Counter-diabatic driving in many-particle systems

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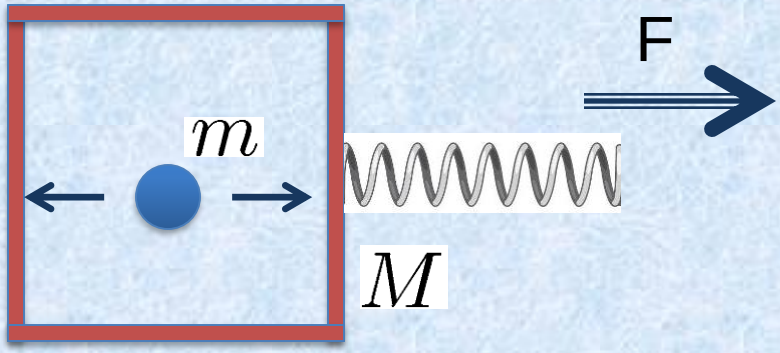
Non-adiabatic response and quantum geometry

Counter-diabatic (CD) driving, fast forward protocols, and quantum speed limit

Variational approach to finding approximate CD driving protocols in complex systems

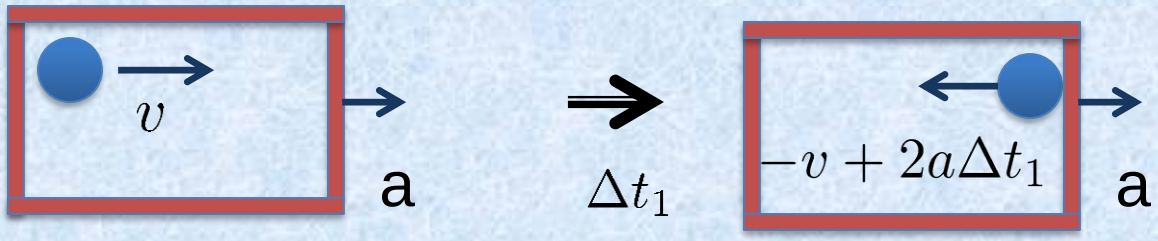
Mass as a non-adiabatic response?



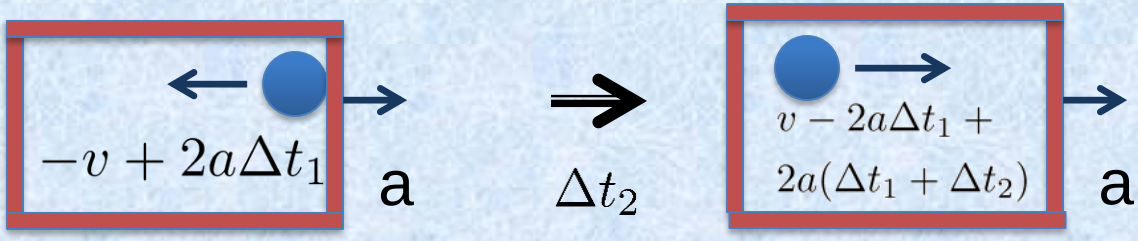
Take container and start slowly accelerating it to velocity V.

Compute the force (or work).

Assume particle is fast compared to the container



$$\Delta p_1 = -2mv + 2a\Delta t_1$$

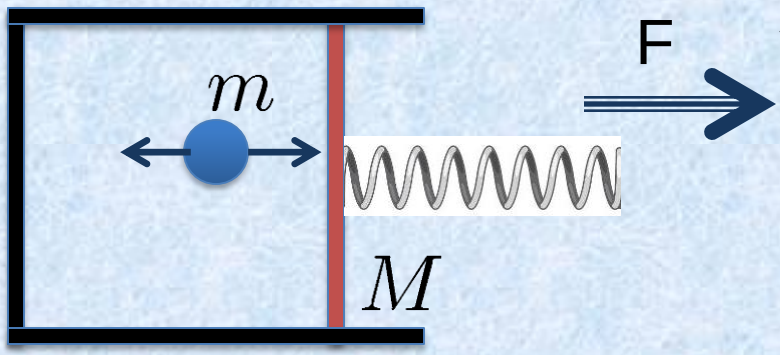


$$\Delta p_2 = 2mv$$
$$+2a(\Delta t_2 - \Delta t_1)$$

$$F = \frac{\Delta p_1 + \Delta p_2}{\Delta t_1 + \Delta t_2} = \frac{2ma\Delta t_2}{\Delta t_1 + \Delta t_2} \approx ma$$

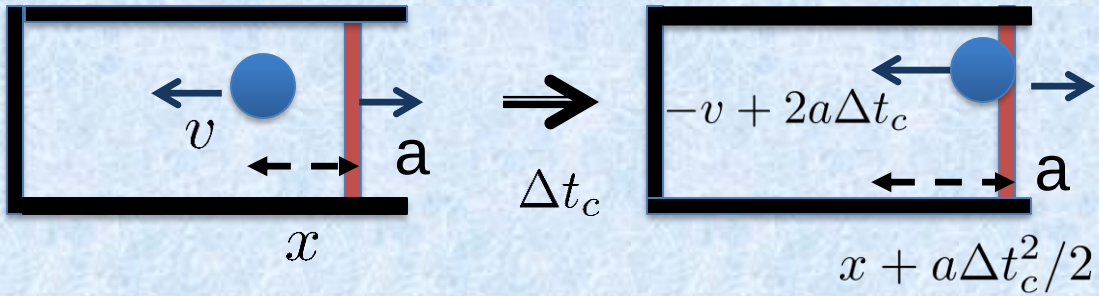
Only valid near the adiabatic limit, where $\Delta t_1 \approx \Delta t_2$

Slightly more complicated setup. Move one wall.



Accelerate one wall

Compute the force under the same assumption.



$$\begin{aligned}\Delta t_c(x) &= \frac{2L-x}{v} + a\frac{\Delta t_c^2}{2v} \\ &\approx \frac{2L-x}{v} + \frac{a(2L-x)^2}{2v^3}\end{aligned}$$

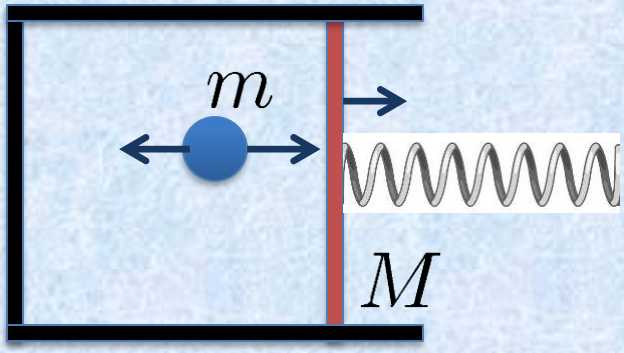
$$T(x) = \Delta t_c + \frac{x + a\Delta t_c^2/2}{v - 2a\Delta t_c} \approx \frac{2L}{v}\left[1 + \frac{a}{2Lv^2}(4L^2 - x^2)\right]$$

$$\Delta p(x) = 2mv - 2ma\Delta t_c \approx 2mv\left[1 - \frac{a(2L-x)}{v^2}\right]$$

$$F(x) = \frac{\Delta p(x)}{T(x)} \approx -\frac{mv^2}{L}\left[1 - \frac{4aL}{v^2} + \frac{ax}{v^2} + \frac{ax^2}{2Lv^2}\right]$$

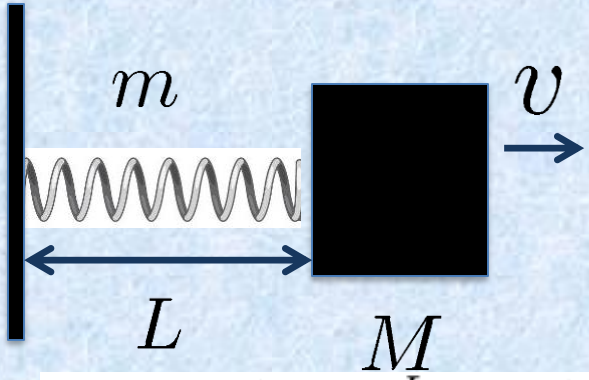
$$\overline{F} = \frac{1}{2L}\int_0^{2L} F(x)dx = -\frac{mv^2}{L} + \frac{ma}{3}$$

The wall gets mass correction from the particle: $m/3$



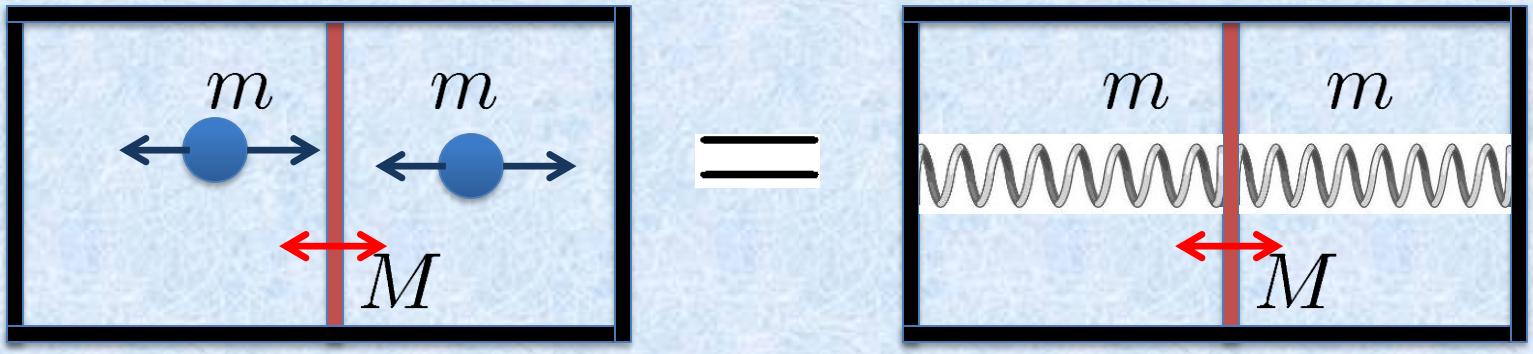
$$M \to M + \frac{m}{3}$$

This is the result for the mass of the elastic band or a spring



Massive (not massless) spring.
Compute kinetic energy

$$K = \frac{Mv^2}{2} + \int_0^L dx \frac{\rho v^2(x)}{2} = \frac{Mv^2}{2} + \frac{\rho v^2}{2} \int_0^L dx \left(\frac{x}{L}\right)^2 = \frac{Mv^2}{2} + \frac{1}{3}\frac{mv^2}{2}$$



