Three coupled qubits in a multi-modal superconducting quantum circuit

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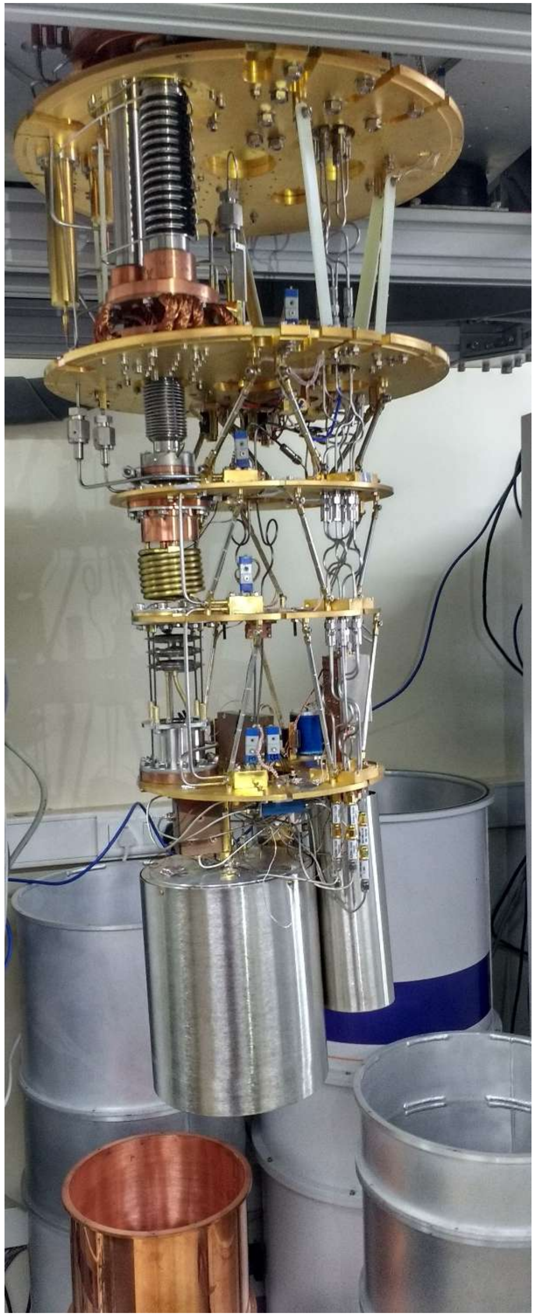
21st July 2017



Acknowledgements

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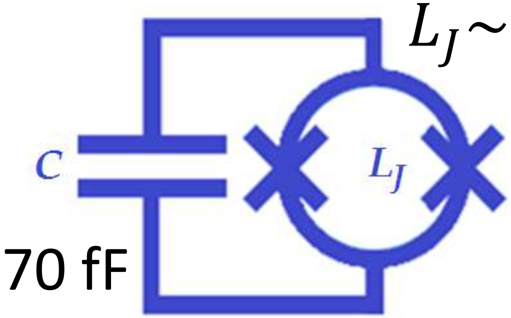
Cosmic Raj

Prof. Kedar Damle

TIFR nanofabrication facility

The 3D Transmon qubit



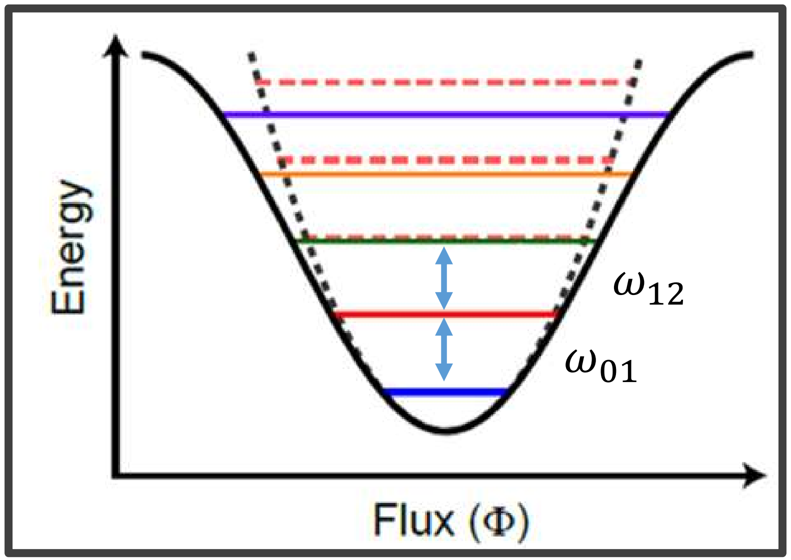
$L_J \sim$15-20nH

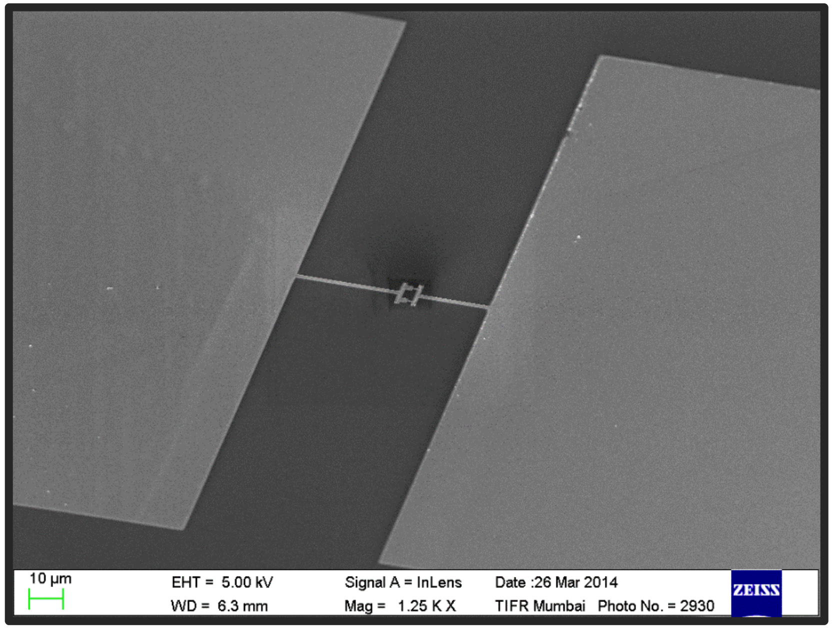
$C \sim$50-70 fF

JJ shunted by a large capacitor

$E_J >> E_C$

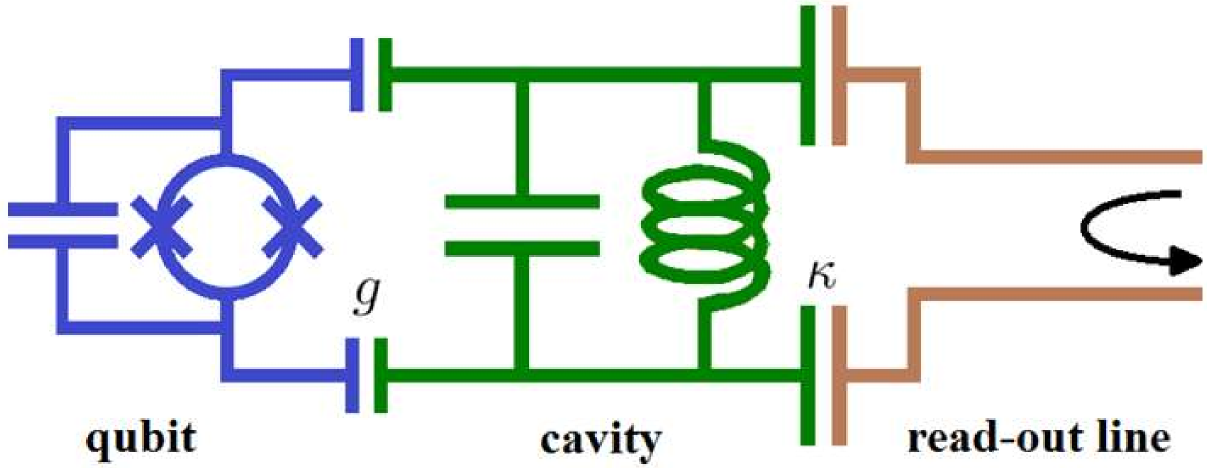
$$H_{\text{Transmon}} = \frac{\Phi^2}{2L_J(\Phi)} + \frac{Q^2}{2C} - \delta\Phi^4$$



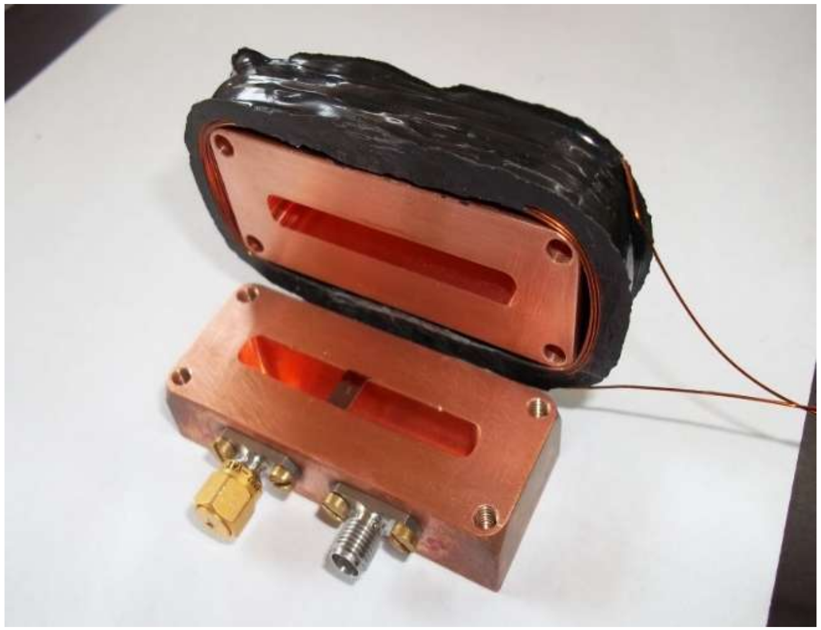


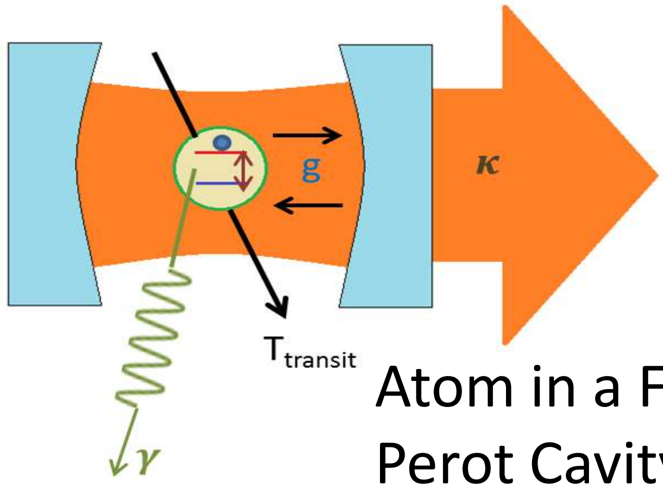
J. Koch *et al.*, *Phys. Rev. A* **76**, 04319 (2007)

