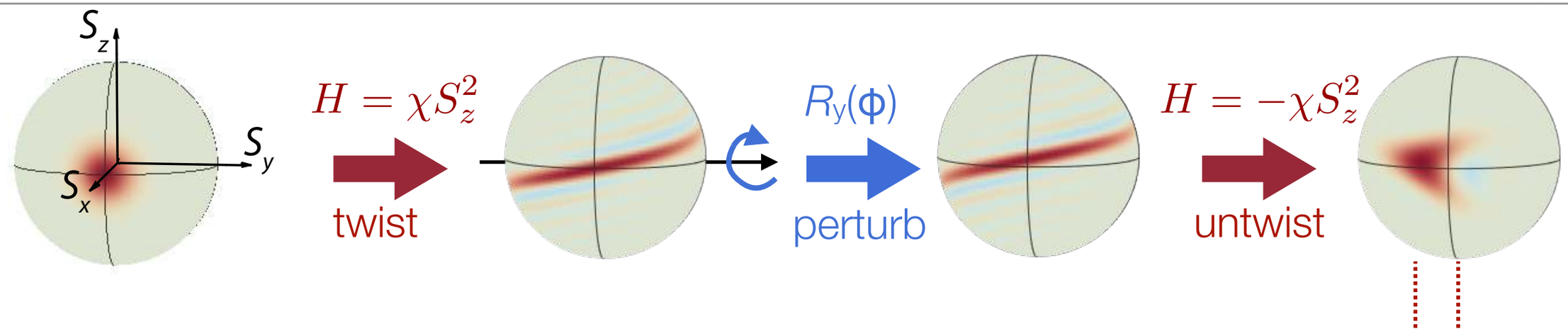
E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



Measure $\langle S_y \rangle = GS\phi$

One-Axis Twisting Echo

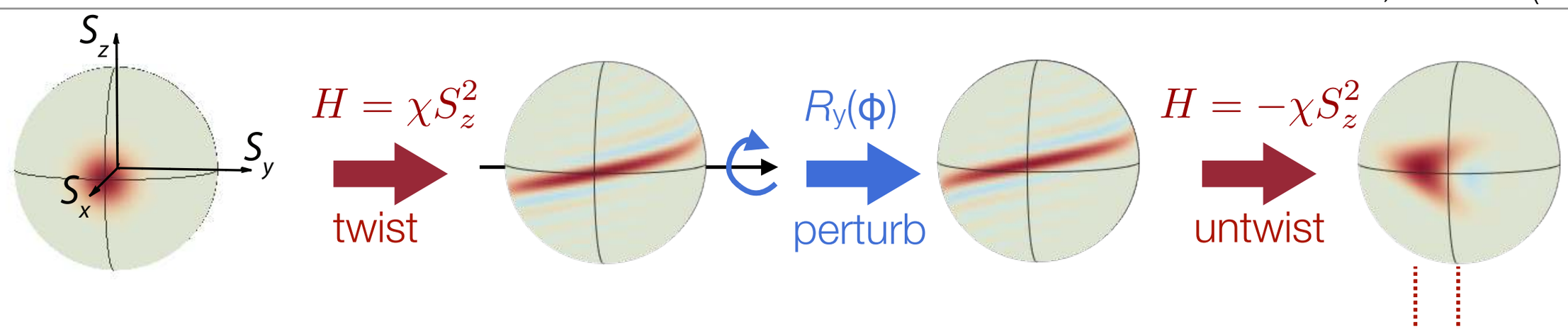
E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



Measure $\langle S_y \rangle = GS\phi$

One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



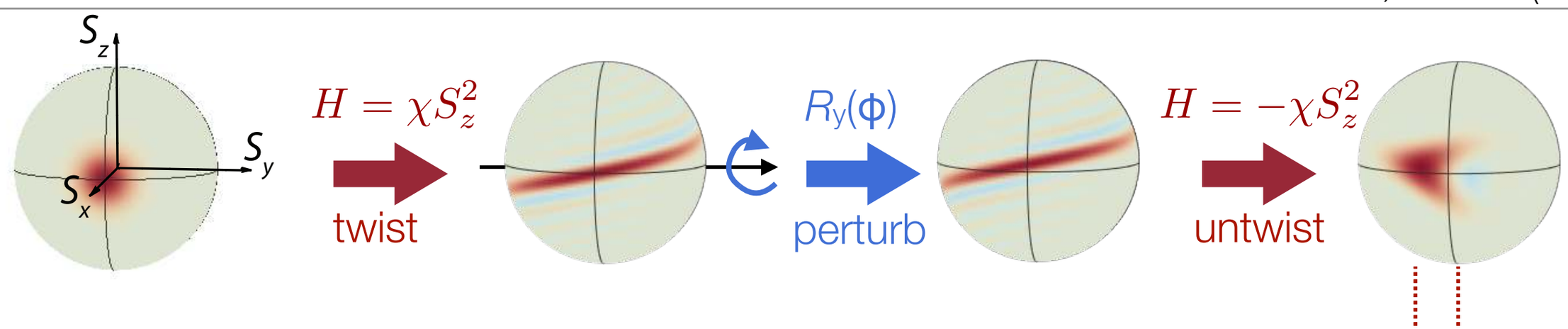
Signal amplified by factor *G*

Measure $\langle S_y \rangle = G S \phi$

Quantum noise is that of the *initial* state

One-Axis Twisting Echo

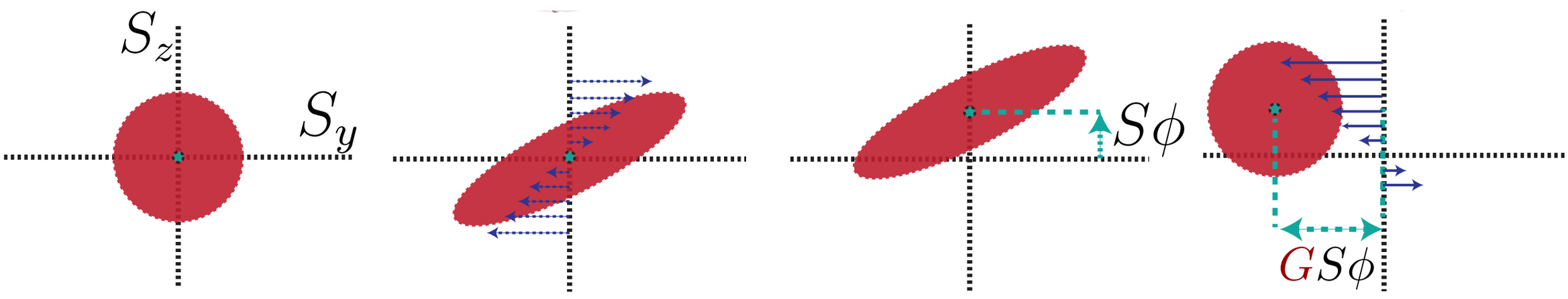
E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



Signal amplified by factor *G*

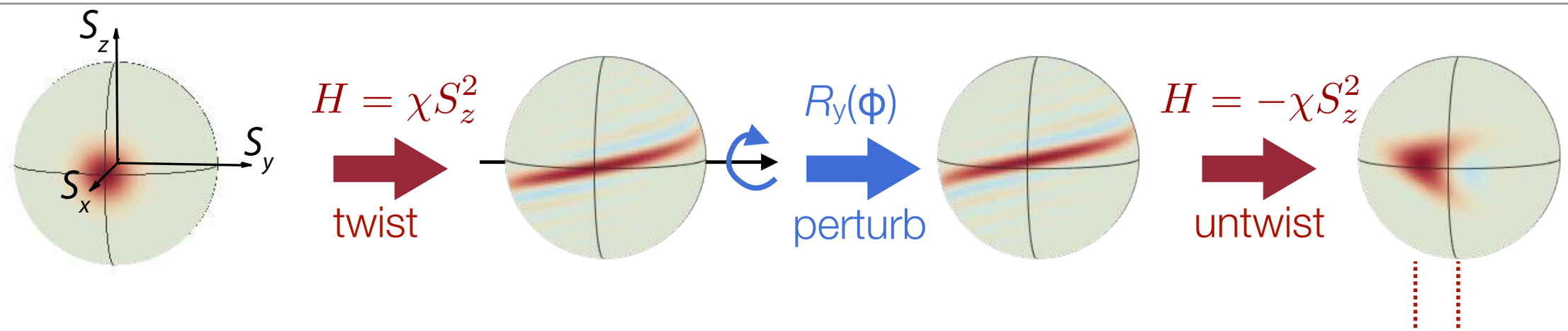
Measure $\langle S_y \rangle = G S \phi$

Quantum noise is that of the *initial* state



One-Axis Twisting Echo

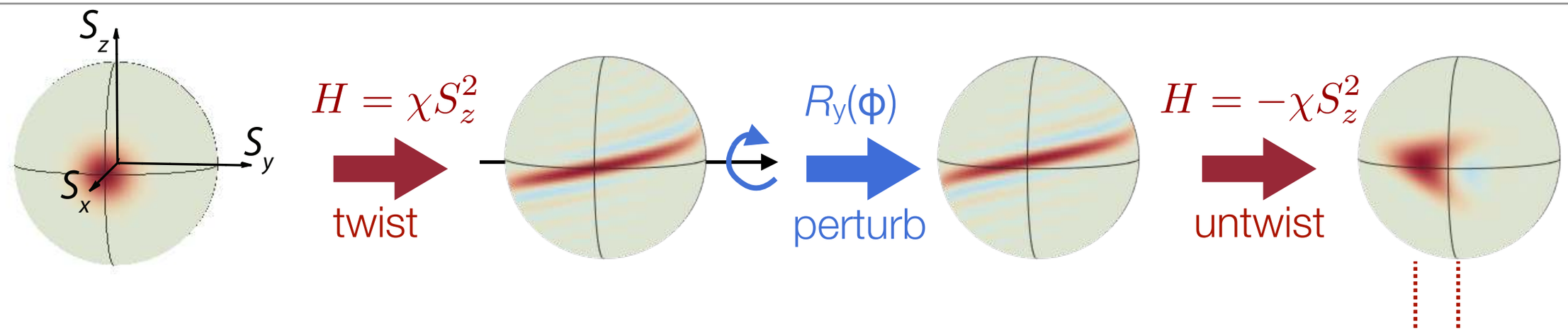
E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



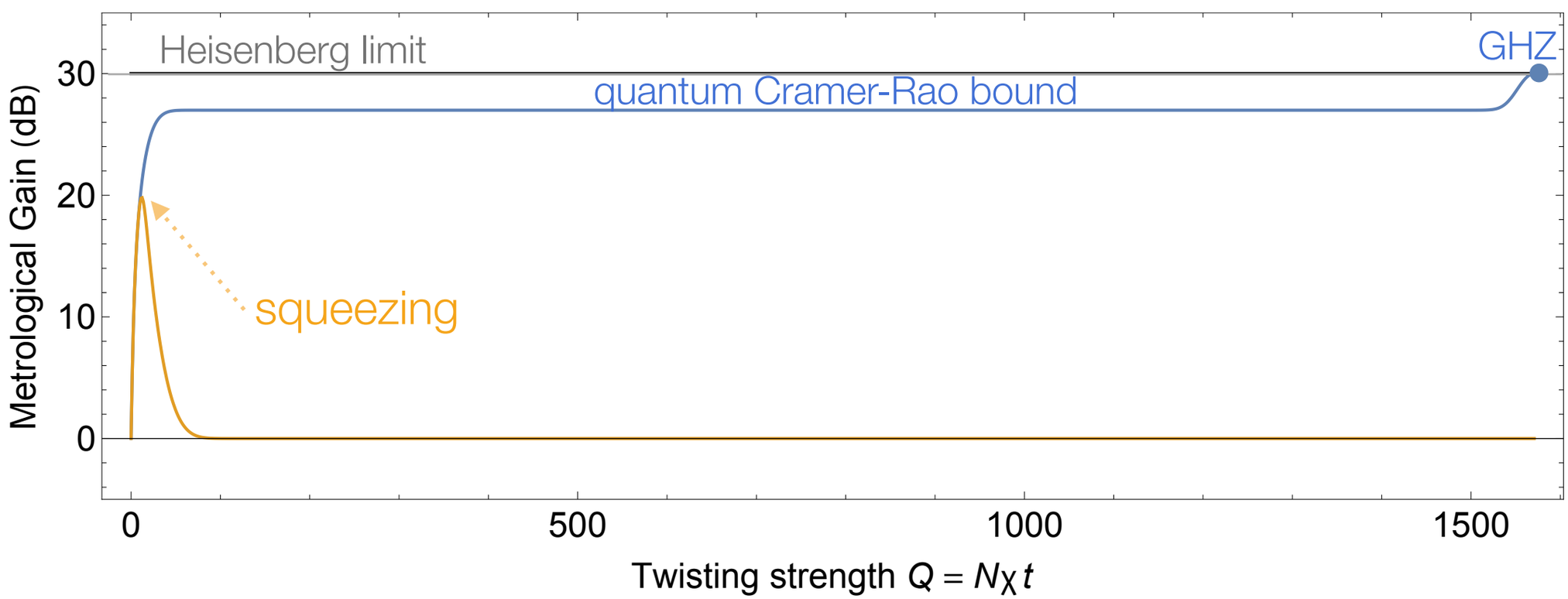
Measure $\langle S_y \rangle = GS\phi$

One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).

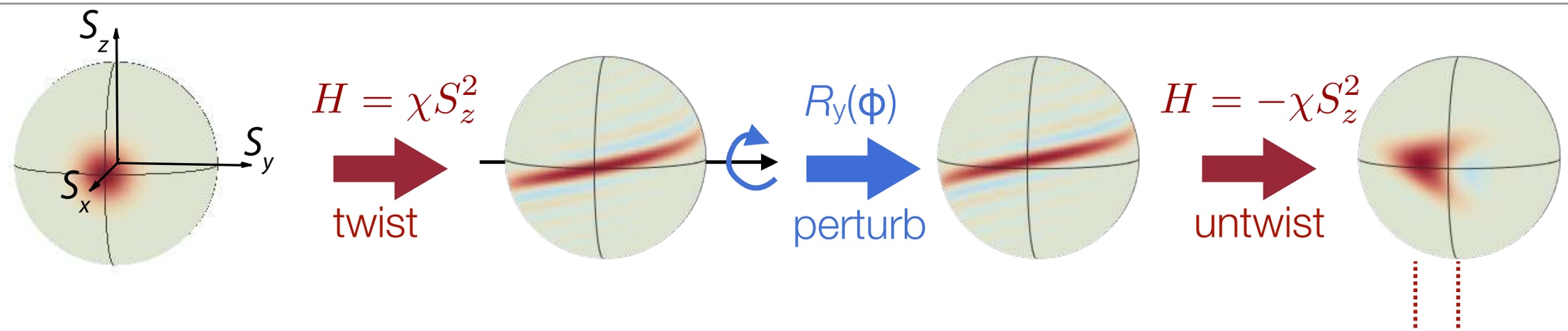


Measure $\langle S_y \rangle = G S \phi$

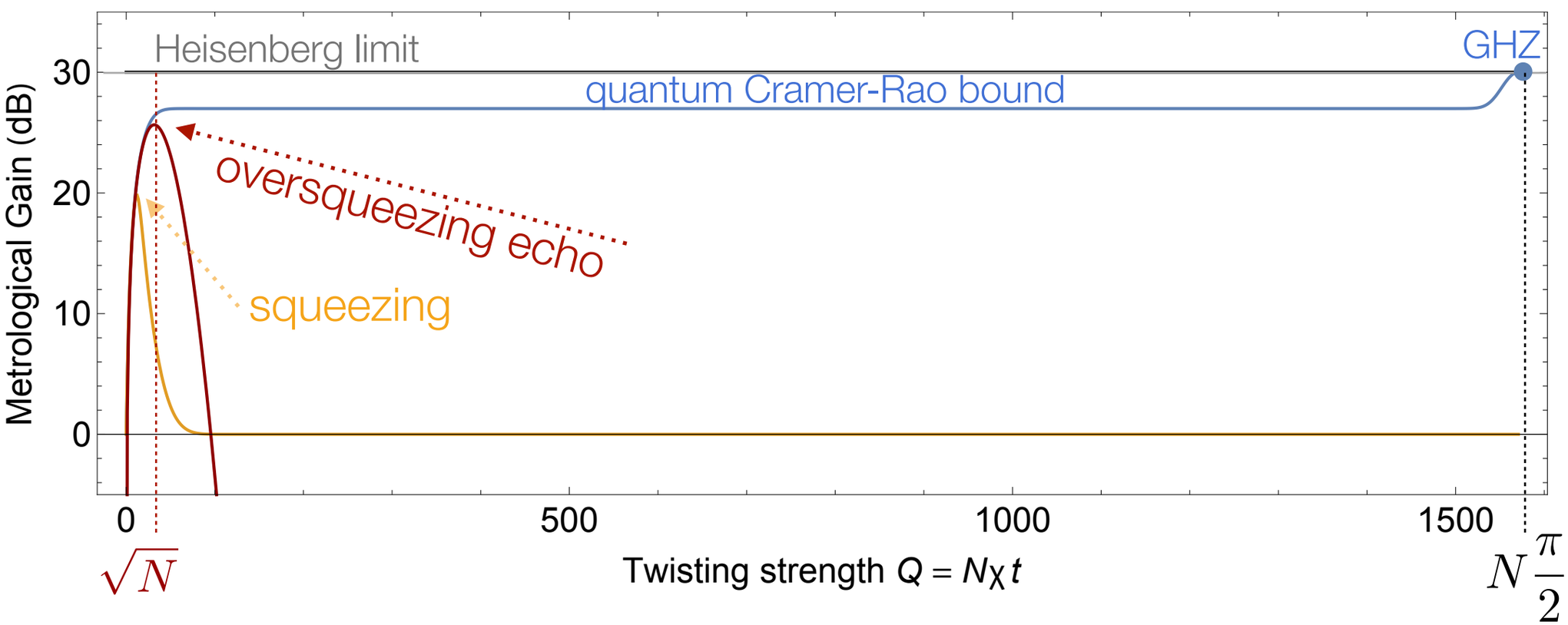


One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).

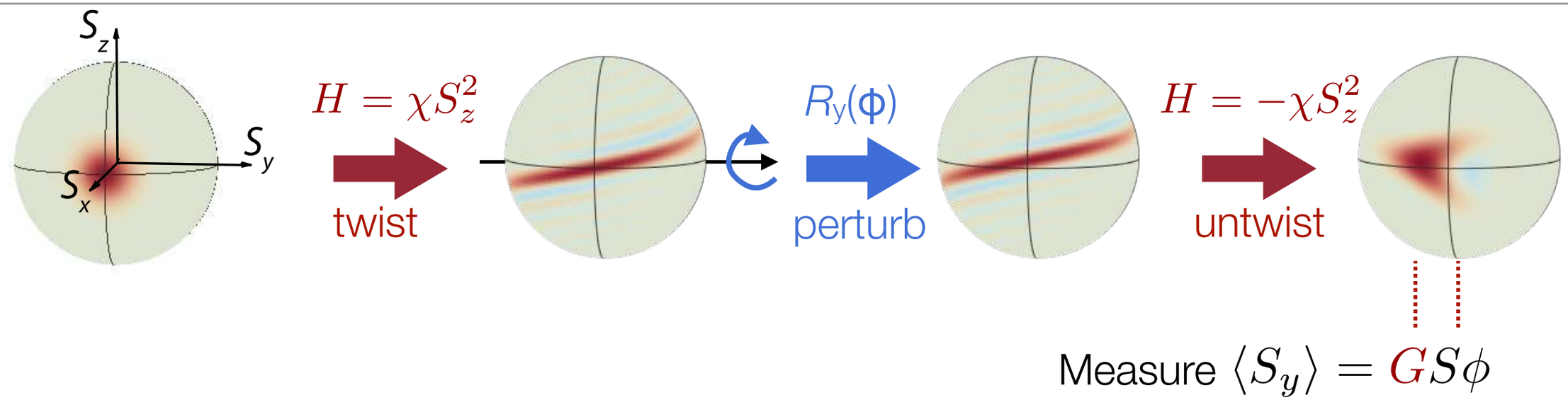


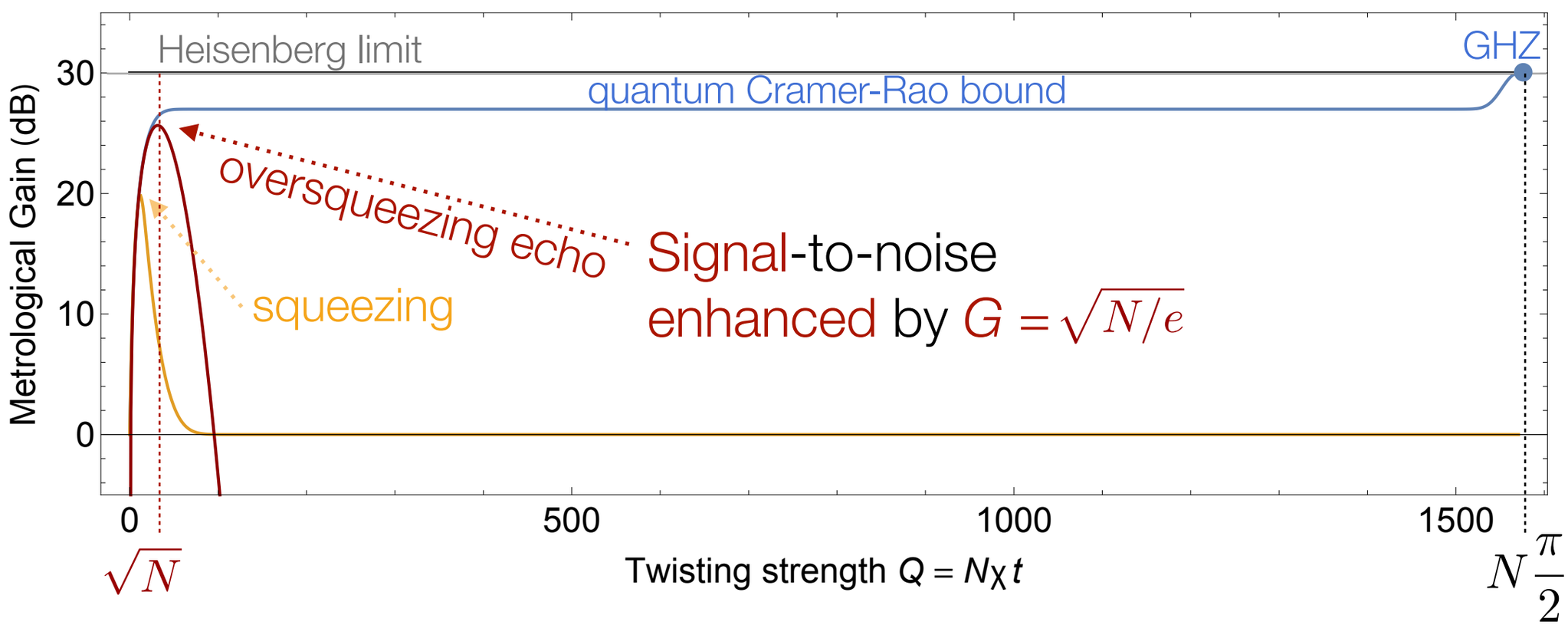
Measure $\langle S_y \rangle = G S \phi$



One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).



