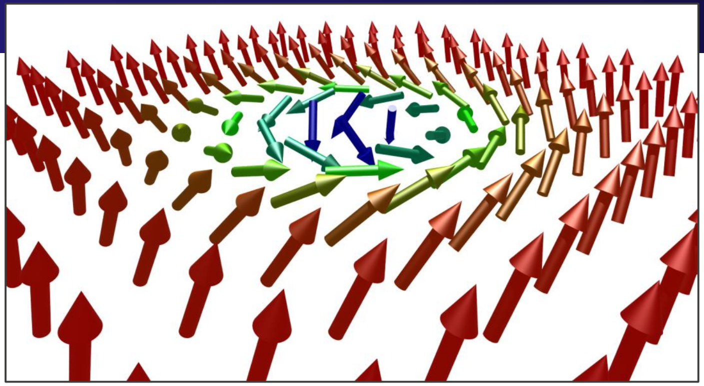
Possible skyrmion lattice in inversion-symmetric magnets



Shizeng Lin

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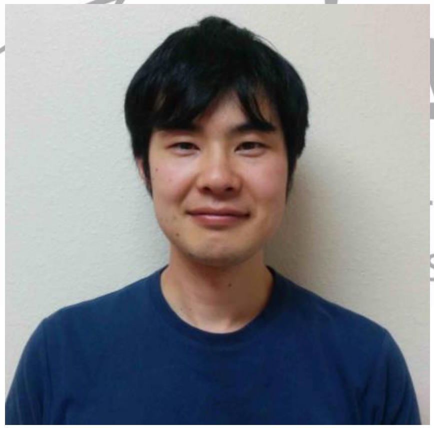
Jul. 19, 2017

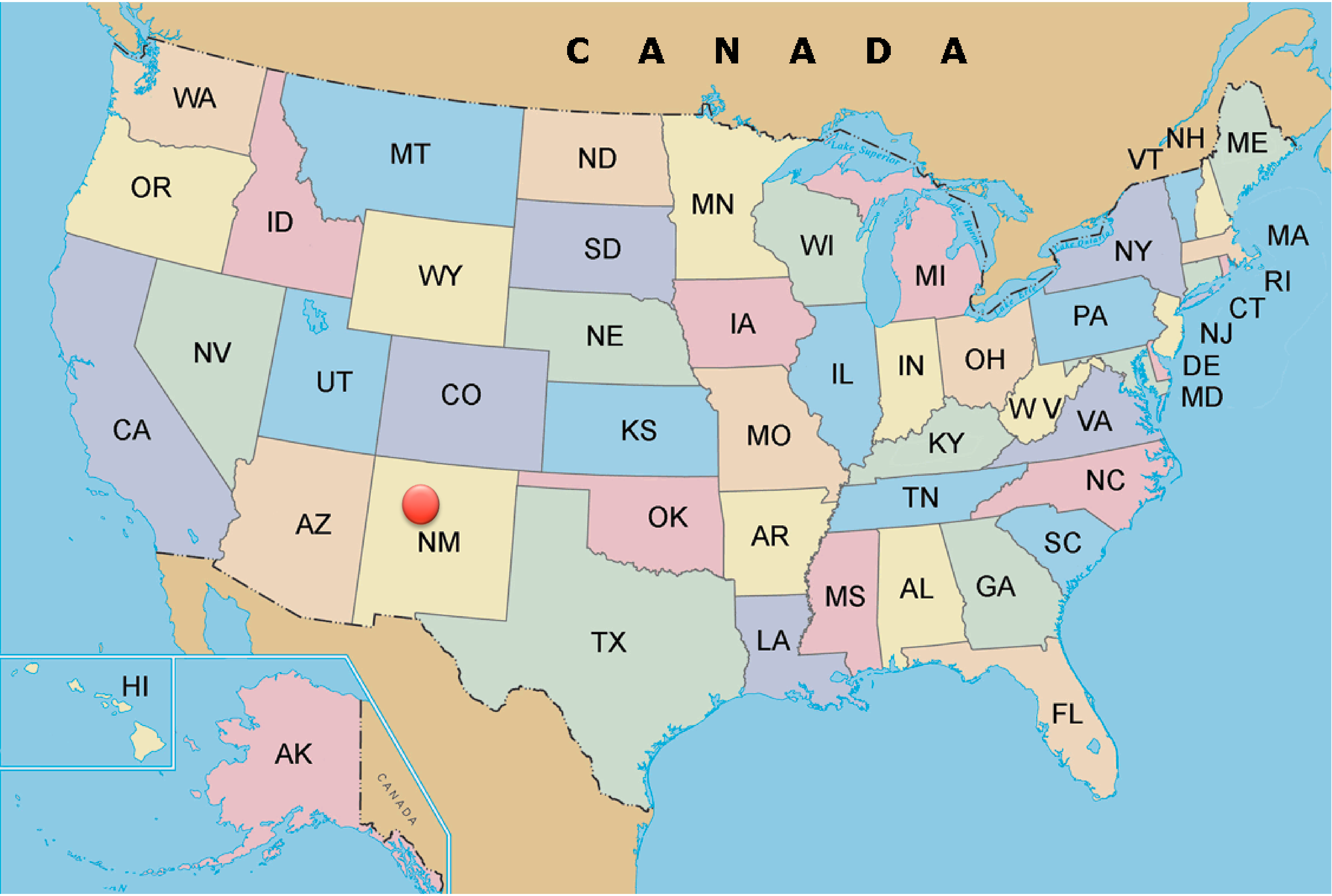
International Centre for Theoretical Sciences, Bengaluru, India

Collaborators



Satoru Hayami

Cristian D. Batista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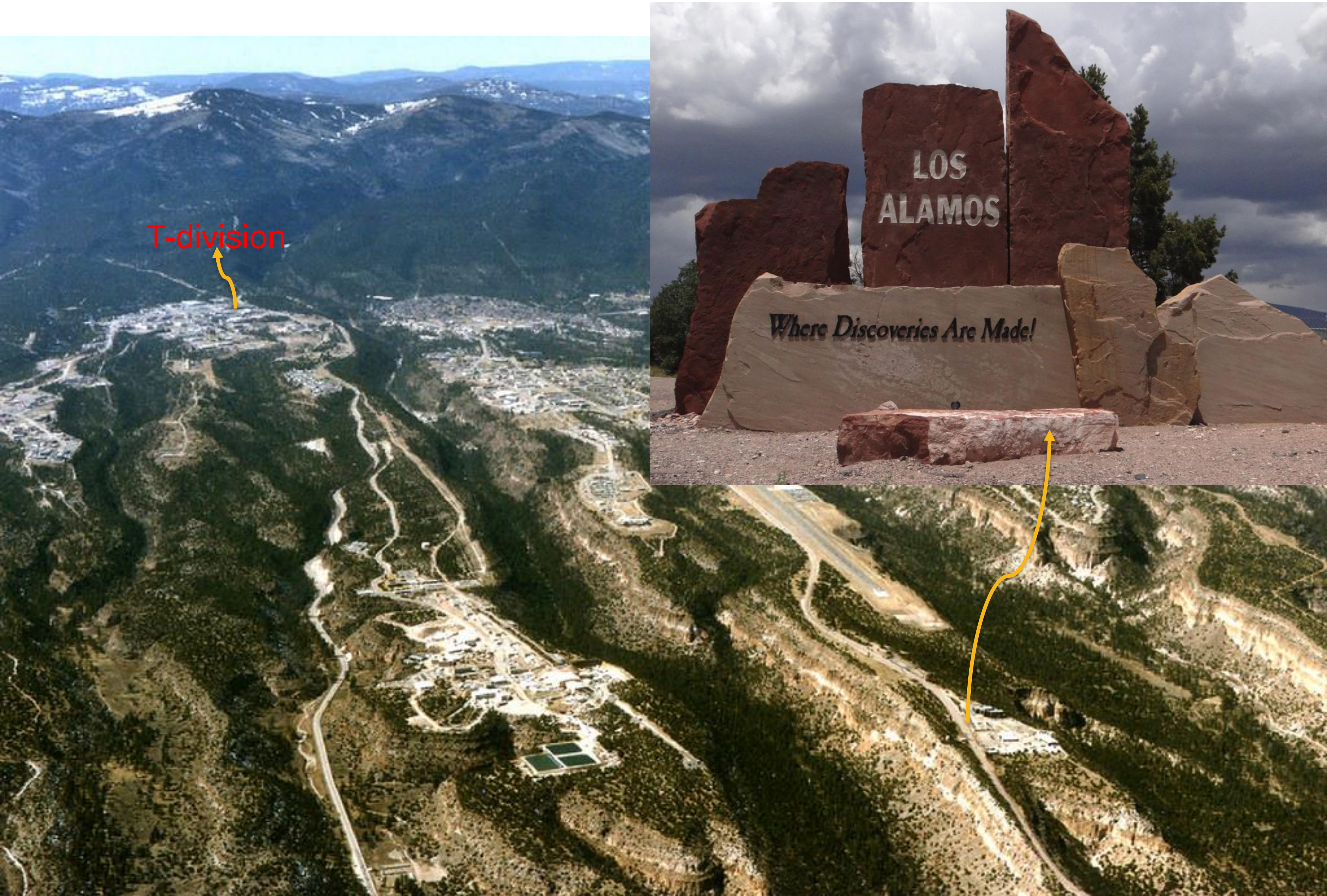
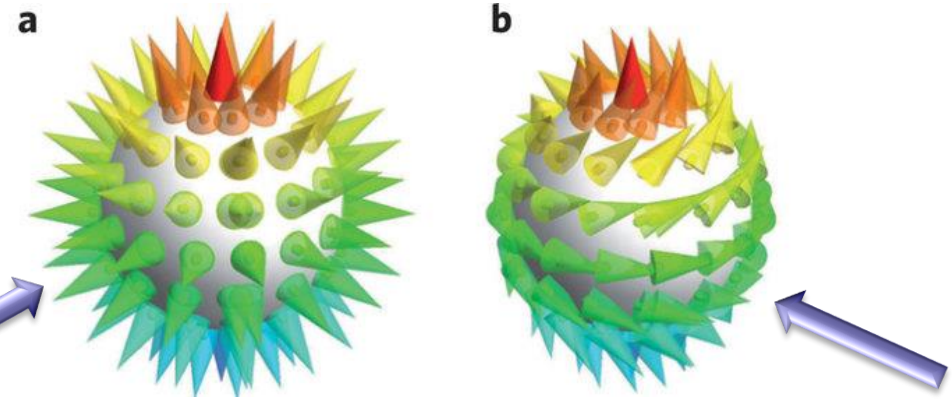


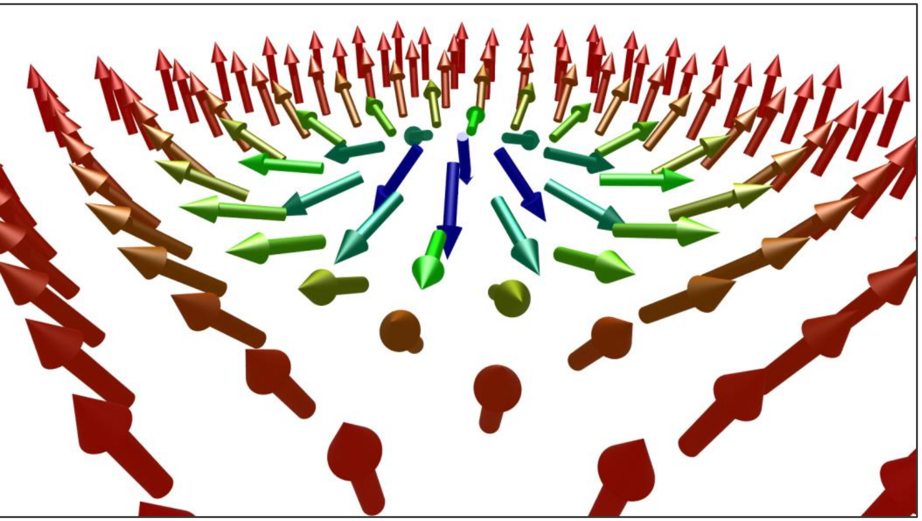
Outline

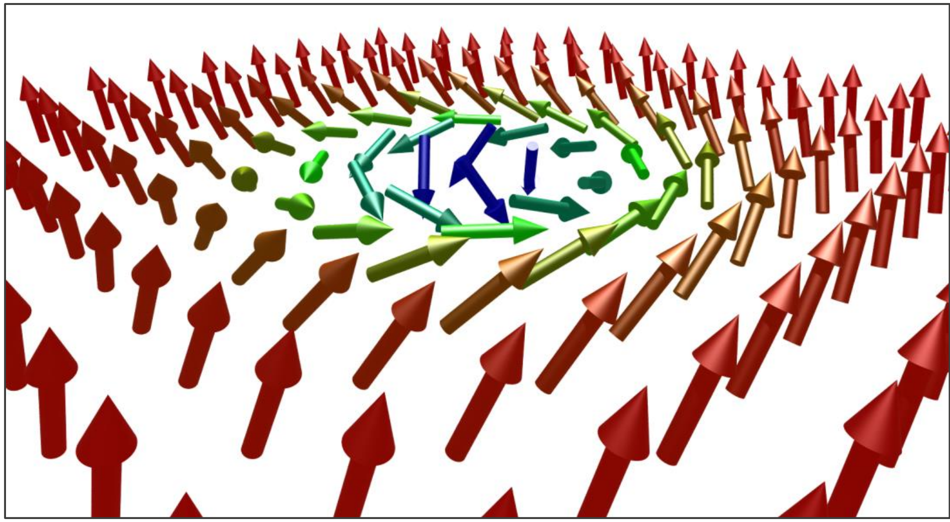
- **Introduction: Skyrmion in chiral magnets**
- **Design principles for skyrmions in inversion-symmetric magnets with competing interaction**
- **Skyrmion phase diagram**
- **Optimal conditions and experimental signatures**
- **Conclusions**

Magnetic skyrmions

$$Q = \frac{1}{4\pi}\int \mathbf{S}\cdot(\partial_x \mathbf{S}\times\partial_y \mathbf{S})\, dr^2$$
$$= \pm 1$$







Hedgehog:

Skyrmion solution in chiral magnets

Skyrmion stabilized by the Dzyaloshinskii-Moriya interaction

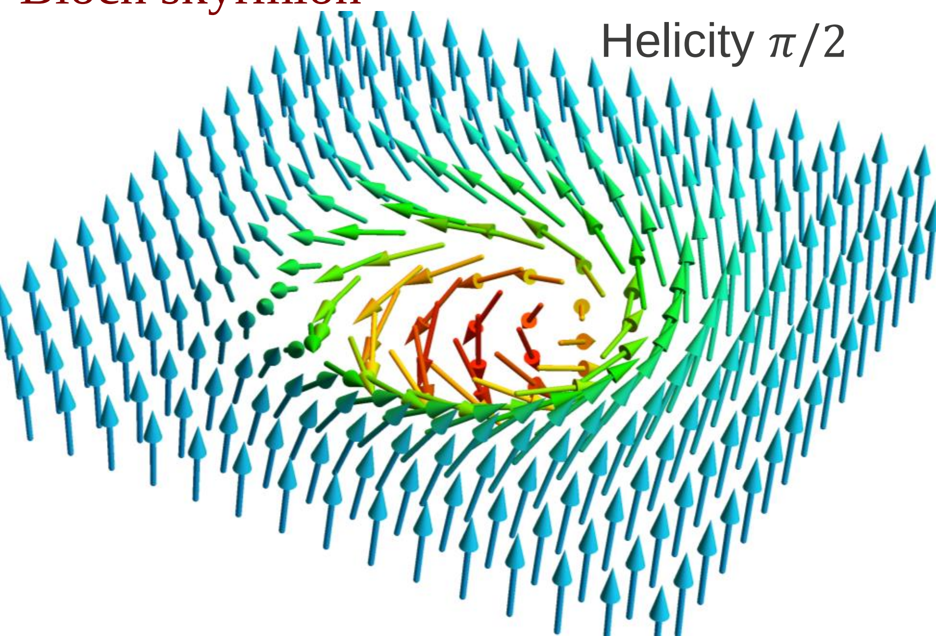
Bulk
$$\mathcal{H} = \int \left[\frac{J}{2} (\nabla \mathbf{S})^2 + D\mathbf{S} \cdot (\nabla \times \mathbf{S}) - \mathbf{H} \cdot \mathbf{S} \right] dr^3$$
with $J>0$, $|\mathbf{S}|=1$

Interface
$$\mathcal{H} = \int \left[\frac{J}{2} (\nabla \mathbf{S})^2 + D\mathbf{S} \cdot [(\hat{z} \times \nabla) \times \mathbf{S}] - \mathbf{H} \cdot \mathbf{S} \right] dr^3$$

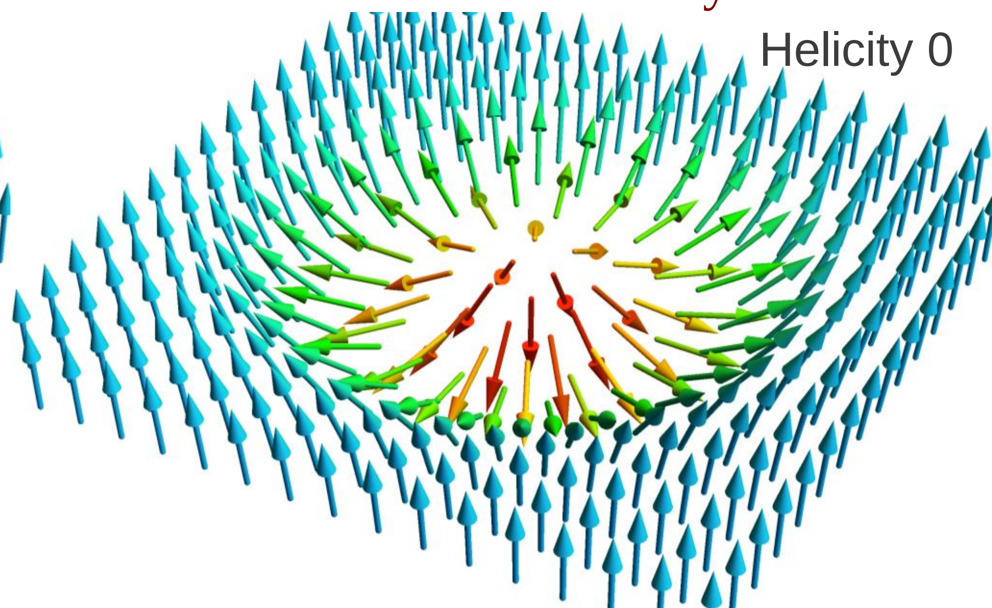
Goldstone modes: spatial translational motion

Bogdanov and Yablonskii, Sov. Phys. JETP 68, 101 (1989).

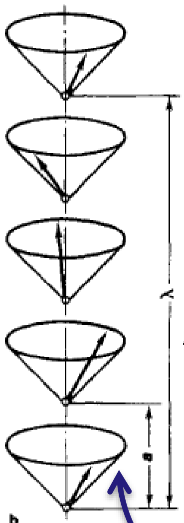
Bloch skyrmion



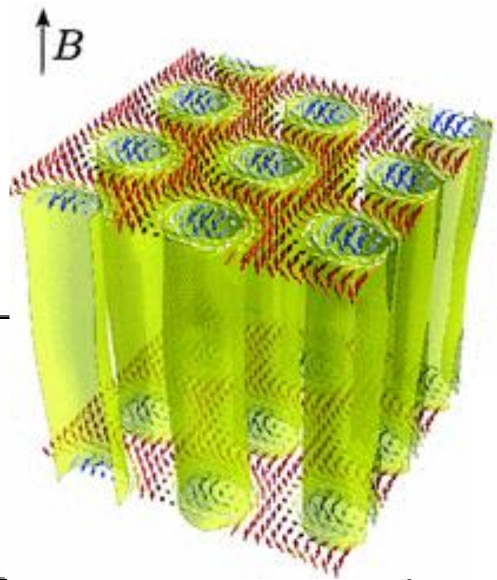
Néel skyrmion



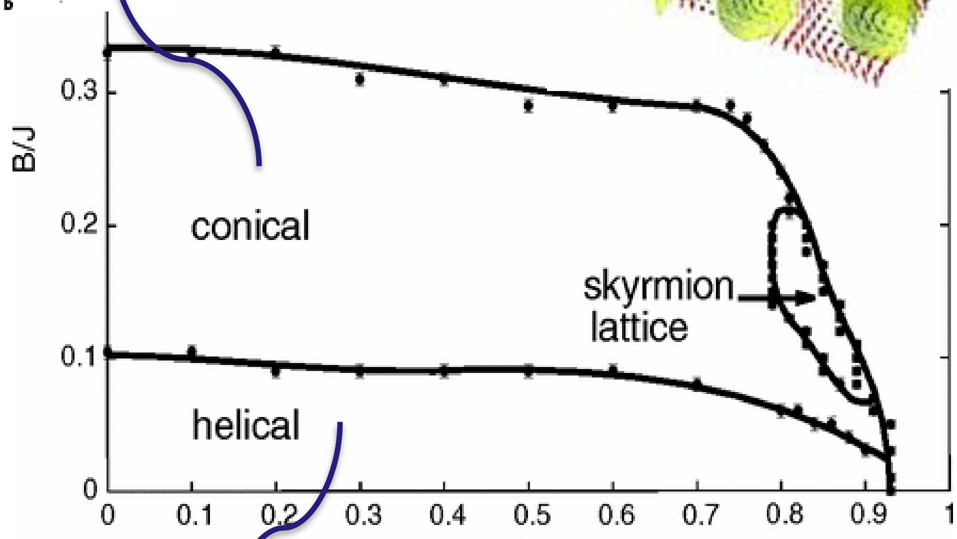
Theoretical T-B phase diagram



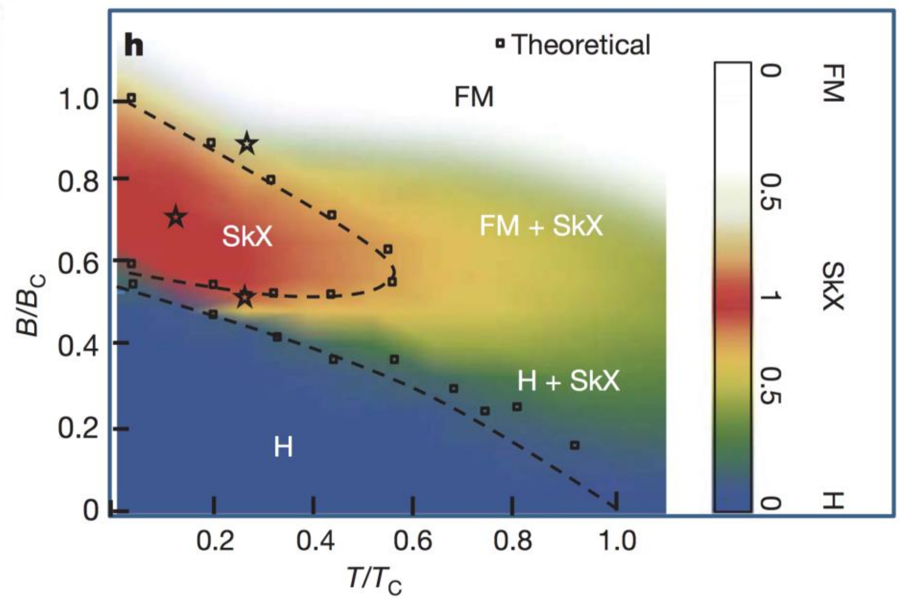
3D



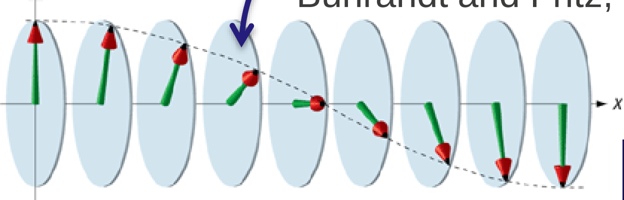
In 3D, a skyrmion is a line-like object.



