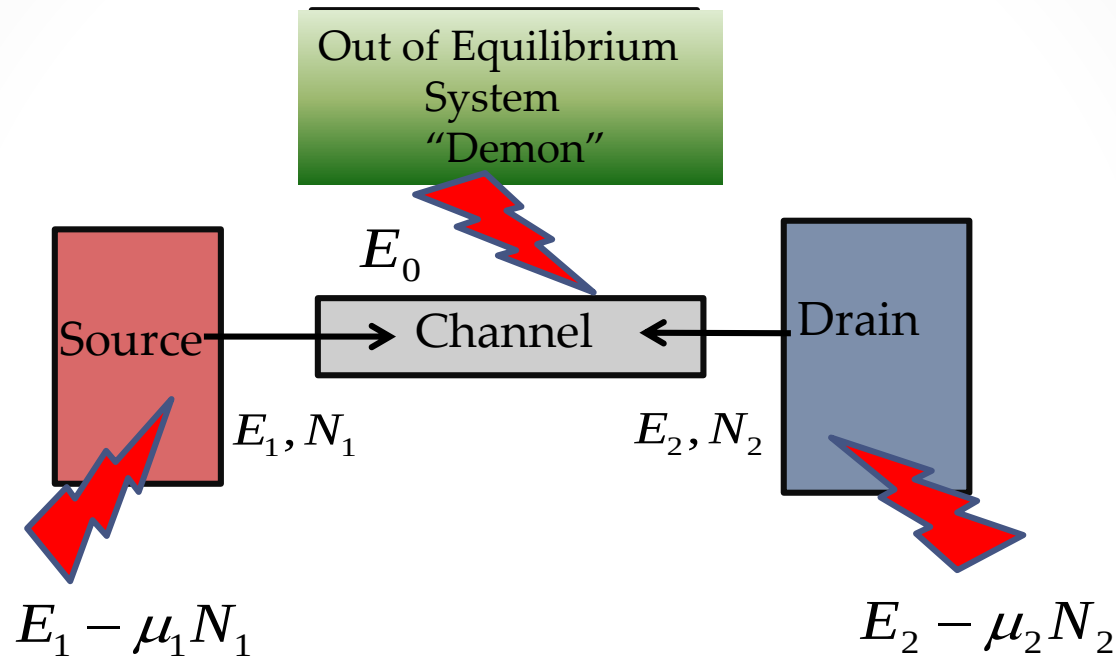


Information driven quantum dot thermal machines



Bhaskaran Muralidharan
Dept. of Electrical Engineering,
Indian institute of technology Bombay

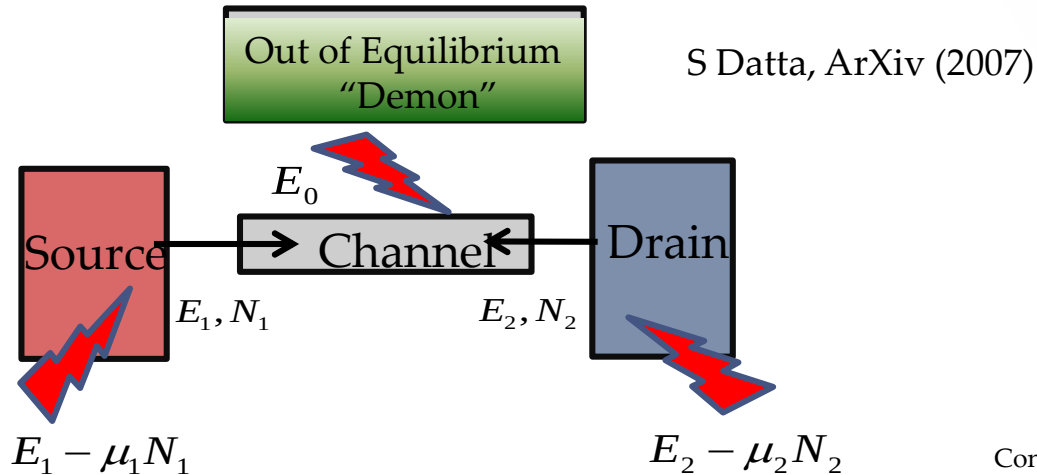
SEMINAR TALK

ICTS workshop on Open Quantum Systems

21/07/2017

Outline

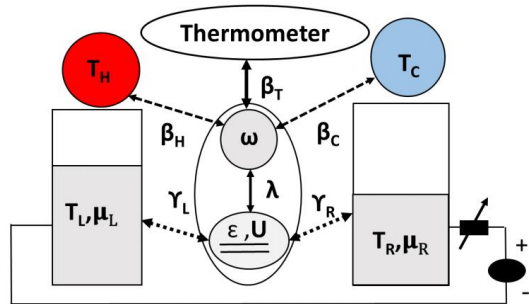
- Fundamentals and Set ups
- Quantum dot heat engines



Non-equilibrium Phonons

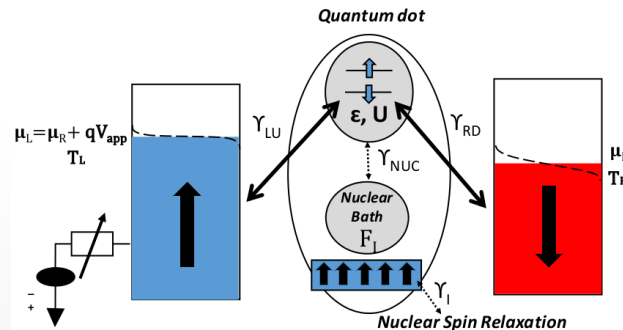
Concurrence Engine

B. De and B. Muralidharan,
Phys. Rev. B, 94, 165416, (2016).

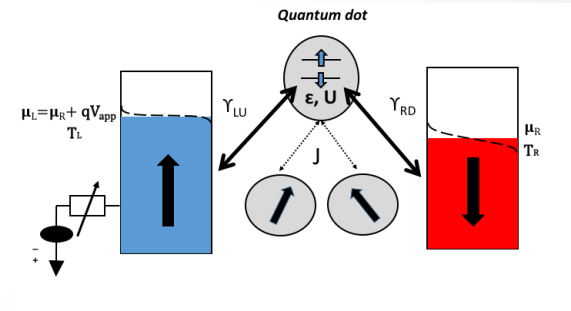


Classical Information driving

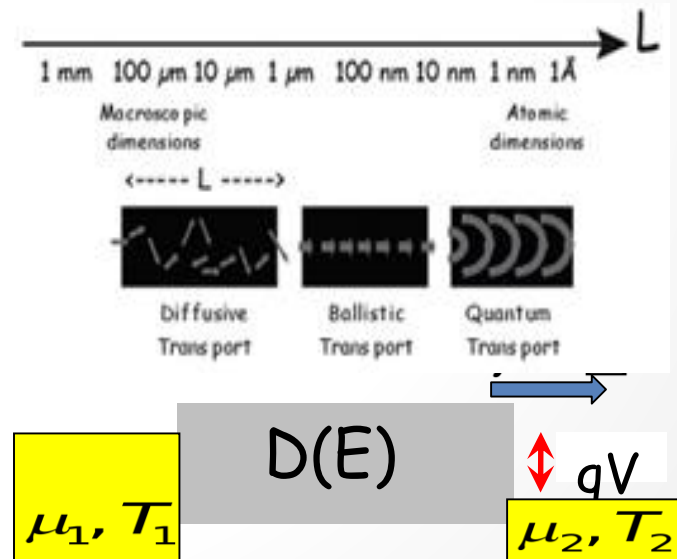
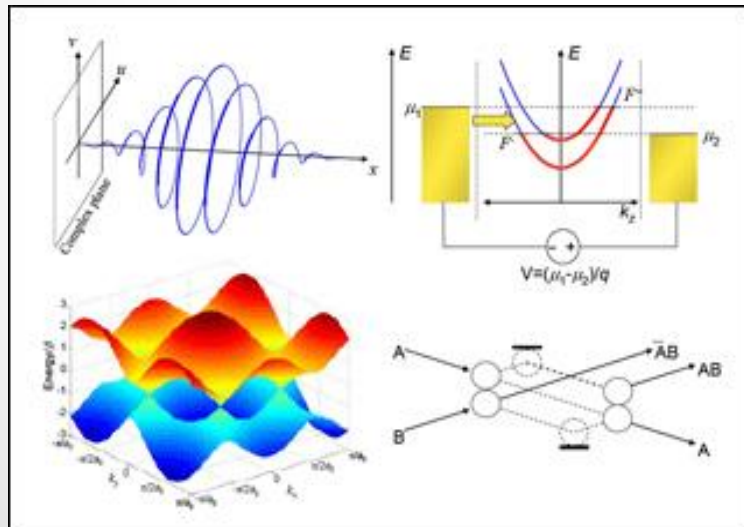
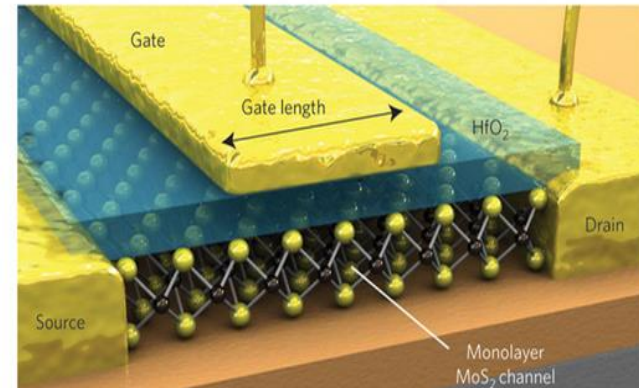
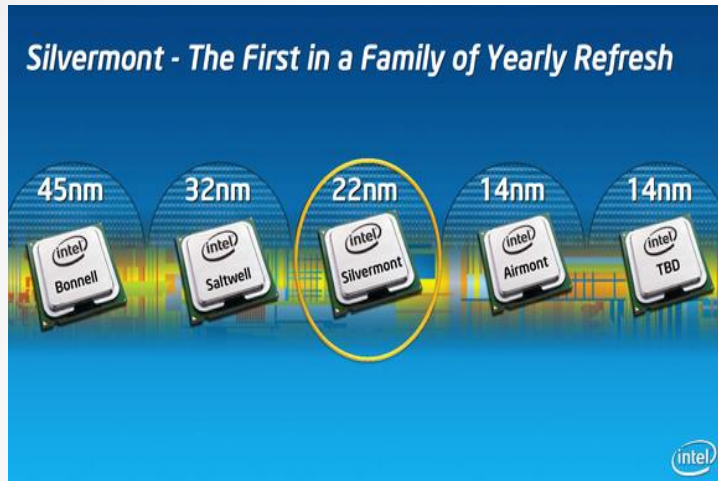
A. Shah, S. Vinjanampathy and B. Muralidharan,
ArXiv: 1706.04299, (2017).



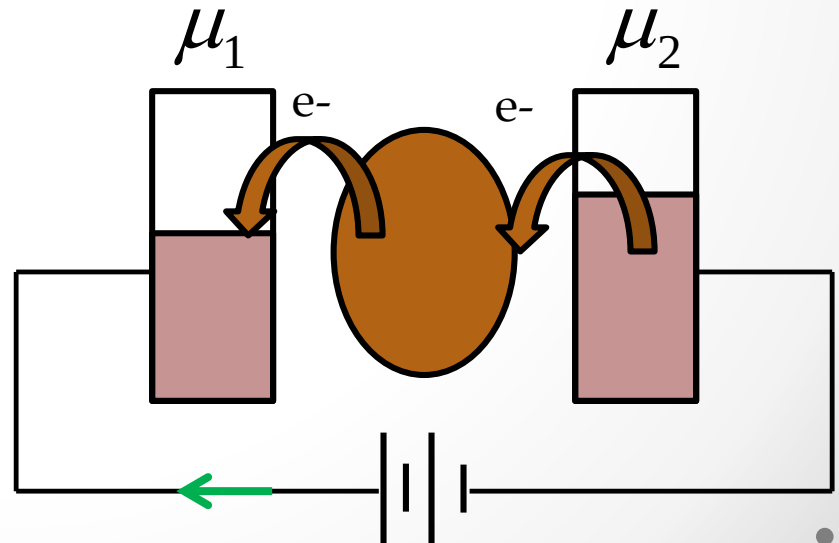
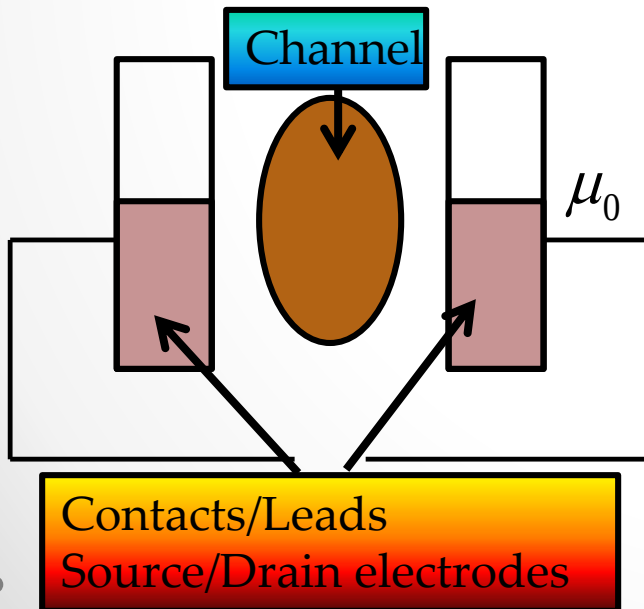
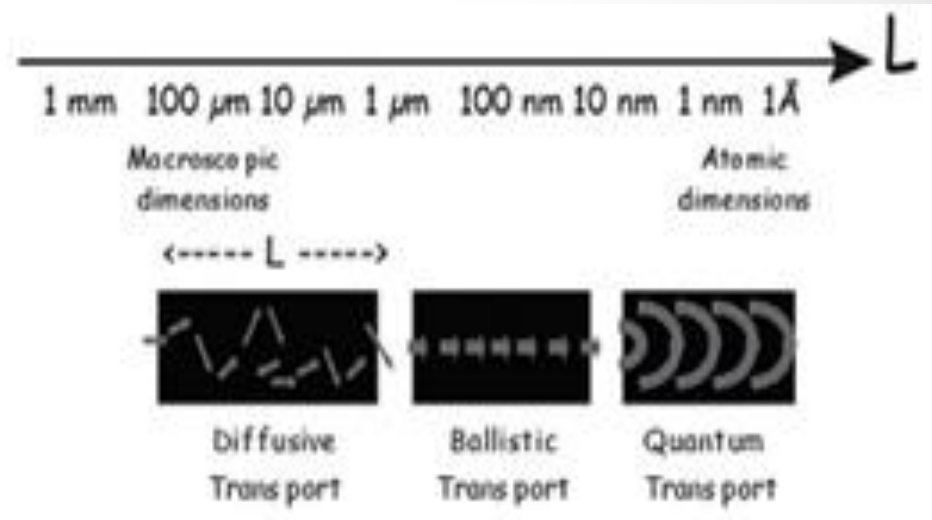
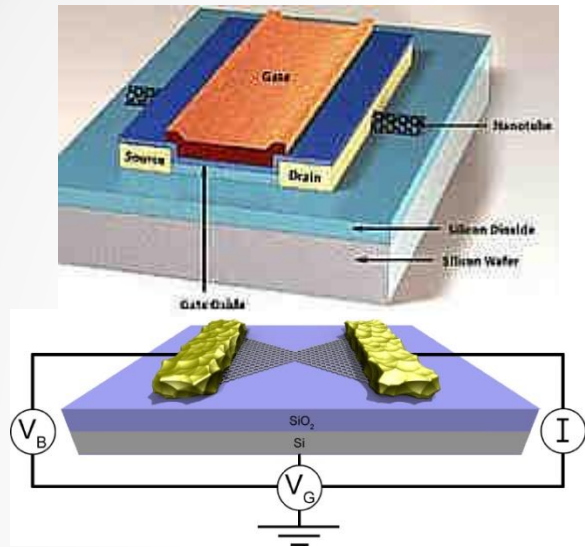
B. Muralidharan and M. Grifoni,
Phys. Rev. B, 88, 045402, (2013).



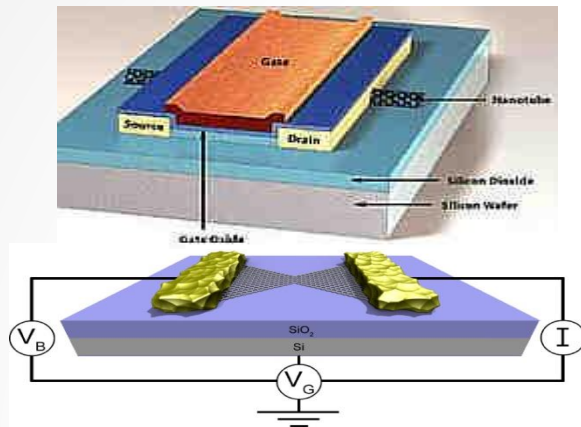
Central Theme of quantum transport: currents at the atomic level



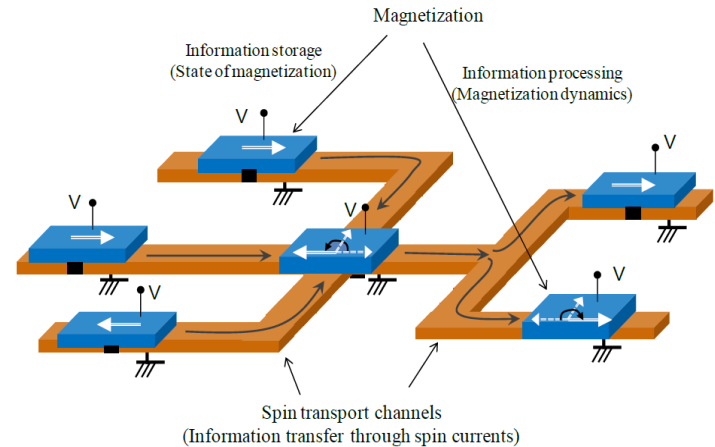
Nano-Devices



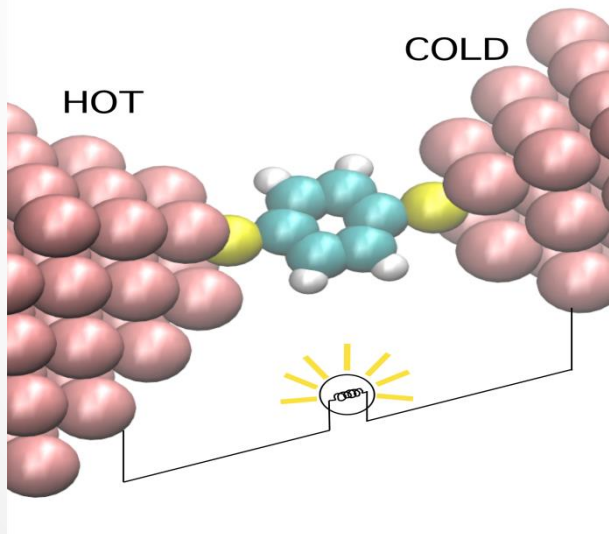
Focus Areas



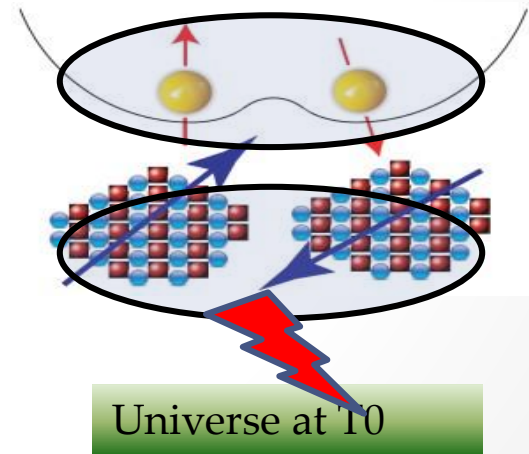
Charge based logic and memories



Spin based logic and memories

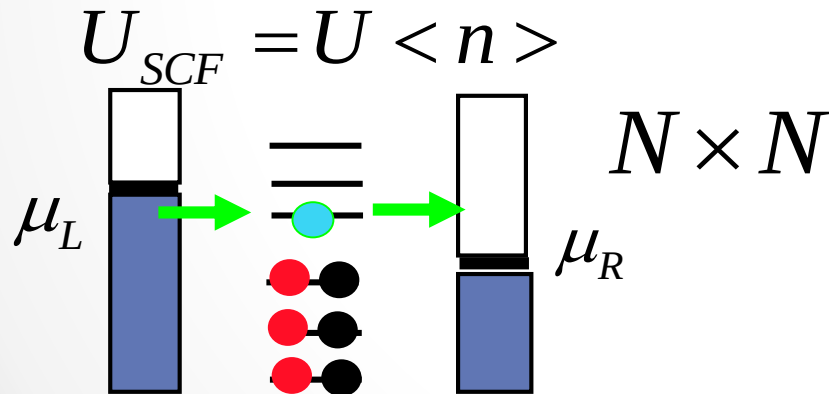
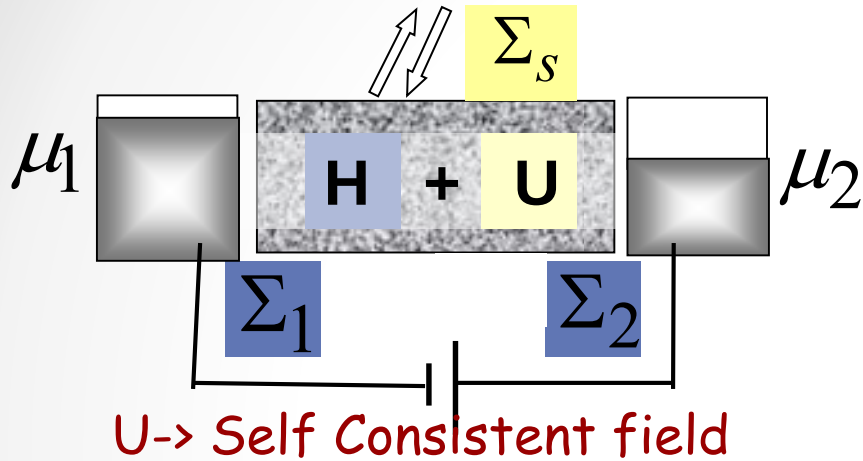