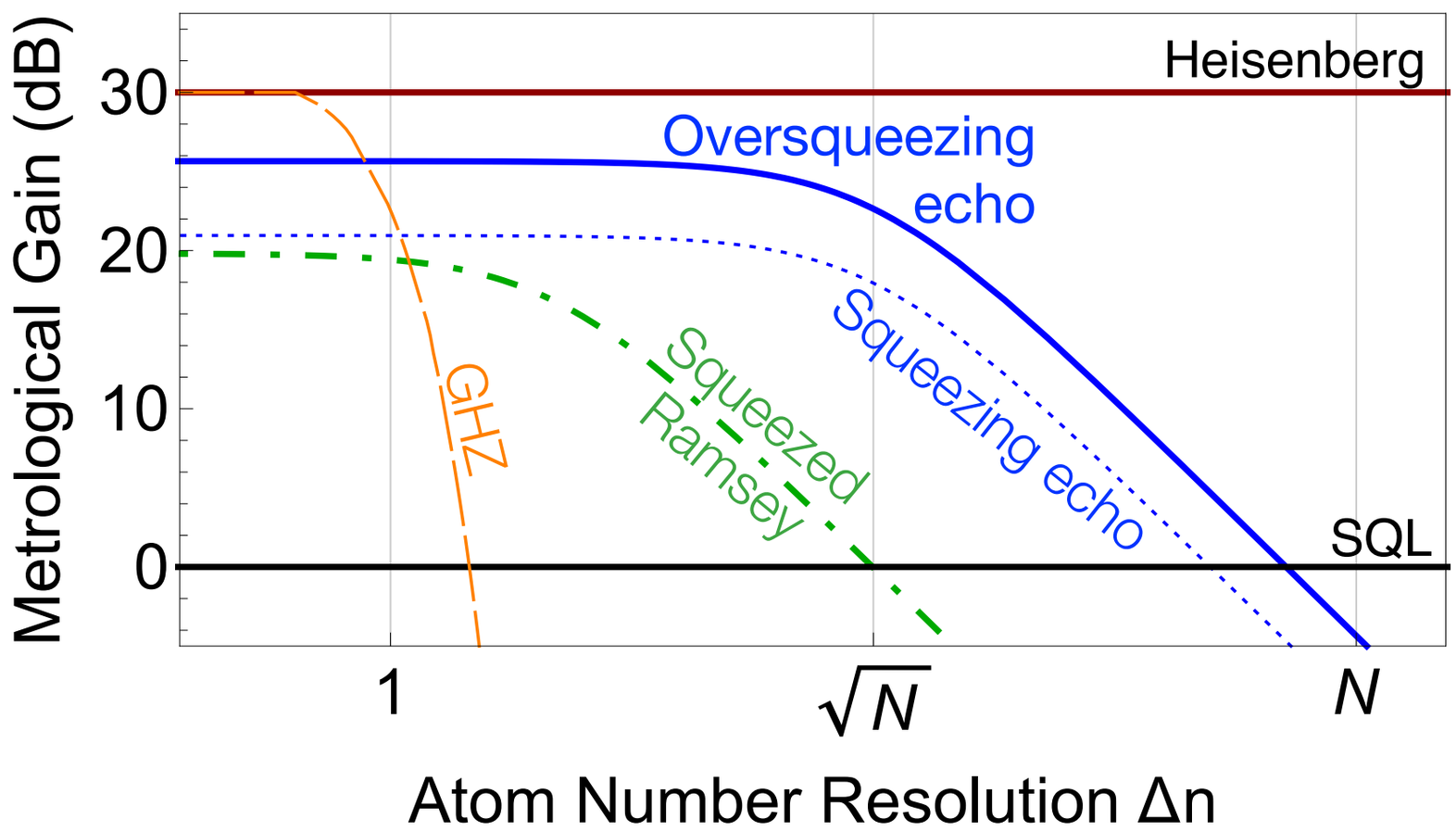


One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).

- Heisenberg scaling $\Delta\phi = \sqrt{e}/N$ reached after short twisting time

- Measurement resolution $\Delta S_{\mathrm{meas}} \lesssim \sqrt{N}/2$ suffices!

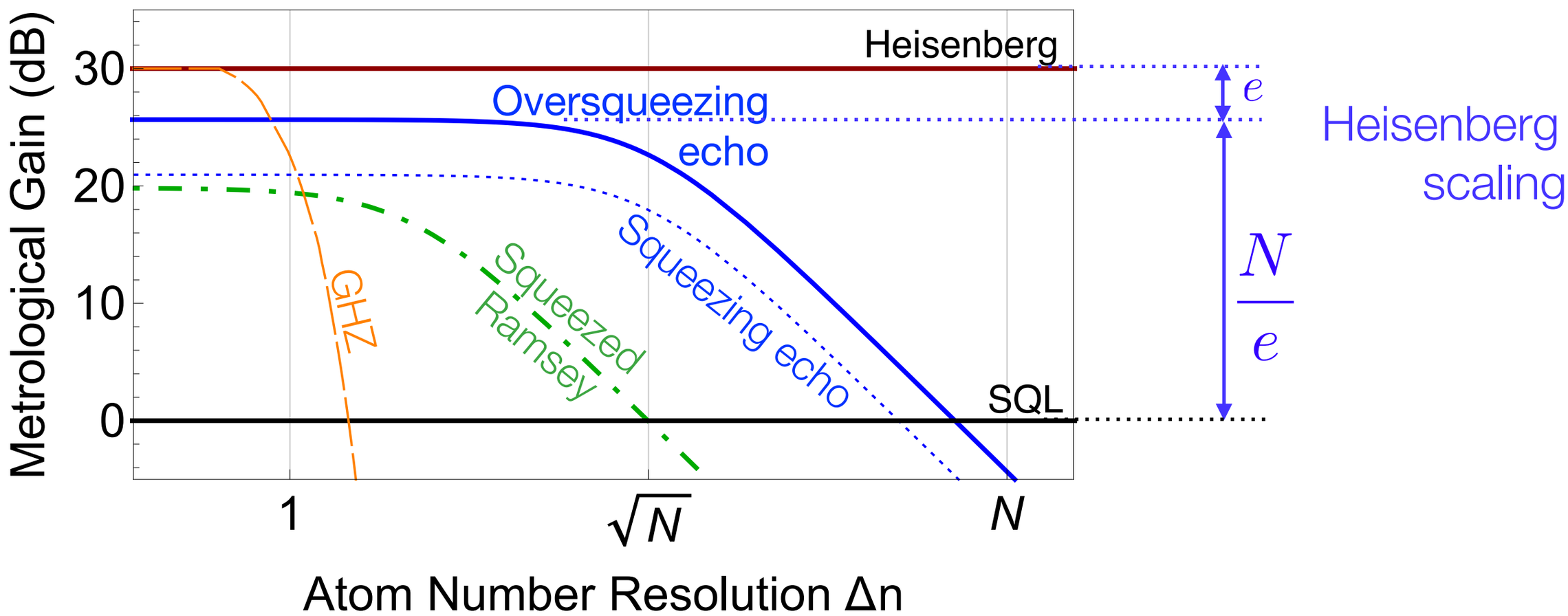


One-Axis Twisting Echo

E. Davis, G. Bentsen, & MS-S,
PRL **116**, 053601 (2016).

- Heisenberg scaling $\Delta\phi = \sqrt{e}/N$ reached after short twisting time

- Measurement resolution $\Delta S_{\mathrm{meas}} \lesssim \sqrt{N}/2$ suffices!

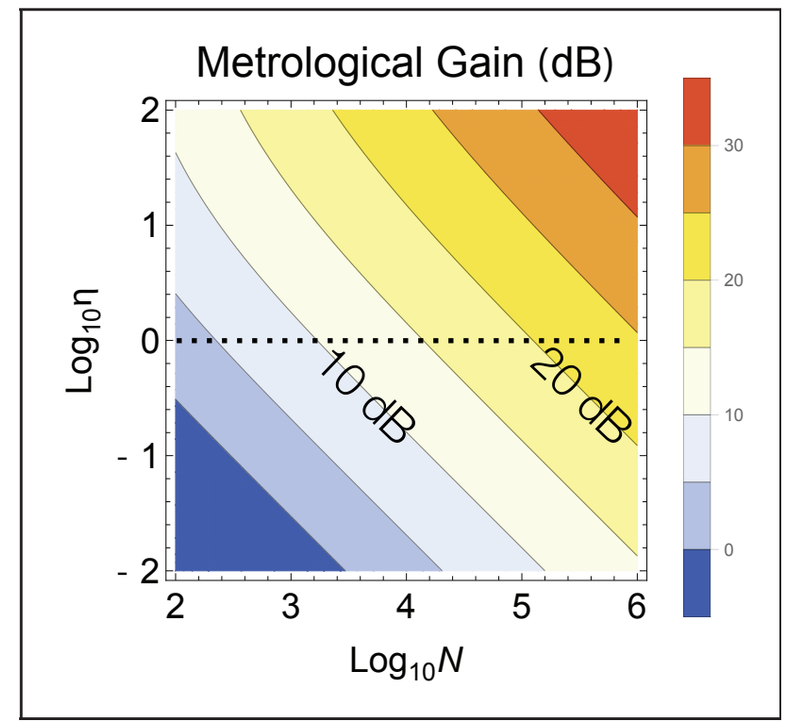


Dissipative Twisting Echo

$$\Delta\phi = \frac{\Delta S_y}{dS_y/d\phi}$$

← broadening from dissipation $\propto \sqrt{Q}$

← amplification from "untwisting" $\propto Q$



Q = # of photons scattered into cavity

Q/(Nη) = # of photons scattered into free space

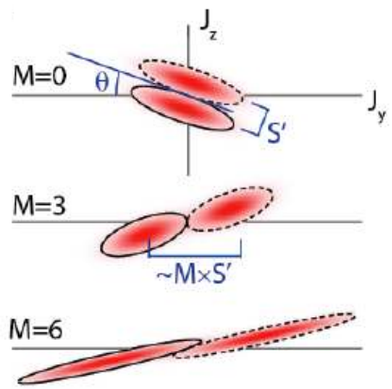
$$\Rightarrow \quad N\Delta\phi^2 \sim \frac{1}{Q} + \frac{Q}{N\eta} \geq \frac{1}{\sqrt{N\eta}}$$

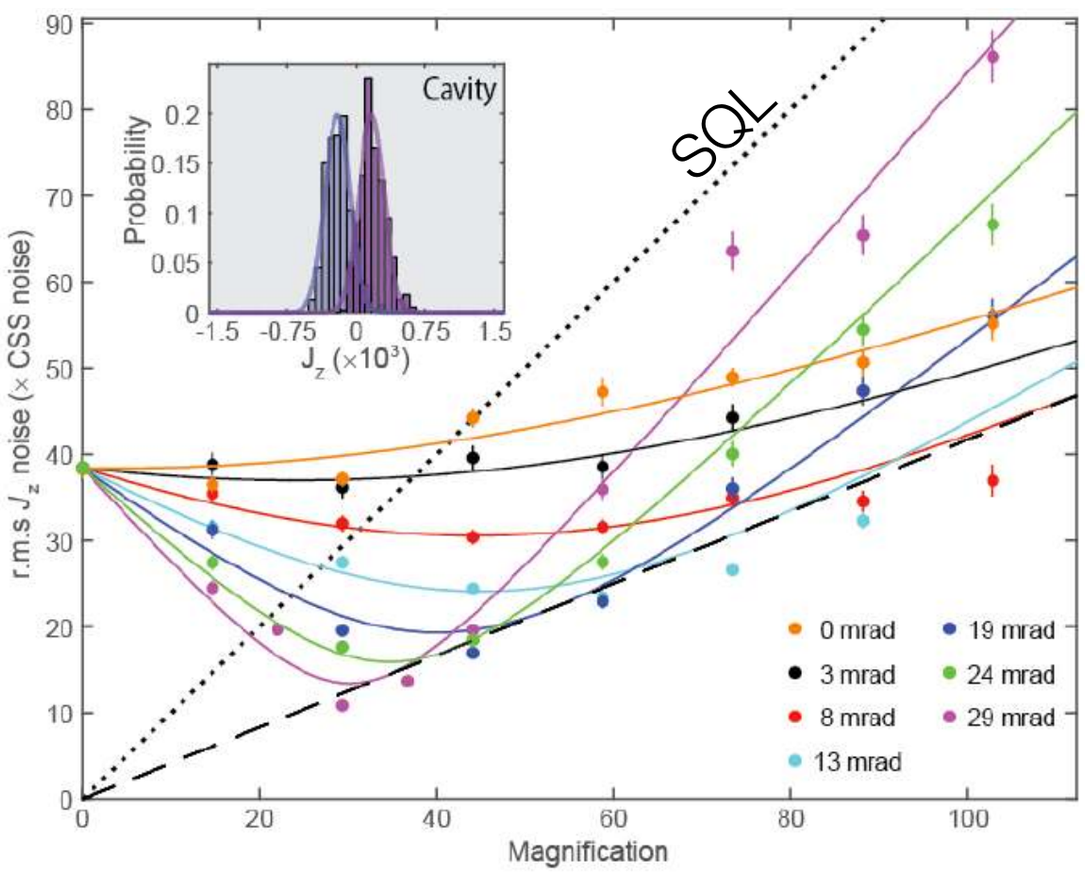
Scales just as spin squeezing by quantum non-demolition measurement…
but no need for sensitive, coherence-preserving measurement

Interaction-Based Readout: Demonstrations

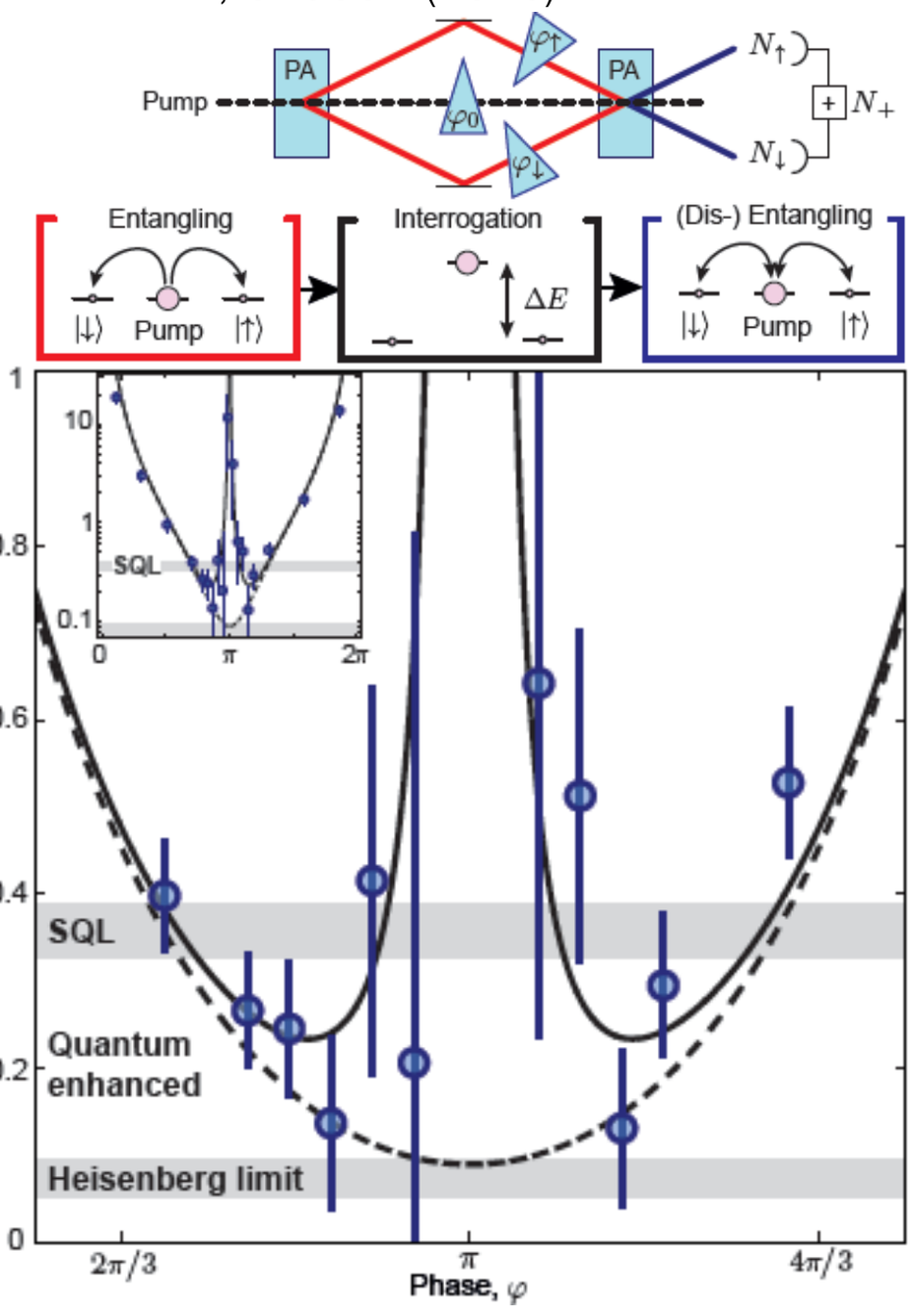
Hosten, Krishnakumar, Engelson & Kasevich, *Science* (2016).

8 dB squeezing detected with low-resolution fluorescence imaging





Linnemann, Strobel, Muessel, Schulz, Lewis-Swan, Kheruntsyan, & Oberthaler, *PRL* 117, 013001 (2016).

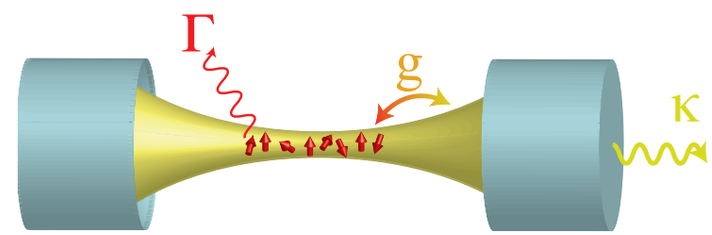


Oversqueezing Echo: Implementations?

Requirement: coherent interactions of switchable sign

Ion Penning trap

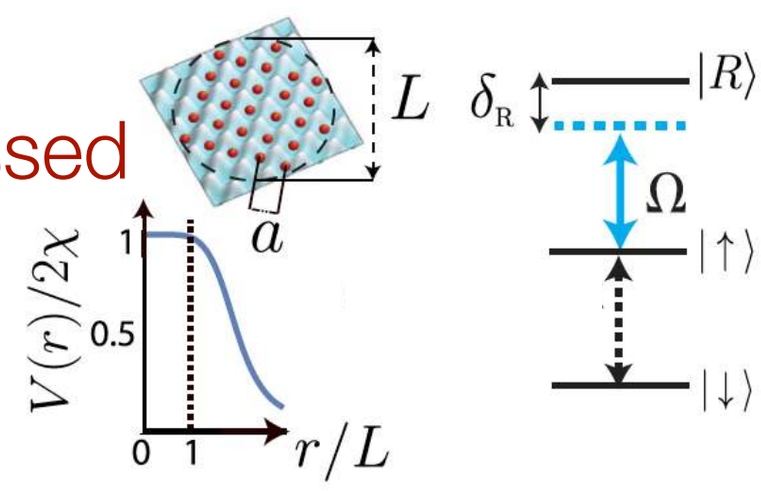
Gärttner, Bohnet, Safavi-Naini, Wall, Bollinger, & Rey, *Nat. Phys.* (2017).



Atoms in a cavity:

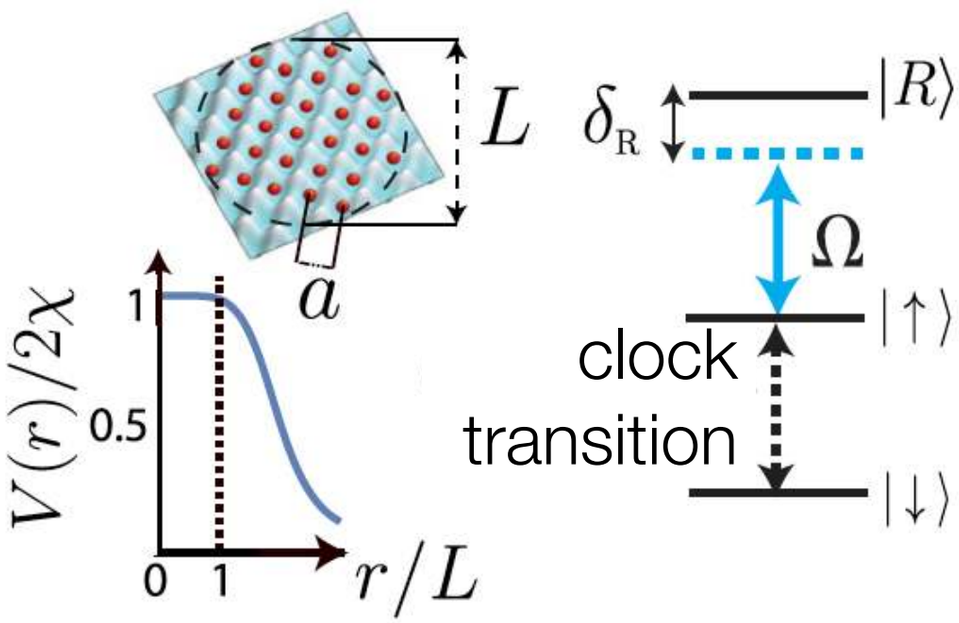
Photon-mediated interactions

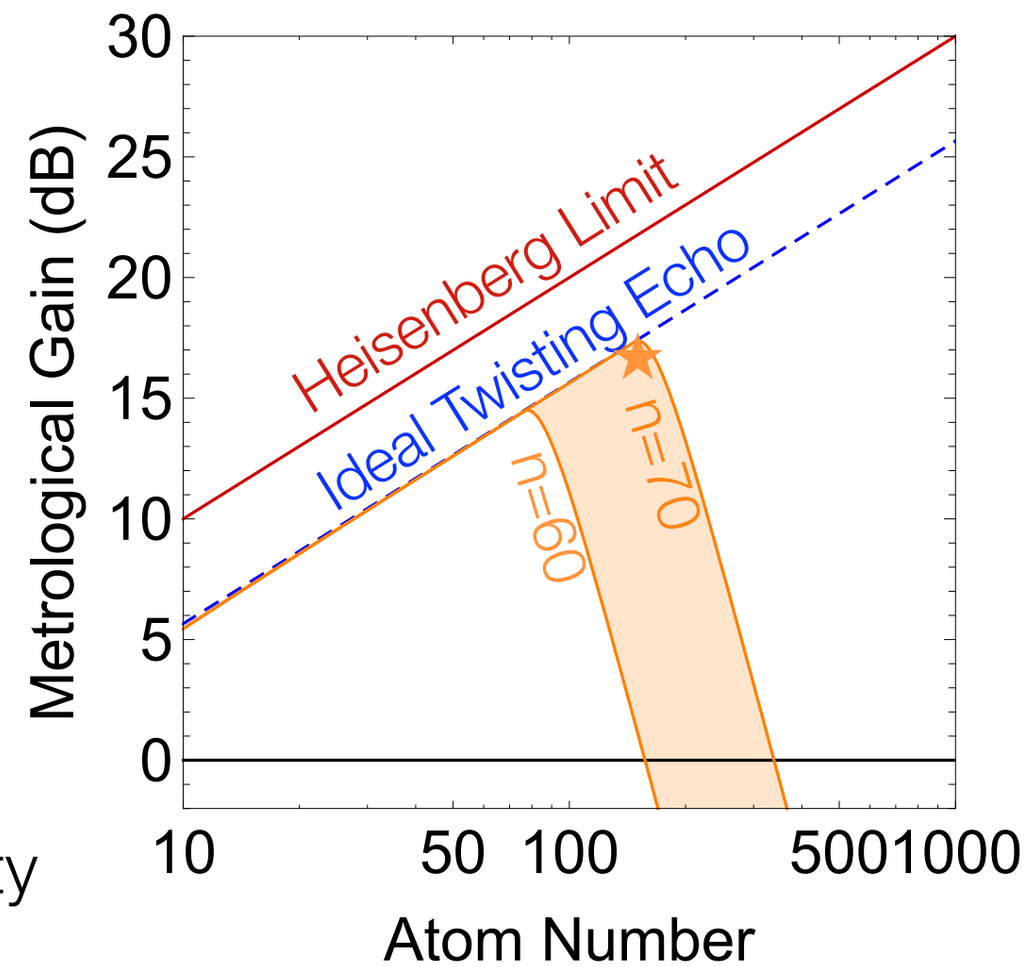
Rydberg-dressed atoms



Prospects for Oversqueezing Echo

…by Rydberg dressing



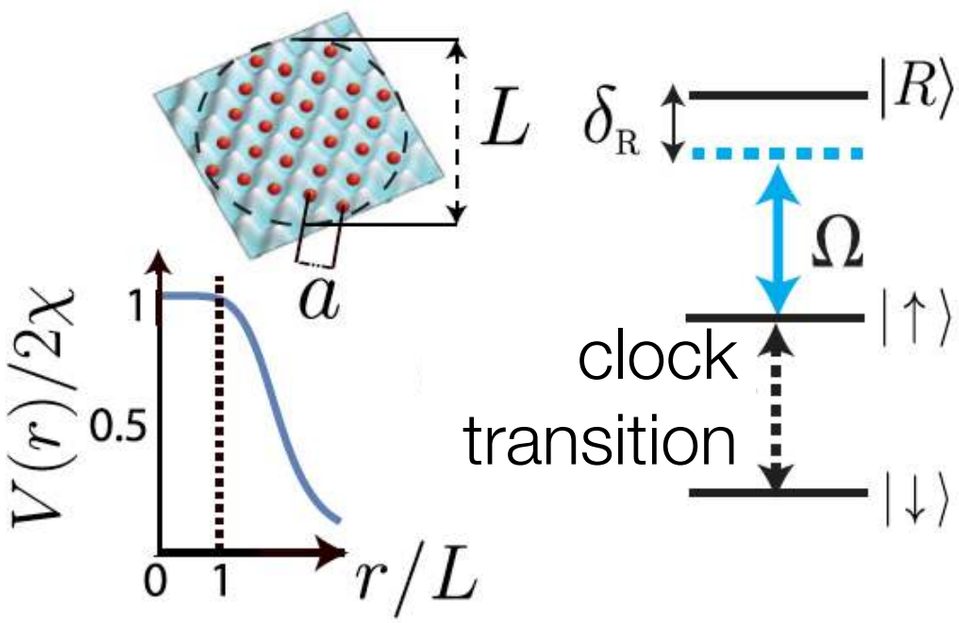


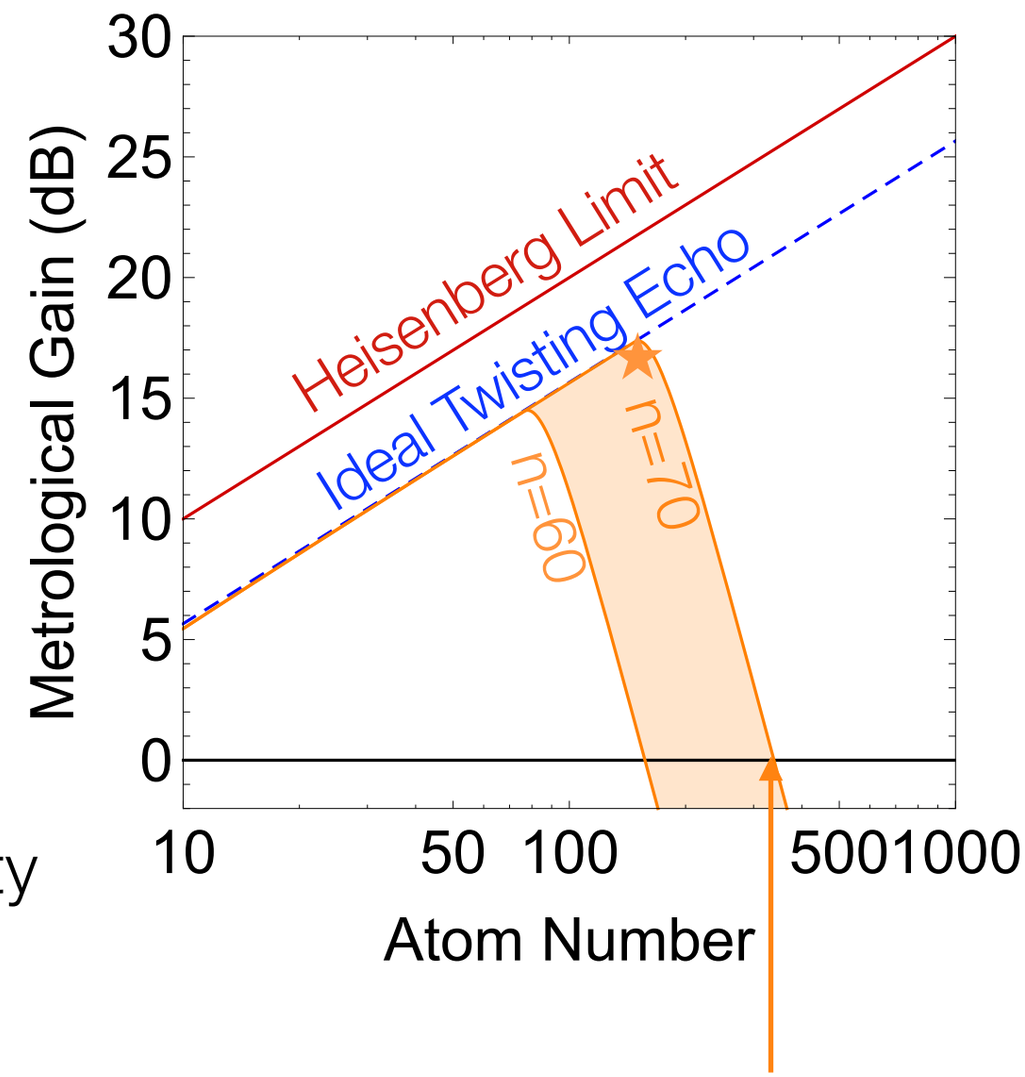
★200 entangled atoms with sensitivity matching ~10^4 unentangled atoms

Inspiration: "Spin squeezing in a Rydberg lattice clock," Gil, Mukherjee, Bridge, Jones & Pohl, *PRL* (2014).

Prospects for Oversqueezing Echo

…by Rydberg dressing



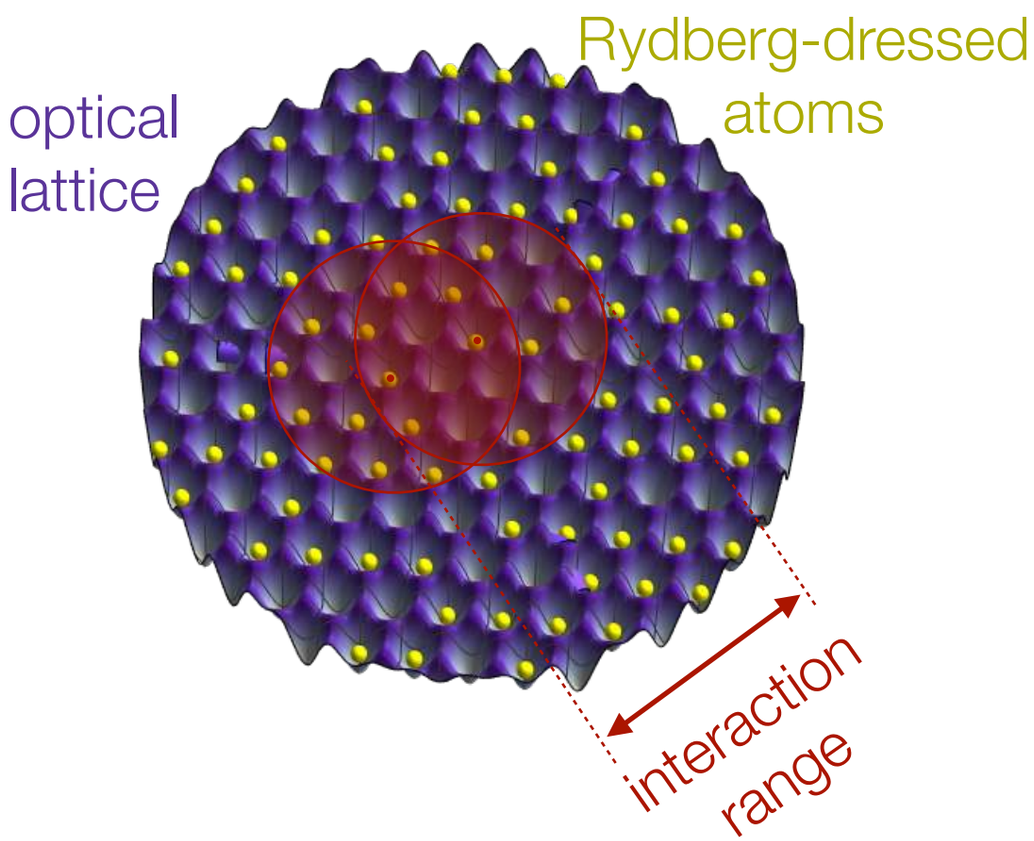


★200 entangled atoms with sensitivity matching ~10^4 unentangled atoms

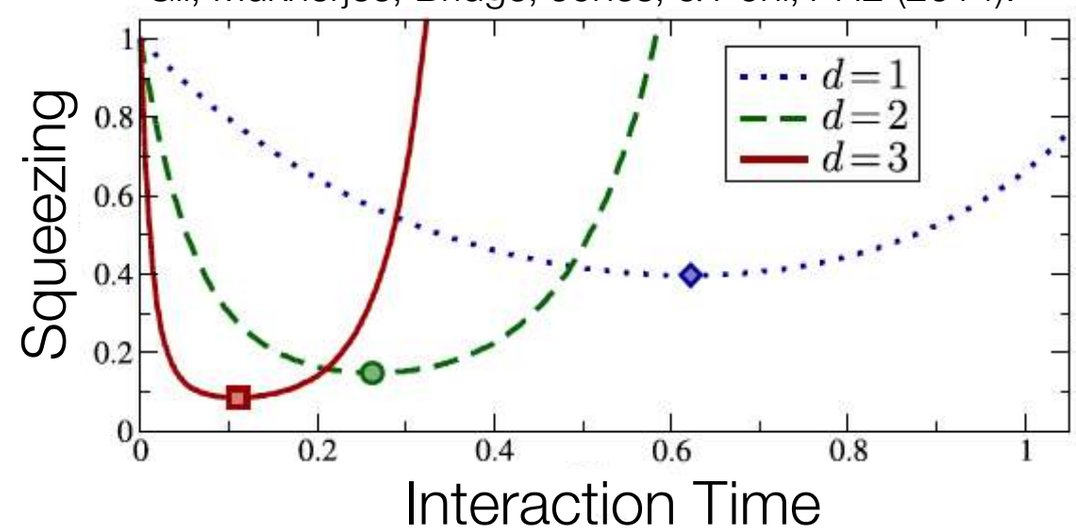
Inspiration: "Spin squeezing in a Rydberg lattice clock," Gil, Mukherjee, Bridge, Jones & Pohl, *PRL* (2014).

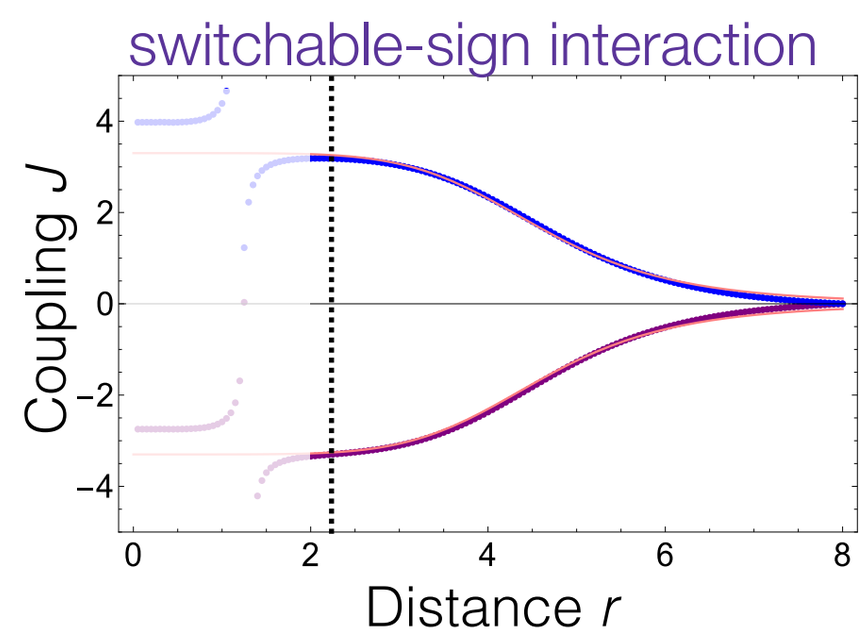
Beyond Global Interactions

Harnessing complex many-body entangled states for metrological gain?



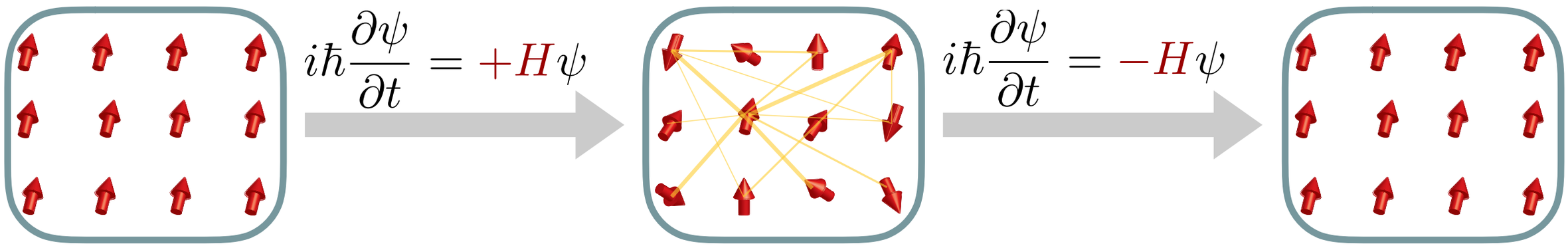
Gil, Mukherjee, Bridge, Jones, & Pohl, *PRL* (2014).



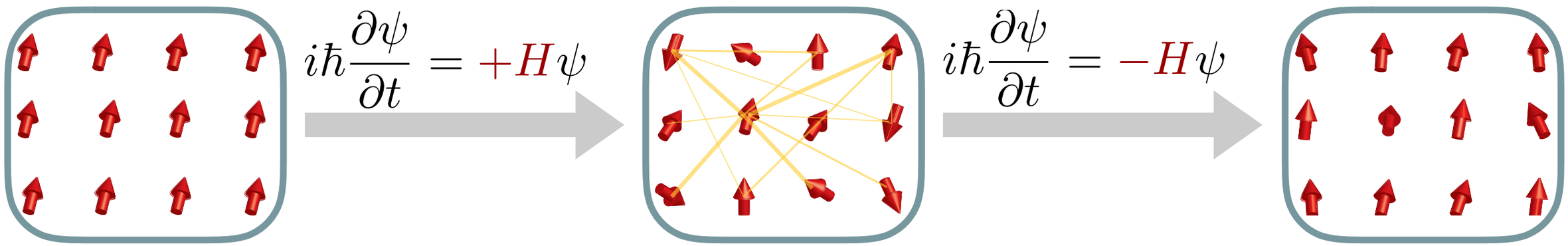


Access hidden entanglement by echo spectroscopy?

Interaction Echo



Interaction Echo

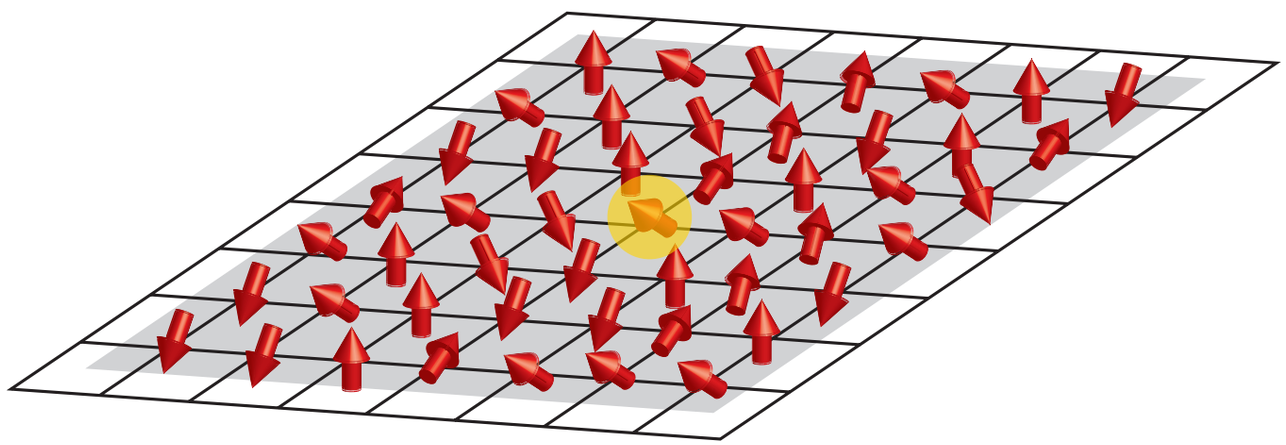


Motivations

- ✓ Exploiting sensitivity to perturbations for precision measurements
- Probing the spread of entanglement via sensitivity to perturbations

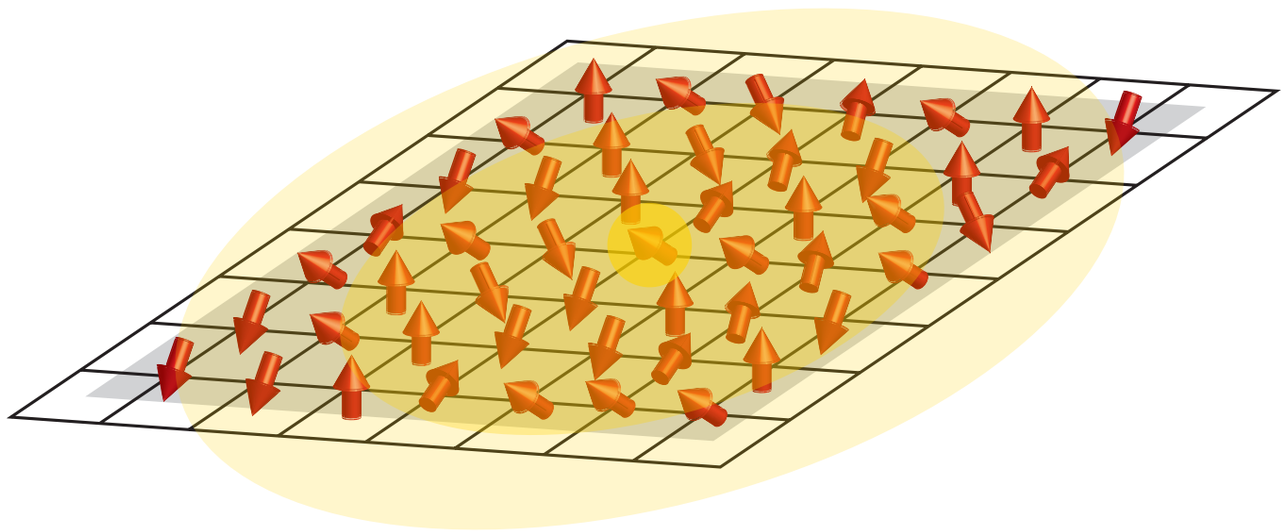
Quantum Information Scrambling

How fast can an initially localized quantum bit become entangled with all degrees of freedom, i.e., scrambled?



Quantum Information Scrambling

How fast can an initially localized quantum bit become entangled with all degrees of freedom, i.e., scrambled?



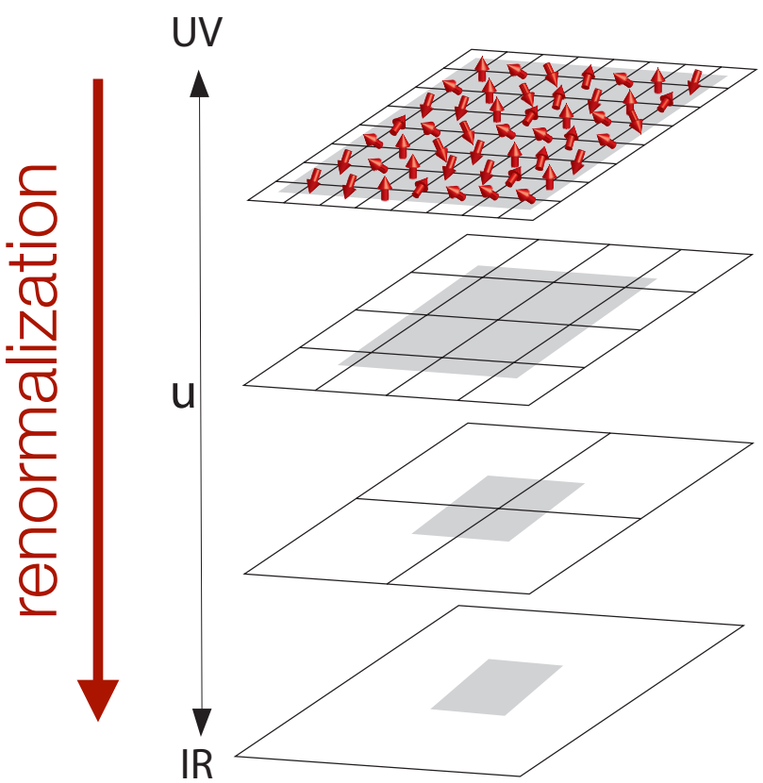
Inspiration: information problem in black holes

Hayden, Preskill, Maldacena, Shenker, Susskind, Stanford …

Gauge/Gravity Duality

Quantum many-body system
d spatial dimensions

Spacetime geometry
d+1 spatial dimensions



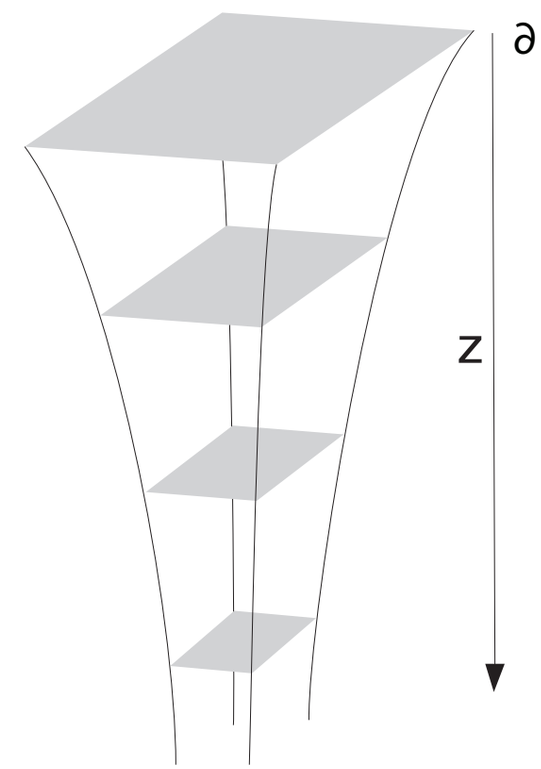
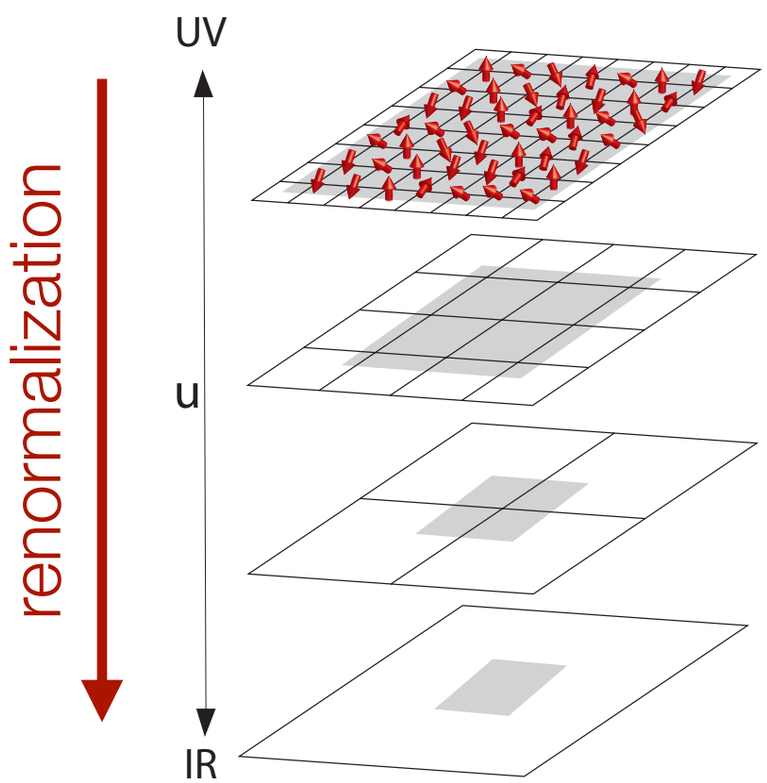


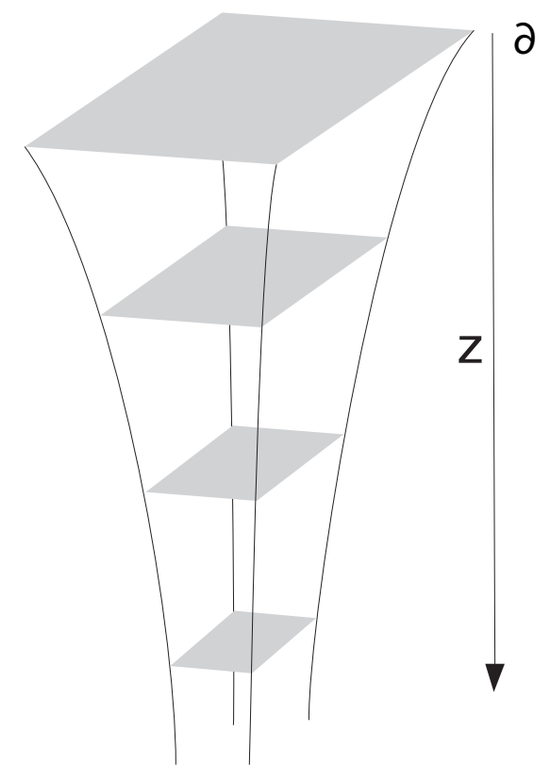
Figure adapted from Ramallo, *arXiv*:1310.4319v3[hep-th].