2D

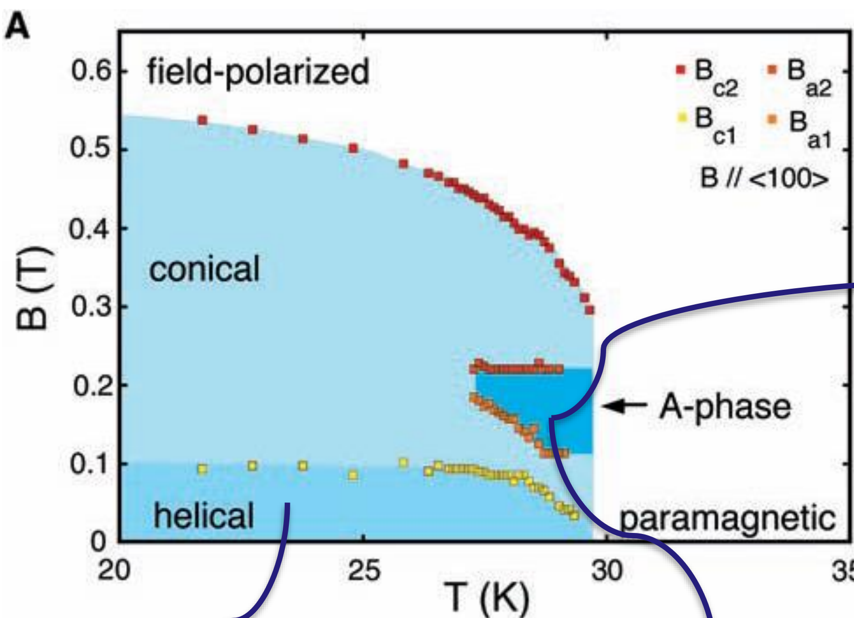


Buhrandt and Fritz, Phys. Rev. B 88 (2013)

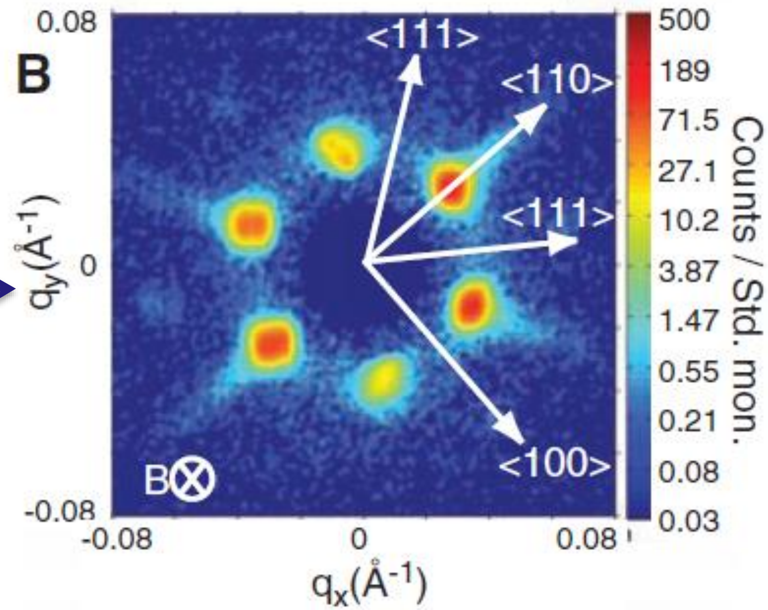


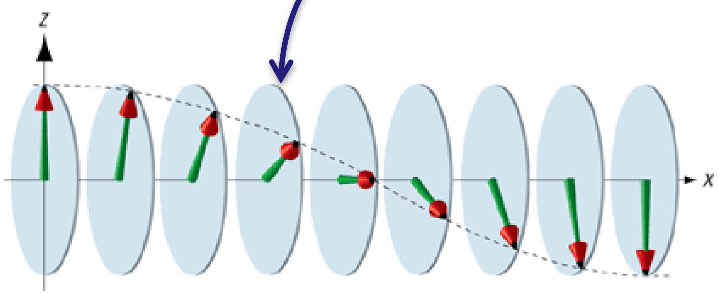
X. Z. Yu et al. Nature 465, 901 (2010).

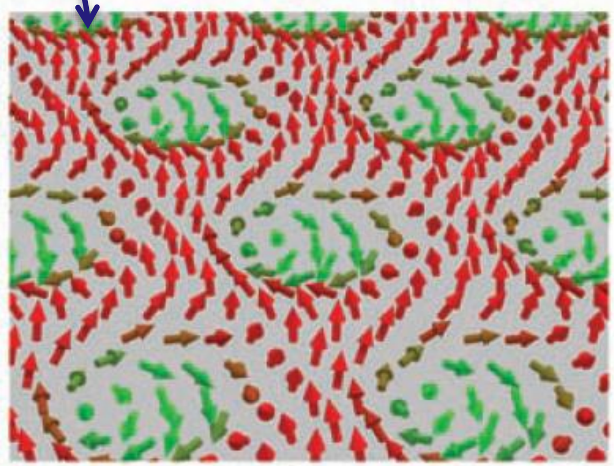
First experimental observation

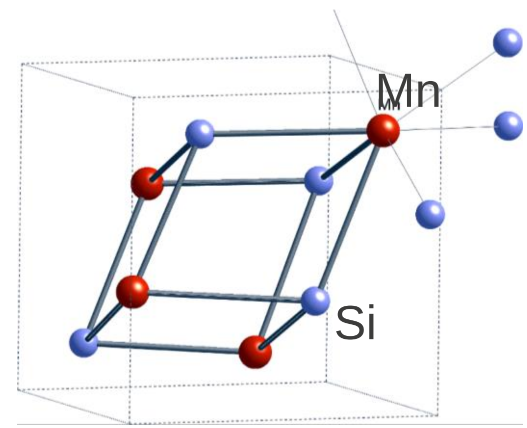


SANS





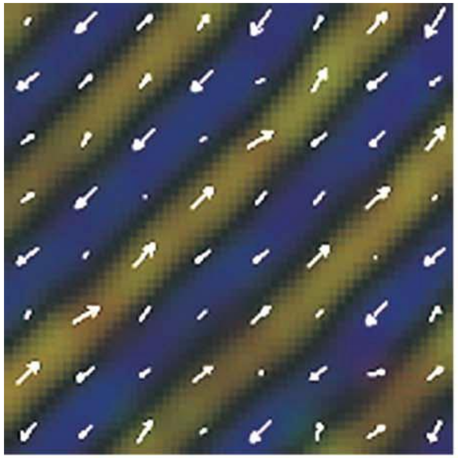




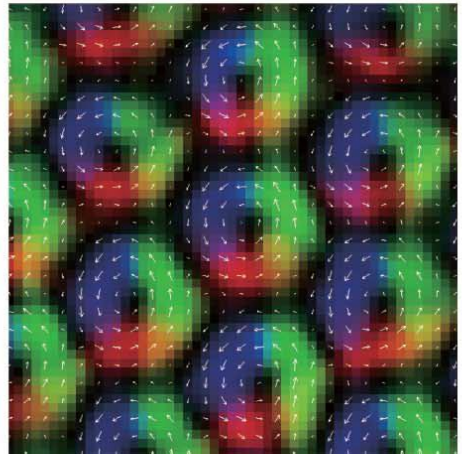
Bulk chiral magnet: MnSi

S. Mühlbauer, et al. Science 323, 915 (2009).

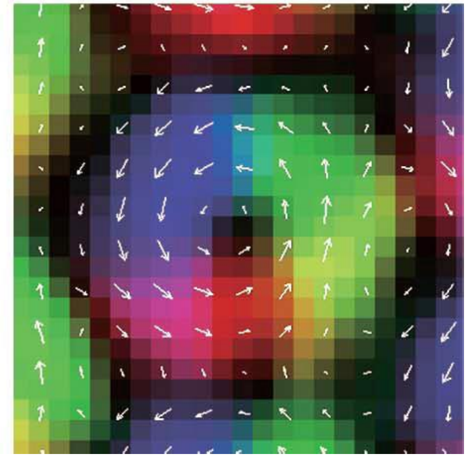
Real space observation of skyrmions in thin films



Helical



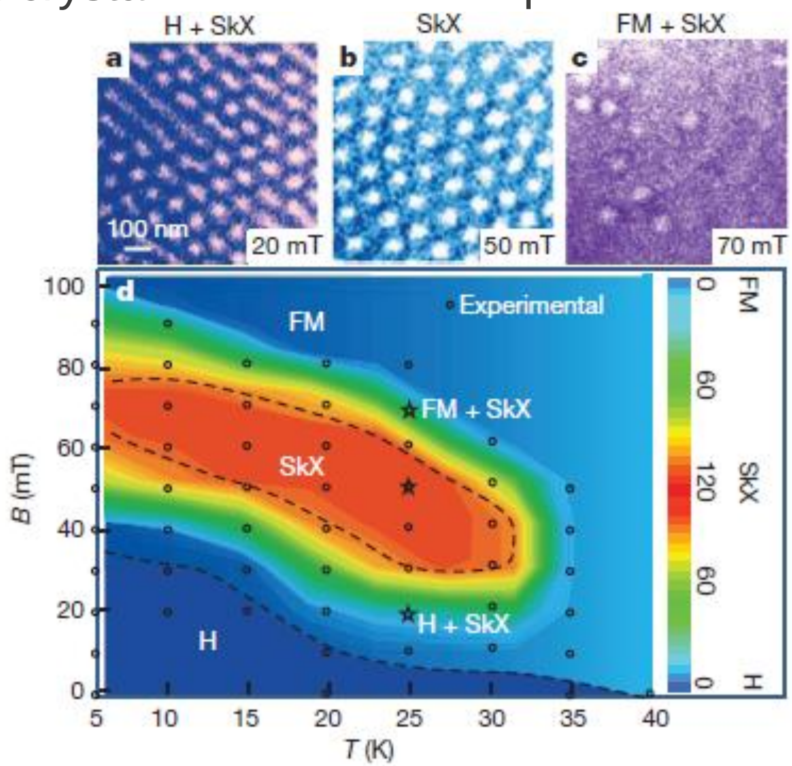
Skyrmion crystal



Close-up

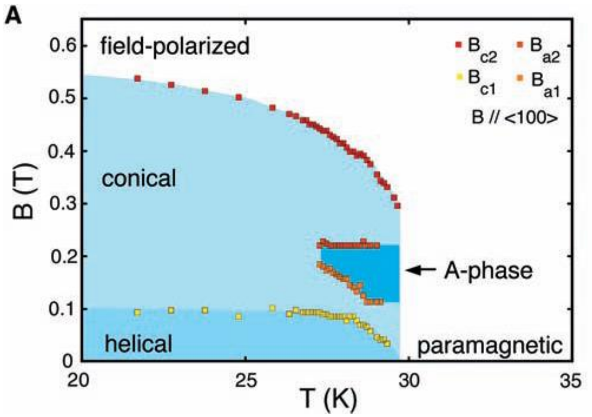
- Lorentz transmission electron microscopy; measure the in-plane component of spin.
- Materials: $Fe_{0.5}Co_{0.5}Si$

X. Z. Yu et al. Nature 465, 901 (2010).

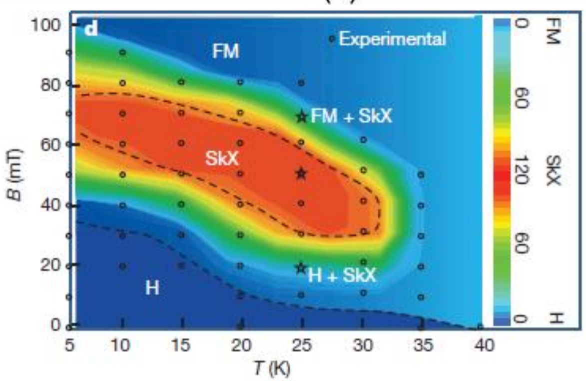


Universal phase diagram for skyrmion in chiral magnets

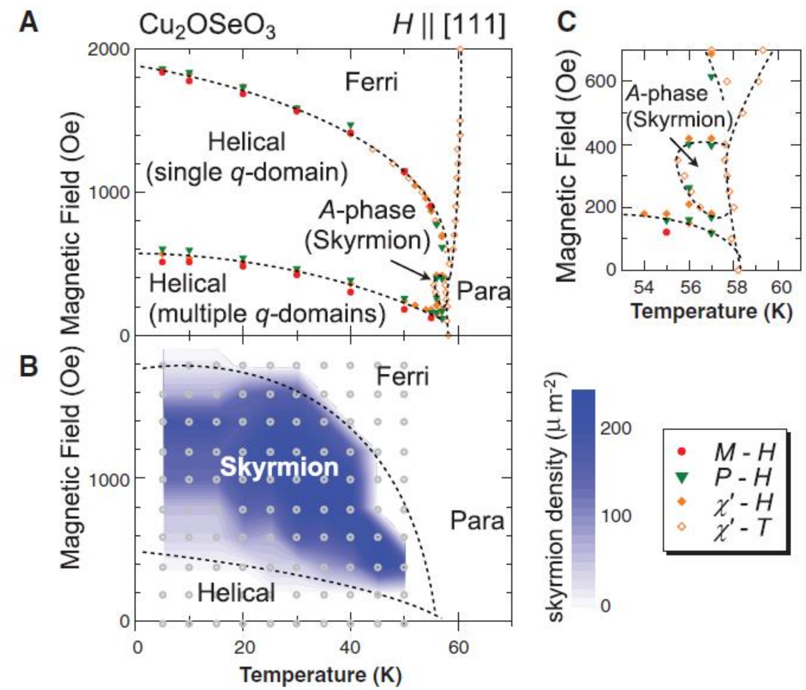
Bulk



Thin films



metals



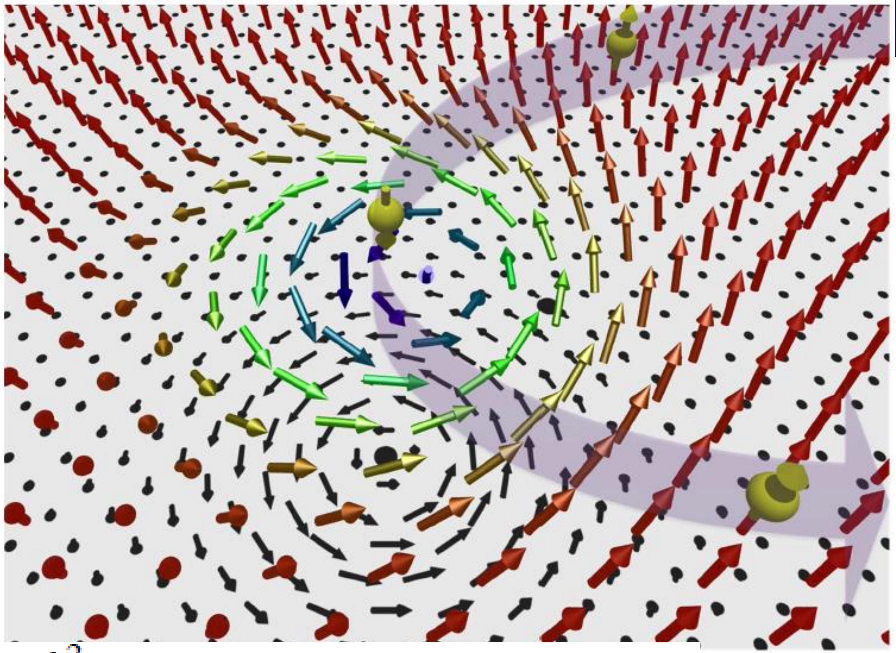
insulators

The skyrmion phase diagram is similar, suggesting that the physics is governed by a universal low energy theory.

$$\mathcal{H} = \int \left[\frac{J}{2}(\nabla \mathbf{S})^2 + D\mathbf{S}\cdot(\nabla\times\mathbf{S}) - \mathbf{H}\cdot\mathbf{S} \right] dr^3$$

Bak and Jensen, J. Phys. C,13, L881 (1980).

Emergent electromagnetism in metallic magnets



- Topological charge

$$Q=\frac{1}{4\pi}\int dr^2(\mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n})=\pm 1$$

- Conduction electrons Hamiltonian

$$S_{\rm el}=\int dt\,dx^3\left[i\hbar\psi^\dagger\dot{\psi}-\mathcal{H}\right]\text{with}\,\mathcal{H}=\frac{\hbar^2}{2m}\nabla\psi^\dagger\nabla\psi-J_H\,S\psi^\dagger\boldsymbol{\sigma}\cdot\mathbf{n}\,\psi$$

- Large Hund's coupling (adiabatic limit) $J_{\rm H}>>E_{\rm F}$

$$S_{el}=\int \mathrm{d}r^3\mathrm{d}t[\mathrm{i}\hbar\chi^\dagger\dot{\chi}+eA_0-\frac{1}{2m}[(\mathrm{i}\hbar\nabla-\frac{e}{c}\mathbf{A}^*)\chi^\dagger][(-\mathrm{i}\hbar\nabla-\frac{e}{c}\mathbf{A})\chi]-\frac{\hbar^2}{8m}(\nabla\mathbf{n})^2+J_HS$$

- Emergent electromagnetic fields

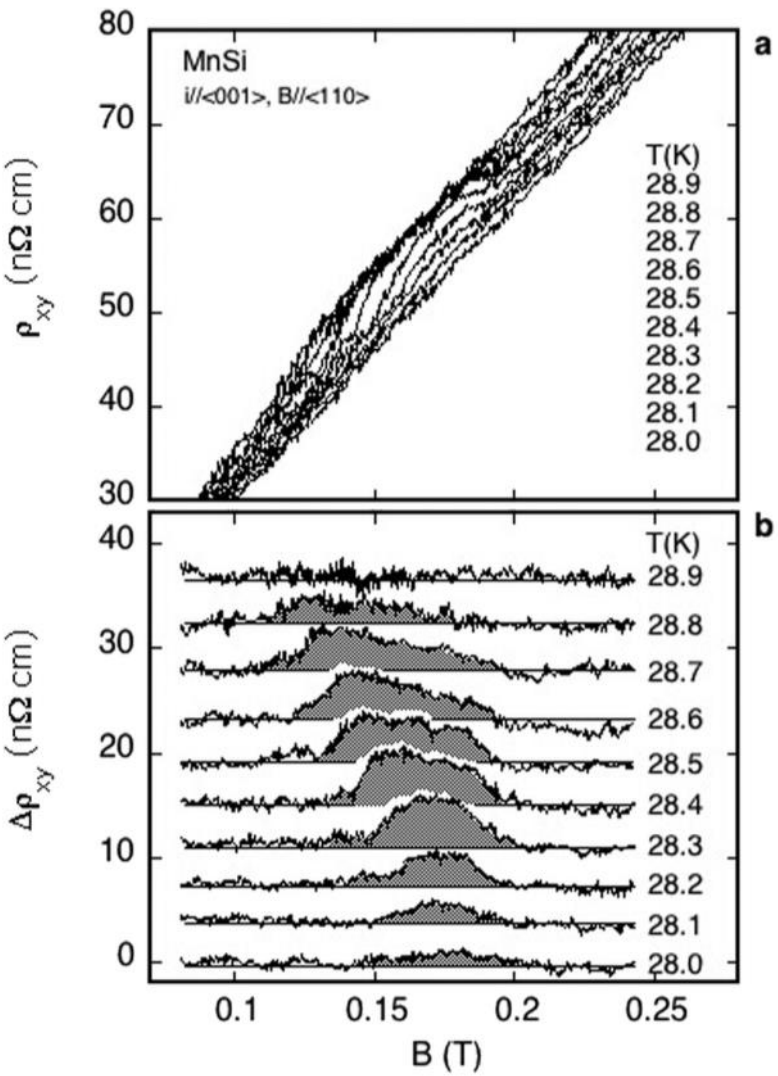
$$(\nabla\times\mathbf{A})_z=B_z=\frac{\hbar c}{2e}[\mathbf{n}\cdot(\partial_x\mathbf{n}\times\partial_y\mathbf{n})],\;\nabla A_0=\mathbf{E}=\frac{\hbar}{2e}[\mathbf{n}\cdot(\nabla\mathbf{n}\times\partial_t\mathbf{n})]$$