Circuit QED Architecture



g ~ 70-100 MHz
κ ~ 5 MHz





Atom in a Fabry Perot Cavity

D. I. Schuster Ph.D. thesis, Yale University (2007)
A. Wallraff et. Al. *Nature* **431**, 162-167 (2004)
A. Blais et al. (2004). Phys. Rev. A **69**: 062320

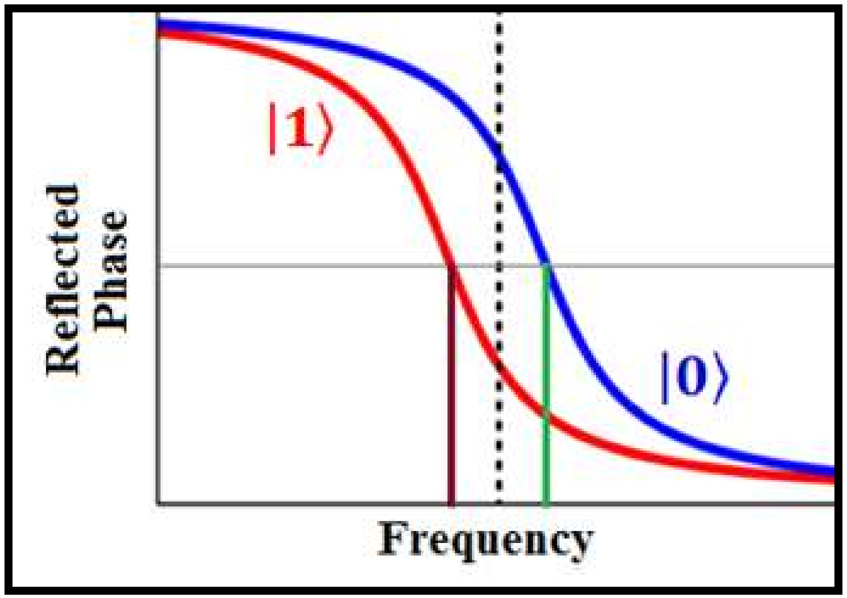
Measurement

Jaynes-Cummings (JC) Hamiltonian in the dispersive regime

$$H_{JC} = \frac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar(\omega_{cav} + \chi\hat{\sigma}_z)\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

$$\chi = \frac{g^2}{\omega_{01} - \omega_{cav}} - \frac{g^2}{\omega_{12} - \omega_{cav}}$$

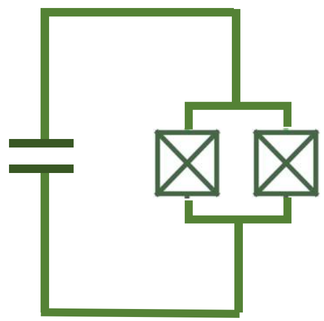
$$|\Delta| = |\omega_{01} - \omega_{cav}| \gg g$$



Probe qubit state dependent cavity frequency

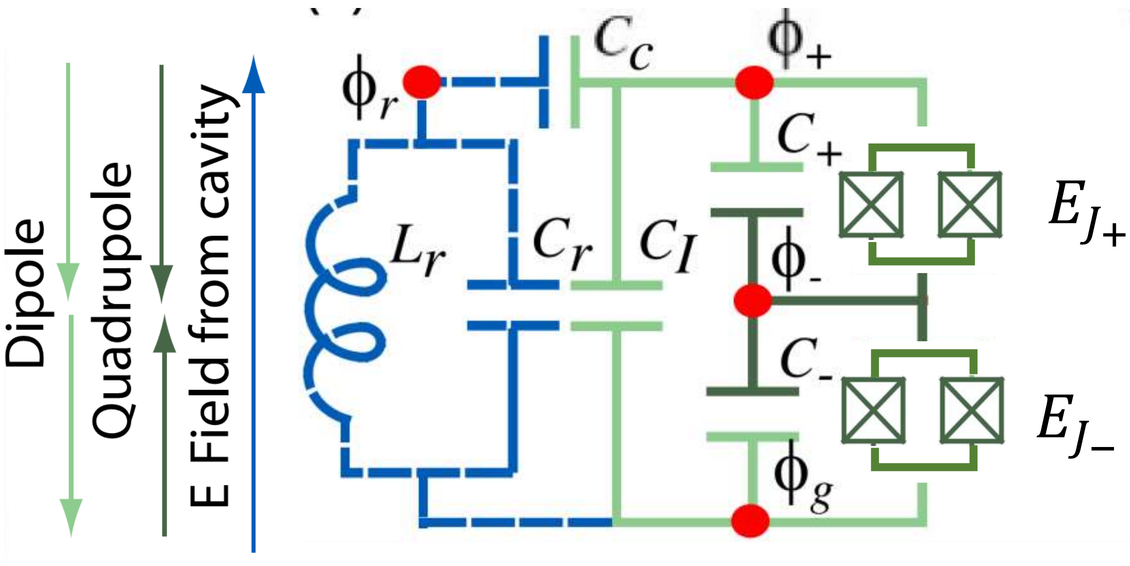
Multimode circuits I

One normal mode



Transmon

Two normal modes

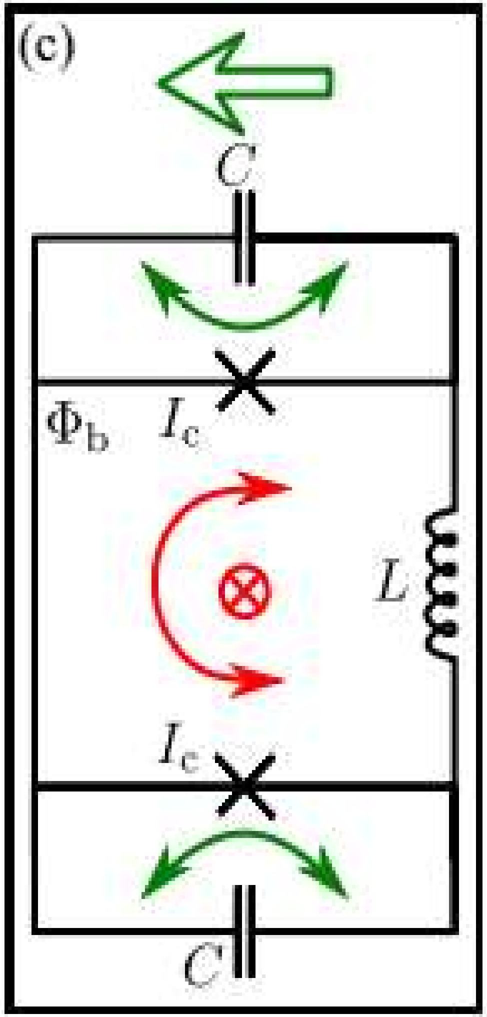


Tunable coupling qubit

Coupling between the modes $\propto \sigma_z\sigma_z$

J. M. Gambetta et al. Phys. Rev. Lett. **106**, 030502 (2011)
A. J. Hoffman et. al. Phys. Rev B **84** 184515 (2011)
S. J. Srinivasan et. al. Phys Rev Lett **106** 083601 (2011)

Multimode circuits II



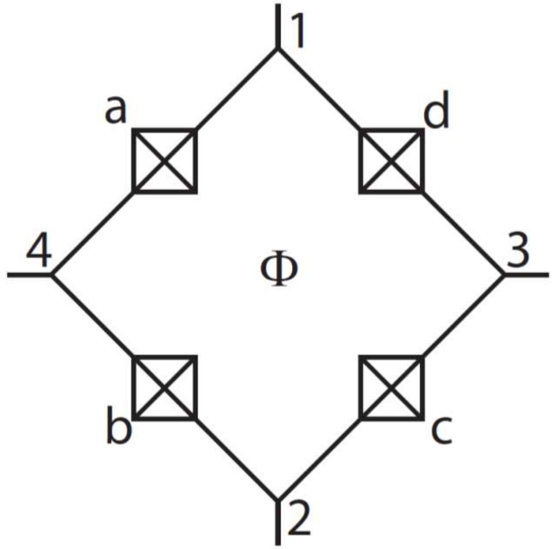
Symmetric: electric dipole
Antisymm mode: magnetic dipole

Positioned at current
antinode of $\lambda/4$ resonator

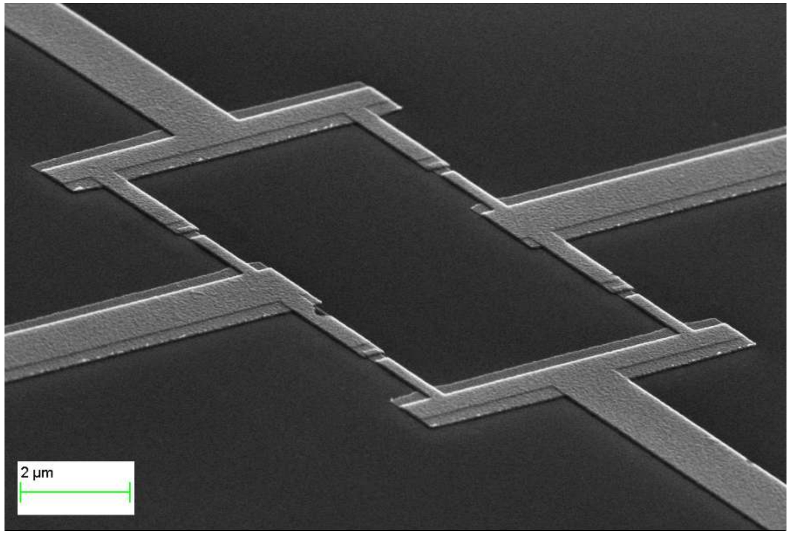
$$H = \frac{\hbar\omega_s\sigma_Z^s}{2} + \frac{\hbar\omega_a\sigma_Z^a}{2} - \frac{\hbar g_{ZZ}\sigma_Z^s\sigma_Z^a}{2}$$

É. Dumur et al Phys. Rev. B **92**, 020515(R) (2015)
É. Dumur et al IEEE Transactions on Applied Superconductivity **26** (2016)
F. Lecocq et al, Phys. Rev. Lett. **108** 107001 (2012)