Gauge/Gravity Duality

Quantum many-body system
d spatial dimensions

Spacetime geometry
d+1 spatial dimensions





Can we realize quantum many-body systems in table-top experiments that are holographically dual to black holes? *How would we know?*

Figure adapted from Ramallo, *arXiv*:1310.4319v3[hep-th].

Fast Scrambling Conjecture

T = Temperature

$\mathbb{S}$ = Entropy

Fast Scrambling Conjecture

Conjecture: black holes are the fastest scramblers in nature

- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_S = \tau \log(\mathbb{S})$

T = Temperature

$\mathbb{S}$ = Entropy

Fast Scrambling Conjecture

Conjecture: black holes are the fastest scramblers in nature

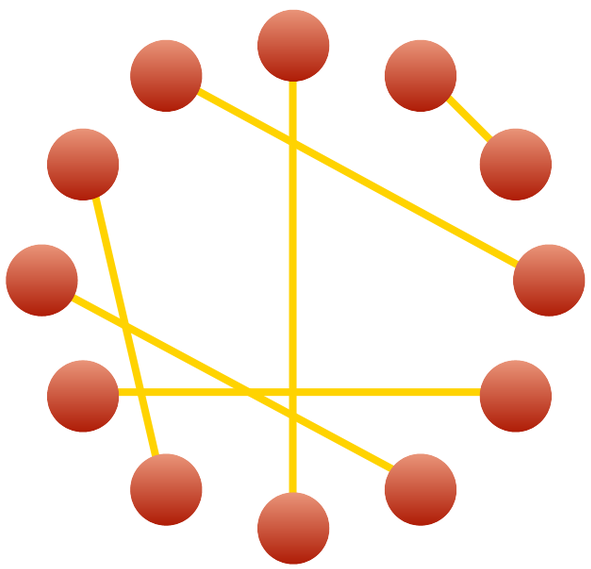
- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_S = \tau \log(\mathbb{S})$

T = Temperature

$\mathbb{S}$ = Entropy

Intuition: random circuit model

- $\mathbb{S}$ = number of qubits
- τ = interaction time
- Time $t_S \gtrsim \tau \log_2(\mathbb{S})$ to connect all pairs



Fast Scrambling Conjecture

Conjecture: black holes are the fastest scramblers in nature

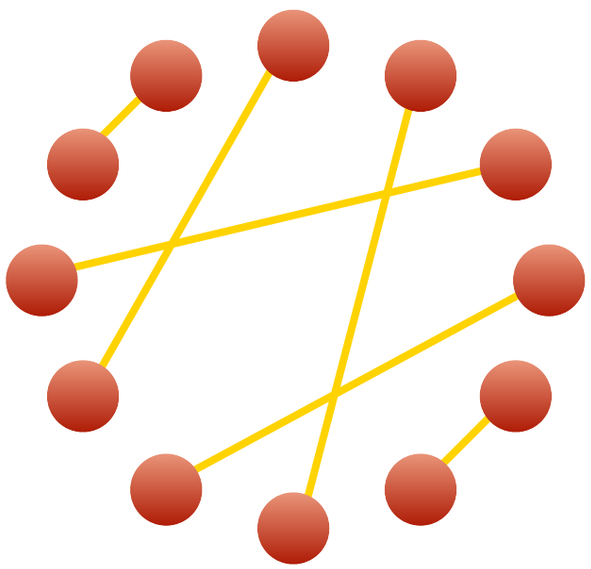
- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_S = \tau \log(\mathbb{S})$

T = Temperature

$\mathbb{S}$ = Entropy

Intuition: random circuit model

- $\mathbb{S}$ = number of qubits
- τ = interaction time
- Time $t_S \gtrsim \tau \log_2(\mathbb{S})$ to connect all pairs



Fast Scrambling Conjecture

Conjecture: black holes are the fastest scramblers in nature

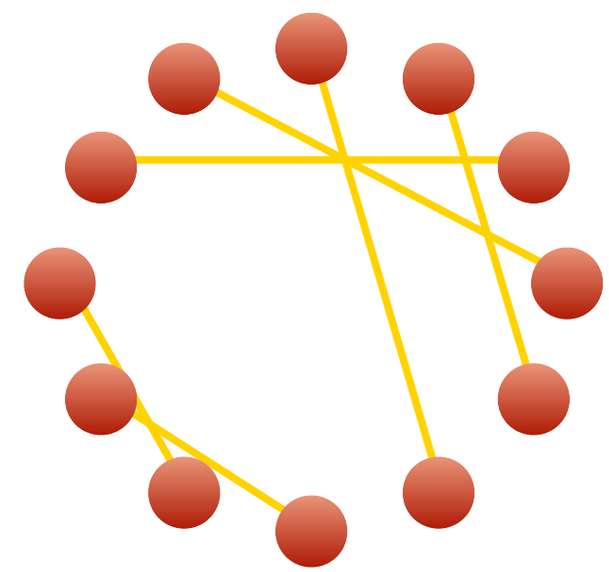
- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_S = \tau \log(\mathbb{S})$

T = Temperature

$\mathbb{S}$ = Entropy

Intuition: random circuit model

- $\mathbb{S}$ = number of qubits
- τ = interaction time
- Time $t_S \gtrsim \tau \log_2(\mathbb{S})$ to connect all pairs



Fast Scrambling Conjecture

Conjecture: black holes are the fastest scramblers in nature

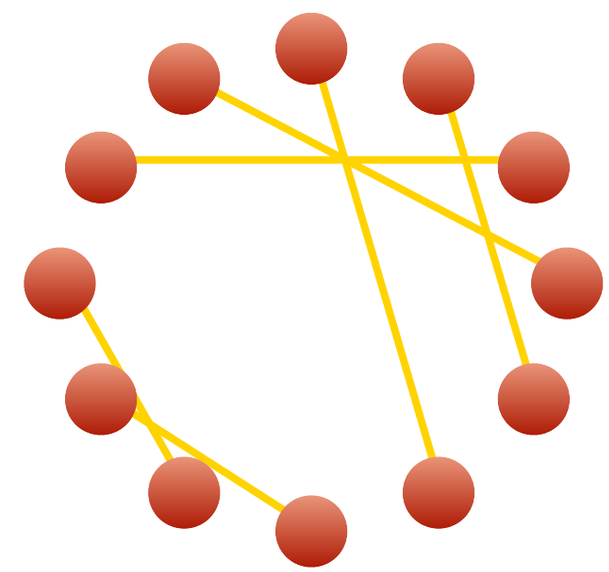
- Relaxation time $\tau = 1 / (2\pi T)$
- Scrambling time $t_S = \tau \log(\mathbb{S})$

T = Temperature

$\mathbb{S}$ = Entropy

Intuition: random circuit model

- $\mathbb{S}$ = number of qubits
- τ = interaction time
- Time $t_S \gtrsim \tau \log_2(\mathbb{S})$ to connect all pairs



Candidates for fast scrambling: non-local, chaotic spin models

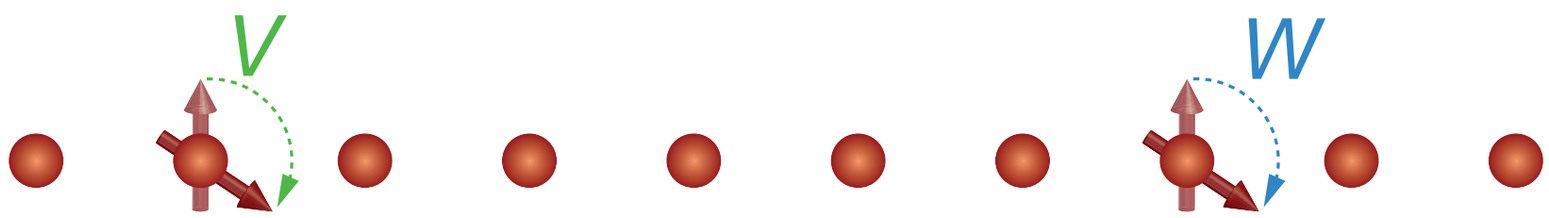
Quantifying Scrambling

How to define chaos in a quantum many-body system?

Quantifying Scrambling

How to define chaos in a quantum many-body system?

Quantum many-body butterfly effect: growth of commutator $[V, W_t]$ between initially commuting operators vs. their separation in time t



How fast does $W_t = e^{-iHt}\, W\, e^{iHt}$ fail to commute with V due to interactions H?

Shenker & Stanford, *JHEP* 2014:067.
Maldacena, Shenker, & Stanford, *JHEP* 2016:106.
Hosur, Qi, Roberts, & Yoshida, *JHEP* 2016:4.

Measuring Fast Scrambling

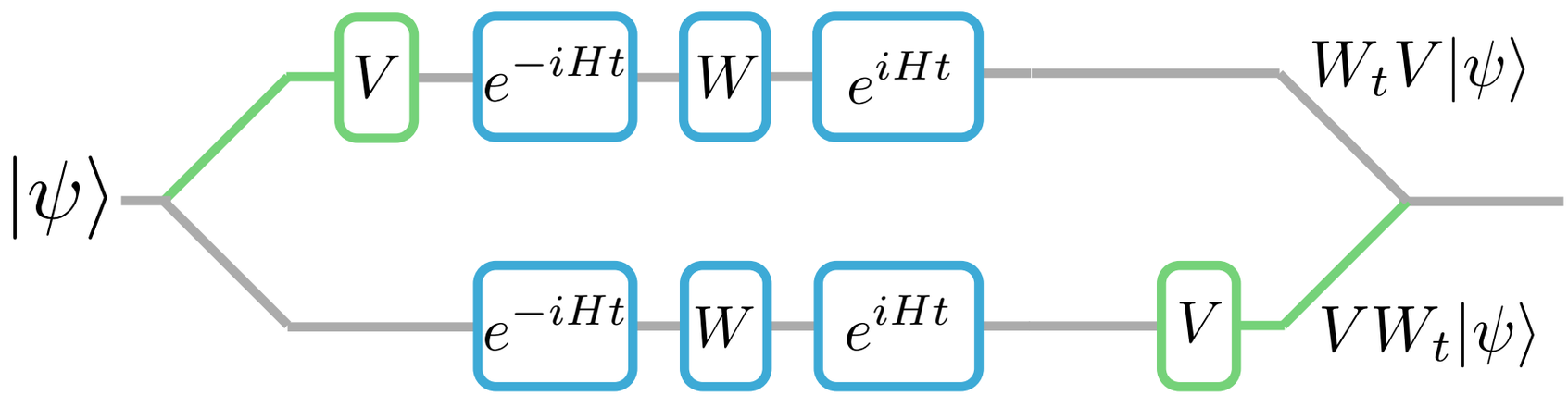
Decay of out-of-time-order correlation function $F(t) \equiv \langle\psi|W_t^\dagger V^\dagger W_t V|\psi\rangle$

indicates growth of commutator: $\mathrm{Re}[F(t)] = 1 - \left\langle |[W_t, V]|^2 \right\rangle /2$

Measuring Fast Scrambling

Decay of out-of-time-order correlation function $F(t) \equiv \langle\psi|W_t^\dagger V^\dagger W_t V|\psi\rangle$

indicates growth of commutator: $\mathrm{Re}[F(t)] = 1 - \left\langle |[W_t, V]|^2 \right\rangle /2$



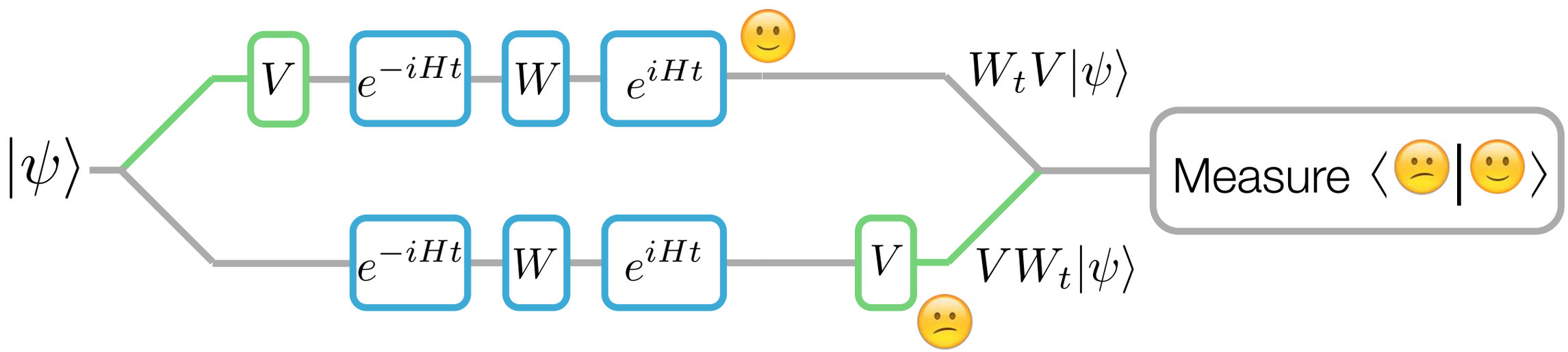
V, W: simple operators, e.g., spin rotations

[*V*,*W*]=0 at *t*=0

Measuring Fast Scrambling

Decay of out-of-time-order correlation function $F(t) \equiv \langle\psi|W_t^\dagger V^\dagger W_t V|\psi\rangle$

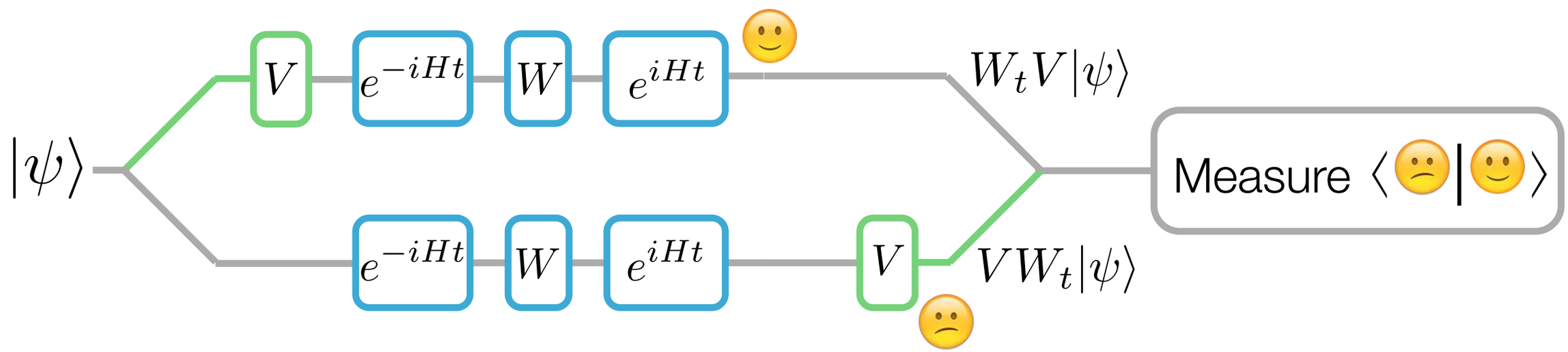
indicates growth of commutator: $\mathrm{Re}[F(t)] = 1 - \left\langle |[W_t, V]|^2 \right\rangle /2$



Measuring Fast Scrambling

Decay of out-of-time-order correlation function $F(t) \equiv \langle\psi|W_t^\dagger V^\dagger W_t V|\psi\rangle$

indicates growth of commutator: $\mathrm{Re}[F(t)] = 1 - \left\langle |[W_t, V]|^2 \right\rangle /2$



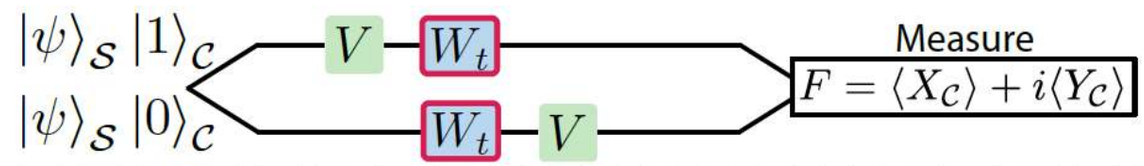
Tools for measuring *F*

- Time reversal ($H \rightarrow -H$)
- Many-body interferometry

Measuring Out-of-Time-Order Correlations

Many-body interferometry

…with control qubit



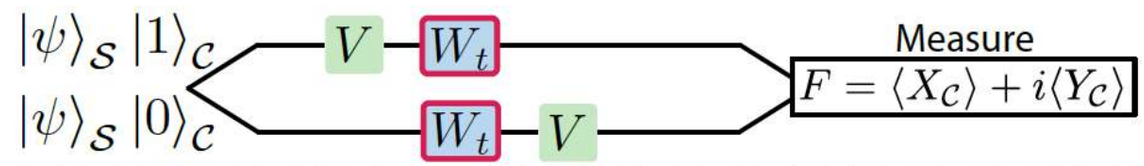
B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

…with 2 copies of same system ····· Yao, Grusdt, Swingle, …, & Demler, *arXiv:*1607.01801.

Measuring Out-of-Time-Order Correlations

Many-body interferometry

…with control qubit

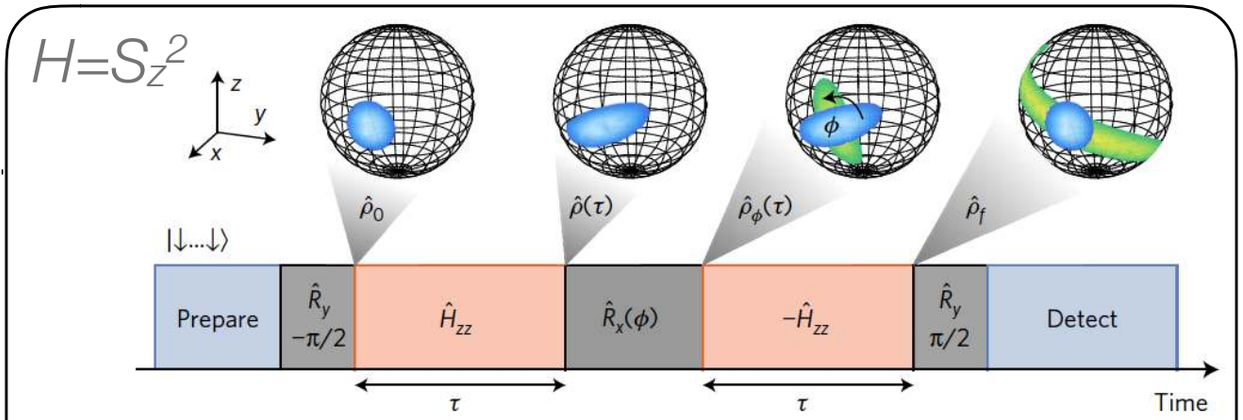


B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

…with 2 copies of same system ····· Yao, Grusdt, Swingle, …, & Demler, *arXiv:*1607.01801.

Echo spectroscopy

…for simple initial states



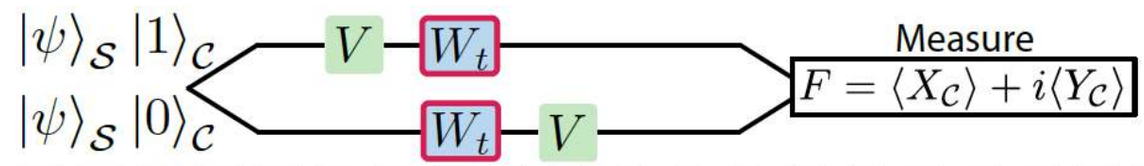
Gärttner, Bohnet, Safavi-Naini, Wall, Bollinger, & Rey, *Nat. Phys.* (2017).

Wei, Ramanathan & Cappellaro, *arXiv* (2016).
Li et al. *arXiv* (2016).

Measuring Out-of-Time-Order Correlations

Many-body interferometry

…with control qubit

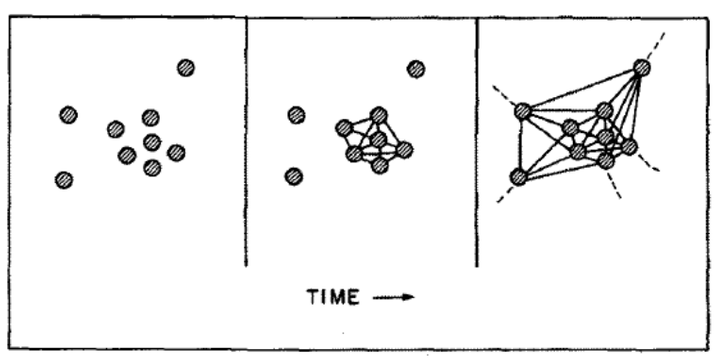


B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

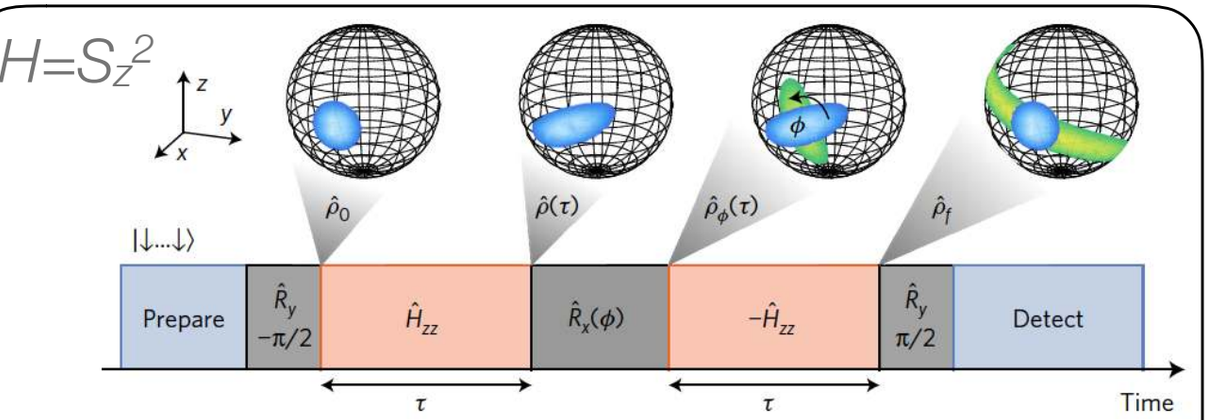
…with 2 copies of same system ····· Yao, Grusdt, Swingle, …, & Demler, *arXiv:*1607.01801.

