A wave-wave interaction mechanism for near-inertial waves

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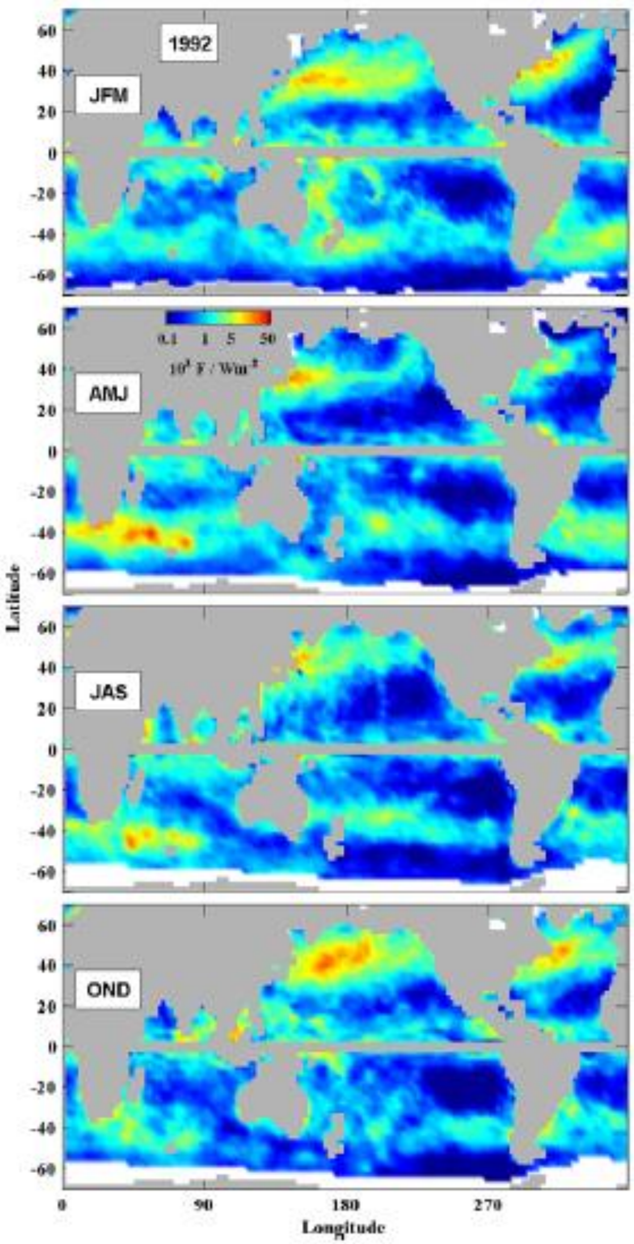
Wind-to-near-inertial-motions



M.H. Alford (2003)

- Wind work on ocean inertial motion
- Wind stress from reanalysis and ocean surface near-inertial motion from slab model (Pollard & Millard 1970)
- Enhanced near-inertial wind work during storms
- Total work done of the same order as tidal dissipation by conversion to internal tides

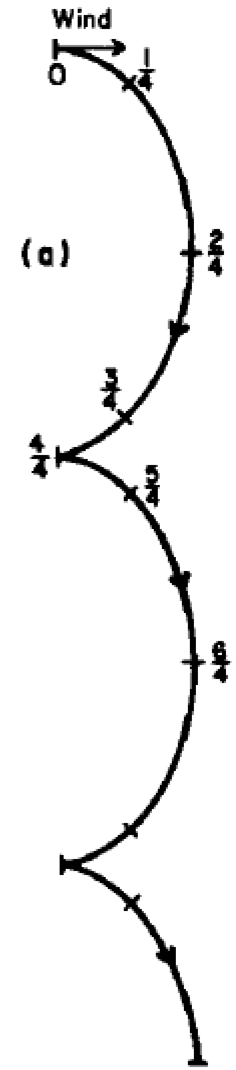
MIXED LAYER NEAR-INERTIAL OSCILLATIONS – GENERATION BY WIND STRESS

- Sudden wind stress in the x direction (westerly wind) – Gill (1984)