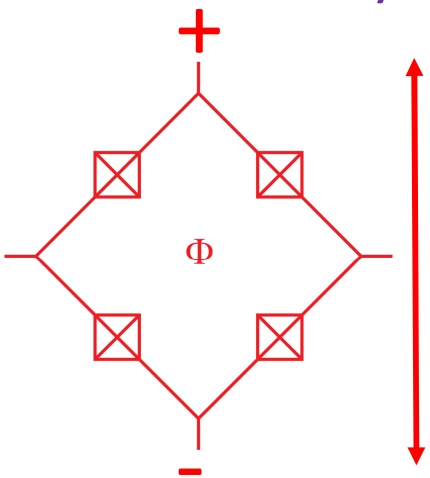
Trimon



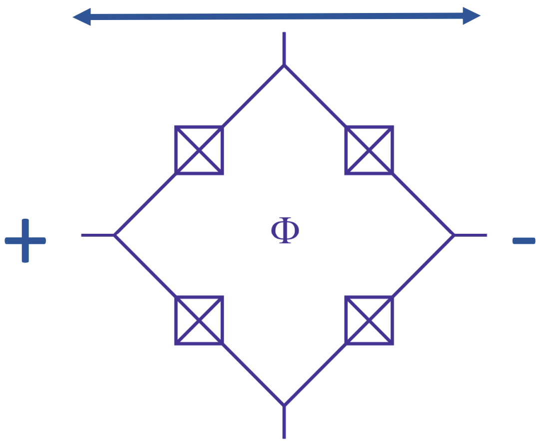
Josephson
Ring
Modulator
(JRM)



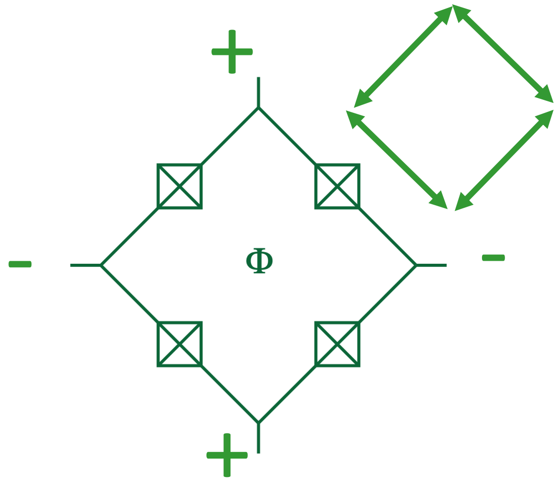
Symmetry of the circuit: 3 normal modes



A: dipolar mode



B: dipole orthogonal to A

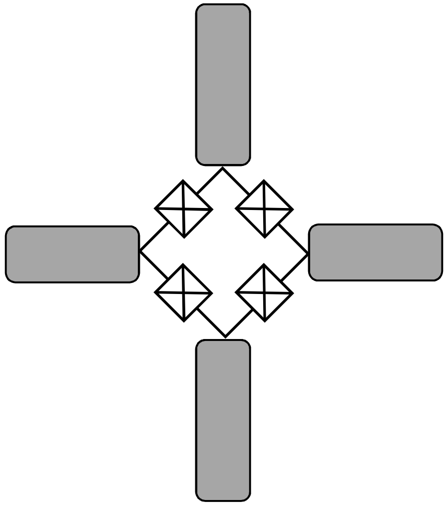


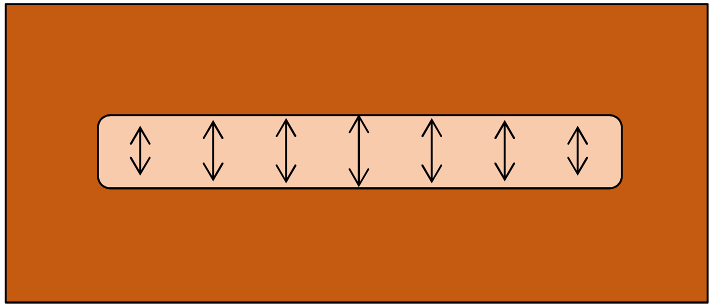
C: quadrupolar mode

N. Bergeal et al, Nat. Phys. **6** 296 (2010)

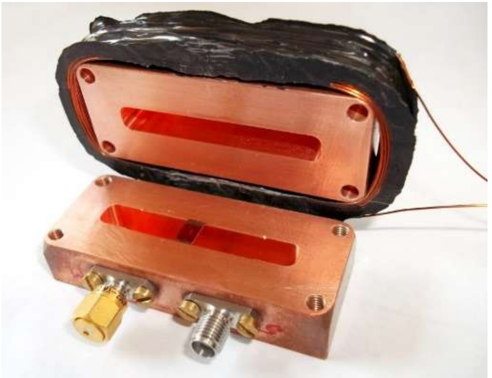
Tanay Roy et al: Phys Rev Applied **7**, 054025 (2017)

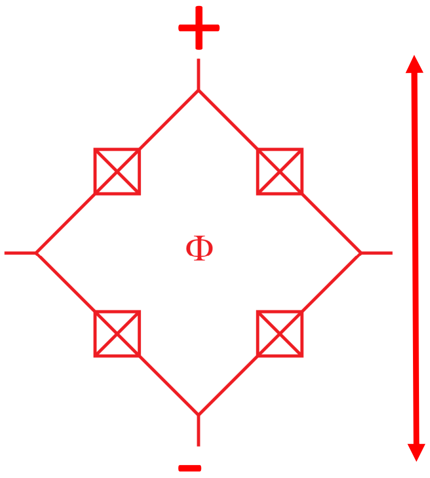
Trimon



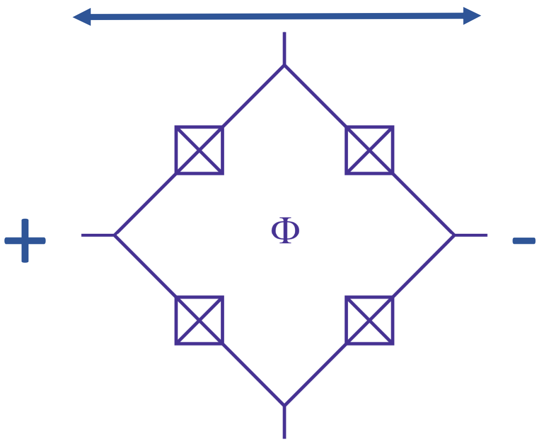


TE$_{101}$ mode

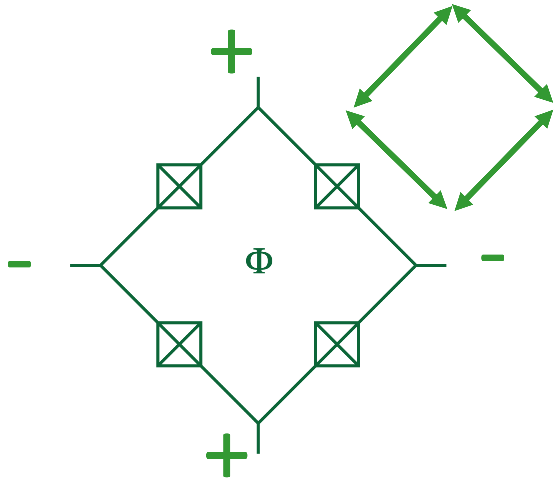




A: dipolar mode



B: dipole orthogonal to A

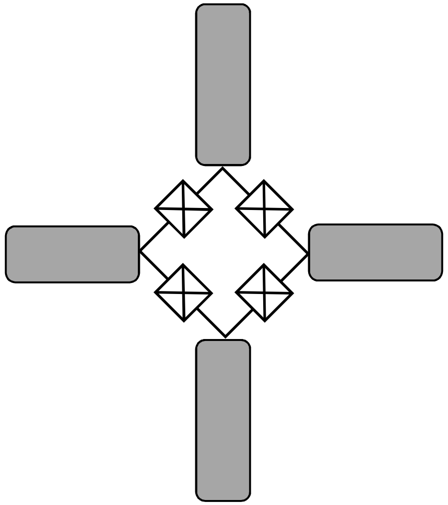


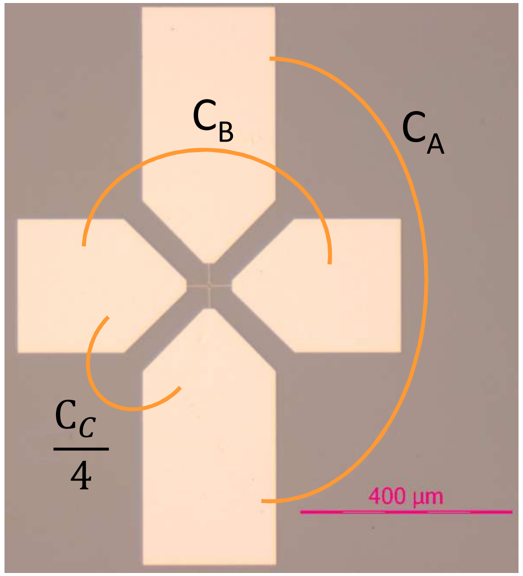
C: quadrupolar mode

N. Bergeal et al, Nat. Phys. **6** 296 (2010)

Tanay Roy et al: Phys Rev Applied **7**, 054025 (2017)

Trimon Hamiltonian





Operate at zero flux Φ=0

$$H_{ring} = -4E_J\left[\cos\left(\frac{\Phi_A}{2\varphi_0}\right)\cos\left(\frac{\Phi_B}{2\varphi_0}\right)\cos\left(\frac{\Phi_C}{\varphi_0}\right)\cos\left(\frac{\Phi}{4\varphi_0}\right) + \sin\left(\frac{\Phi_A}{2\varphi_0}\right)\cancel{\sin\left(\frac{\Phi_B}{2\varphi_0}\right)\sin\left(\frac{\Phi_C}{\varphi_0}\right)}\sin\left(\frac{\Phi}{4\varphi_0}\right)\right]$$

Expand up to quartic order

$$H_{circuit} = \frac{\Phi_A^2}{2L_J(\Phi)} + \frac{Q_A^2}{2C_A} - \delta_A\Phi_A^4 + \frac{\Phi_B^2}{2L_J(\Phi)} + \frac{Q_B^2}{2C_B} - \delta_B\Phi_B^4 + \frac{2\Phi_C^2}{L_J(\Phi)} + \frac{Q_C^2}{2C_C} + \delta_C\Phi_C^4$$

$$-[J'_{AB}\Phi_A^2\Phi_B^2 + J'_{BC}\Phi_B^2\Phi_C^2 + J'_{CA}\Phi_C^2\Phi_A^2]$$

Transmon-like qubit

Pairwise cross Kerr coupling

System Hamiltonian

$$H_{coupled\ qubit\ system}$$

$$= \frac{1}{2}\omega^A\sigma_Z^A + \frac{1}{2}\omega^B\sigma_Z^B + \frac{1}{2}\omega^C\sigma_Z^C$$

3 qubits

$$+ a^\dagger a\,(\omega_{cavity} + \chi^A\sigma_Z^A + \chi^B\sigma_Z^B + \chi^C\sigma_Z^C)$$

