Power-law Decays in Isolated Many-Body Quantum Systems

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How **fast** can isolated interacting quantum systems evolve?

Dynamics

How does the evolution depend on the initial state, **perturbation**?

How does the dynamics depend on the **time scale**?

Is the dynamics affected by **critical points**?

How does the dynamics depend on the **Hamiltonian**? (interactions, chaos)

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Quantum chaos and thermalization in isolated systems of interacting particles
Borgonovi, Izrailev, LFS, Zelevinsky
Physics Reports **626**, 1 (2016)

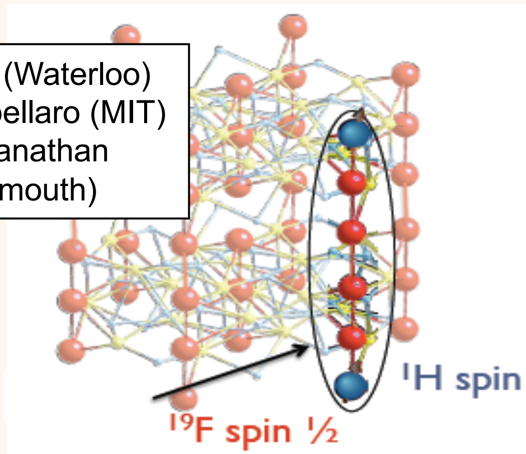
D'Alessio et al,
Advances in Physics **65**, 239 (2016)
Zelevinsky et al,
Physics Reports **276**, 85 (1996)

Coherent Evolution in Experiments

NMR and NV centers

Solid state NMR: nuclear positions are fixed;
They are collectively addressed with magnetic pulses;
Very slow relaxation

Cory (Waterloo)
Cappellaro (MIT)
Ramanathan (Dartmouth)

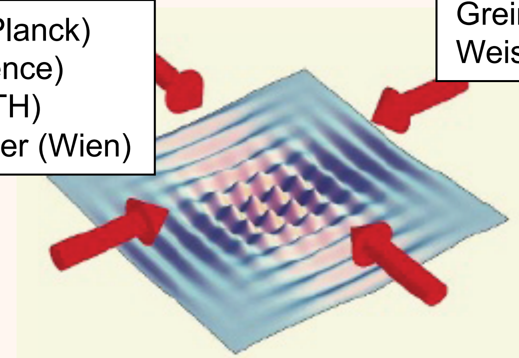


Ultracold Gases

Dynamics under designed potentials.

Bloch (Max Planck)
Fallani (Florence)
Esslinger (ETH)
Schmiedmayer (Wien)

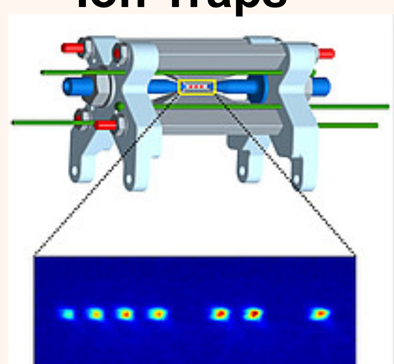
Greiner (Harvard)
Weiss (Penn Sate)



- highly controllable systems – interactions, level of disorder, 1,2,3D (simple models)
- quasi-isolated -- study evolution for very long time

Ion Traps

Ions trapped via electric and magnetic fields.
Laser used to induce couplings.
Isolated from an external environment.



Blatt (Innsbrück)

Monroe (Maryland)

SYSTEM MODELS
1D spin-1/2

Hardcore bosons, qubits

Integrable spin ½ models

Noninteracting Integrable system:

XX model (1D)

$$H=\sum_{n=1}^{L-1}\frac{J}{4}(\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

Interacting Integrable system:

XXZ model (1D)

$$H=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

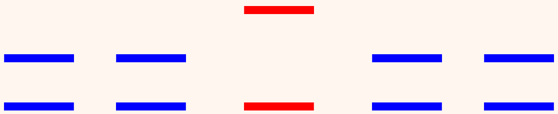
Chaotic Models

$$H=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

Breaking the integrability of the 1D XXZ model

Defect model

LFS,
JPA (2004)



$$H=\frac{Jd}{2}\sigma_{L/2}^z+\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

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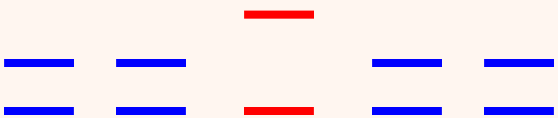

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Disordered model

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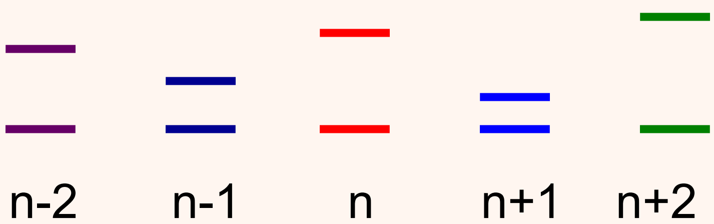

$$H=\sum_{n=1}^{L}\frac{h_n}{2}\sigma_n^z+\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

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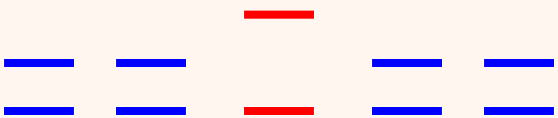

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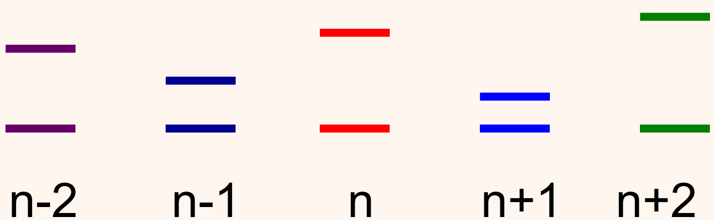
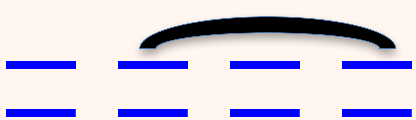


$$H=\sum_{n=1}^{L}\frac{h_n}{2}\sigma_n^z+\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$

NNN model



$$H_{NN}+H_{NNN}=\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)+$$
$$+\lambda\sum_{n=1}^{L-2}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+2}^z+\sigma_n^x\sigma_{n+2}^x+\sigma_n^y\sigma_{n+2}^y)$$

FULL RANDOM MATRICES

QUANTUM CHAOS

Full random matrices:
Matrices filled with random numbers and respecting the symmetries of the system.

Wigner in the 1950's used random matrices to study the spectrum of heavy nuclei
(atoms, molecules, quantum dots)

Full Random Matrices vs XXZ model

FULL RANDOM MATRIX

0.23	-0.09	1.13	-0.22	0.59	0.58	-0.46	-0.43	-0.46	-1.12	0.90	-0.
-0.09	-0.02	-0.04	-0.58	0.65	0.05	-0.20	-0.14	0.06	-0.50	1.29	-0.
1.13	-0.04	0.17	0.55	1.31	0.36	-0.24	0.05	0.49	0.65	-1.18	-0.
-0.22	-0.58	0.55	0.79	-0.20	-0.03	-0.68	0.16	1.58	0.15	-0.56	0.
0.59	0.65	1.31	-0.20	-0.79	-0.19	-1.15	0.59	1.14	1.21	-0.25	0.
0.58	0.05	0.36	-0.03	-0.19	0.59	1.46	0.96	-0.66	0.05	-0.30	0.
-0.46	-0.20	-0.24	-0.68	-1.15	1.46	-0.80	0.61	0.07	0.15	-0.11	0.
-0.43	-0.14	0.05	0.16	0.59	0.96	0.61	0.68	-0.59	-0.40	-0.47	-0.
-0.46	0.06	0.49	1.58	1.14	-0.66	0.07	-0.59	0.82	-0.31	-0.08	0.
-1.12	-0.50	0.65	0.15	1.21	0.05	0.15	-0.40	-0.31	0.02	-0.95	0.
0.90	1.29	-1.18	-0.56	-0.25	-0.30	-0.11	-0.47	-0.08	-0.95	-0.41	0.
-0.92	-0.42	-0.40	0.15	0.92	0.88	0.28	-0.08	0.42	0.58	0.03	-0.
-0.66	0.07	0.47	0.28	-0.44	-0.14	0.14	-0.34	0.47	1.97	-0.48	0.
0.70	0.62	0.28	-0.06	-0.19	-0.24	0.11	0.47	0.42	0.39	-0.13	1.
0.54	0.40	0.31	1.04	$0.\times 10^{-3}$	0.25	-0.56	0.37	-0.54	-0.37	0.33	-0.
-0.98	-0.47	1.00	1.26	-0.46	-0.45	0.12	-0.08	1.19	-0.23	0.13	0.

Basis is ill defined

Time-reversal invariant systems
with rotational symmetry:
<u>Hamiltonians are real and symmetric</u>
Gaussian Orthogonal Ensemble (GOE)

Full Random Matrices vs XXZ model

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SPIN-1/2 MODEL

$\lvert 1111\rangle$	$\lvert 1110\rangle$	$\lvert 1101\rangle$	$\lvert 1011\rangle$	$\lvert 0111\rangle$	$\lvert 1100\rangle$	$\lvert 1010\rangle$	$\lvert 1001\rangle$	$\lvert 0110\rangle$	$\lvert 0101\rangle$	$\lvert 0011\rangle$	$\lvert 0001\rangle$	$\lvert 0010\rangle$	$\lvert 0100\rangle$	$\lvert 1000\rangle$	$\lvert 0000\rangle$
$\frac{3J\Delta}{4}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	$+\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	$\frac{J}{2}$	$-\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0	0	0	0	0	0	0	0
0	0	$\frac{J}{2}$	$-\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	$+\frac{J}{2}$	$+\frac{J\Delta}{4}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$+\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{J}{2}$	$-\frac{3J\Delta}{4}$	$\frac{J}{2}$	$\frac{J}{2}$	0	0	0	0	0	0	0
0	0	0	0	0	0	$\frac{J}{2}$	$-\frac{J\Delta}{4}$	0	$\frac{J}{2}$	0	0	0	0	0	0
0	0	0	0	0	0	$\frac{J}{2}$	0	$-\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0	0
0	0	0	0	0	0	0	$\frac{J}{2}$	$\frac{J}{2}$	$-\frac{3J\Delta}{4}$	$\frac{J}{2}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	$\frac{J}{2}$	$\frac{J\Delta}{4}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$+\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$\frac{J}{2}$	$-\frac{J\Delta}{4}$	$\frac{J}{2}$	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$\frac{J}{2}$	$-\frac{J\Delta}{4}$	$\frac{J}{2}$	0
0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{J}{2}$	$+\frac{J\Delta}{4}$	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{3J\Delta}{4}$

↑↓↑↑↓↓↑↓ ↑↓↑↓↑↓↑↓

Quantum Chaos: Level Repulsion

Full random matrices:
Matrices filled with random numbers and respecting the symmetries of the system.

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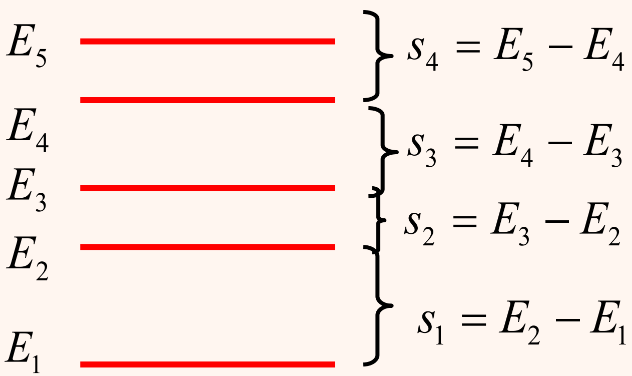
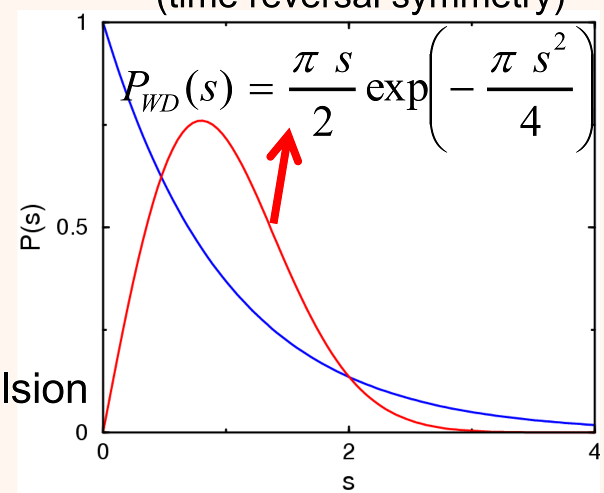
Level spacing distribution



Wigner-Dyson distribution
(time reversal symmetry)

Level repulsion



(i) Time-reversal invariant systems with rotational symmetry :
Hamiltonians are real and symmetric
Gaussian Orthogonal Ensemble (GOE)

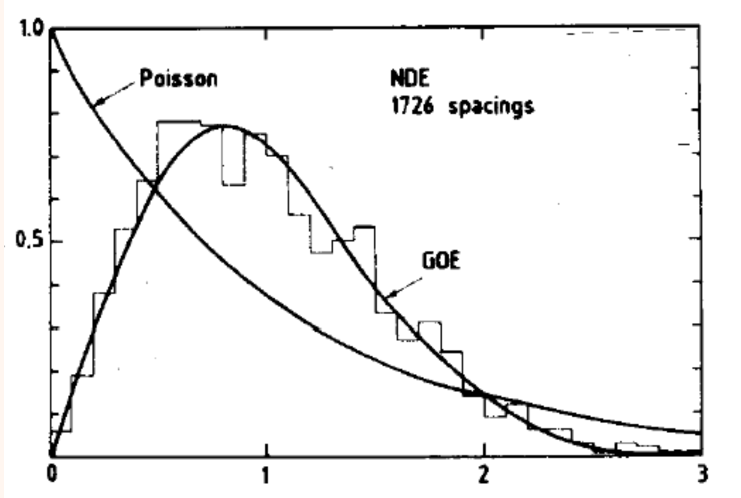
Level repulsion = quantum chaos

(ii) Systems without invariance under time reversal (atom in an external magnetic field)
Gaussian Unitary Ensemble (GUE)
Hamiltonians are Hermitian)

(iii) Time-reversal invariant systems,
half-integer spin, broken rotational symmetry
Gaussian Sympletic Ensemble (GSE)

Level spacing distribution

Nearest neighbor spacing distribution for the "**Nuclear Data Ensemble**" comprising 1726 spacings s = S/D with D the mean level spacing and S the actual spacing.



T. Guhr, A. Mueller-Gröeling, and H. A. Weidenmüller, Phys. Rep. **299**, 189 (1998).

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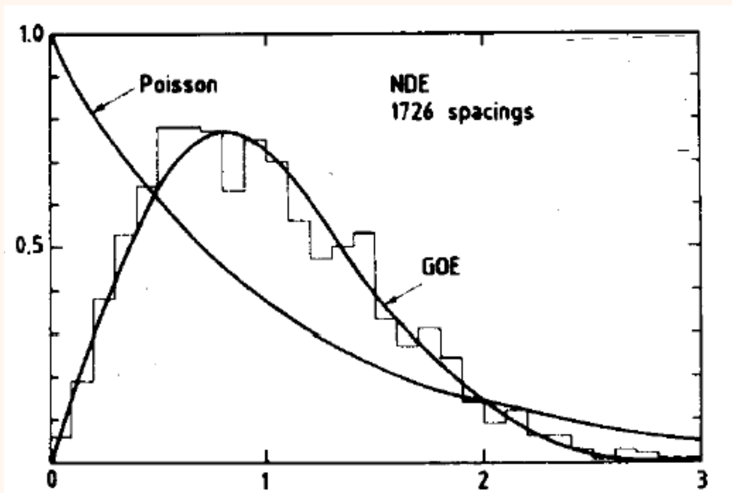

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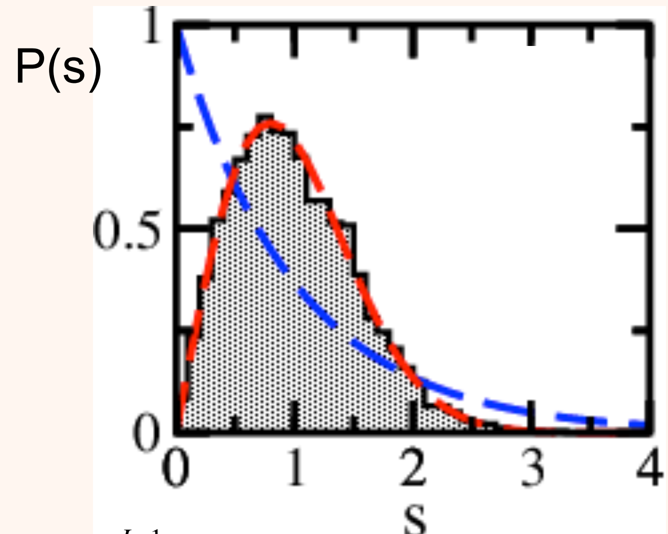
Chaotic



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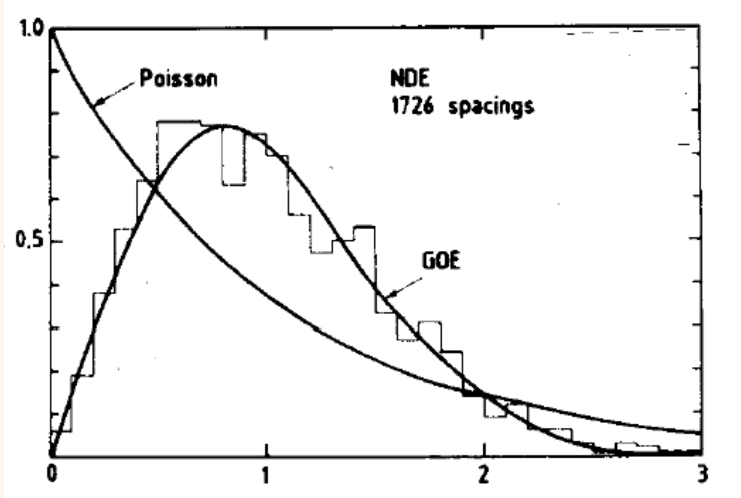