Nanoscale energy transfer



Single spin manipulation for quantum information processing

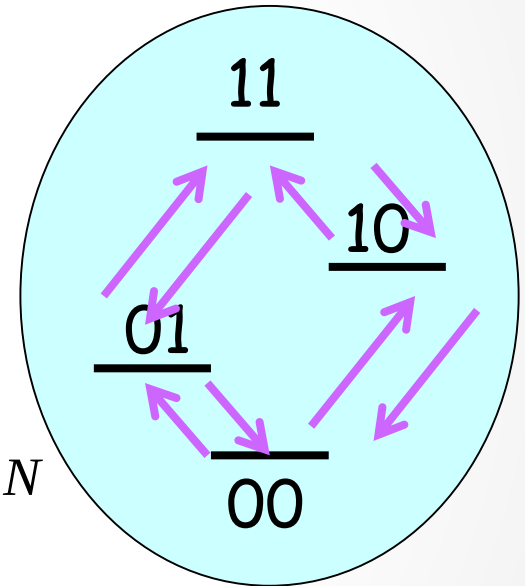
Regimes of transport



SCF Regime

Works for $\Gamma \geq U$

2^N many electron levels



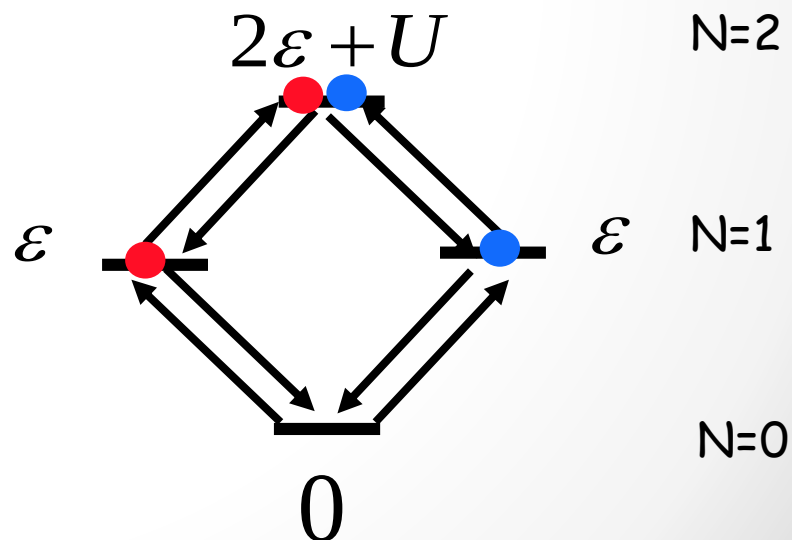
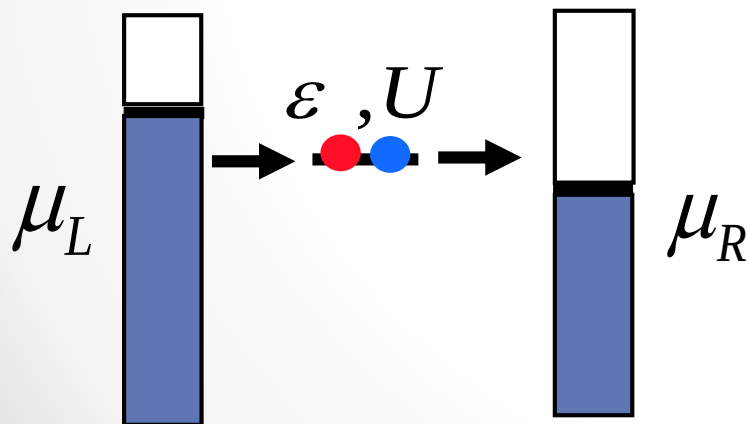
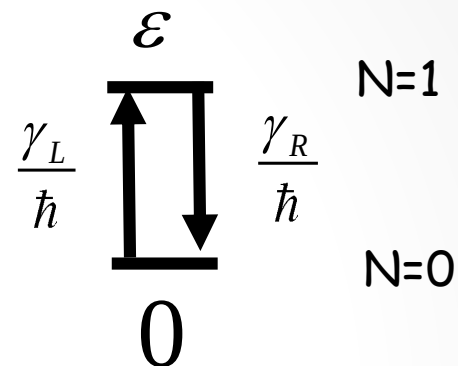
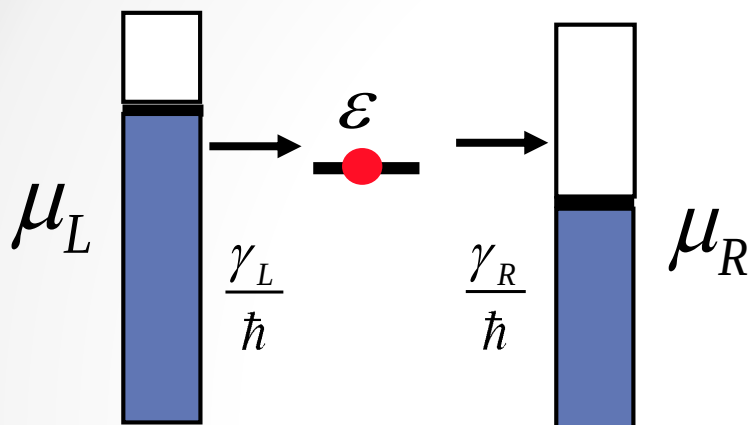
$2^N \times 2^N$

Fock space approach
CWJ Beenakker (1991)

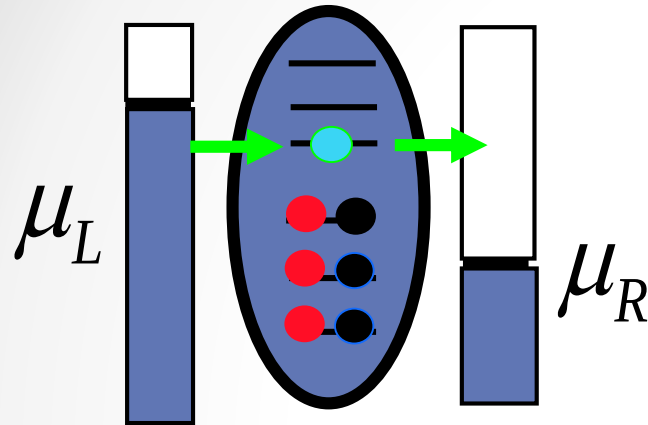
CB Regime

Works for $\Gamma \ll U$

Fock space picture



Generalized Viewpoint



$$N \times N$$

$$N = n$$

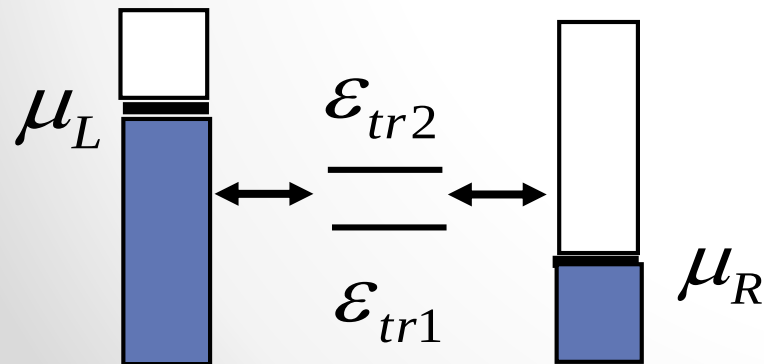
$$N = n + 1$$



$$N = n$$

$$2^N \times 2^N$$

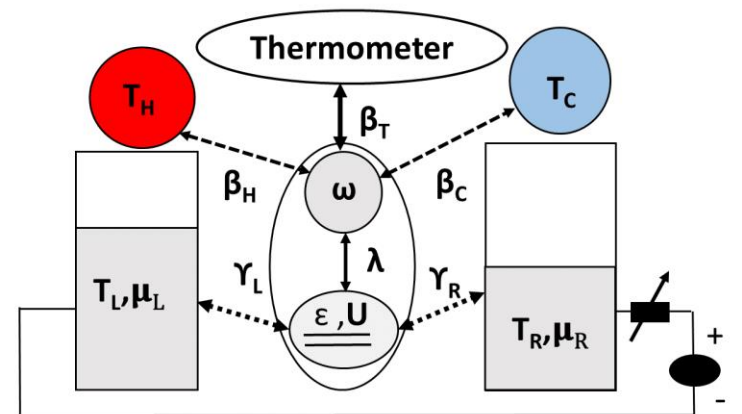
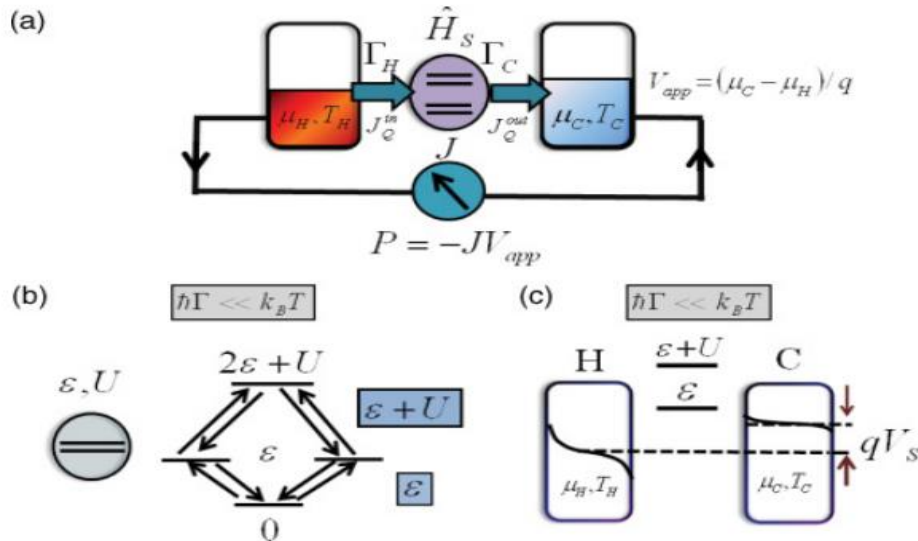
$$\{P_i\}$$



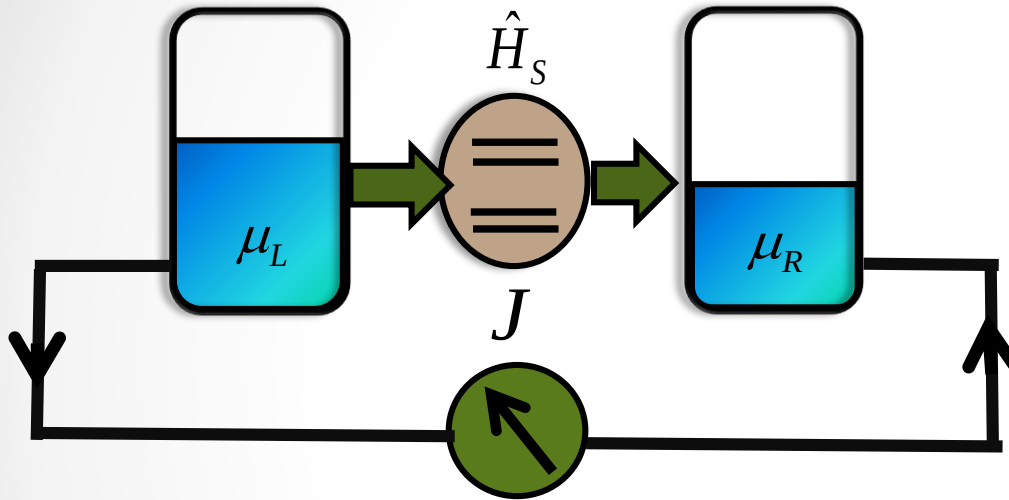
Given a bias point a set of Fock states are probabilistically distributed!

These may be viewed as transition Energies in the one-particle picture

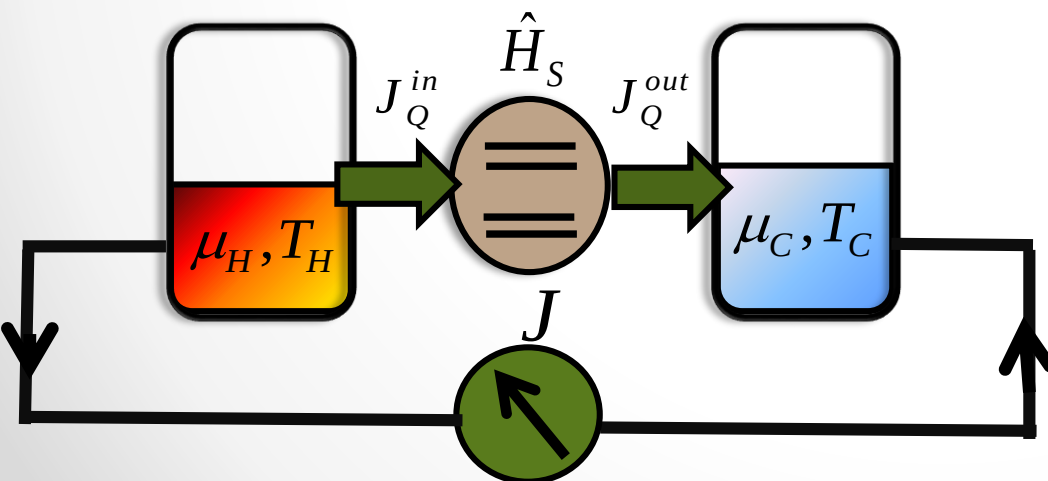
Part II: Quantum Dot Heat Engines



PRELIMINARIES



Electronic transport in the Presence of electrochemical Potential gradients.



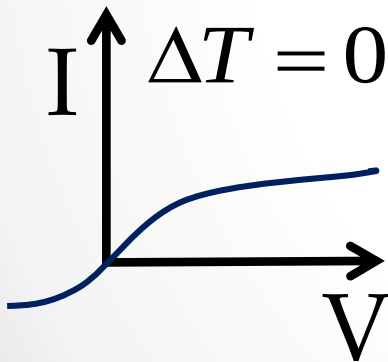
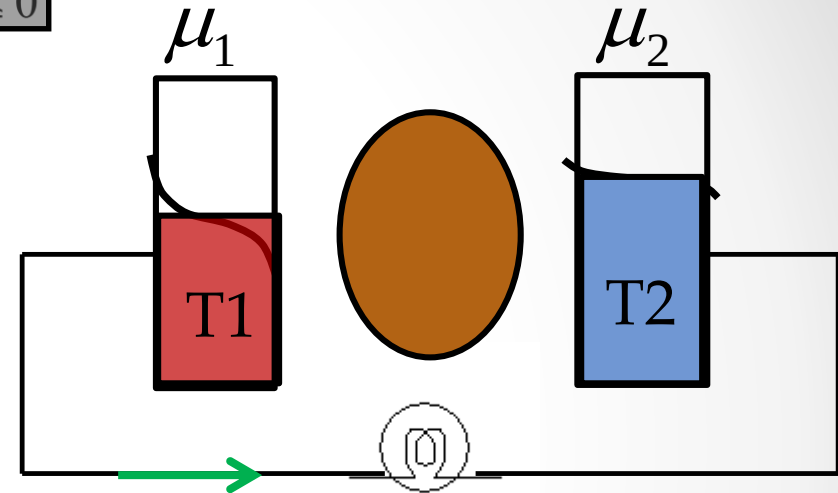
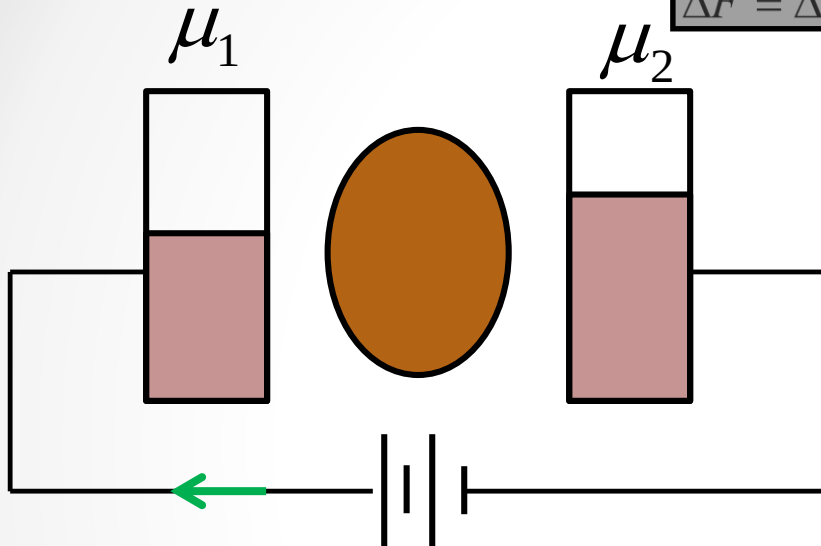
Thermoelectric transport in the presence of electrochemical potential and temperature gradients.

ENERGY CONVERSION!!

Nanoscale Devices/Heat Engines?

$$\Delta S_{tot} \geq 0$$

$$\Delta F = \Delta E - T\Delta S \leq 0$$

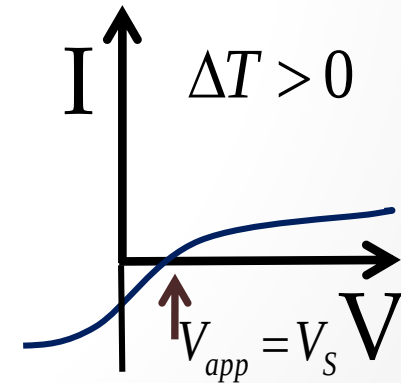


$$E = TS - pV + \mu N + HM$$

$$F = E - TS$$

$$\dot{S} = \sum_n F_n J_n$$

$$\dot{S} = \Delta\left(\frac{1}{T}\right)J_Q - \frac{1}{T}V_{app}J - \frac{1}{T}\Delta\mu_s J_M$$



Onsager-Callen theory

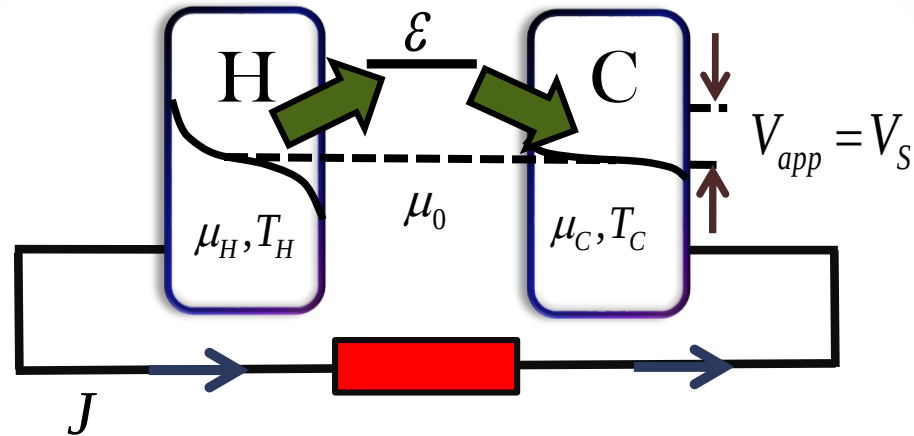
$$\mu_1 N_1 + \mu_2 N_2 \geq 0$$

$$N_1 = -N_2$$

$$\frac{E - \mu_1}{T_1} \geq \frac{E - \mu_2}{T_2}$$

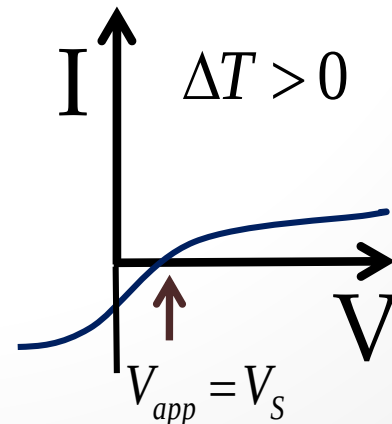
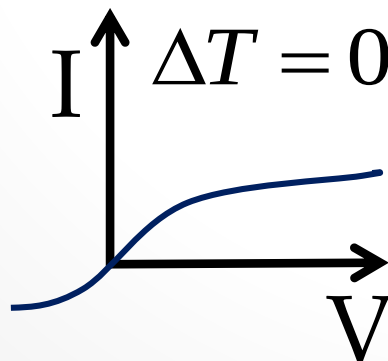
$$N_2 = -N_1$$