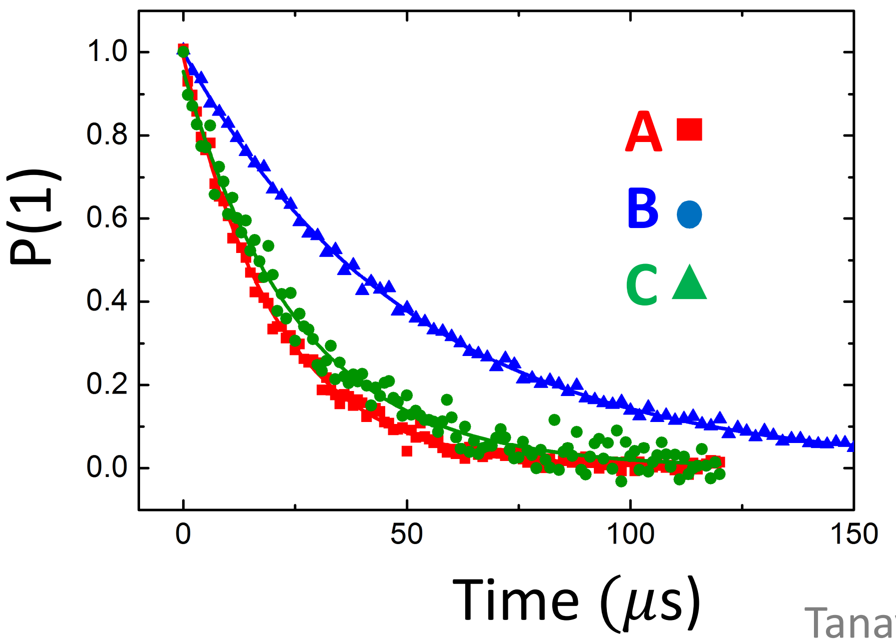
Coupling to the cavity (measurability)

$$+ \frac{1}{2}[\ J_{AB}\sigma_Z^A\sigma_Z^B + J_{BC}\sigma_Z^B\sigma_Z^C + J_{CA}\sigma_Z^C\sigma_Z^A\ ]$$

Pairwise $\sigma_Z\sigma_Z$ coupling

Trimon results

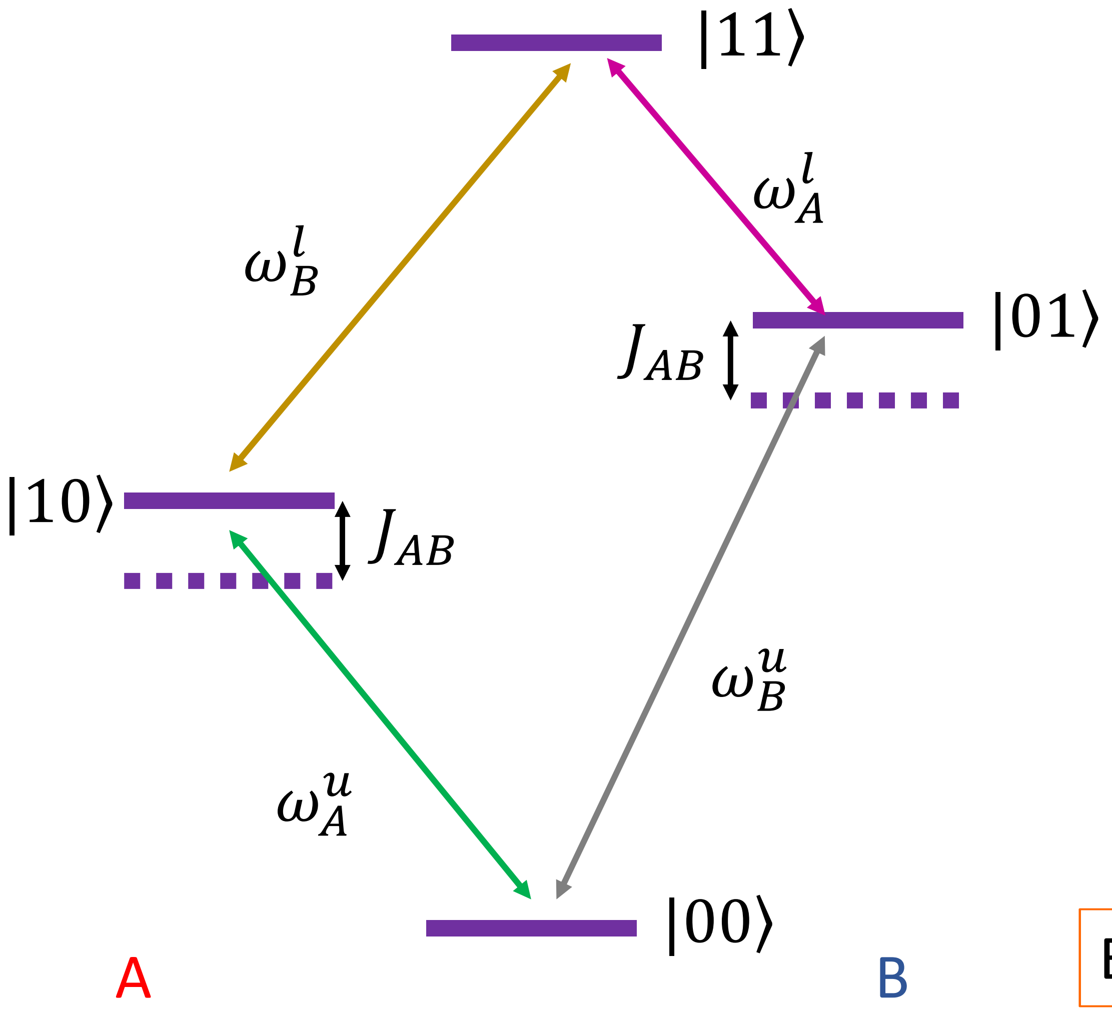
Mode	$\omega^u/2\pi$ (GHz)	$\alpha/2\pi$ (MHz)	T_1 (μs)	T_2^E (μs)	$\chi/2\pi$ (MHz)	J_{ij}/π (MHz)
A	5.5585	-111.0	20.6	39.7	-0.332	J_{AB}/π = 201.2
B	6.1470	-116.0	51.4	64.8	-0.376	J_{BC}/π = 253.0
C	7.0180	-138.6	26.2	32.3	-0.386	J_{CA}/π = 232.0



Device: RT15
Cavity resonance: 7.23 GHz

Tanay Roy et al: Phys Rev Applied **7**, 054025 (2017)