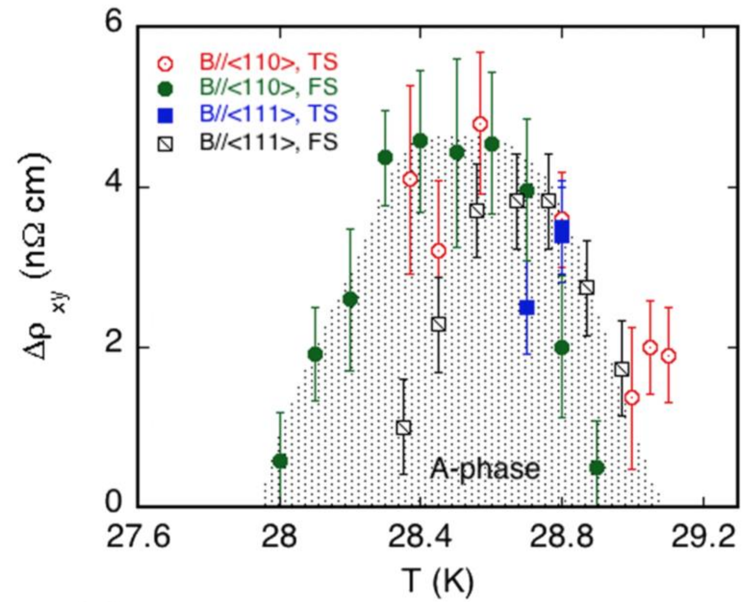
- For skyrmion size 10 nm, B ~ 100 T !!!
- Longitudinal and topological Hall conductivity

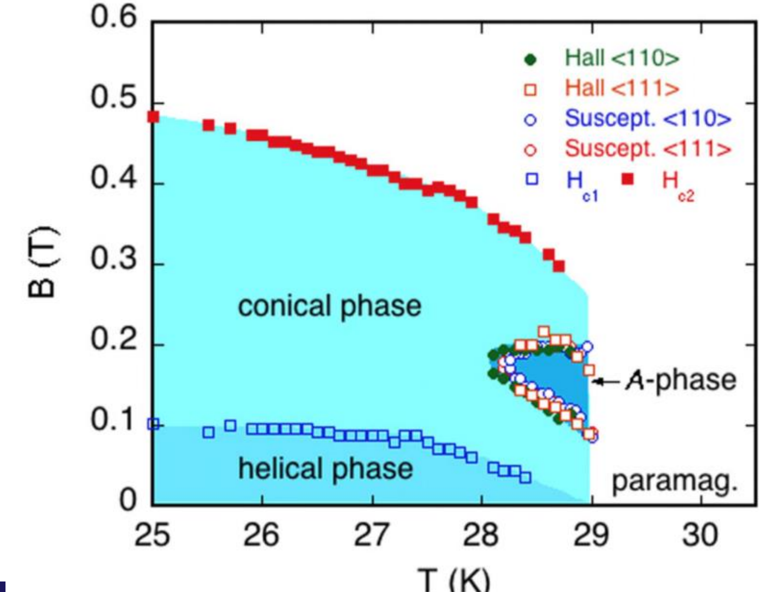
$$s_{\parallel}=\frac{e^2r_nt}{m}\frac{1}{1+(w_ct)^2},\;s_{\wedge}=\frac{e^2r_nt}{m}\frac{w_ct}{1+(w_ct)^2}\text{ with } w_c=eB_z/mc$$

Topological Hall effect

Neubauer et al., PRL 102, 186602 (2009)

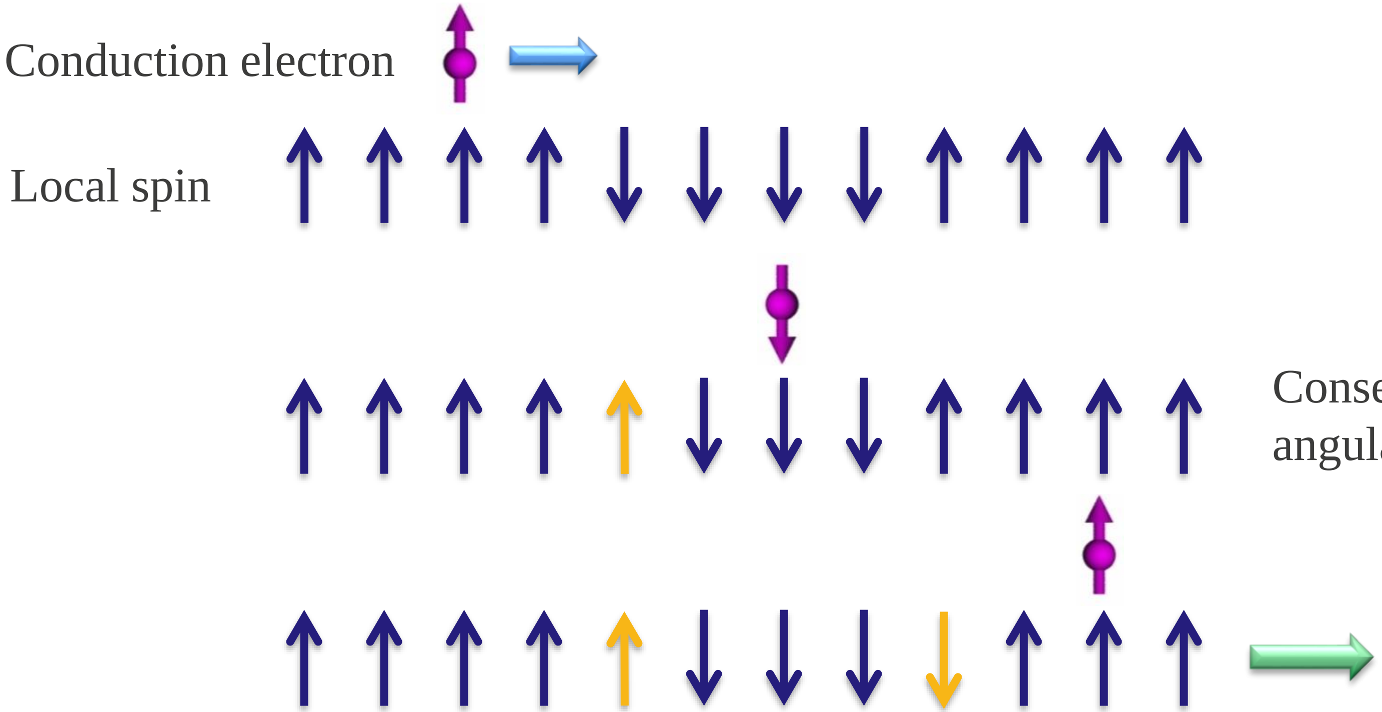






Spin transfer torque: simple picture

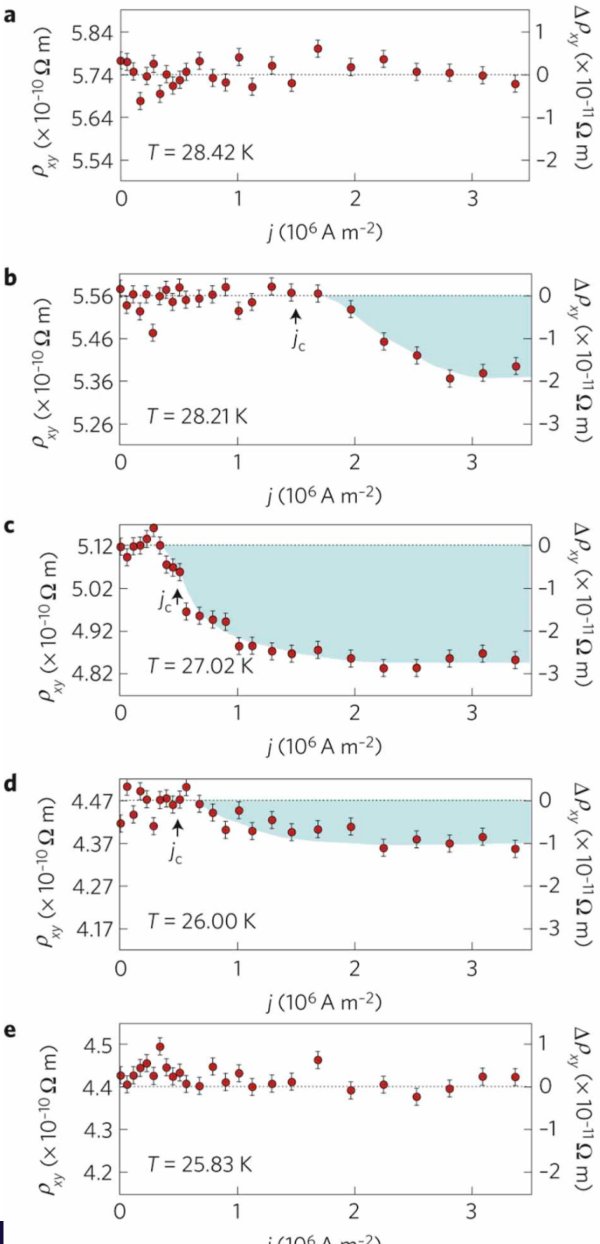
- Spin configuration in radial direction of a skyrmion = magnetic bubble domain



- 2013 Oliver E. Buckley Condensed Matter Physics Prize Recipient: John Slonczewski and Luc Berger

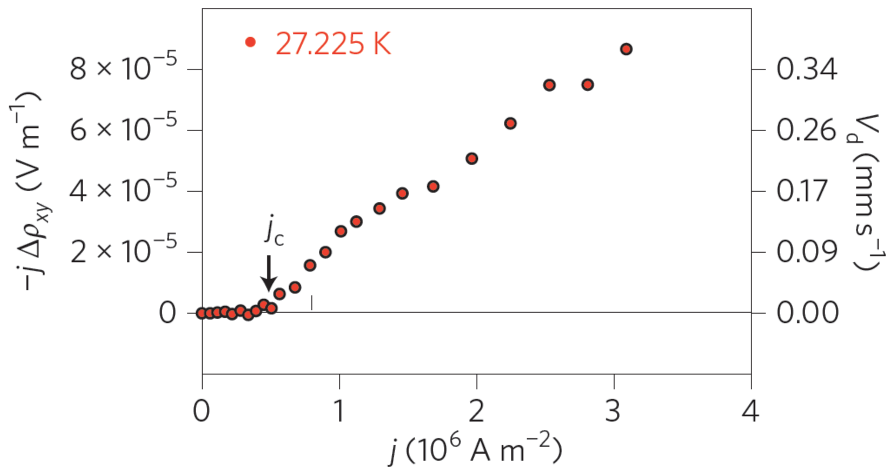
"For predicting spin-transfer torque and opening the field of current-induced control over magnetic nanostructures."

Driven skyrmions by current

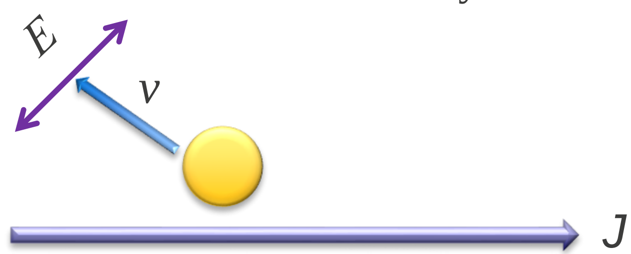


Material: MnSi

Schulz, et. al., Nature Physics 8, 301 (2012).



Hall resistance measurement and skyrmion velocity

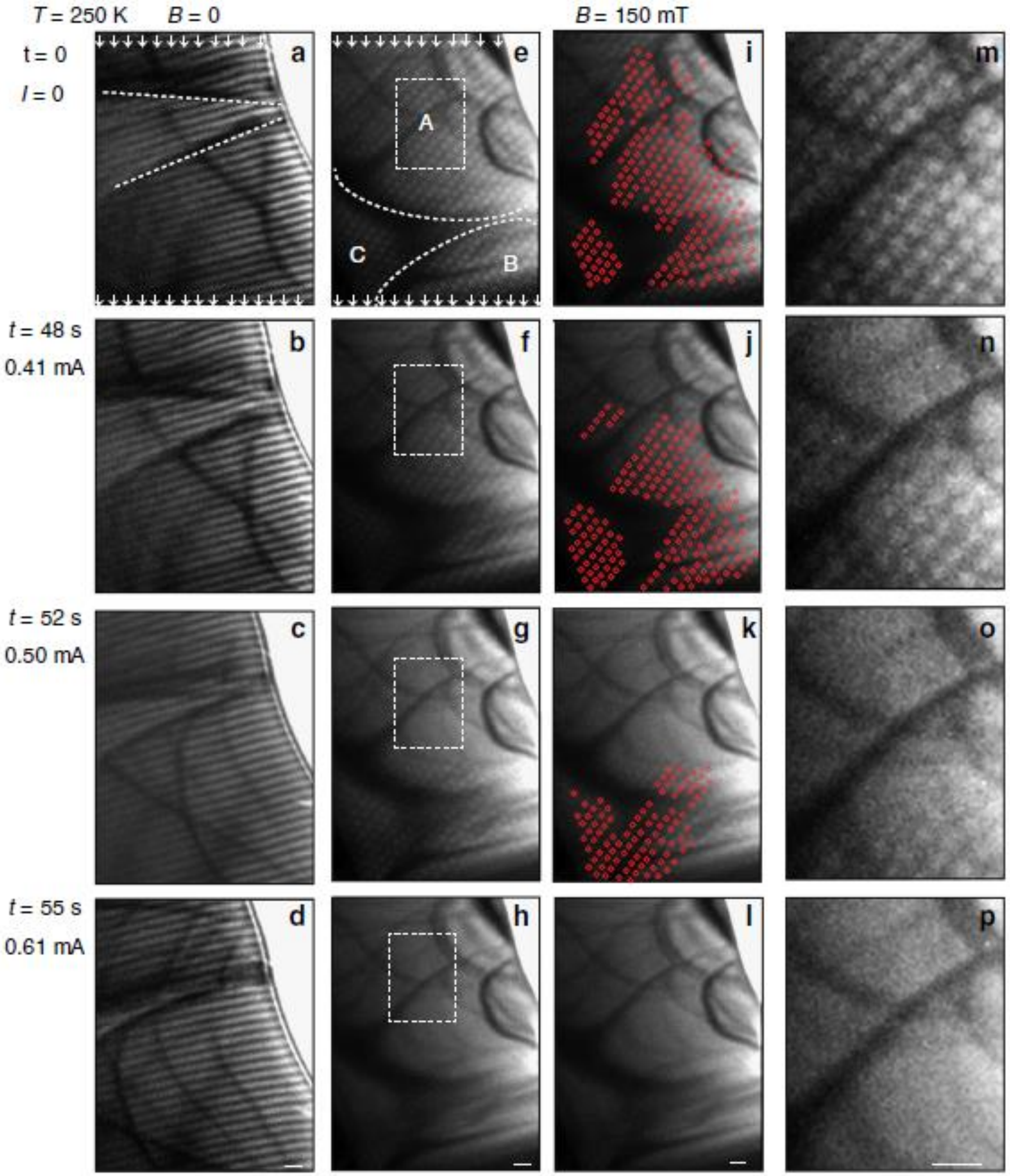


- Hall resistance is due to the emergent electric field induced by the motion of skyrmion.
- The depinning current is extremely low J_c~10^6 A/m^2. For magnetic domain walls J_c~10^{11} A/m^2.

Jonietz, et. al., Science 330, 1648 (2010).

Yu, et. al, Nature Communications 3, 988 (2012).

Real time observation of skyrmion motion



Material: FeGe

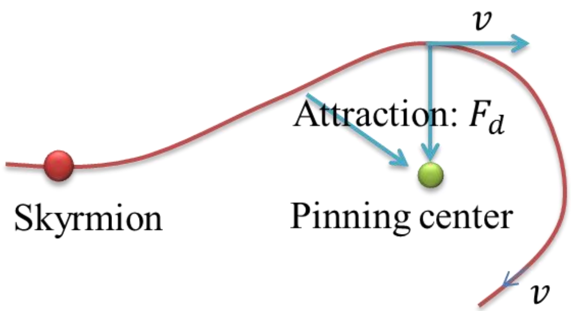
Yu, et. al, Nature Communications 3, 988 (2012).

Equation of motion for a rigid skyrmion

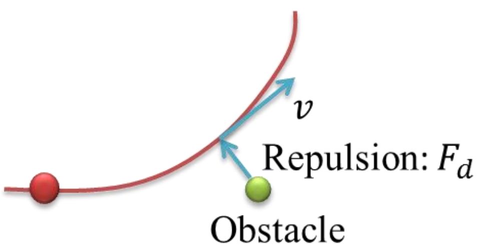
Driving by spin-transfer torque

$$\eta \mathbf{v}_i = \underbrace{\alpha \hat{\mathbf{z}} \times \mathbf{v}_i}_{\text{Magnus force}} + \hat{\mathbf{z}} \times \mathbf{J} + \sum_j \mathbf{F}_{\mathrm{d}}\left(\mathbf{r}_j - \mathbf{r}_i\right) + + \sum_j \mathbf{F}_{\mathrm{ss}}\left(\mathbf{r}_j - \mathbf{r}_i\right)$$

For skyrmion $\alpha/\eta \gg 1$, skyrmions are not easily trapped by pinning centers.



(a) Pinning with $\alpha/\eta \gg 1$

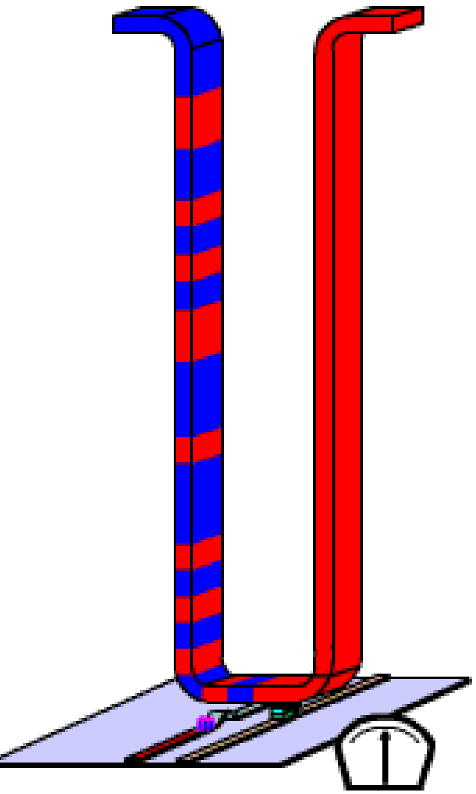


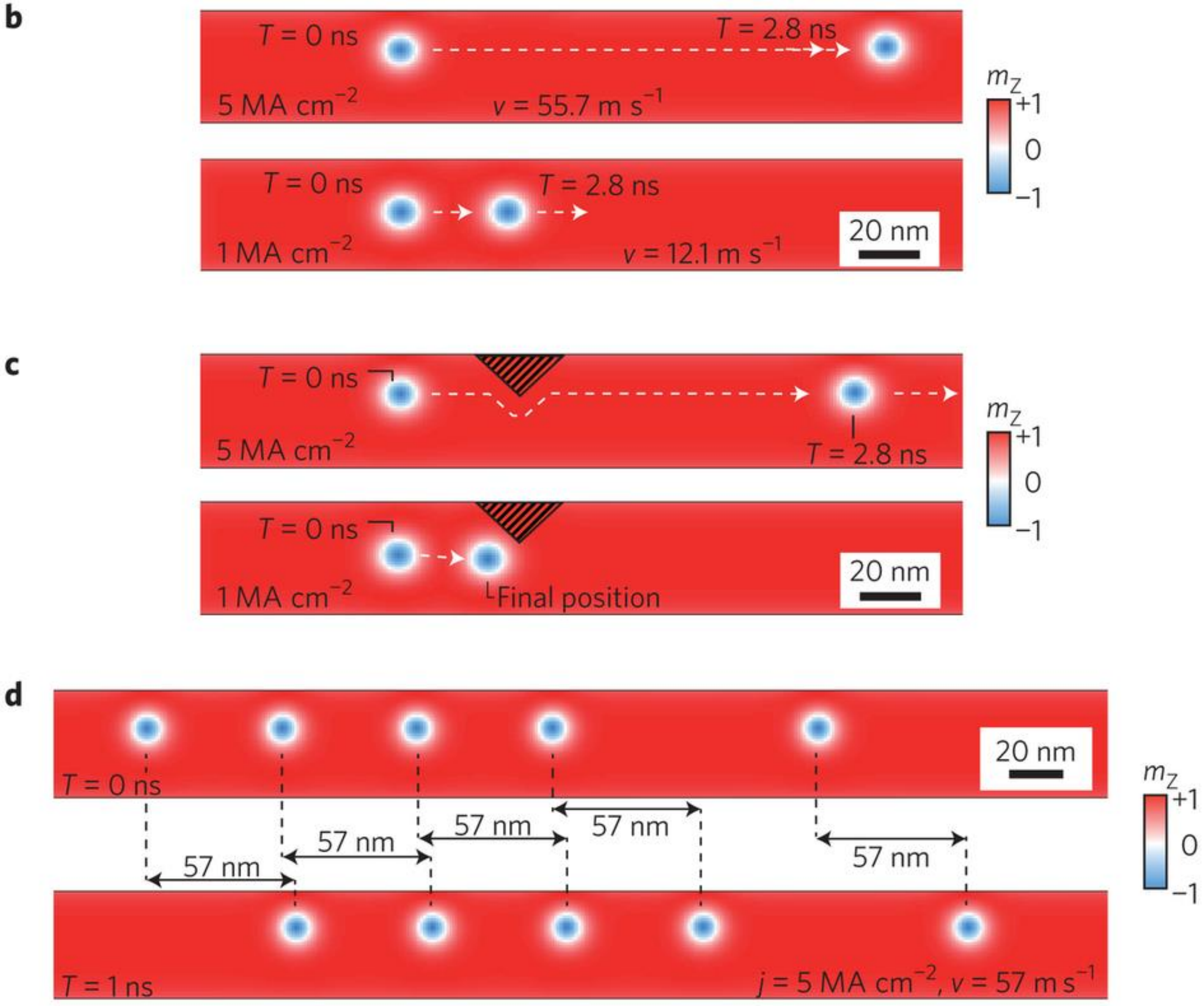
(b) Obstacle with $\alpha/\eta \gg 1$

SZL, Reichhardt, Batista, and Saxena, Phys. Rev. B **87**, 214419 (2013)