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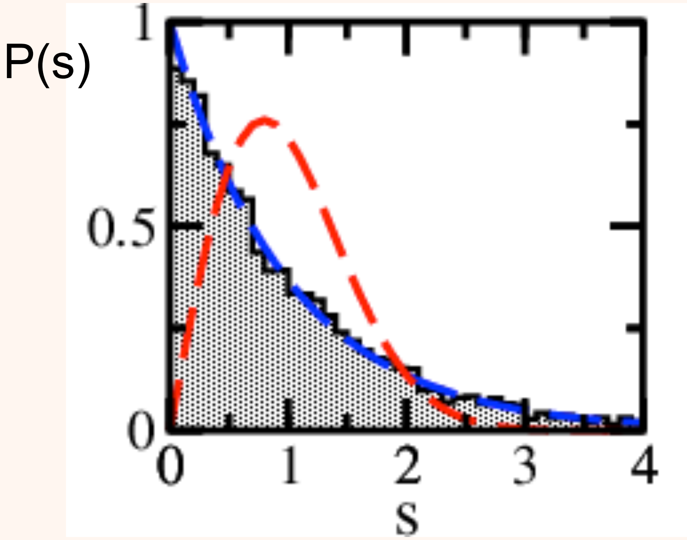
Integrable

XXZ model



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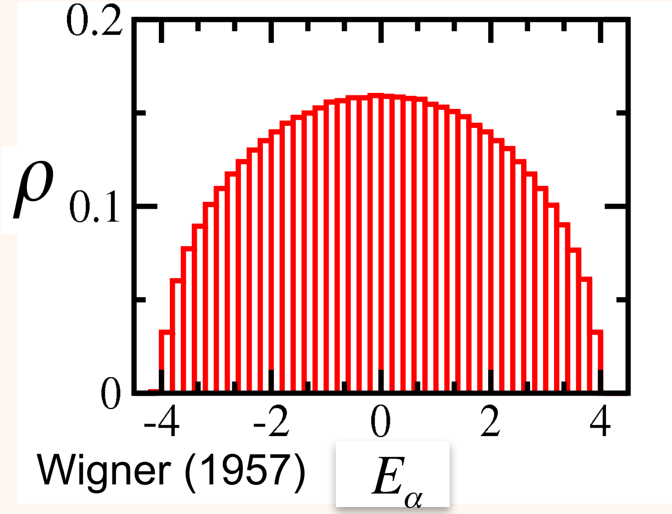
Full Random Matrices vs Two-Body Interaction

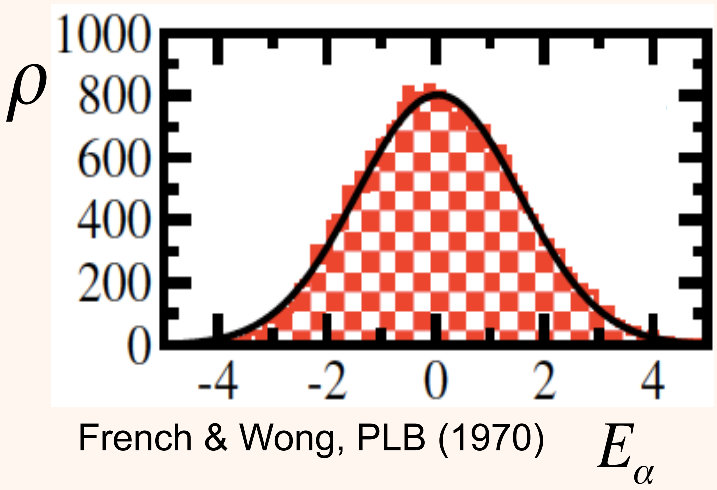
Density of States (Energy Distribution)

Full random matrices: semicircular



Wigner (1957)

Two-body interactions: Gaussian



French & Wong, PLB (1970)

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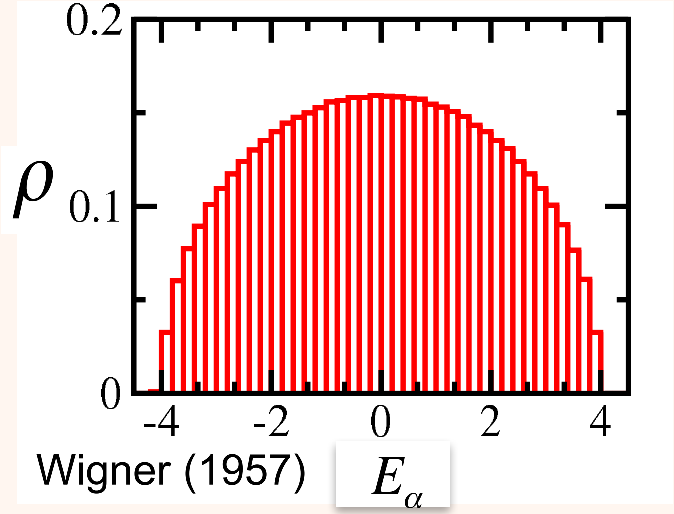

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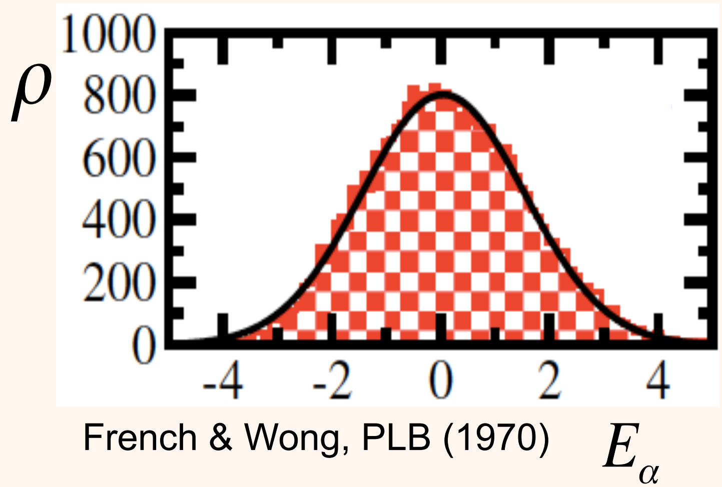

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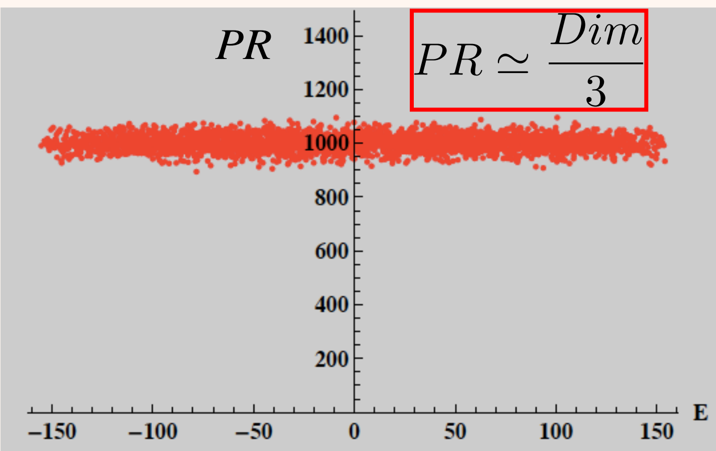

$$\left|\psi^{(\alpha)}\right\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \dots \end{pmatrix}$$

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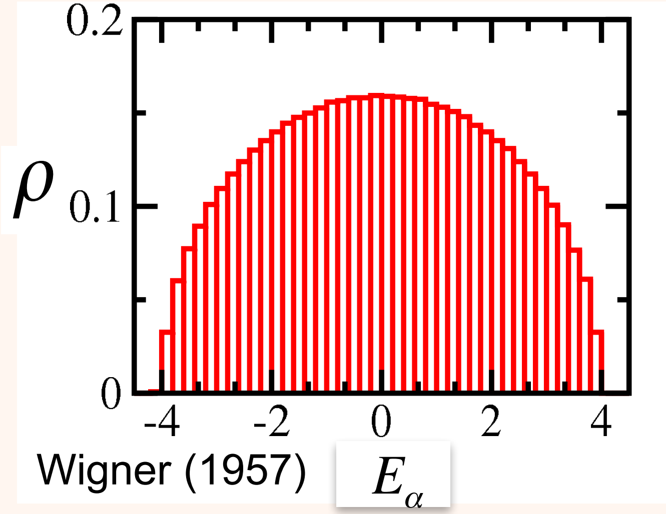
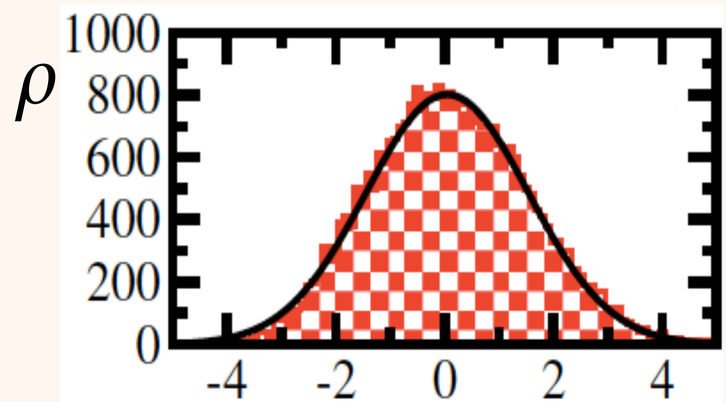


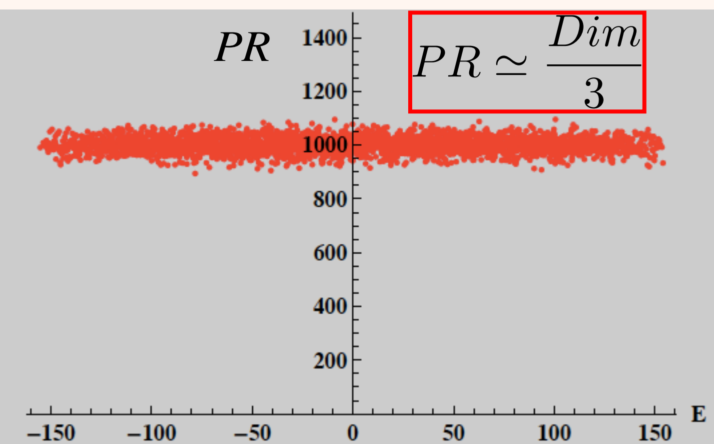
Wigner (1957)

Two-body interactions: Gaussian



French & Wong, PLB (1970)

Participation Ratio



$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} |c_i^{(\alpha)}|^4}$$

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

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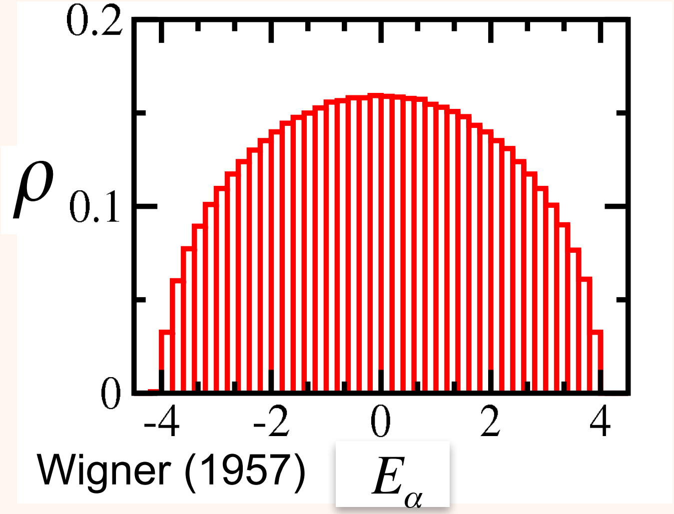
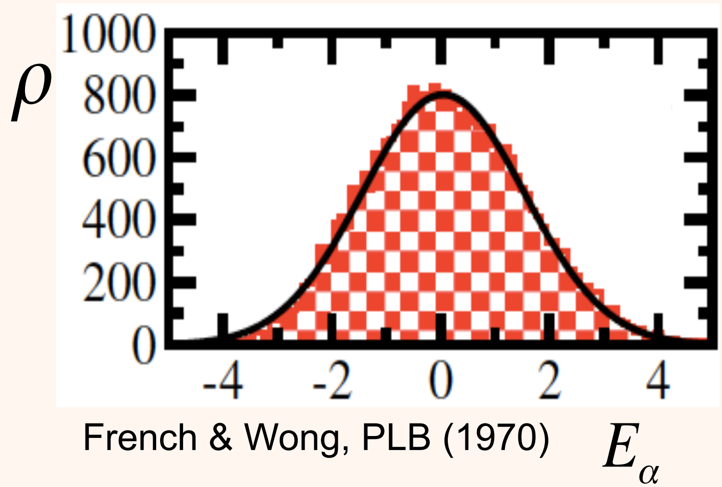
Density of States (Energy Distribution)

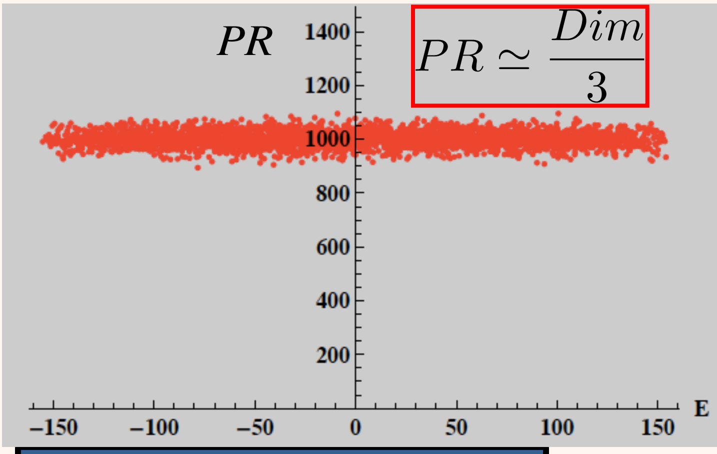
Full random matrices: semicircular



Two-body interactions: Gaussian

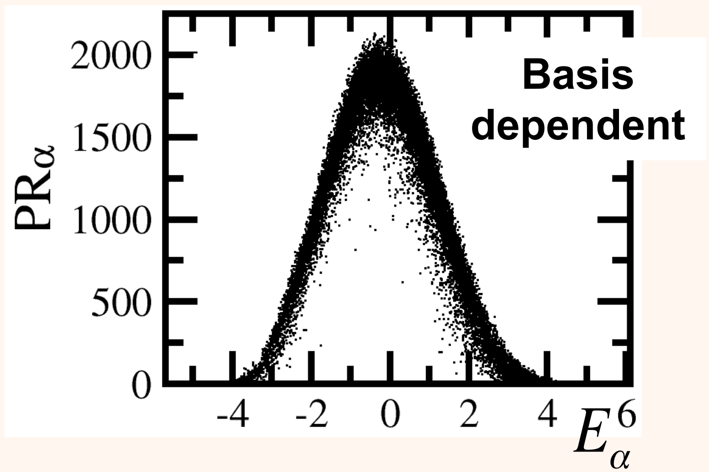


Participation Ratio



$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} |c_i^{(\alpha)}|^4}$$

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$



DYNAMICS

Survival Probability (Fidelity)

Overlap between the initial state and the evolved state

$$F(t) = \left| \langle \Psi(0) | \Psi(t) \rangle \right|^2 = \left| \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 e^{-iE_{\alpha}t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

$$| \Psi(0) \rangle = | ini \rangle = \sum_{\alpha} C_{\alpha}^{ini} | \alpha \rangle$$

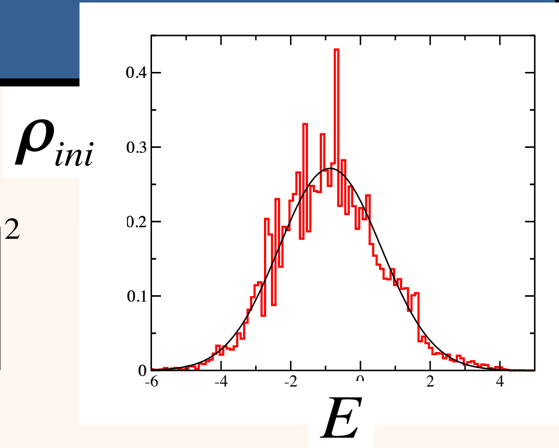
$$| \Psi(t) \rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} | \alpha \rangle$$

Eigenvalues and eigenstates of the final Hamiltonian

Survival Probability (Fidelity)

Overlap between the initial state and the evolved state

$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left| \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$



$$\rho_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \delta(E - E_{\alpha})$$

Fourier transform
- of the weighted energy distribution of the initial state
- of the LDOS (local density of states), strength function