Pairwise $\boldsymbol{\sigma_z \sigma_z}$ coupling



2 qubit subspace

$$\omega_A^u - \omega_A^l = 2J_{AB}$$

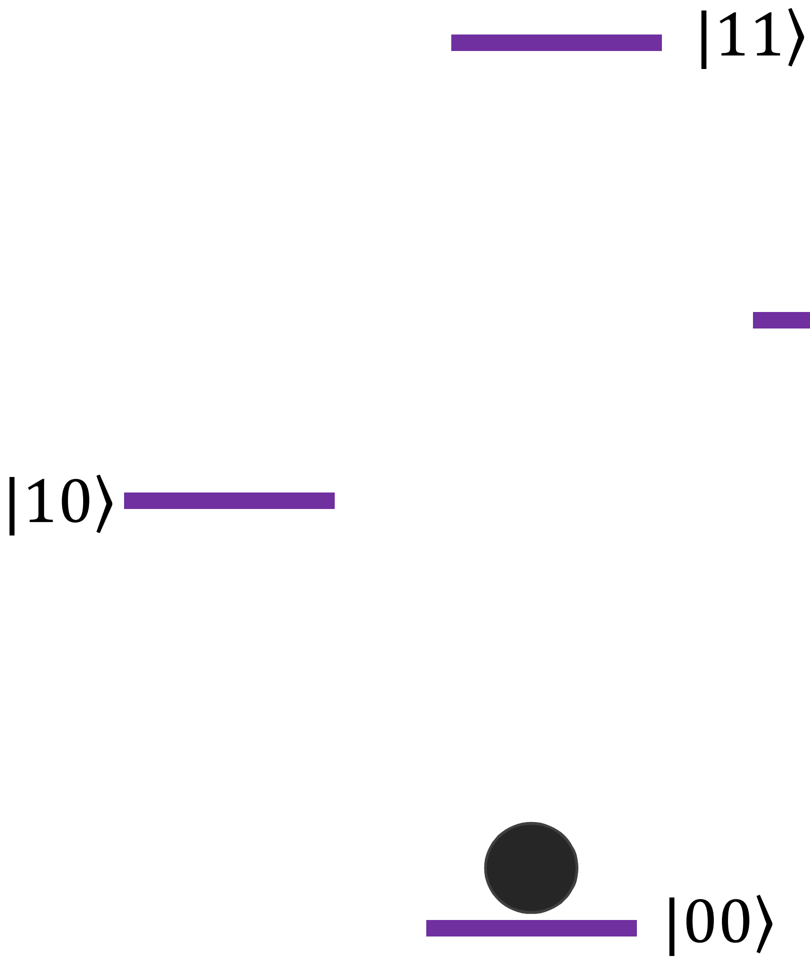
$$\omega_B^u - \omega_B^l = 2J_{AB}$$

Address specific transitions

Not individual qubits

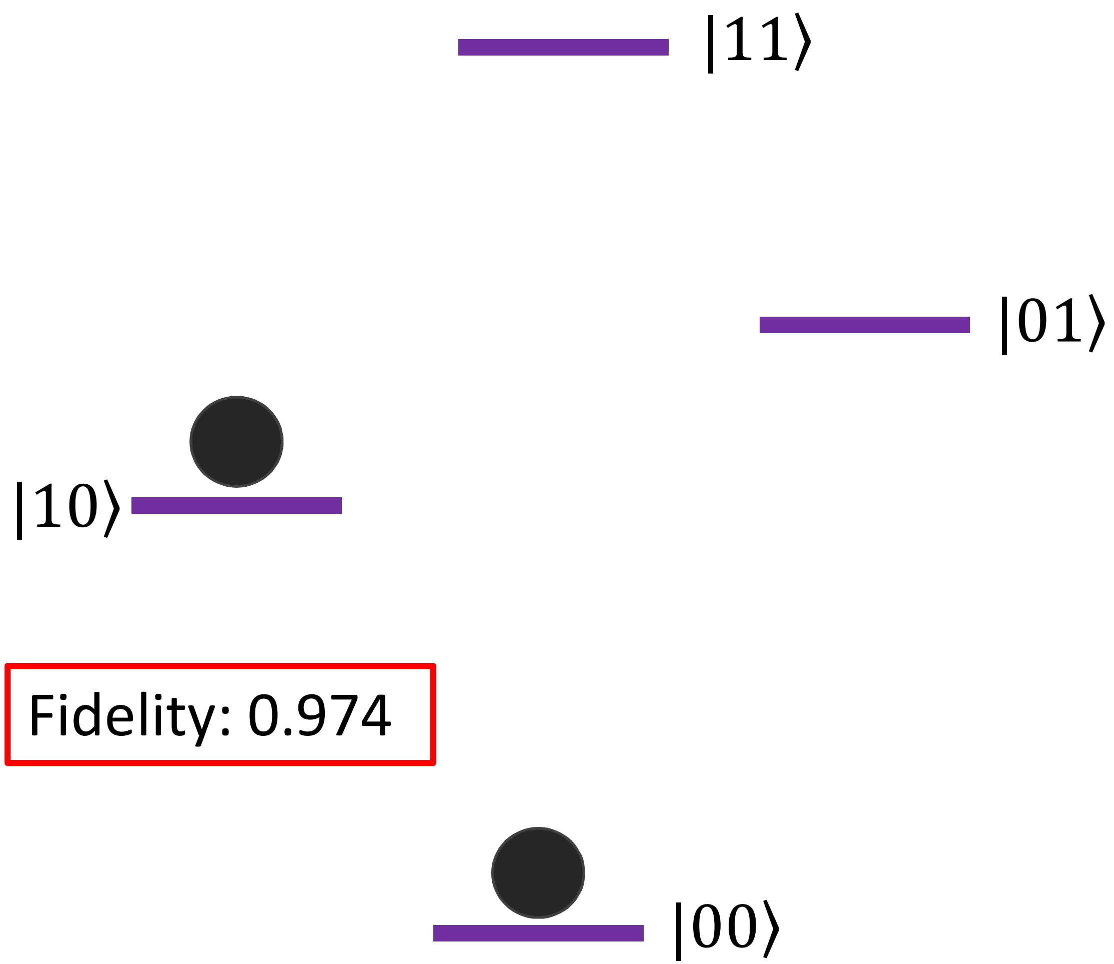
Enables single-pulse CNOT

Bell State preparation



$|0\rangle \otimes |0\rangle$

Bell State preparation



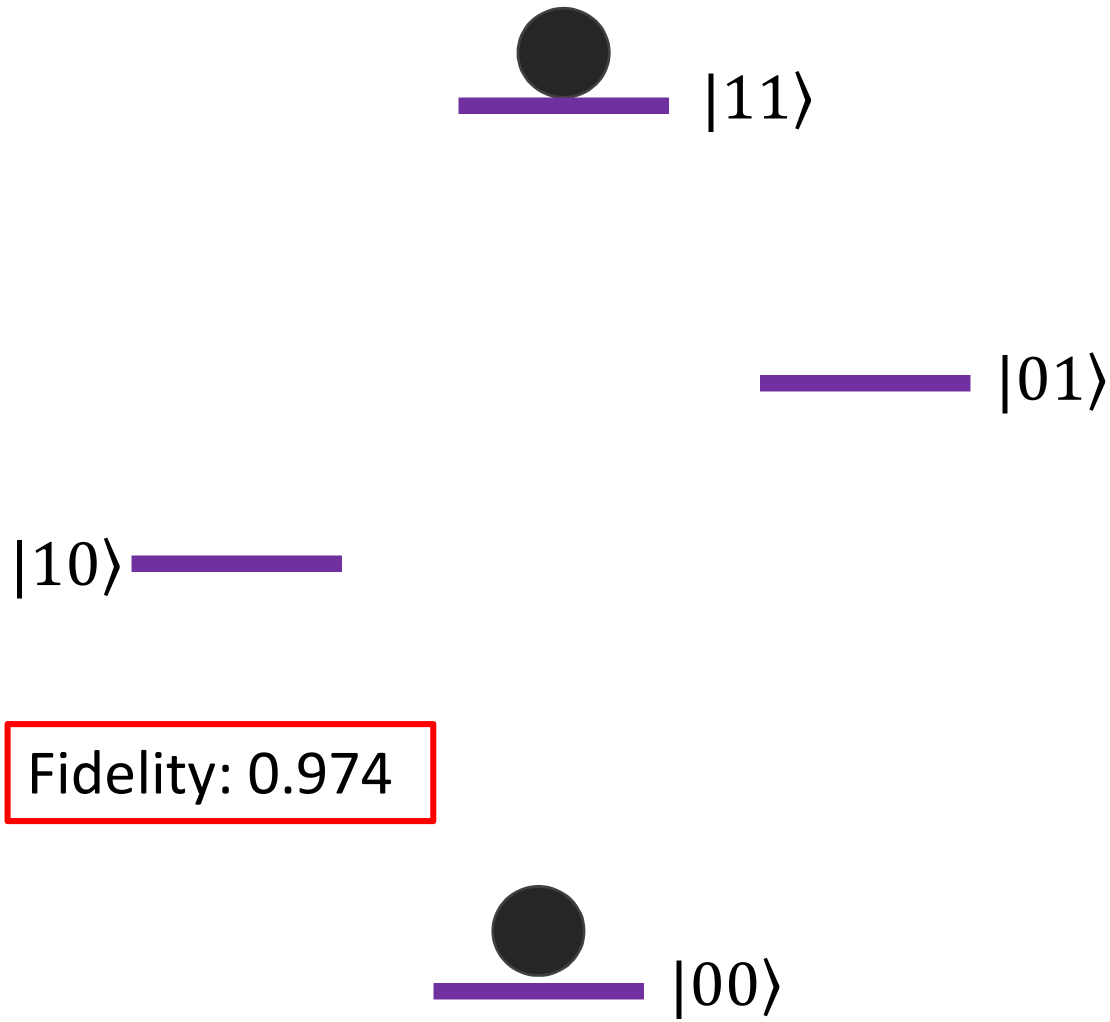
Fidelity: 0.974

$$|0\rangle \otimes |0\rangle$$

$$\downarrow \ \pi/2 \text{ at } \omega_A^u$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

Bell State preparation



Fidelity: 0.974

$$|0\rangle \otimes |0\rangle$$

$$\Big\downarrow \ \pi/2 \text{ at } \omega_A^u$$

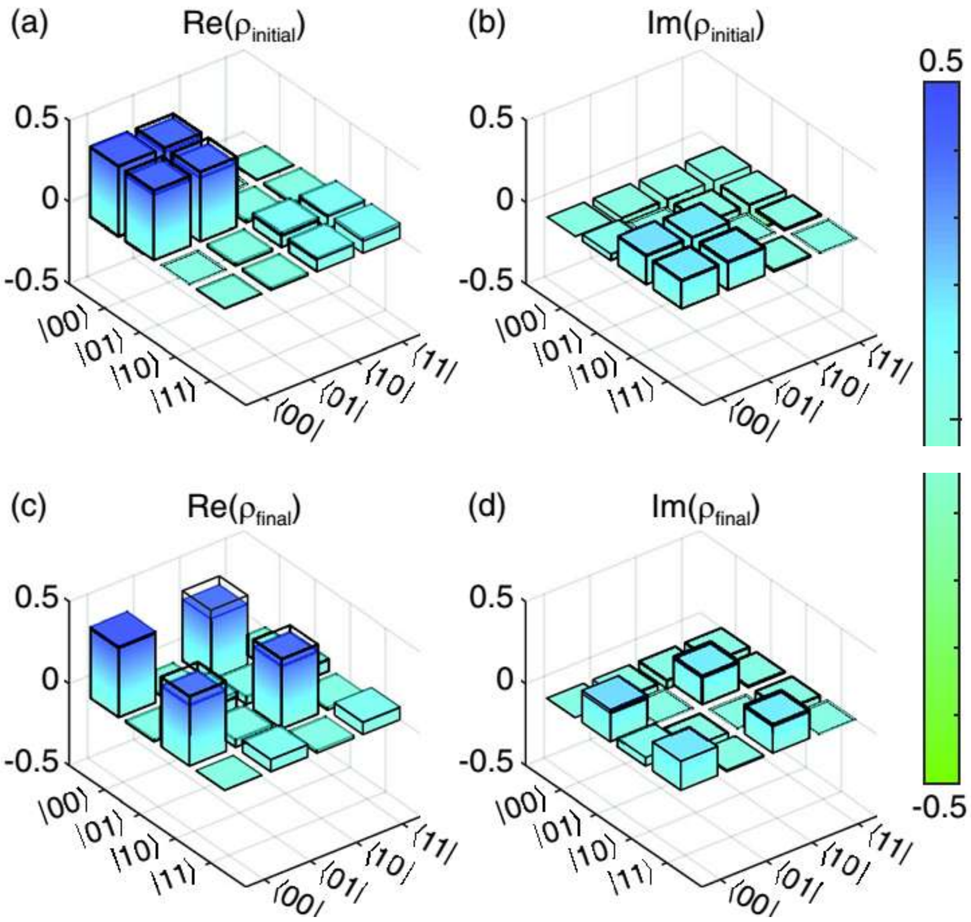
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$\Big\downarrow \ \mathrm{CNOT}_{\mathrm{BA}}$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

SWAP operation

$$\mathrm{SWAP} = \mathrm{CNOT}_{AB}\ \mathrm{CNOT}_{BA}\ \mathrm{CNOT}_{AB}$$



Initial state:

$$|A\rangle = \cos\left(\frac{\pi}{8}\right)|0\rangle + i\ \sin\left(\frac{\pi}{8}\right)|1\rangle$$

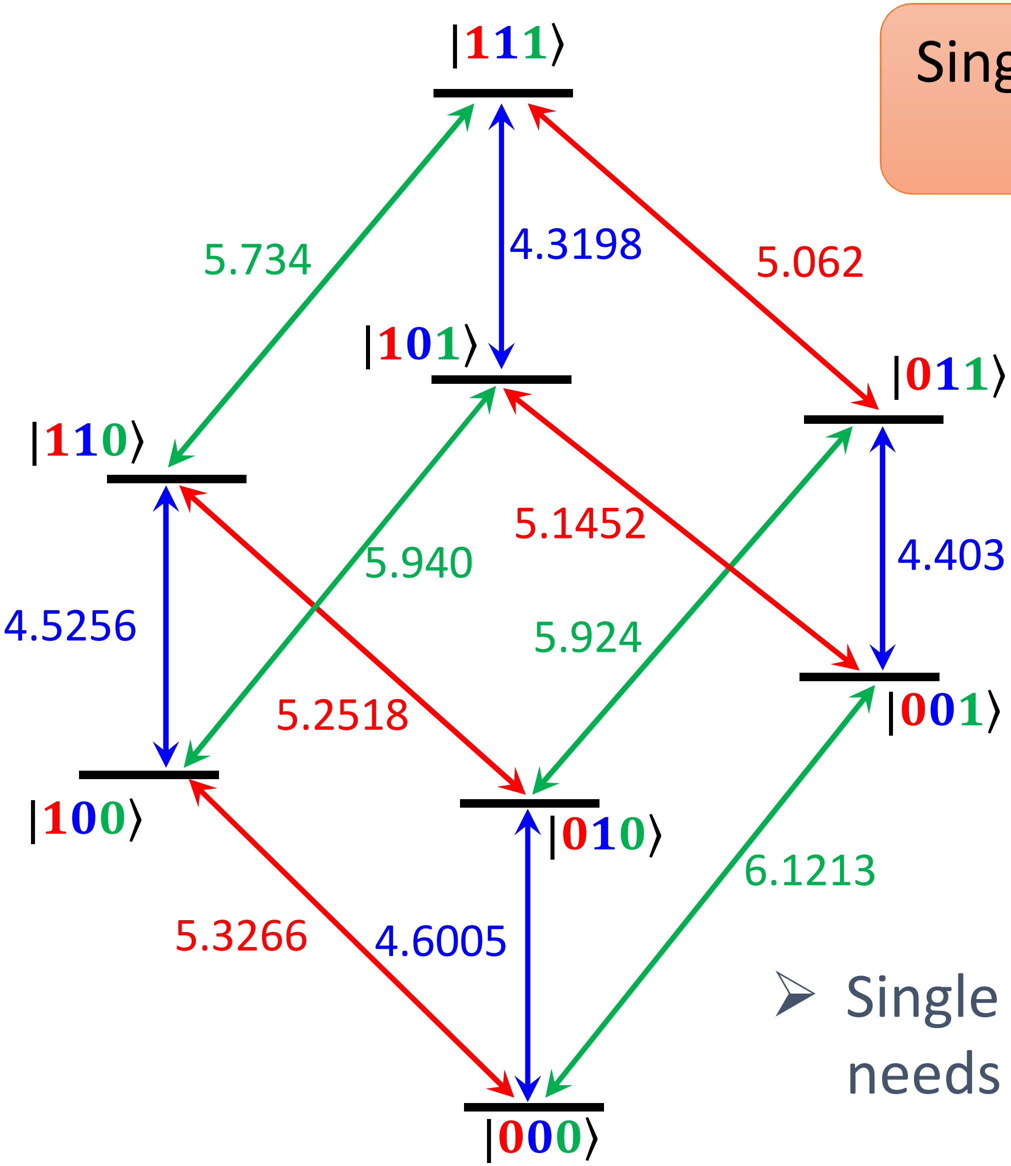
$$|B\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