Thermodynamics of a resonant state

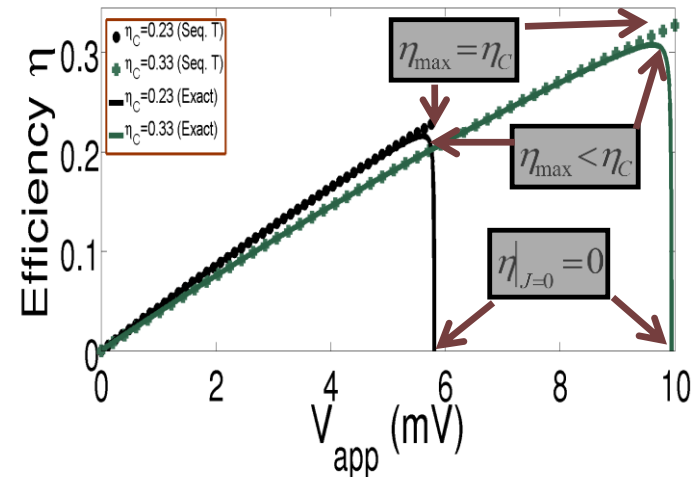
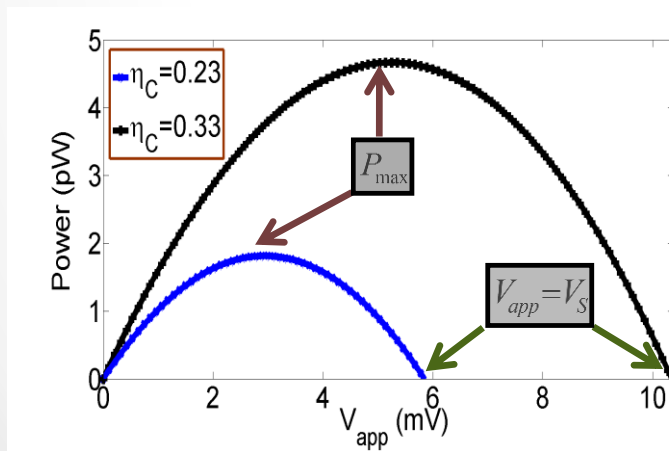
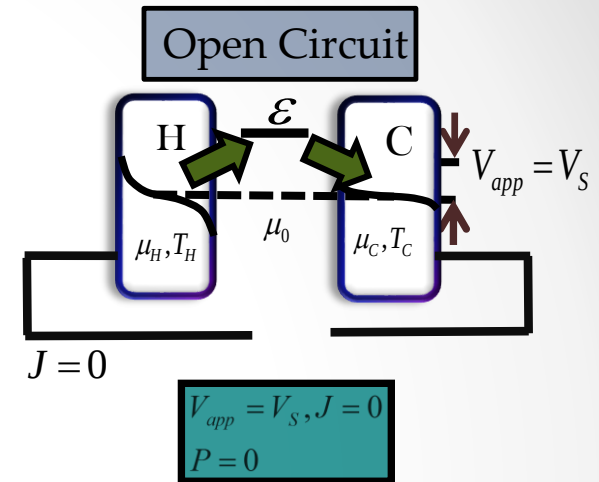
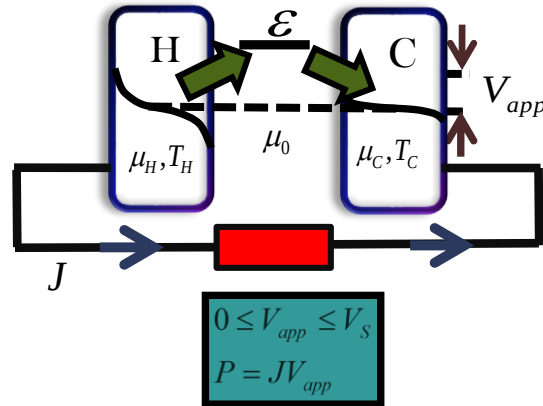
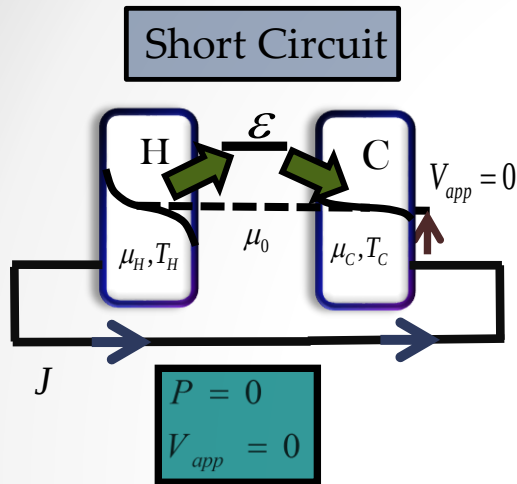


$$\begin{aligned}
 E &= TS - pV + \mu N + HM \\
 \dot{S} &= \sum_n F_n J_n \\
 \dot{S} &= \Delta \left(\frac{1}{T} \right) J_Q - \frac{1}{T} V_{app} J - \frac{1}{T} \Delta \mu_s J_M
 \end{aligned}$$

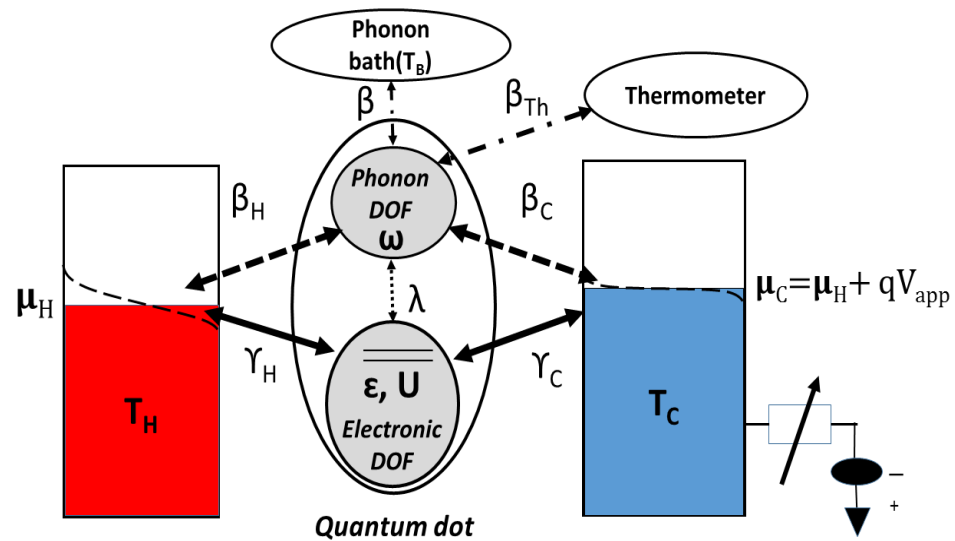
Callen-Onsager theory



Thermoelectric operation: Power



Role of Non-equilibrium phonons



The system Hamiltonian

$$\hat{H}_D = \left(\sum_{\sigma} \epsilon_{\sigma} \hat{n}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow} \right) + \hbar \omega_{\nu} \hat{n}_{\nu} \\ + \sum_{\sigma} \lambda_{\nu} \hbar \omega_{\nu} \hat{n}_{\sigma} (\hat{b}_{\nu}^{\dagger} + \hat{b}_{\nu}),$$

The dot Hamiltonian

$$\hat{H}_C = \sum_{\alpha \in H, C} \sum_{k\sigma'} \epsilon_{\alpha k \sigma'} \hat{n}_{\alpha k \sigma'} + \sum_{\alpha \in H, C} \sum_p \hbar \omega_{\alpha p} \hat{n}_{\alpha p}$$

The contact Hamiltonian

$$\hat{H}_B = \sum_t \hbar \omega_t \hat{B}_t^{\dagger} \hat{B}_t$$

The Bath Hamiltonian

$$\hat{H}_{CD} = \sum_{k\sigma', \sigma} \tau_{\alpha k \sigma' \sigma}^{el} \hat{c}_{\alpha k \sigma'}^{\dagger} \hat{d}_{\sigma} + h.c. \\ + \sum_{\nu, p} \tau_{\alpha p \nu}^{ph} (\hat{a}_{\alpha p}^{\dagger} + \hat{a}_{\alpha p}) (\hat{b}_{\nu}^{\dagger} + \hat{b}_{\nu})$$

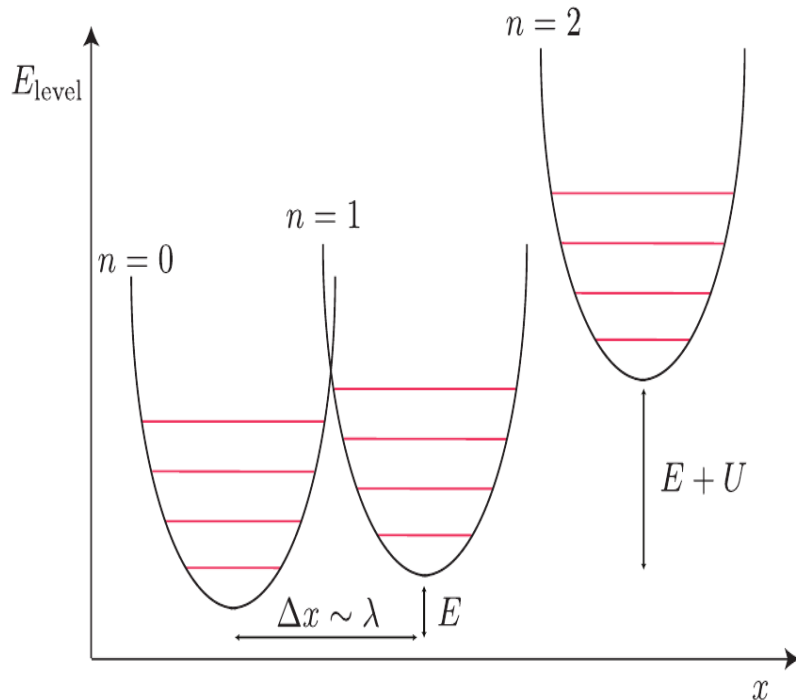
The Contact-dot tunneling Hamiltonian

$$\hat{H}_{BD} = \sum_{\nu, t} \tau_{t\nu}^{ph} (\hat{B}_t^{\dagger} + \hat{B}_t) (\hat{b}_{\nu}^{\dagger} + \hat{b}_{\nu})$$

The Bath-dot phonon transition Hamiltonian

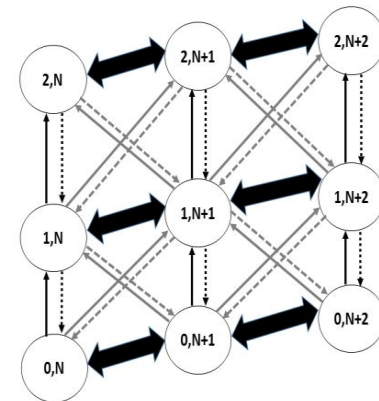
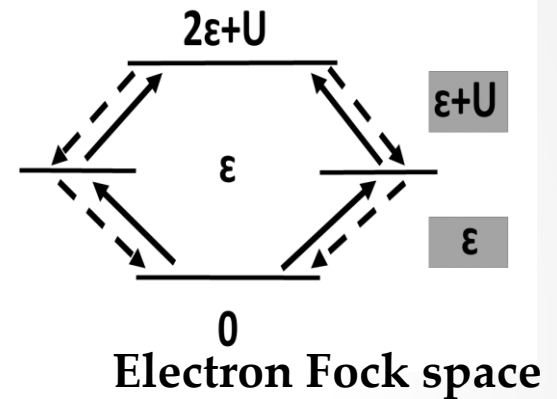
$$\hat{H} = \hat{H}_D + \hat{H}_C + \hat{H}_B + \hat{H}_{CD} + \hat{H}_{BD}$$

The Lang-Firsov transformation



$$\tilde{\epsilon}_{\sigma} = \epsilon_{\sigma} - (\lambda^2 \hbar \omega_{\nu}),$$

$$\tilde{U} = U - (2\lambda^2 \hbar \omega_{\nu}).$$



Electron-phonon Fock space

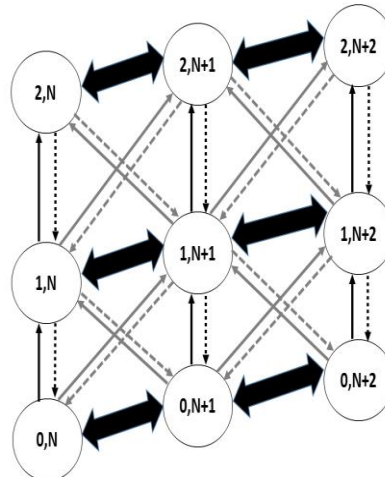
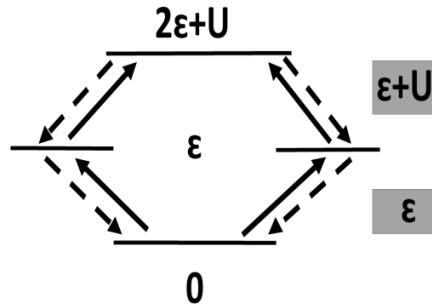
Formulation

$$R_{(n,q) \rightarrow (n+1,q')}^{el} = \sum_{\alpha \in H,C} \gamma_{\alpha} |\langle n, q | \hat{d}_{\sigma}^{\dagger} | n+1, q' \rangle|^2 \times f_{\alpha} \left(\frac{E_{(n+1,q')} - E_{(n,q)} - \mu_{\alpha}}{k_B T_{\alpha}} \right)$$

**Electron tunneling
from contact to dot**

$$R_{(n,q) \rightarrow (n,q+1)}^{ph} = \sum_{r' \in H,C,B} \beta_{r'} (q+1) \exp\left(-\frac{\hbar \omega_{r'}}{k_B T_{r'}}\right)$$

**Bath assisted phonon
absorption**



$$\gamma_{\alpha} = \sum_{k\sigma'} \frac{2\pi}{\hbar} |\tilde{\tau}_{\alpha k \sigma'}^{el}|^2 \delta(E - \epsilon_{\alpha k \sigma'})$$

$$\beta_{r'} = \sum_r \frac{2\pi}{\hbar} |\tilde{\tau}_{r\nu}^{ph}|^2 \delta(\omega - \omega_r)$$

$$R_{(n,q) \rightarrow (n-1,q')}^{el} = \sum_{\alpha \in H,C} \gamma_{\alpha} |\langle n, q | \hat{d}_{\sigma}^{\dagger} | n-1, q' \rangle|^2 \times \left[1 - f_{\alpha} \left(\frac{E_{(n,q)} - E_{(n-1,q')} - \mu_{\alpha}}{k_B T_{\alpha}} \right) \right]$$

**Electron tunneling
from dot to contact**

$$R_{(n,q) \rightarrow (n,q-1)}^{ph} = \sum_{r' \in H,C,B} \beta_{r'} (q+1).$$

**Bath assisted phonon
relaxation**

Currents

$$\frac{dP_{(n,q)}}{dt} = \sum_{k'=0}^{N'_q-1} \left[R_{(n\pm 1, q') \rightarrow (n, q)}^{el} P_{(n\pm 1, q')} - R_{(n, q) \rightarrow (n\pm 1, q')}^{el} P_{(n, q)} \right] + \left[R_{(n, q\pm 1) \rightarrow (n, q)}^{ph} P_{(n, q\pm 1)} - R_{(n, q) \rightarrow (n, q\pm 1)}^{ph} P_{(n, q)} \right].$$

Master equation

$$J_{el\alpha}^Q = \sum_{n, q=0}^{N_e, N_p-1} \sum_{k'}^{N'_p-1} \left[E_{(n\pm 1, q')} - \mu_\alpha \right] \times R_{(n\pm 1, q') \rightarrow (n, q)}^{el\alpha} P_{(n\pm 1, q')} - \left[E_{(N_e, N_p)} - \mu_\alpha \right] \times R_{(n, q) \rightarrow (n\pm 1, q)}^{el\alpha} P_{(n, q)}$$

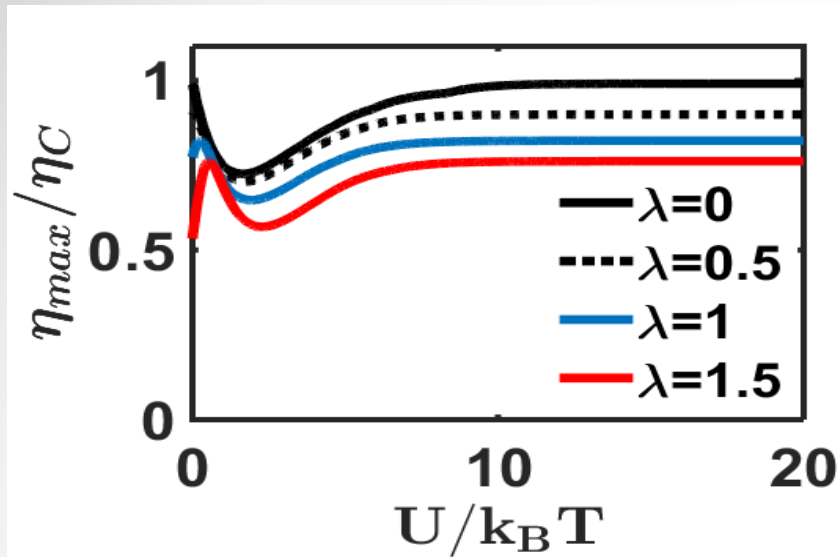
Electron heat current

$$J_\alpha = \sum_{n, q=0}^{N_e, N_p-1} \sum_{q'=0}^{N'_q-1} -q \left[R_{(n\pm 1, q') \rightarrow (n, q)}^{el\alpha} P_{(n\pm 1, q')} - R_{(n, q) \rightarrow (n\pm 1, q')}^{el\alpha} P_{(n, q)} \right]$$

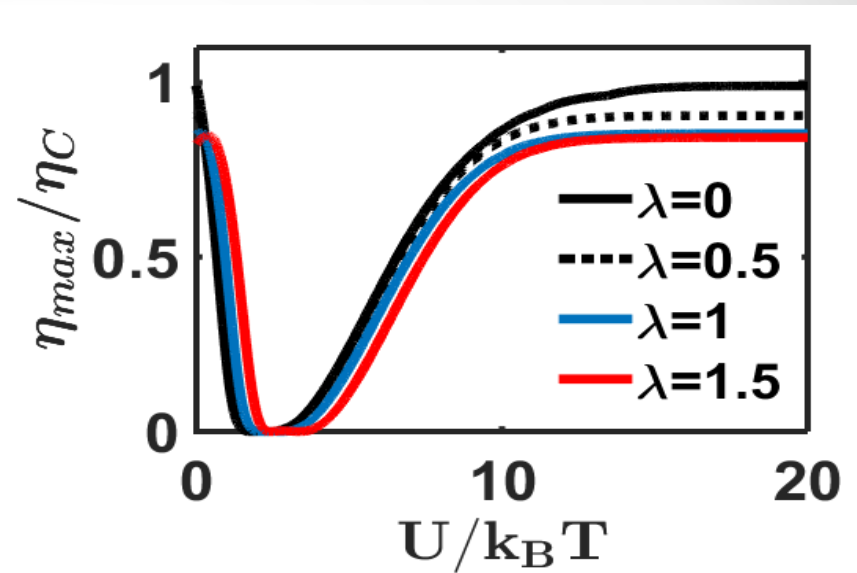
**Electron charge
current**

$$J_{ph, r'}^Q = \sum_{n, q=0}^{N_e, N_p-1} \hbar\omega_r \left[R_{(n, q) \rightarrow (n, q\pm 1)}^{ph\alpha} P_{(n, q)} - R_{(n, q\pm 1) \rightarrow (n, q)}^{ph\alpha} P_{(n, q\pm 1)} \right]$$

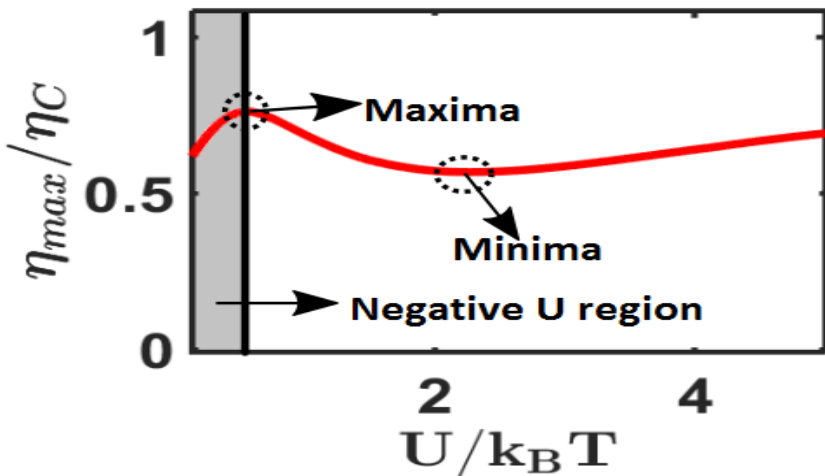
**Phonon heat
current**



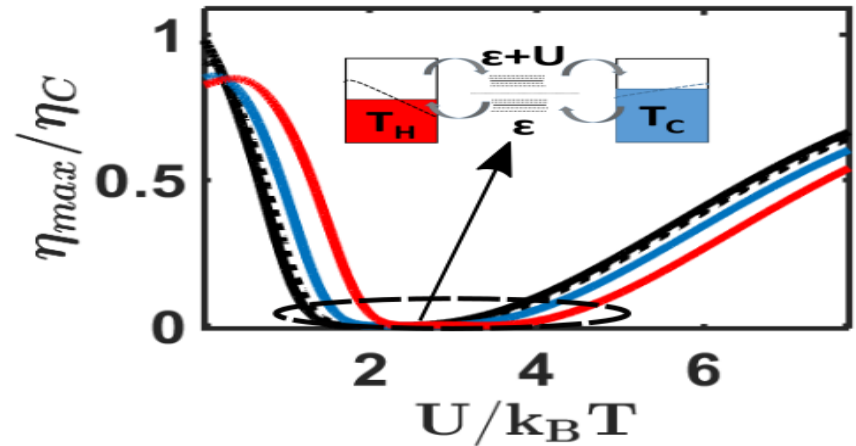
n-type quantum dot



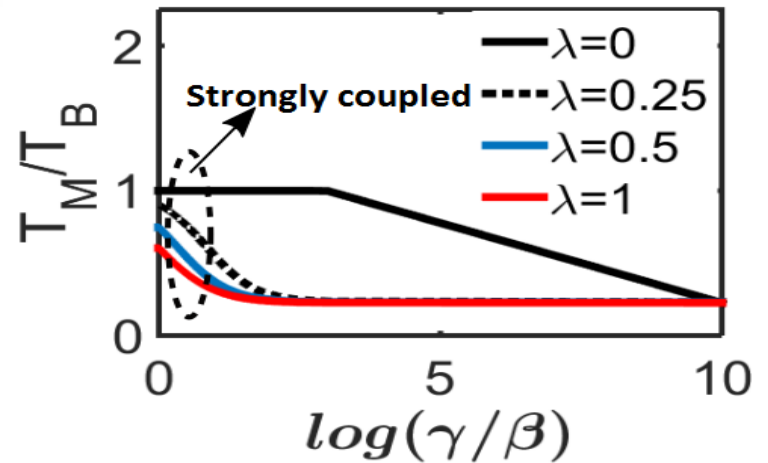
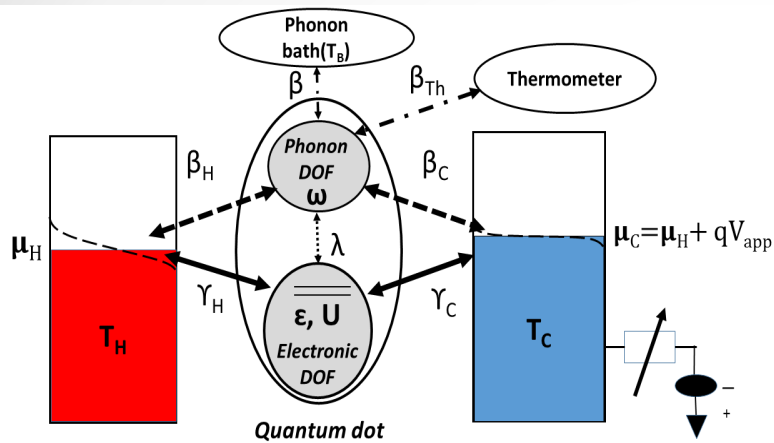
p-type quantum dot