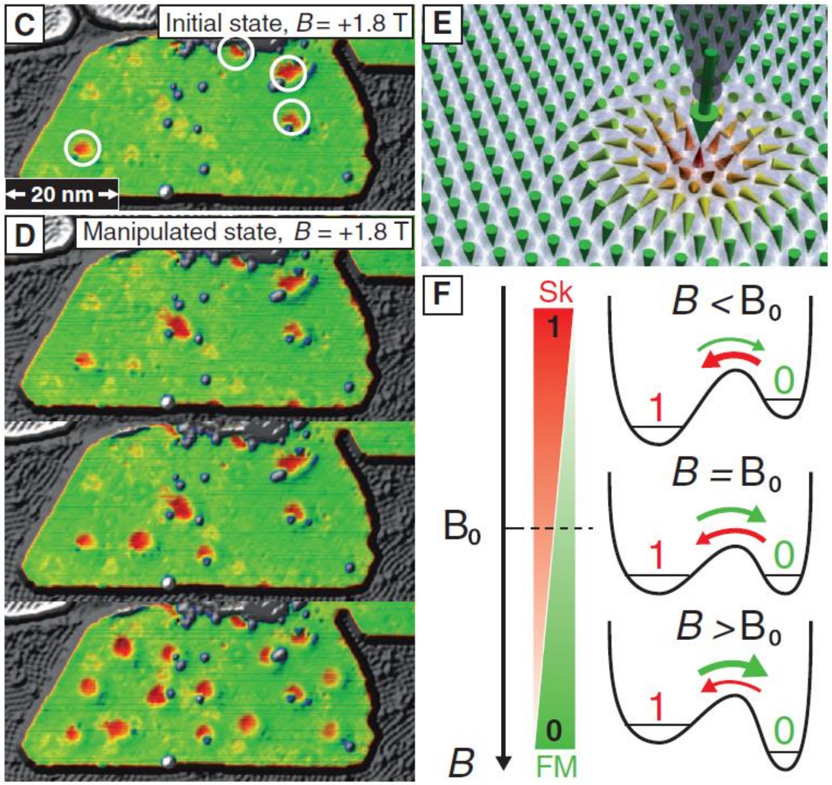
Skyrmion racetrack memory

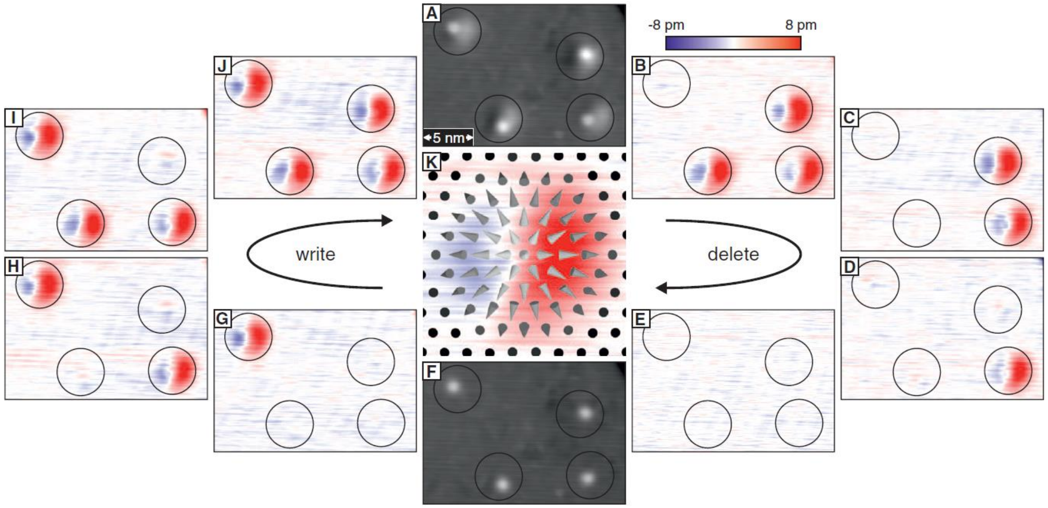
Fert et al., Nature Nanotechnology 8, 152–156 (2013)





Writing and deleting skyrmions





- ❑ Controlled creation and removal of a skyrmion by a spin polarized STM tip.
- ❑ Demonstration of application of skyrmions in information storage.

N. Romming *et al*. Science 341, 636 (2013).

RESEARCH ARTICLE

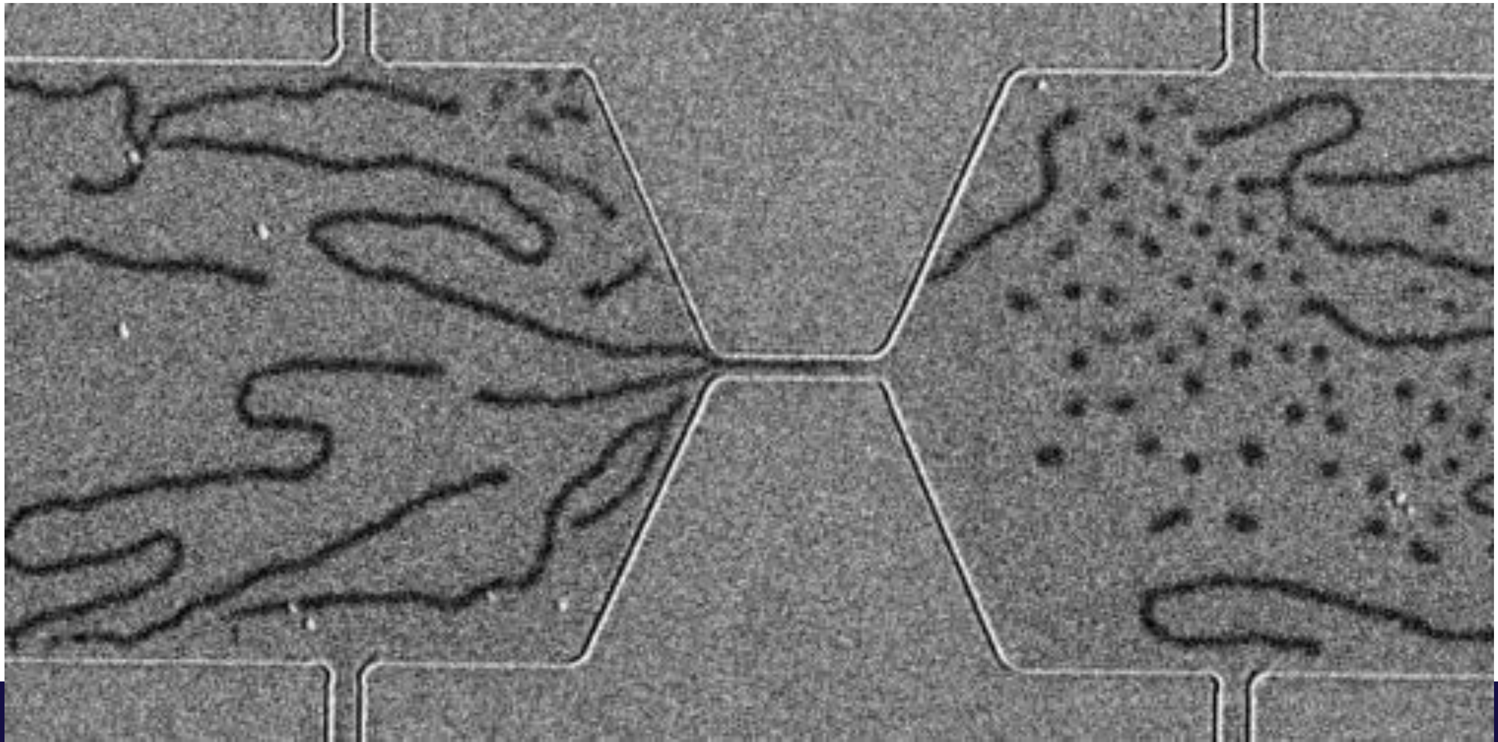
Jiang et al., Science 349, 6245 (2015)

MAGNETISM

Ta(5 nm)/$Co_{20}Fe_{60}B_{20}$(CoFeB)(1.1 nm)/TaO_x(3 nm)

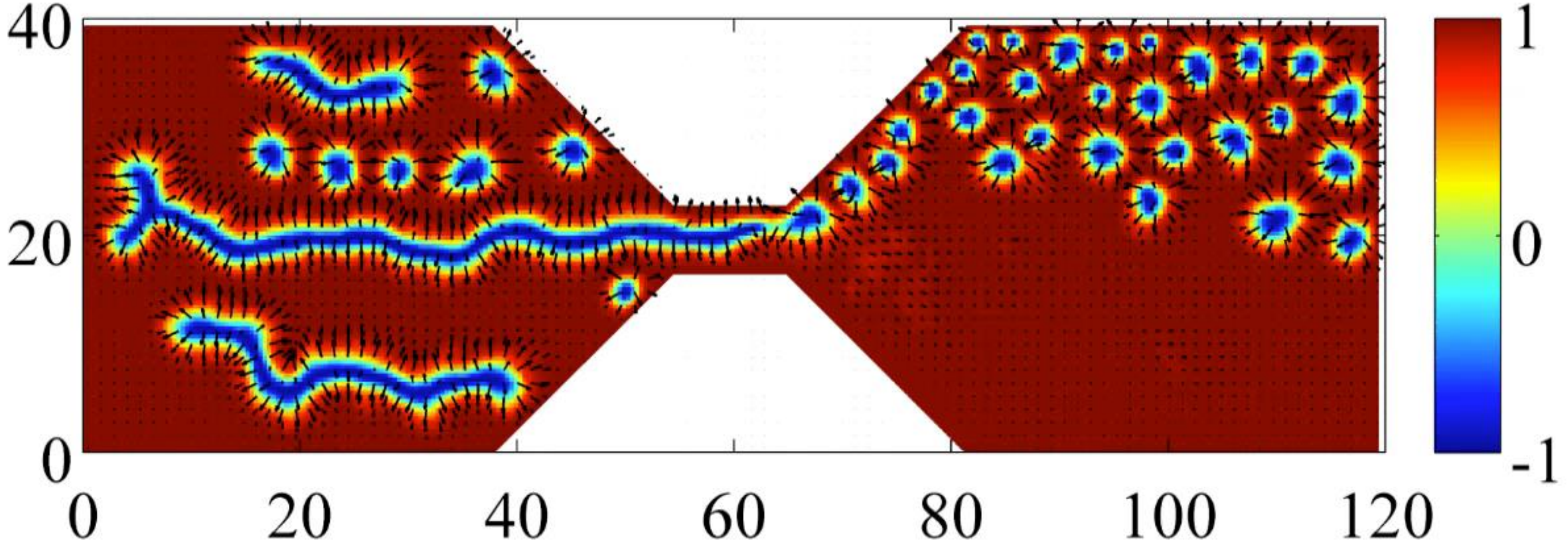
Blowing magnetic skyrmion bubbles

Wanjun Jiang,[1] Pramey Upadhyaya,[2] Wei Zhang,[1] Guoqiang Yu,[2]
M. Benjamin Jungfleisch,[1] Frank Y. Fradin,[1] John E. Pearson,[1] Yaroslav Tserkovnyak,[3]
Kang L. Wang,[2] Olle Heinonen,[1,4,5,6] Suzanne G. E. te Velthuis,[1] Axel Hoffmann[1*]



Theory: Edge instability of chiral stripe domains

SZL, Phys. Rev. B 94, 020402(R) (2016).

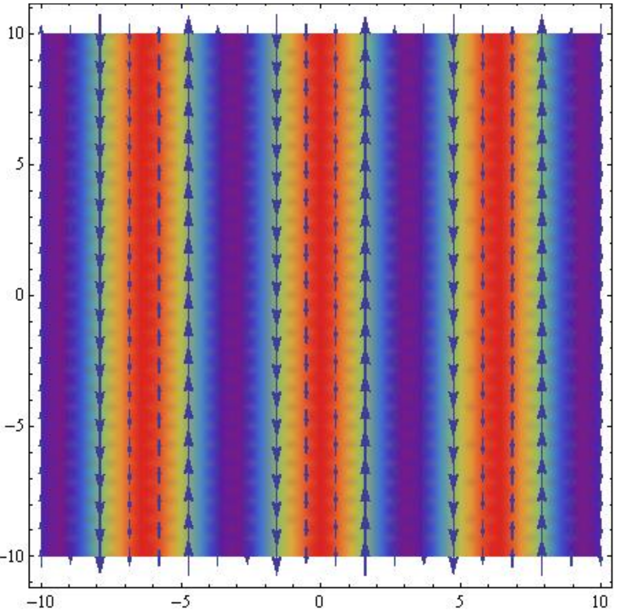


- Can we have thermodynamically stable skyrmion phase in inversion-symmetric magnets, such as rare earth magnets?

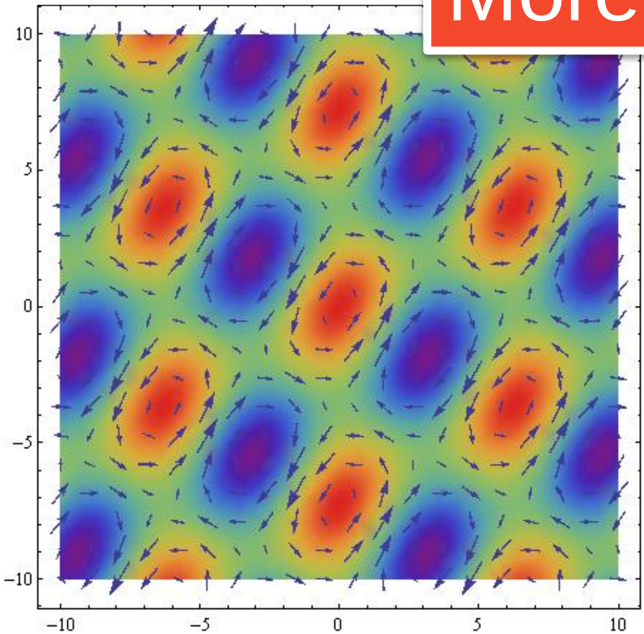
- What is the stabilization mechanism?

Triangular skyrmion lattice as superposition of triple-Q helix (particle & wave duality)

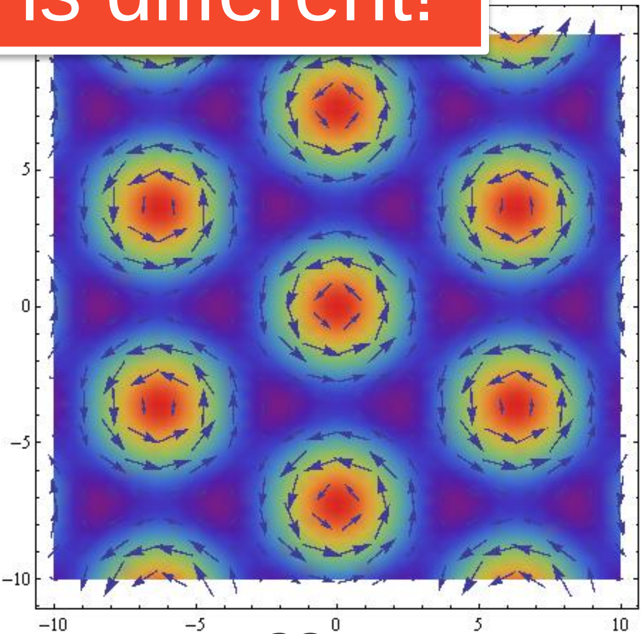
More is different!



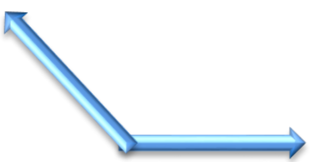
1Q state

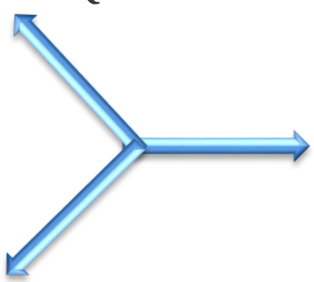


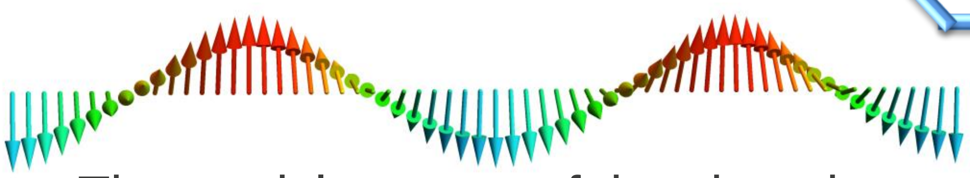
2Q state



3Q state







The particle nature of the skyrmion manifests as high harmonics (higher order peaks in spin structure factor).

Design principles

- Competing magnetic interaction

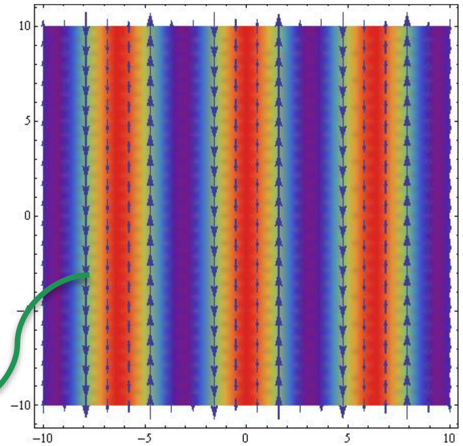
$$\mathcal{H} \propto -\sum_{q} J(\mathbf{q})\mathbf{S}(\mathbf{q}) \cdot \mathbf{S}(-\mathbf{q})$$

- $J(\mathbf{q})$ is maximized at a nonzero $\mathbf{q}$.
- Optimal lattice structure should have C_6 symmetry in the ab plane.

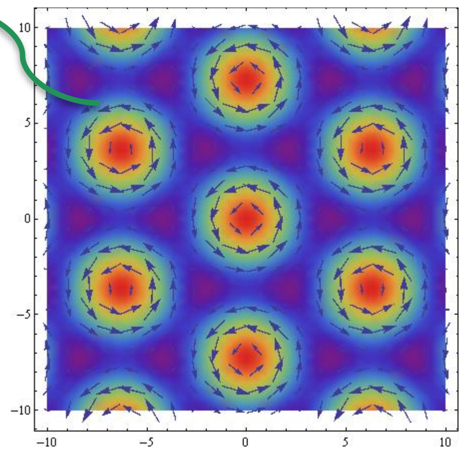
- However, superposition of triple Q helices violates the condition $|S| = constant$ and costs energy.

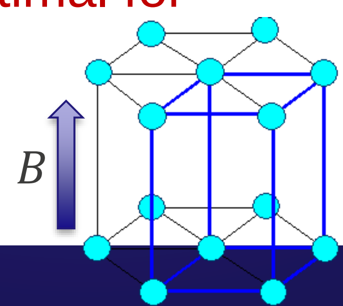
- Solution: easy axis anisotropy
- The single-Q helix has higher domain wall energy

- Hexagonal crystal structure is optimal for stabilizing the skyrmion lattice.