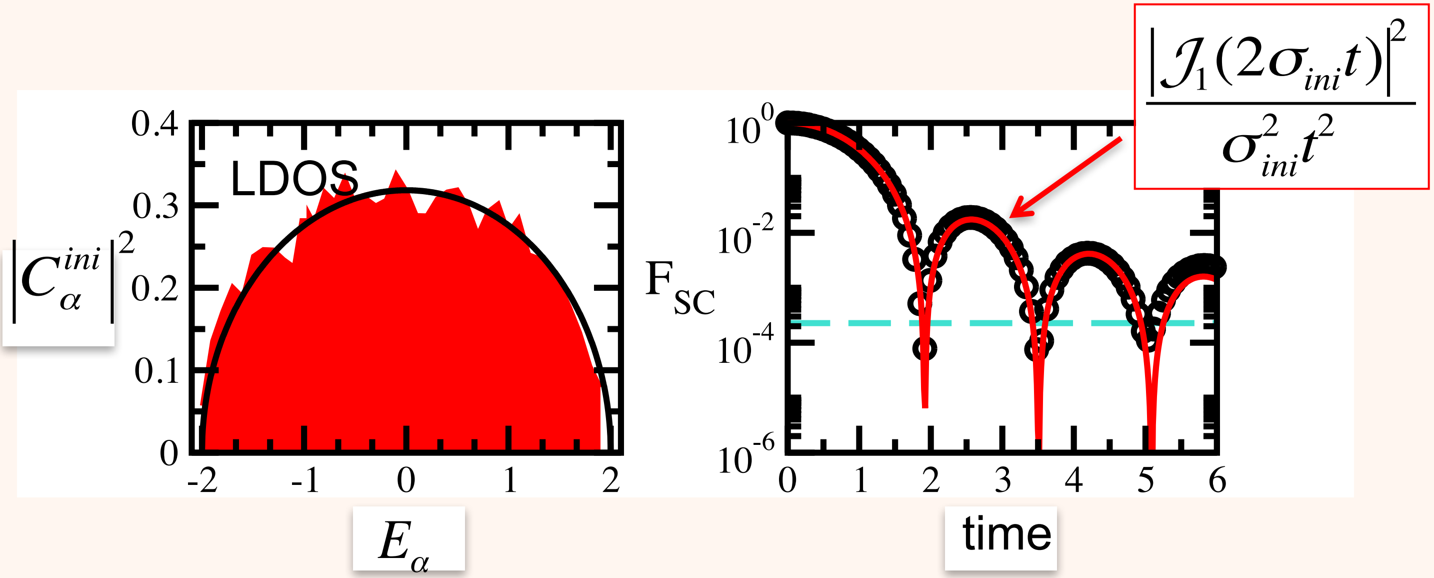
$$|\Psi(0)\rangle = |ini\rangle = \sum_{\alpha} C_{\alpha}^{ini} |\alpha\rangle \qquad |\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha}t} |\alpha\rangle$$

Eigenvalues and eigenstates of the final Hamiltonian

Dynamics under full random matrices

Distribution of $\left|C_\alpha^{ini}\right|^2$ for initial state projected into random matrices: semicircular

$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left| \sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$



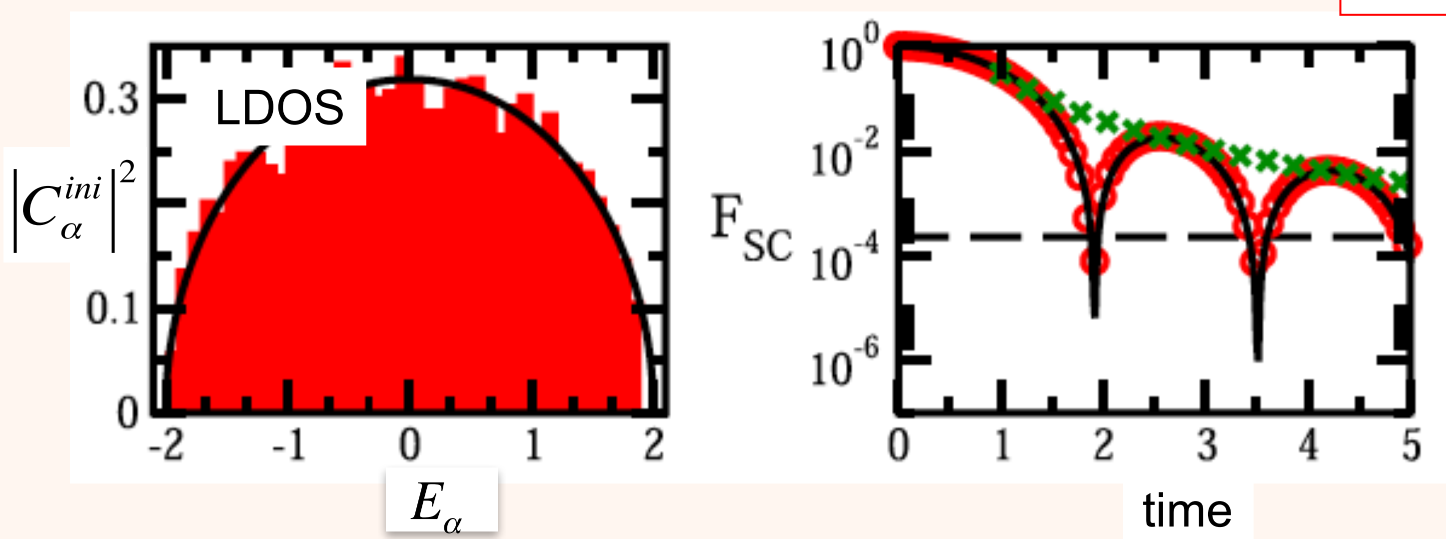
Torres & LFS
PRA **89** (2014)

Torres, Vyas, LFS
NJP **16** (2014)

Dynamics under full random matrices

Distribution of $\left|C_\alpha^{ini}\right|^2$ for initial state projected into random matrices: semicircular

$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int_{-\infty}^{\infty} P_{ini}(E) e^{-iEt}\, dE\right|^2 \longrightarrow \boxed{\frac{\left|\mathcal{J}_1(2\sigma_{ini} t)\right|^2}{\sigma_{ini}^2 t^2}}$$



$$\boxed{\frac{1}{t^3}}$$

Torres & LFS
PRA **89** (2014)

Torres, Vyas, LFS
NJP **16** (2014)

Analytical Results: Full Random Matrices

$$F(t) = \left| \langle \Psi(0) | \Psi(t) \rangle \right|^2 = \left| \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 e^{-iE_{\alpha} t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

LDOS

$$\rho_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 \delta(E - E_{\alpha})$$

$$F(t) = \sum_{\alpha \neq \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha} - E_{\beta})t} + \sum_{\alpha} |C_{\alpha}^{ini}|^4$$

$$\overline{F} = \sum_{\alpha} |C_{\alpha}^{ini}|^4 \simeq \frac{3}{Dim}$$

Analytical Results: Full Random Matrices

$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t}\right|^2 \cong \left|\int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

LDOS

$$\rho_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \delta(E - E_{\alpha})$$

$$F(t) = \sum_{\alpha \neq \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha} - E_{\beta})t} + \boxed{\sum_{\alpha} |C_{\alpha}^{ini}|^4}$$

$$F(t) = \int G(E) e^{-iEt} dE + \boxed{\overline{F}}$$

Spectral autocorrelation function

$$G(E) = \sum_{\alpha \neq \beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 \delta(E - (E_{\alpha} - E_{\beta}))$$

Mehta's Book

Alhassid & Levine
PRA **46**, 4650 (1992)

Analytical Results: Full Random Matrices

$$F(t) = \sum_{\alpha \neq \beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} + \sum_\alpha |C_\alpha^{ini}|^4$$

$$F(t) = \frac{1-\overline{F}}{Dim-1} Dim \boxed{\frac{\mathcal{J}_1^2(2\sigma_{ini}t)}{\sigma_{ini}^2 t^2}} - \frac{1-\overline{F}}{Dim-1} \boxed{b_2\left(\frac{\sigma_{ini}t}{2Dim}\right)} + \boxed{\overline{F}}$$

Bessel and power-law decay

Two-level form-factor

saturation

Torres, García-García & LFS, arXiv:1704.06272

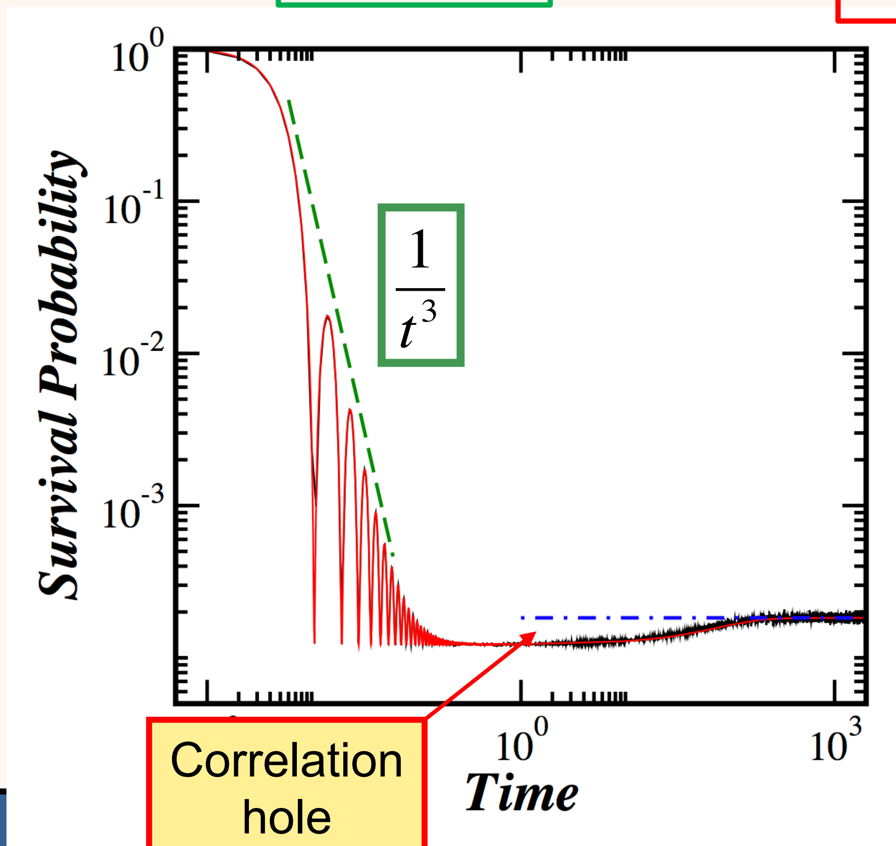
Mehta's book

where $b_2(\overline{t}) = [1-2\overline{t}+\overline{t}\ln(1+2\overline{t})]\Theta(1-\overline{t})+\{-1+\overline{t}\ln[(2\overline{t}+1)/(2\overline{t}-1)]\}\Theta(\overline{t}-1)$ and Θ is the Heaviside step function.

Analytical Results: Full Random Matrices

$$F(t) = \frac{1-\overline{F}}{Dim-1} Dim \frac{\mathcal{J}_1^2(2\sigma_{ini}t)}{\sigma_{ini}^2 t^2} - \frac{1-\overline{F}}{Dim-1} b_2\left(\frac{\sigma_{ini}t}{2Dim}\right) + \overline{F}$$

Two-level form-factor



Torres, García-García & LFS, arXiv:1704.06272

Correlation hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique
Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France
(Received 27 November 1985)

VOLUME 58, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1987

Chaos and Dynamics on 0.5–300-ps Time Scales in Vibrationally Excited Acetylene: Fourier Transform of Stimulated-Emission Pumping Spectrum

J. P. Pique,[a] Y. Chen, R. W. Field, and J. L. Kinsey
Department of Chemistry and George Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 October 1986)

Chemical Physics 146 (1990) 21–38
North-Holland

Correlations in anticrossing spectra and scattering theory. Analytical aspects

T. Guhr and H.A. Weidenmüller
Max-Planck-Institut für Kernphysik, 6900 Heidelberg, FRG

Received 12 December 1989

Experimental results of anticrossing spectroscopy in molecules, in particular the correlation hole, are discussed in a theoretical model. The laser measurements are modelled in terms of the scattering matrix formalism originally developed for compound nucleus scattering. Random matrix theory is used in the framework of this model. The correlation hole is analytically derived for small singlet–triplet coupling. In the case of the data on methylglyoxal this limit is realistic if the spectrum is indeed a superposition of several pure sequences as one can conclude from the analysis of the measurements.

PHYSICAL REVIEW E, VOLUME 65, 026214

Signatures of the correlation hole in total and partial cross sections

T. Gorin* and T. H. Seligman
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(Received 3 August 2001; published 24 January 2002)

In a complex scattering system with few open channels, say a quantum dot with leads, the correlation properties of the poles of the scattering matrix are most directly related to the internal dynamics of the system. We may ask how to extract these properties from an analysis of cross sections. In general this is very difficult, if we leave the domain of isolated resonances. We propose to consider the cross correlation function of two different elastic or total cross sections. For these we can show numerically and to some extent also analytically a significant dependence on the correlations between the scattering poles. The difference between uncorrelated and strongly correlated poles is clearly visible, even for strongly overlapping resonances.

PHYSICAL REVIEW A VOLUME 46, NUMBER 8 15 OCTOBER 1992

Spectral autocorrelation function in the statistical theory of energy levels

Y. Alhassid
Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06511 and the A.W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511

R. D. Levine
The Fritz Haber Research Center for Molecular Dynamics, The Hebrew University, Jerusalem 91904, Israel
(Received 11 October 1991; revised manuscript received 5 May 1992)

Large anti-de Sitter black holes and the correlation hole

DYNAMICS

Realistic System

Quench Dynamics

Integrable

XXZ model

$$H_{ini} = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

$$|\Psi(0)\rangle = |ini\rangle$$

Chaotic

NNN model

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y) + $$

$$+ \lambda \sum_{n=1}^{L-2} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+2}^z + \sigma_n^x \sigma_{n+2}^x + \sigma_n^y \sigma_{n+2}^y)$$

quench parameter

Survival Probability

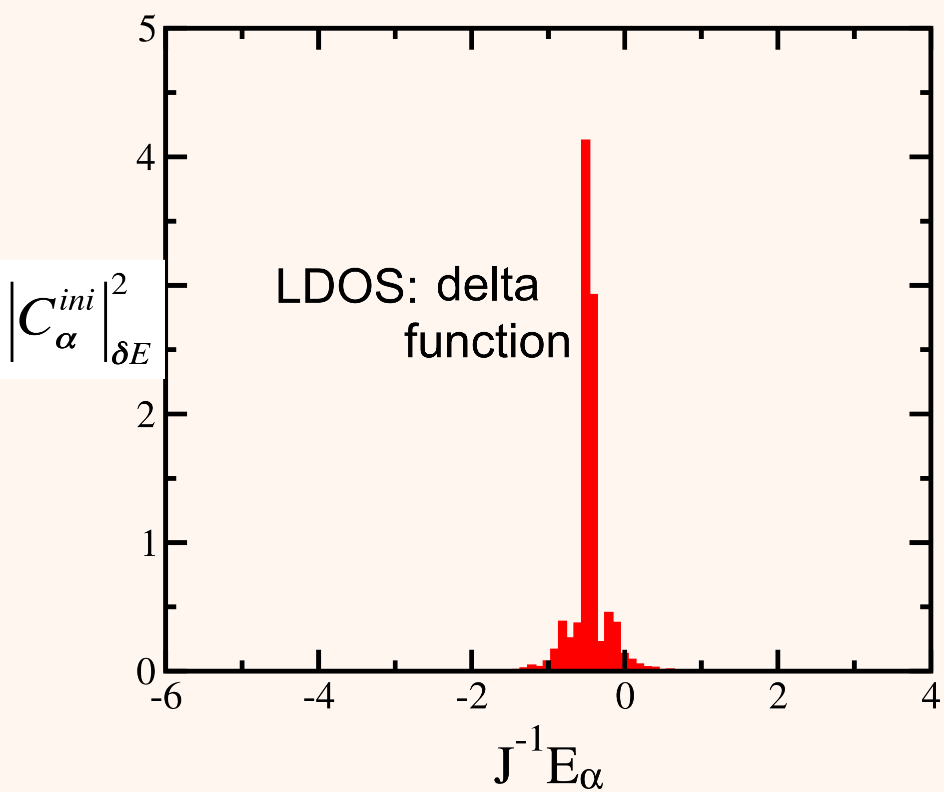
$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left| \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t} \right|^2 \cong \left| \int_{-\infty}^{\infty} \rho_{ini}(E) e^{-iEt} \, dE \right|^2$$