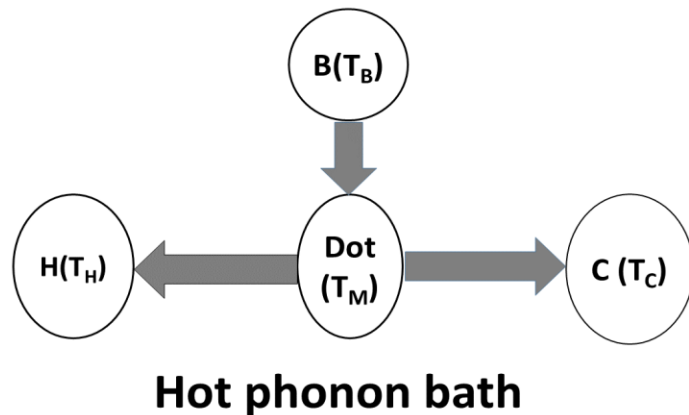
Negative U region



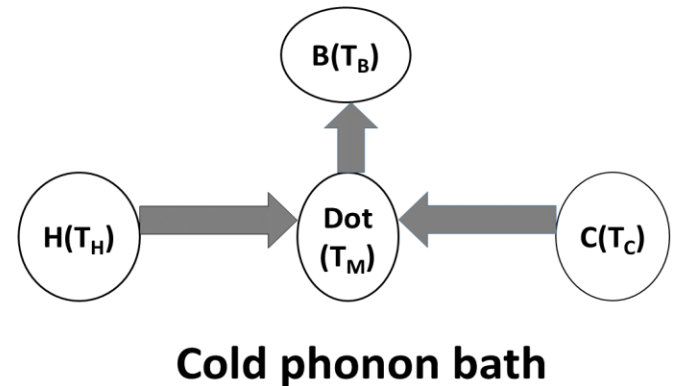
Particle-hole symmetry

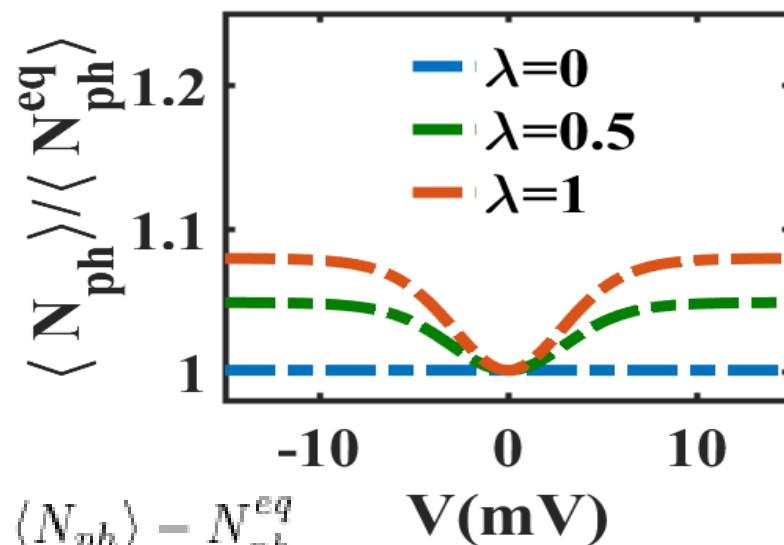
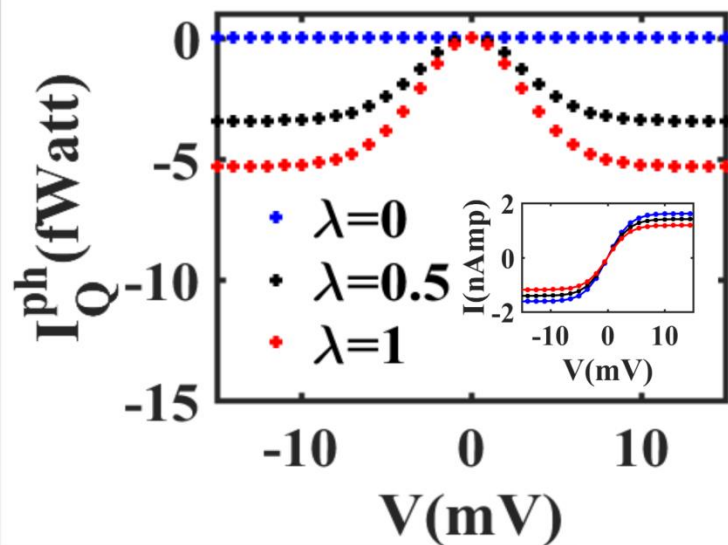


Circuit diagram of the set-up

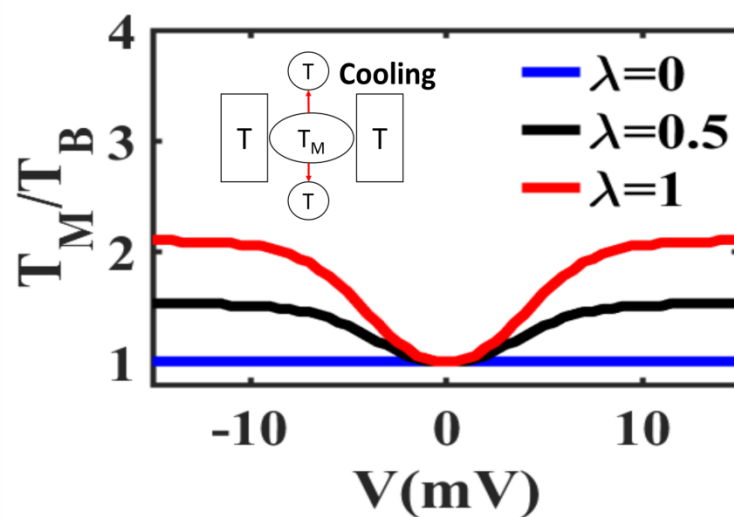
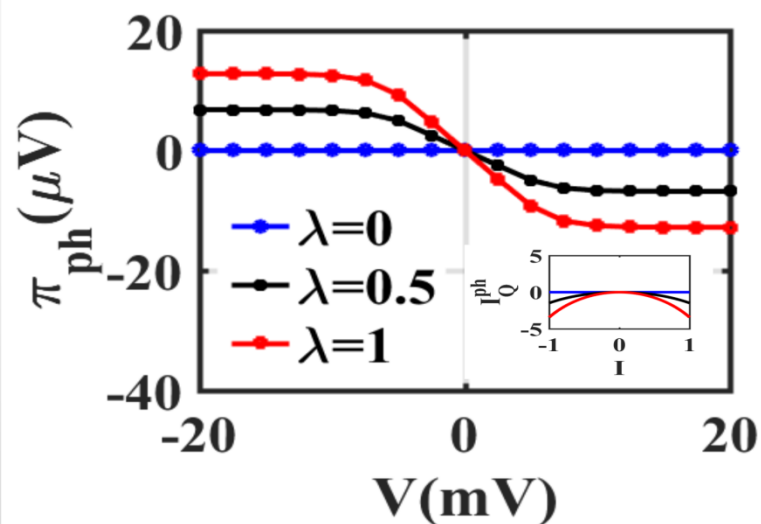


Deviation of dot temperature with interaction and bath coupling





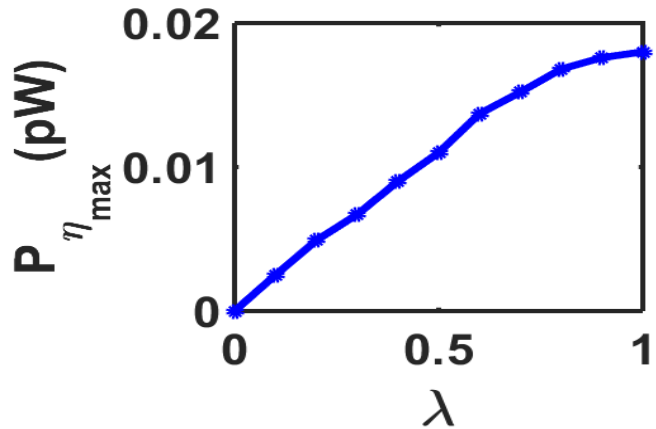
$$RE_{ph}^{\alpha_2} = \beta_{\alpha_2} \frac{\langle N_{ph} \rangle - N_{ph}^{\text{eq}}}{1 + N_{ph}^{\text{eq}}}.$$



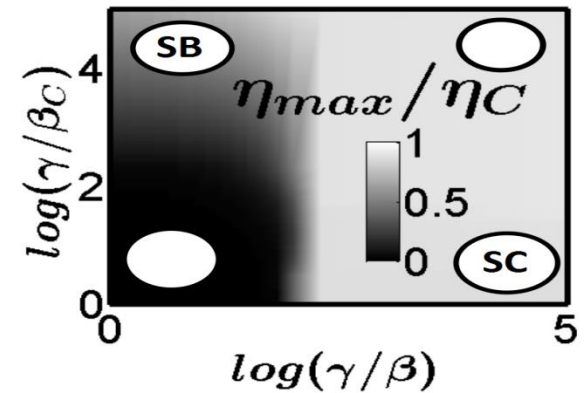
Phonon Peltier coefficient

Peltier cooling

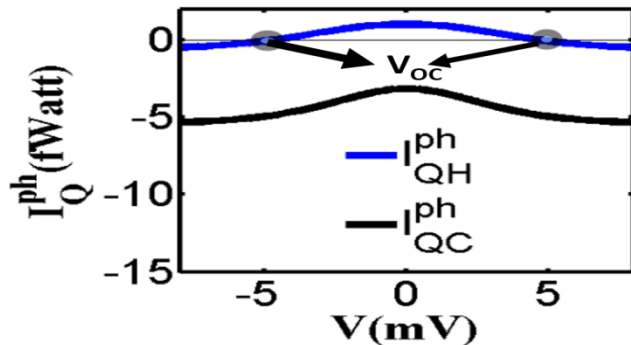
Applications



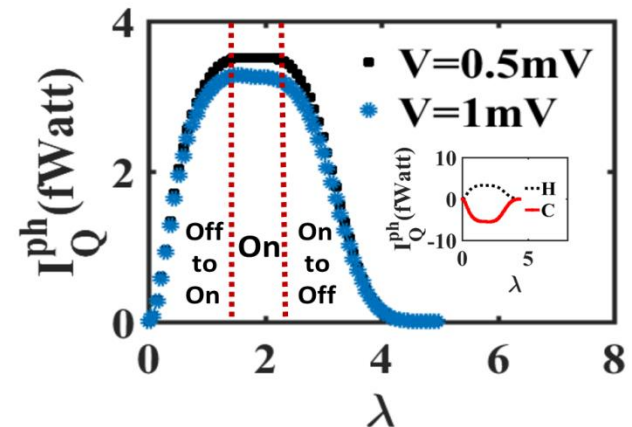
Power-efficiency trade off



Optimization of efficiency

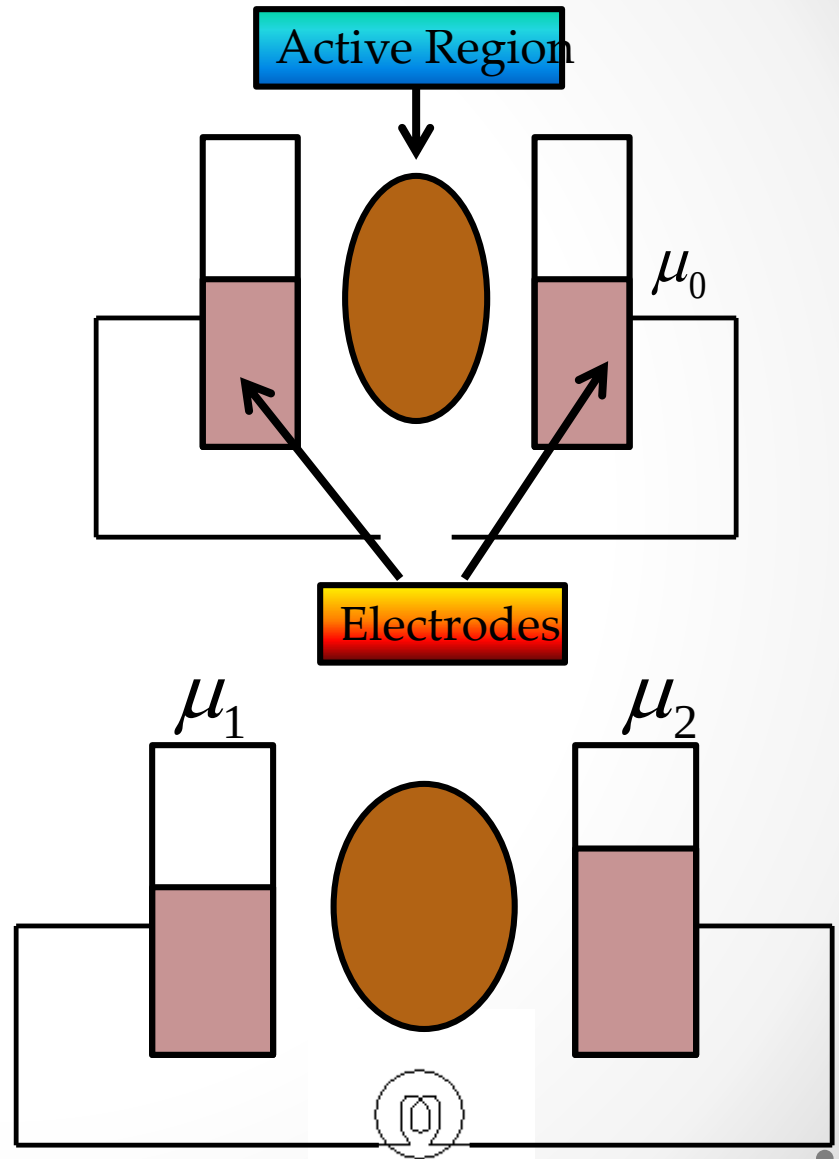
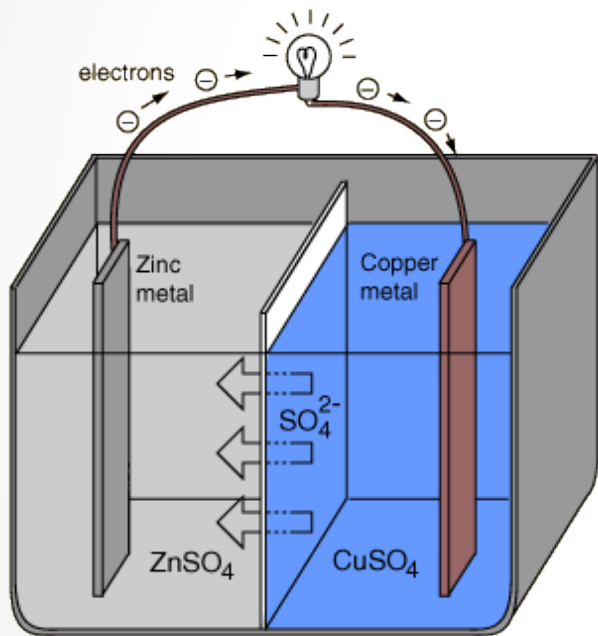


Cooling through Hot bath

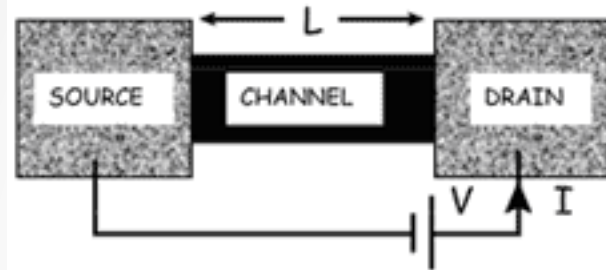


Electron induced phonon switching

Battery Operation Schematic



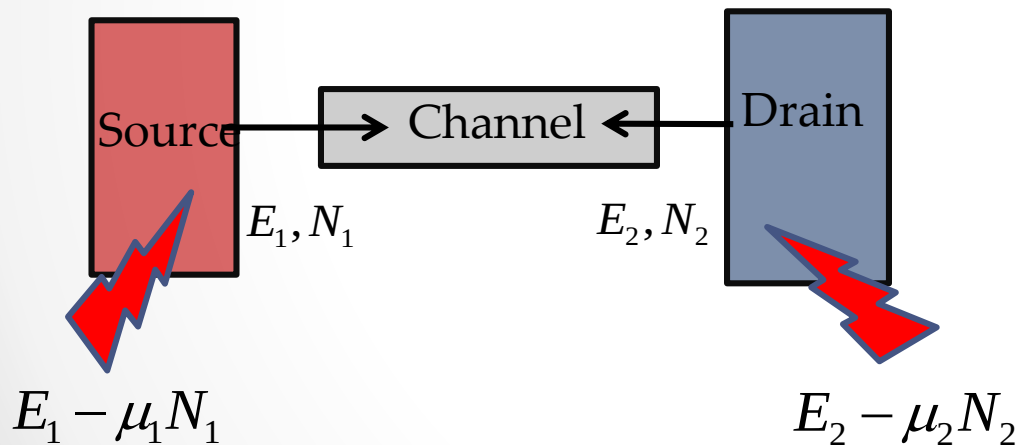
Nanoscale Devices-The energy picture



$$N_1 + N_2 = 0$$

$$E_1 + E_2 = 0$$

$$\frac{E_1 - \mu_1 N_1}{T_1} + \frac{E_2 - \mu_2 N_2}{T_2} \leq 0$$

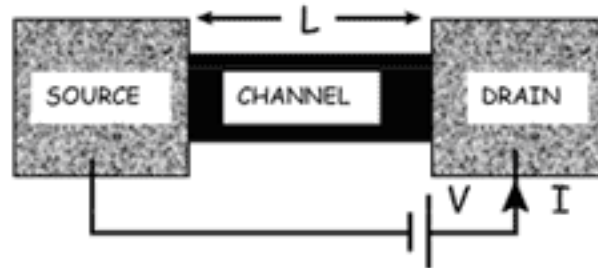


$$\frac{E_1 - \mu_1 N_1}{T_1} + \frac{E_2 - \mu_2 N_2}{T_2} \leq 0$$

$$-\Delta S_1 - \Delta S_2 \leq 0$$

Heat Engines with demons

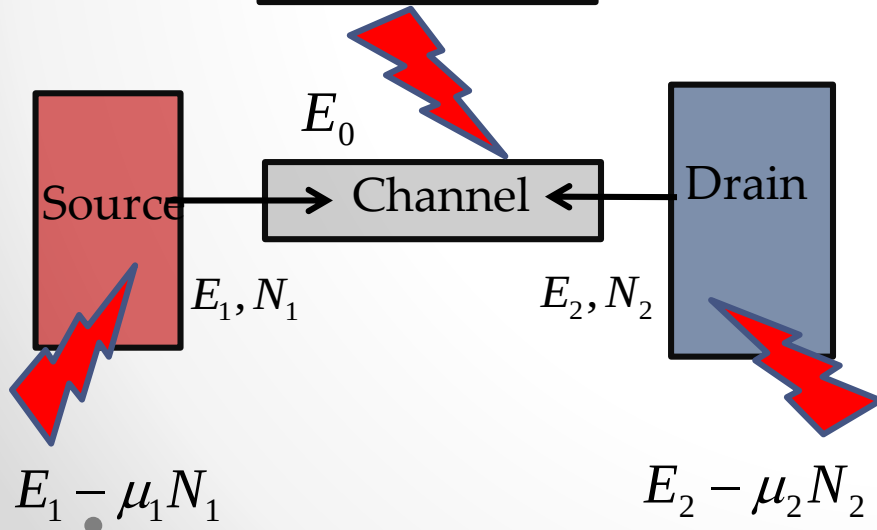
$$\begin{aligned}
 N_1 + N_2 &= 0 \\
 E_1 + E_2 + E_0 &= 0 \\
 \frac{E_1 - \mu_1 N_1}{T_1} + \frac{E_2 - \mu_2 N_2}{T_2} + \frac{E_0}{T_0} &\leq 0
 \end{aligned}$$