$$\frac{\partial(U_E + iV_E)}{\partial t} + if(U_E + iV_E)) = \frac{\tau_x^S}{\rho}$$

$$U_E + iV_E = -i(\tau_x^S/\rho f)(1 - exp(-ift))$$

- Steady state plus inertial oscillation



- Pollard & Millard (1970) slab mixed layer model – add damping term

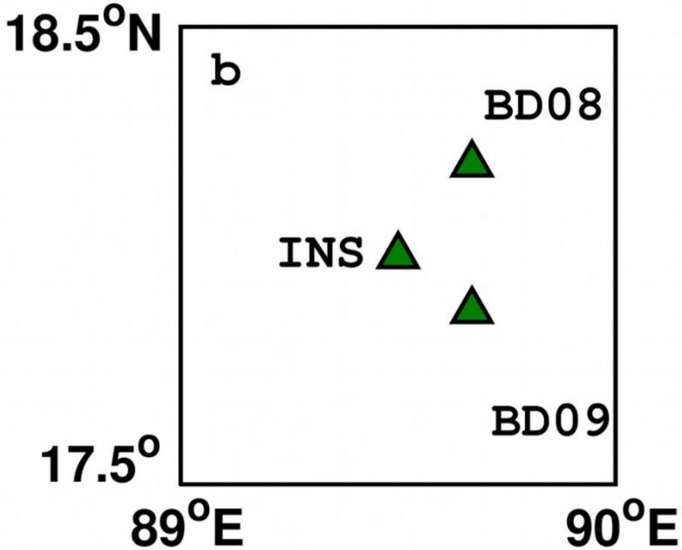
$$\frac{dZ}{dt} + (r_d + if)Z = \frac{T}{H}$$

$$Z = U_E + iV_E \qquad T = \rho^{-1}(\tau_x^S + i\tau_y^S)$$

$$H - mixed\ layer\ thickness$$

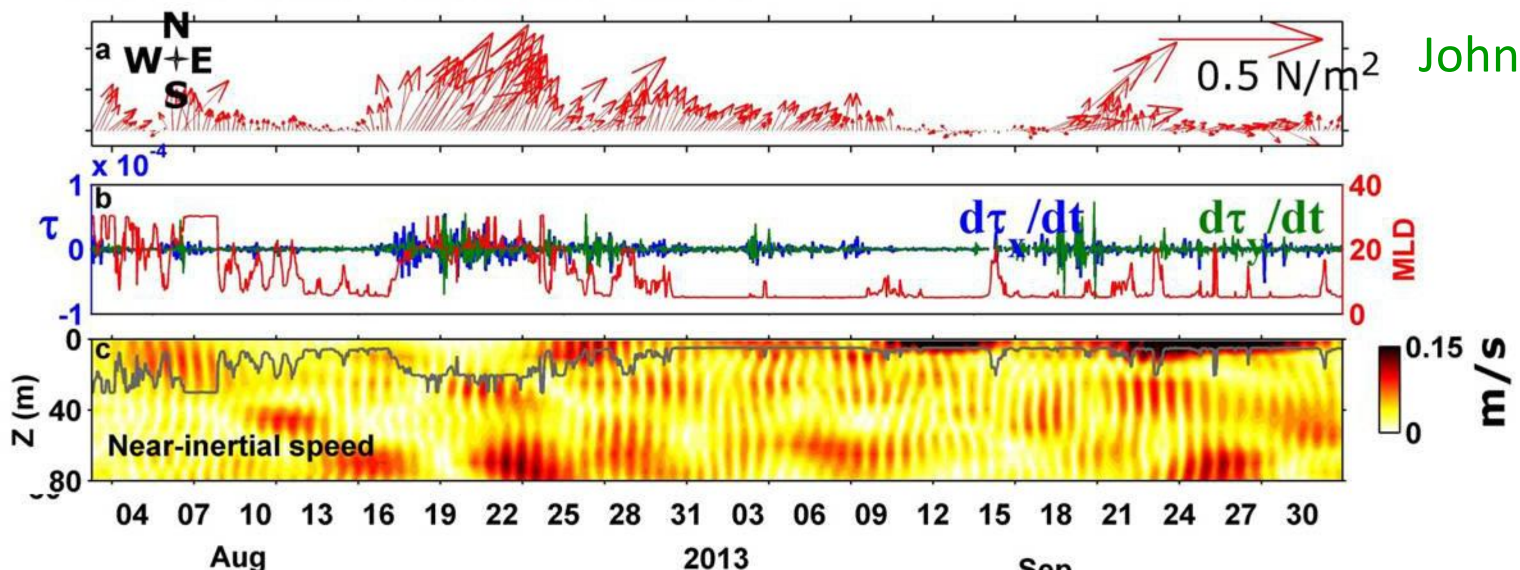
$$r_d^{-1} - decay\ time - arbitrarily\ chosen\ to\ fit\ observations$$