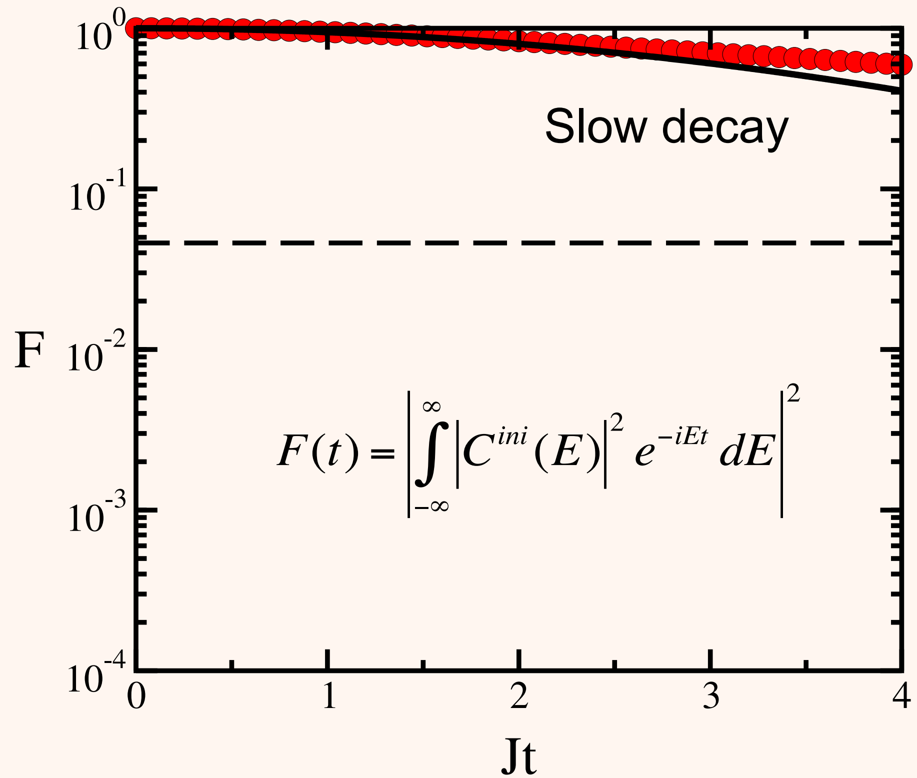
LDOS

Perturbation increases Fidelity decays faster

$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

$$\lambda = 0.2$$





Lea F. Santos, Yeshiva University

L=16, 8 up spins, T=7.1 $\Delta = 0.5$

Exponential decay

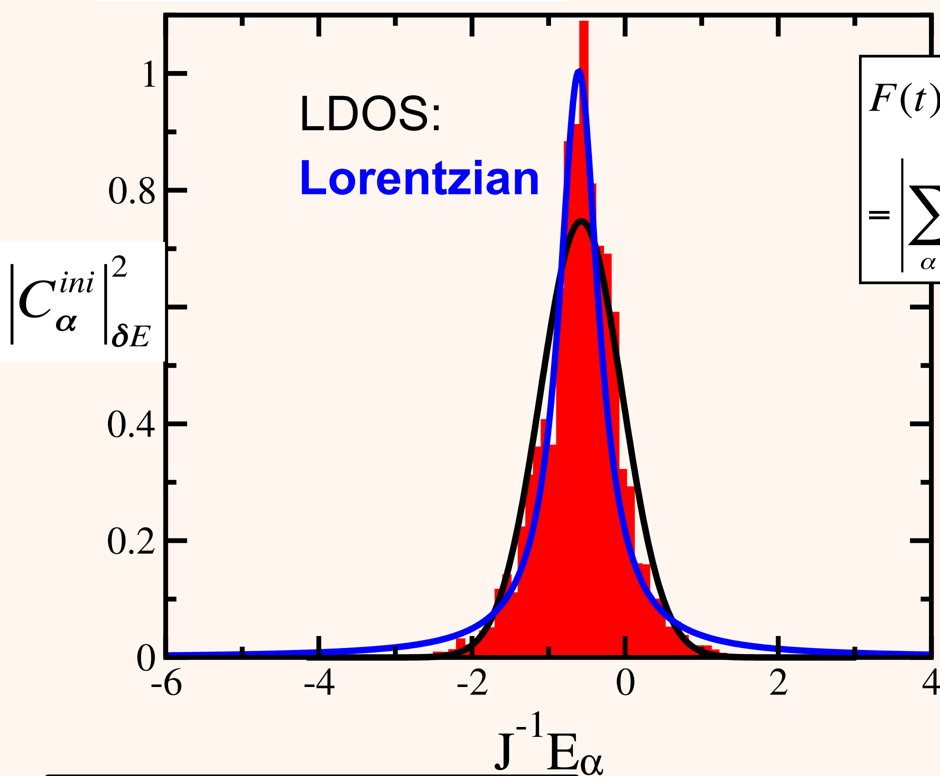
$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

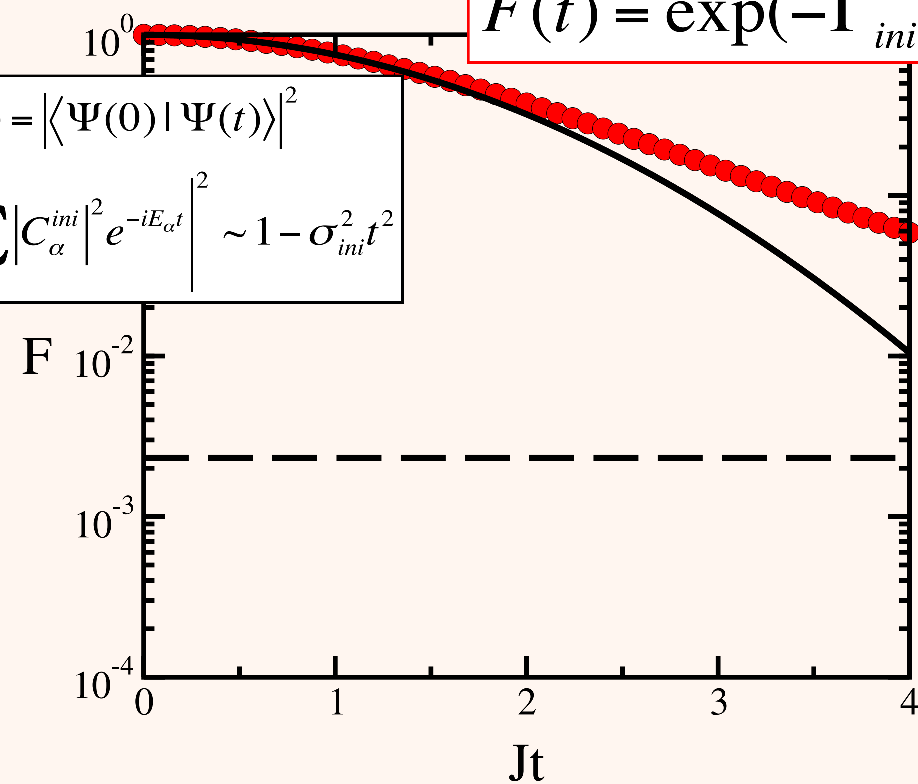
$$\lambda = 0.45$$

$$\frac{1}{2\pi} \frac{\Gamma_{ini}}{(E_{ini} - E)^2 + \Gamma_{ini}^2 / 4}$$

$$F(t) = \exp(-\Gamma_{ini} t)$$

$$F(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \sim 1 - \sigma_{ini}^2 t^2$$





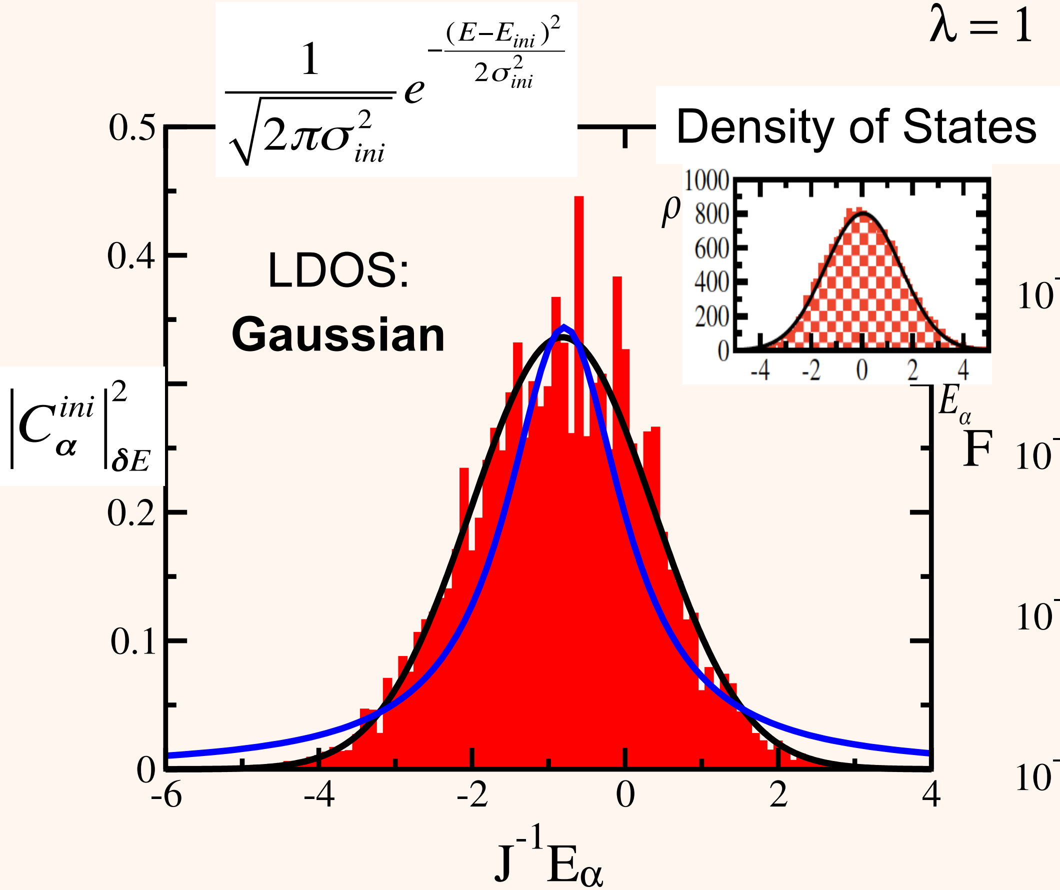
L=16, 8 up spins, T=7.1

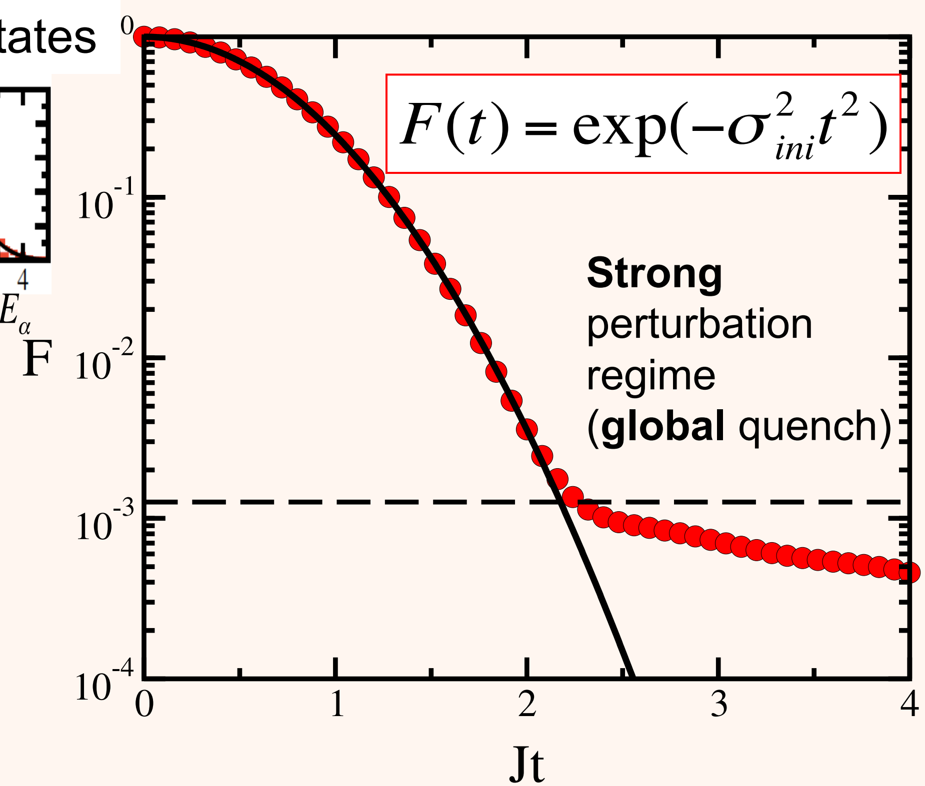
$\Delta = 0.5$

Faster than exponential: Gaussian

$$H_{initial} = H_{XXZ} \xrightarrow{quench} H_{final} = H_{XXZ} + \lambda H_{NNN}$$

$$\lambda = 1$$





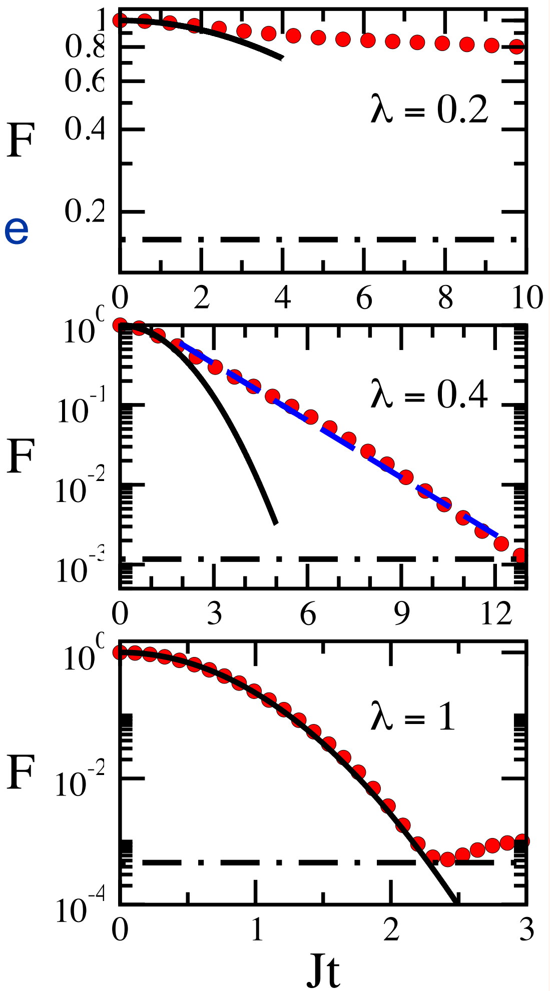
L=16, 8 up spins, T=7.1 $\Delta = 0.5$

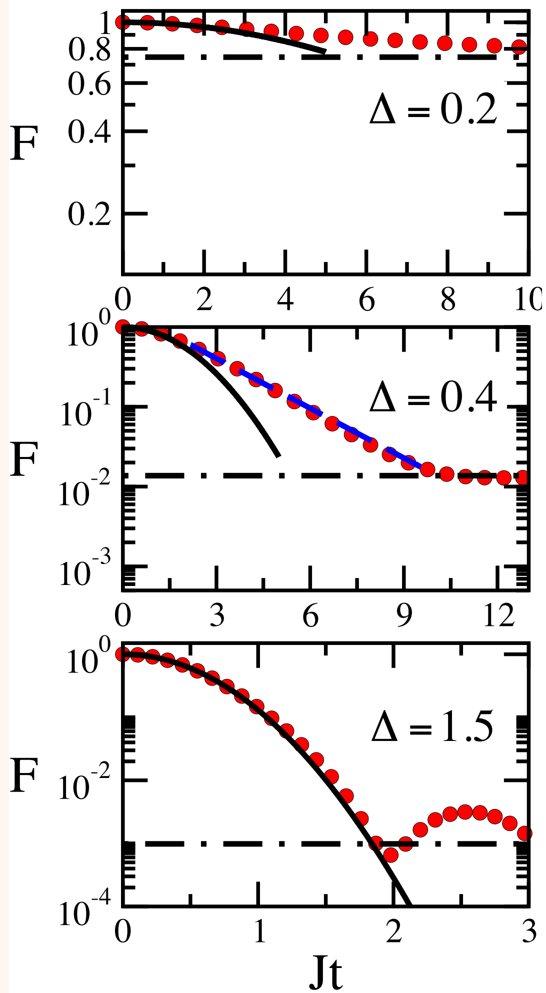
Exponential and Gaussian F(t)
H^{final}: Chaotic or Integrable

$H_{XXZ} \xrightarrow{\lambda} H_{XXZ} + \lambda H_{NNN}$

$H_{XX} \xrightarrow{\Delta} H_{XXZ}$

Integrable
to
chaotic





Integrable
to
integrable

$$\sum_{n=1}^{L-1} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

$$\downarrow \Delta$$

$$\sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

Torres & LFS
PRA **89** (2014)

Torres, Vyas, LFS
NJP **16** (2014)

Torres & LFS
PRA **90** (2014)

L=18, 6 up spins

Strong Perturbation

$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$

quench parameter

$\Delta \to \infty$ to $\Delta \to finite$

Very strong perturbation

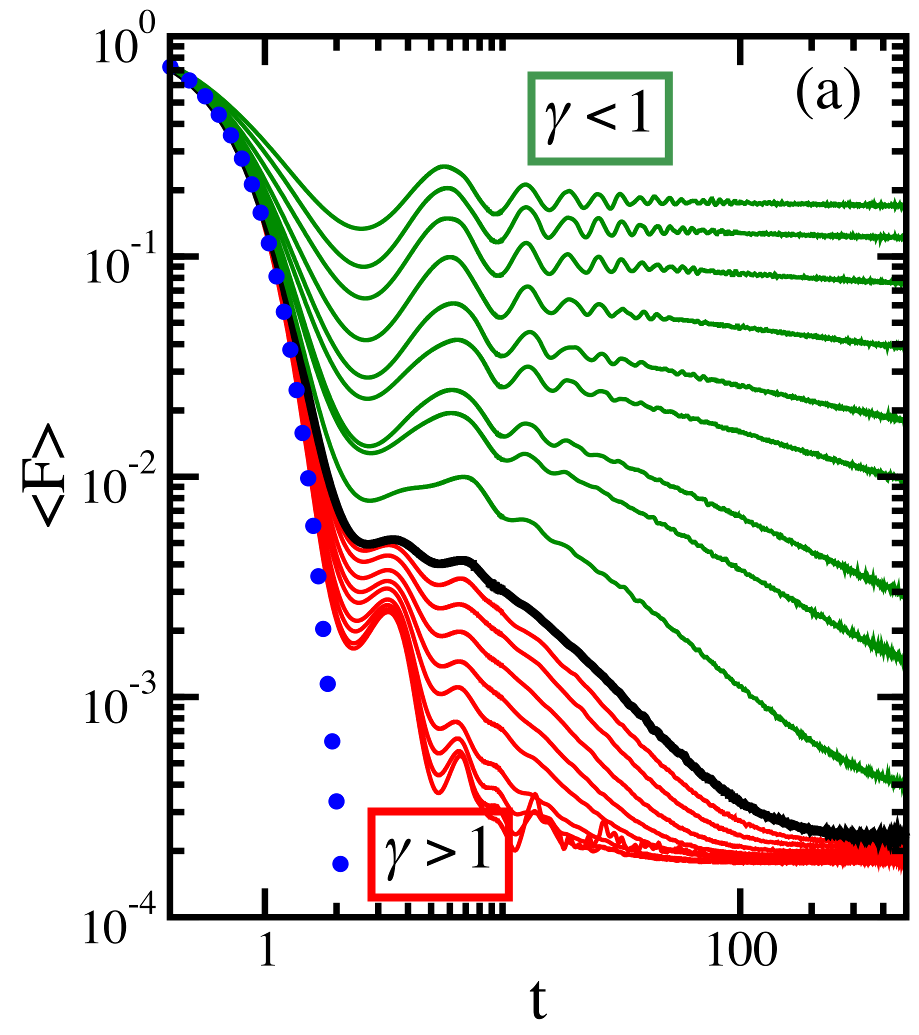
$$H_{ini} = \sum_{n=1}^{L-1} \frac{J\Delta}{4} \sigma_n^z \sigma_{n+1}^z \longrightarrow H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

$$|\Psi(0)\rangle = |ini\rangle$$

Disorder strength: h
[-h,h]

