Enhancing the $\mathcal{PT}$-transition threshold

Non-Hermitian Physics - PHHQP XVIII (June 4 – June 13, 2018)

Kaustubh S. Agarwal
Indiana University-Purdue University Indianapolis
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Collaborators :
Dr. Yogesh N. Joglekar
Prof. Rajeev K. Pathak

Phys. Rev. A 97 (4), 042107

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Outline

- 1D $\mathcal{PT}$-symmetric discrete lattice model
- Enhanced $\mathcal{PT}$-threshold in 2D lattice model
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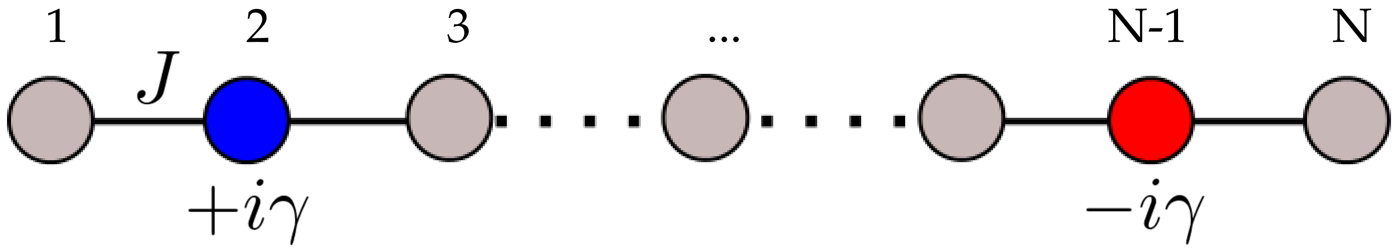
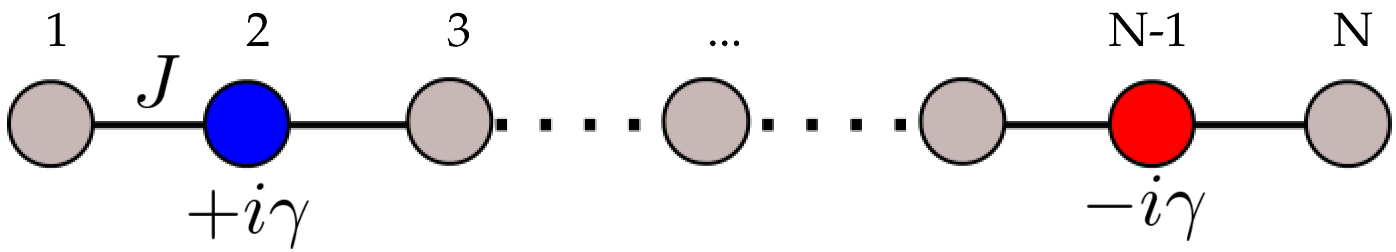
1D $\mathcal{PT}$-symmetric lattice model: Waveguides

Finite, discrete, 1D tight-binding lattice with uniform nearest neighbour coupling and one pair of balanced gain-loss sites, assuming open boundary conditions.



$$H = -J(\sum_{m=1}^{N-1} |m\rangle \langle m+1| + h.c.)$$
$$+ i\gamma(|m_0\rangle \langle m_0| - |\bar{m_0}\rangle \langle \bar{m_0}|)$$