"domain wall"





Example: Interactions in rare-earth magnets

- RKKY interaction: depends on topology of the Fermi surface

$$J(\mathbf{q}) = \frac{2\mu_B^2}{V} \sum_{nn'\mathbf{k}} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}-\mathbf{q}}}{\varepsilon_{n'}(\mathbf{k}-\mathbf{q}) - \varepsilon_n(\mathbf{k})},$$

- Single-ion anisotropy induced by the crystal field

$$\mathcal{H}_{\mathrm{cf}} = \sum_i \Big[\sum_{l=2,4,6} B_l^0 O_l^0(\mathbf{J}_i) + B_6^6 O_6^6(\mathbf{J}_i) \Big] \quad \text{for Hexagonal crystal}$$

- Zeeman interaction
- Dipolar interaction, $E_{dipole} < 1\ \mathrm{K}$
- Hyperfine interaction with nuclear spin, $E_{Nuclear} \sim \mu\mathrm{eV}$

The Hamiltonian we will consider

$$\mathcal{H} = -\frac{N}{2} \sum_q J(\mathbf{q}) \mathbf{S}(\mathbf{q}) \cdot \mathbf{S}(-\mathbf{q}) - \frac{A}{2} \sum_i S_{i,z}^2 - H_z \sum_i S_{i,z}$$

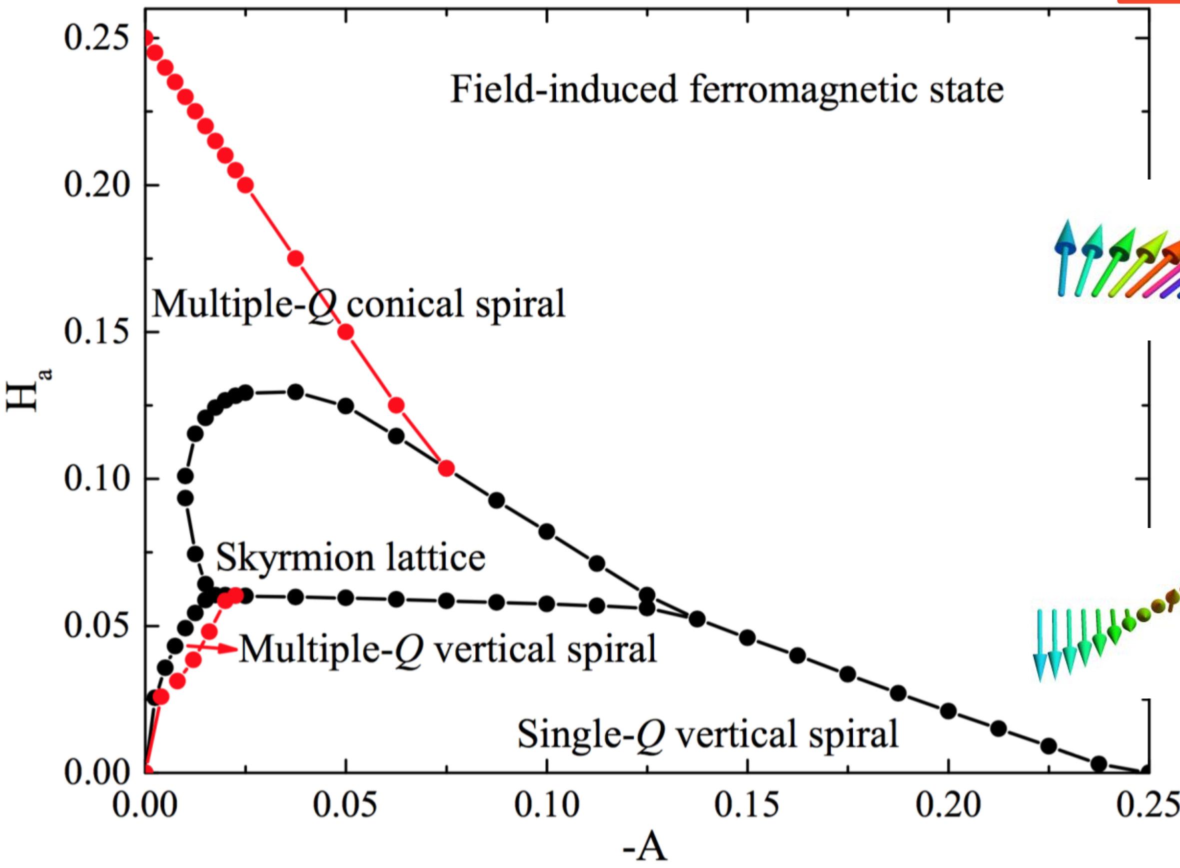
Long wavelength limit $Q \ll 1$

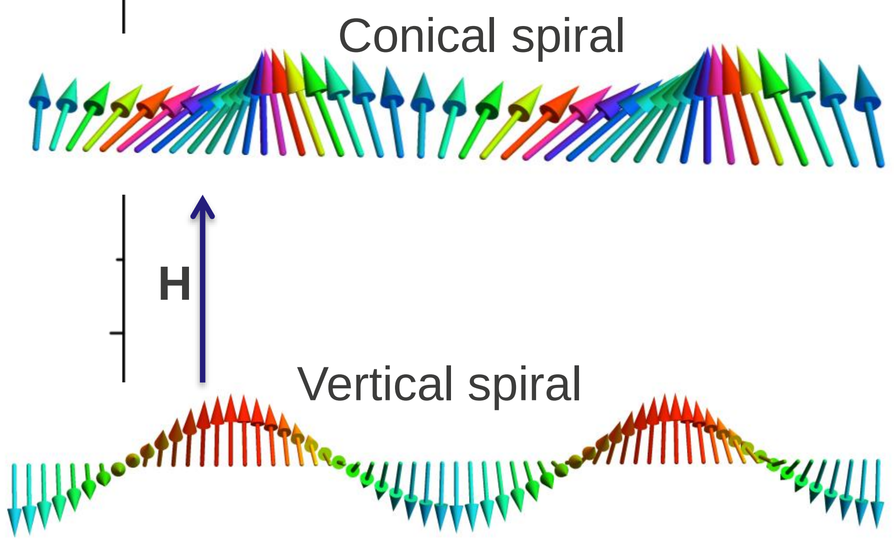
$$\mathcal{H} = -\frac{N}{2}\sum_q J(\mathbf{q})\mathbf{S}(\mathbf{q})\cdot\mathbf{S}(-q) - \frac{A}{2}\sum_i S_{i,z}^2 - H_z\sum_i S_{i,z}$$

$$\mathcal{H} = \int\left[-\frac{1}{2}\left(\nabla\mathbf{S}\right)^2 + \frac{1}{2}\left(\nabla^2\mathbf{S}\right)^2 - \mathbf{H}\cdot\mathbf{S} + AS_z^2\right]dr^2$$

A new class of the Ginzburg-Landau theory for skyrmions in inversion-symmetric magnet with competing interactions.

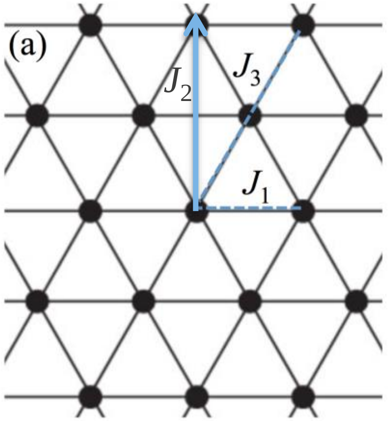
Variational calculations at T=0 in 2D



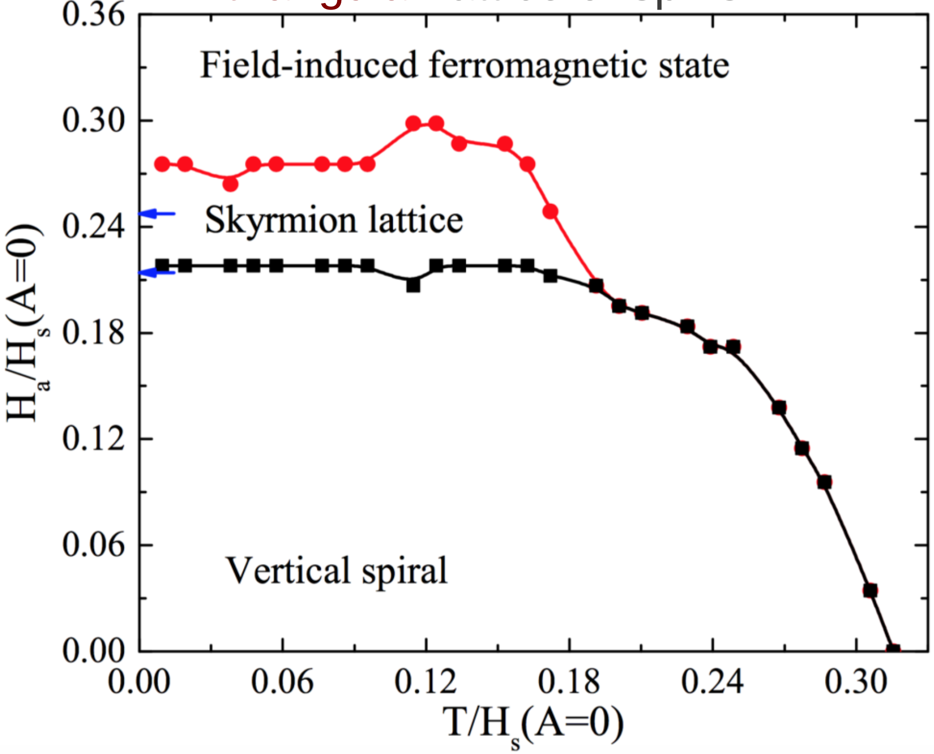


Monte Carlo simulation in 2D

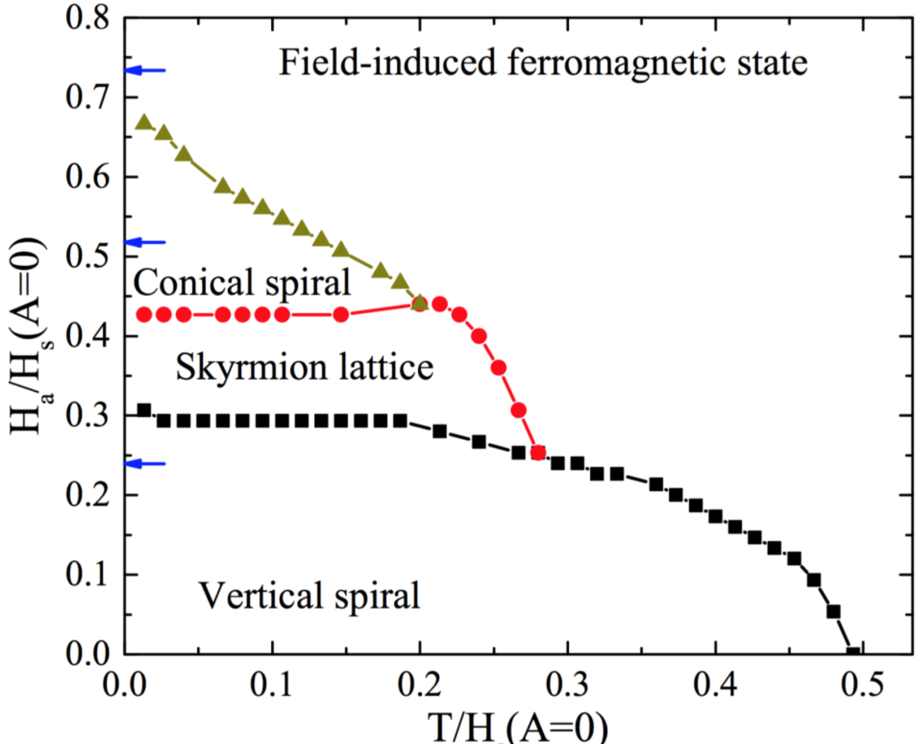
Introducing competing interactions to make $J(q)$ maximal at a nonzero q.



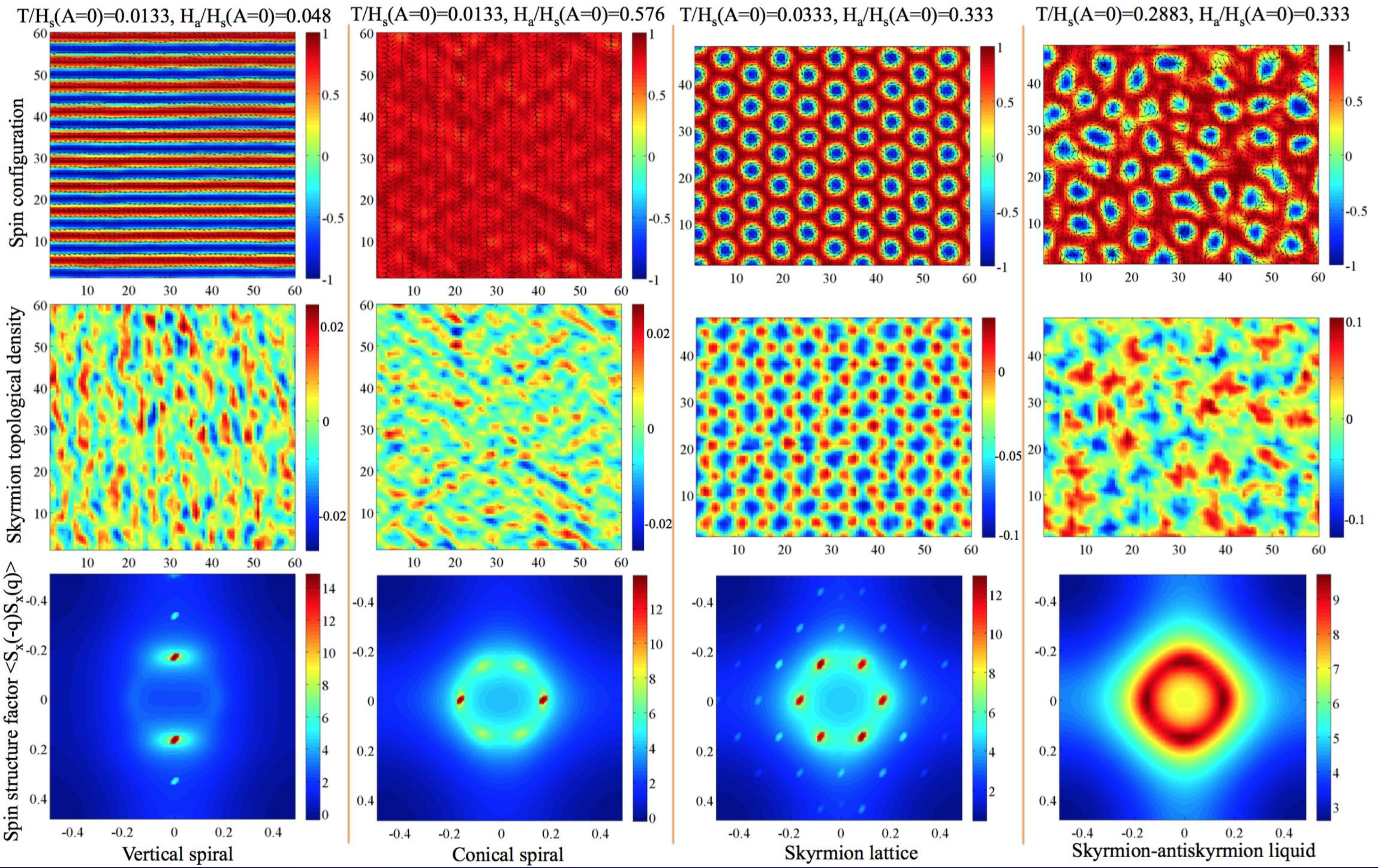
J_1-J_3 Heisenberg model on a triangular lattice of spins



J_1-J_2-J_3 Heisenberg model on a square spin lattice of spins

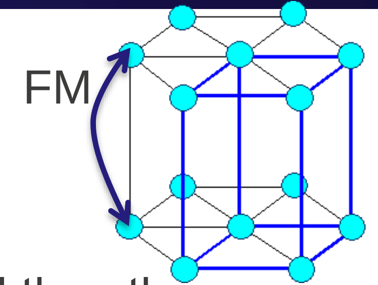


Spin configurations

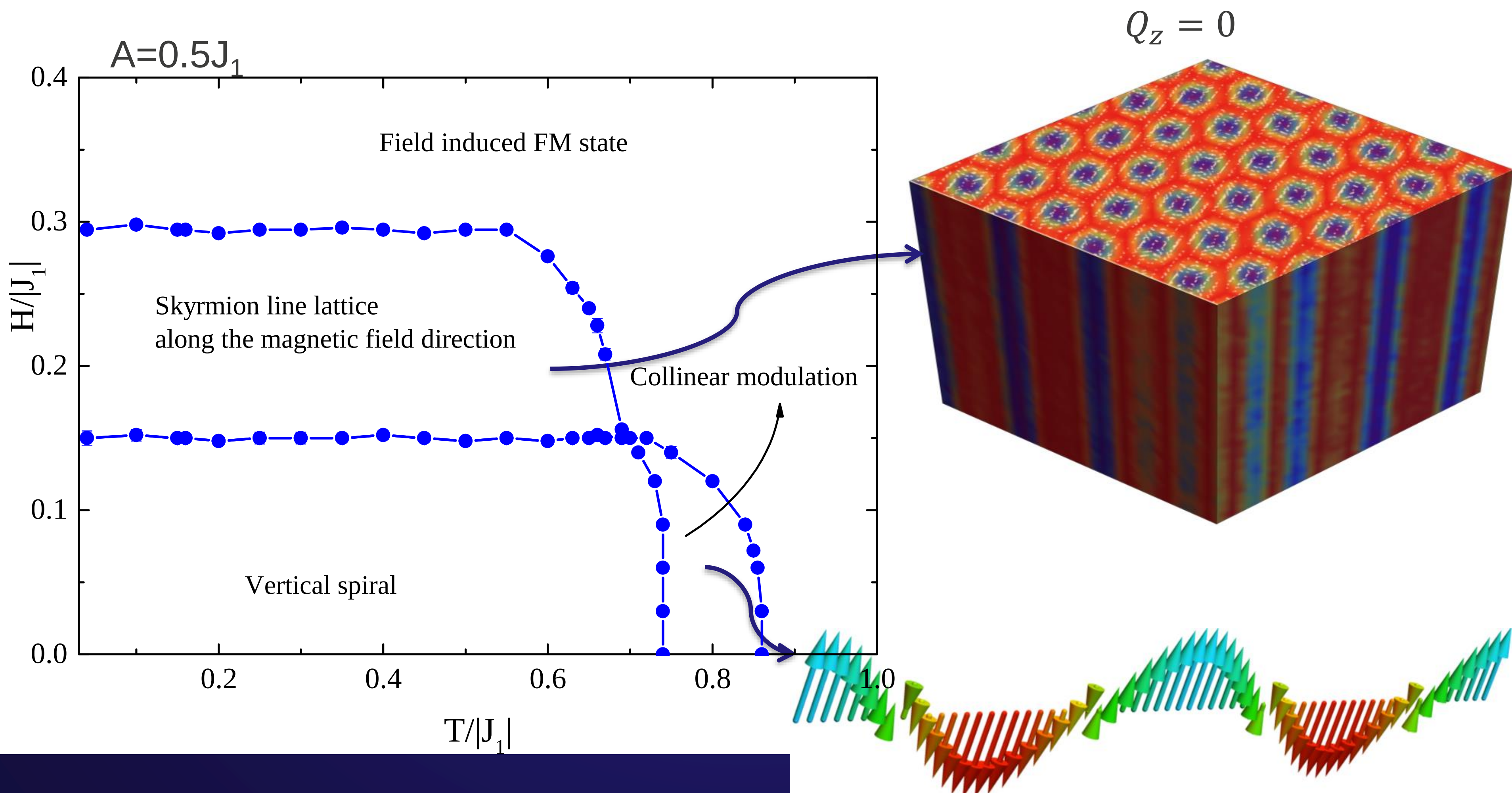


3D case

$$\mathcal{H} = -\frac{N}{2}\sum_{q} J(\mathbf{q})\mathbf{S}(\mathbf{q}) \cdot \mathbf{S}(-q) - \frac{A}{2}\sum_{i} S_{i,z}^{2} - H_z \sum_{i} S_{i,z}$$



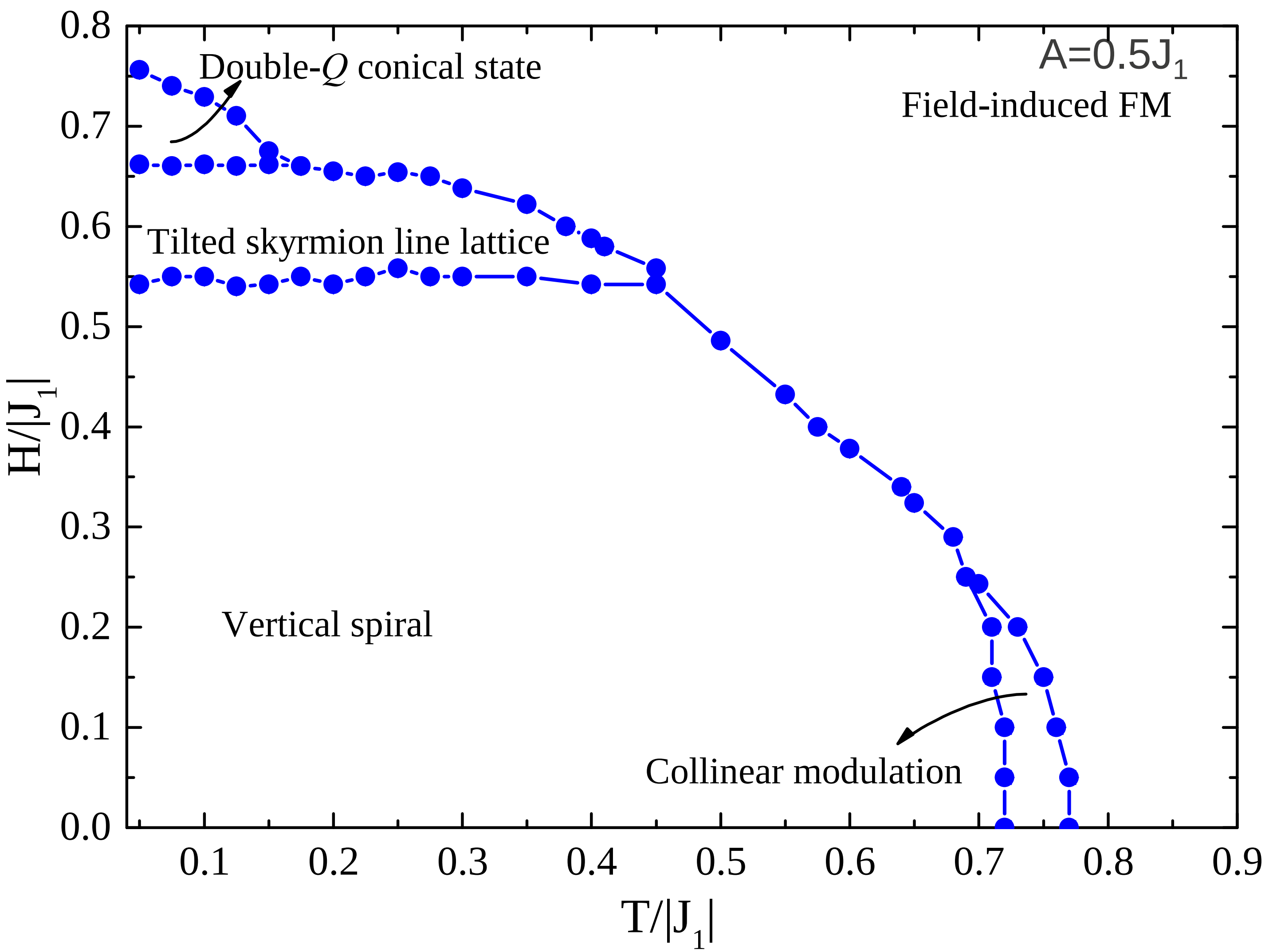
We consider hexagonal lattice. $J(\mathbf{q})$ is maximal at **Q**=(2π/5, 0, 0) and the other five **q**'s related by C_6 symmetry.