Can we understand this more formally.
Use Quantum Mechanics

Consider an arbitrary unitary transformation of the wave function

$$|\psi(\vec{\lambda})\rangle = U^{\dagger}(\vec{\lambda})|\Psi\rangle$$

Infinitesimal transformations are like the Schrödinger equation

$$i\hbar\partial_{\lambda_\alpha}|\psi(\vec{\lambda})\rangle = -\mathcal{A}_\alpha|\psi\rangle,\ \mathcal{A}_\alpha = i\hbar U^{\dagger}\partial_{\lambda_\alpha}U,\ \mathcal{A}_\alpha^{\dagger} = \mathcal{A}_\alpha$$

Hamiltonian equations of motion in a moving frame

$$i\partial_t|\psi\rangle = (U^{\dagger}HU - \dot{\lambda}_a\mathcal{A}_a)|\psi\rangle$$

Special instantaneous frame, where U diagonalizes the instantaneous Hamiltonian. Convenient near the adiabatic limit

$$|\psi\rangle = |\psi_0(\vec{\lambda})\rangle \ \Rightarrow\ A_\alpha = \langle\mathcal{A}_\alpha\rangle$$

Berry connection is the expectation value of the gauge potential

Moving frame Hamiltonian and connections to geometry

$$\mathcal{H}_m = \mathcal{H} - \dot{\lambda}_\alpha \mathcal{A}_\alpha$$

Only the gauge potential term is responsible for transitions

Gauge potential and the Berry connection

$$\langle n_0|\mathcal{A}_\lambda|m_0\rangle = i\hbar\langle n_0|U^\dagger \partial_\lambda U|m_0\rangle = i\hbar\langle n(\lambda)|\partial_\lambda|m(\lambda)\rangle$$

$$\mathcal{A}_\lambda = i\hbar\partial_\lambda \quad \langle 0|\mathcal{A}_\lambda|0\rangle = A_\lambda$$

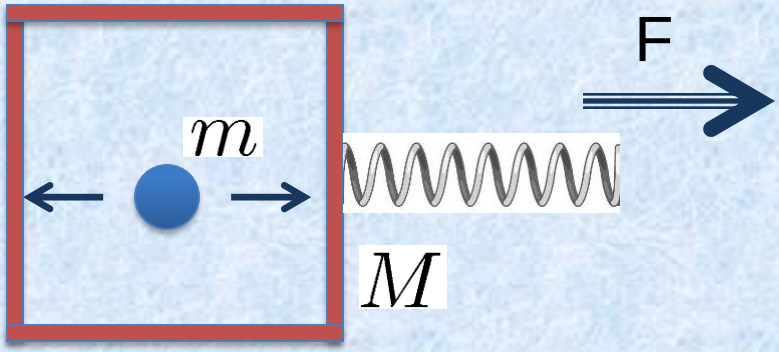
$$F_{\alpha\beta} = -i\langle 0|[\mathcal{A}_\alpha, \mathcal{A}_\beta]|0\rangle = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

Berry curvature.

$$g_{\alpha\beta} = \frac{1}{2}(\langle \partial_\alpha\psi|\partial_\beta\psi\rangle_c + \langle \partial_\beta\psi|\partial_\alpha\psi\rangle_c) = \frac{1}{2}\langle 0|\mathcal{A}_\alpha\mathcal{A}_\beta + \mathcal{A}_\beta\mathcal{A}_\alpha|0\rangle_c$$

Metric tensor. Defines the Riemannian metric structure on the ground state manifold. Can be used to geometrically classify phases and phase transitions (M. Kolodrubetz, V. Gritzev, A. P., 2013)

Example: Galilean Transformation



$$H = \frac{p^2}{2m} + V(x - X_0(t))$$

$$U = \exp\left[-\frac{i}{\hbar} p X_0(t)\right]$$

$$U^\dagger H U = \frac{p^2}{2m} + V(x)$$

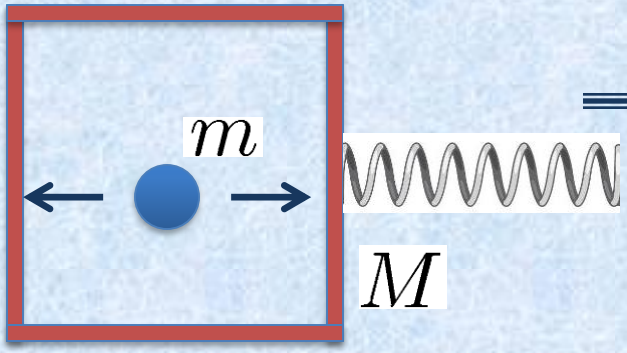
$$\mathcal{A}_{X_0} = i\hbar U^\dagger \partial_{X_0} U = p$$

Moving frame Hamiltonian

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = H - \dot{X}_0 p$$

Recover Galilean transformation

Idea: use perturbation theory with respect to the Galilean term



F

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = H - \dot{X}_0 p$$

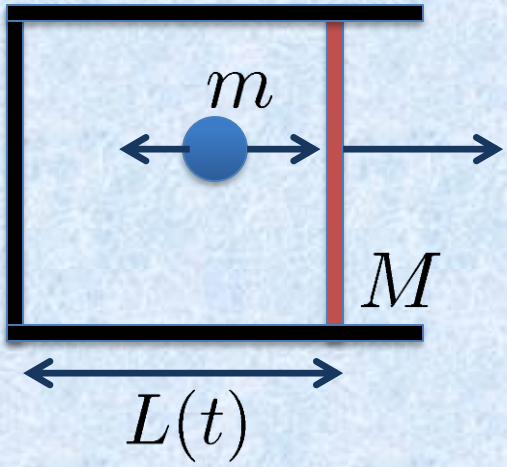
Compute leading correction to the energy due to the Galilean term (consider the ground state)

$$\Delta E_1 = \langle 0| - \dot{X}_0 p|0\rangle = 0$$

$$\Delta E_2 = \dot{X}_0^2 \sum_{n\neq 0} \frac{\langle 0|p|n\rangle\langle n|p|0\rangle}{E_n - E_0} = \dot{X}_0^2 \hbar^2 \sum_{n\neq 0} \frac{\langle n|\partial_X|0\rangle^2}{(E_n - E_0)} = m\dot{X}_0^2 \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{16n^2}{(4n^2-1)^3} = \frac{m\dot{X}_0^2}{2}$$

Recover the mass term as the leading non-adiabatic correction to the energy.

Can recover the classical Hamiltonian equations of motion from the nonadiabatic response (can set bare mass M to zero).



$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi n x}{L}$$

$$\partial_L \phi_n(x) = -\frac{1}{2L}\phi_n(x) - \sqrt{\frac{2}{L}} \frac{\pi n x}{L^2} \cos \frac{\pi n x}{L} = \mathcal{D}\phi_n(x)$$

$$\mathcal{D} = \frac{px + xp}{2L}$$

Dilation operator

Moving frame

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = \frac{p^2}{2mL^2(t)} - \dot{L}\mathcal{D}$$

Can absorb L^2 into time dilatation:

$$dt = L^2 d\tau, \quad H \to L^2 H$$

Leading non-adiabatic correction.

$$\Delta E_2 = \dot{L}^2 \sum_{m \neq n} \frac{\langle n|\mathcal{D}|m\rangle\langle m|\mathcal{D}|n\rangle}{E_m - E_n} = m\dot{L}^2 \frac{16}{\pi^2} \sum_{m \neq m} \frac{n^2 m^2}{(m^2 - n^2)^3} = \frac{m\dot{X}_0^2}{2}\left(\frac{1}{3} - \frac{1}{2\pi^2 n^2}\right)$$

Recover “quantum” dilatation mass: the classical result plus a quantum correction. Mass is related to the quantum geometric tensor.

Quantum Coriolis force through non-adiabatic response

(M. Schroer, M. Kindel, … K. Lenhert, M. Kolodrubetz, A.P. PRL 2014

$$\vec{\lambda} = (h_\theta, h_\phi), \quad M_\phi = \langle -\partial_\phi H \rangle = \frac{\sin(\theta)}{2}\langle \sigma_y \rangle = F_{\phi\theta}(\theta,\phi)\dot{\theta} + O(\dot{\theta}^2)$$

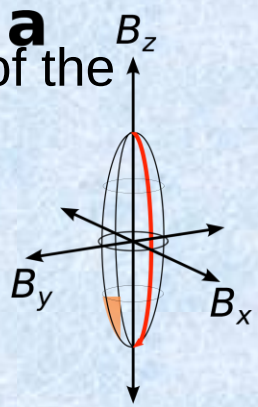
$$H_{\text{eff}} = -\frac{\epsilon}{2}\sigma^z - \frac{\Omega}{2}\sigma^x$$