NEAR-INERTIAL CURRENTS - GENERATION & DISSIPATION



- Ocean Moored Buoy Network in the Northern Indian Ocean (OMNI) – Venkatesan *et al.* (2013).

- We present data (1hr. resolution) obtained at 17.88deg. N, 89.67deg. E.



Johnston *et al.* (2016)

- Obvious downward propagation in early August (relatively large MLD); Fresh water influx in September – small MLD & strong stratification below – not so obvious downward propagation.

Decay mechanisms for mixed layer near-inertial currents

- Radiation of downward-propagating NIOs – Gill (1984), D'Asaro (1989), Young & Ben Jelloul (1997)
- Nonlinear Interactions transferring energy to other frequencies – Henyey *et al*. (1986)
- Turbulent Dissipation - Hebert & Moum (1993)
- High-frequency internal waves – Polton et al. (2008)

✓ Relative importance of each mechanism is unclear. In this talk, we will propose a potential nonlinear interaction mechanism

Internal wave modes – weakly nonlinear solutions

$$\frac{\partial^2}{\partial t^2}(\nabla^2\psi) + f^2\frac{\partial^2\psi}{\partial z^2} = \frac{g}{\rho^*}\frac{\partial}{\partial x}[J(\psi,\rho)] - \frac{\partial}{\partial t}[J(\psi,\nabla^2\psi)] + f\frac{\partial}{\partial z}[J(\psi,v)]$$

$$\frac{\partial\rho}{\partial t} = -J(\psi,\rho),$$

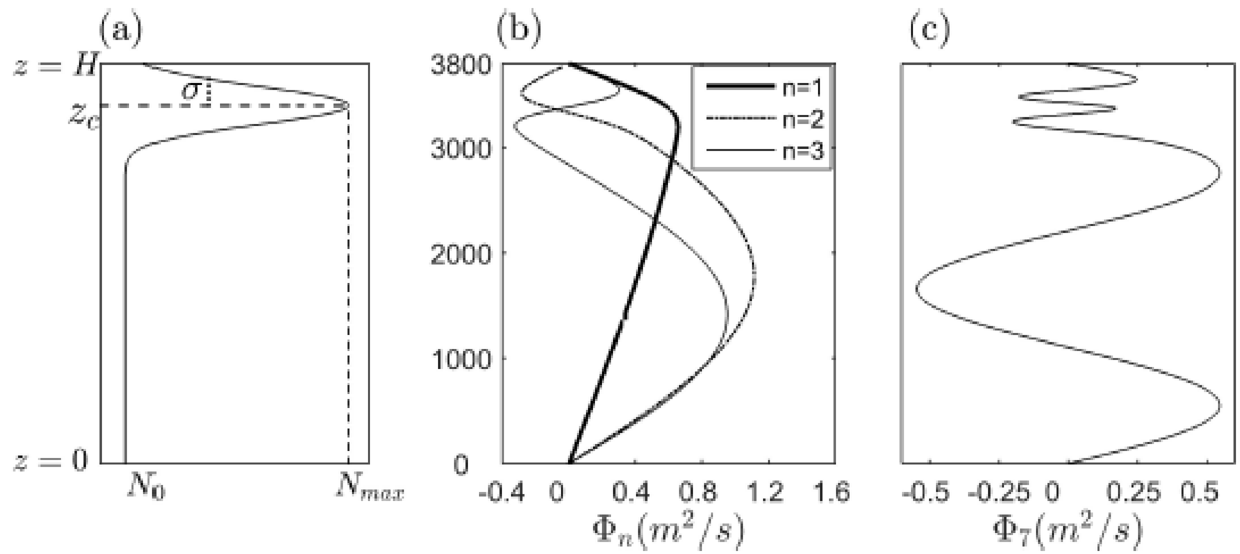
$$\frac{\partial v}{\partial t} + J(\psi,v) = f\frac{\partial\psi}{\partial z},$$