Fidelity: 0.983

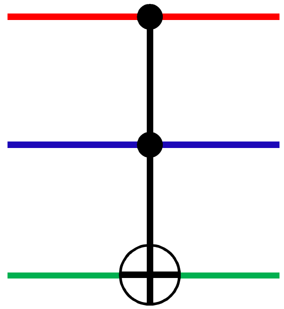
Fidelity after SWAP: 0.971

Tanay Roy et al: Phys Rev Applied **7**, 054025 (2017)

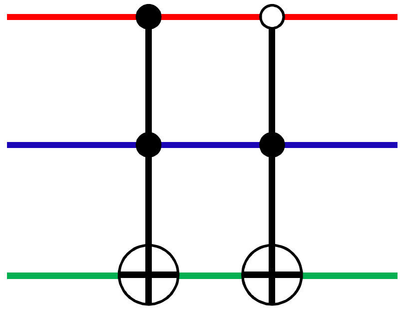
Three-qubit Gates



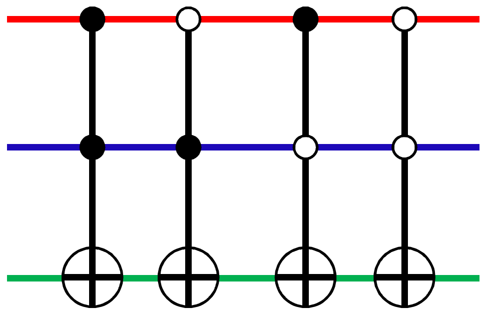
Single pulse general CCNOT gate



- CNOT gate: 2 pulses

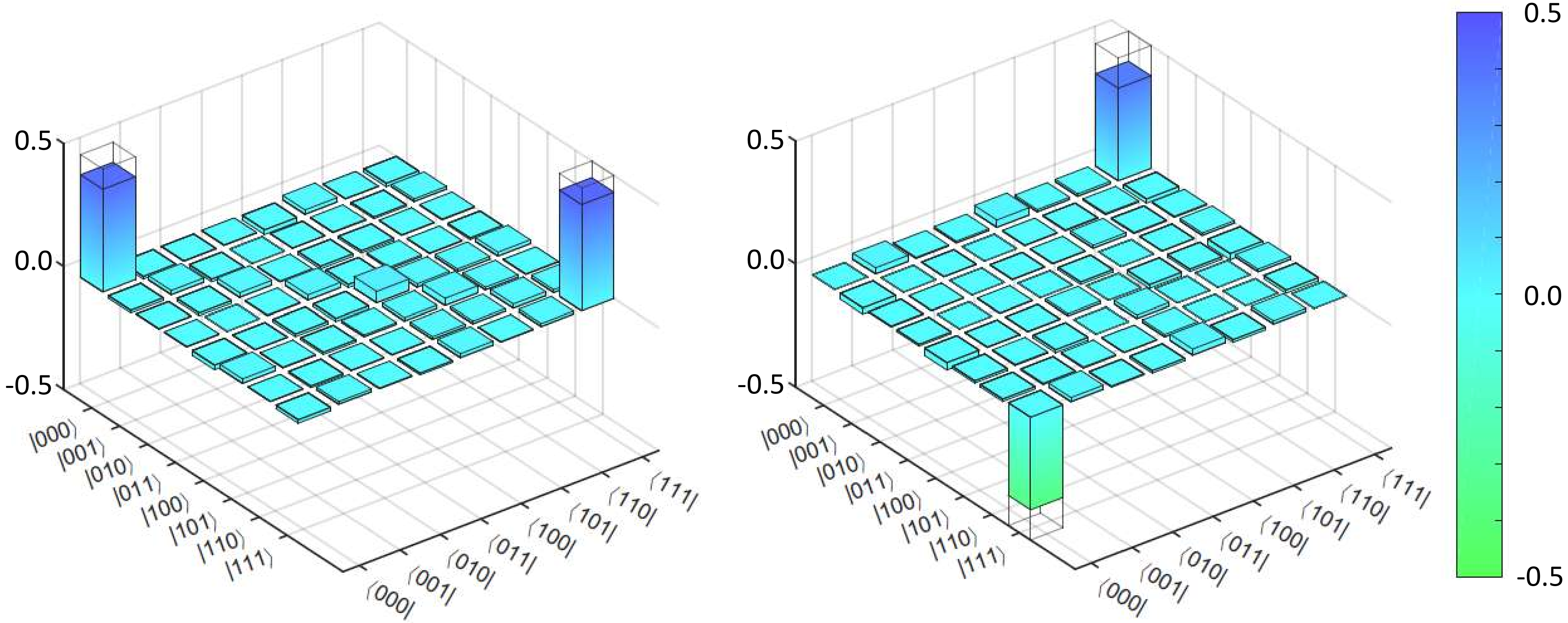


- Single qubit rotation needs 4 pulses



GHZ state

$$|000\rangle \xrightarrow[65\text{ ns}]{R_{x,A}(\pi/2)} \frac{|000\rangle - i|100\rangle}{\sqrt{2}} \xrightarrow[78\text{ ns}]{\text{CCNOT}_B} \frac{|000\rangle - i|110\rangle}{\sqrt{2}} \xrightarrow[288\text{ ns}]{\text{CCNOT}_C} \frac{|000\rangle - i|111\rangle}{\sqrt{2}}$$



$\mathcal{F}$= 0.896

*See poster by Tanay Roy

Generalized multimode circuit analysis

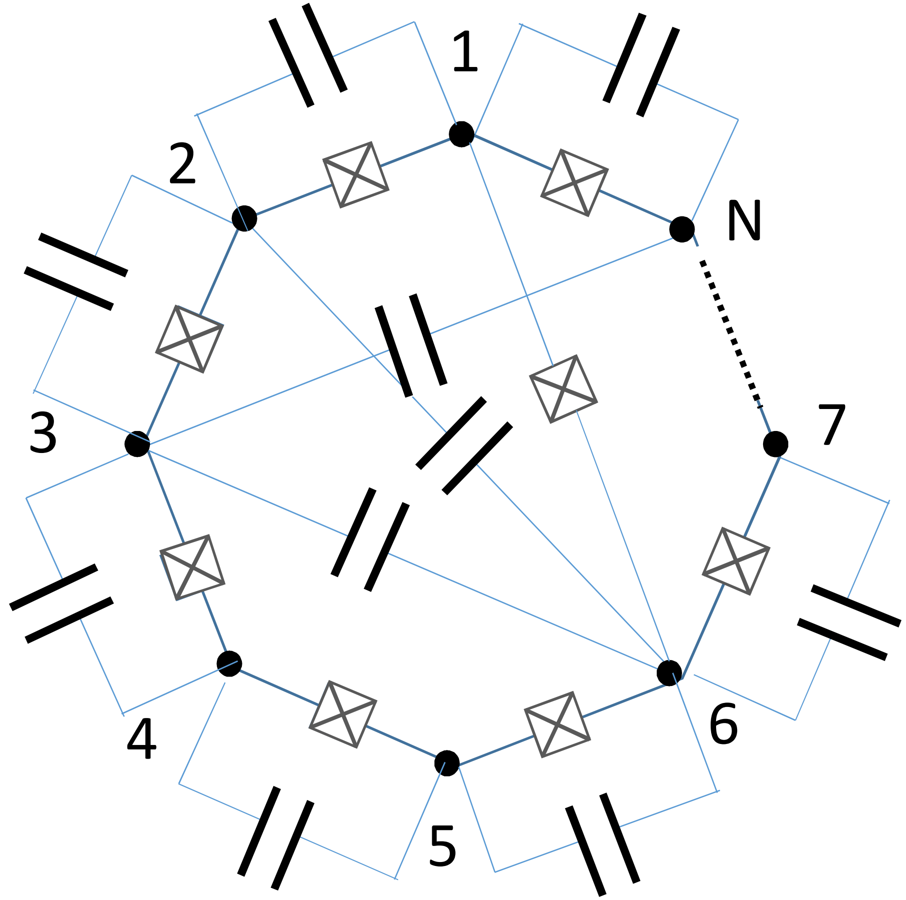
$$\mathcal{L} = \frac{1}{2}\sum_{i,k=1}^{N}\left(\frac{d\phi_i}{dt}C_{ik}\frac{d\phi_k}{dt} + E_{J\,ik}\cos\left[\frac{\phi_i - \phi_k}{\varphi_0}\right]\right)$$

ϕ_i : Node flux variables

Construct E_J and Capacitance matrices

Simultaneously diagonalize the matrices: normal modes

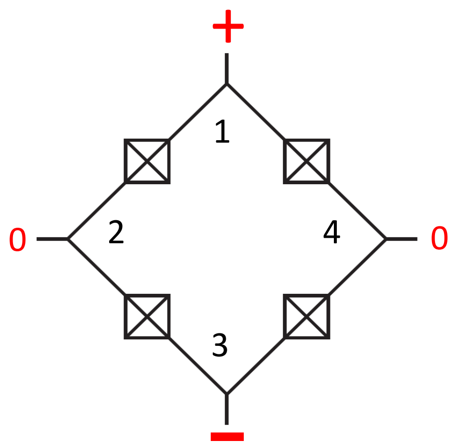
Non-linearity: Anharmonicity and cross-Kerr couplings



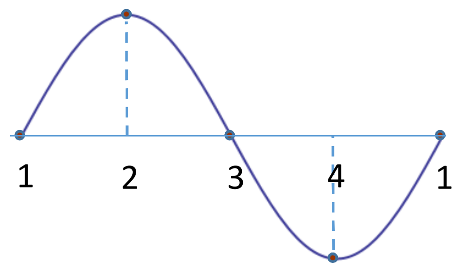
Nigg et al, Phys Rev Lett. **108** 240502 (2012)
Bourasa et al, Phys Rev A **86** 013814 (2012)