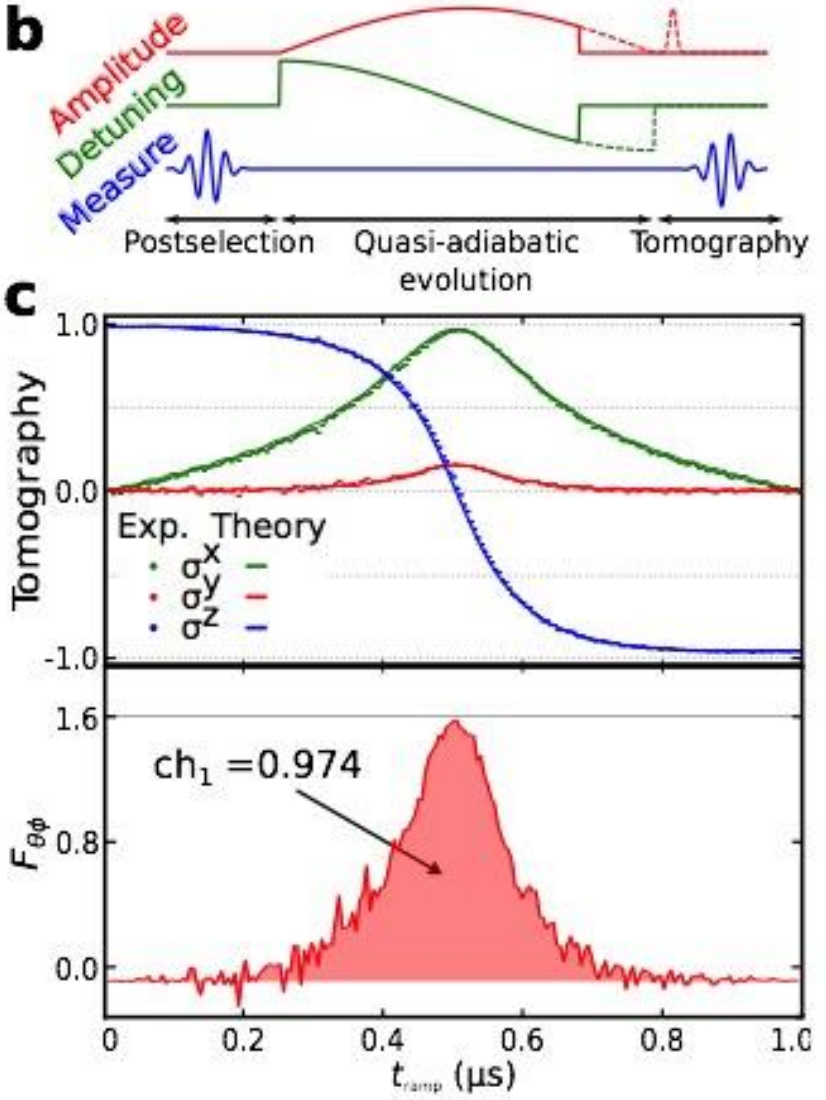
1) Preparation in the (up) ground state at zero detuning

2) Fast ramp of the frequency and slow ramp of the amplitude of the pulse

3) Sudden switching off of the amplitude

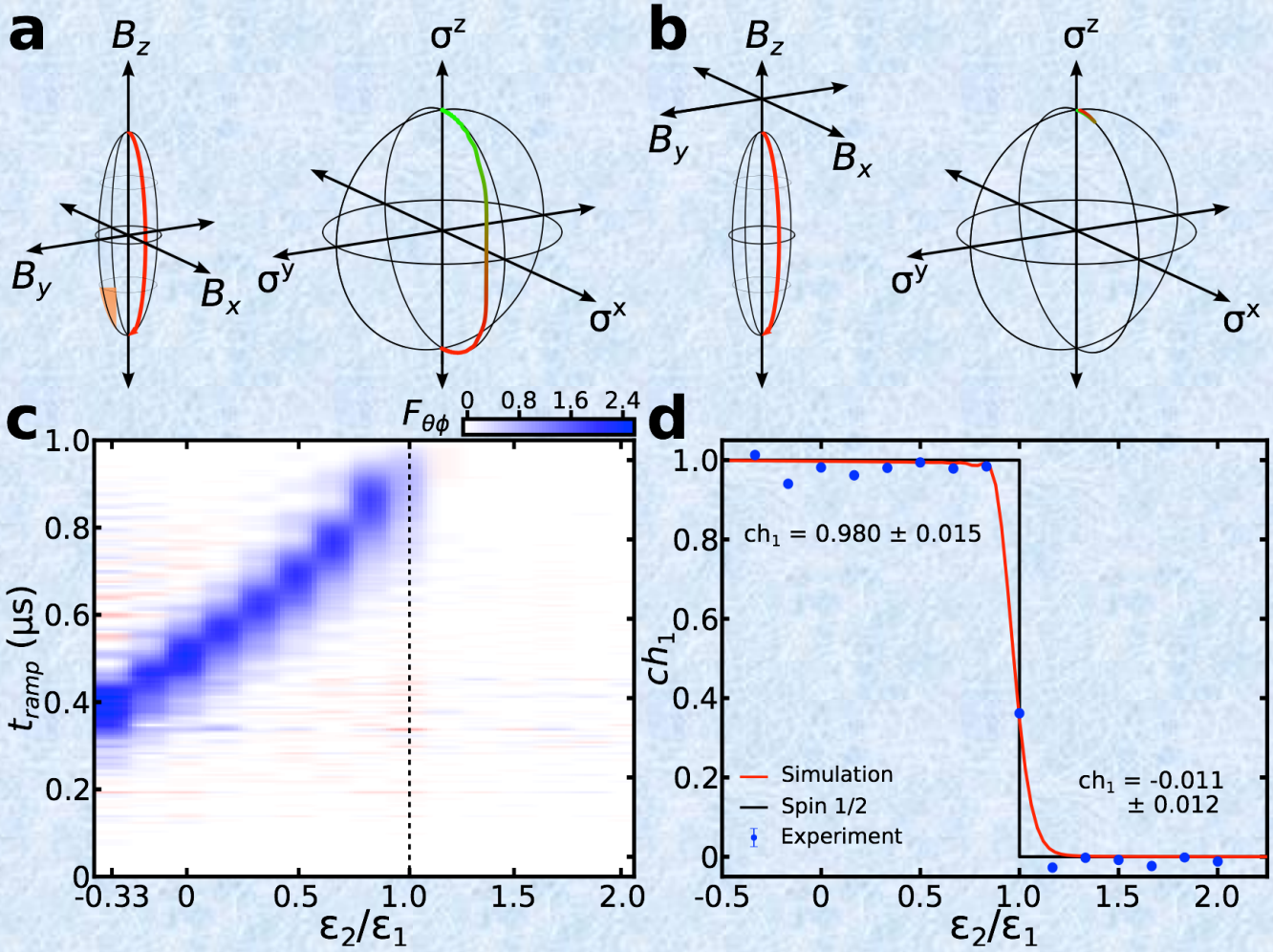
4) Tomography





$$H = -\frac{1}{2}\left[\epsilon\sigma^z + \Omega\sigma^x\cos(\phi) + \Omega\sigma^y\sin(\phi)\right]$$

$$\epsilon = \epsilon_1\cos(\theta) + \epsilon_2,\ \Omega = \Omega_1\sin(\theta)$$



Related 2 coupled qubits experiments with a richer topological phase diagram (P. Rouchan, …, J. Martinis 2014)

Beyond adiabatic response. Shortcuts to adiabaticity.

(M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+), also Bloch-Siegert shifts in NMR, fast-forward technique,…).

Moving frame Hamiltonian

$$\tilde{H}_m = \tilde{H} - \dot{\lambda}\tilde{\mathcal{A}}_\lambda, \quad \tilde{H} = \mathrm{diag}(E_1, E_2, \ldots)$$

Idea: introduce counter-adiabatic (CA) term

$$\tilde{H} \to \tilde{H} + \dot{\lambda}\tilde{\mathcal{A}}_\lambda \quad \tilde{H}_m = \tilde{H}$$

Moving frame follow eigenstates of $\tilde{H}$. Back to the lab frame:

$$H_{CD} = H + \dot{\lambda}\mathcal{A}_\lambda$$

CD driving intuitively:

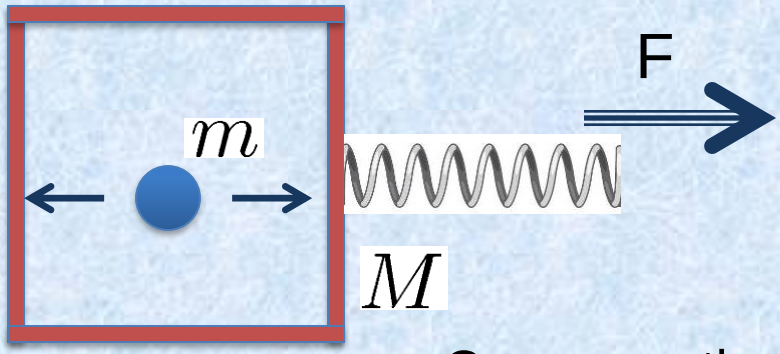
- Have to introduce extra parameters
- Do not follow instantaneous ground state
- Use only local (physical) counter terms, i.e. do not address individual water molecules

No CD term

CD term

Beyond adiabatic response. Shortcuts to adiabaticity.

(M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+)



$$H = \frac{p^2}{2m} + V(x - X_0(t))$$

Suppose we want to move a box in space without exciting a particle inside (without heating).

Can move the box slowly but it takes time. If move too slow will likely decohere due to a bath.

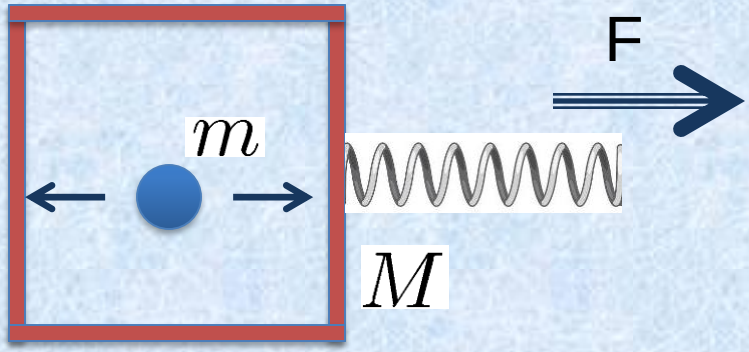
Recall a moving Hamiltonian

$$\tilde{H} = U^\dagger H U - \dot{X}_0 \mathcal{A}_{X_0} = \frac{p^2}{2m} + V(x) - \dot{X}_0 p$$

Can compensate the last term by adding the counter term

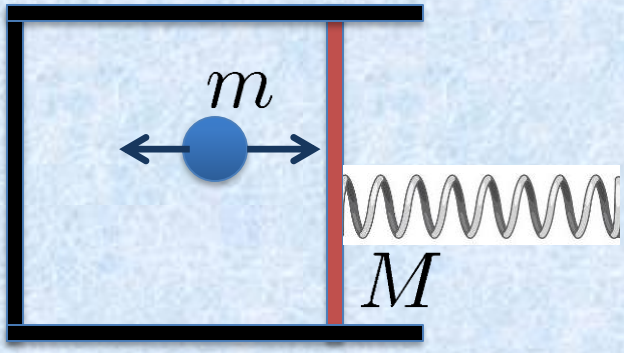
$$H \to H + \dot{X}_0 p$$

The moving frame the Hamiltonian is diagonal (time-independent). Can move arbitrarily fast. This is not what the waiter does!