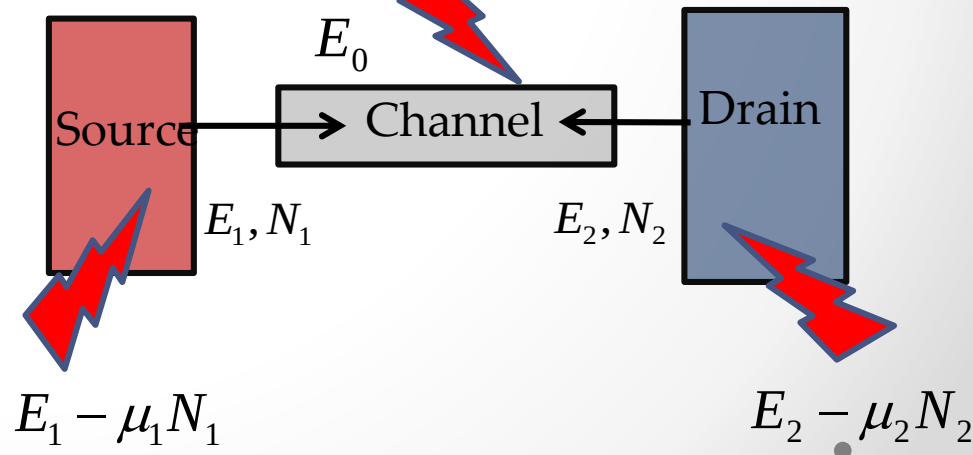
$$\begin{aligned}
 N_1 + N_2 &= 0 \\
 E_1 + E_2 + E_0 &= 0 \\
 \frac{E_1 - \mu_1 N_1}{T_1} + \frac{E_2 - \mu_2 N_2}{T_2} - \Delta S_0 &\leq 0
 \end{aligned}$$

$$S = k \ln W$$

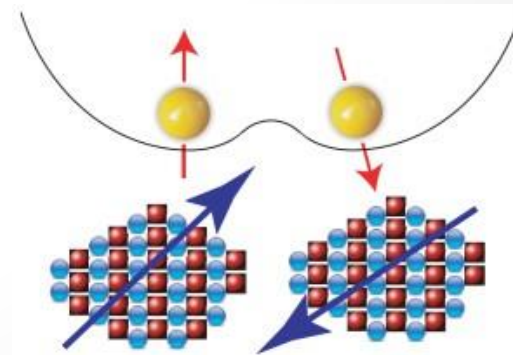
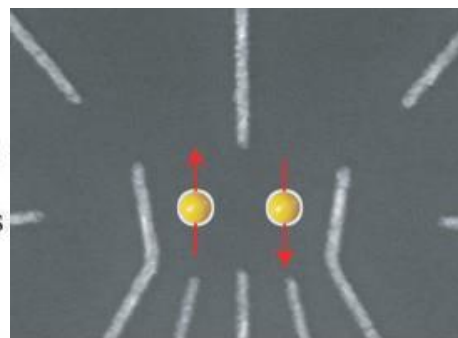
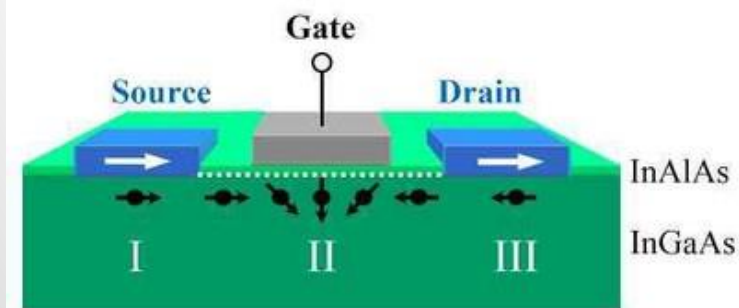
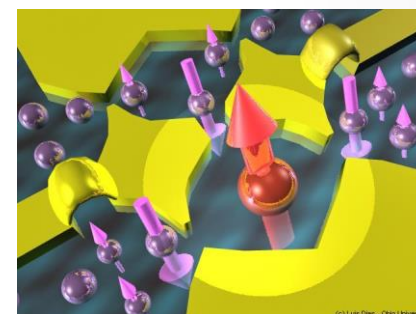
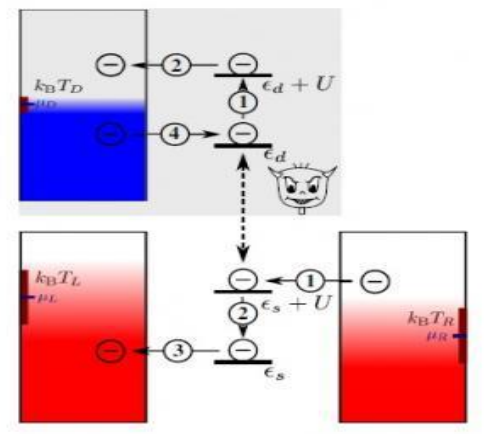
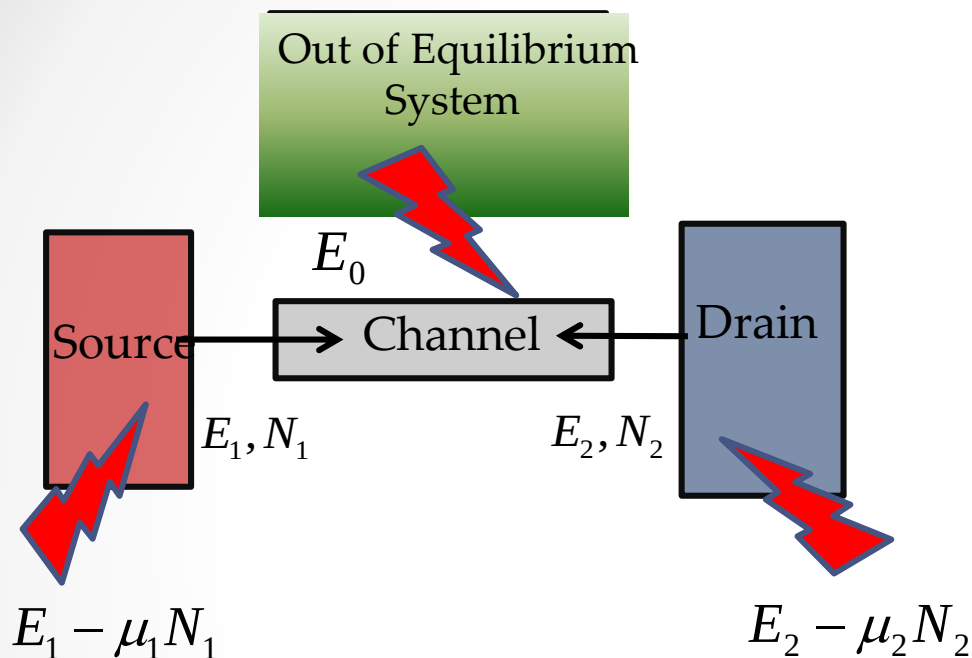
Reservoir/Bath
at T_0



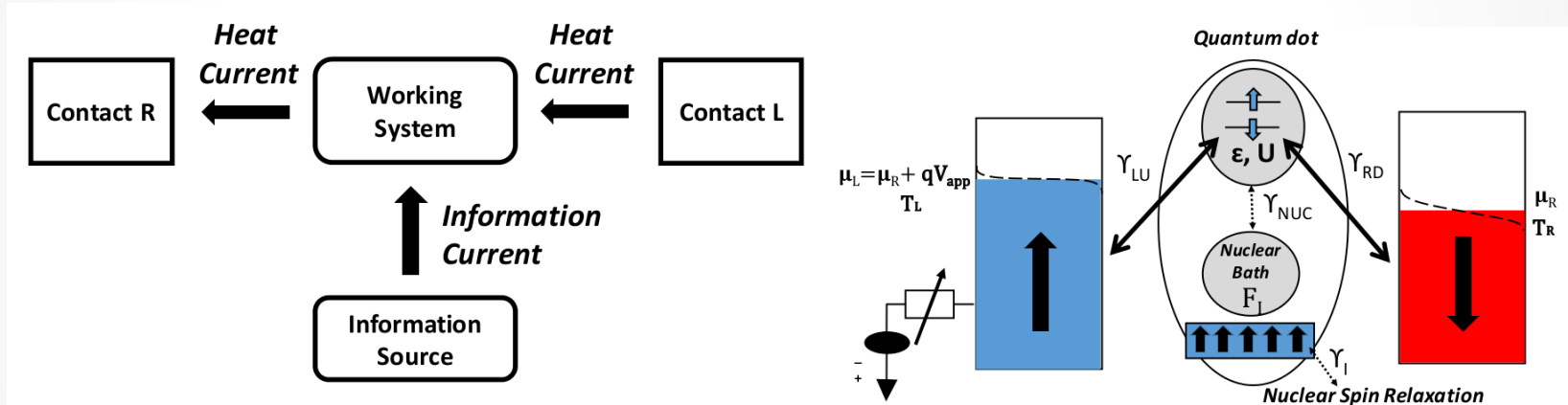
Out of Equilibrium
System
"Demon"



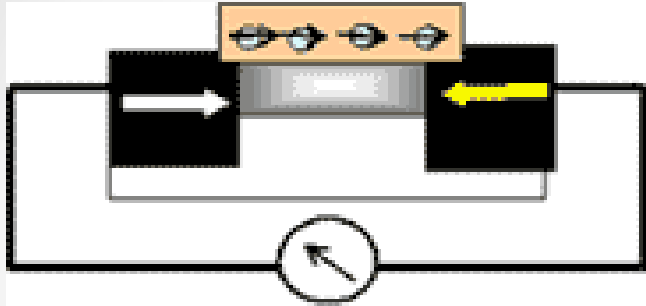
Examples of nano-device “demons”



Part III: Information driven thermal machine



Landauer writing and erasure



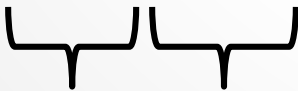
$$N_1 + N_2 = 0$$

$$E_1 + E_2 + E_0 = 0$$

$$\frac{E_1 - \mu_1 N_1}{T_1} + \frac{E_2 - \mu_2 N_2}{T_2} - \Delta S_0 \leq 0$$

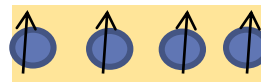
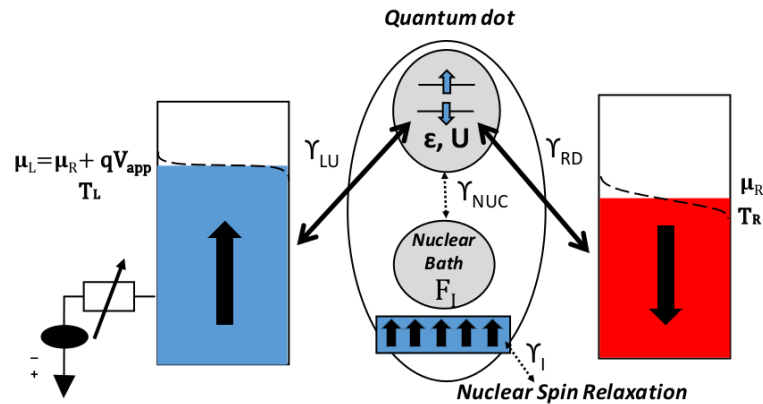
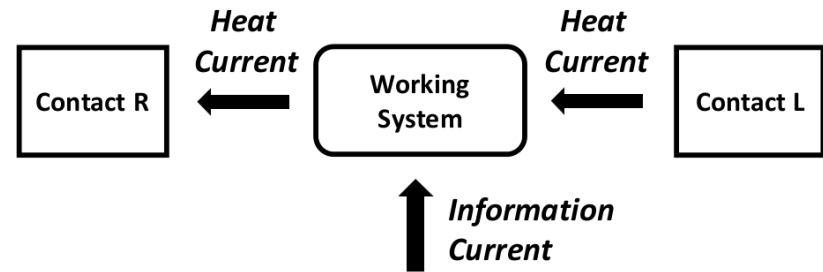
$$\Delta S_{tot} \geq 0$$

$$\Delta F = \Delta E - T\Delta S \leq 0$$

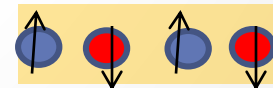


ENERGY

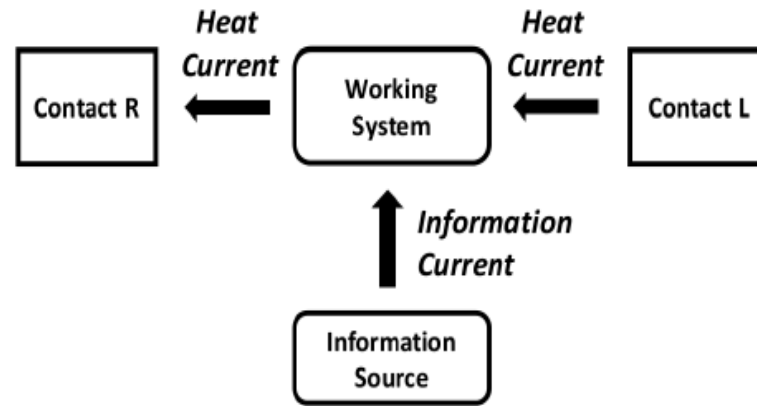
INFORMATION



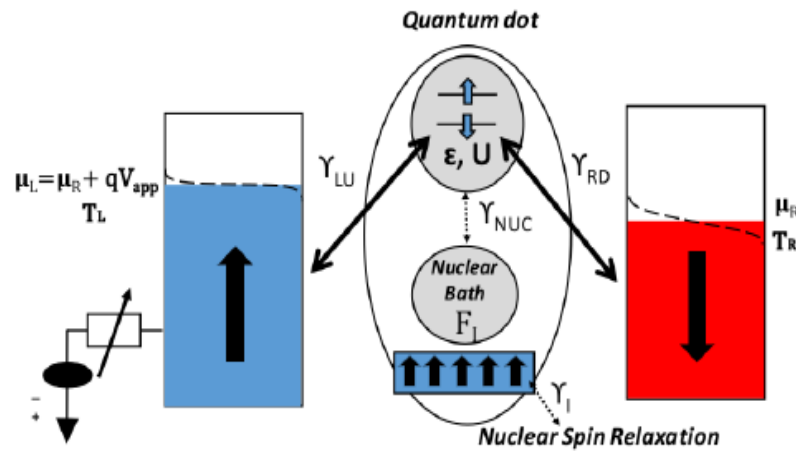
v/s



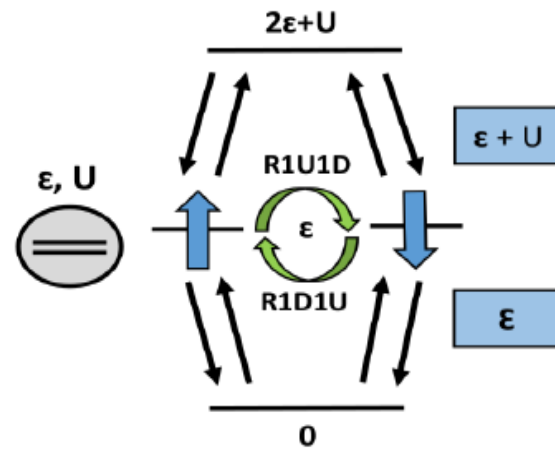
Hyperfine thermal machine



(a)

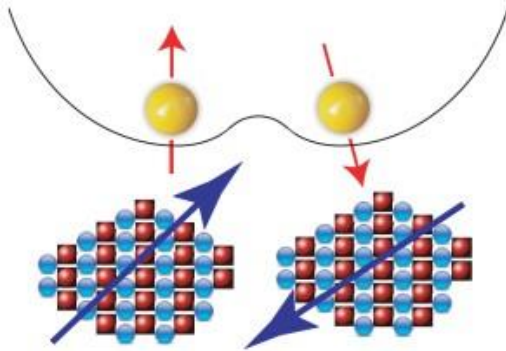


(b)

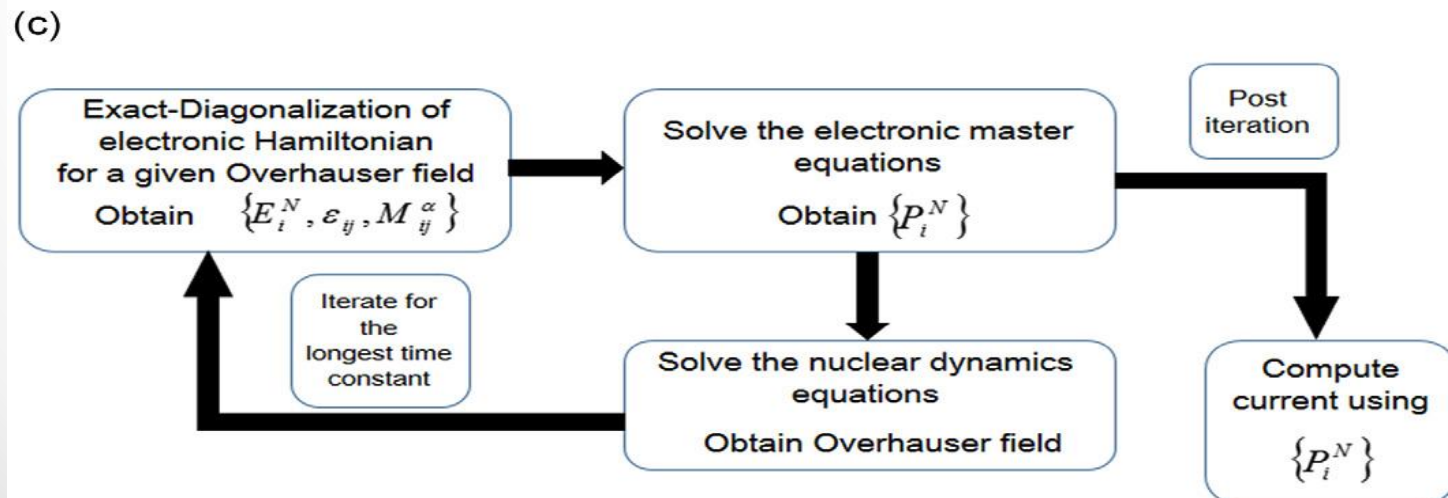
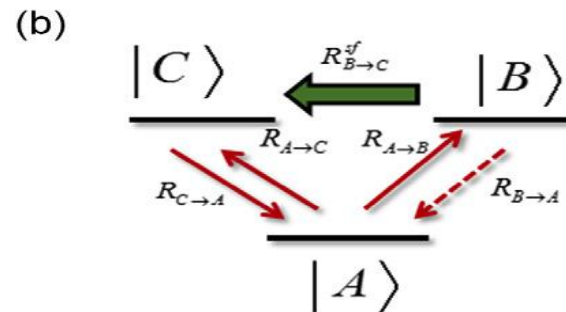
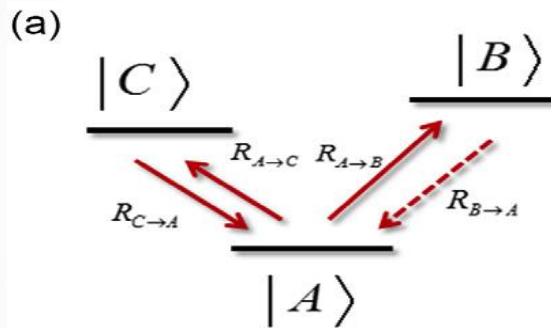


(c)

Hyperfine interaction basics



$$\hat{H}_{HF}(\mathbf{r}_n) = \sum_n A_{eff} \psi^*(\mathbf{r}_n) \psi(\mathbf{r}_n) \mathbf{a}_0^3 \left[\hat{\mathbf{S}}_z \otimes \hat{\mathbf{I}}_z^n + \left\{ \frac{\hat{\mathbf{I}}_+^n \otimes \hat{\mathbf{S}}_- + \hat{\mathbf{S}}_+ \otimes \hat{\mathbf{I}}_-^n}{2} \right\} \right]$$



SQD: Basic Formulation

$$\frac{dP_0^0}{dt} = -R_{(0,0) \rightarrow (1,\uparrow)} P_0^0 - R_{(0,0) \rightarrow (1,\downarrow)} P_0^0$$

$$+ R_{(1,\uparrow) \rightarrow (0,0)} P_{\uparrow}^1 + R_{(1,\downarrow) \rightarrow (0,0)} P_{\downarrow}^1$$

$$\frac{dP_{\uparrow}^1}{dt} = -R_{(1,\uparrow) \rightarrow (0,0)} P_{\uparrow}^1 - R_{(1,\uparrow) \rightarrow (2,0)} P_{\uparrow}^1$$

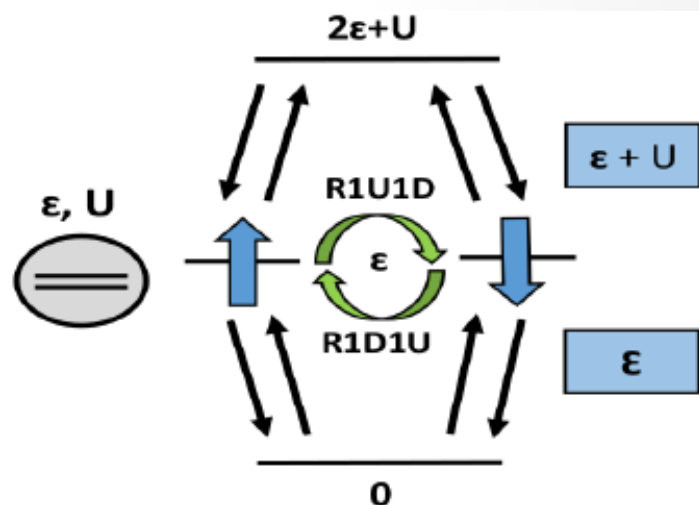
$$+ R_{(2,0) \rightarrow (1,\uparrow)} P_0^2 + R_{(0,0) \rightarrow (1,\uparrow)} P_0^0$$

$$+ R_{(1,\downarrow) \rightarrow (1,\uparrow)}^{sf} P_{\downarrow}^1 - R_{(1,\uparrow) \rightarrow (1,\downarrow)}^{sf} P_{\uparrow}^1$$

$$\frac{dP_{\downarrow}^1}{dt} = -R_{(1,\downarrow) \rightarrow (0,0)} P_{\downarrow}^1 - R_{(1,\downarrow) \rightarrow (2,0)} P_{\downarrow}^1$$

$$+ R_{(2,0) \rightarrow (1,\downarrow)} P_0^2 + R_{(0,0) \rightarrow (1,\downarrow)} P_0^0$$