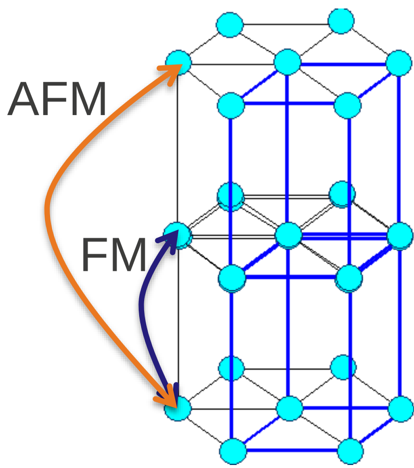


Competing interaction along the z axis

J(**q**) is maximal at **Q**=(2π/5, 0, 2π/5) and the other five **q**'s related by C_6 symmetry.

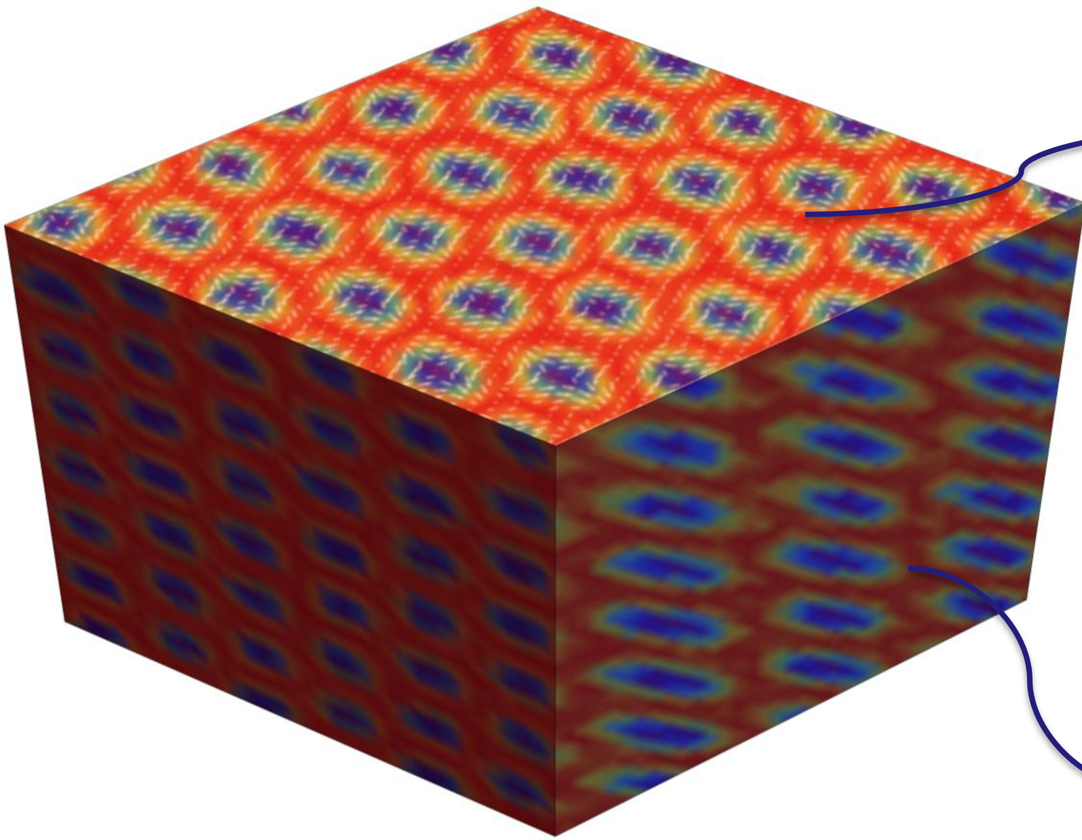
Stabilization of 4Q state, with three Q in the plane and one Q out-of-plane.





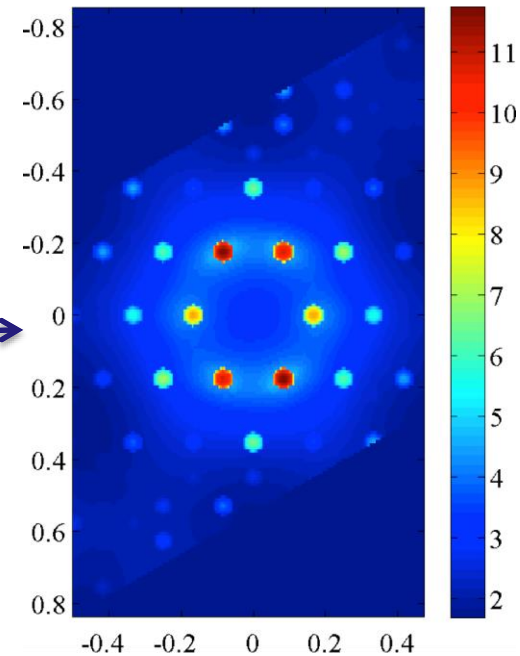
3Q state (titled skyrmion line lattice)

Surface plot of S_z

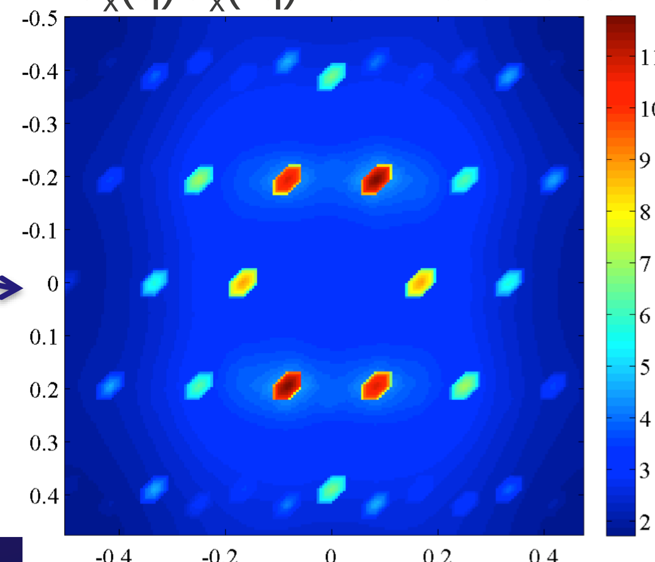


$Q_z = 2\pi/5$

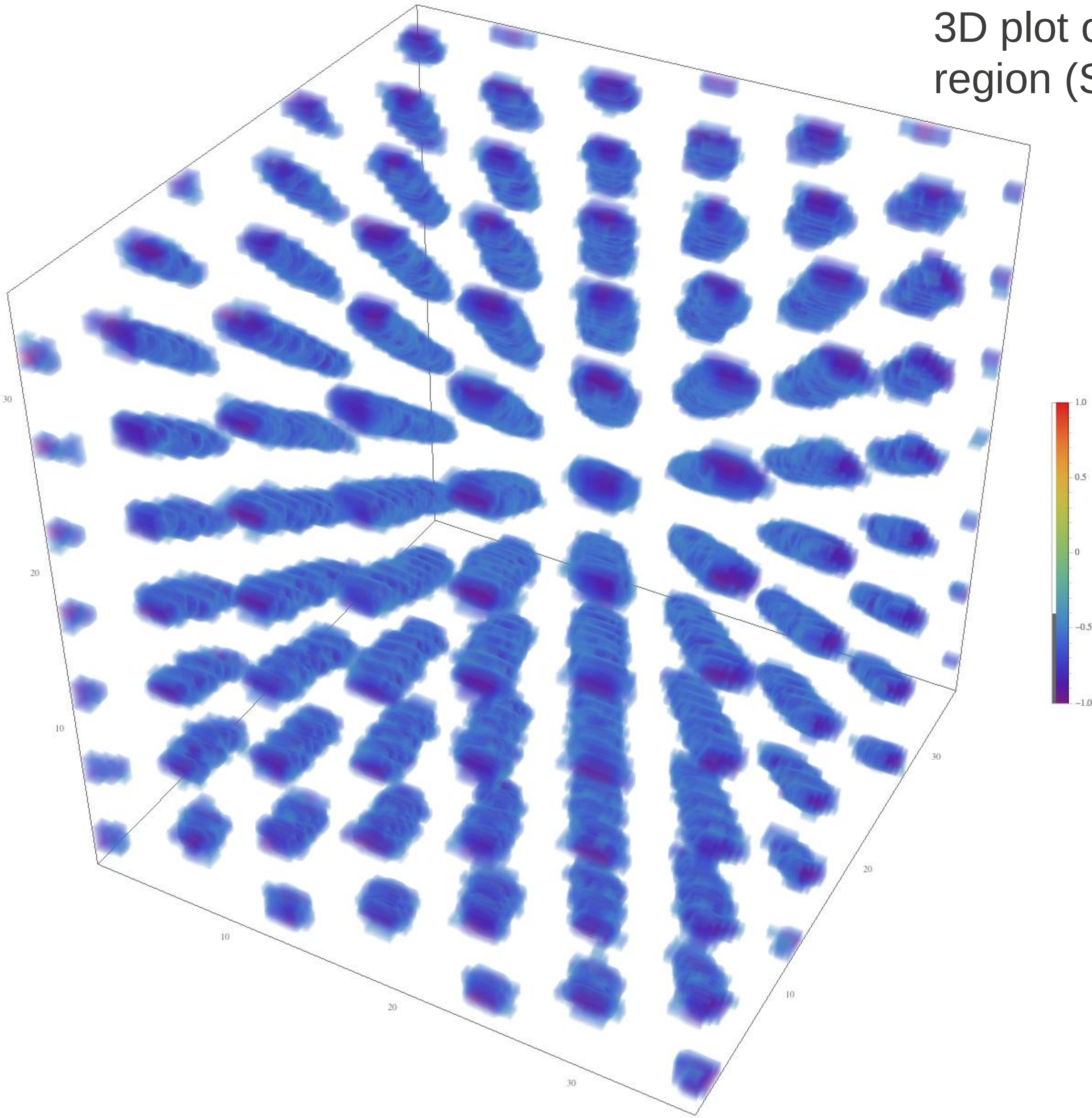
$\langle S_x(q)S_x(-q)\rangle$ at the top surface



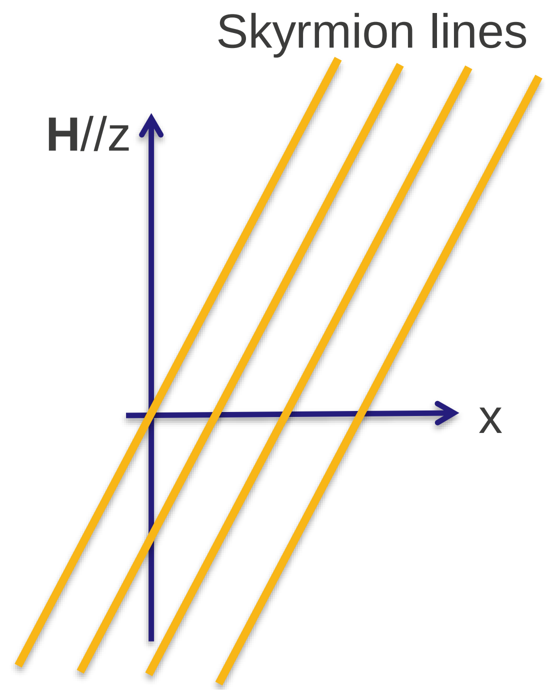
$\langle S_x(q)S_x(-q)\rangle$ at the side surface



Tilted skyrmion line lattice



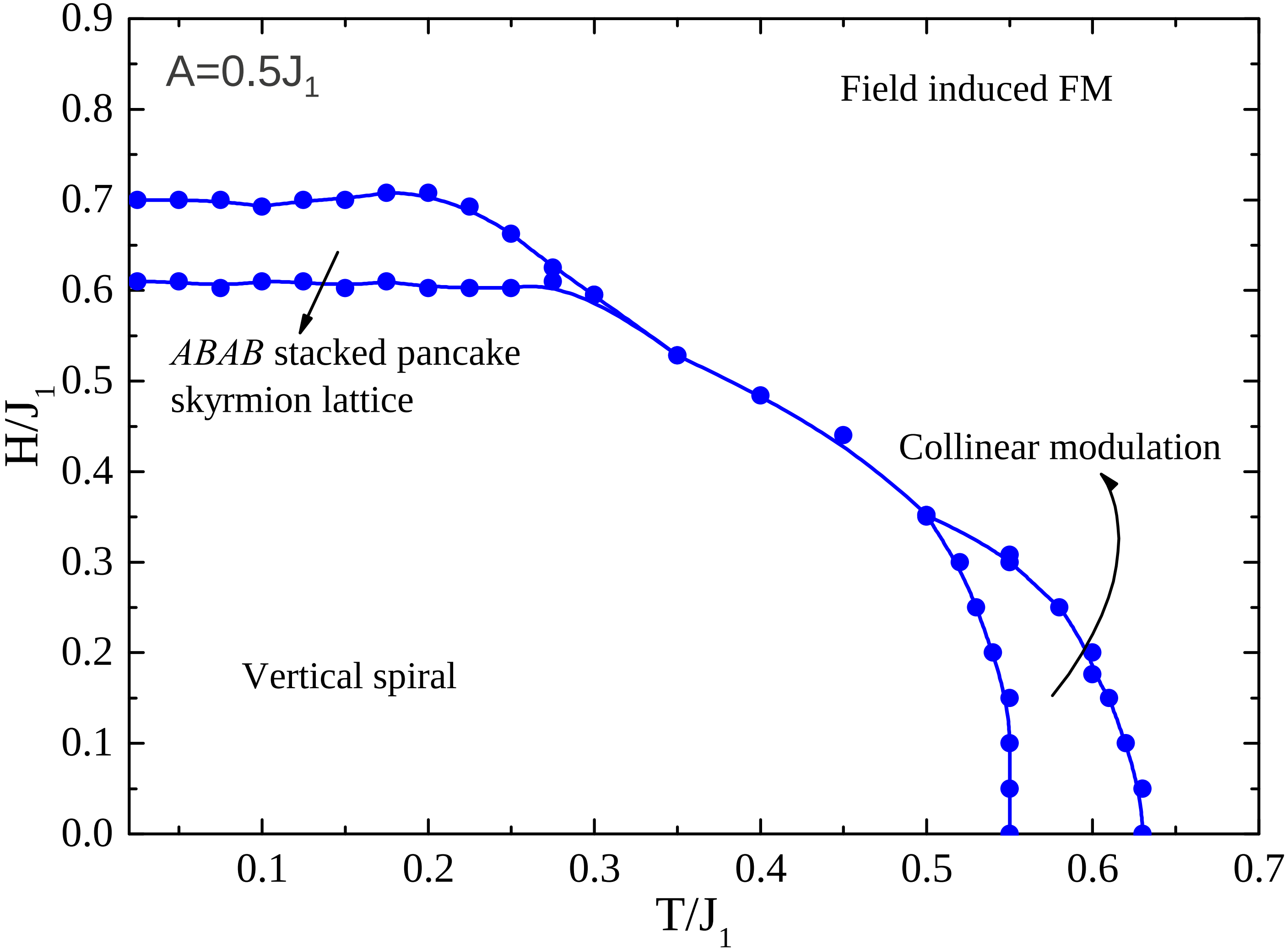
3D plot of skyrmion core region (S_z<-0.7)

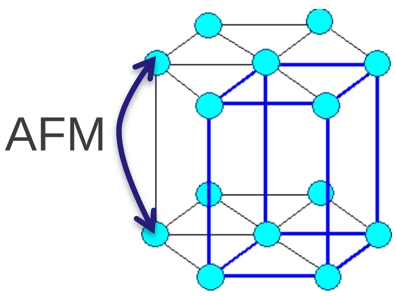


For chiral magnets, skyrmion lines are along the field direction.

AFM interaction along the z axis

J(**q**) is maximal at **Q**=(2π/5, 0, π) and the other five q's related by C_6 symmetry.

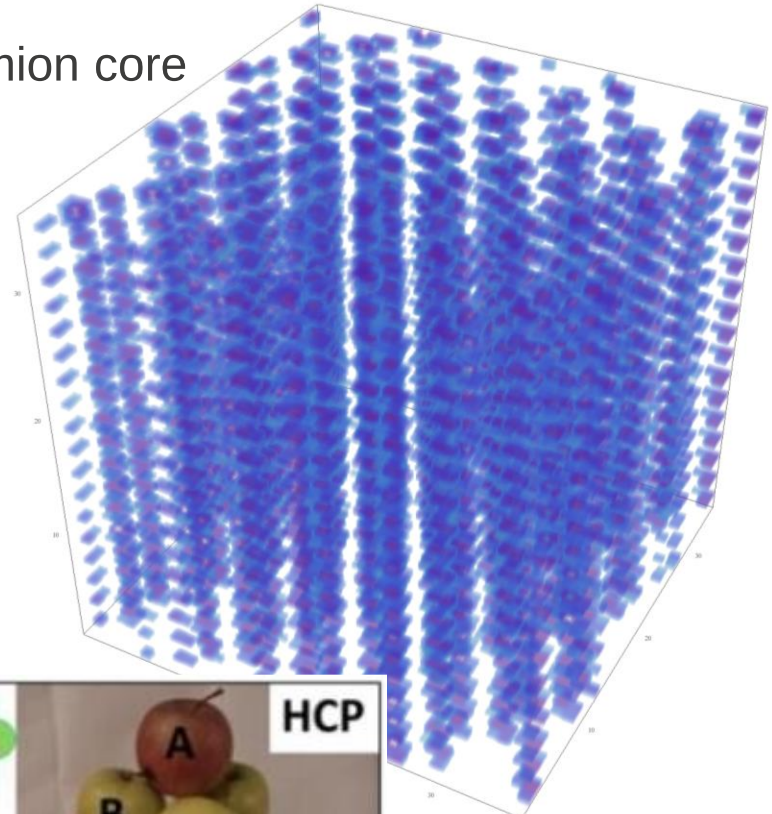




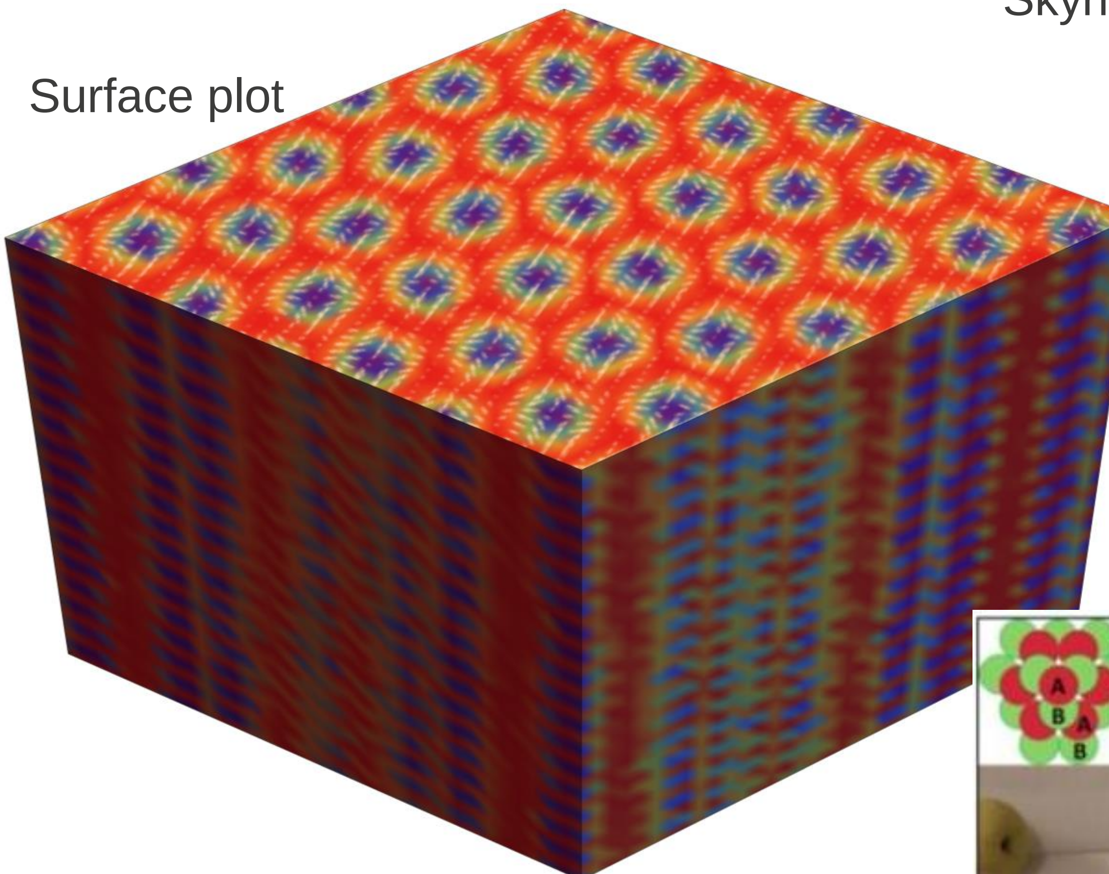
AFM interaction along the z axis ($Q_z = \pi$)

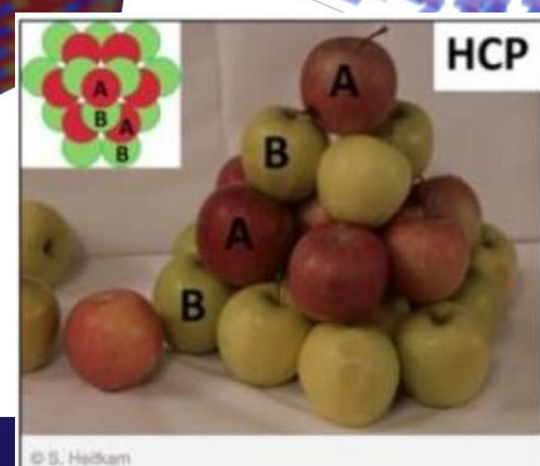
Triangular lattice of skyrmion pancake with π phase shift between layers.