Fully nonlinear, 2D, inviscid, Boussinesq equations

$$(\psi, v, \rho) = (\psi_0, v_0, \rho_0) + \epsilon(\psi_1, v_1, \rho_1) + \epsilon^2(\psi_2, v_2, \rho_2) + \ldots,$$

Subscript

0 – background state
1 – linear wave field
2 – weakly nonlinear solution



$$\psi_1 = \sum_{n=1}^{\infty}\sqrt{\frac{2\omega k_n E_n}{\rho^*(\omega^2 - f^2)\int_0^H(\frac{d\Phi_n}{dz})^2 dz}}\,\Phi_n(z)\cos(k_n x - \omega t + \alpha_n)$$

Weakly nonlinear solutions

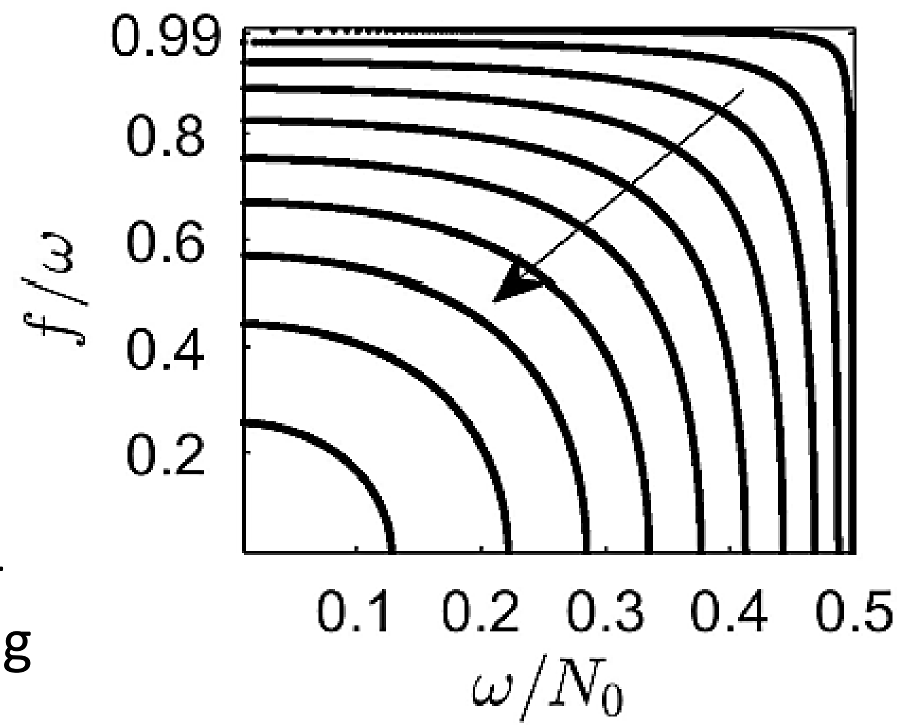
$$\psi_2 = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [h_{mn}(z) \cos((k_m+k_n)x-2\omega t+\alpha_m+\alpha_n)+g_{mn}(z) \cos((k_m-k_n)x+\alpha_m-\alpha_n)]$$