Power-law exponent: energy bounds

$$t^{-\gamma}$$



$PR^{(\alpha)} \propto Dim^{D_2}$

h>J

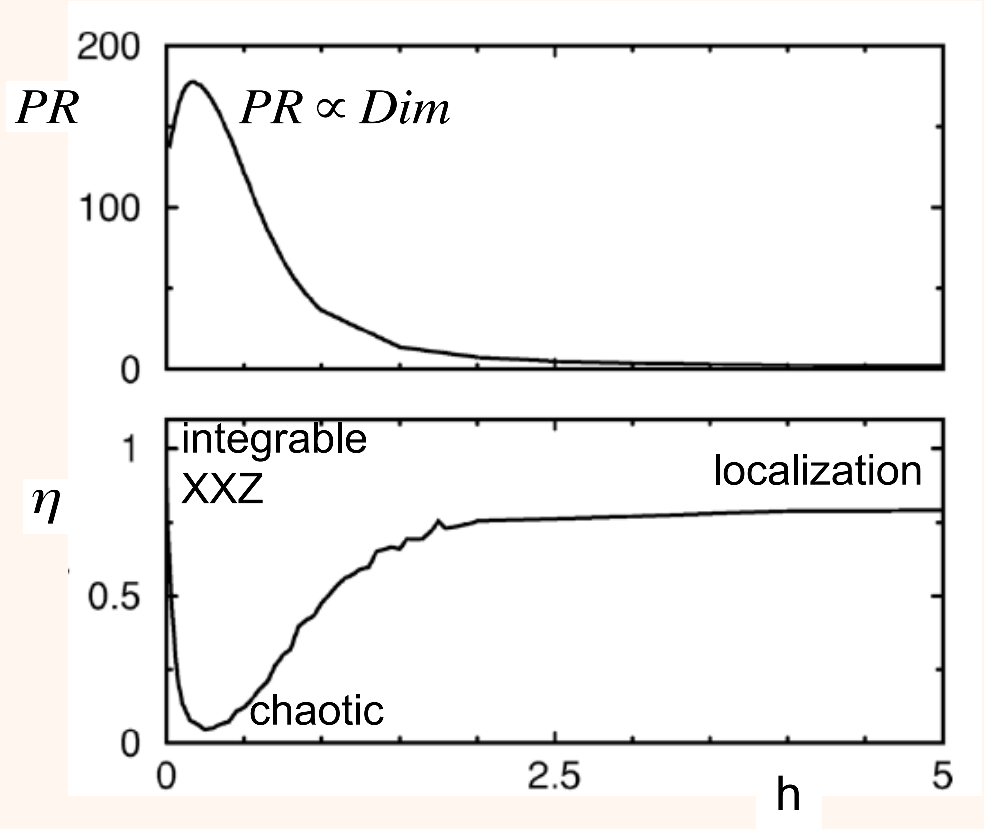
$t^{-\gamma}$

h<J

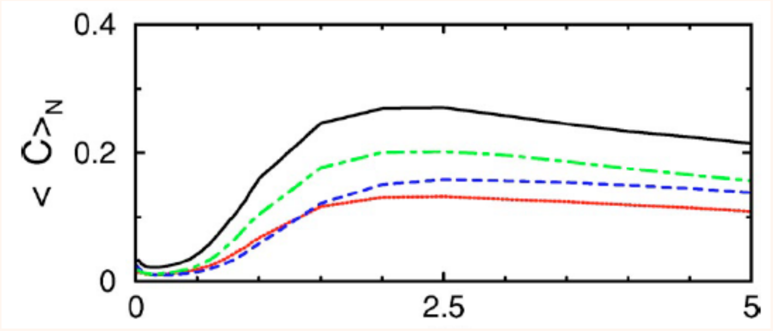
$PR^{(\alpha)} \propto Dim$

Localization and entanglement

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z + \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y)$$



Concurrence

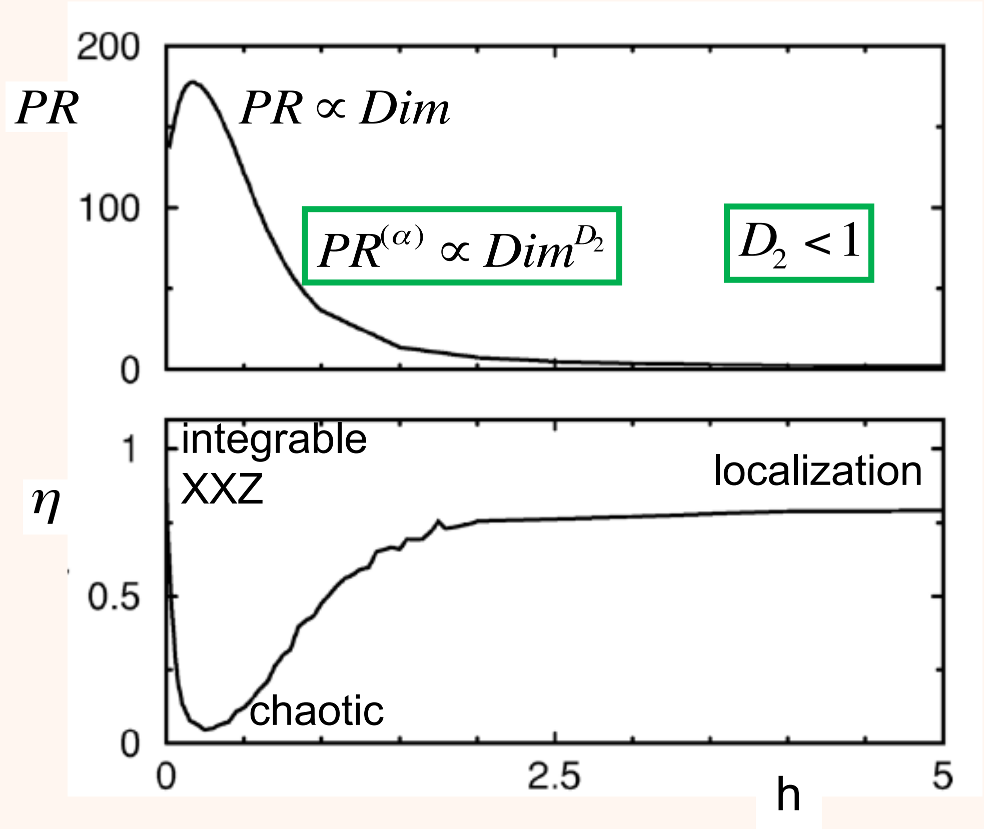


$$\eta = \frac{\int_0^{s_0}[P(s) - P_{WD}(s)]ds}{\int_0^{s_0}[P_P(s) - P_{WD}(s)]ds},$$

LFS, Rigolin, Escobar PRA (2004)

Integrable-chaos-integrable

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$



Participation Ratio

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} |c_i^{(\alpha)}|^4}$$

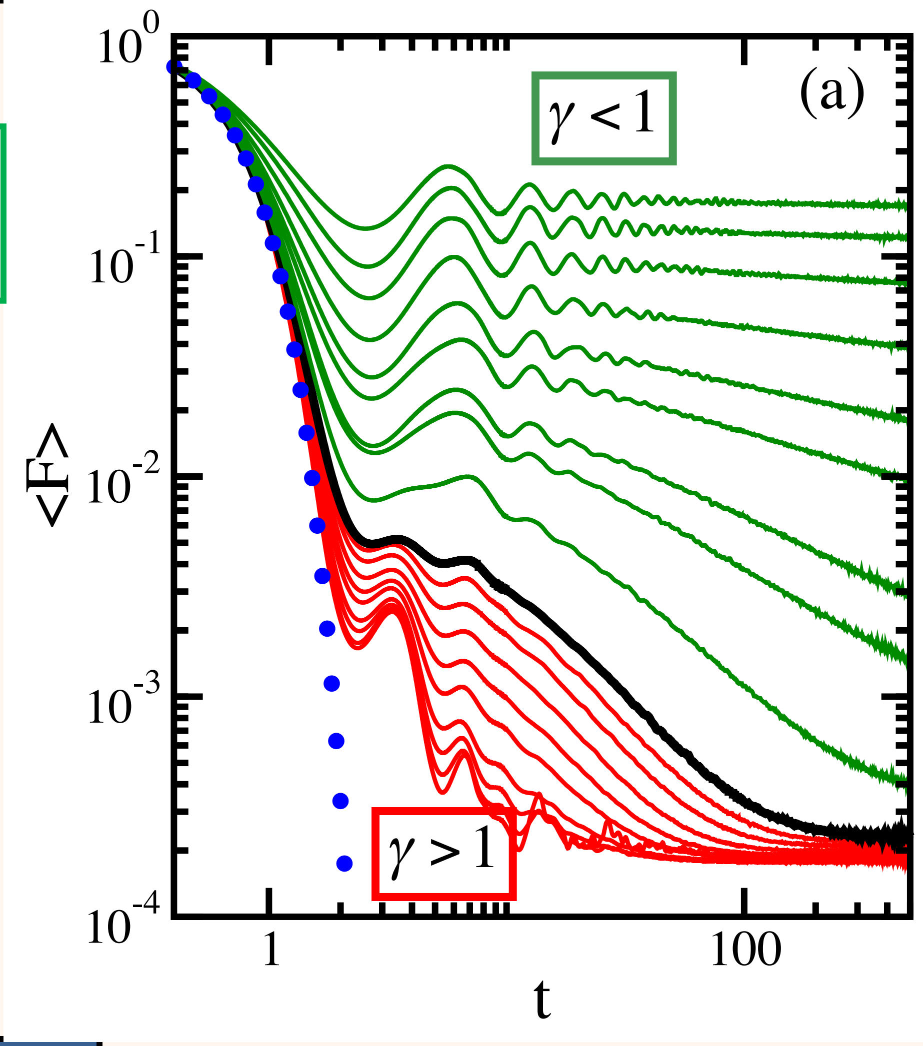
$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

$$\eta = \frac{\int_0^{s_0} [P(s) - P_{WD}(s)] ds}{\int_0^{s_0} [P_P(s) - P_{WD}(s)] ds},$$

Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. (2017)

Power-law exponent: energy bounds

Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. (2017)



$PR^{(\alpha)} \propto Dim^{D_2}$

h>J

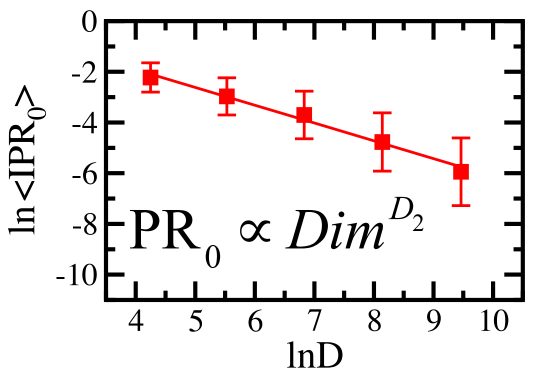
$t^{-\gamma}$

h<J

$PR^{(\alpha)} \propto Dim$

Power-law exponent: correlations

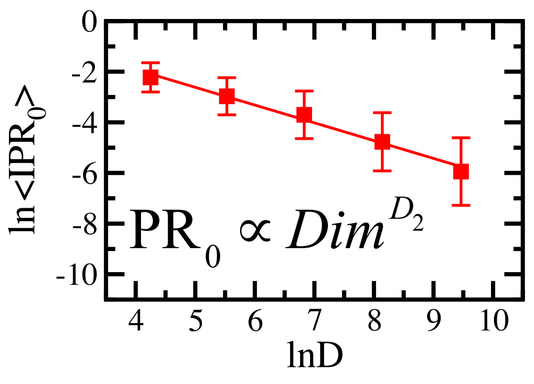
$PR^{(\alpha)} \propto Dim^{D_2}$ $\qquad D_2 < 1$

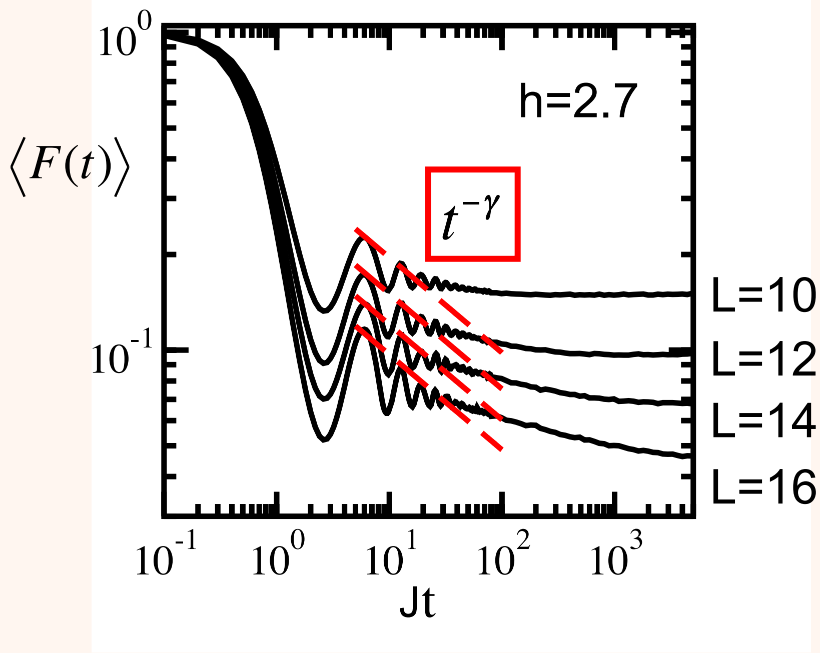


Torres & LFS
PRB **92**, 01420 (2015)

Power-law exponent: correlations

$PR^{(\alpha)} \propto Dim^{D_2}$ $\qquad D_2 < 1$





$$F(t) = \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} = \int G(E) e^{-iEt} dE \to t^{-\gamma}$$

$$G(E) = \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \delta(E - (E_\alpha - E_\beta)) = |E|^{\gamma - 1}$$

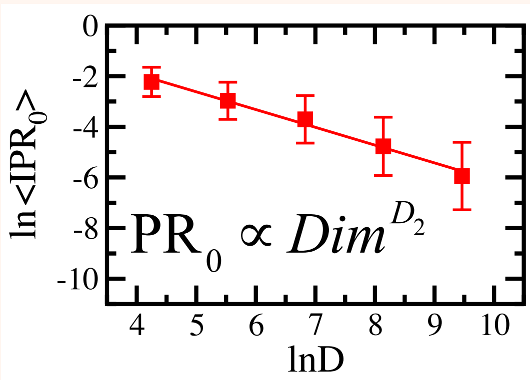
$$PR_q^{(\alpha)} \propto Dim^{(q-1)D_q}$$

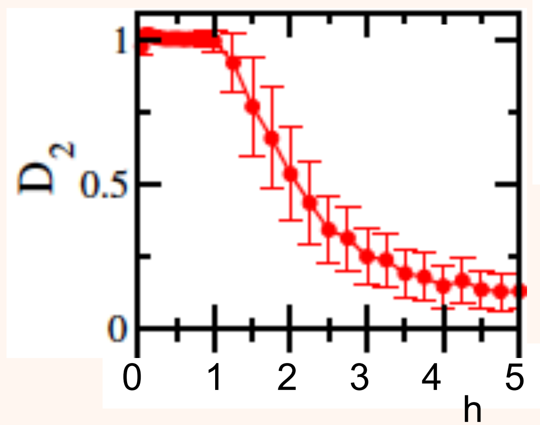
Multifractality = nonlinear dependence of the generalizeddimension on q

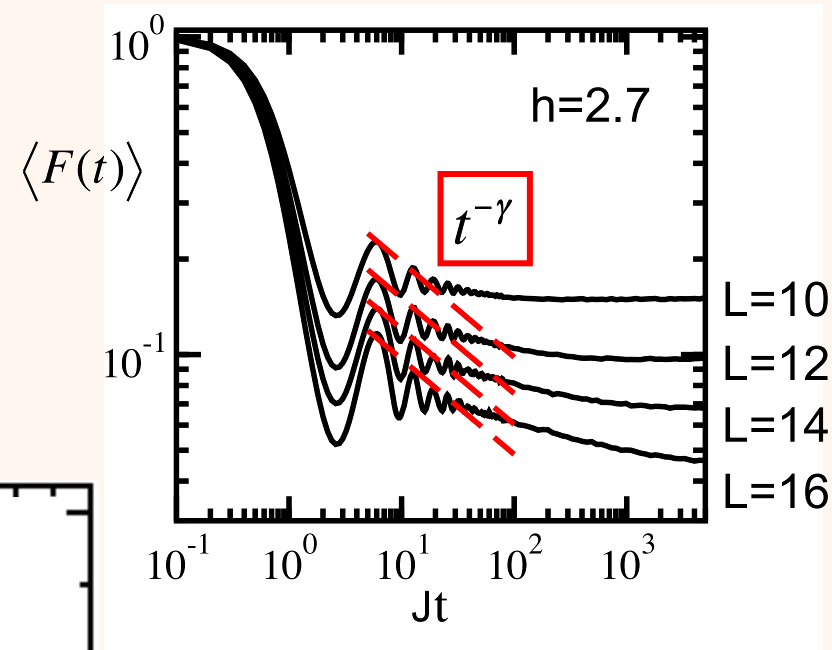
Torres & LFS
PRB **92**, 01420 (2015)

Power-law exponent: correlations

$PR^{(\alpha)} \propto Dim^{D_2}$ $\qquad D_2 < 1$







$$F(t) = \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} = \int G(E) e^{-iEt} dE \to t^{-\gamma}$$