Echo spectroscopy

…for simple initial states



Baum, J., … & Pines, A., *J. Chem. Phys.* **(1985).**



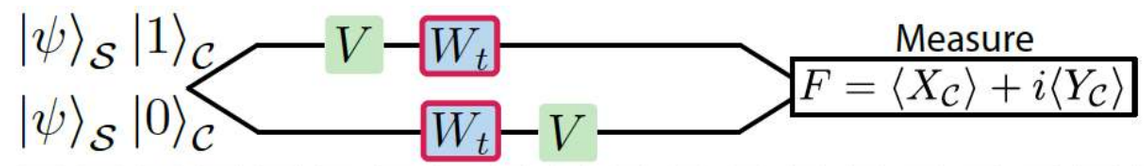
Gärttner, Bohnet, Safavi-Naini, Wall, Bollinger, & Rey, *Nat. Phys.* (2017).

Wei, Ramanathan & Cappellaro, *arXiv* (2016).
Li et al. *arXiv* (2016).

Measuring Out-of-Time-Order Correlations

Many-body interferometry

…with control qubit

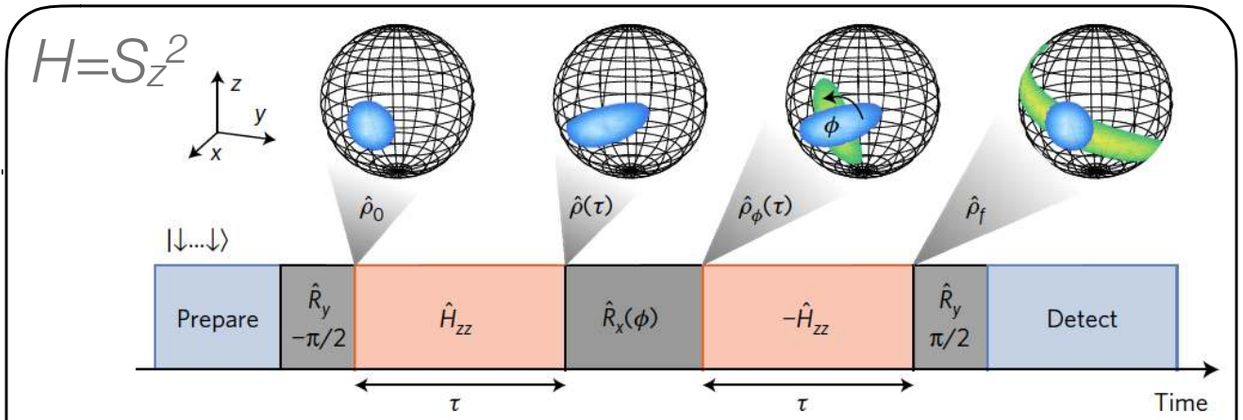


B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

…with 2 copies of same system ····· Yao, Grusdt, Swingle, …, & Demler, *arXiv:*1607.01801.

Echo spectroscopy

…for simple initial states



Gärttner, Bohnet, Safavi-Naini, Wall, Bollinger, & Rey, *Nat. Phys.* (2017).

What do we expect to measure in a chaotic system?

Wei, Ramanathan & Cappellaro, *arXiv* (2016).
Li et al. *arXiv* (2016).

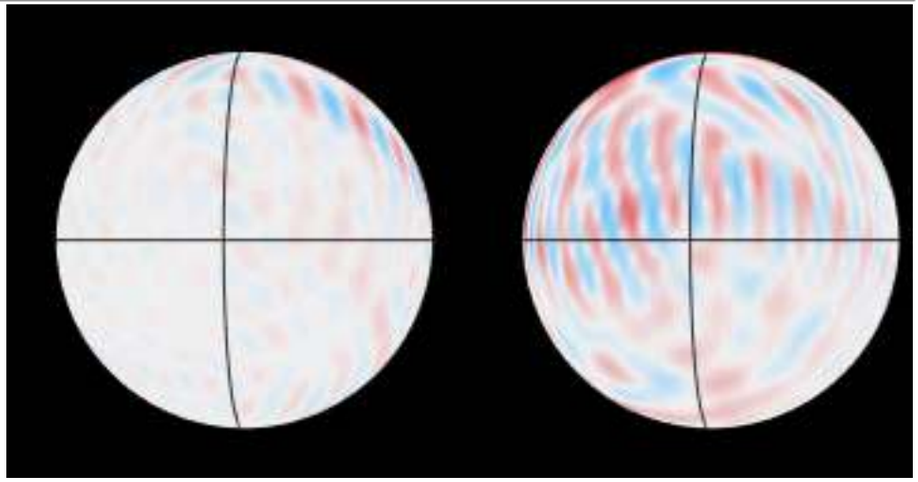
Chaotic Kicked Top

Haake, *Z. Phys. B* (1987).

kick

precession

$$H = \frac{k S_x^2}{2S} \sum_{n=-\infty}^{\infty} \delta(t - n\tau) + \frac{p}{\tau} S_z$$



$N = 2S = 30$, k=3, p=π/2

- Expect $n_{\text{chaos}} \sim \log N$ kicks for initial state of solid angle ~1/N to spread over the entire N-atom Bloch sphere
- Probe with rotations $V = W = e^{i\phi S_z}$ by small angle $\phi = 1/\sqrt{N}$

C.f. kicked rotor: Rozenbaum, Ganeshan & Galitski, *PRL* **118**, 086801 (2017).

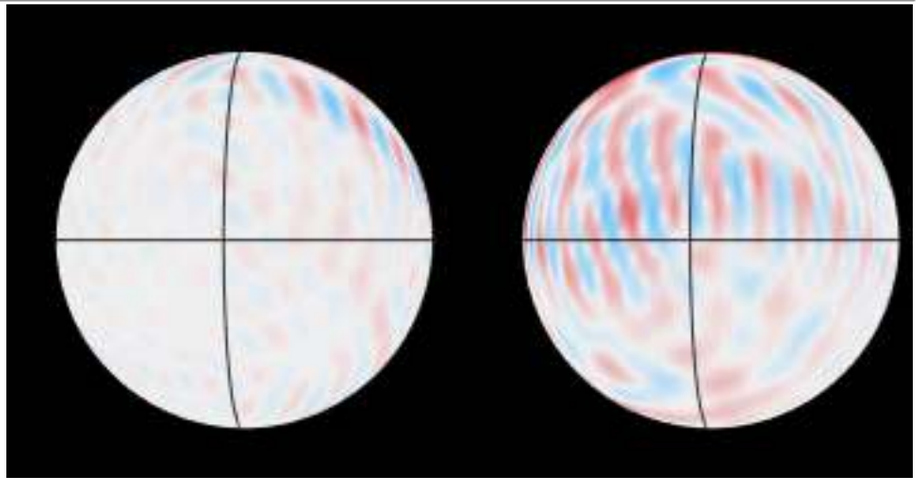
Chaotic Kicked Top

Haake, *Z. Phys. B* (1987).

kick

precession

$$H = \frac{k S_x^2}{2S} \sum_{n=-\infty}^{\infty} \delta(t - n\tau) + \frac{p}{\tau} S_z$$



$N = 2S = 30$, k=3, p=π/2

- Expect $n_{\text{chaos}} \sim \log N$ kicks for initial state of solid angle ~1/N to spread over the entire N-atom Bloch sphere
- Probe with rotations $V = W = e^{i\phi S_z}$ by small angle $\phi = 1/\sqrt{N}$

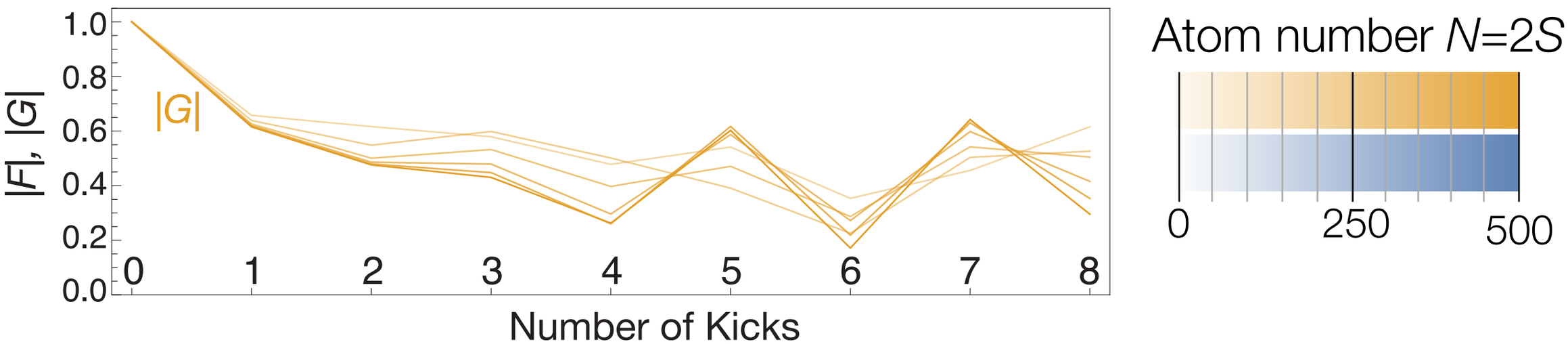
C.f. kicked rotor: Rozenbaum, Ganeshan & Galitski, *PRL* **118**, 086801 (2017).

Scrambling of a Kicked Top

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$

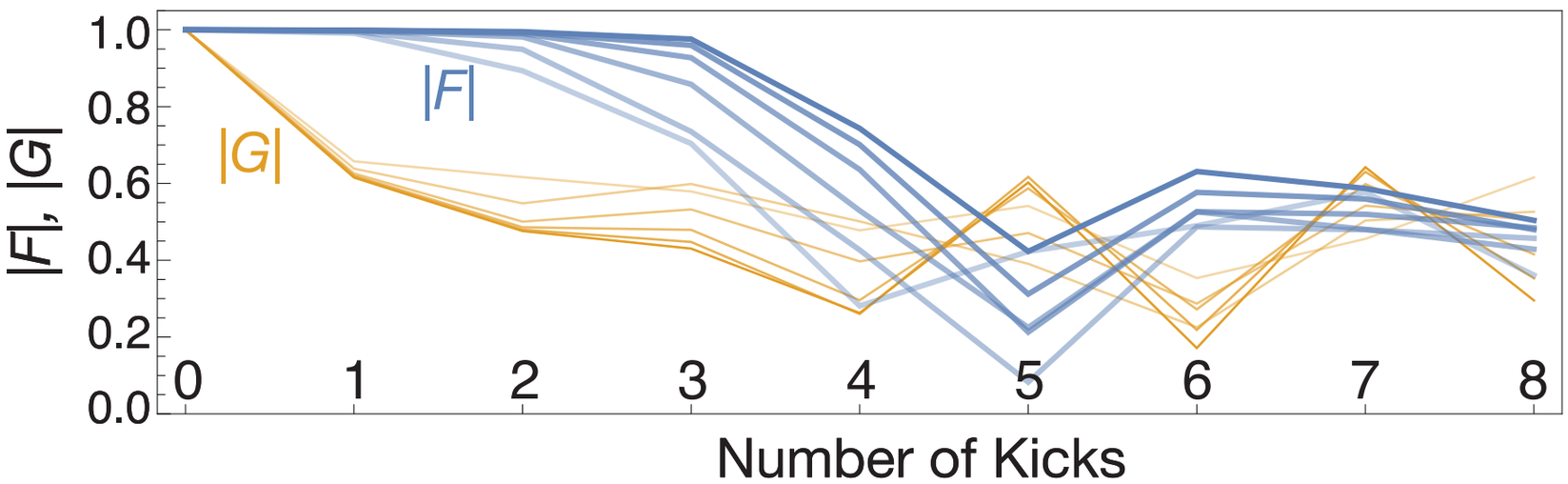


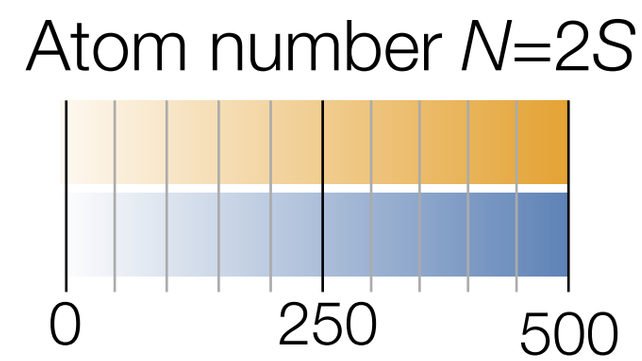
Scrambling of a Kicked Top

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$



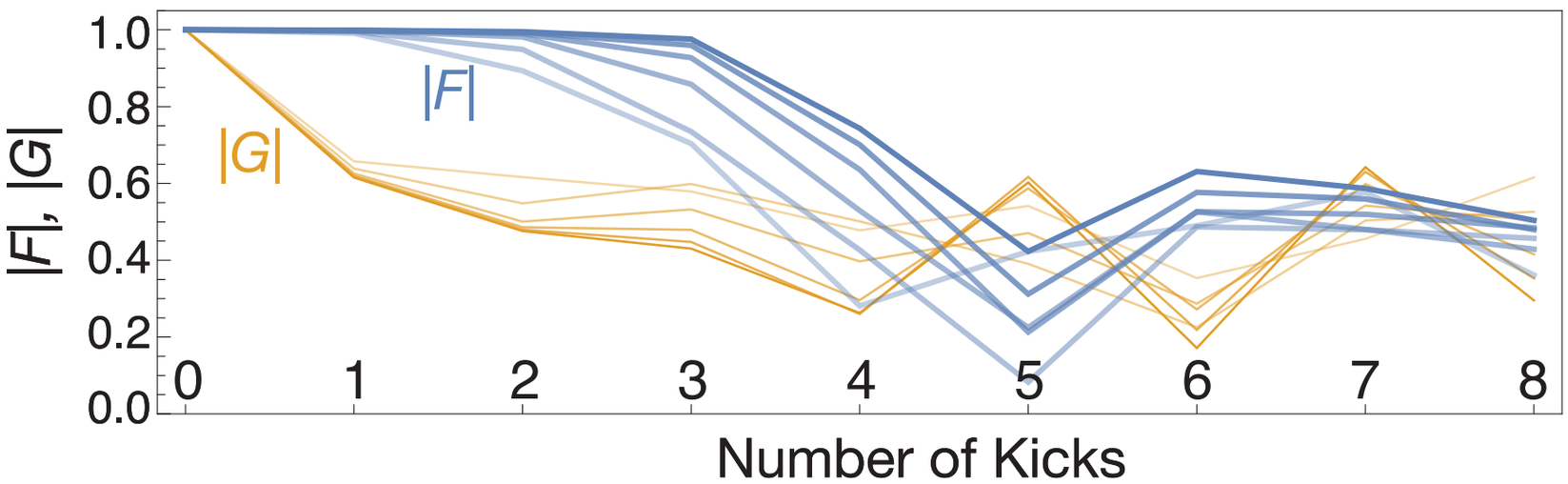


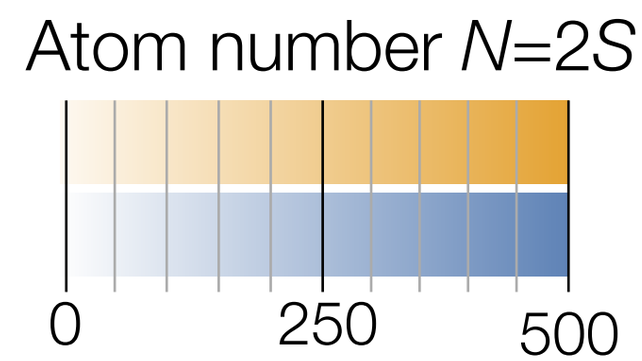
Scrambling of a Kicked Top

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$





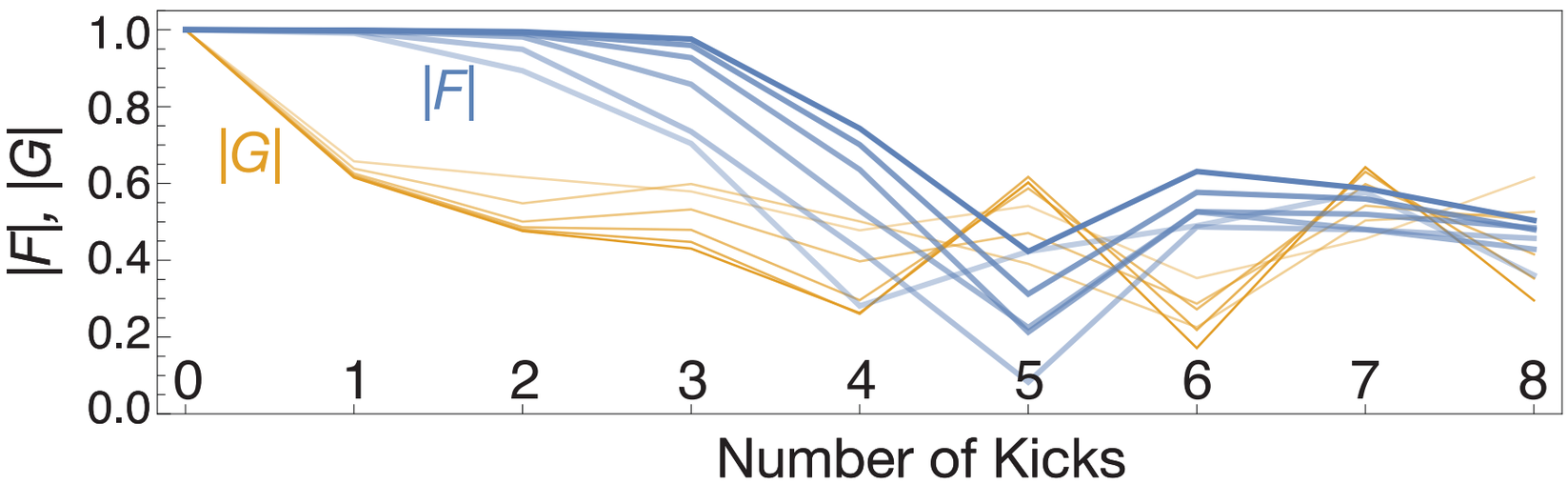
- Scrambling time grows as $t_S \sim \log(N)$ ↔ butterfly effect on Bloch sphere

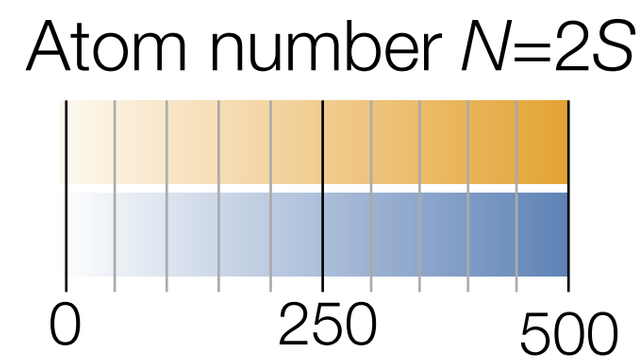
Scrambling of a Kicked Top

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$



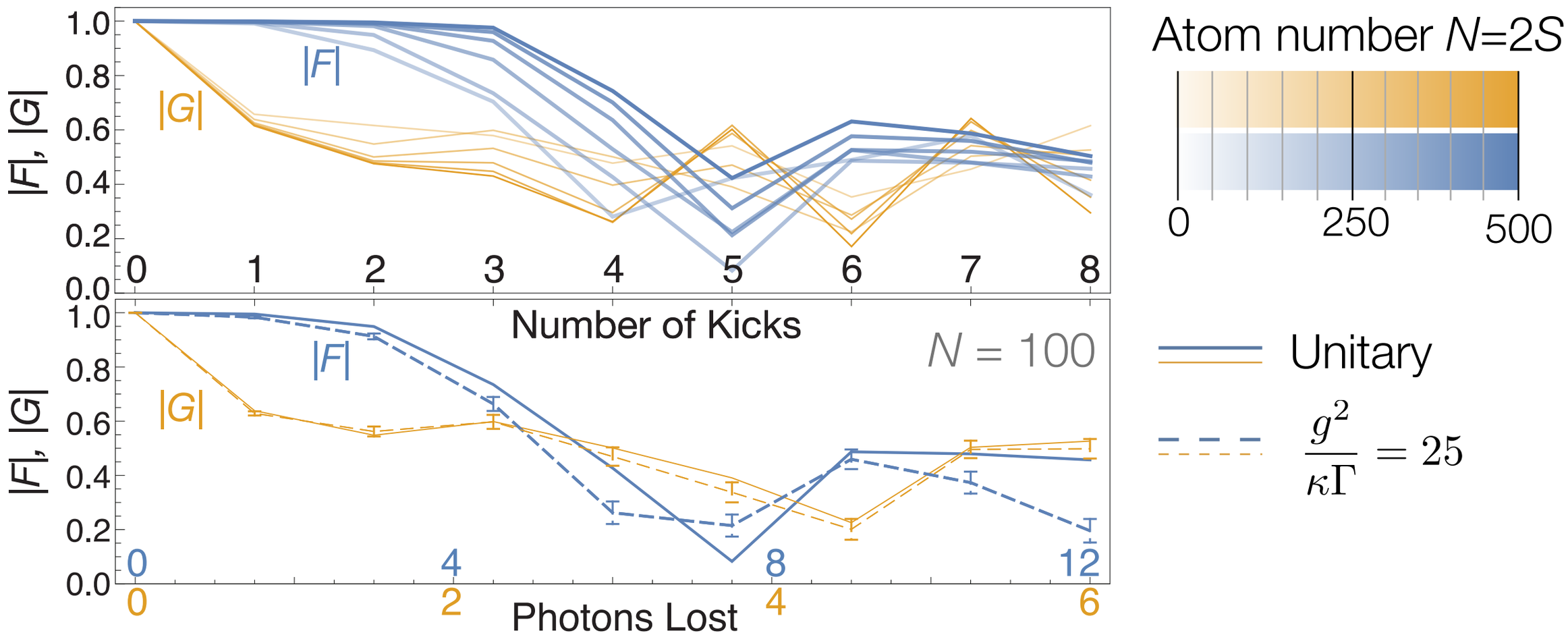


- Scrambling time grows as $t_S \sim \log(N)$ ↔ butterfly effect on Bloch sphere
- Observing with realistic dissipation?