Skyrmion core



Surface plot

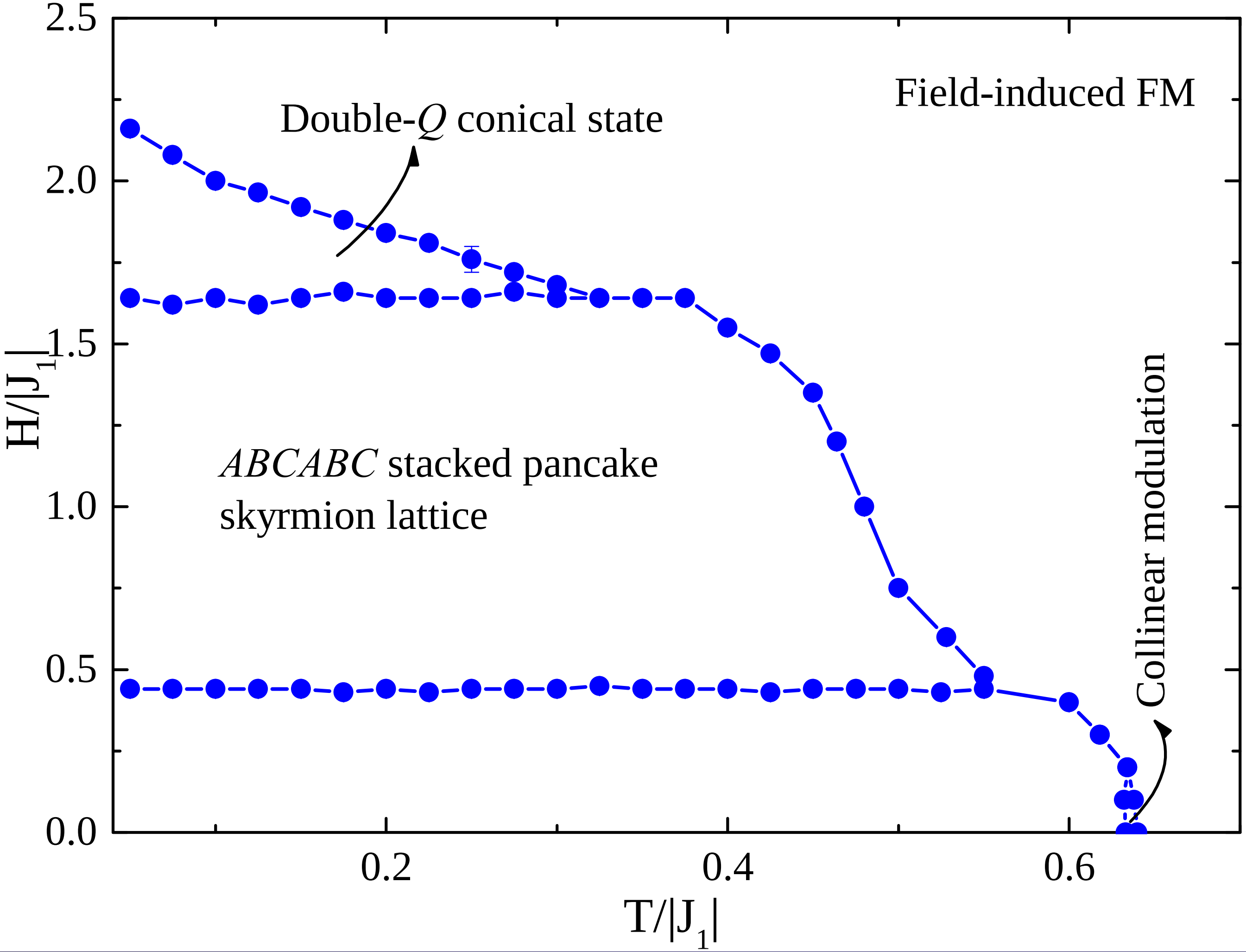


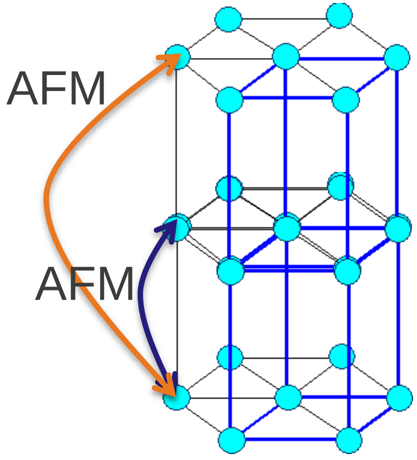


ABAB… stacking of pancake skyrmions

ABCABC stacking of skyrmions

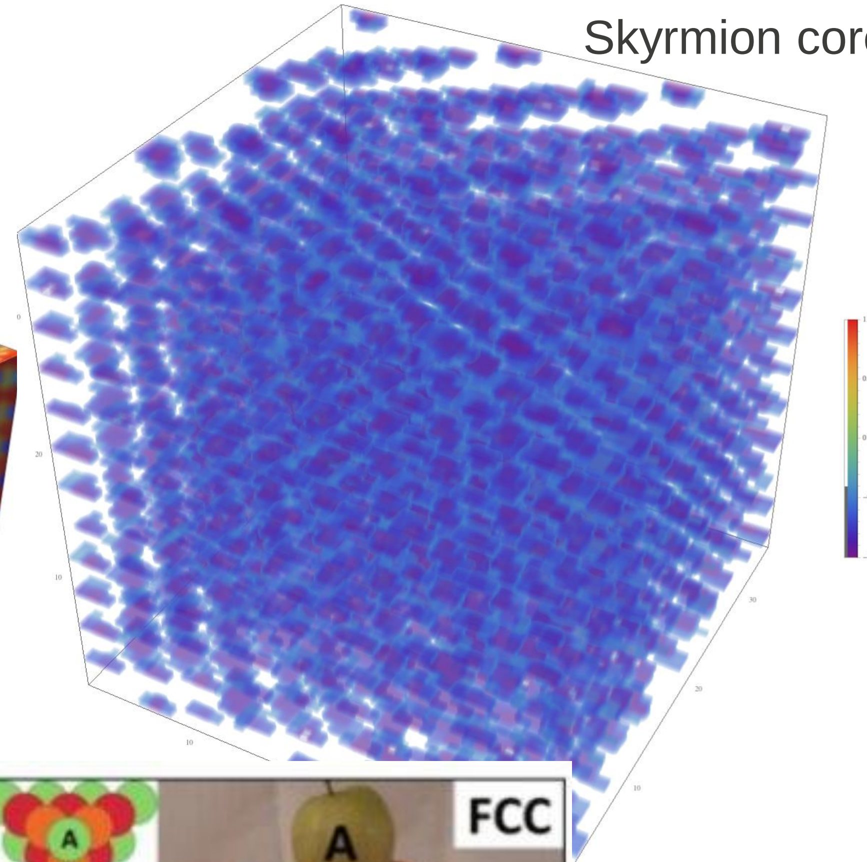
J(**q**) is maximal at **Q**=(2π/5, 0, 2π/3) and the other five **q**'s related by C_6 symmetry.



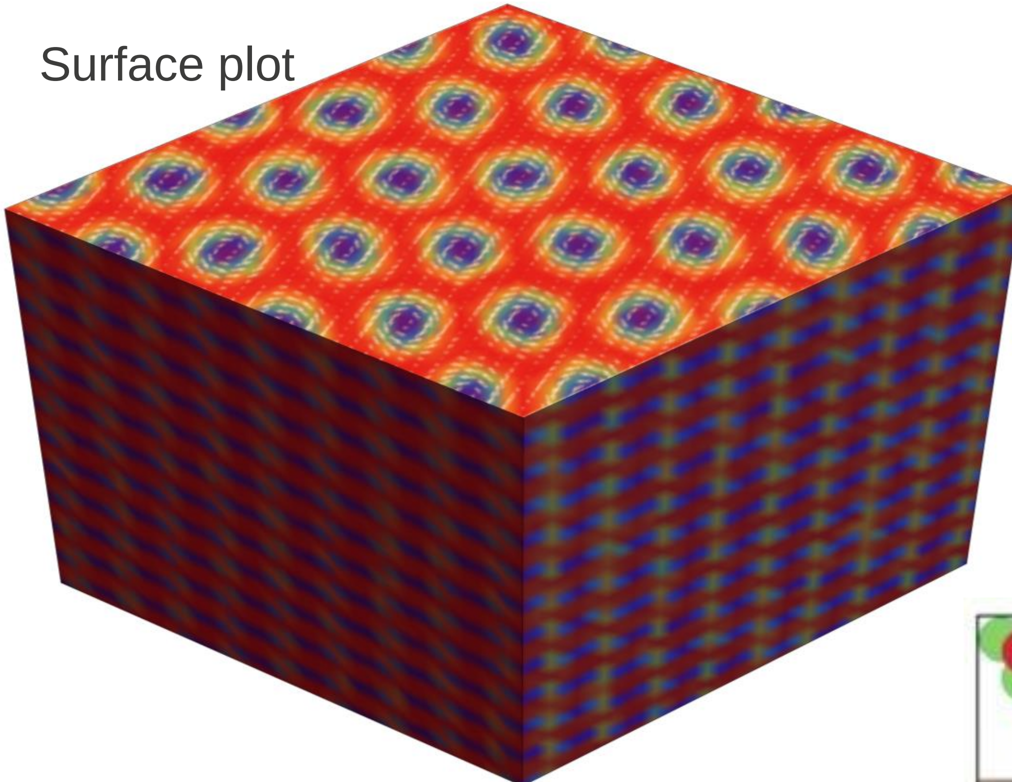


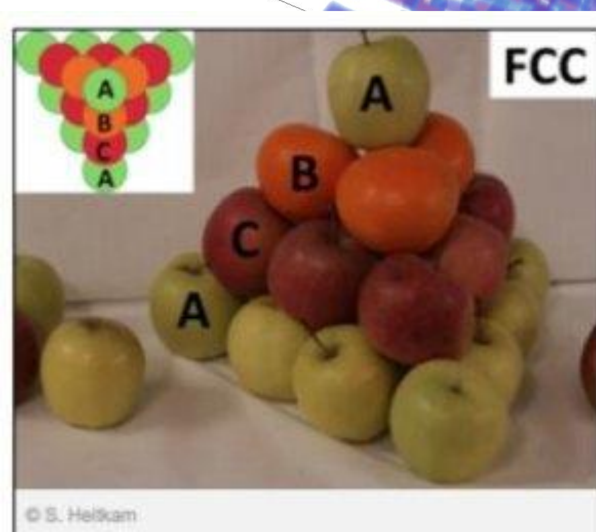
ABCABC stacking of pancake skyrmions ($Q_z = 2\pi/3$)

Skyrmion core



Surface plot





ABCABC… stacking of pancake skyrmions

SZL and Batista, arXiv:1707.05818 (2017)

Skyrmions are particles and they can organize into hexagonal close packed structures (HCP) and face centered cubic (FCC) lattice.

However, the results can be better understood in term of linear superposition of helices. (wave nature of the skyrmion lattice)

Ansatz: skyrmion lattice as linear superposition of three helices

$$S_x(\mathbf{r}) = \frac{I_x}{A}\sum_{\mu=1}^{3} \sin\left(\mathbf{Q}_v \cdot \mathbf{r} + \theta_\mu\right) e_{\mu,x}$$

$$S_y(\mathbf{r}) = \frac{I_y}{A}\sum_{\mu=1}^{3} \sin\left(\mathbf{Q}_v \cdot \mathbf{r} + \theta_\mu\right) e_{\mu,y}$$

$$S_x(\mathbf{r}) = \frac{I_z}{A}\left[\sum_{\mu=1}^{3} \cos\left(\mathbf{Q}_v \cdot \mathbf{r} + \theta_\mu\right) + S_{z0}\right]$$

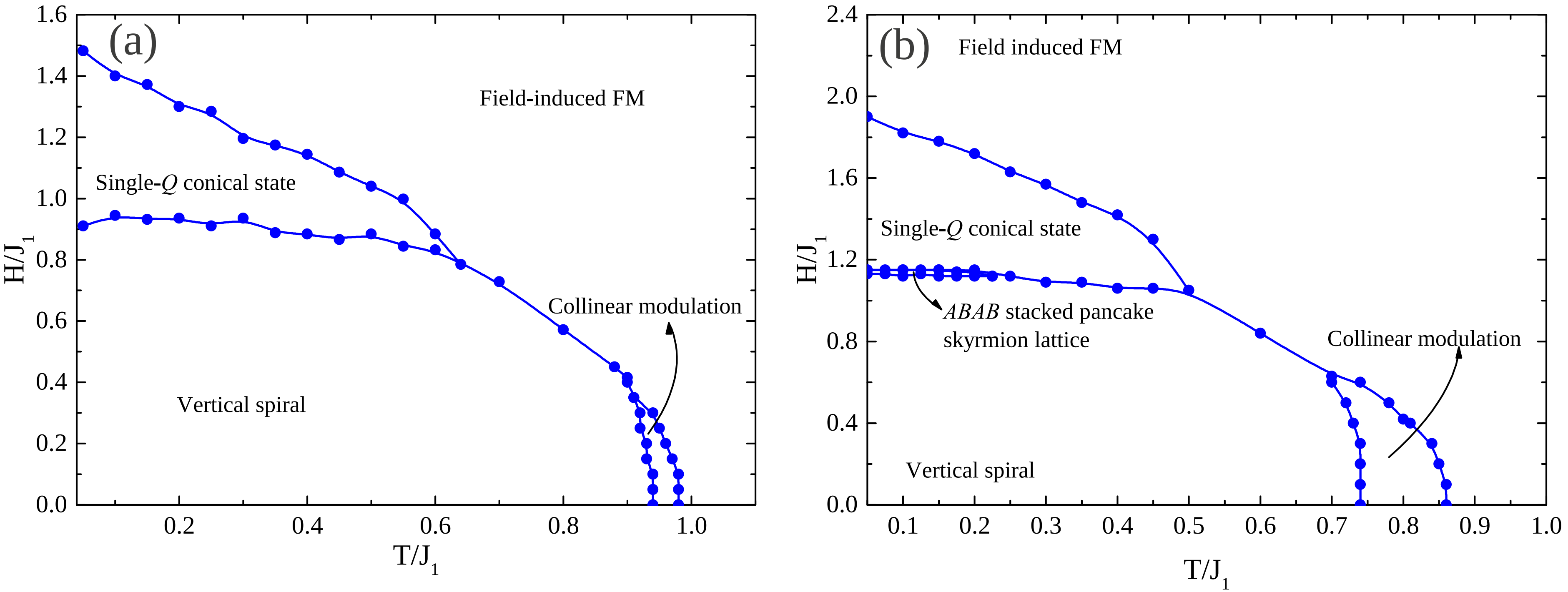
with $\mathbf{Q}_1 = (Q_{ab},\ 0,\ Q_z)$, $\mathbf{Q}_2 = \left(-Q_{ab}/2, \sqrt{3}Q_{ab}/2,\ Q_z\right)$, $\mathbf{Q}_3 = \left(-Q_{ab}/2,\ -\sqrt{3}Q_{ab}/2,\ Q_z\right)$

Optimal condition for skyrmion stabilization: $Q_1 + Q_2 + Q_3 = 2n\pi$

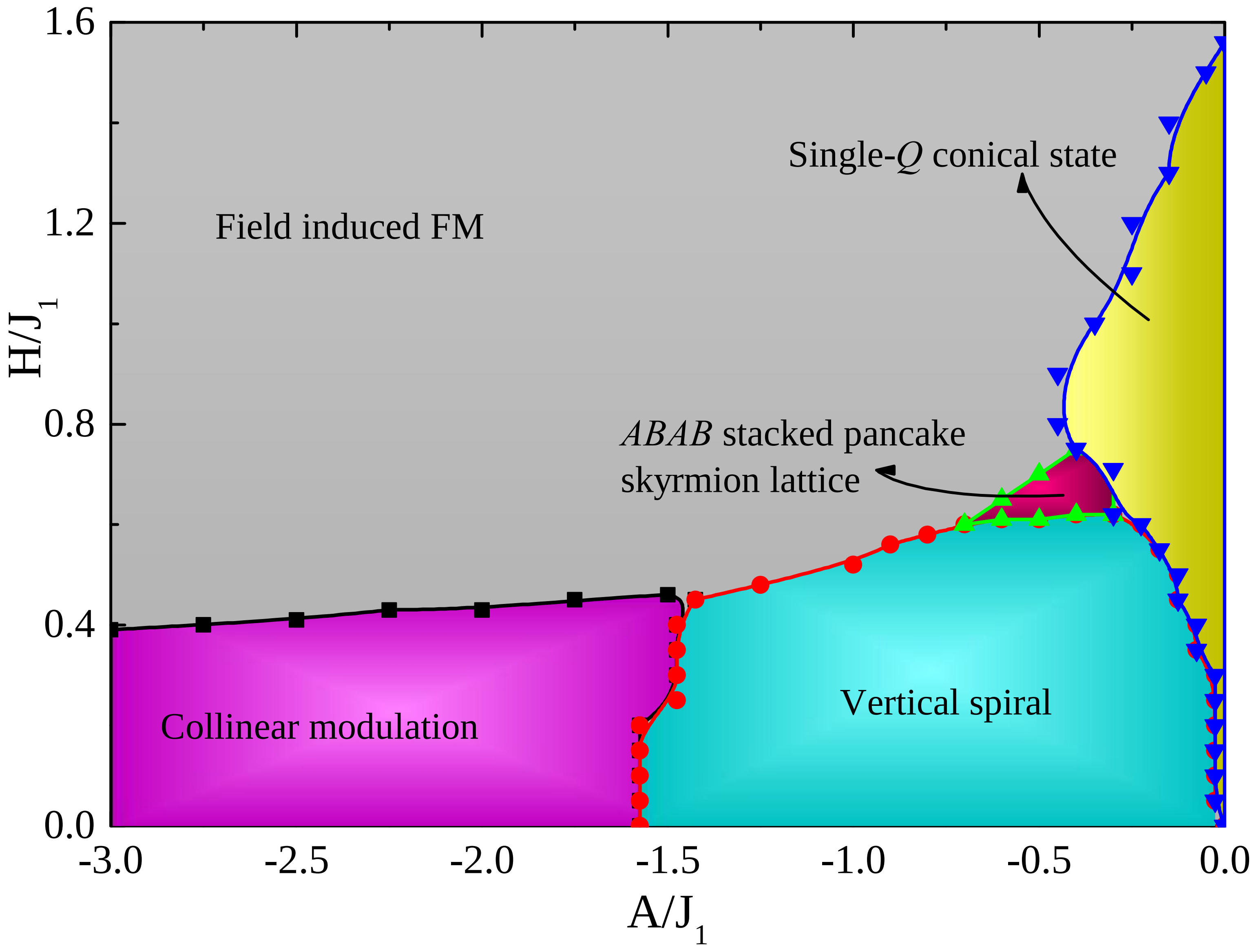
- At $T = 0$, $|S|$ is uniform in space, so $A = \sqrt{S_x^2 + S_y^2 + S_z^2}$. This creates terms higher order in $\mathbf{Q}$, such as $\mathbf{Q}_i + \mathbf{Q}_j$ and costs exchange energy.
- Change of θ_μ corresponds to the translation of skyrmion lattice.
- Skyrmion phase is more stable for $Q_z = 0$ or $Q_z = \frac{2\pi}{3}$. For other $Q_z s$, the skyrmion phase is suppressed.