Mode ‘Shapes’: Trimon

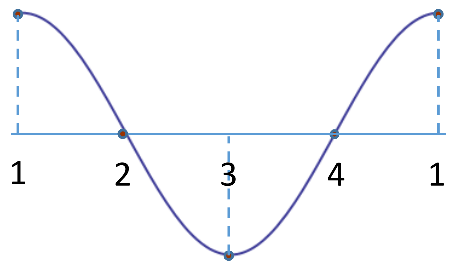
Trimon



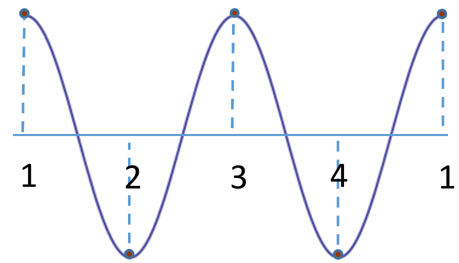
$$\phi_B = \begin{pmatrix} 0 \\ \phi_2 \\ 0 \\ -\phi_4 \end{pmatrix}$$



$$\phi_A = \begin{pmatrix} \phi_1 \\ 0 \\ -\phi_3 \\ 0 \end{pmatrix}$$

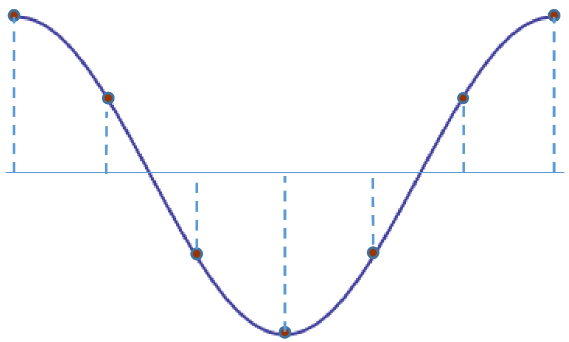


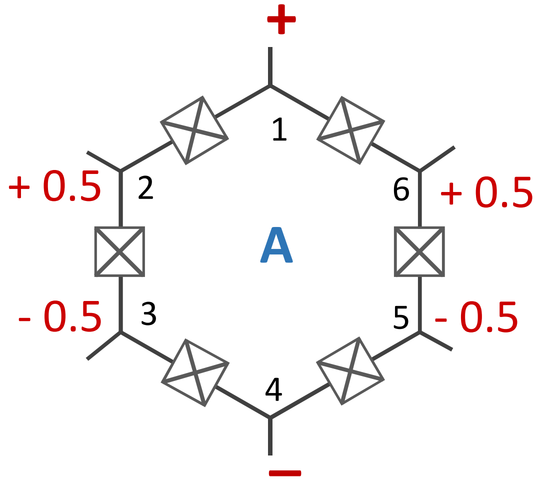
$$\phi_C = \begin{pmatrix} \phi_1 \\ -\phi_2 \\ \phi_3 \\ -\phi_4 \end{pmatrix}$$



Mode 'Shapes': Pentamon

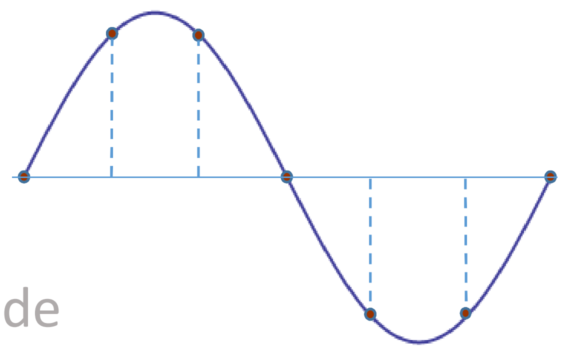
Standing waves on a string with periodic boundry conditions

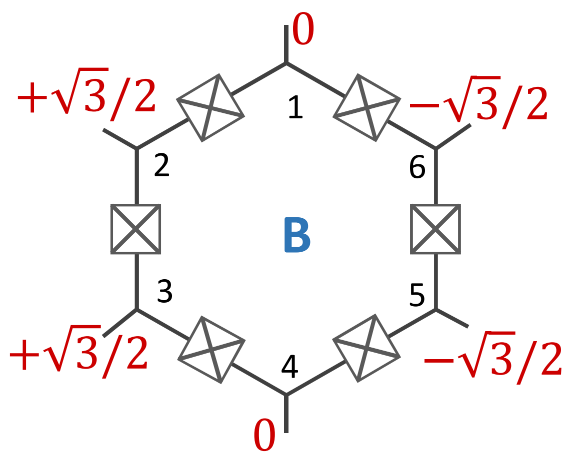




Pentamon:

2 orthogonal dipolar modes



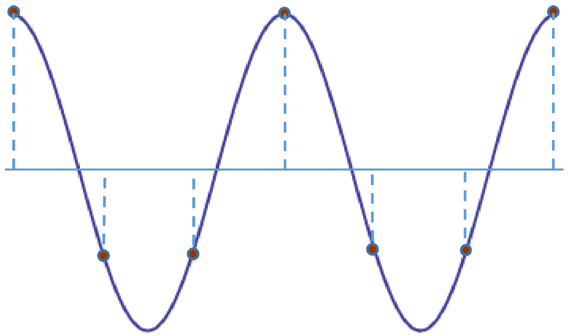


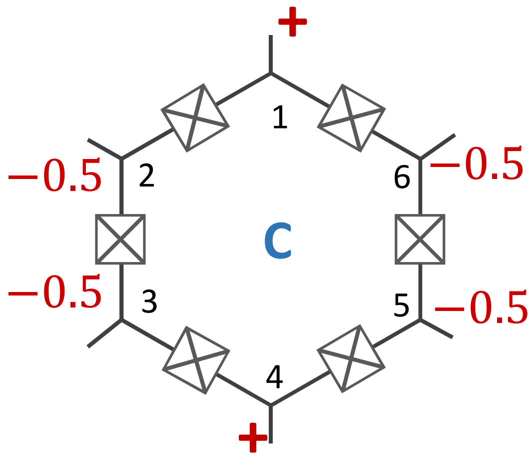
2 orthogonal quadrupolar mode

1 alternating mode

Mode ‘Shapes’: Pentamon

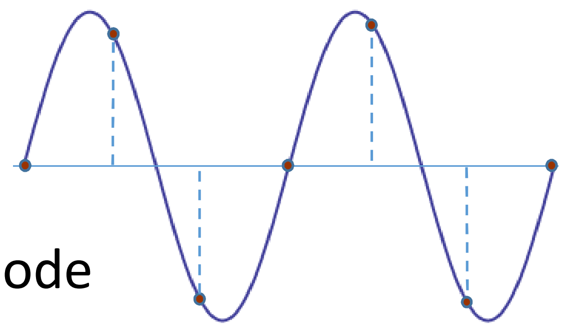
Standing waves on a string with periodic boundry conditions

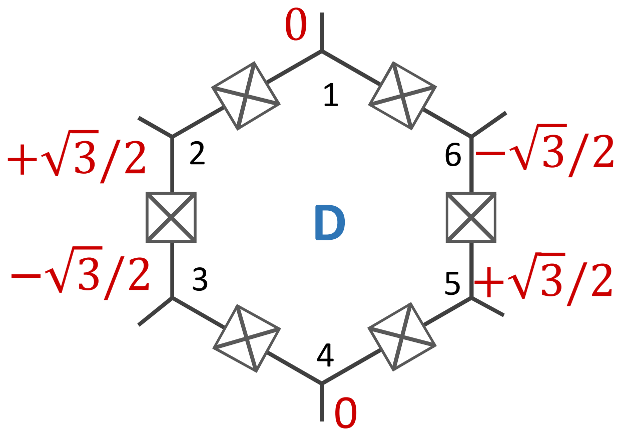




Pentamon:

2 orthogonal dipolar modes



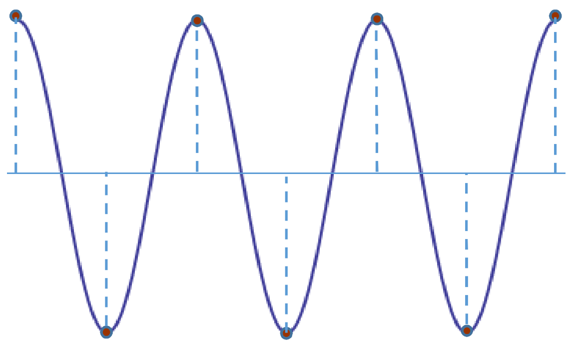


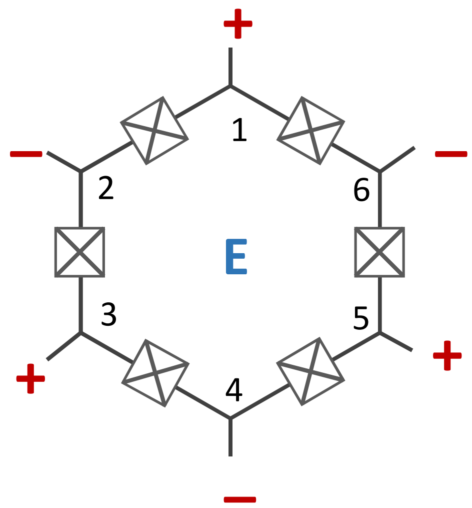
2 orthogonal quadrupolar mode

1 alternating mode

Mode ‘Shapes’: Pentamon

Standing waves on a string with periodic boundry conditions





Pentamon:

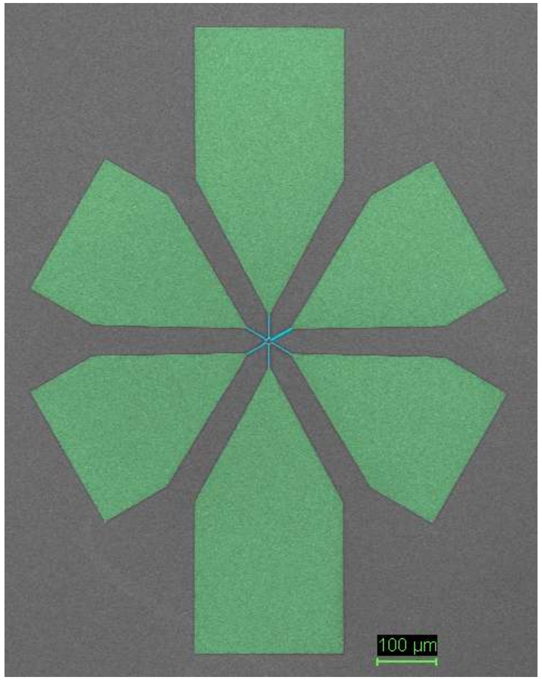
2 orthogonal dipolar modes

2 orthogonal quadrupolar mode

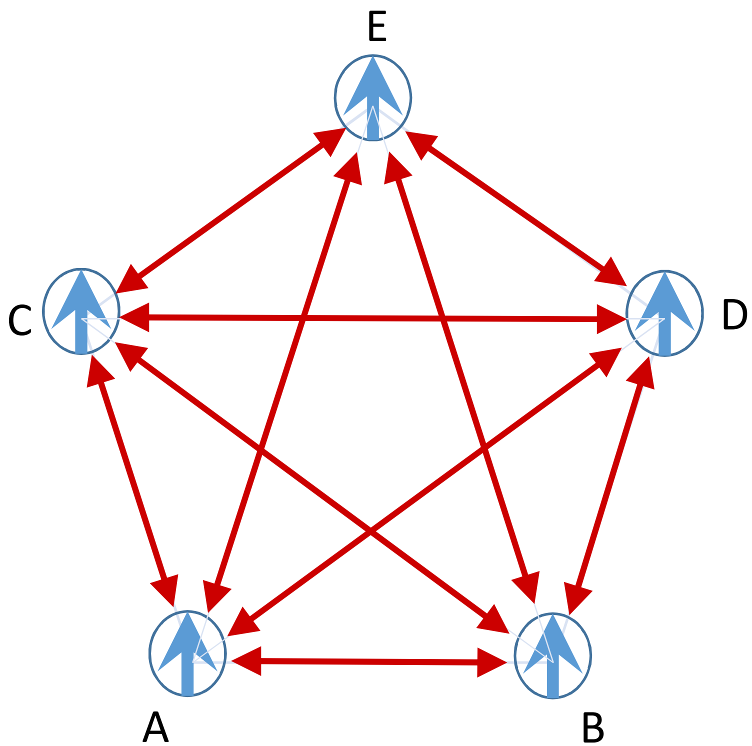
1 alternating mode

$$\mathcal{H}_{Pentamon} = \sum_{\mu=A,B,C,D,E} \frac{1}{2}\omega^{\mu}\sigma_Z^{\mu} + \sum_{\mu,\nu} \frac{1}{2} J_{\mu\nu}\sigma_Z^{\mu}\sigma_Z^{\nu}$$

Pentamon



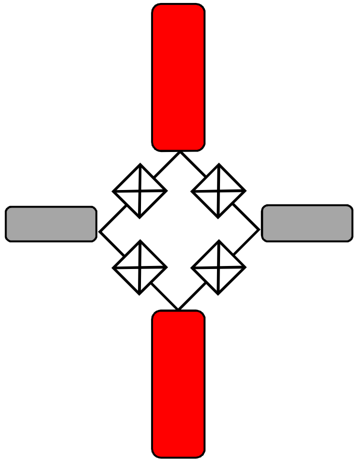
Mode	ω_{01}(GHz)
A	4.8548
B	5.74765
C	6.76706
D	6.8676
E	7.7287

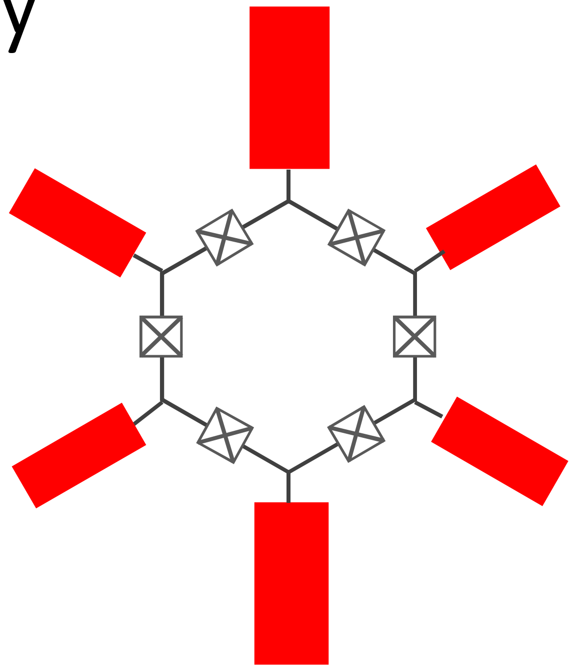


Single qubit gates need 16 pulses

Mode pair	2J (MHz)
AB	59
AC	81.3
AD	138.7
AE	109.7
BC	217.9
BD	126.7
BE	72.1
CD	231.3
CE	98.8
DE	39.7

Summary



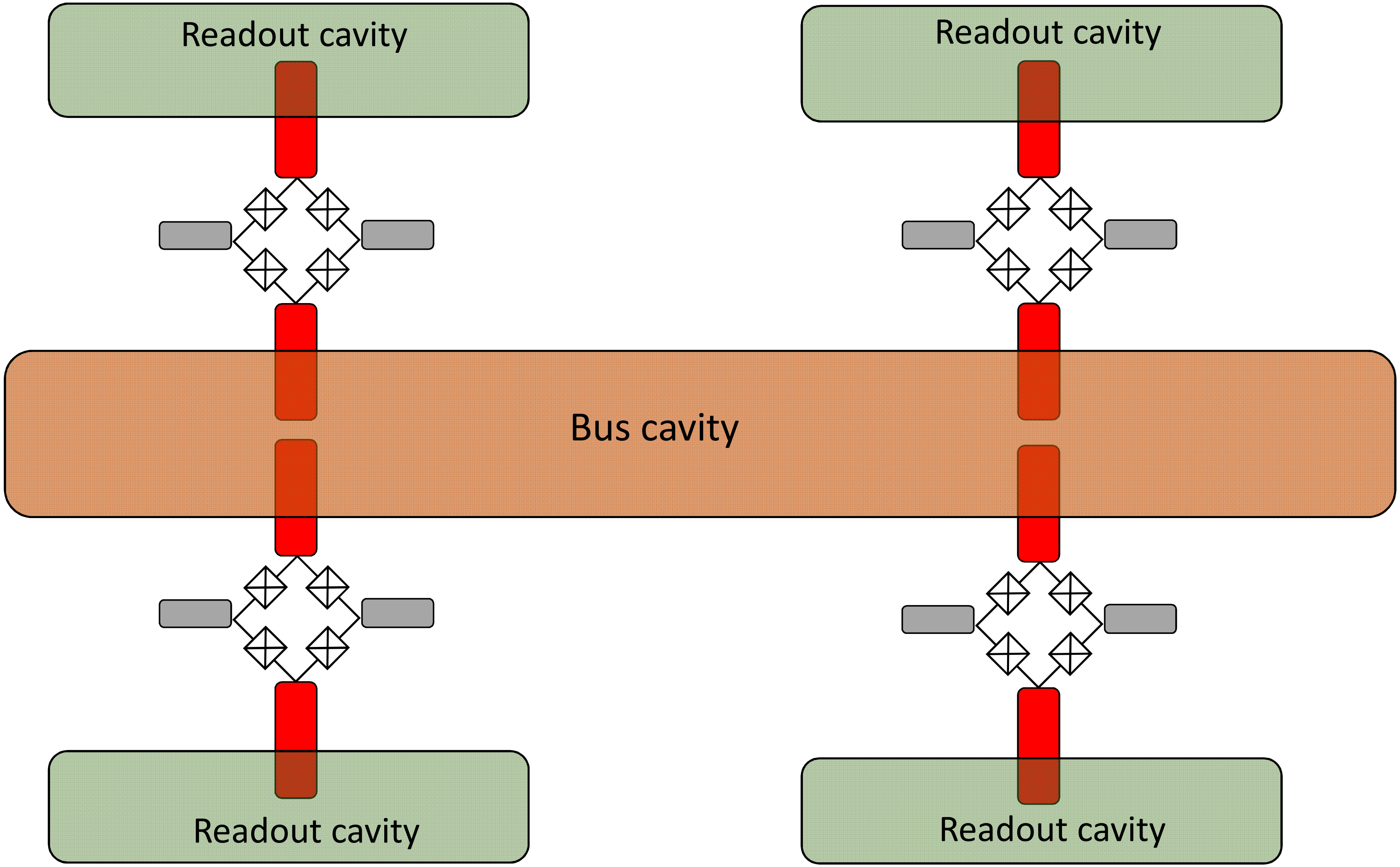


- Trimon(3) and Pentamon(5) coupled qubits with pairwise strong $\sigma_z\sigma_z$ coupling
- System of coupled oscillators: N-1 coupled qubits for N-node cyclic ring
- “Protected” qubit with high fidelity readout

*See poster by Suman Kundu and Sumeru Hazra

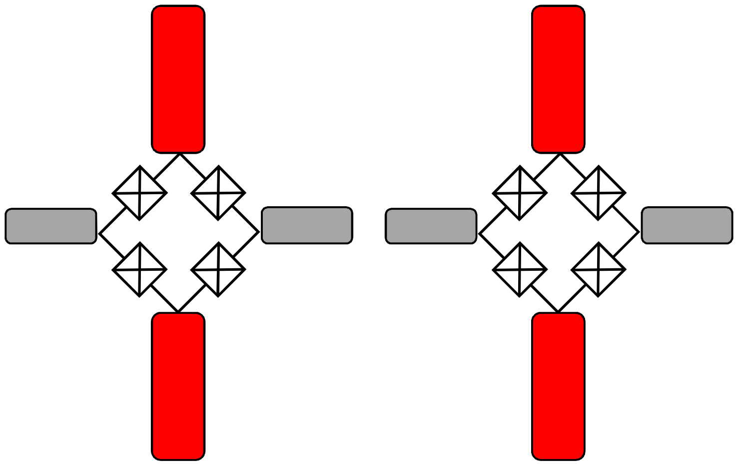
Future Directions

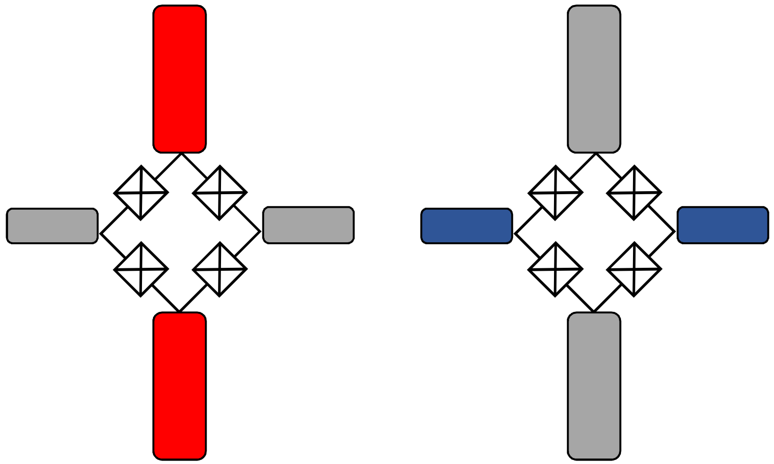
Trimon as a building block



Future Directions

- Trimon as a single protected qubit: tunable coupling by mode-switching





- Spin Chain
  Pairwise Ising interaction

See posters by Tanay Roy and Suman Kundu