Scrambling of a Kicked Top

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

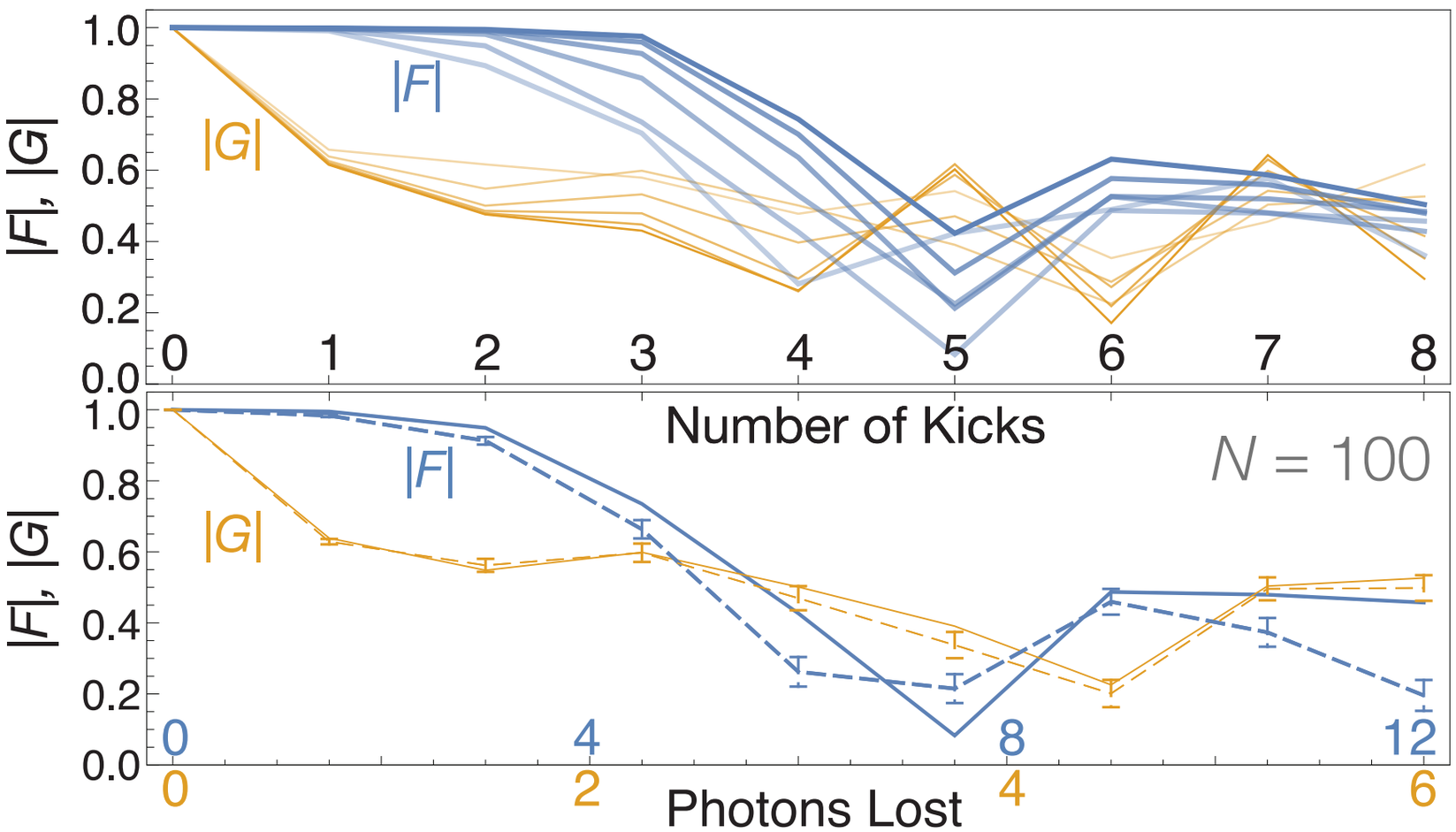
Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$

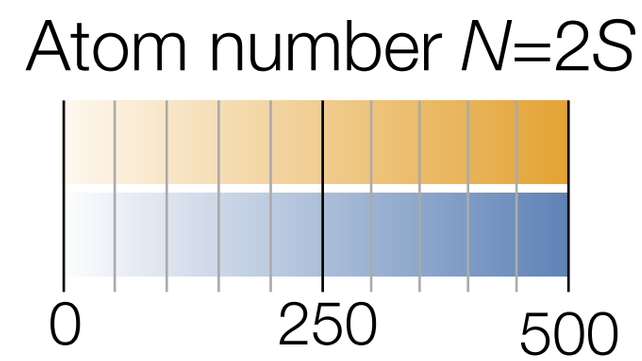


Scrambling of a Kicked Top

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$





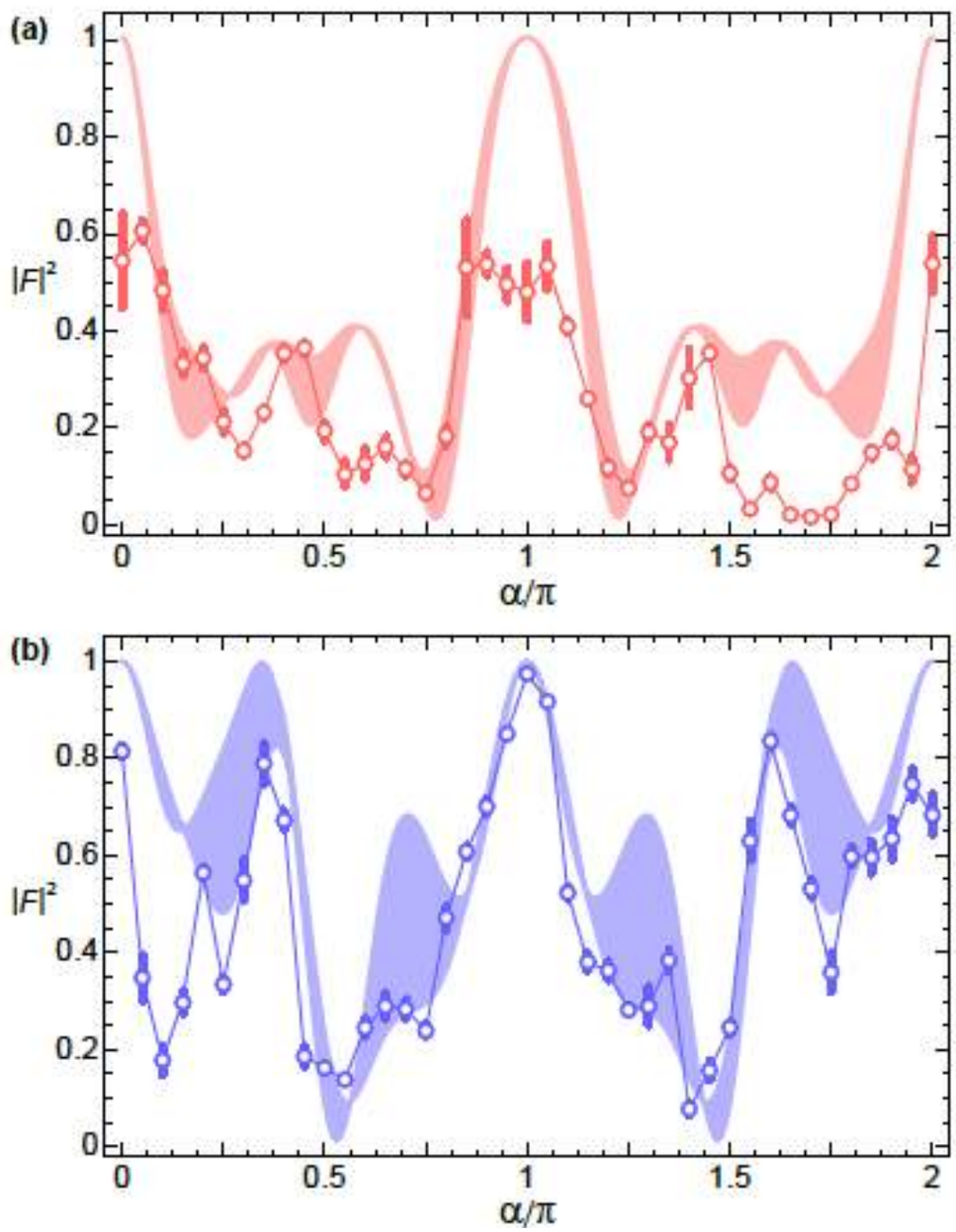
Unitary

$\frac{g^2}{\kappa\Gamma} = 25$

More dissipation?
Hard to calculate!

Experiment: Scrambling in a Kicked Top

Kicked top with pseudo-spin $J = 5$: "spin" states = momentum states of BEC



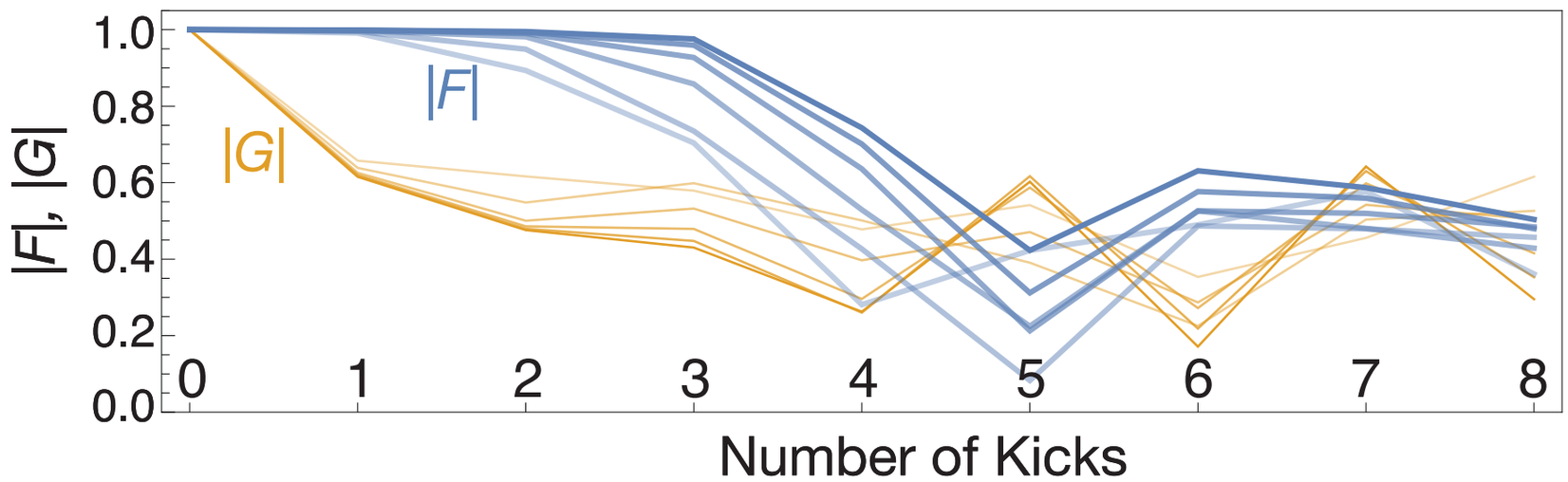
Meier, Ang'ong'a, An, & Gadway, *arXiv*:1705.06714 (2017).

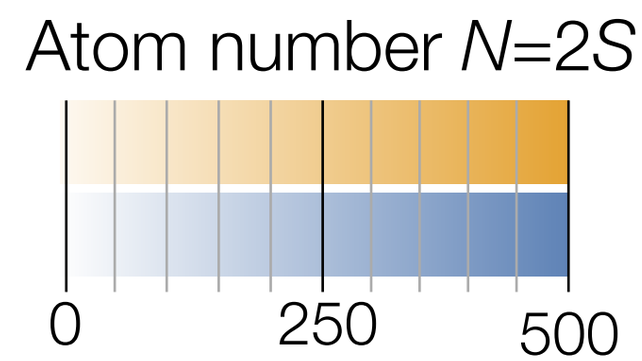
Fast Scrambling?

B. Swingle, G. Bentsen, MS-S, & P. Hayden,
PRA 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$



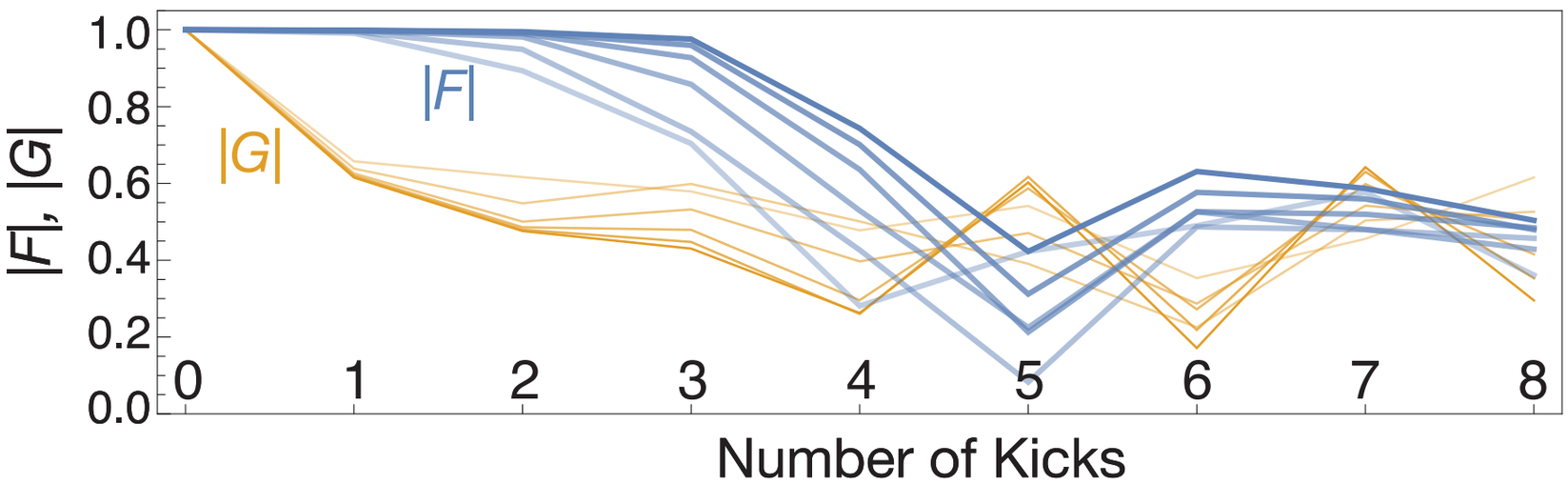


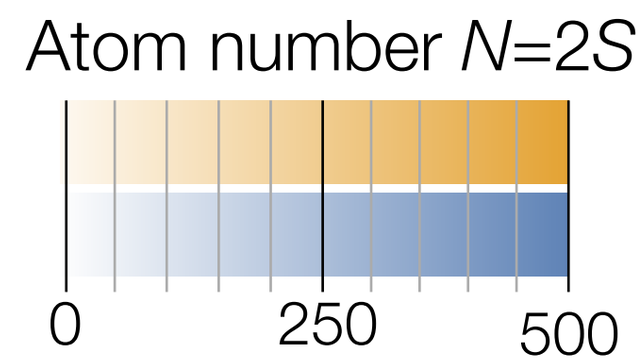
Fast Scrambling?

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$





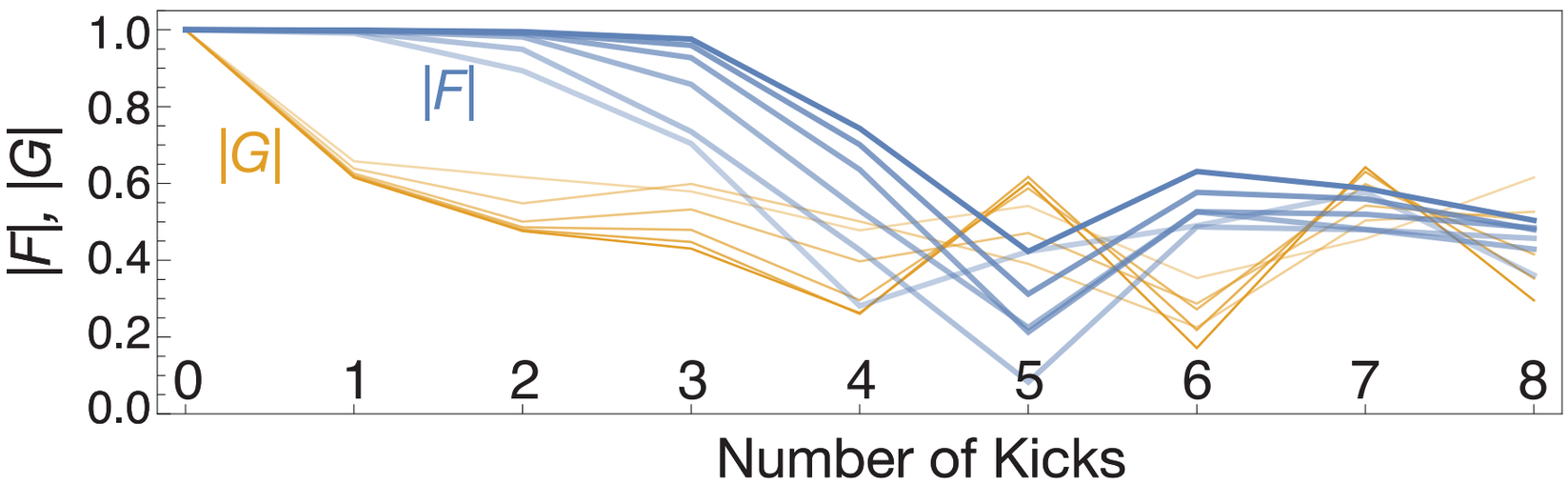
- Scrambling time grows as $t_S \sim \log(N)$ ↔ butterfly effect on Bloch sphere
- Is this a fast scrambler?

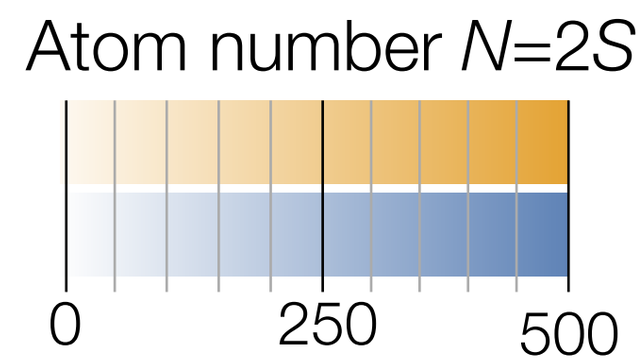
Fast Scrambling?

B. Swingle, G. Bentsen, MS-S, & P. Hayden, *PRA* 040302(R) 2016.

Reference: time-ordered correlation function $G = \langle V_t^\dagger V \rangle$

Scrambling: out-of-time-order correlation $F = \langle W_t^\dagger V^\dagger W_t V \rangle$

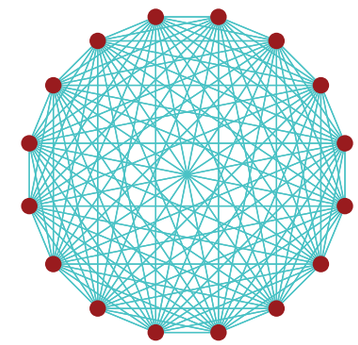




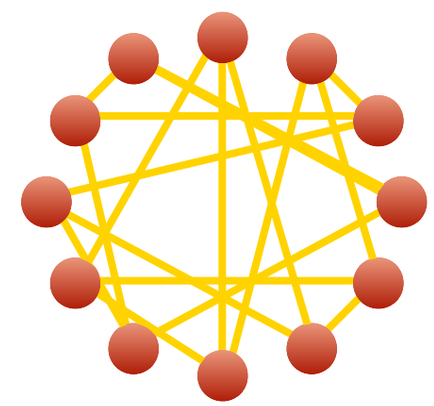
- Scrambling time grows as $t_S \sim \log(N)$ ↔ butterfly effect on Bloch sphere
- Is this a fast scrambler? ✘

Engineering Fast Scrambling?

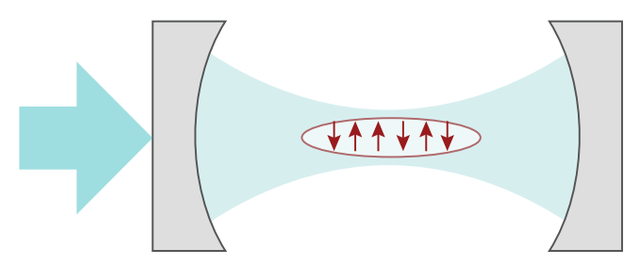
All-to-all interactions restrict us to “single-particle” physics…



…but more complex long-range interactions should allow information to spread fast…

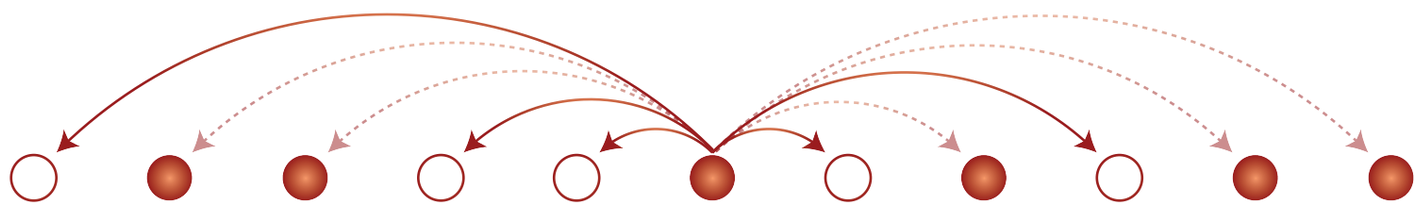


Can a single mode of light mediate more complex interactions?



Exotic XY Models

Photon-mediated interactions for versatile control of of long-range “hopping”:

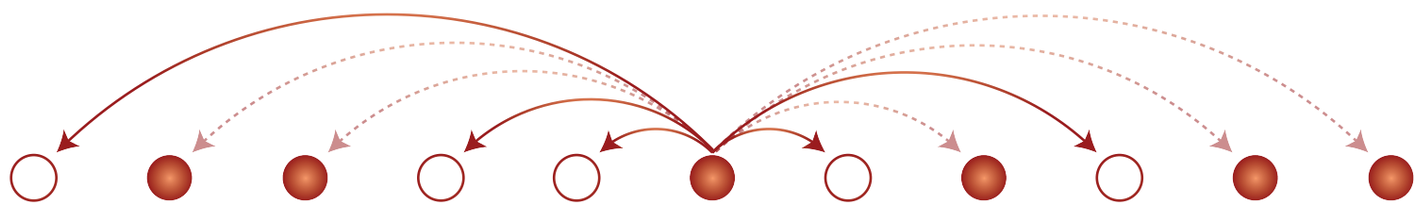


$$H_I = \sum_{i>j} J(i-j)\sigma_+^i\sigma_-^j + \text{h.c.}$$

hard-core bosons = spin excitations: ○ = ⇩; ● = ⬆

Exotic XY Models

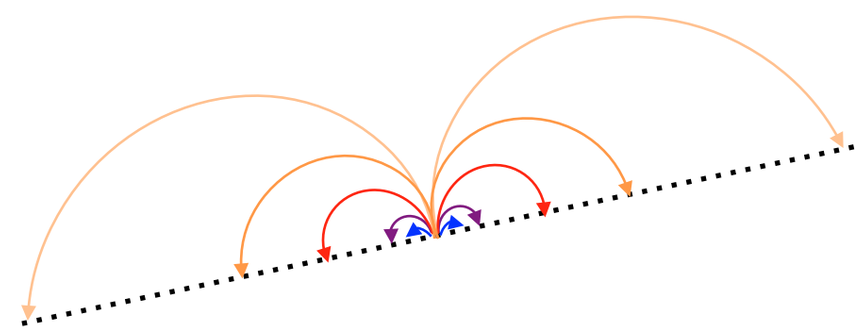
Photon-mediated interactions for versatile control of of long-range “hopping”:



$$H_I = \sum_{i>j} J(i-j)\sigma_+^i\sigma_-^j + \text{h.c.}$$

hard-core bosons = spin excitations: ○ = ⇩; ● = ⬆

Approach:



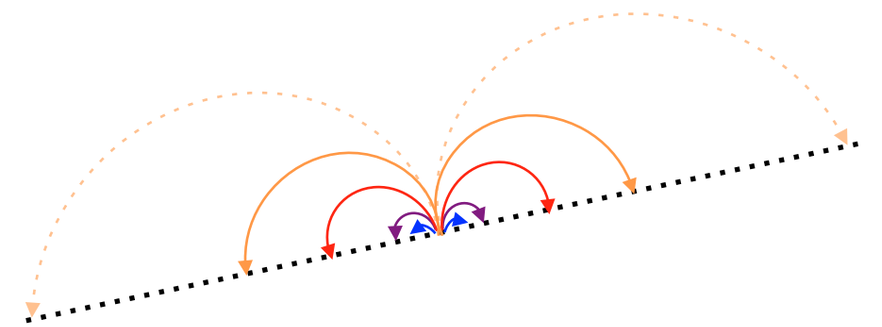
- Suppress hopping with magnetic field gradient
- Restore hopping at *arbitrary* distances *i-j* with modulated control field
- Single-particle dispersion relation = modulation waveform

Hung, Gonzales-Tudela, Cirac & Kimble, *PNAS* (2016).