Uniform Stratification

Weakly nonlinear solution diverges if

$$\frac{\omega^2}{N_0^2} = \frac{(m+n)^2-(m-n)^2(4-f^2/\omega^2)/(1-f^2/\omega^2)}{4(m+n)^2-(m-n)^2(4-f^2/\omega^2)/(1-f^2/\omega^2)}$$

$$(m/3) < n < 3m, \ m \neq n$$

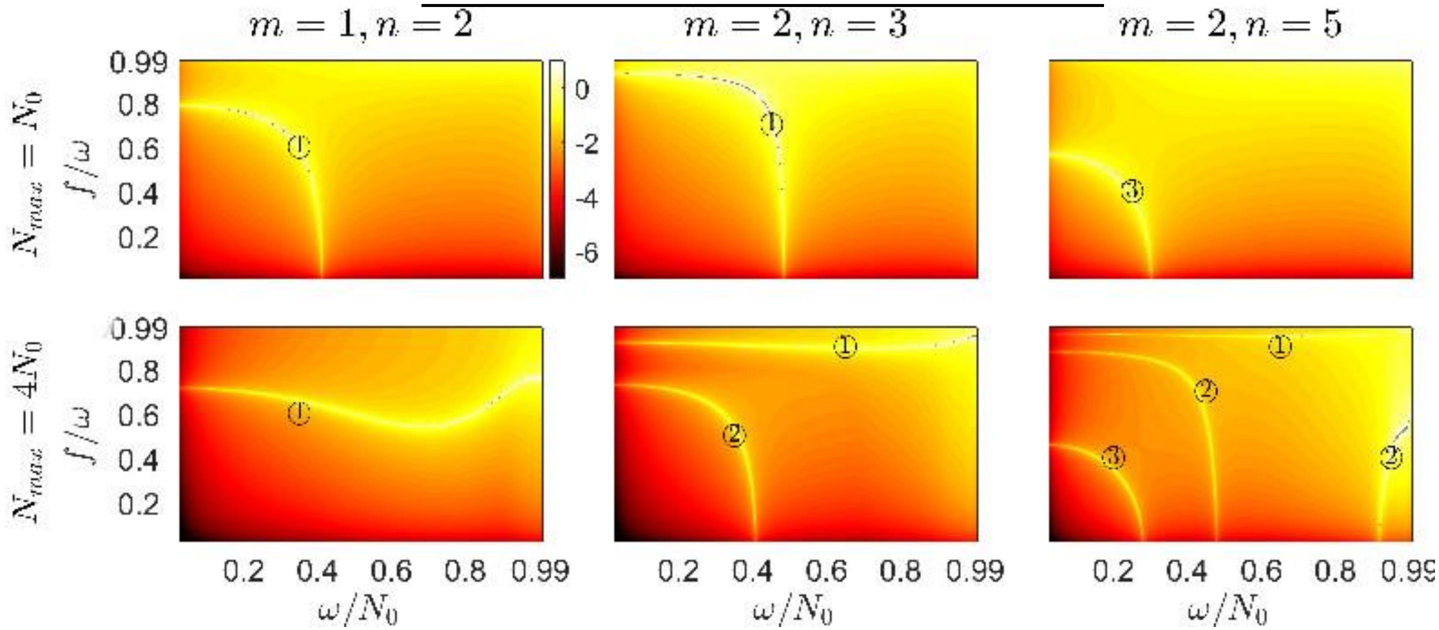


Non-dimensional parameters

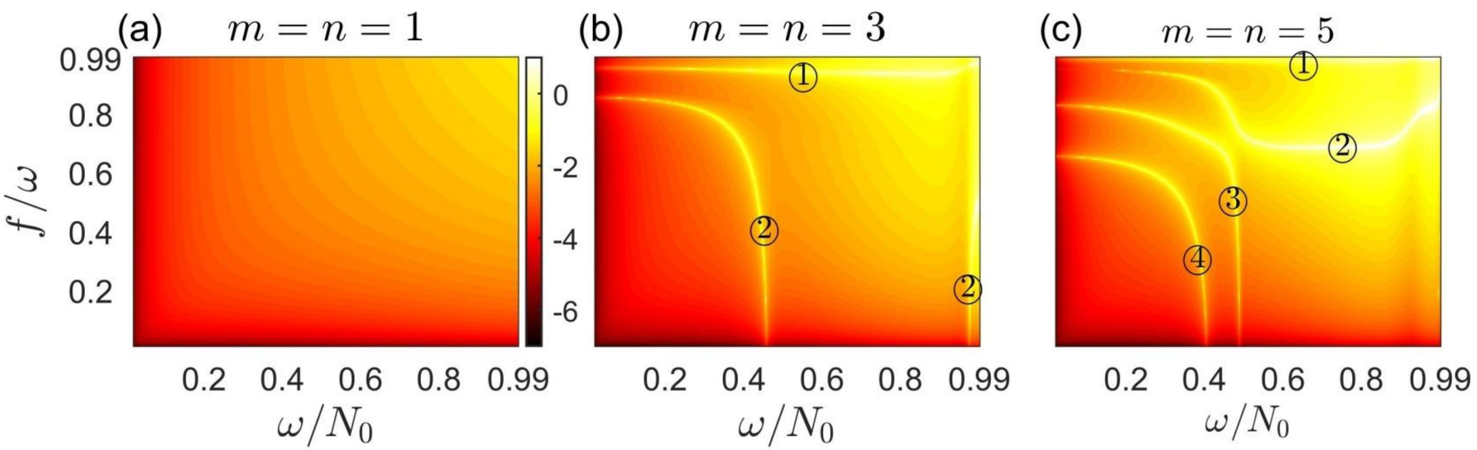
1. ω/N_0
2. f/ω
3. m/n