$$
\begin{aligned}
H_{\rm CD} &= \frac{p^2}{2m} + V(x - X_0) + \dot{X}_0\, p \\
&= \frac{(p + m\dot{X}_0)^2}{2m} + V(x - X_0) - \frac{m\dot{X}_0^2}{2} \\
\sim H_{\rm FF} &= \frac{p^2}{2m} + V(x - X_0) - mx\ddot{X}_0.
\end{aligned}
$$

Fast forward (FF) protocol – uses a linear potential proportional to the acceleration (gravitational field) of the control field. FF protocol is gauge equivalent to CD protocol.



$$\mathcal{D}: \quad x \to Lx,\ p \to p/L$$

$$
\begin{aligned}
H_{\rm CD} &= \frac{p^2}{2m} + \frac{1}{L^2} V(x/L) + \dot{L}\,\frac{px + xp}{2L} \\
\sim H_{\rm FF} &= \frac{p^2}{2m} + \frac{1}{L^2} V(x/L) - \frac{mx^2\ddot{L}}{L}.
\end{aligned}
$$

CD (FF) term is a harmonic potential (Deffner, Jarzynski, Del Campo 2014). Interestingly this problem has a horizon.

$$x = qL,\ p = \eta/L,\ H = \frac{1}{L^2}\left(\frac{p^2}{2m} + V(\xi)\right) \quad \Rightarrow \quad \tau = \int \frac{dt}{L^2(t)}$$

Adiabatic loading in LZ problem (quantum control problem)

$$H = \Delta\sigma_z + h(t)\sigma_x$$

Standard route: go slowly,
especially near the gap minimum

$$p_{\rm ex} \sim \exp\left[-\frac{\Delta^2}{|\dot{h}|}\right]$$

CD route: introduce counter term

$$\mathcal{A}_h = \frac{1}{2}\frac{h}{h^2+\Delta^2}\sigma_y, \quad H_{CD} = \Delta\sigma_z + h(t)\sigma_x + \frac{1}{2}\frac{\Delta\dot{h}}{h^2+\Delta^2}\sigma_y$$

Can perform time dependent rotation around z-axis to map to gauge equivalent fast forward Hamiltonian:

$$H_{\rm FF} = \Delta\sigma_z + h'(t)\sigma_x$$

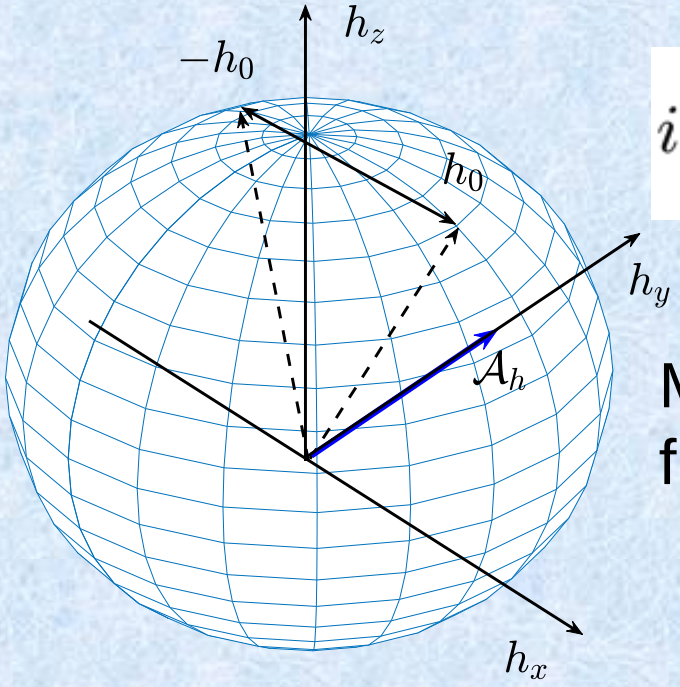
CD driving and the quantum speed limit

$$i\partial_t\psi = H\psi = ||H||\frac{H}{||H||}\psi$$

Can redefine time $$d\tau = dt||H||, \quad \tau = \int d\tau = \int_0^{t_f} dt||H||$$

The problem of minimizing τ *is well defined*

$$i\frac{d\psi}{dt} = i\dot{\lambda}\frac{d\psi}{d\lambda} = (H + \dot{\lambda}\mathcal{A}_\lambda)\psi \qquad i\frac{d\psi}{d\lambda} = \left(\frac{H}{\dot{\lambda}} + \mathcal{A}_\lambda\right)\psi$$



$$i\frac{d\psi}{dh} = \left[\frac{\Delta\sigma_z + h\sigma_x}{\dot{h}} + \frac{1}{2}\frac{\Delta}{h^2+\Delta^2}\sigma_y\right]|\psi\rangle$$

Minimal possible time corresponds to the free rotation around the y-axis.

Minimal CD time is given by the geodesic length (if we use the ground state norm).

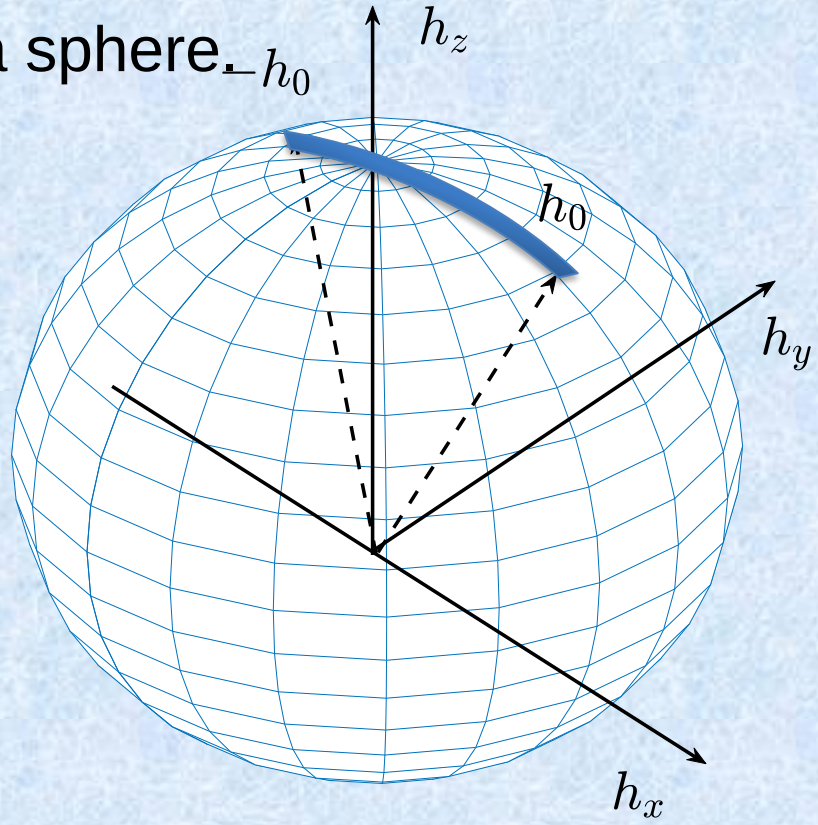
$$\tau_{\min} = \int_{\lambda_i}^{\lambda_f} d\lambda ||\mathcal{A}|| = \int_{\lambda_i}^{\lambda_f} d\lambda \sqrt{\langle \mathcal{A}_\lambda^2 \rangle_c}$$

$$= \int_{\lambda_i}^{\lambda_f} d\lambda \, g_{\lambda\lambda} = \frac{1}{2} \int_{-h_0}^{h_0} dh \, \frac{\Delta}{h^2 + \Delta^2} = \arctan \frac{\Delta}{h}$$

Minimum time = geodesic length on a sphere.

Infinite speed limit

$$i \frac{d\psi}{d\lambda} = \mathcal{A}_\lambda \psi$$



Gauge potential becomes the Hamiltonian and coupling is time.

This is the fastest possible protocol with the geodesic length defining the speed limit (could be exponentially long in glassy systems or if target excited states).

Many-particle example

$$H = -J\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \sum_j V(j,\lambda) c_j^\dagger c_j$$

$$H_{CD} = H + \dot{\lambda}\mathcal{A}_\lambda$$

Mathematically $\mathcal{A}_\lambda$ is the connection, must satisfy the equation for parallel transport

$$[D_\lambda H, H] \equiv [\partial_\lambda H + i[\mathcal{A}_\lambda, H], H] = 0$$

$$\mathcal{A}_\lambda = i\sum_{i\neq j} \alpha_{ij} c_i^\dagger c_j, \quad \alpha_{ij} = -\alpha_{ji}$$

Gauge potential is imaginary, in general long range, hopping

Exact solution for a constant electric field $V_j = \lambda \sum_j j c_j^\dagger c_j$

$$\mathcal{A}_\lambda = -i\frac{J}{\lambda^2}\sum_j (c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$$

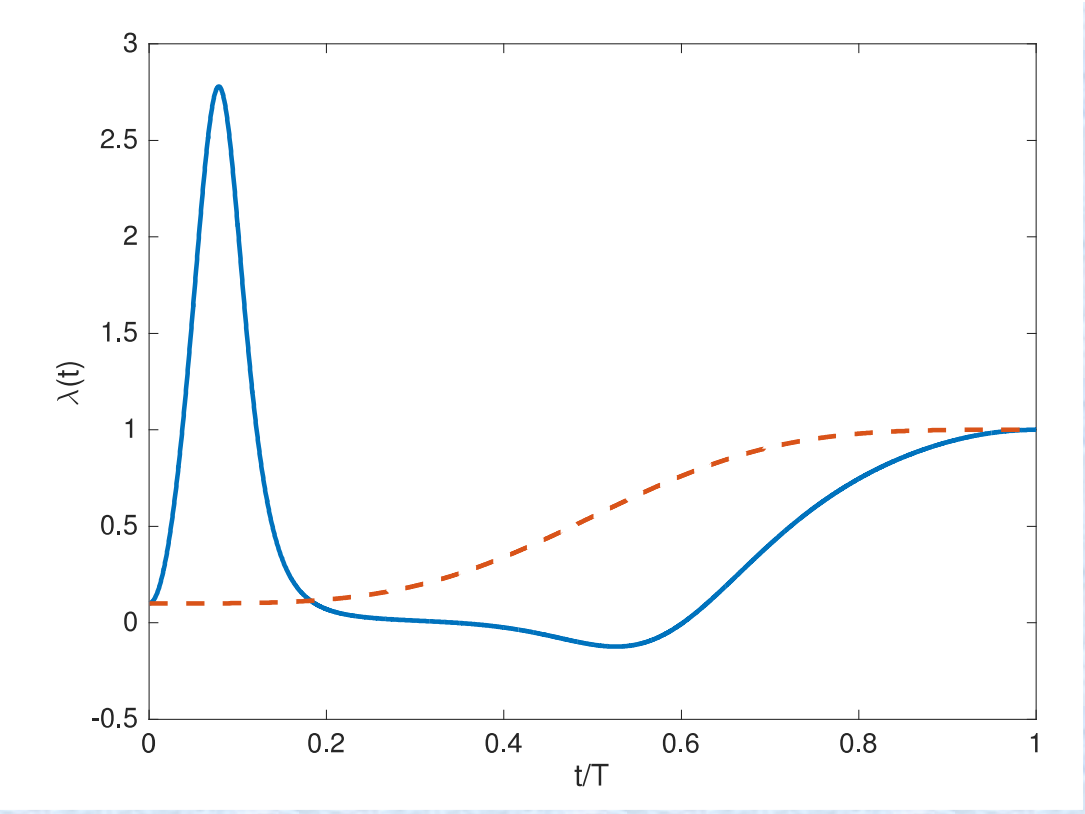
CD term is the current operator

Counter-diabatic Hamiltonian (set J=1)