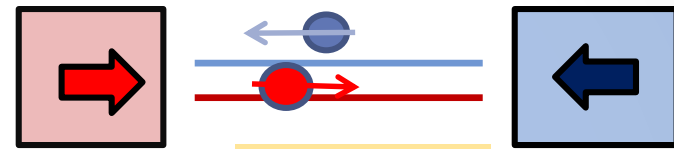
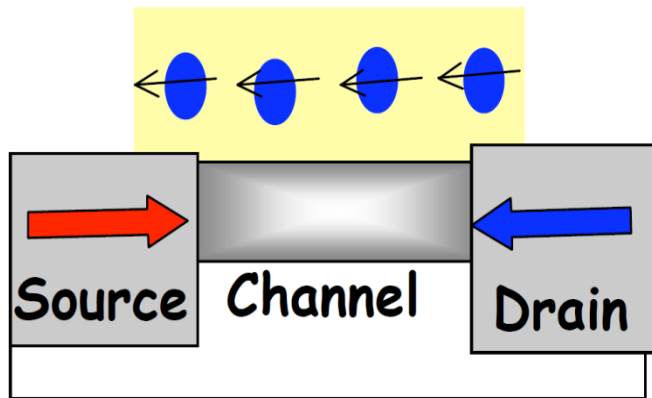
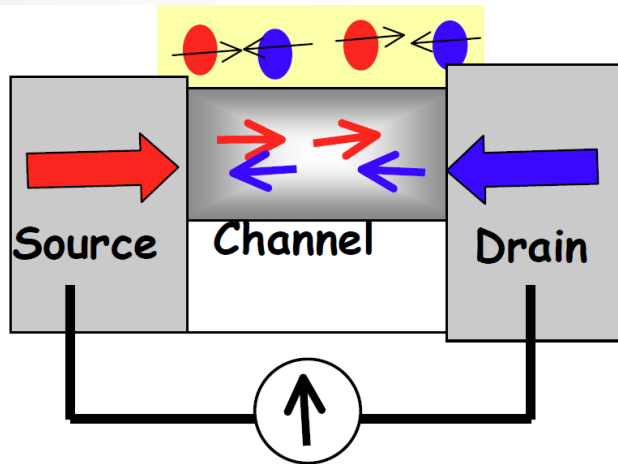
$$+ R_{(1,\uparrow) \rightarrow (1,\downarrow)}^{sf} P_{\uparrow}^1 - R_{(1,\downarrow) \rightarrow (1,\uparrow)}^{sf} P_{\downarrow}^1$$



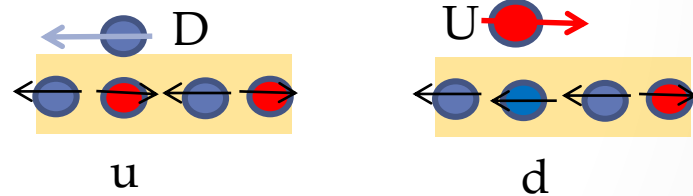
$$\frac{dF_I}{dt} = \gamma^{nuc}[(P_{\uparrow}^1 - P_{\downarrow}^1) - (P_{\uparrow}^1 + P_{\downarrow}^1)F_I] - \gamma_I F_I$$

$$\frac{dS_I}{dt} = k_b \frac{1}{2} \ln \frac{1 + F_I}{1 - F_I} (\gamma^{nuc}[(P_{\uparrow}^1 - P_{\downarrow}^1) - (P_{\uparrow}^1 + P_{\downarrow}^1)F_I] - \gamma_I F_I)$$

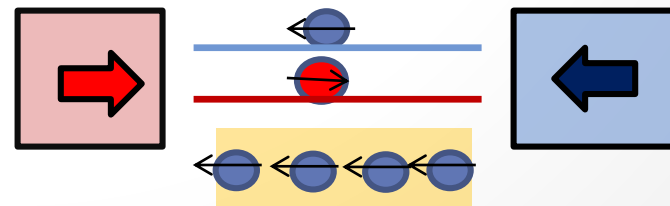
nano-device “demons”+ info battery



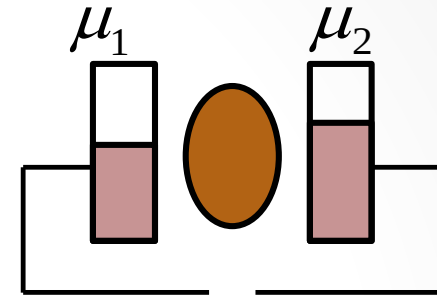
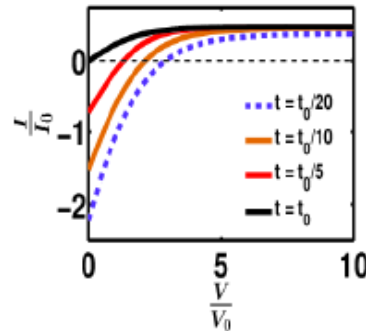
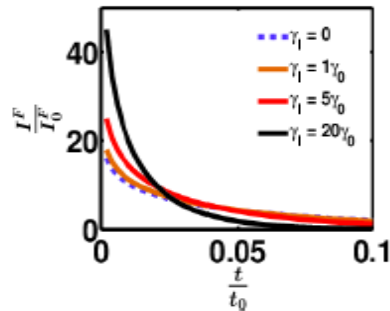
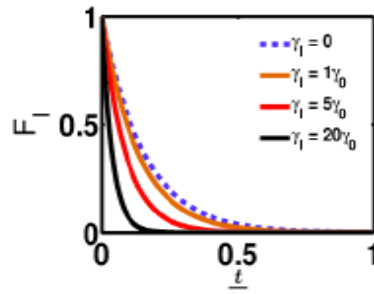
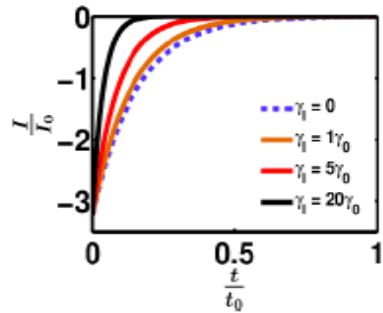
$$H = J \vec{S}_1 \cdot \vec{S}_2$$



$$u+D \longleftrightarrow d+U$$



nano-device “demons”+ info battery



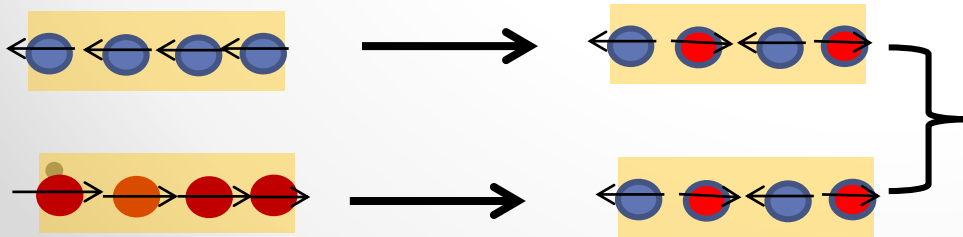
$$\begin{aligned} S_i &= 0 \\ S_f &= Nk \ln 2 \\ \Delta S &= Nk \ln 2 \\ \Delta W &\leq NkT \ln 2 \end{aligned}$$

$$\begin{aligned} S_i &= 0 \\ S_f &= Nk \ln 2 \\ \Delta S &= Nk \ln 2 \end{aligned}$$

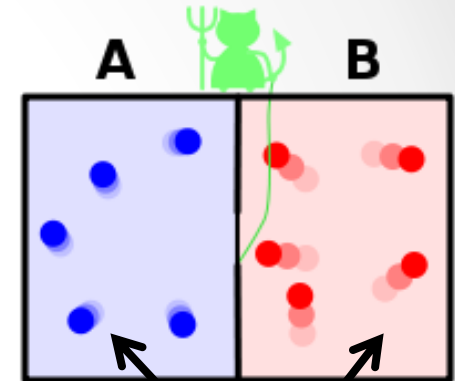
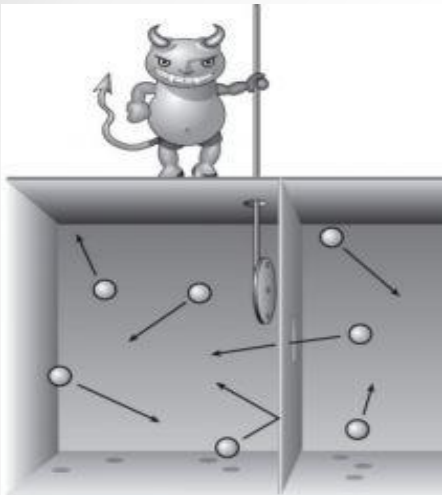
Current Stops to flow eventually!

Where does the energy come from?!

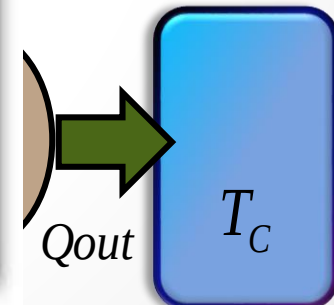
State of the Demons



Connection with Maxwell's Demon



Reservoirs
at T_1 and T_2



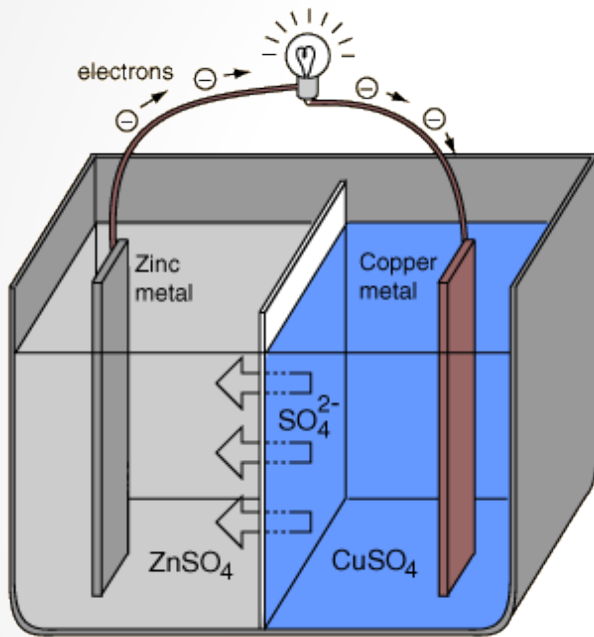
W

Does it no violate an
Common sense?

If not, what is the cau

Demon exorcism by

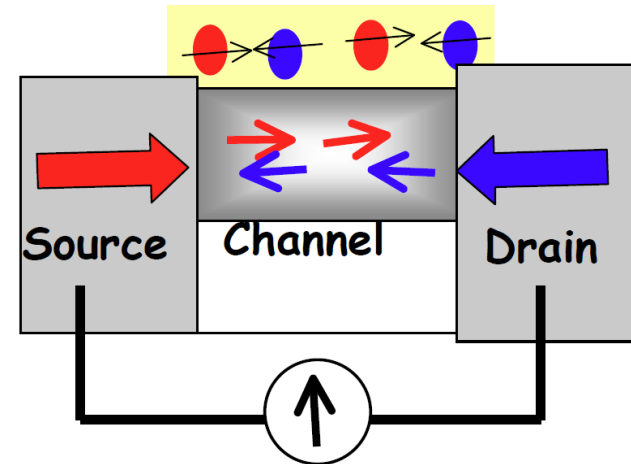
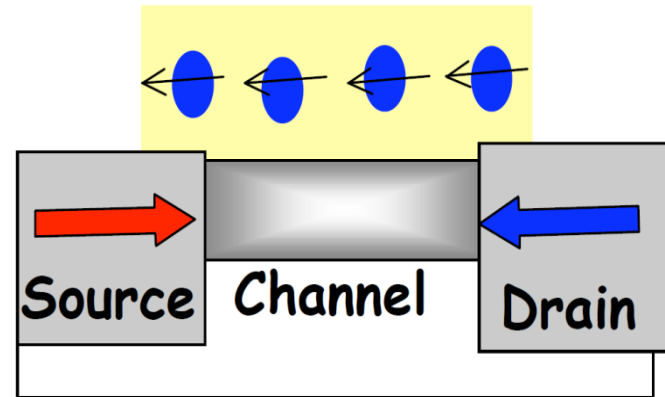
Connection with Maxwell's demon



$$\Delta S_{\text{tot}} \geq 0$$

$$\Delta F = \Delta E - T\Delta S \leq 0$$

Energy

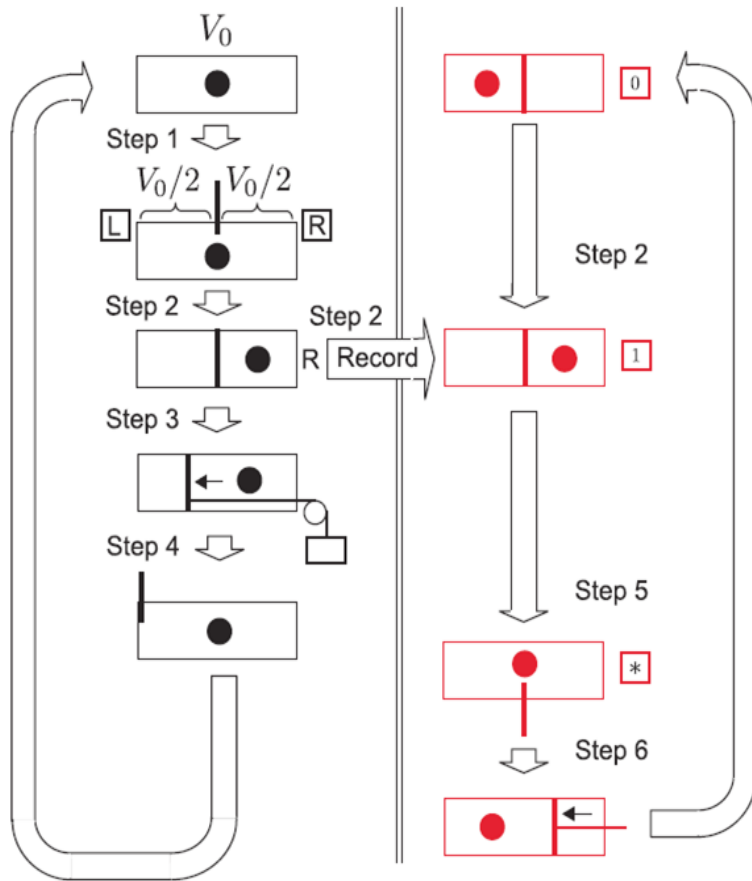


$$\Delta S_{\text{tot}} \geq 0$$

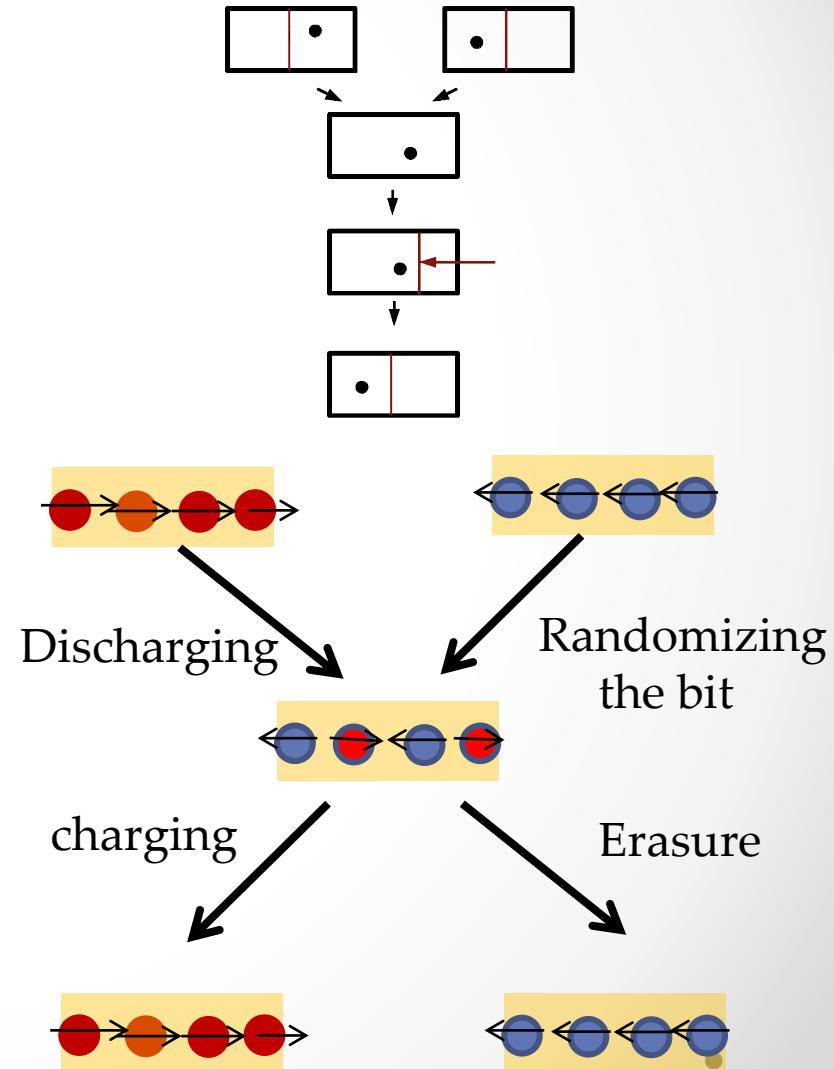
$$\Delta F = \Delta E - T\Delta S \leq 0$$

Information

Connection with Maxwell's demon/Landauer principle



State of the Demons

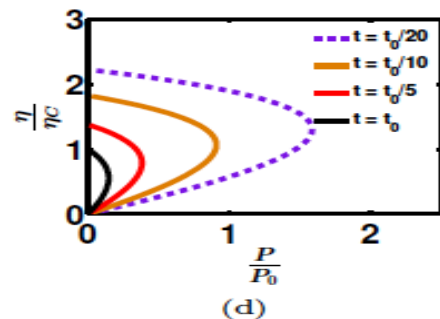
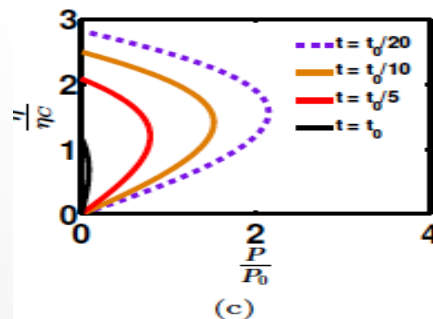
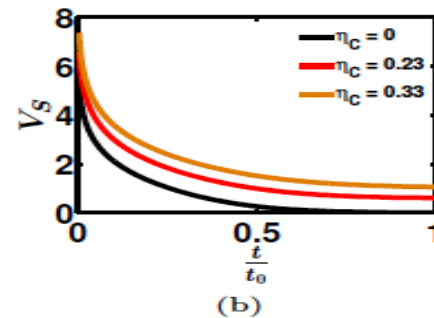
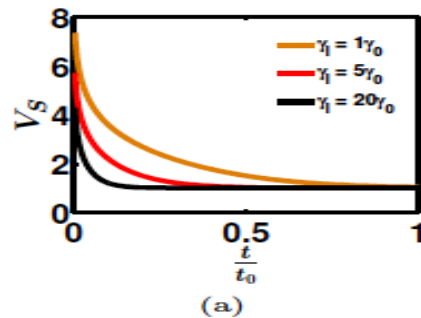
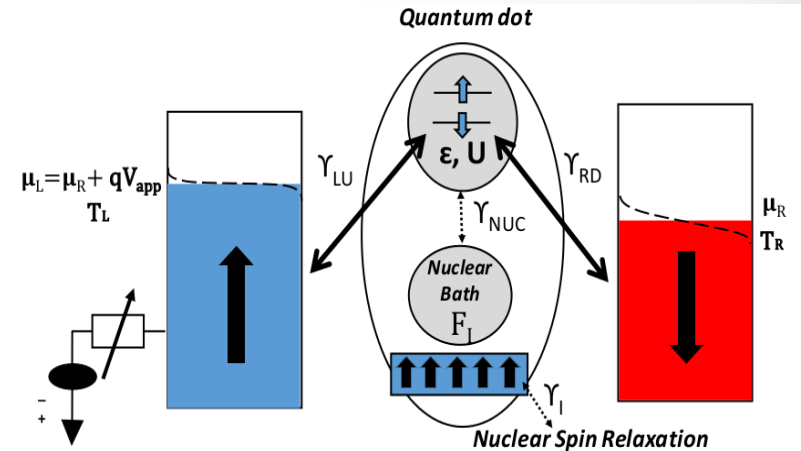
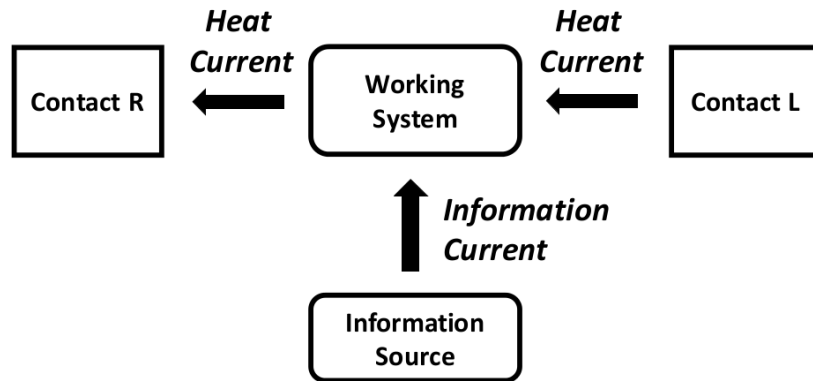


$$\Delta S = Nk \ln 2$$

$$\Delta W \leq NkT \ln 2$$

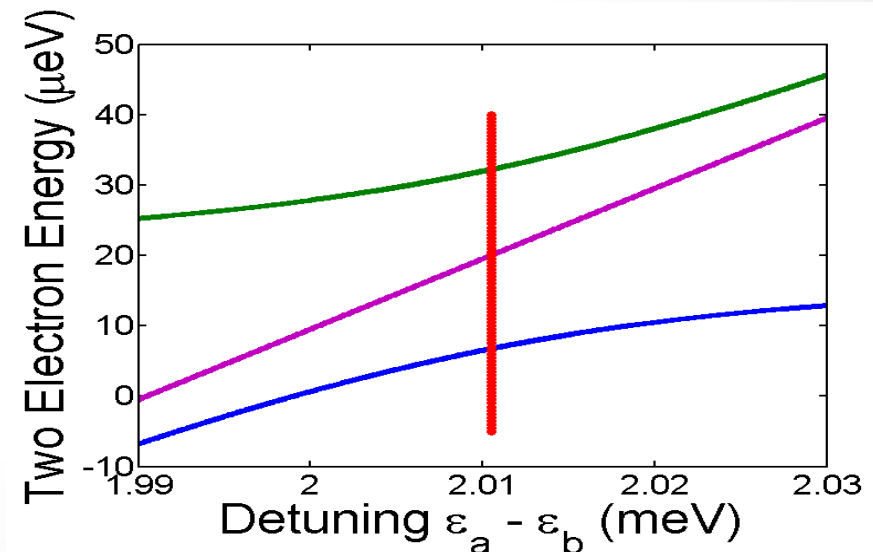
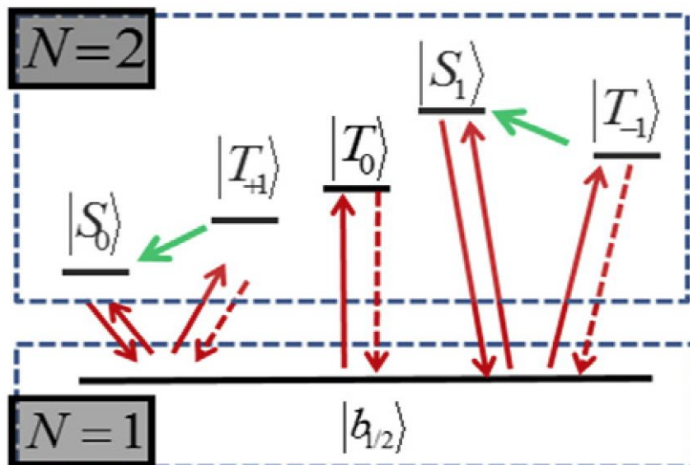
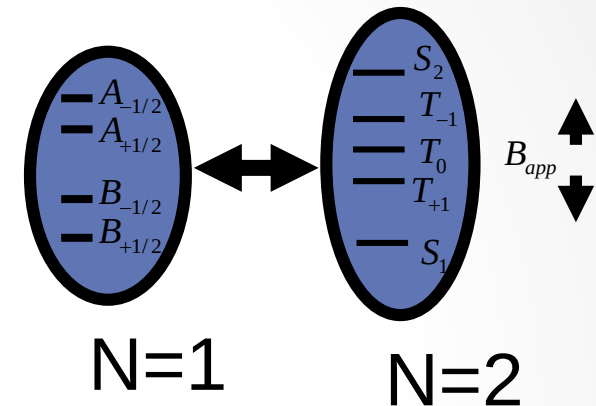
$$E_{\text{Erase}} \geq NkT \ln 2$$