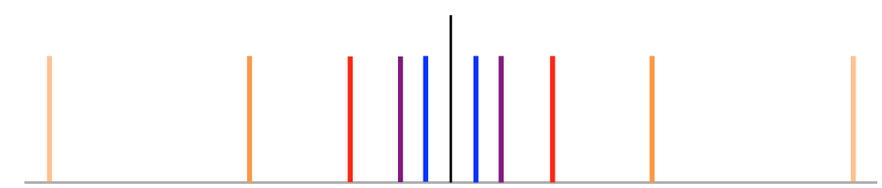
Dispersion Engineering

Efficiently spread information over long distances

by coupling i^{th} spin to $i\pm1$, $i\pm2$, $i\pm4$, $i\pm8$,…, $i\pm2^l$



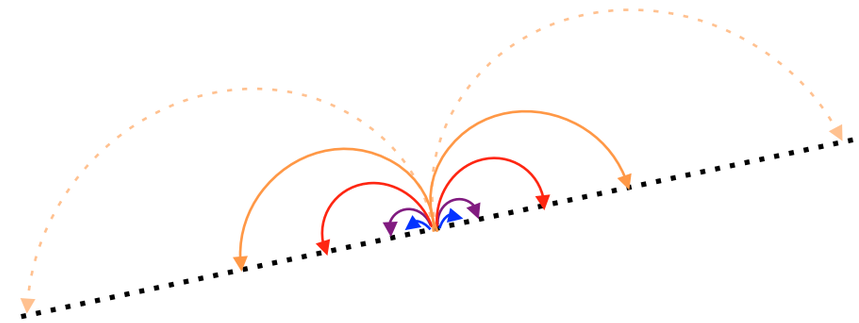
control field spectrum

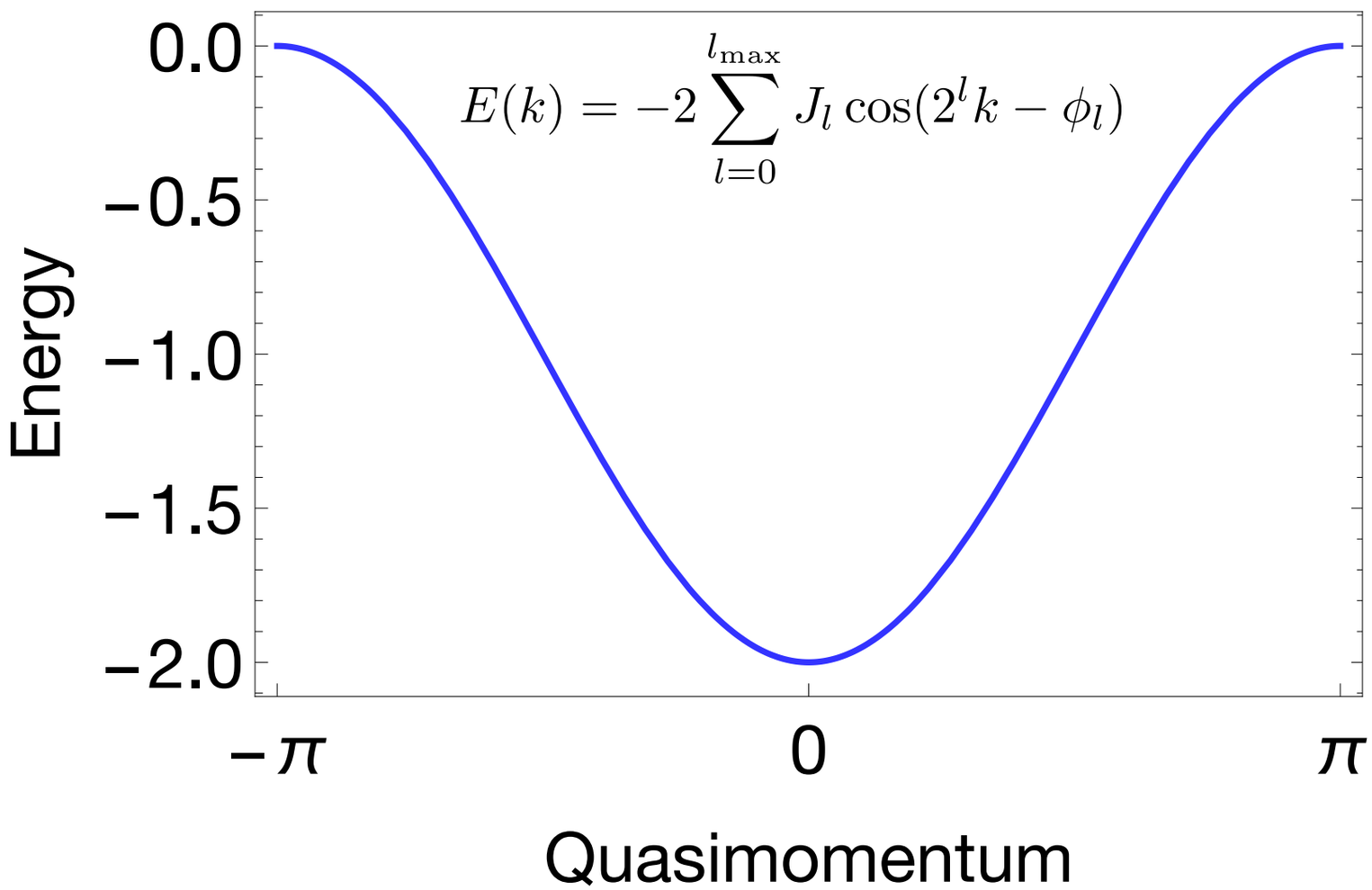


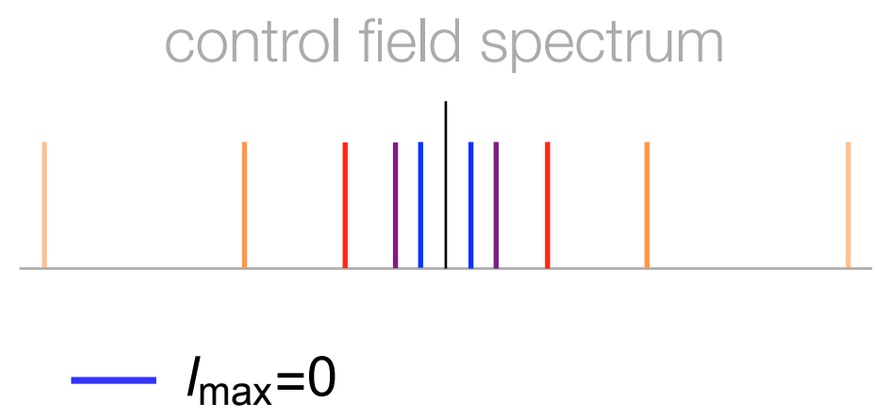
Dispersion Engineering

Efficiently spread information over long distances

by coupling i^{th} spin to $i\pm1$, $i\pm2$, $i\pm4$, $i\pm8$,…, $i\pm2^l$



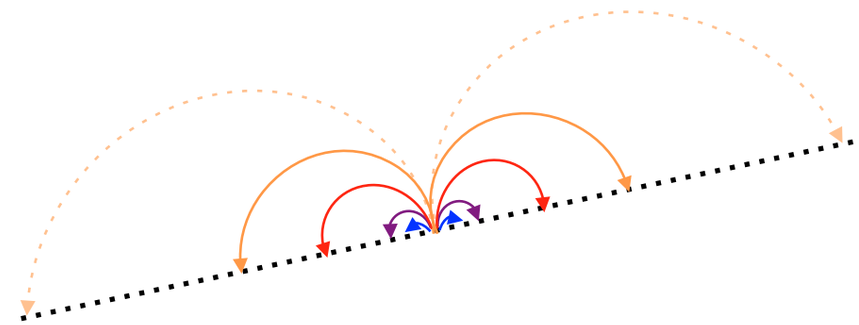


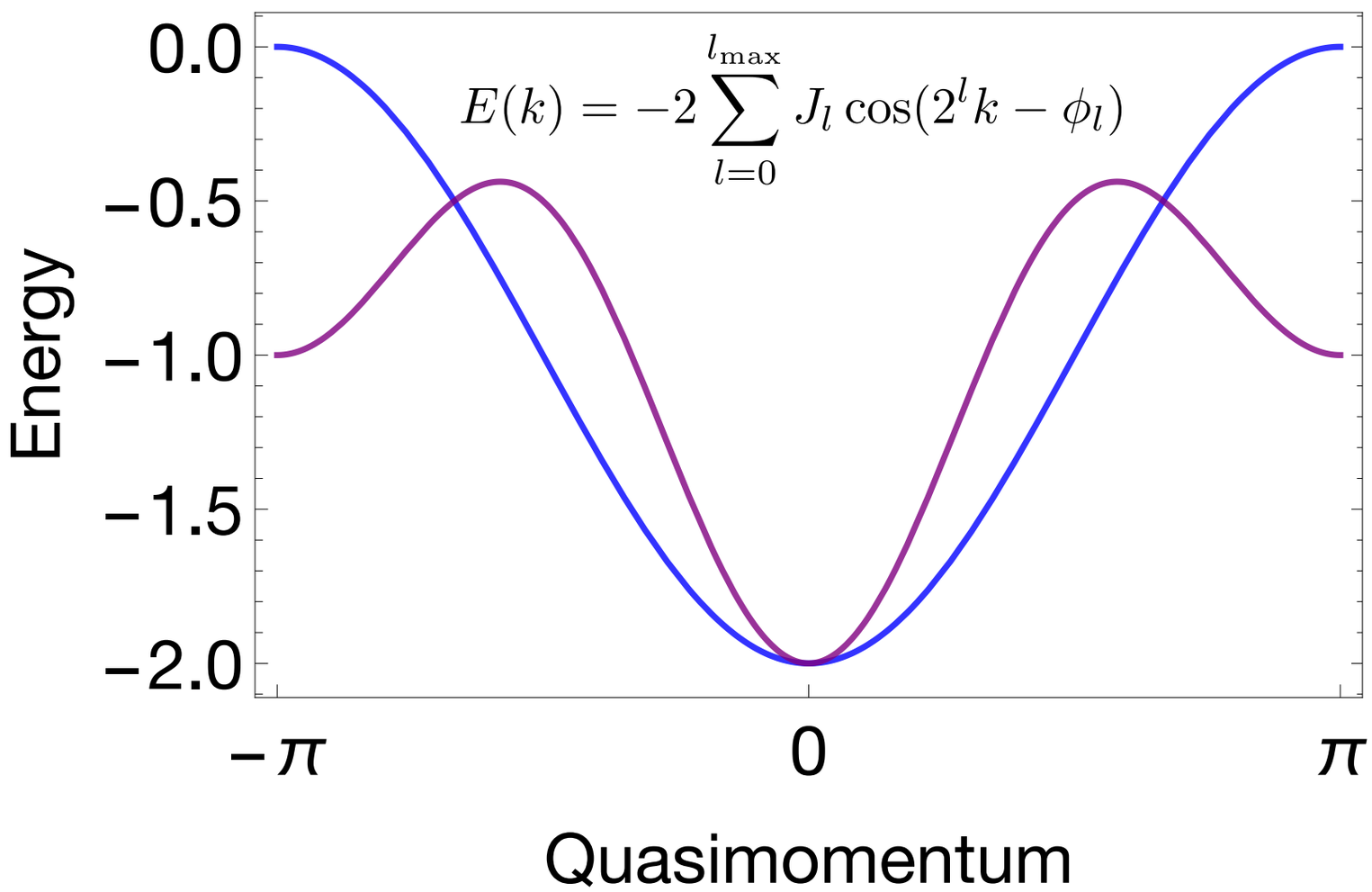


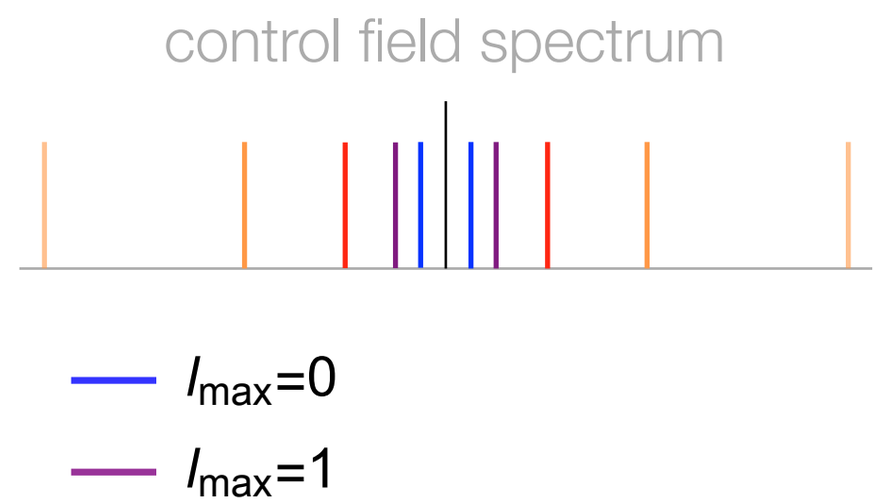
Dispersion Engineering

Efficiently spread information over long distances

by coupling i^{th} spin to $i\pm1$, $i\pm2$, $i\pm4$, $i\pm8$,…, $i\pm2^l$



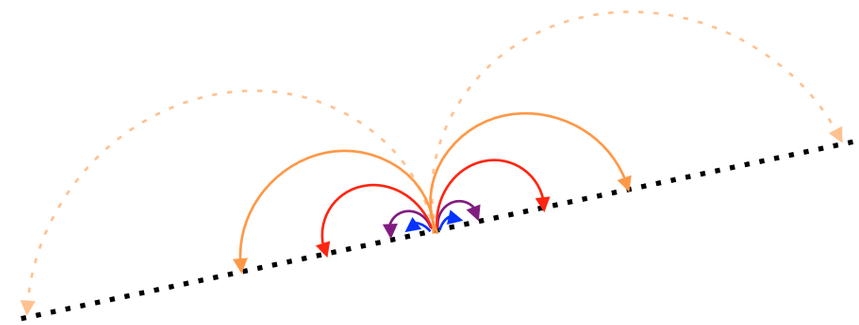


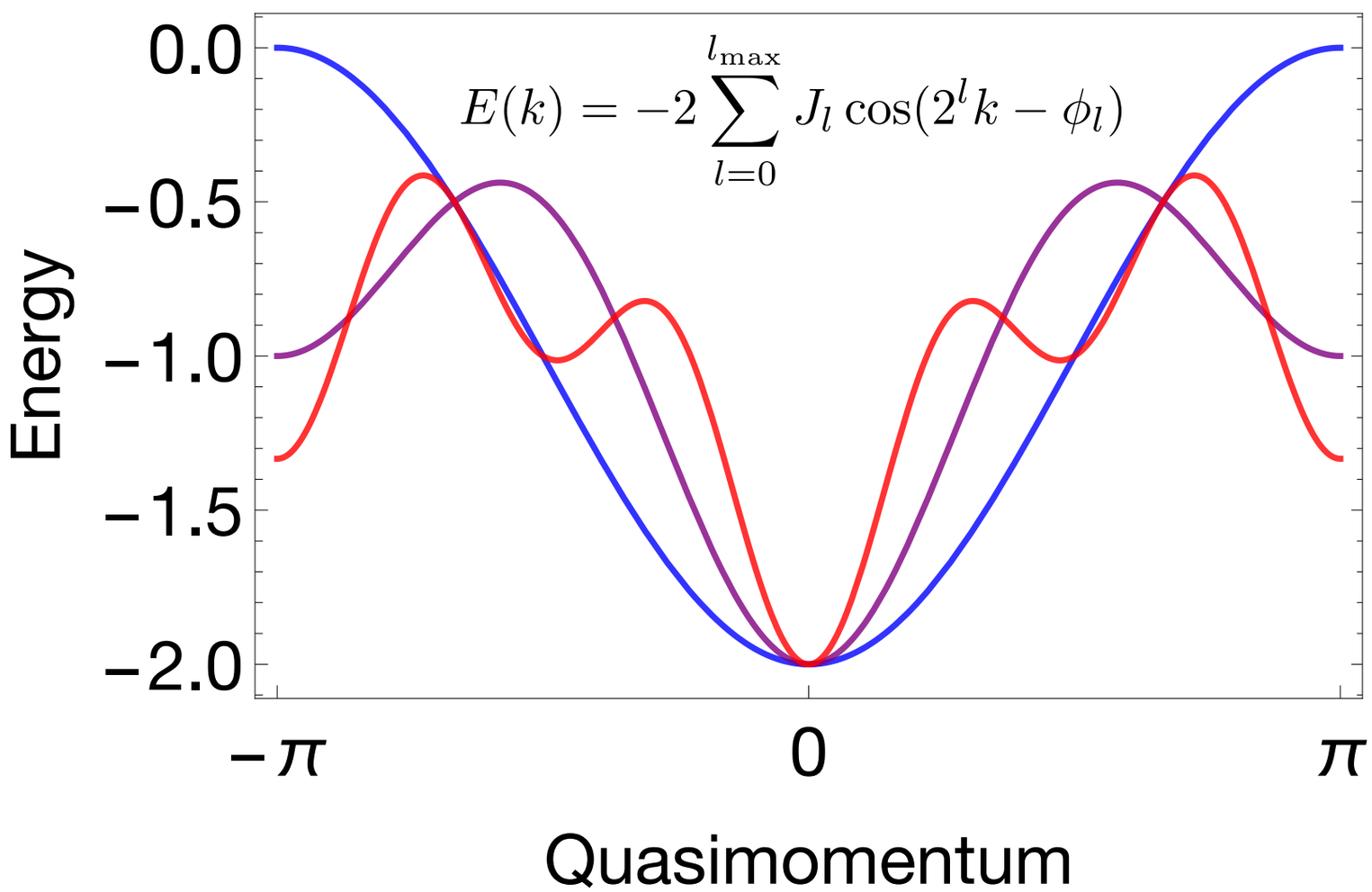


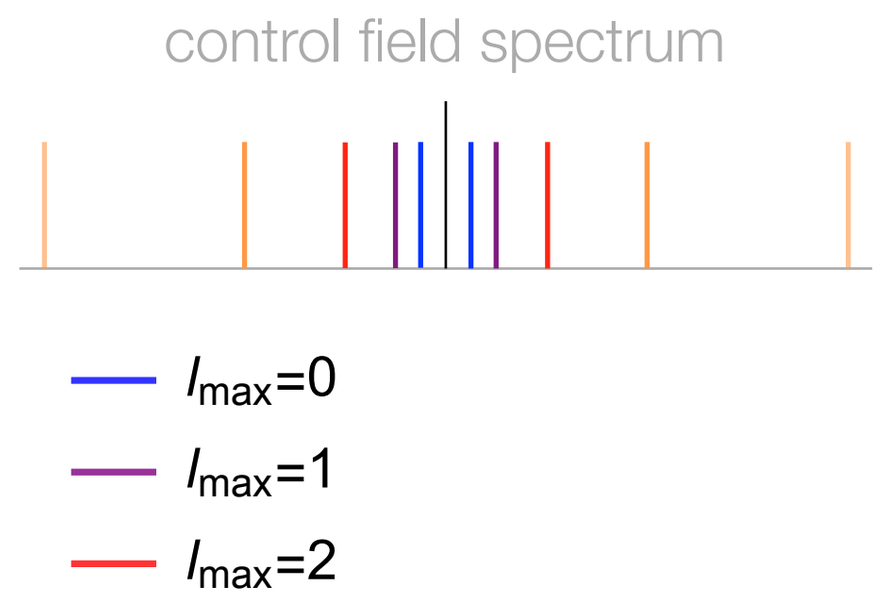
Dispersion Engineering

Efficiently spread information over long distances

by coupling i^{th} spin to $i\pm1$, $i\pm2$, $i\pm4$, $i\pm8$,…, $i\pm2^l$



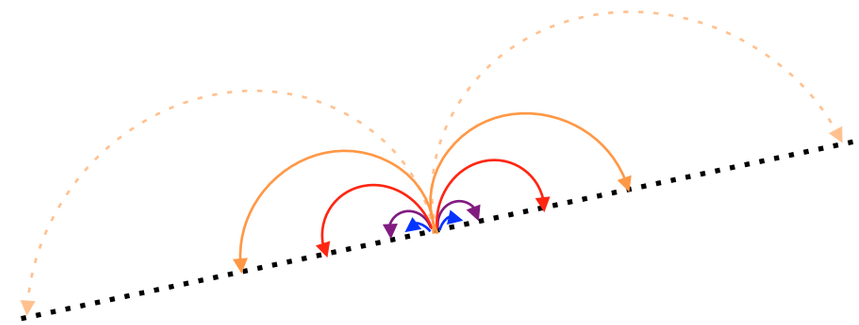


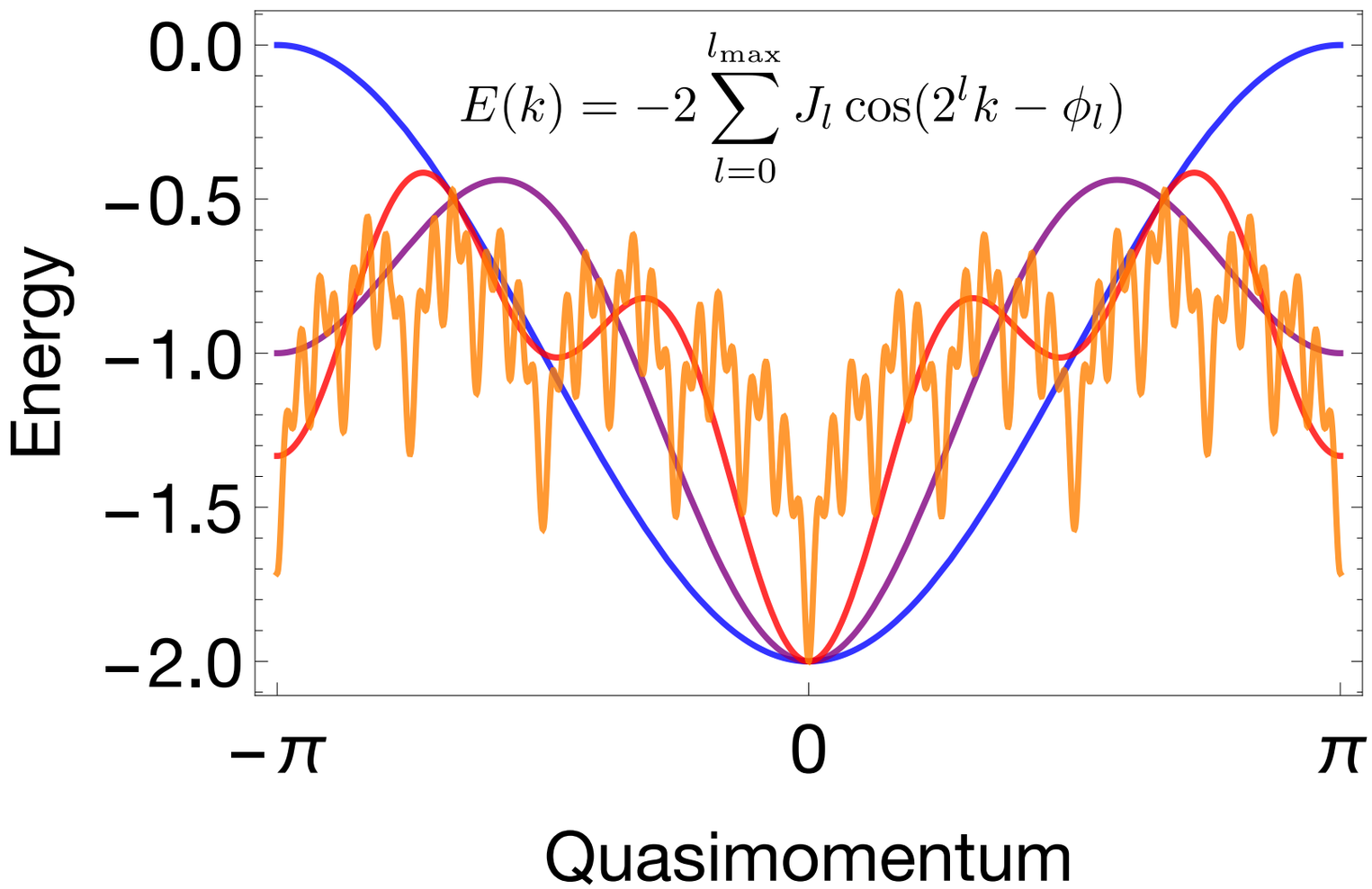


Dispersion Engineering

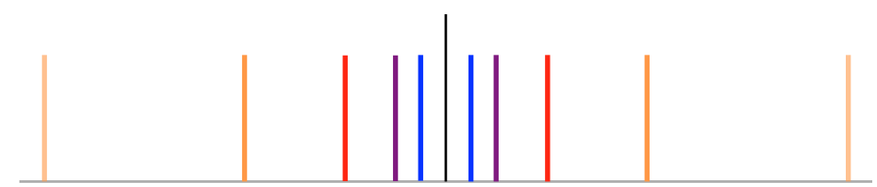
Efficiently spread information over long distances