$$H_{CD} = -\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \lambda \sum_j j c_j^\dagger c_j - \frac{i\dot{\lambda}}{\lambda^2} \sum_j (c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$$

Can eliminate complex hopping by the gauge (Pierls) transformation and get fast forward Hamiltonian

$$H_{\mathrm{FF}} = -\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \frac{\lambda(t)}{\sqrt{1+\dot{\mu}^2}} \left(1 - \frac{\mu\ddot{\mu}}{1+\dot{\mu}^2}\right) \sum_j j c_j^\dagger c_j, \quad \mu = 1/\lambda$$



Gauge potential from the variational principle

$$[\partial_\lambda H + i[\mathcal{A}_\lambda, H], H] = 0$$

$$G_\lambda(\mathcal{X}) = \partial_\lambda H + i[\mathcal{X}, H] \qquad \mathcal{D}(\mathcal{X}) = \mathrm{Tr}(G_\lambda^2)$$

Equivalent to the minimization problem (Euler-Lagrange equations)

$$\frac{\delta \mathcal{D}(\mathcal{X})}{\delta \mathcal{X}} = 0 \;\Rightarrow\; \mathcal{A}_\lambda = \mathcal{X}$$

Back to the spin example

$$H = -h\cos\theta\sigma_z - h\sin\theta\sigma_x, \qquad \mathcal{X} = x\sigma_y$$

$$G_\theta(x) = (h\sin\theta - 2xh\sin\theta)\sigma_z - (h\cos\theta - 2xh\cos\theta)\sigma_x$$

$$\mathcal{D}(x) = 4h^2(1-2x)^2, \qquad x_{\min} = \frac{1}{2}, \;\Rightarrow\; \mathcal{A}_\theta = \frac{1}{2}\sigma_y$$

Two coupled spins (qubits)

$$H = -h\cos\theta(\sigma_1^z + \sigma_2^z) - h\sin\theta(\sigma_1^x + \sigma_2^x) - J\sigma_1^z\sigma_2^z$$

Try a non-interacting trial gauge potential

$$\mathcal{X} = \frac{x}{2}(\sigma_1^y + \sigma_2^y).$$

$$G_\theta(x) = \partial_\theta\mathcal{H} + i[\mathcal{X},\mathcal{H}] = h\sin\theta(1-x)(\sigma_1^z+\sigma_2^z) - h\cos\theta(1-x)(\sigma_1^x+\sigma_2^x) + xJ_z(\sigma_1^x\sigma_2^z+\sigma_1^z\sigma_2^x).$$

$$\frac{\mathcal{D}(x)}{4} = 2h^2(1-x)^2 + 2x^2J_z^2 \;\Rightarrow\; x_{\min} = \frac{h^2}{h^2+J^2} \Rightarrow \mathcal{A}_\theta \approx \frac{1}{2}\frac{h^2}{h^2+J^2}(\sigma_1^y+\sigma_2^y)$$

Enlarge the variational ansatz adding two spin terms

$$\mathcal{X} = \frac{\alpha}{2}(\sigma_1^y+\sigma_2^y) + \frac{\beta}{2}(\sigma_1^y\sigma_2^x+\sigma_1^x\sigma_2^y) + \frac{\gamma}{2}(\sigma_1^y\sigma_2^z+\sigma_1^z\sigma_2^y)$$

$$\begin{aligned} G(\mathcal{X}) &= h\sin\theta(1-\alpha)(\sigma_1^z+\sigma_2^z) + (\gamma J_z - h\cos\theta(1-\alpha))(\sigma_1^x+\sigma_2^x) \\ &+ (\alpha J_z - \beta h\sin\theta + \gamma h\cos\theta)(\sigma_1^x\sigma_2^z+\sigma_1^z\sigma_2^x) \\ &+ 2\beta h\cos\theta\,\sigma_1^x\sigma_2^x - 2\gamma h\sin\theta\,\sigma_1^z\sigma_2^z - 2h(\beta\cos\theta - \gamma\sin\theta)\sigma_1^y\sigma_2^y. \end{aligned}$$

Finding the gauge potential is equivalent to solving a 3x3 linear equations

$$\begin{pmatrix} h^2+J_z^2 & -J_z h\sin\theta & 2J_z h\cos\theta \\ -J_z h\sin\theta & h^2(3\cos^2\theta+1) & -3h^2\cos\theta\sin\theta \\ 2J_z h\cos\theta & -3h^2\cos\theta\sin\theta & h^2(3\sin^2\theta+1)+J_z^2 \end{pmatrix} \begin{pmatrix} \alpha^* \\ \beta^* \\ \gamma^* \end{pmatrix} = \begin{pmatrix} h^2 \\ 0 \\ J_z h\cos\theta \end{pmatrix}$$

Find the exact gauge potential for two spins (approximate for many spins)

$$\mathcal{A}_\theta(\theta=0) = \frac{1}{2}\frac{h^2}{h^2-J_z^2}(\sigma_1^y+\sigma_2^y) - \frac{1}{2}\frac{J_z h}{h^2-J_z^2}(\sigma_1^y\sigma_2^z+\sigma_1^z\sigma_2^y).$$

Singularities indicate that one can not adiabatically cross exact degeneracies.

Free electrons in generic external potential

$$H = H_0 + \sum_j V_j(\lambda)(c_j^\dagger c_j + h.c.), \quad \mathcal{X} = i\sum_j \alpha_j(c_j^\dagger c_{j+1} + h.c.)$$

$$G_\lambda = \sum_j \left(\partial_\lambda V_j - 2J(\alpha_j - \alpha_{j-1})\right) c_j^\dagger c_j$$

$$+ J\sum_j (\alpha_j - \alpha_{j-1})(c_{j+1}^\dagger c_{j-1} + c_{j-1}^\dagger c_{j+1}) + \sum_j (V_{j+1} - V_j)\alpha_j (c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}).$$

Result of the minimization

$$-3\Delta\alpha + (\nabla_j V)^2 \alpha = \nabla_j(\partial_\lambda V)$$

Smooth potentials, continuum limit

$$-3\partial_x^2\alpha + (\partial_x V)^2\alpha = \partial_x(\partial_\lambda V)$$

This gauge potential defines the best local co-moving frame. Does not require diagonalization of the Hamiltonian. Maps quantum to classical problem

$$H_{\rm CD} = H_0 + \dot\lambda \mathcal{A}^*_\lambda = -J\sum_j \left[c^\dagger_{j+1} c_j \left(1 - i\frac{\alpha_j \dot\lambda}{J}\right) + c^\dagger_j c_{j+1}\left(1 + i\frac{\alpha_j\dot\lambda}{J}\right)\right] + \sum_j V_j(\lambda) c^\dagger_j c_j$$

Perform a phase (Pierls) transformation: $c_j \to c_j \mathrm{e}^{-i\phi_j}$

$$H_{\rm FF} = -\sum_i J_{eff}(i)\left(c^\dagger_{i+1}c_i + c^\dagger_i c_{i+1}\right) + \sum_i V_{eff}(i) c^\dagger_i c_i,$$

$$V_{eff}(x,t) = V(\lambda,x) - \ddot\lambda \int_a^x \frac{\alpha(x')}{1+\dot\lambda^2(\alpha(x'))^2}\mathrm{d}x', \quad J_{eff}(x,t) = \sqrt{1+\dot\lambda^2(\alpha(x))^2},$$