➢ High-mode interactions at near-inertial frequencies result in strong low-mode superharmonics

Nonuniform stratifications

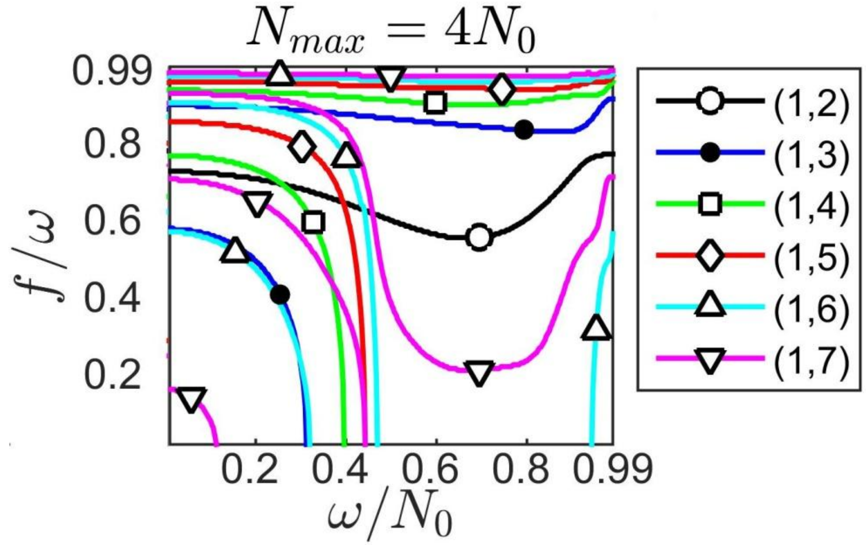


Self-interactions



- ➢ Divergence of weakly nonlinear solution due to triadic resonance
- ➢ Nonuniform stratification increases the number of triadic resonances
- ➢ Appearance of several resonances at the near-inertial frequencies