by coupling i^{th} spin to $i\pm1$, $i\pm2$, $i\pm4$, $i\pm8$,…, $i\pm2^l$





control field spectrum



$l_{\max}$=0

$l_{\max}$=1

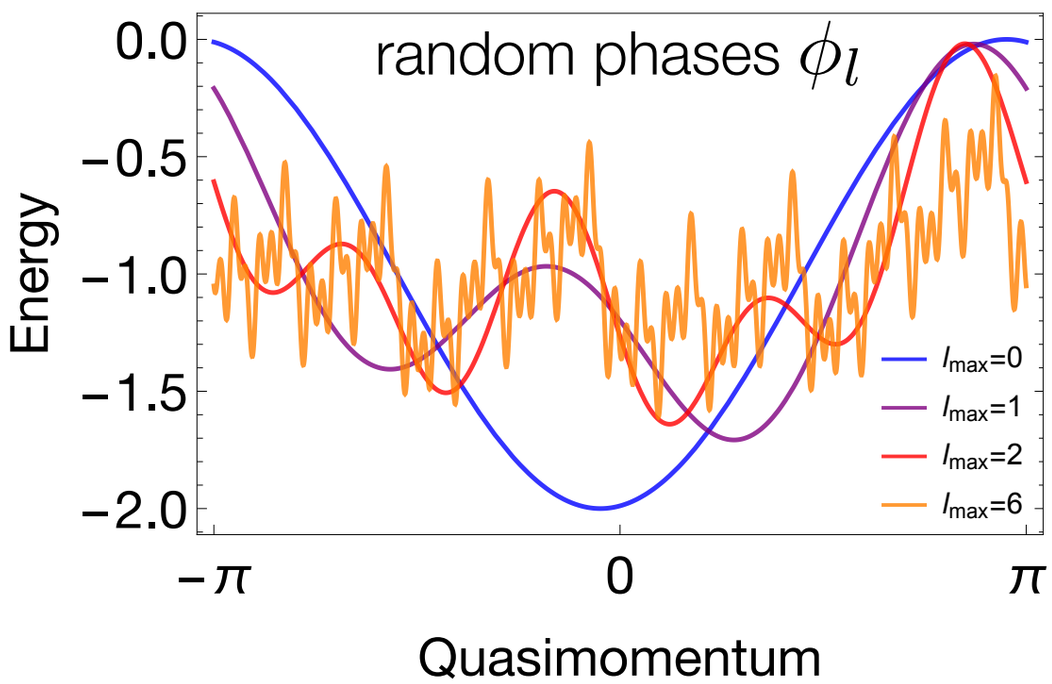
$l_{\max}$=2

$l_{\max}$=6

Dispersion relation is a fractal!

“Chaotic” dispersion?

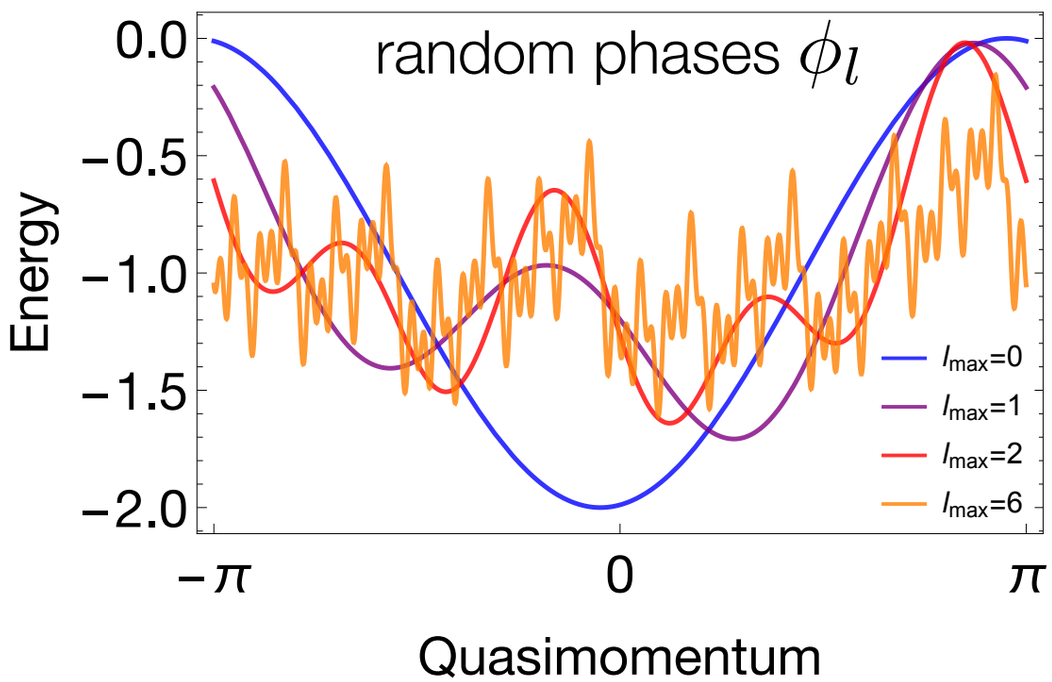
$$E(k) = -2\sum_{l=0}^{l_{\max}} J_l \cos(2^l k - \phi_l)$$



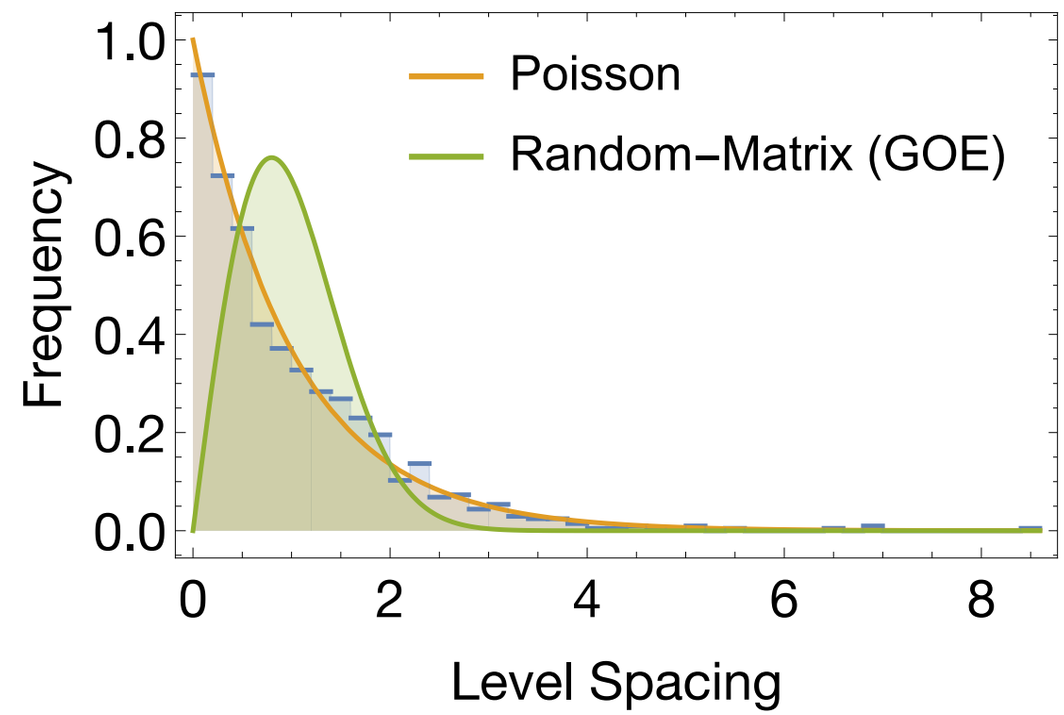
Looks crazy but must be **integrable**, since quasimomentum is conserved

“Chaotic” dispersion?

$$E(k) = -2\sum_{l=0}^{l_{\max}} J_l \cos(2^l k - \phi_l)$$



Energy Level Statistics



Looks crazy but must be **integrable**, since quasimomentum is conserved

$\Rightarrow$

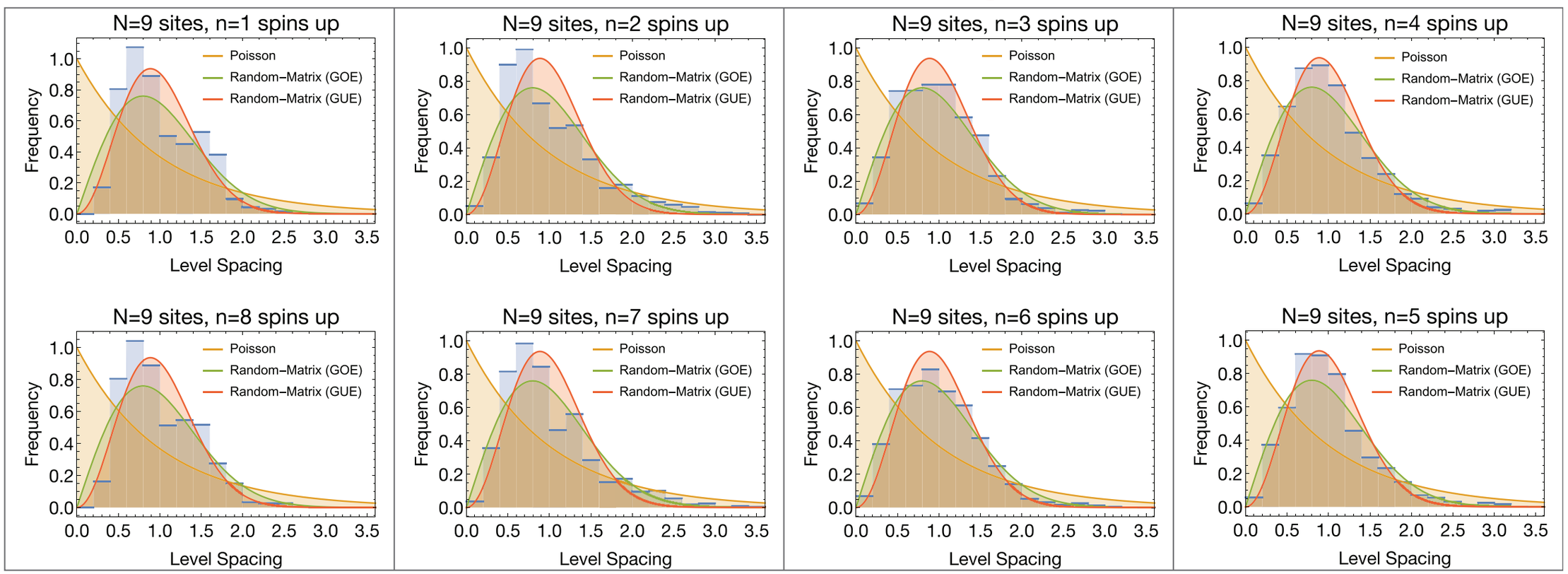
Poisson distribution of level spacings

Engineered Chaos

Break integrability with disorder potential: $H = \sum_i h_i \sigma_z^i + \sum_{i-j=2^l} J e^{i\phi_l} \sigma_+^i \sigma_-^j + \text{h.c.}$

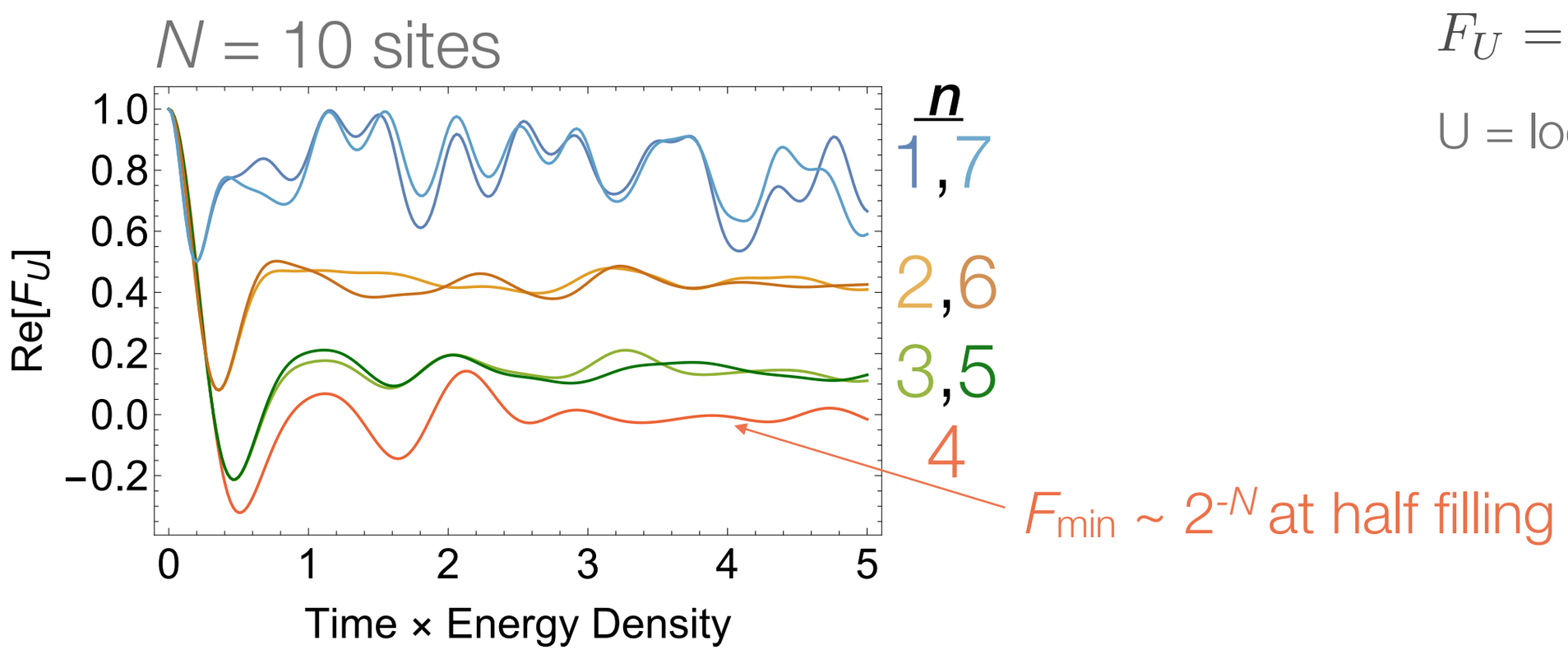
Signature of chaos: level repulsion

single particle n=1 | n=2 | n=3 | strongly interacting n=4



single hole

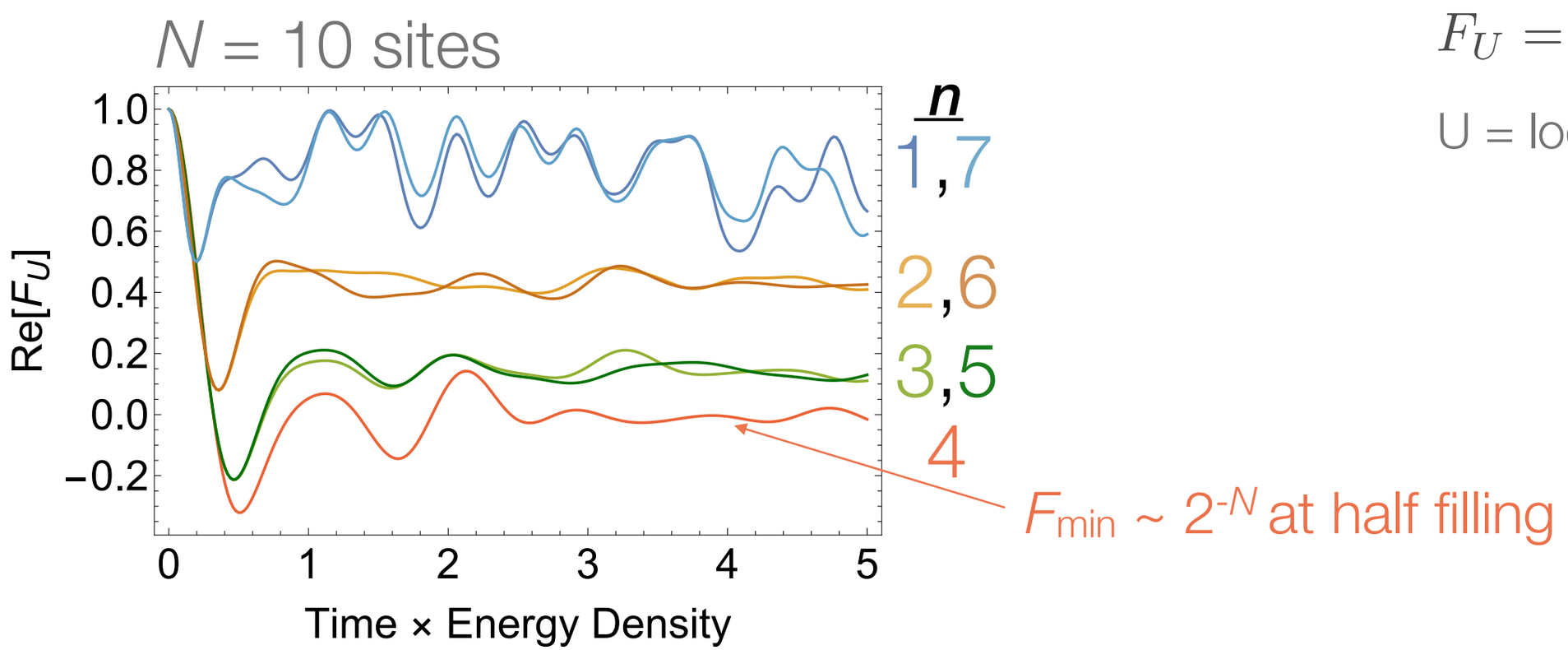
Scrambling?



$$F_U = \langle U_t^\dagger U_0^\dagger U_t U_0 \rangle$$

U = local phase shift

Fast Scrambling?



$$F_U = \langle U_t^\dagger U_0^\dagger U_t U_0 \rangle$$

U = local phase shift

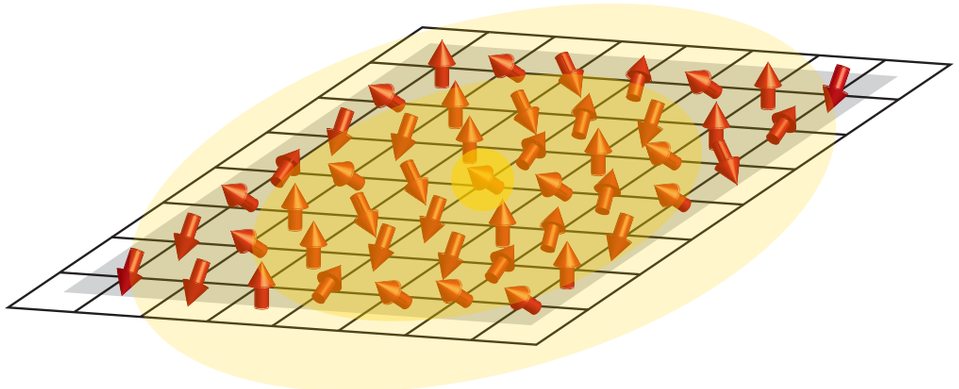
Scrambling time: $t_S \overset{?}{\sim} \tau \log(N)$; $\tau \overset{?}{=} 1 / (2\pi T)$

Extending numerics will help a little…

Quantum simulations will help more!

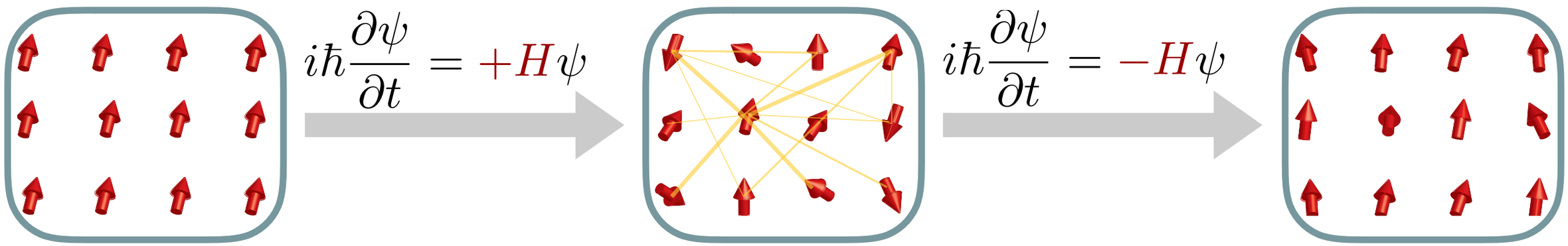
Scrambling Summary & Prospects

- Ultimate form of thermalization in a quantum many-body system
- Butterfly effect in commutator ⇔ spreading of entanglement
- Now experimentally accessible
- Which systems scramble fully and how fast? Many open questions!



Summary

Echoes of entanglement



- Quantifying many-body quantum chaos & spreading of entanglement

  ⇒ Identifying strongly correlated systems dual to black holes?

- Accessing “hidden” entanglement for precision measurements

  ⇒ Robustness to noise in quantum metrology

Acknowledgements

Research Group

Emily Davis
Gregory Bentsen
Tracy Li
Tori Borish
Ognjen Markovic
Jacob Hines
Alex Kwiatkowski

Collaborators

Brian Swingle
Patrick Hayden
Norman Yao
Dragos Potirniche

Past visitors

Anna Wang
Thomas Reimann
Sebastian Scherg