Information flow: Larger than Carnot operation

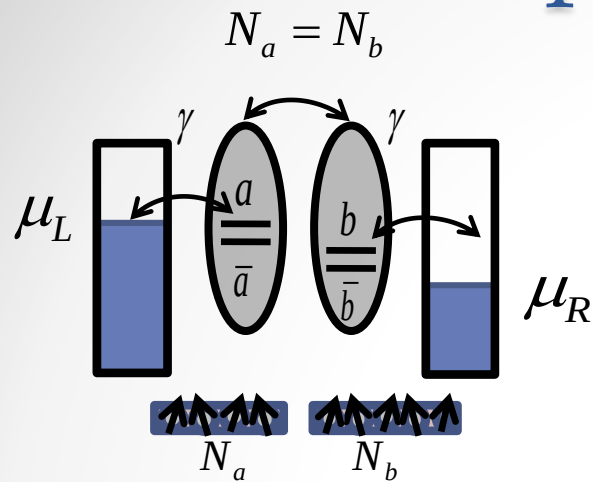


PSB Application: Double Resonance due to Two-Electron states

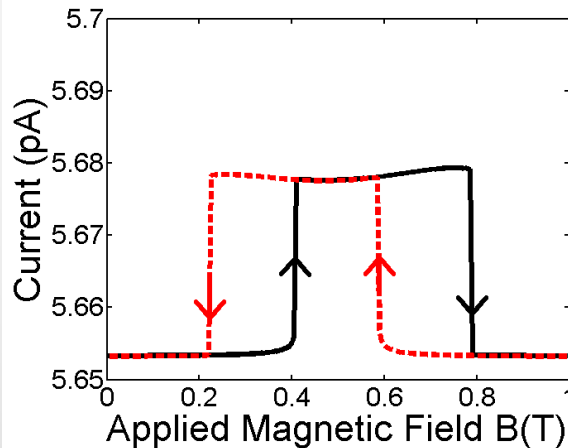
- ❑ Two-electron states at $\mathbf{B} = 0$
 - ❑ Three $S = 1$ states (T)
 - ❑ Three $S = 0$ states (S)
- ❑ Triplets are blocking states
- ❑ **TWO RESONANCES:**
 - ❑ $T_{+1} - S_1$
 - ❑ $T_{-1} - S_2$
- ❑ T_+ and T_- move in opposite directions under $\mathbf{B}$ and $\mathbf{F}_1$.



Applied to PSB

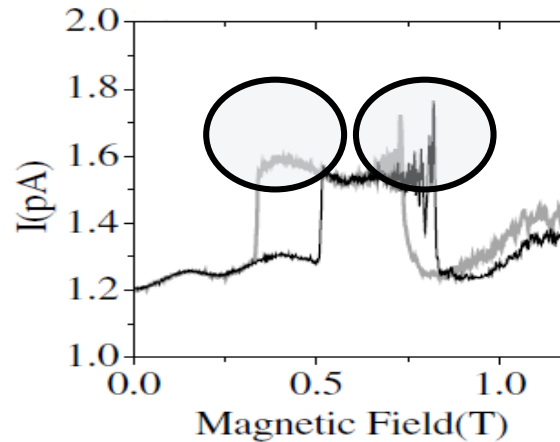


THEORY

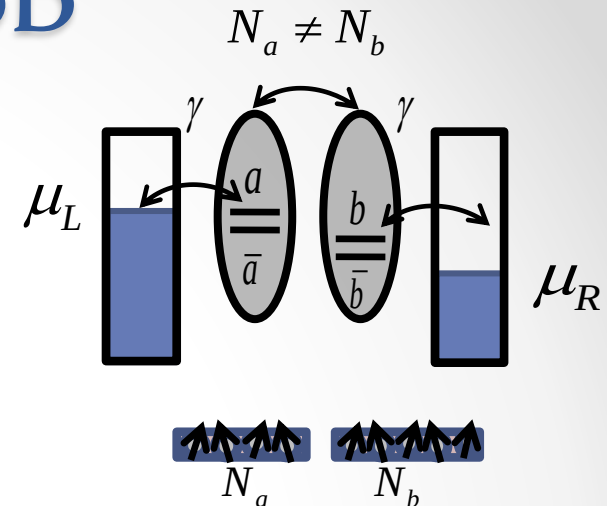


Dual
Resonance

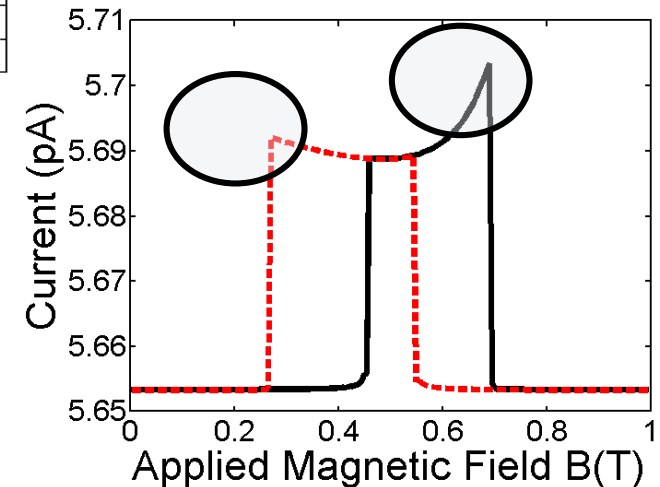
K. Ono et al. PRL 04
EXPERIMENT



Fin
Structure



THEORY

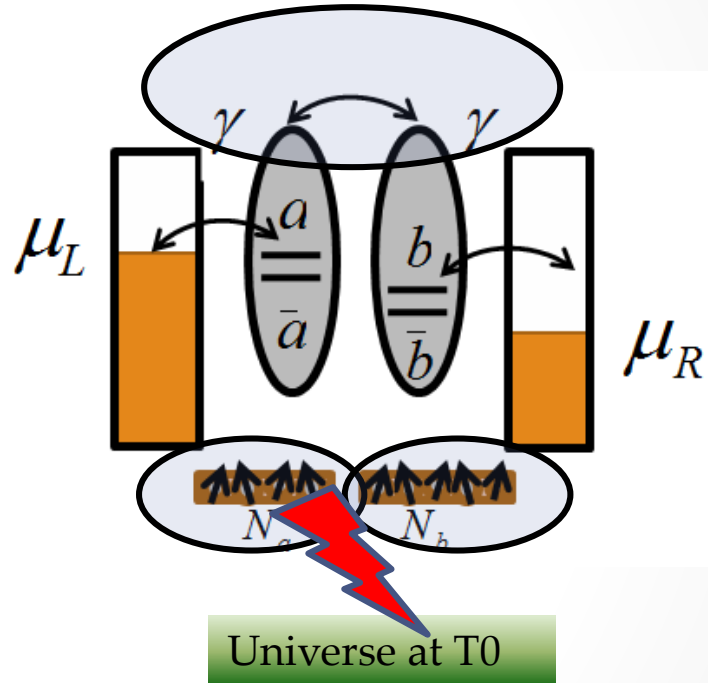
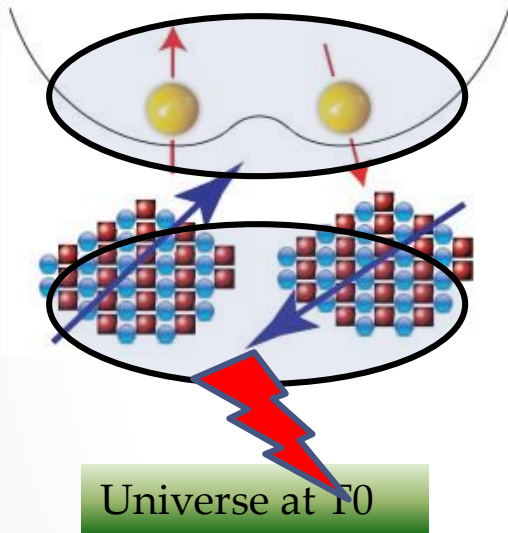


Un-identical
baths

Other “complicated” nanoscale demons

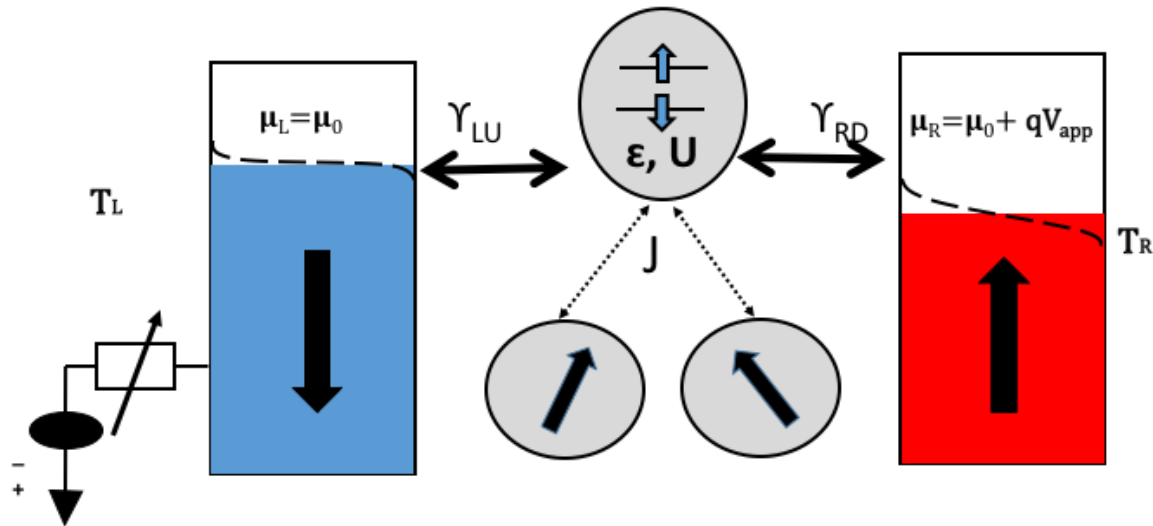
The case of Entanglement and quantum information:

$$S = \text{Tr}[\rho \ln \rho]$$

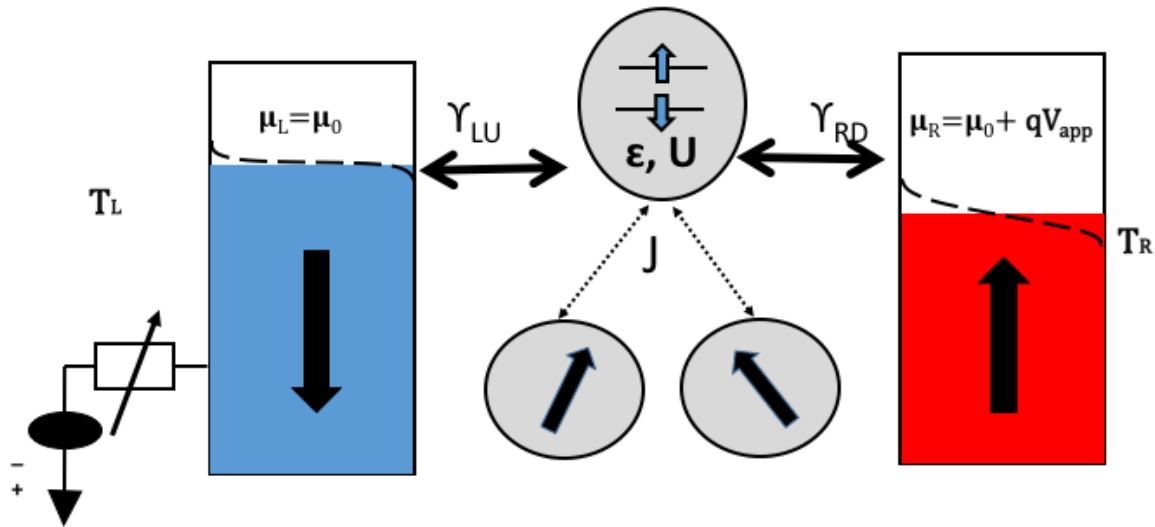


$$|2, i\rangle = \alpha_{0/1} |a\bar{b}\rangle + \beta_{0/1} |\bar{a}b\rangle + \xi_{0/1} |a\bar{a}\rangle + \delta_{0/1} |b\bar{b}\rangle$$

Part IV: Concurrency Thermal Machines



Set up and Many-body states

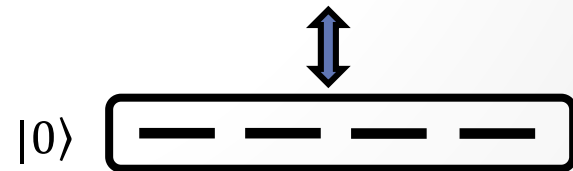
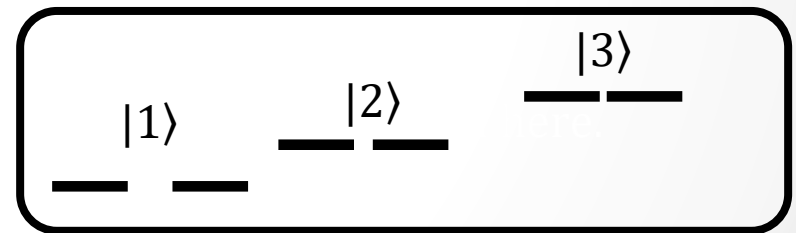


$$H_S = \sum_{\sigma=\uparrow,\downarrow} \hbar \omega_{\sigma} \hat{n}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow} + \hbar J \mathbf{S}_{dot} \cdot (\mathbf{S}_1 + \mathbf{S}_2)$$

$$H_T = \hbar \sum_{\alpha k \sigma} \left(t_{\alpha k \sigma} c_{\alpha k \sigma}^{\dagger} d_{\sigma} + \text{H.C.} \right)$$

$$H_C = \sum_{\alpha k \sigma} (\epsilon_{\alpha k \sigma} \hat{n}_{\alpha k \sigma}) = \hbar \sum_{\alpha k \sigma} \left(\omega_{\alpha k \sigma} c_{\alpha k \sigma}^{\dagger} c_{\alpha k \sigma} \right)$$

$$H_1 = \frac{\hbar}{4\pi} P \int d\xi \sum_{\alpha} (1 - 2f_{\alpha}(\xi)) \times \left[\frac{\gamma_{\alpha} \gamma_{\alpha}^{\dagger}}{\epsilon_1 - \epsilon_0 - \xi} - \frac{\bar{\gamma}_{\alpha} \bar{\gamma}_{\alpha}^{\dagger}}{\epsilon_2 - \epsilon_1 - \xi} \right]$$



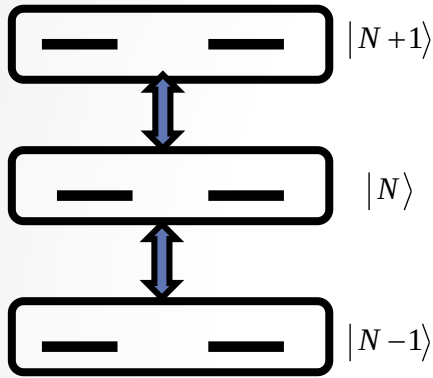
$$(N = 2, n_d = 0)$$

Density Matrix master equations

For constant density of states we define:

$$\Gamma_{\alpha\sigma_\alpha} = \frac{2\pi D_{\alpha k\sigma_\alpha} |t_\alpha|^2}{\hbar}$$

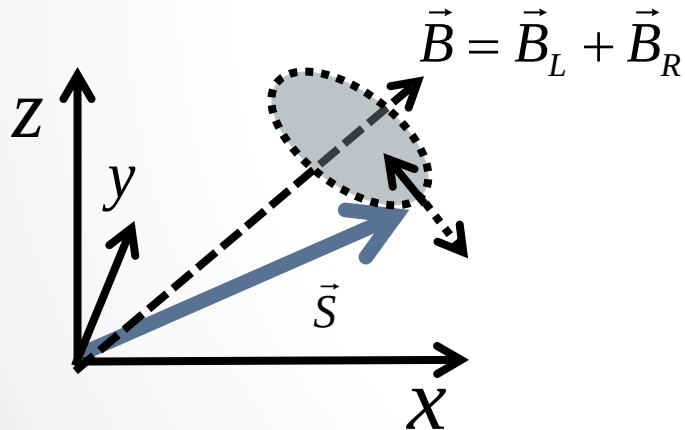
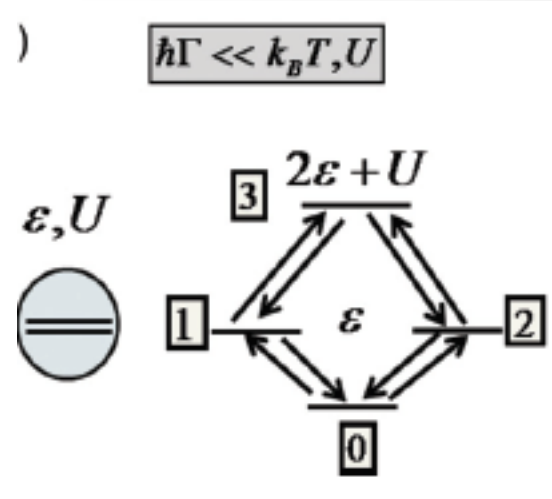
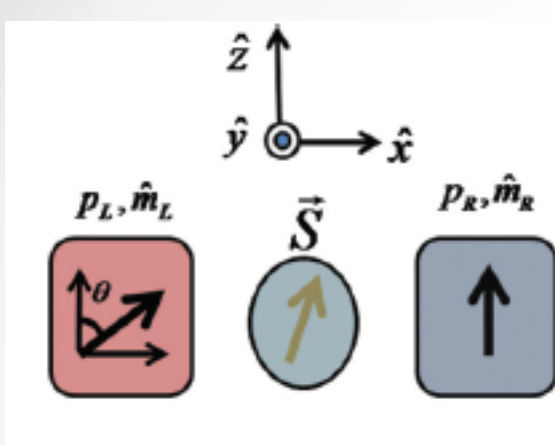
The equations then become:



$$\dot{\rho}_{nm}^{E_N N}(t) = -\sum_{\alpha\sigma_\alpha} \left\{ \sum_{l,l' \in |N-1\rangle} \sum_{j \in |E_N, N\rangle} \sum_{h,h' \in |N+1\rangle} \right\} \frac{\Gamma_{\alpha\sigma_\alpha}}{2} \left\{ \begin{array}{l} \text{Real transitions} \\ \text{Virtual transitions} \\ \text{Energy non-conserving} \end{array} \right.$$

$$\left\{ \begin{array}{l} \left[f_\alpha(\varepsilon_h - \varepsilon_j) + \frac{i}{\pi} \int d\varepsilon_k \frac{f_\alpha(\varepsilon_k)}{\varepsilon_k - \varepsilon_h + \varepsilon_j} \right] M_{nh}^{\alpha\sigma_\alpha} (M_{hj}^{\alpha\sigma_\alpha})^* \rho_{jm}^{E_N N} \\ + \left[(1 - f_\alpha(\varepsilon_j - \varepsilon_l)) - \frac{i}{\pi} \int d\varepsilon_k \frac{(1 - f_\alpha(\varepsilon_k))}{\varepsilon_k - \varepsilon_j + \varepsilon_l} \right] (M_{nl}^{\alpha\sigma_\alpha})^* M_{lj}^{\alpha\sigma_\alpha} \rho_{jm}^{E_N N} \\ + \left[f_\alpha(\varepsilon_h - \varepsilon_j) - \frac{i}{\pi} \int d\varepsilon_k \frac{f_\alpha(\varepsilon_k)}{\varepsilon_k - \varepsilon_h + \varepsilon_j} \right] \rho_{nj}^{E_N N} M_{jh}^{\alpha\sigma_\alpha} (M_{hm}^{\alpha\sigma_\alpha})^* \\ + \left[(1 - f_\alpha(\varepsilon_j - \varepsilon_l)) + \frac{i}{\pi} \int d\varepsilon_k \frac{(1 - f_\alpha(\varepsilon_k))}{\varepsilon_k - \varepsilon_j + \varepsilon_l} \right] \rho_{nj}^{E_N N} (M_{jl}^{\alpha\sigma_\alpha})^* M_{lm}^{\alpha\sigma_\alpha} \\ - 2(1 - f_\alpha(\varepsilon_h - \varepsilon_j)) M_{nh'}^{\alpha\sigma_\alpha} \rho_{h'h}^{E_N N+1} (M_{hm}^{\alpha\sigma_\alpha})^* \\ - f_\alpha(\varepsilon_j - \varepsilon_l) M_{nl'}^{\alpha\sigma_\alpha} \rho_{l'l}^{E_N N-1} (M_{lm}^{\alpha\sigma_\alpha})^* \end{array} \right.$$

Example of Density Matrix-Coherences



$$\vec{J}_\alpha^S = \left[\underbrace{\frac{\hbar}{2q} J_\alpha^q p_\alpha \hat{m}_\alpha}_{\text{Injection}} - \underbrace{\frac{\vec{S} - p_\alpha^2 (\hat{m}_\alpha \cdot \vec{S}) \hat{m}_\alpha}{\tau_{r,\alpha}}}_{\text{Relaxation}} + \underbrace{\vec{S} \times \vec{B}_\alpha}_{\text{Precession}} \right]$$

Koenig and Martinek PRL (2004).