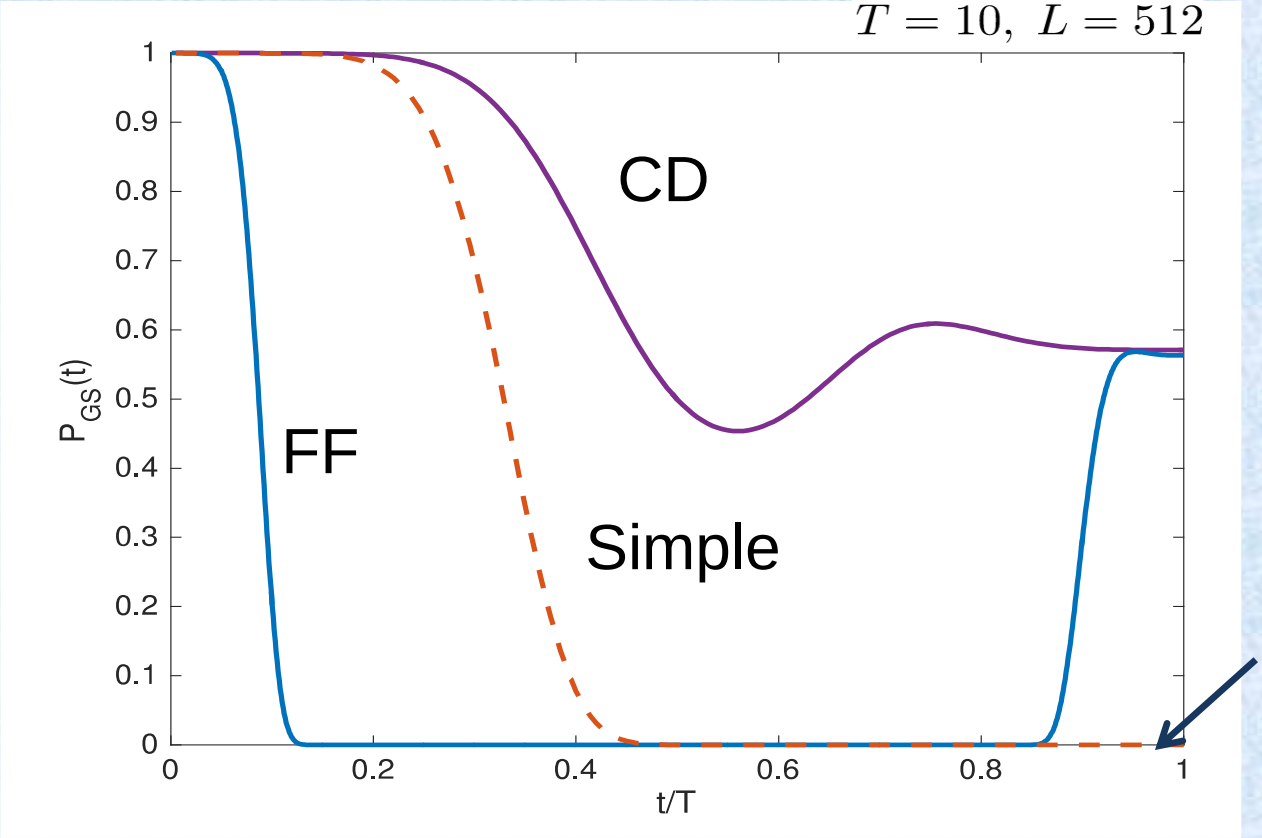
Imaginary CD protocol is only sensitive to velocity. Real CD (FF) protocol also knows about acceleration

Small velocity: potential renormalization (slowing particles in front)

Large velocity: need to locally renormalize hopping: = local time rescaling or local refraction index

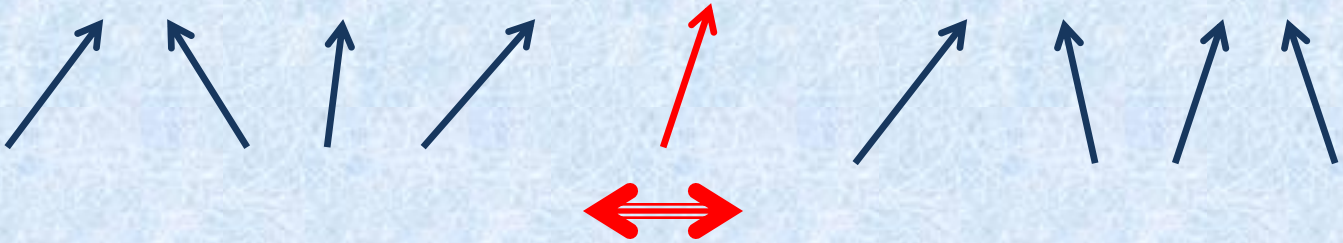
Example: inserting Eckart's potential (fighting Anderson orthogonality catastrophe). Half filling, 512 sites

$$V(j,\lambda) = \frac{\lambda(t)}{\cosh^2(j/\xi)} \qquad \lambda(t) = 2\sin^2\left(\frac{\pi}{2}\sin^2(\pi t/2T)\right)$$



Like throwing a stone into quantum water (gas) without generating ripples. Same ideas apply to quantum control and fighting dissipation.

Interacting spin system



Try to minimize white noise dissipation using CD driving.

$$H = \sum_j \sigma_z^j \sigma_z^{j+1} + 0.8\,\sigma_z^j + 0.9\,\sigma_x^j + h_x(t)\sigma_x^{L/2}$$

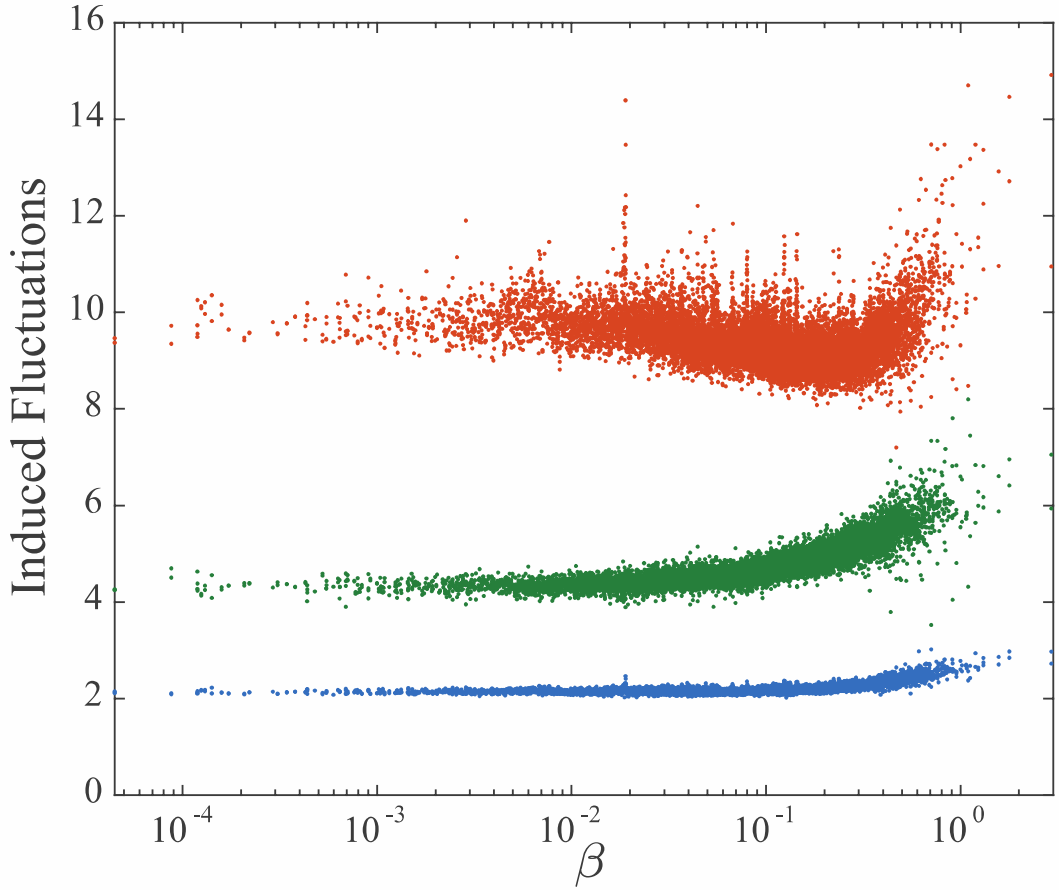
Gauge potential

$$\mathcal{A} = \alpha\sigma_y^{L/2} + \beta\sigma_y^{L/2}(\sigma_z^{L/2+1} + \sigma_z^{L/2-1}) + \gamma\sigma_y^{L/2}(\sigma_x^{L/2+1} + \sigma_x^{L/2-1}) + \dots$$

The expansion seems to quickly converge in gapped phases.

Good measure of dissipation – energy variance spread within FGR (works also for infinite temperature states)

White noise: $$\frac{d\delta E^2}{dt} \propto \langle n| \left([H_0, G]\right)^2 |n\rangle$$



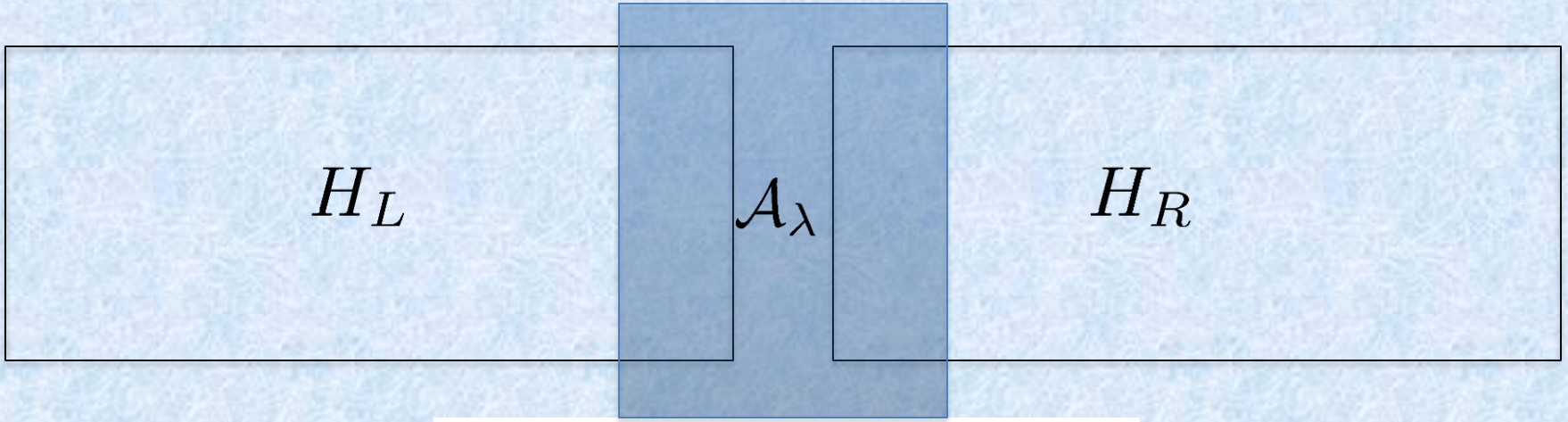
Thermodynamic limit protocol 15 site chain.

No CD protocol

Single site CD protocol

Two site CD protocol

Potential implementations for state preparation



$$H = H_L + H_R + \lambda H_{\text{int}}$$

Instead of adiabatic turning on fast CD driving

$$i\partial_\lambda \psi = \mathcal{A}_\lambda \psi$$

Gauge potential is similar to the entanglement Hamiltonian.