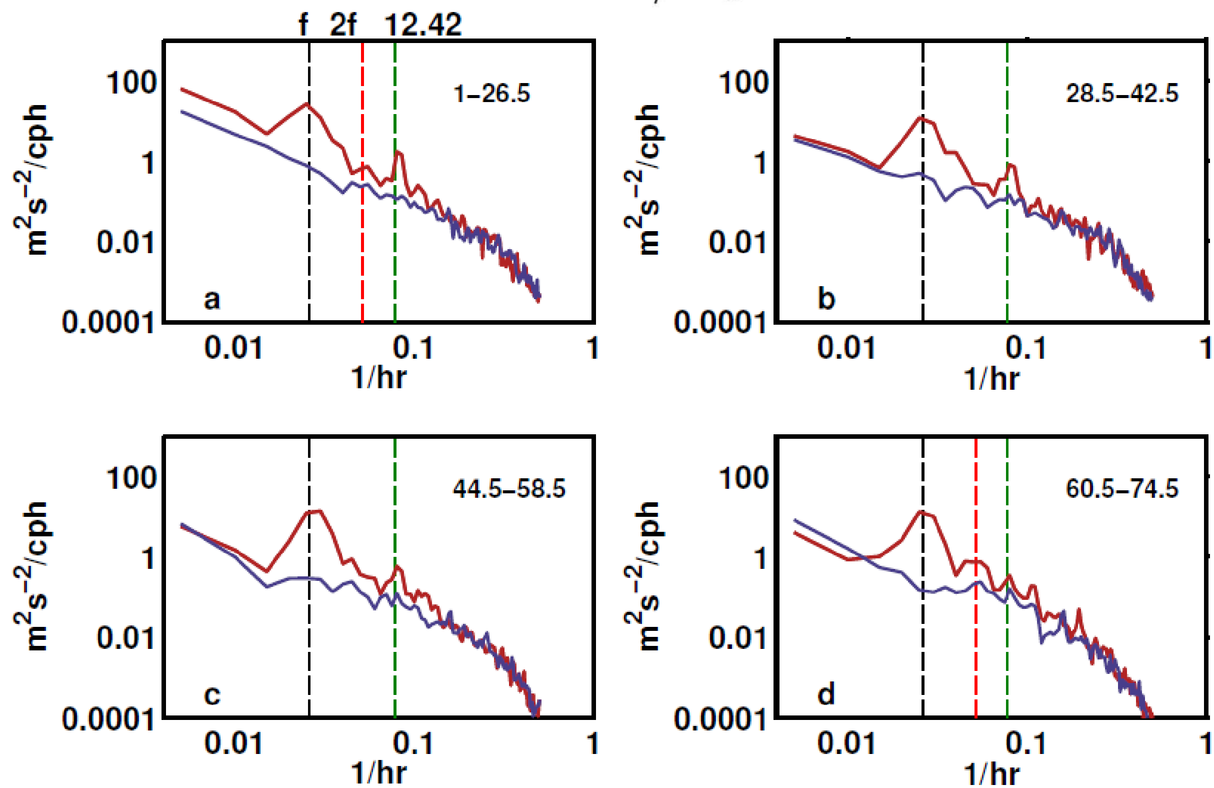
Nonuniform stratifications – Distribution of modes



- Presence of several modes – triadic resonance at near-inertial frequency highly likely

- 2015 WHOI Mooring – Thanks to *Dipanjan Chaudhuri* for the figure

- Energy at 2f may be present at subsurface depths also



Ongoing work

- Rate of energy transfer to superharmonics (with Dheeraj Varma, IIT Madras)

- Numerical Simulations (with Vamsi Chalamalla, IIT Delhi)

- Field Data (with Dipanjan Chaudhuri, IISc)

Dheeraj Varma & Manikandan Mathur, J. Fluid Mech. 2017