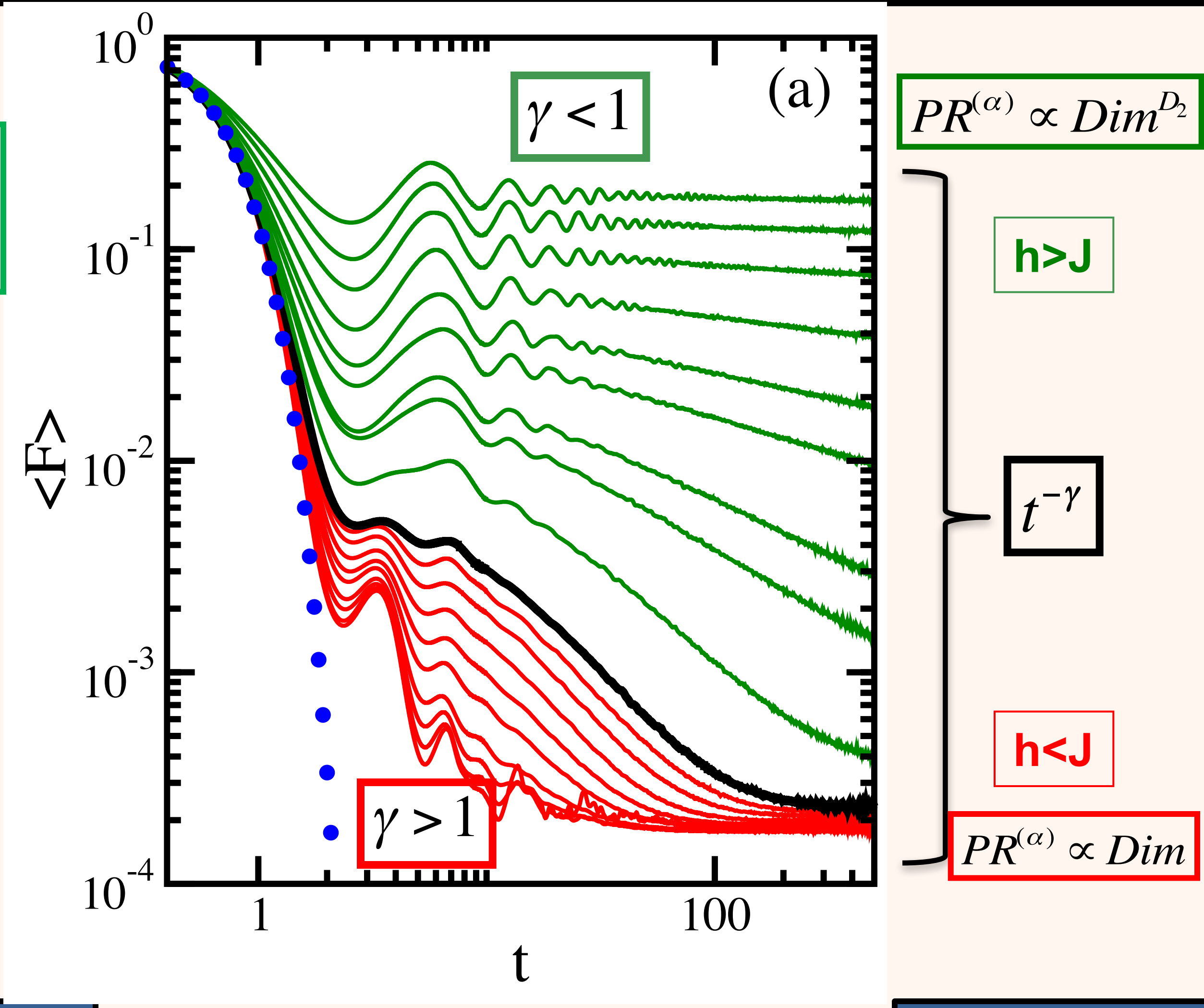
$$G(E) = \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 \delta(E - (E_\alpha - E_\beta)) = |E|^{\gamma-1}$$

Generalized dimension
Multifractal dimension

Torres & LFS
PRB **92**, 01420 (2015)

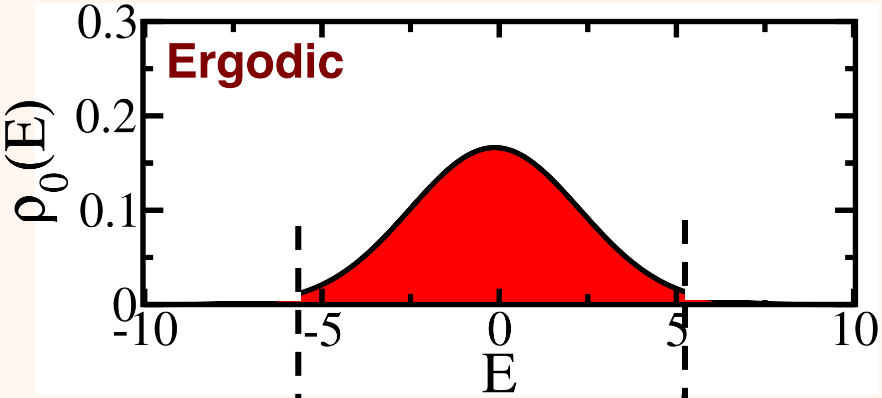
Power-law exponent: energy bounds

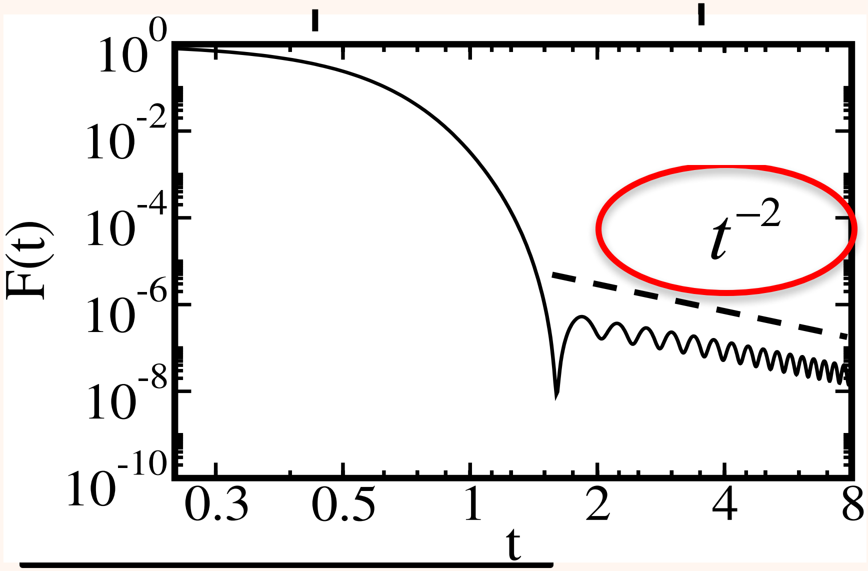
Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. (2017)



Ergodically filled LDOS
Power-law decay caused by energy bounds

Khalfin (JETP, 1958)





$$F(t) = \frac{e^{-\sigma_0^2 t^2}}{4\mathcal{N}^2}\left|\left[\mathrm{erf}\left(\frac{E_0 - E_{\mathrm{low}} + i\sigma_0^2 t}{\sqrt{2}\sigma_0}\right) - \mathrm{erf}\left(\frac{E_0 - E_{\mathrm{up}} + i\sigma_0^2 t}{\sqrt{2}\sigma_0}\right)\right]\right|^2.$$

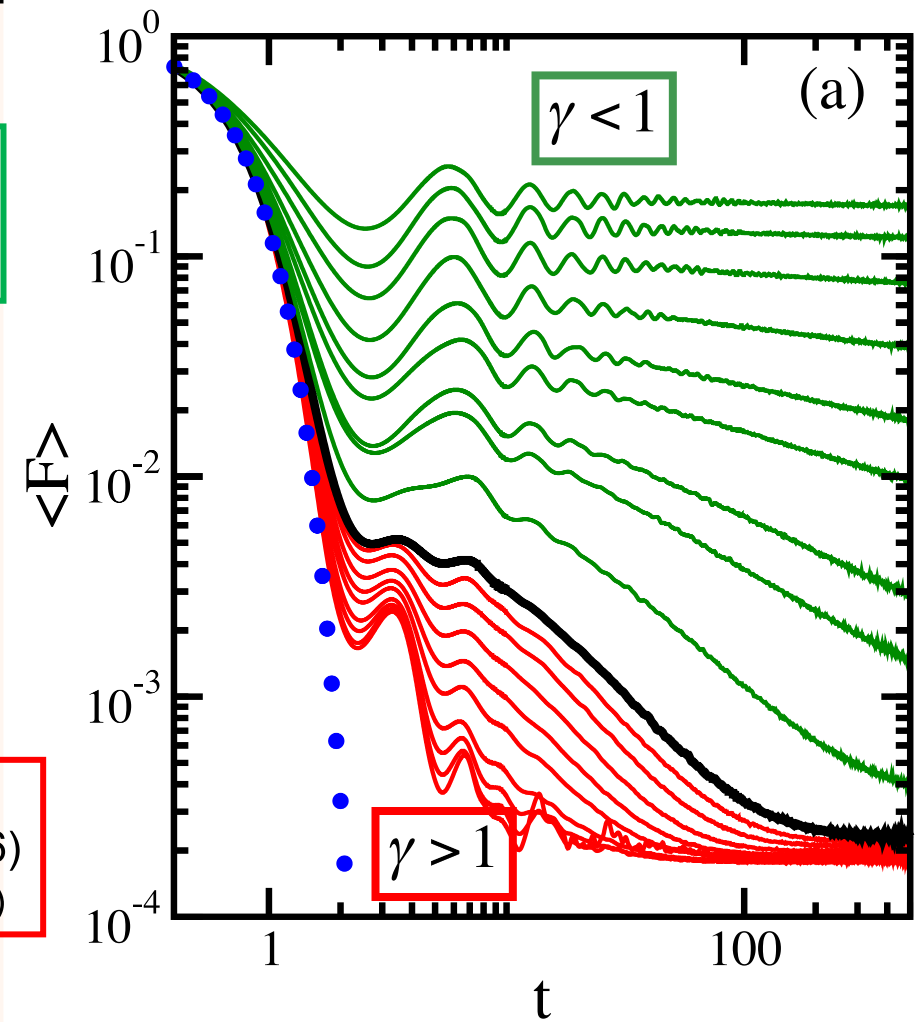
$$F(t) = \left|\frac{1}{\sqrt{2\pi\sigma_{ini}^2}} \int_{Elow}^{Eup} e^{-(E-E_{ini})^2/2\sigma_{ini}^2} e^{-iEt}\, dE\right|^2$$

$$\xrightarrow[t\to\infty]{} \propto \frac{1}{t^2}$$

Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

Power-law exponent: energy bounds

Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. (2017)



$PR^{(\alpha)} \propto Dim^{D_2}$

h>J

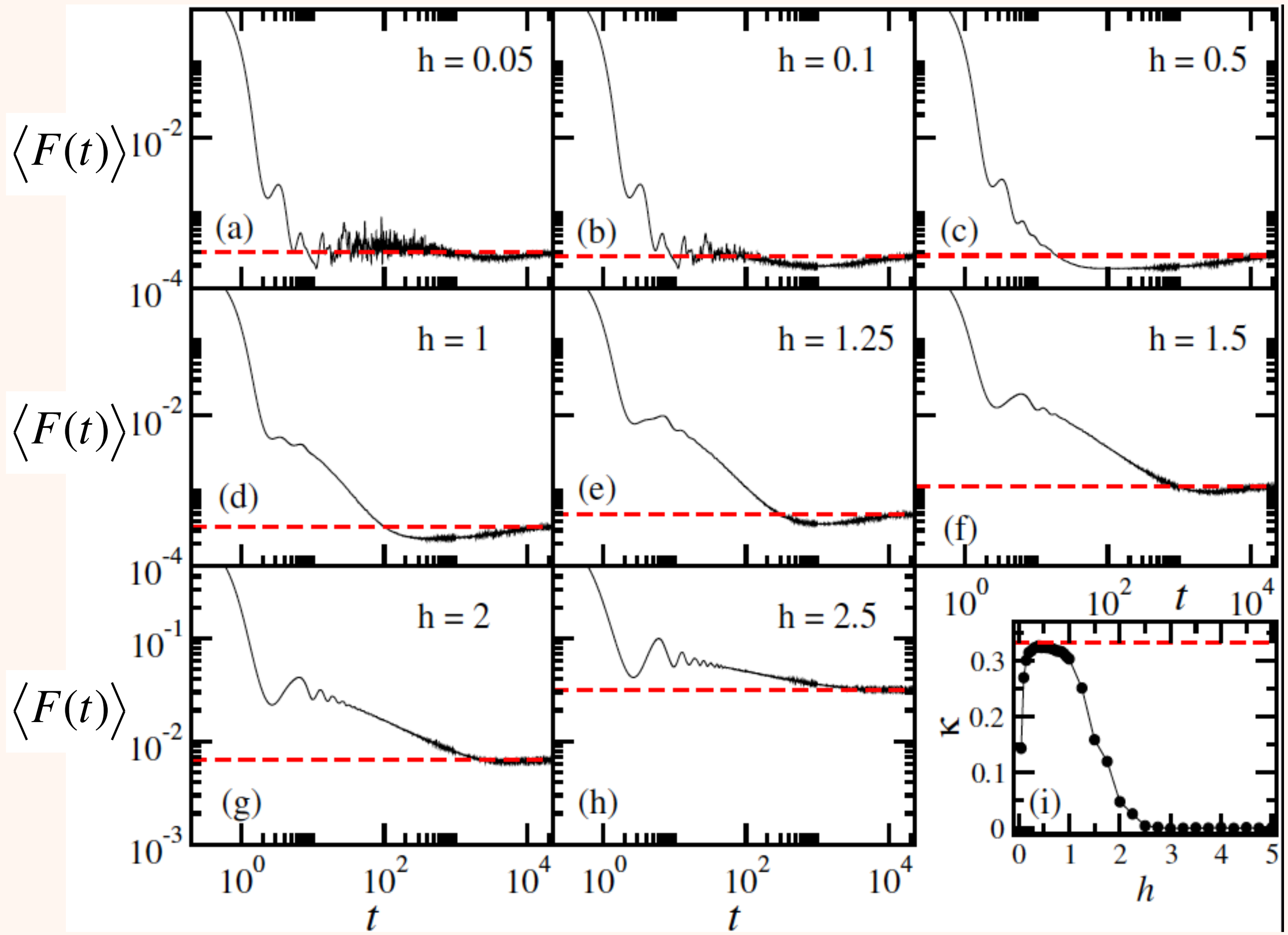
$t^{-\gamma}$

h<J

$PR^{(\alpha)} \propto Dim$

Távora, Torres, LFS
PRA **94**, 041603R (2016)
PRA **95**, 013604 (2017)

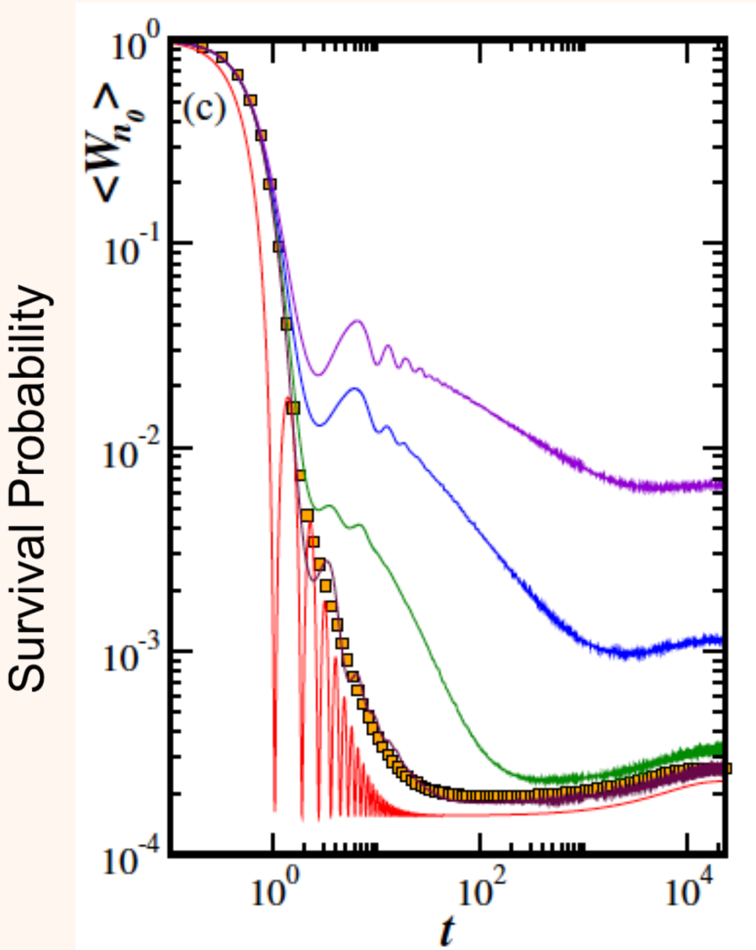
Correlation hole



Torres & LFS
Ann. Phys. (2017)
Philos. Trans. A (2017)

Analytical results and fitting

A: only one fitting constant !