Will not resolve individual eigenstates but can potentially prepare good excited states

$$[\partial_\lambda H + i[\mathcal{A}_\lambda, H], H] = 0$$

Solving this equation defines flow equation method to disentangle the Hamiltonian (S. Kehrein, …)

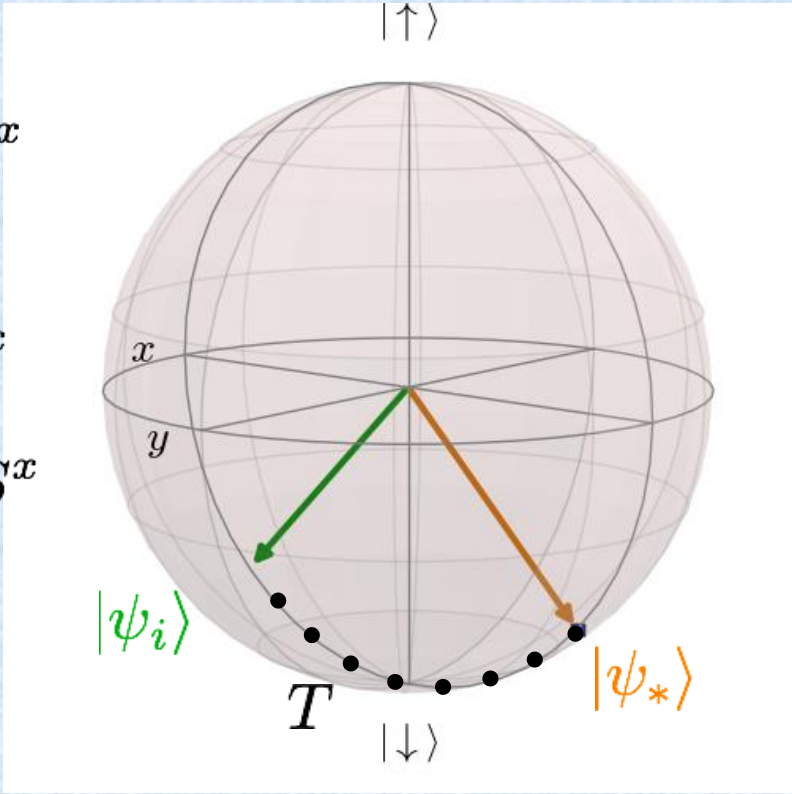
Quantum State Preparation Problem Using Reinforcement Learning (**M. Bukov**, A. Day, D. Sels, P. Weinberg, A.P., P. Mehta)

BU

→ Hamiltonian: $H(t) = S^z + h(t)S^x$

initial state: $|\psi_i\rangle$: GS of $H_i = S^z - S^x$

target state: $|\psi_*\rangle$: GS of $H_* = S^z + S^x$

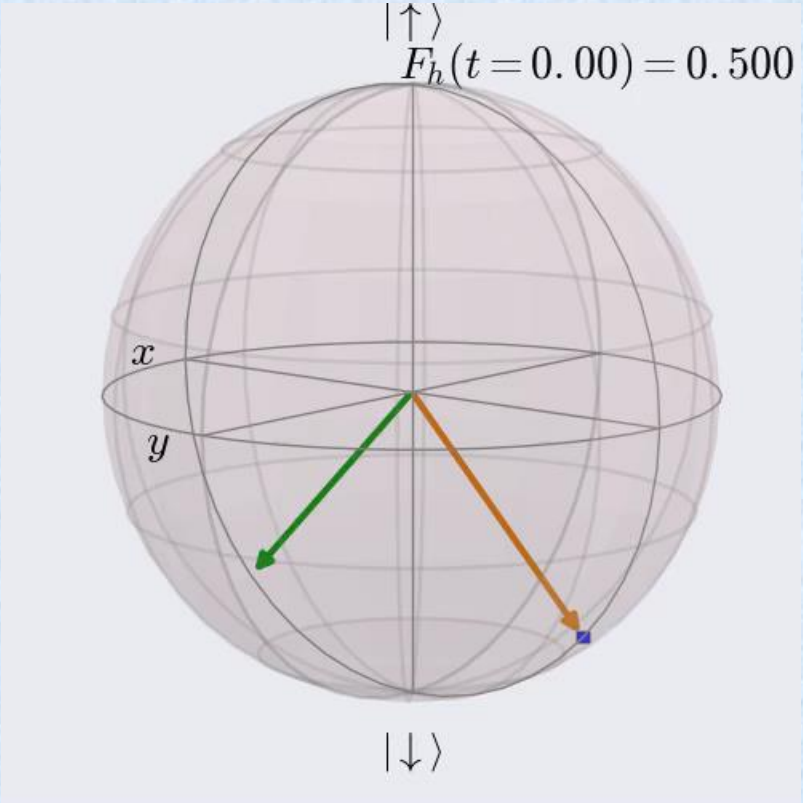


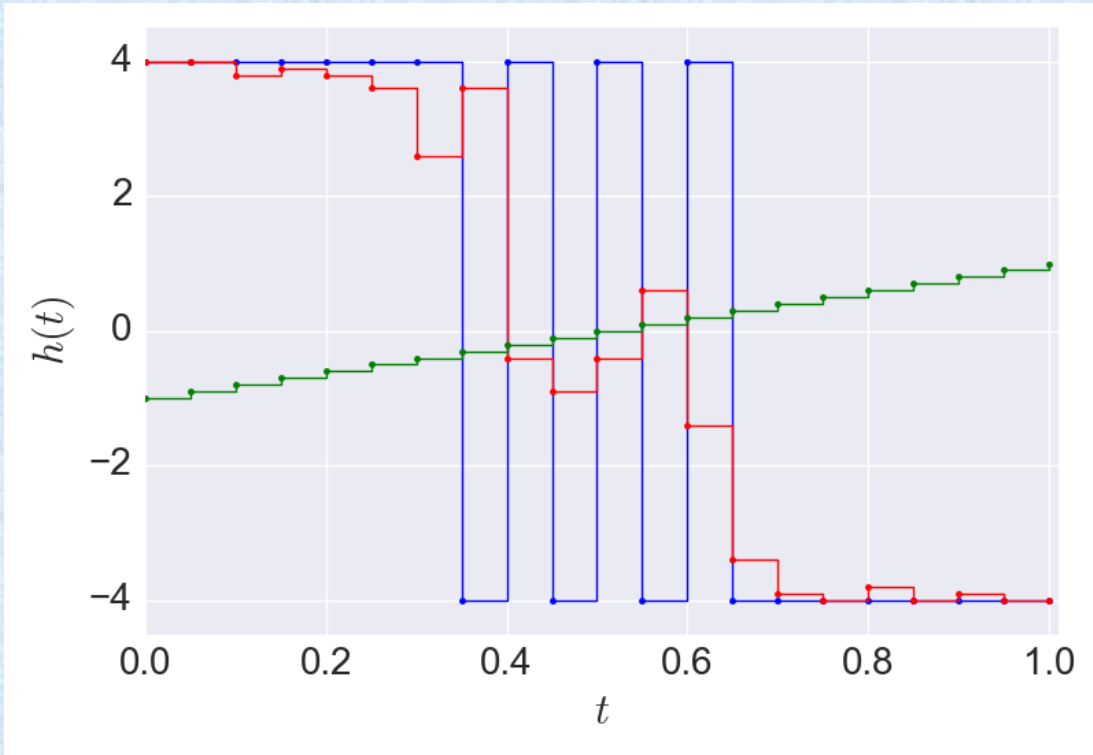
GOAL: find protocol $h(t),\ h(0) = -1,\ h(t) \in [-4, 4]$

such that $|\psi(t=0)\rangle = |\psi_i\rangle,\ |\psi(t=T)\rangle = |\psi_*\rangle$

measure: fidelity $F_h(T) = |\langle\psi(T)|\psi_*\rangle|^2$

$$1 = T < T_{\min} \approx 1.8, \quad \tau = \int_{-h_0}^{h_0} dh \sqrt{g_{hh}} \approx 1.6$$





At short times computer finds the optimal FF geodesic protocol (corresponding to the speed limit CD protocol)

Glass Transition in (ramp) Time

consider map $h(t) \mapsto -F_h(T)$

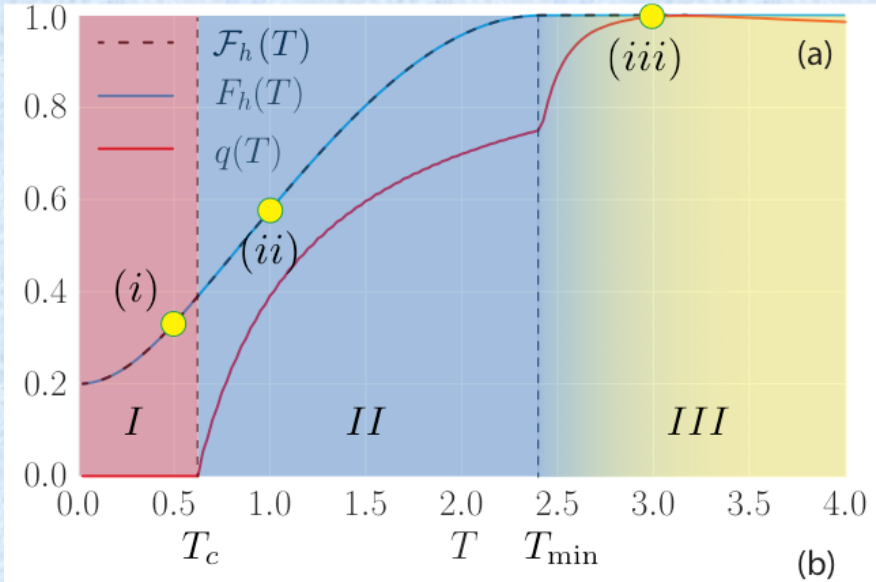
optimisation problem in fidelity landscape