For a strong interlayer competing interaction J_z

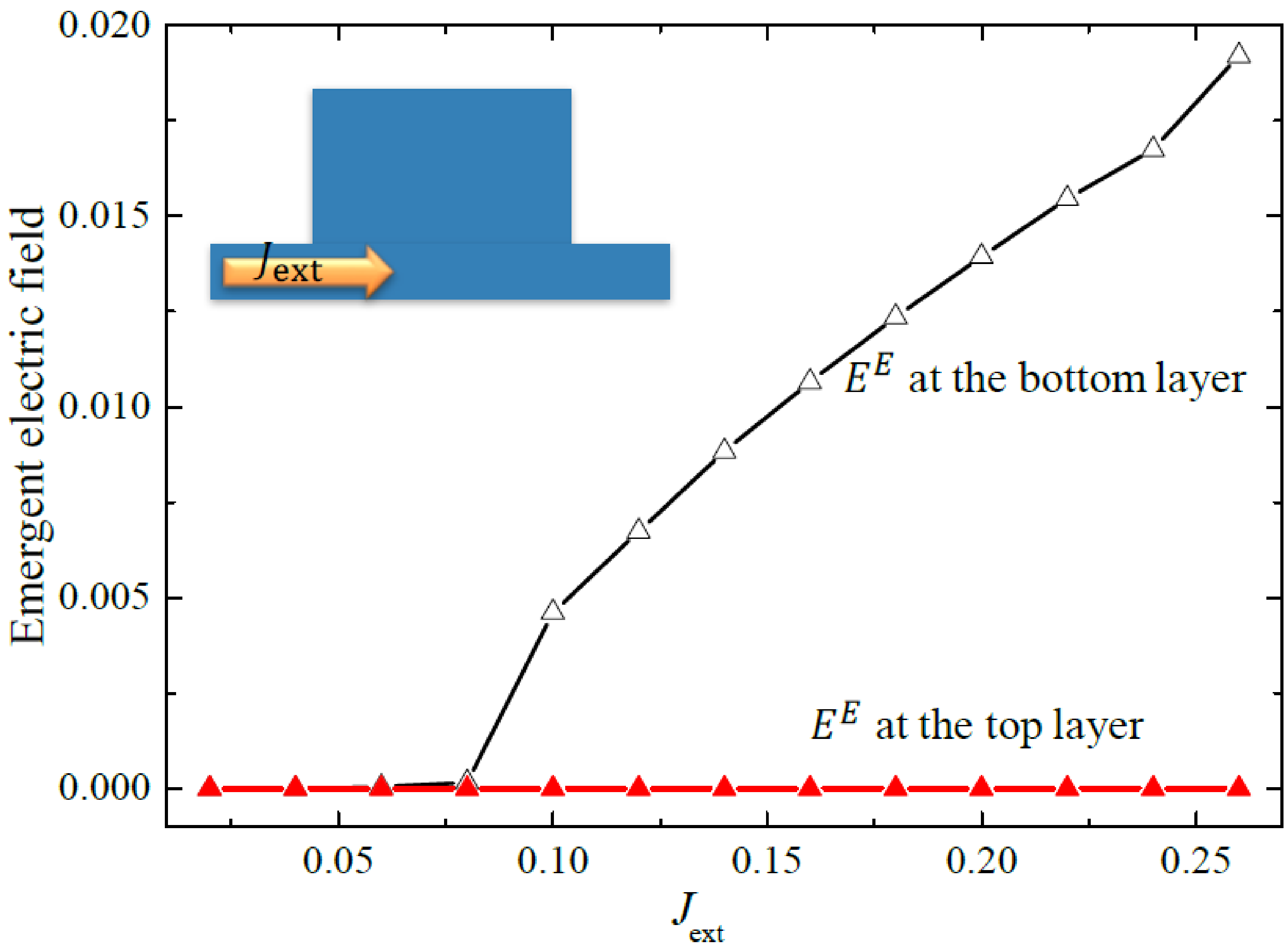
The skyrmion phase is suppressed for $Q_z \neq 0$ or $Q_z \neq 2\pi/3$, because of the higher order coupling terms, $2Q_z,\ 3Q_z,\ 4Q_z, \cdots$, which is not optimal in $J(\mathbf{Q})$.



The easy axis anisotropy has to be right



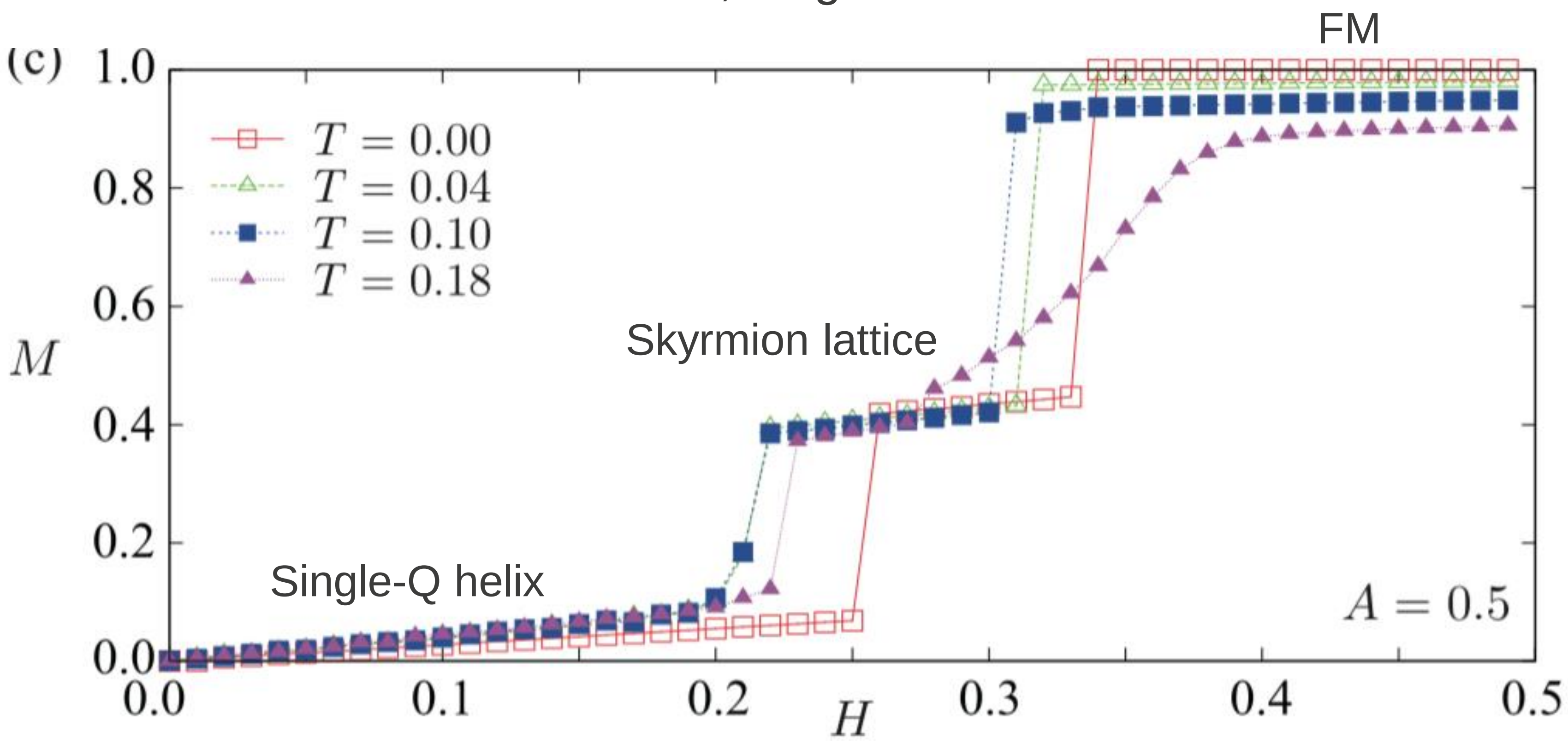
Decoupling of skyrmion motion in different layers



Particle nature of skyrmions

Possible experimental signatures

- Bulk measurements: M-H curve, magnetoresistance



- Small angle neutron scattering

- Real space imaging, eg. MFM, Lorentz TEM,….

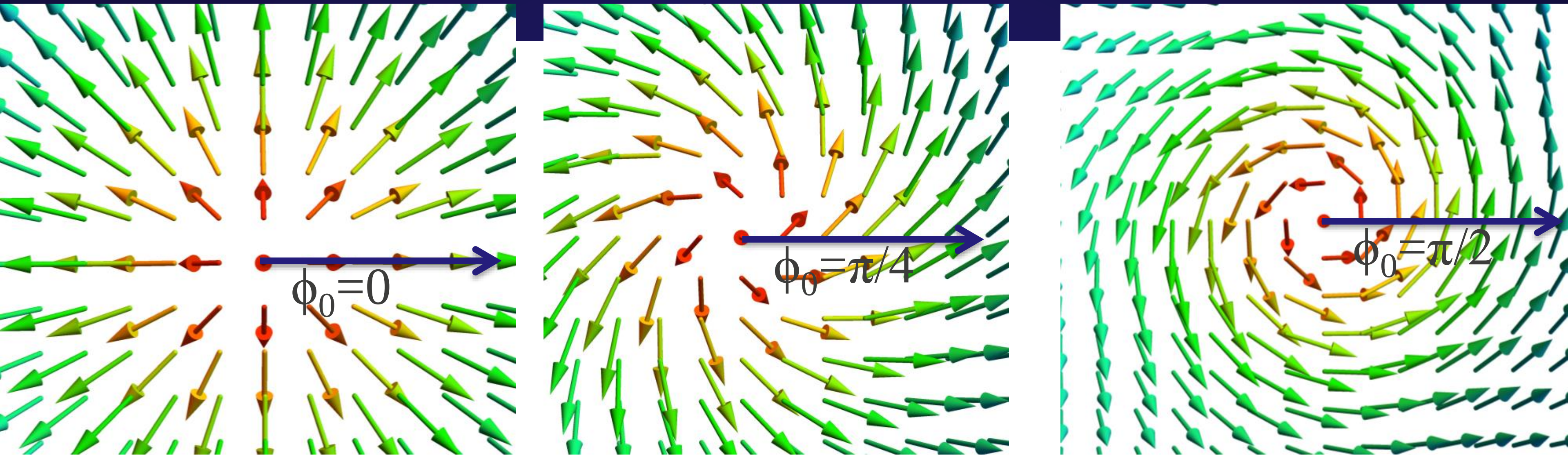
Novel properties

Effective Ginzburg-Landau energy

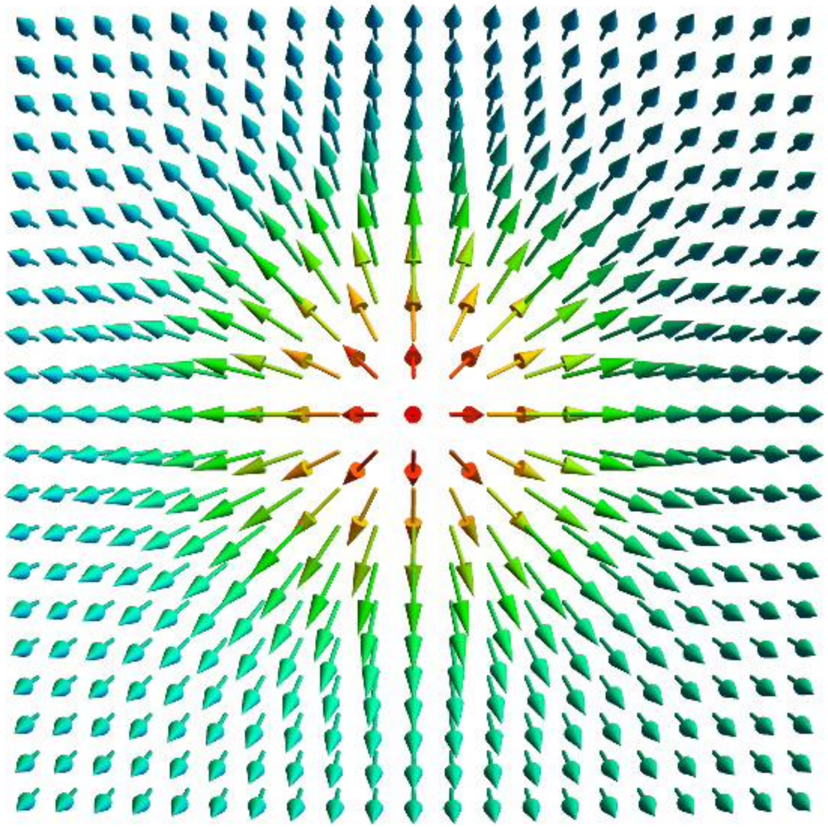
$$\mathcal{H}=\int\left[-\frac{1}{2}\left(\nabla\mathbf{S}\right)^2+\frac{1}{2}\left(\nabla^2\mathbf{S}\right)^2-\mathbf{H}\cdot\mathbf{S}+AS_z^2\right]\mathrm{d}r^3$$

SZL and Hayami, PRB 93, 064430 (2016)

- **Symmetries:**
  - Inversion symmetry ➔ Skyrmion and antiskyrmion are created equal!
  - Translational invariant ➔ Translational motion of skyrmion
  - Spin rotation along the field axis ➔ Helicity as a Goldstone mode of skyrmion



Helicity



Change of helicity or rotation of skyrmion

Emergent "spin" – "orbit" coupling for skyrmions

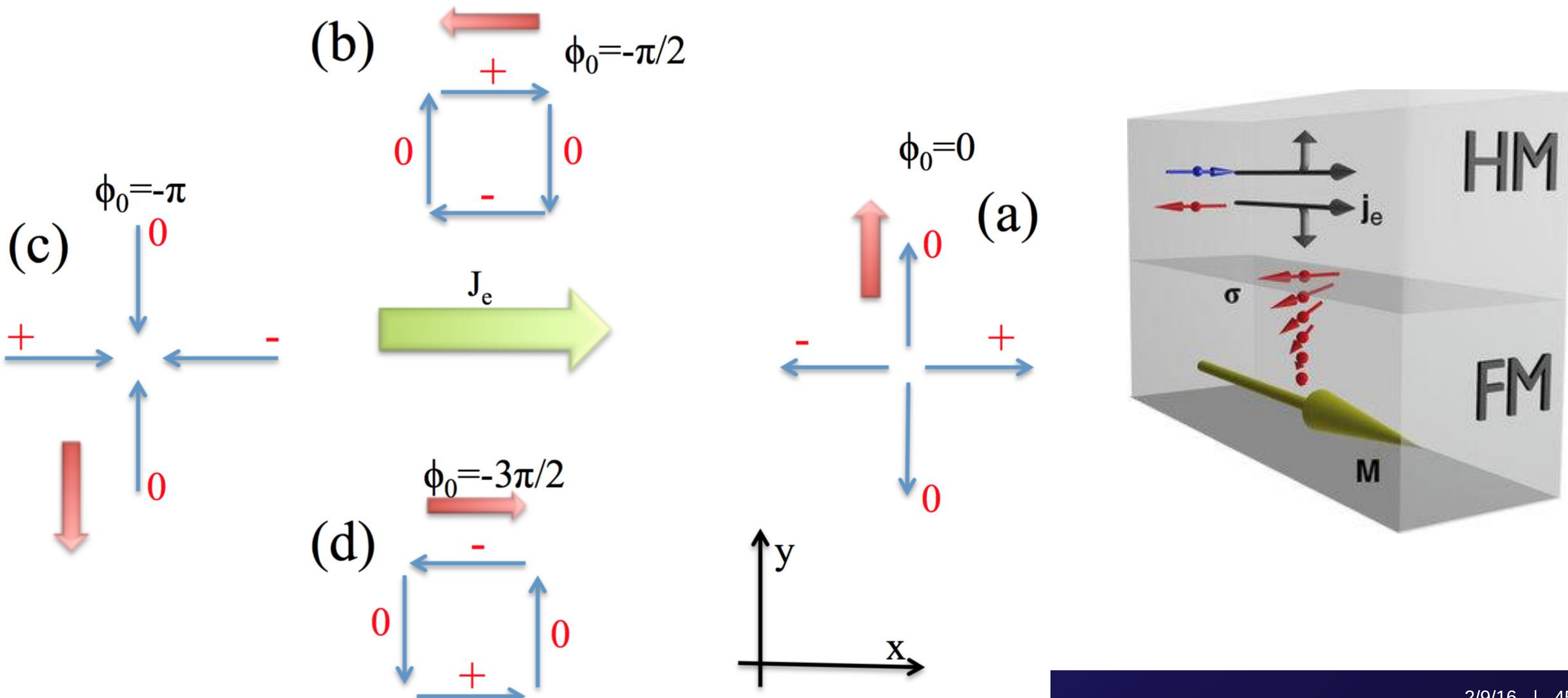
- The torque due to spin Hall effect couples to the helicity of the skyrmion

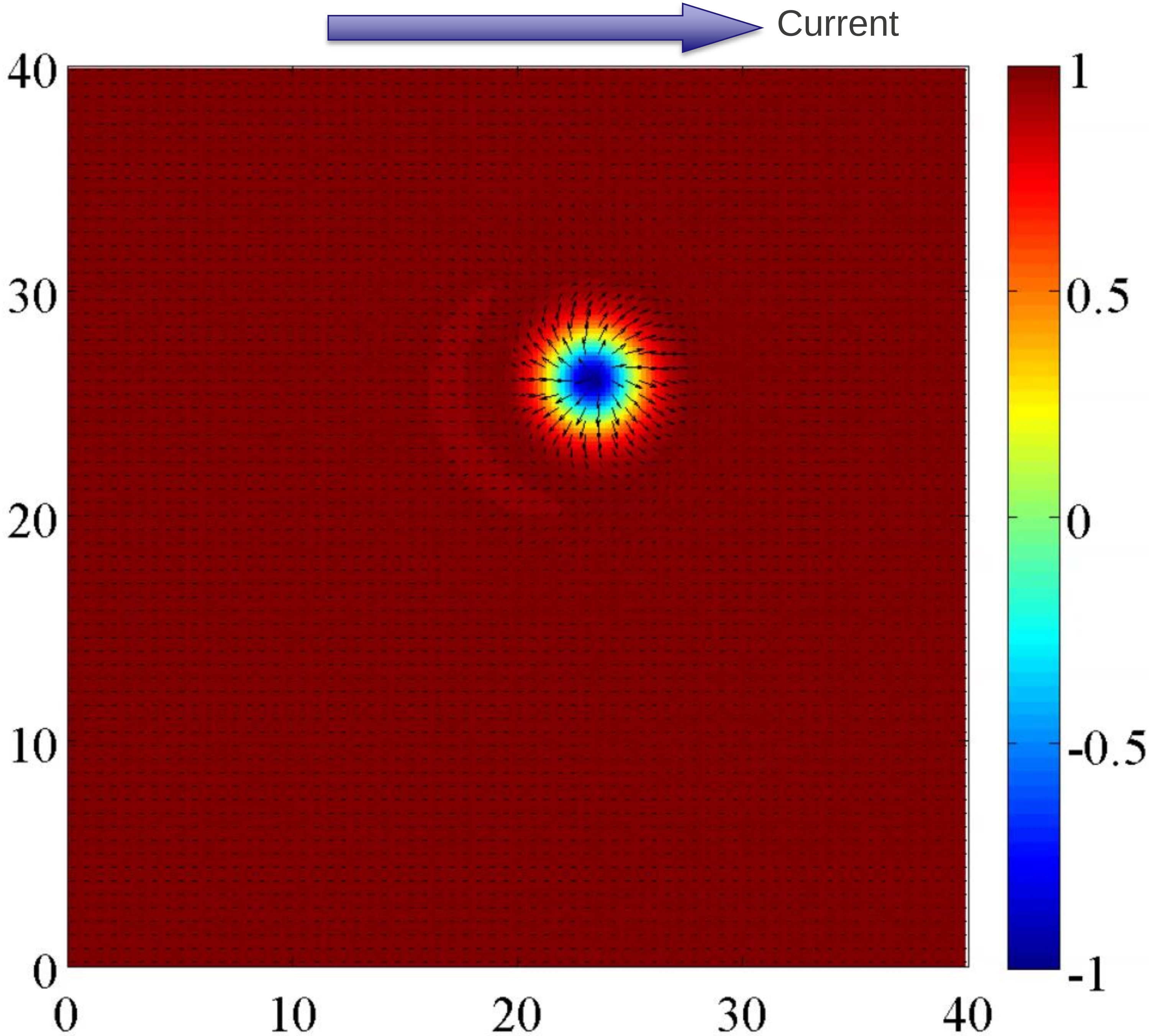
$$\partial_t \mathbf{S} = -\gamma \mathbf{S}\times\mathbf{H}_{\mathrm{eff}} + \alpha\mathbf{S}\times\partial_t\mathbf{S} + \frac{\hbar\gamma}{2e}\frac{g\theta_{sh}}{2d}\mathbf{S}\times\left(\mathbf{S}\times\left(\hat{\mathbf{z}}\times\mathbf{J}_e\right)\right)$$

Liu et al., Science 336, 555 (2012)
Ryu et al., Nat. Nanotechnol. 8, 527 (2013)
Emori et al., Nat. Mater. 12, 611 (2013).

The effective field due to spin current

$H_{\mathrm{eff,t}} \propto \mathbf{S}\times\vec{y}$ for current along the x direction




Current
40
30
20
10
0
0
10
20
30
40
1
0.5
0
-0.5
-1

Conclusions

- **Conditions to stabilize skyrmions in inversion-symmetric magnets (rare-earth or frustrated magnets)**
  - Competing interaction which maximizes $J(\mathbf{q})$ at a nonzero $\mathbf{Q}$
  - Moderate easy-axis anisotropy
  - Hexagonal crystal structure is better.
  - $\mathbf{Q}$ lies in the basal plane.

Thank you!