B. Muralidharan and M Grifoni,
Phys. Rev. B, 88, 045402, (2013).

Density Matrix master equations

$$v_{\alpha k, k'}^+ = \langle k, \cdot | t_{\alpha}^* d_{\alpha}^{\dagger} | k', \cdot \rangle \quad (v_{\alpha k, k'}^+)^{\dagger} = v_{\alpha k', k}^-$$

$$v_{\alpha k, k'}^- = \langle k, \cdot | t_{\alpha} d_{\alpha} | k', \cdot \rangle \quad \gamma_{\alpha k, k'}^{\pm} = \left(\frac{2\pi D_{\alpha}}{\hbar} \right)^{1/2} v_{\alpha k, k'}^{\pm}$$

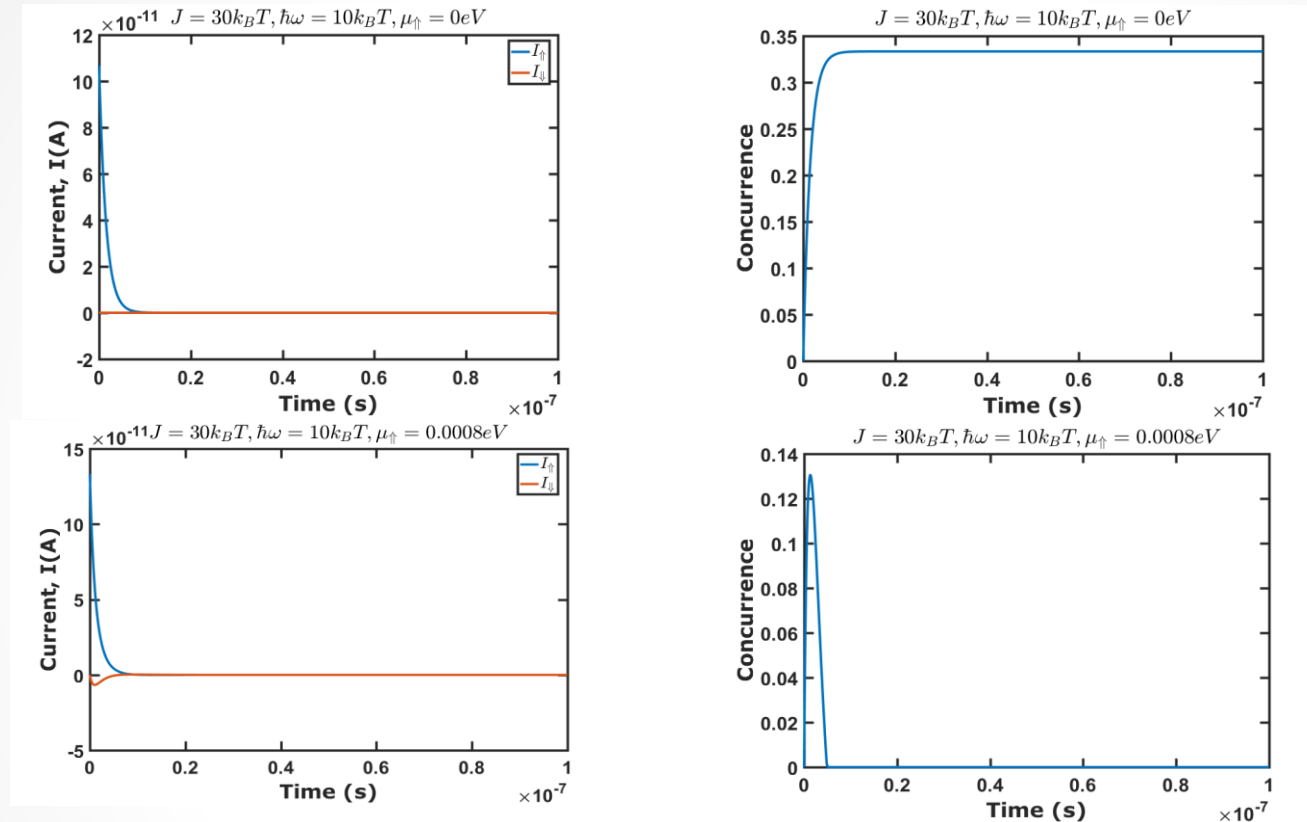
$$H_1 = \frac{\hbar}{4\pi} P \int d\zeta \sum_{\alpha} (1 - 2f_{\alpha}(\zeta)) \times \left[\frac{\gamma_{\alpha} \gamma_{\alpha}^{\dagger}}{\varepsilon_1 - \varepsilon_0 - \zeta} - \frac{\gamma_{\alpha}^{\dagger} \gamma_{\alpha}}{\varepsilon_2 - \varepsilon_1 - \zeta} \right]$$

$$\dot{\rho}_0 = \frac{i}{\hbar} [\rho_0, H_0] + \sum_{\alpha} \sum_{k \in \{a, b, c\}} (1 - f_{\alpha}(E_k - E_0)) \gamma_{\alpha, 0, k}^{-} \rho_k \gamma_{\alpha, k, 0}^{+} - \frac{1}{2} f_{\alpha}(E_k - E_0) \left\{ \rho_0, \gamma_{\alpha, 0, k}^{-} \gamma_{\alpha, k, 0}^{+} \right\}$$

$$\dot{\rho}_2 = \frac{i}{\hbar} [\rho_2, H_2] + \sum_{\alpha} \sum_{k \in \{a, b, c\}} f_{\alpha}(E_2 - E_k) \gamma_{\alpha, 2, k}^{+} \rho_k \gamma_{\alpha, k, 2}^{-} - \frac{1}{2} (1 - f_{\alpha}(E_2 - E_k)) \left\{ \rho_2, \gamma_{\alpha, 2, k}^{+} \gamma_{\alpha, k, 2}^{-} \right\}$$

$$\begin{aligned} \dot{\rho}_k = & \frac{i}{\hbar} [\rho_k, H_k] + \sum_{\alpha} f_{\alpha}(E_k - E_0) \gamma_{\alpha, k, 0}^{+} \rho_0 \gamma_{\alpha, 0, k}^{-} - \frac{1}{2} (1 - f_{\alpha}(E_k - E_0)) \left\{ \rho_k, \gamma_{\alpha, k, 0}^{+} \gamma_{\alpha, 0, k}^{-} \right\} \\ & + \sum_{\alpha} (1 - f_{\alpha}(E_2 - E_k)) \gamma_{\alpha, k, 2}^{-} \rho_2 \gamma_{\alpha, 2, k}^{+} - \frac{1}{2} f_{\alpha}(E_2 - E_k) \left\{ \rho_k, \gamma_{\alpha, k, 2}^{-} \gamma_{\alpha, 2, k}^{+} \right\} \end{aligned}$$

Battery/Power Source Functionality

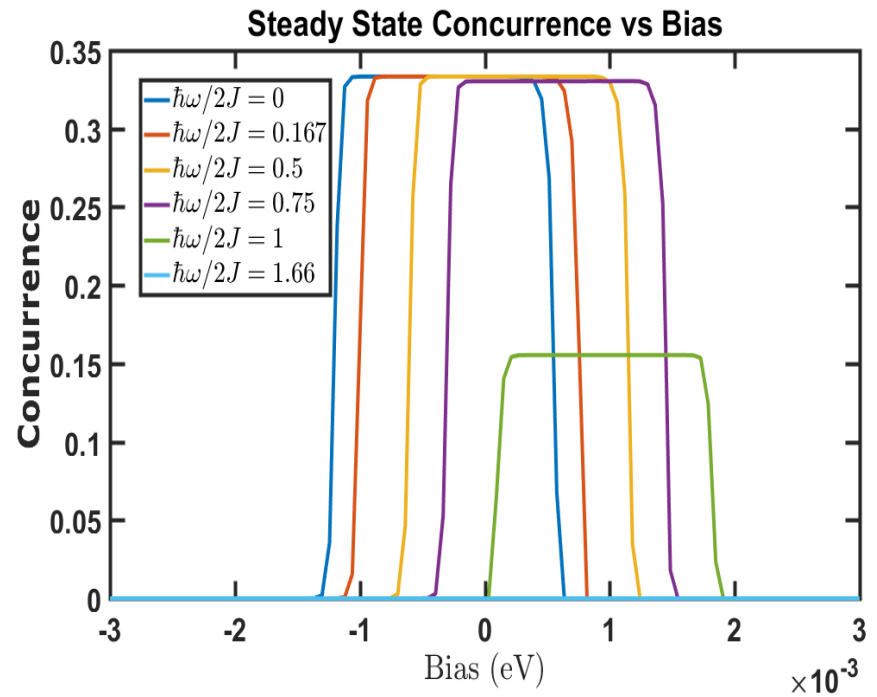
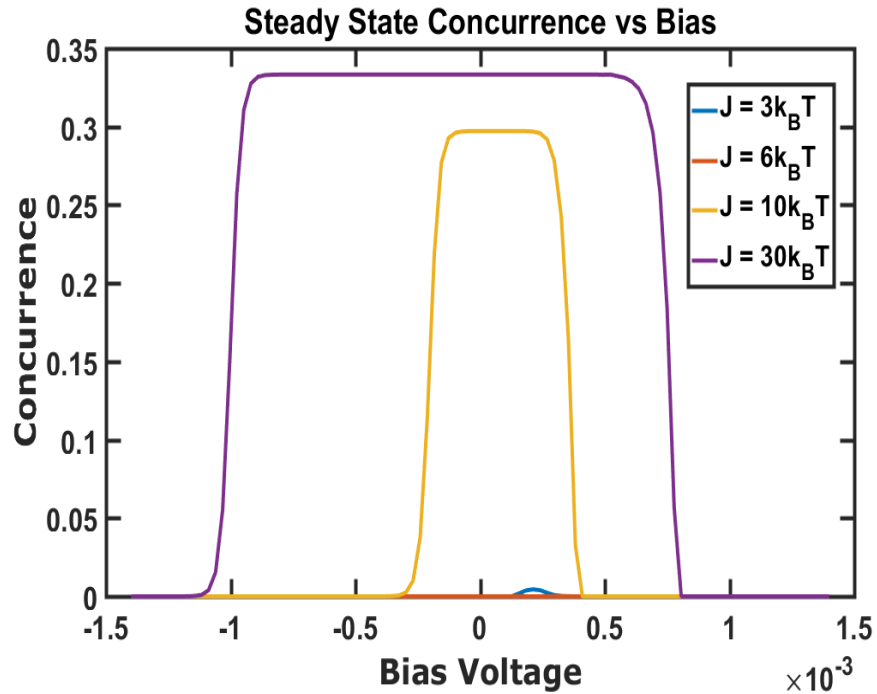


$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4),$$

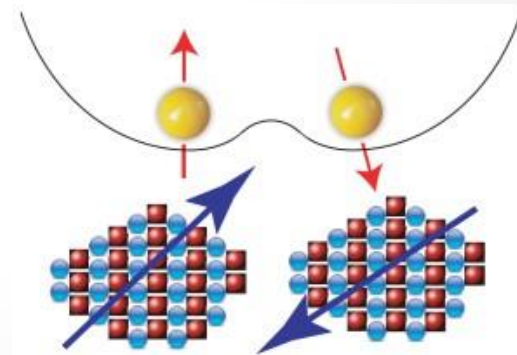
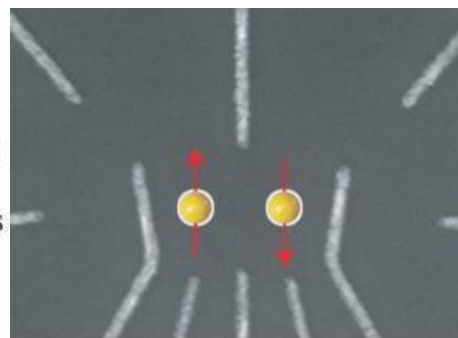
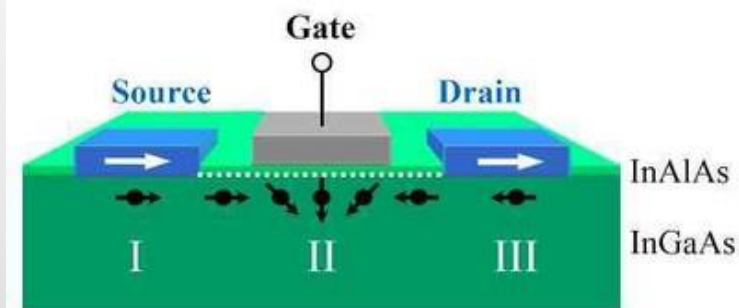
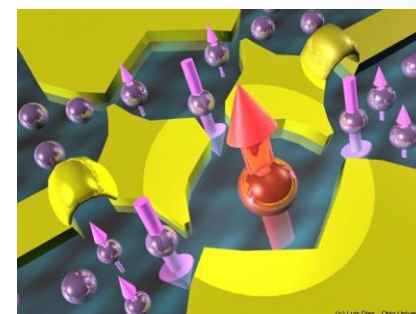
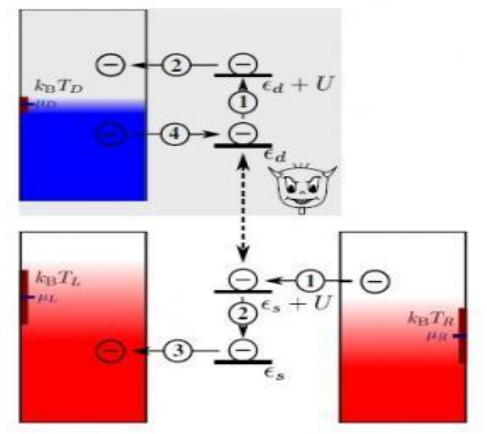
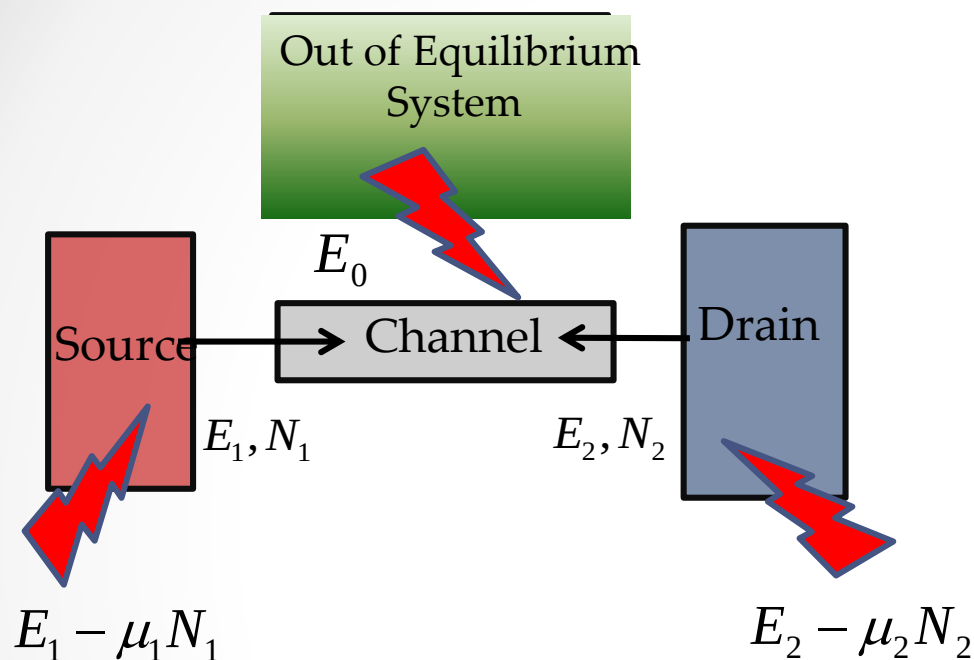
where λ_i are the eigenvalues, in decreasing order, of the Hermitian operator

$$R = \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$$

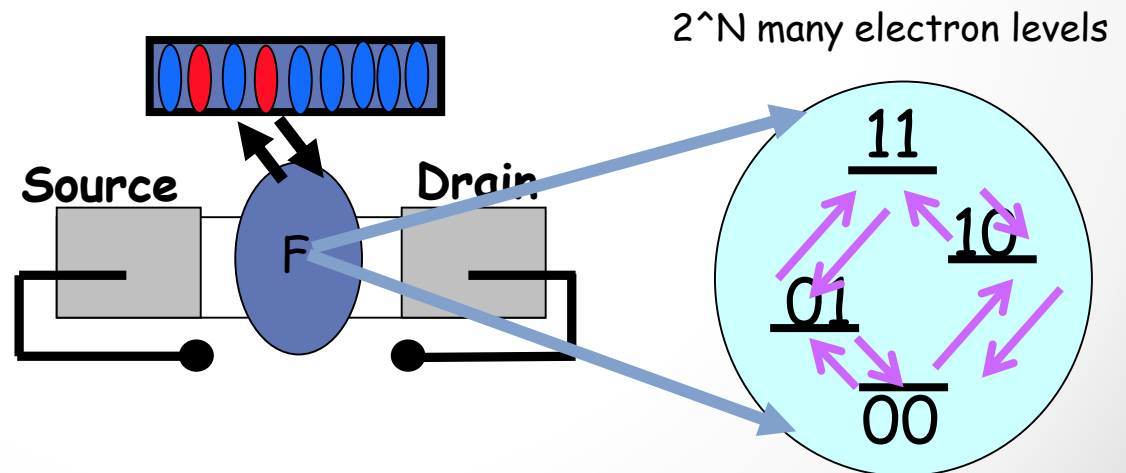
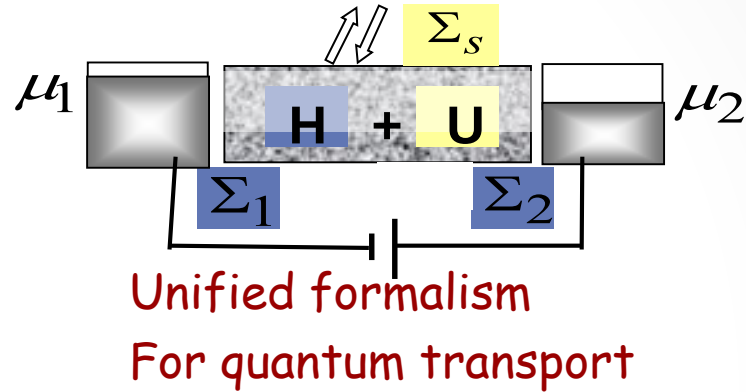
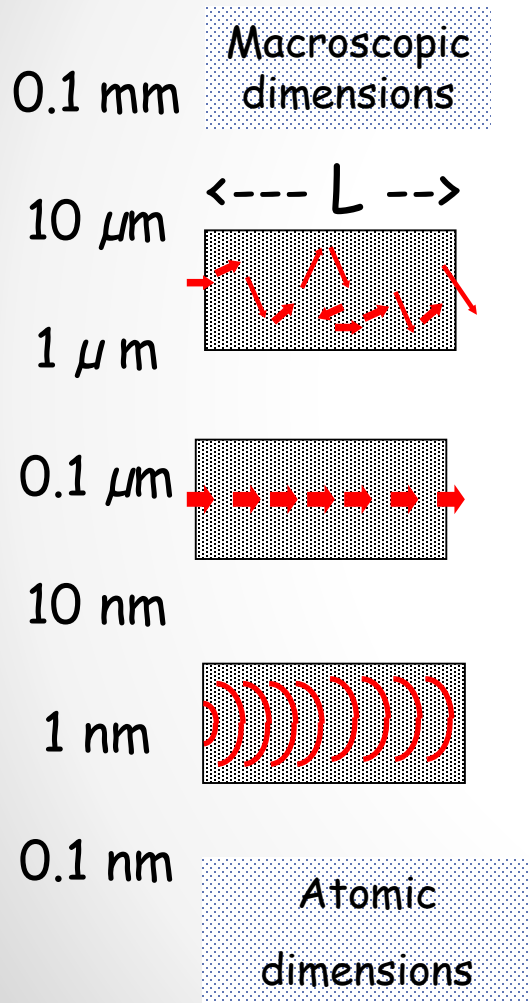
Concurrence Switch



nano-device with “demons”



World of Quantum transport



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- Bitan De
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