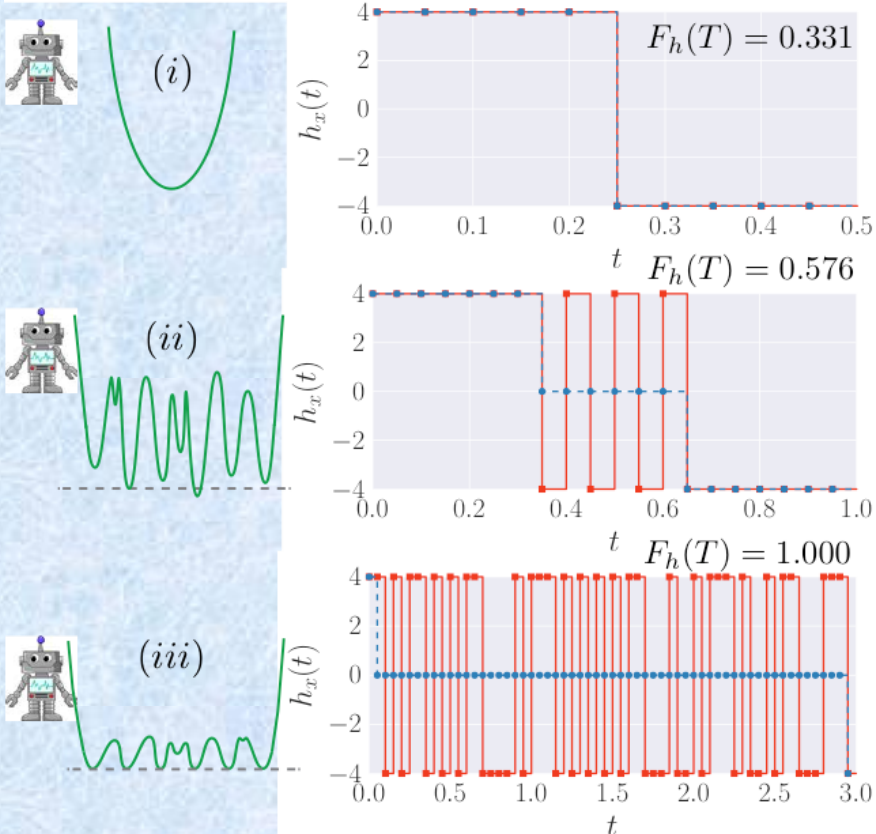
$$H(t) = S^z + h(t)S^x$$

bang-bang protocols

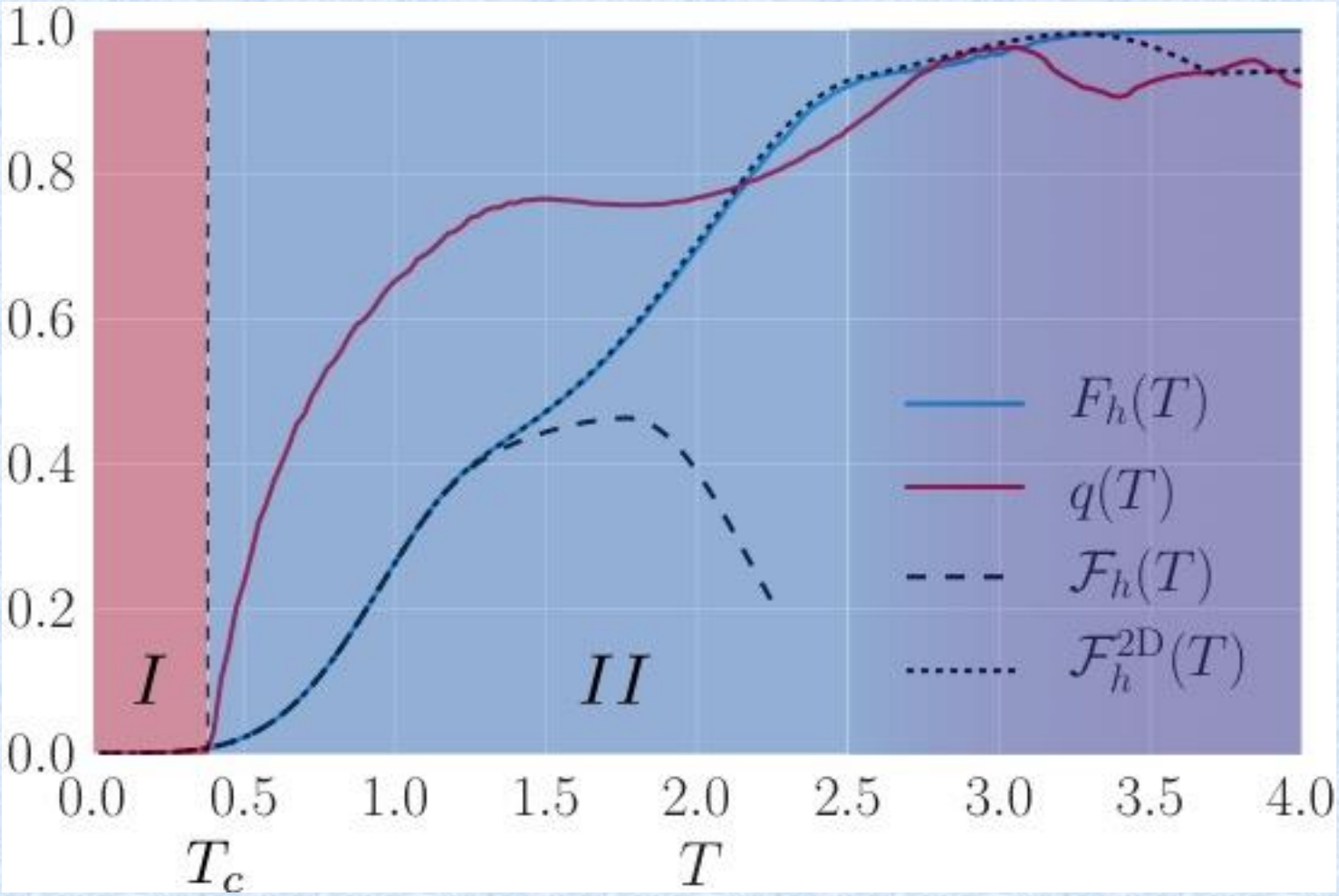
$$\bar{h}(t) = \frac{1}{\#_{\text{real}}} \sum_{\alpha} h^{\alpha}(t)$$

$$q_{EA}(T) \sim \overline{\sum_t \{h(t) - \bar{h}(t)\}^2}$$





Glassy phase survives multi-spin optimization.
Connection to FF and CD protocols is an open problem

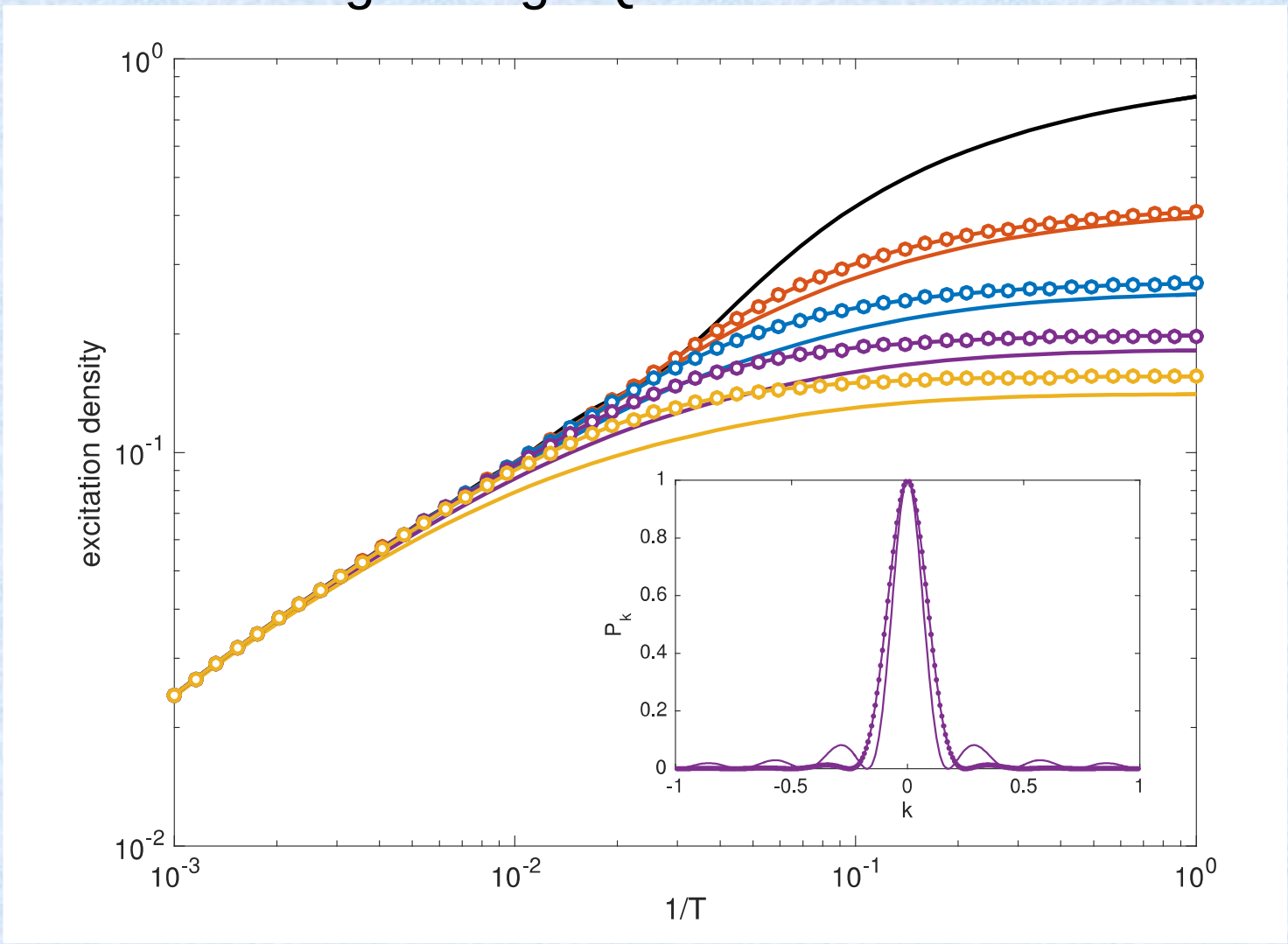


$$H = -\sum_{j=1}^{L} (S_z^j S_z^{j+1} + h_z S_z^j + h_x(t) S_x^j)$$

Summary

- Deep connections between non-adiabatic response and geometry.
- Counter-diabatic (and fast forward) driving for quantum state preparation and suppression of dissipation.
- Glass transition in the optimization problem. Optimal solutions are hard to find and unstable to tiny perturbations. One should aim at finding robust (variational) protocols.

CD driving through QCP in the TFI model



Dots: variational solutions with string lengths 1-4; lines: exact truncated solutions. Variational solution can be generalized to a nonintegrable chain.