$$F(t) = \frac{1-\overline{F}}{Dim-1}\left[Dim \frac{e^{-\sigma_{ini}^2 t^2} + A \frac{1-e^{-\sigma_{ini}^2 t^2}}{\sigma_{ini}^2 t^2}}{1+A} + b_2\left(\frac{\sigma_{ini} t}{Dim}\right)\right] + \overline{F}$$

$$\frac{\exp(-\sigma^2 t^2)}{t^2}$$

Correlation hole

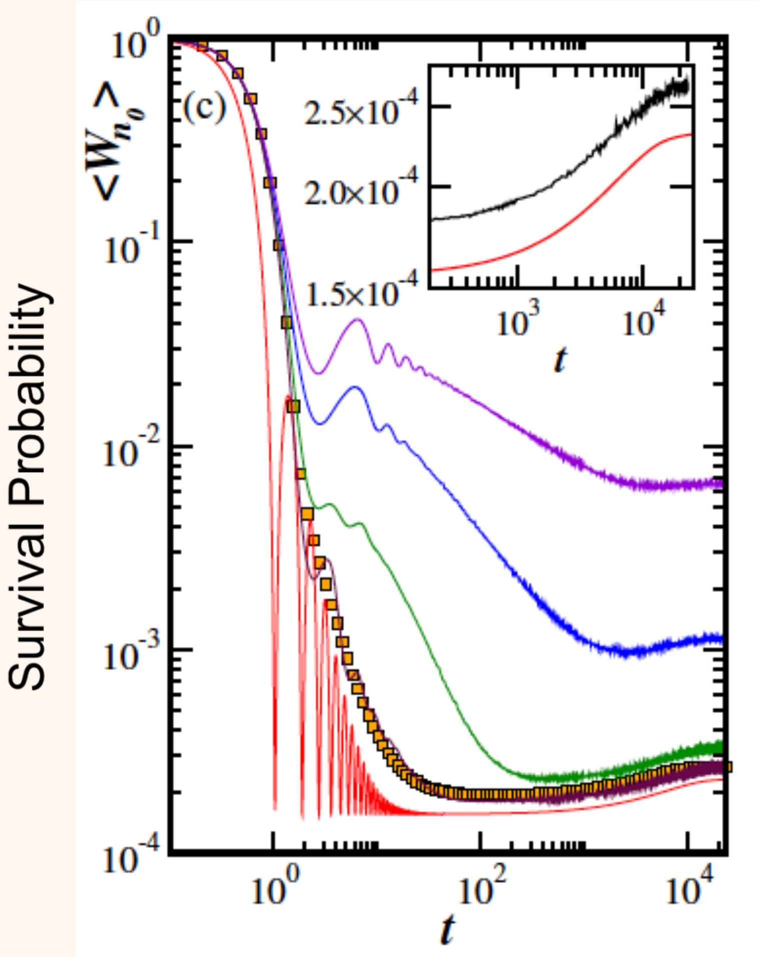
saturation

h=0.5

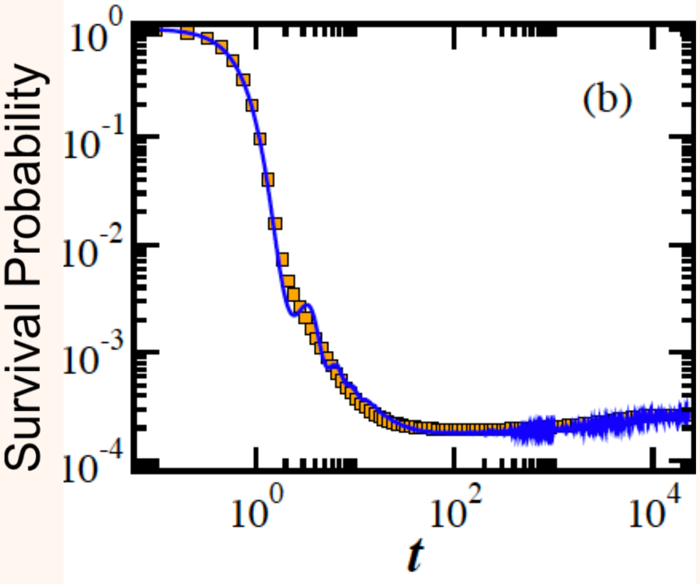
$b_2(t)$: Two-level form factor

Torres, García-García & LFS, arXiv:1704.06272

Correlation hole in realistic system

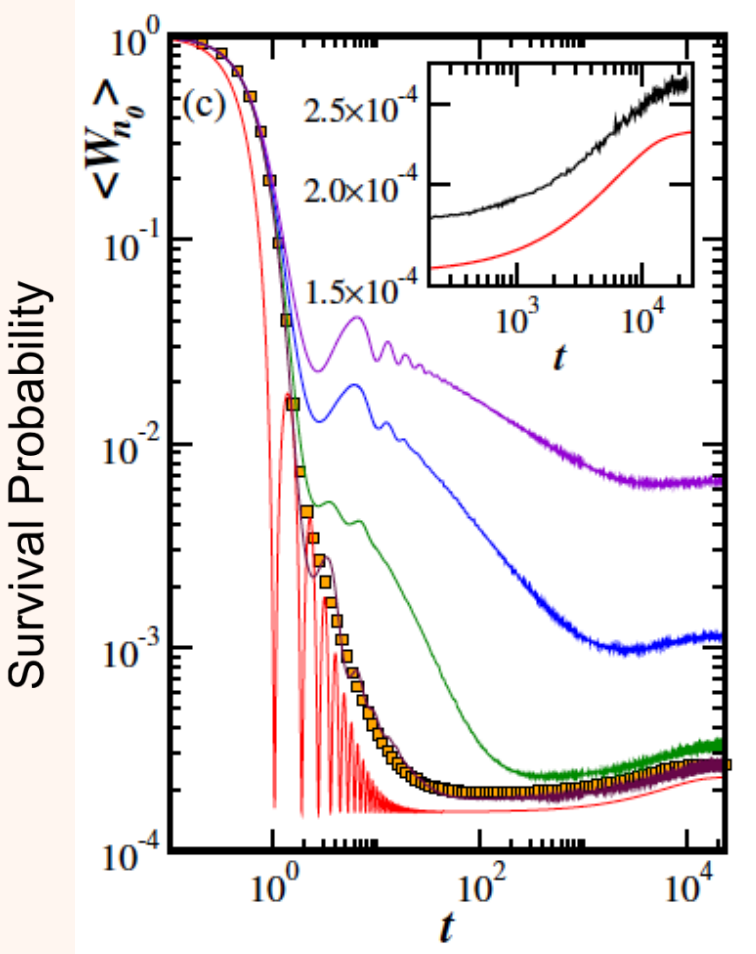


Chaotic disordered model with h=0.5

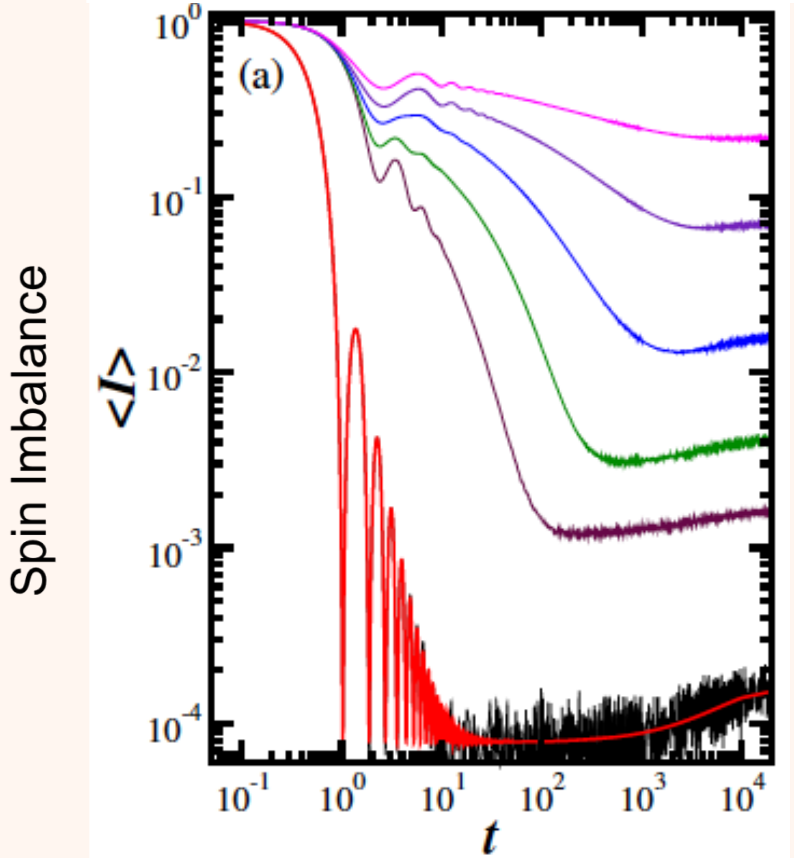


Torres, García-García & LFS, arXiv:1704.06272

Correlation hole for observables

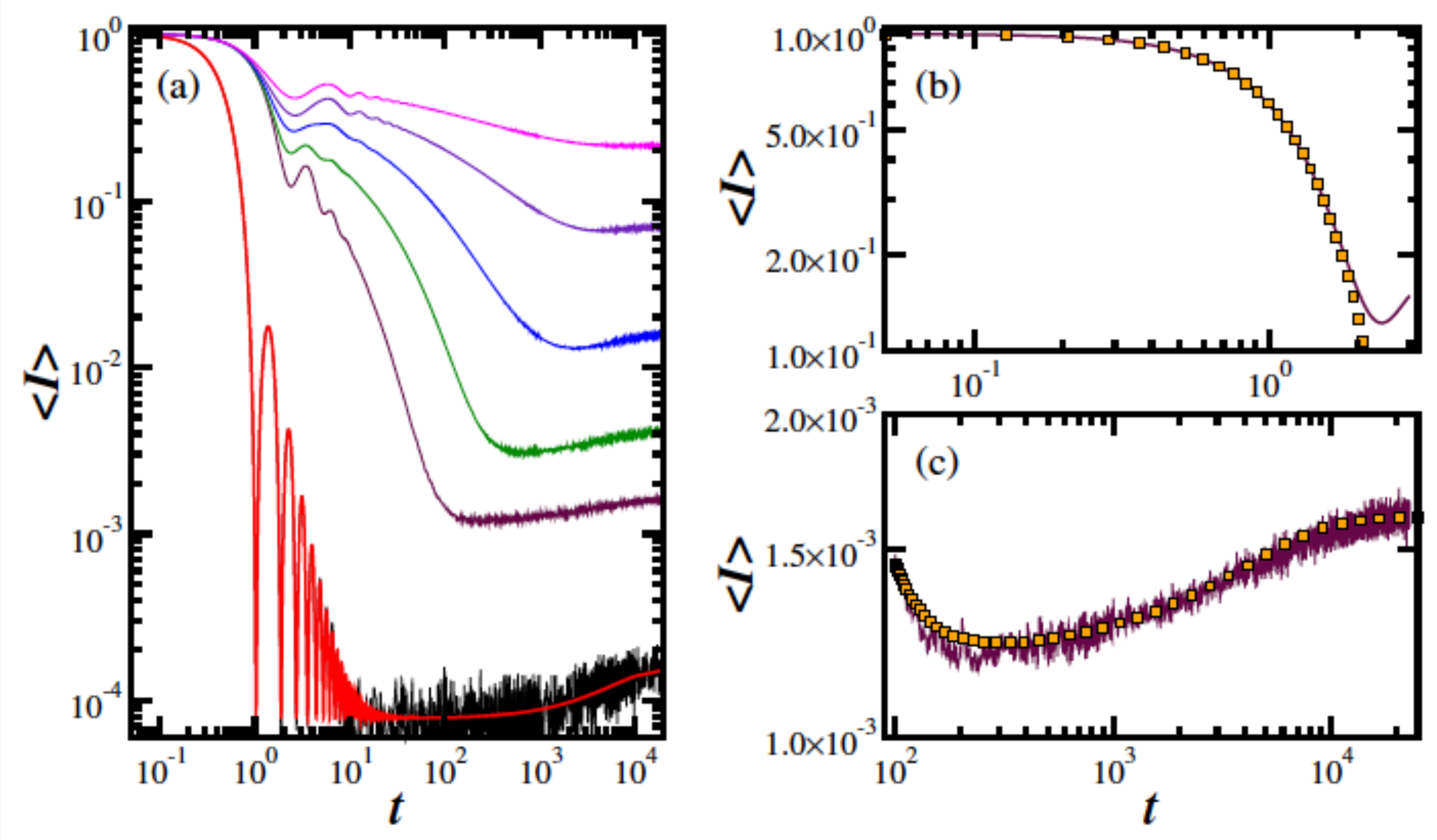


$$I(t) = \frac{4}{L}\sum_{k=1}^{L}\langle\Psi(0)|S_k^z(0)S_k^z(t)|\Psi(0)\rangle.$$



Torres, García-García & LFS,
arXiv:1704.06272

Spin Imbalance



Torres, García-García & LFS,
arXiv:1704.06272

ENTROPIES

Short-time   Intermediate-time   Long-time   Saturation

Short-time Intermediate-time Long-time Saturation

Strong perturbation

Survival Probability/
Imbalance
Exponential/Gaussian

Entropies
Linear

Short-time	Intermediate-time	Long-time	Saturation

Strong perturbation

Survival Probability/ Imbalance **Exponential/Gaussian**	Survival Probability/ Imbalance **Power-law**
Entropies **Linear**	Entropies **Logarithmic**

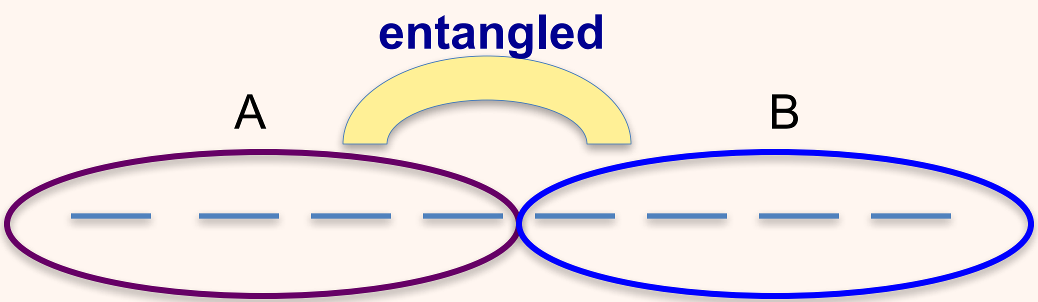
Short-time	Intermediate-time	Long-time Saturation
Strong perturbation		**Level repulsion**
Survival Probability/ Imbalance **Exponential/Gaussian**	Survival Probability/ Imbalance **Power-law**	Survival Probability/ Imbalance **Correlation hole**
Entropies **Linear**	Entropies **Logarithmic**	Entropies **Correlation bulge**

Entanglement Entropy

Entanglement Entropy: von Neumann entropy of the reduced density matrix

$$Sv(t) = -Tr[\boldsymbol{\rho}_A(t)\ln\boldsymbol{\rho}_A(t)]$$

$$\rho_A = Tr_B[\rho]$$



Shannon Information Entropy

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

$$\left|\psi^{(\alpha)}\right\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \ldots \end{pmatrix}$$

Participation Ratio

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} | c_i^{(\alpha)} |^4}$$

Shannon Entropy

$$Sh^{(\alpha)} = -\sum_{i=1}^{D} |c_i^{(\alpha)}|^2 \ln |c_i^{(\alpha)}|^2$$

[Shannon entropy =
Rényi entropy for q=1]

C. E. Shannon,
Bell Sys. Tech. J. **27**, 379 (1948)

Shannon Information Entropy

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

$$\left|\psi^{(\alpha)}\right\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ \dots \end{pmatrix}$$

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Shannon Entropy

$$Sh^{(\alpha)} = -\sum_{i=1}^{D} |c_i^{(\alpha)}|^2 \ln |c_i^{(\alpha)}|^2$$

[Shannon entropy = Rényi entropy for q=1]

In the energy eigenbasis:

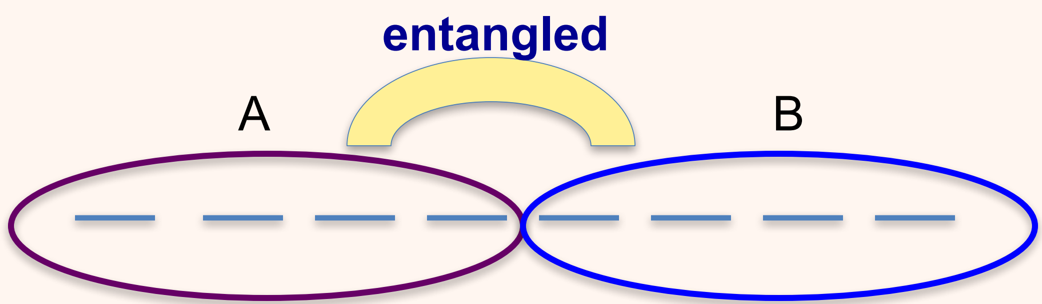
Shannon entropy = Diagonal entropy

A. Polkovnikov
Ann. Phys. **326**, 486 (2011)

Evolution of Entropies

Entanglement Entropy: von Neumann entropy of the reduced density matrix

$$Sv(t) = -Tr[\boldsymbol{\rho}_A(t) \ln \boldsymbol{\rho}_A(t)]$$



Shannon Entropy:

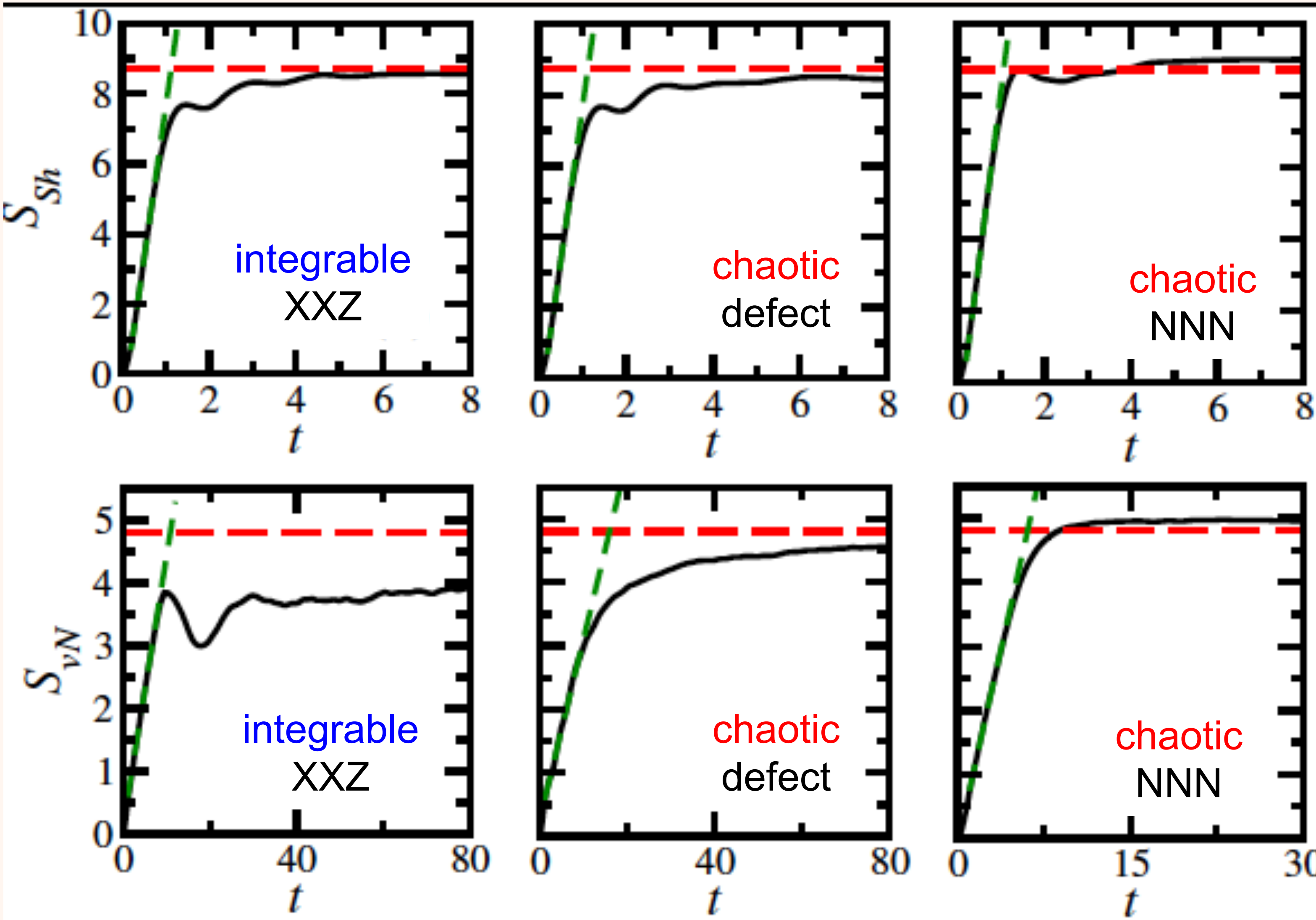
$$Sh(t) = -\sum_n W_n(t) \ln W_n(t)$$

$$W_n(t) = \left| \langle \phi_n | e^{-iHt} | \Psi(0) \rangle \right|^2$$

$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

Torres et al,
Entropy **18**, 359 (2016)

Integrable and Chaotic Models



$|\Psi(0)\rangle$

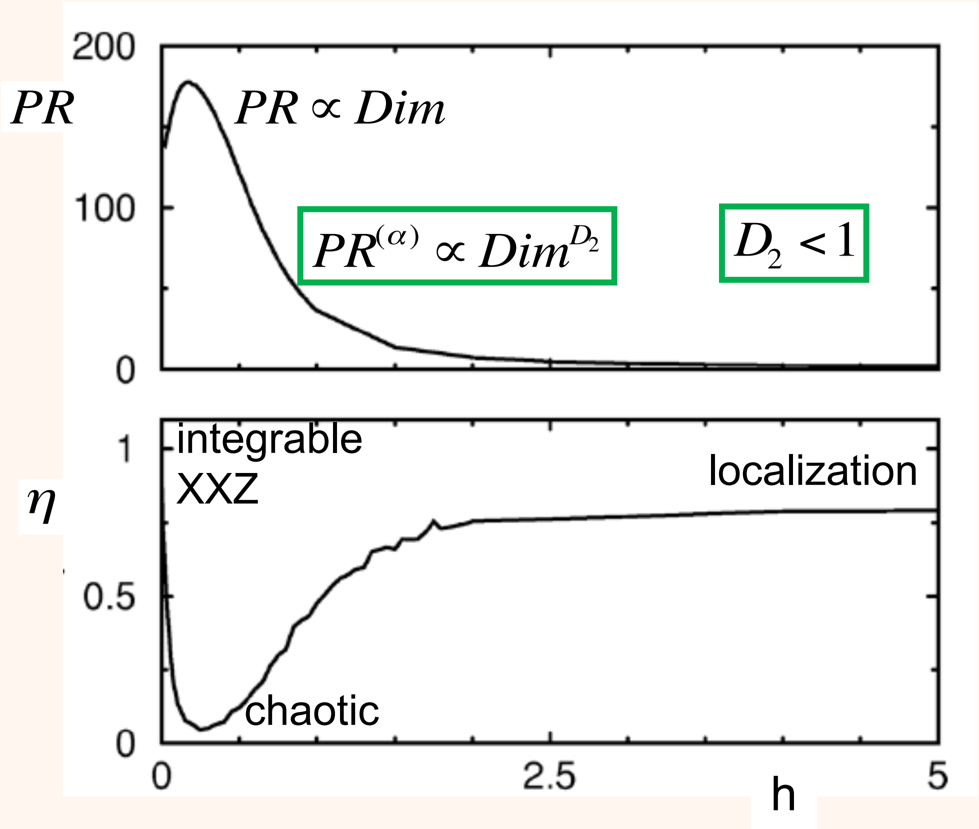
Néel state

↑↓↑↓↑↓↑↓

Torres et al,
Entropy (2016)

Integrable-chaos-integrable

$$H=\sum_{n=1}^{L}\frac{h_n}{2}\sigma_n^z+\sum_{n=1}^{L-1}\frac{J}{4}(\Delta\sigma_n^z\sigma_{n+1}^z+\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)$$



Intermediate level statistics: h>J

Nonergodic delocalized states:

$$PR^{(\alpha)} \propto Dim^{D_2}$$

LFS, Rigolin, Escobar PRA (2004)

Torres & LFS,
PRB **92**, 01420 (2015)
Ann. Phys. (2017)

Entropies: log behavior

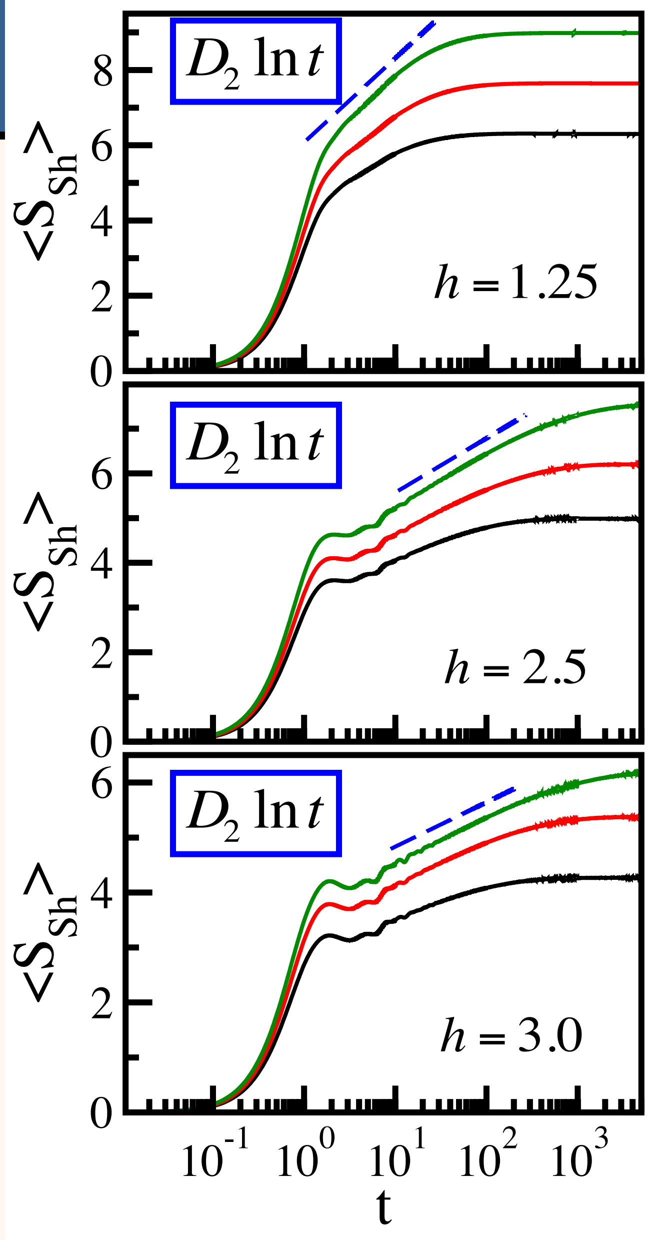
Intermediate level statistics: h>J

Nonergodic delocalized states: $PR^{(\alpha)} \propto Dim^{D_2}$

$D_2 < 1$

$$Sh(t) = -\sum_n W_n(t) \ln W_n(t)$$

$$W_n(t) = \left| \langle \phi_n | e^{-iHt} | \Psi(0) \rangle \right|^2$$



$PR_q^{(\alpha)} \propto Dim^{(q-1)D_q}$

Multifractality = nonlinear dependence of the generalized dimension on q

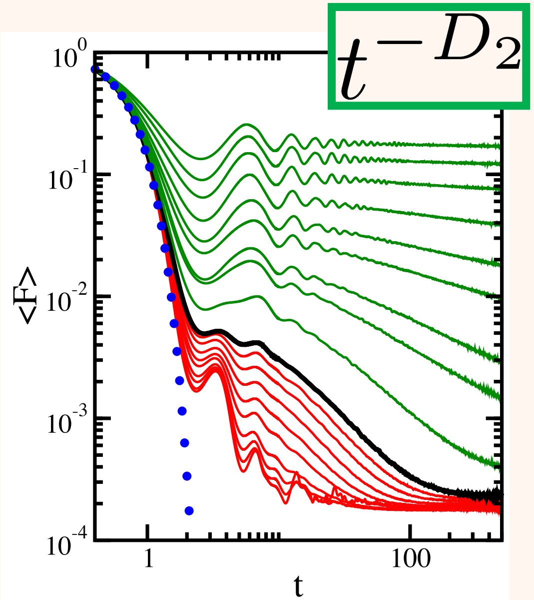
Torres & LFS
Ann. Phys. (2017)

Entropies: log behavior

Intermediate level statistics: h>J

Nonergodic delocalized states: $PR^{(\alpha)} \propto Dim^{D_2}$

$D_2 < 1$



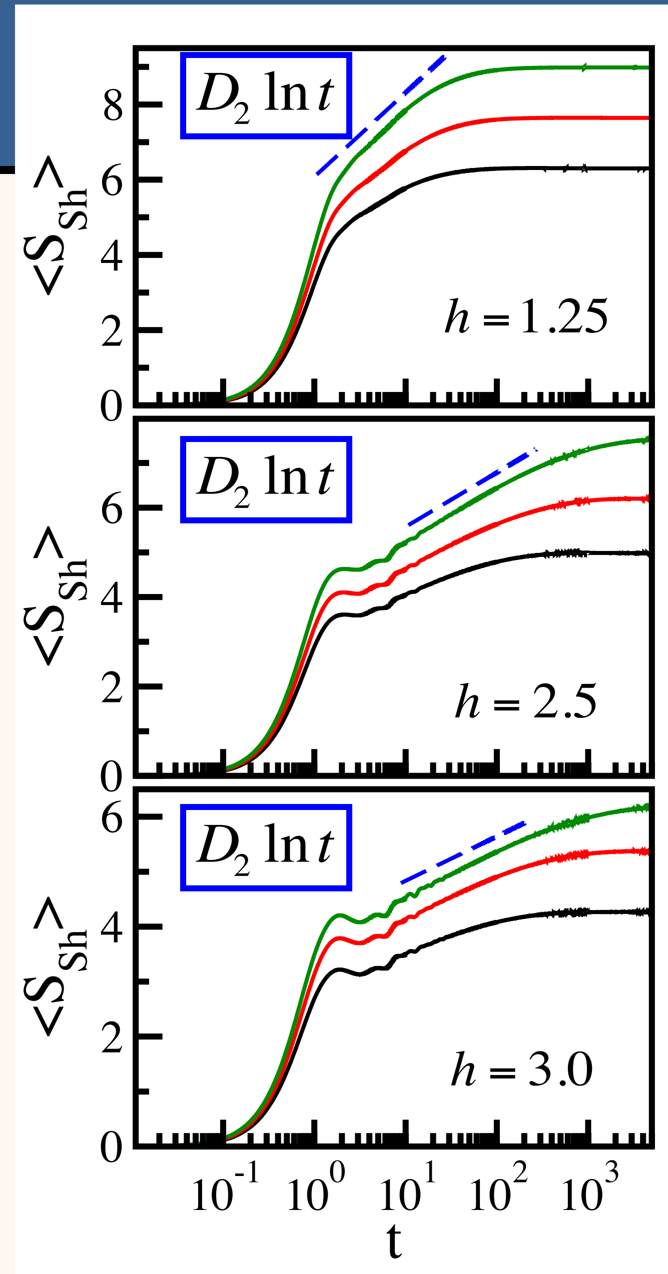
$$Sh(t) = -\sum_n W_n(t) \ln W_n(t)$$

$$W_n(t) = \left| \langle \phi_n | e^{-iHt} | \Psi(0) \rangle \right|^2$$

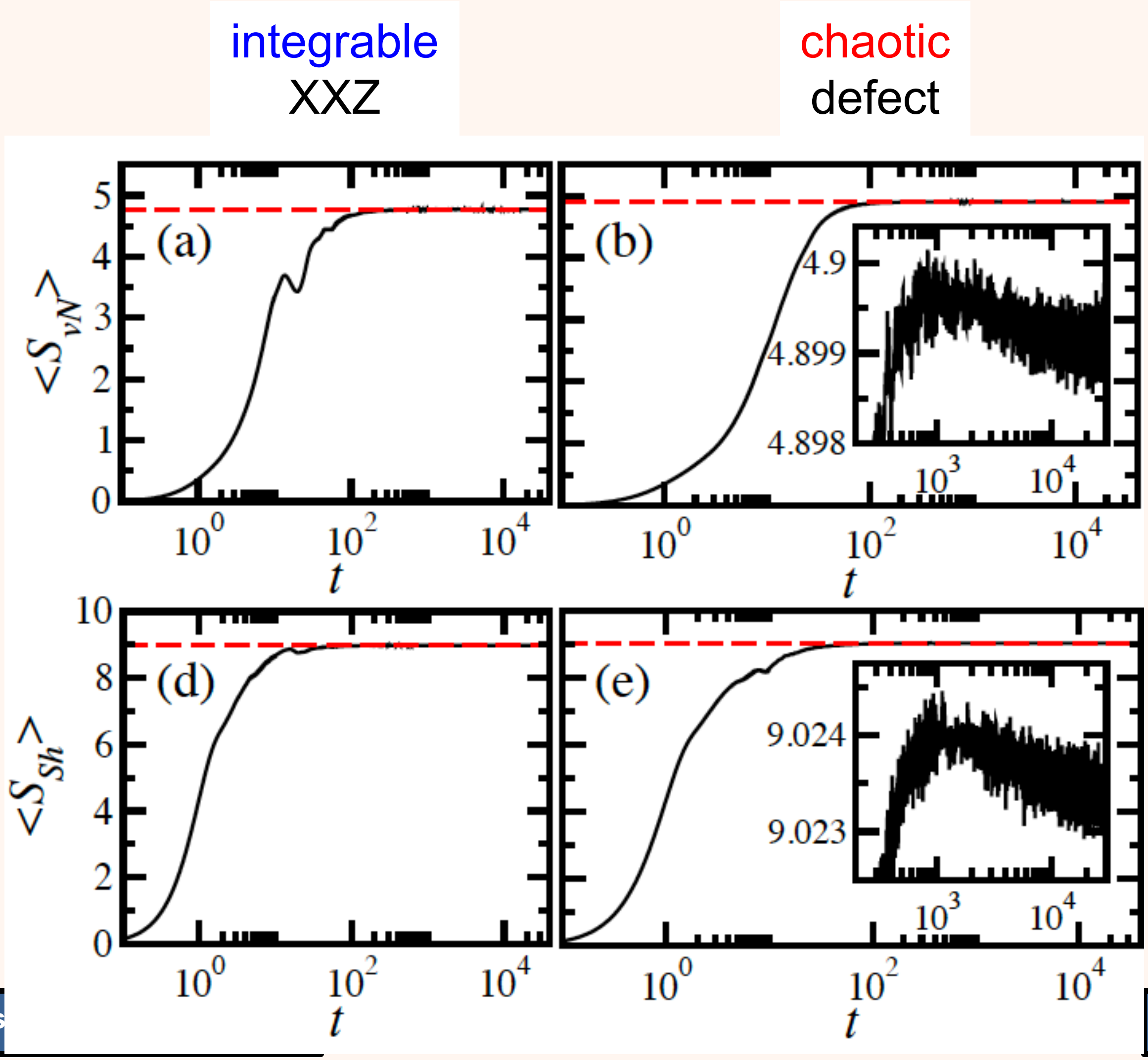
$PR_q^{(\alpha)} \propto Dim^{(q-1)D_q}$

Multifractality = nonlinear dependence of the generalized dimension on q

Torres & LFS
Ann. Phys. (2017)



Correlation "bulge"



Torres & LFS
Philos. Trans. A (2017)

Summary

- Exponential/Gaussian decays appear in integrable and chaotic models.
  indicates **delocalized** initial states.
  determined by the **shape** and width of the LDOS.

- Power-law decay at longer times captures the **filling** of the LDOS.
  caused by energy **bounds** or **correlations**.
  A criterion to anticipate **thermalization** from the dynamics.

- **Correlation hole** emerges before saturation.
  is an unambiguous signature of **level repulsion**.
  is an indicator of the chaos-integrable transition.
  is an indicator of the delocalized-localized transition.

- **Analytical expressions** from full random matrices serve as bounds and references for the analysis of realistic models.