1D $\mathcal{PT}$-symmetric lattice model: Waveguides

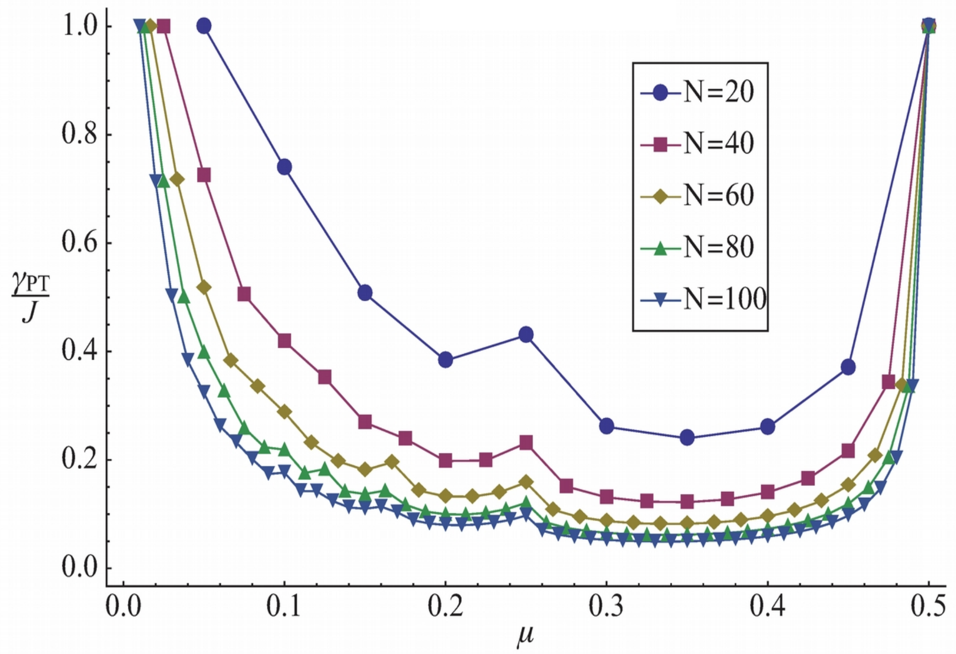


$$H = -J\left(\sum_{m=1}^{N-1} |m\rangle\langle m+1| + h.c.\right)$$
$$+ i\gamma\left(|m_0\rangle\langle m_0| - |\bar{m}_0\rangle\langle \bar{m}_0|\right)$$

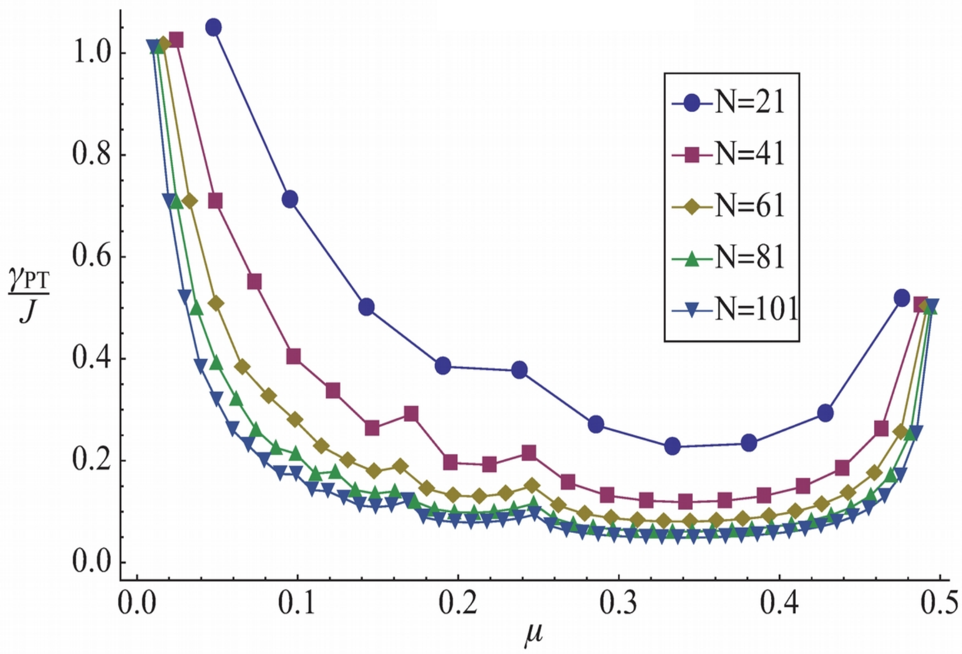
What is the PT-symmetry breaking threshold?

Y.N. Joglekar et al. PRA 83, 030103(R) (2010)

Even N



Odd N



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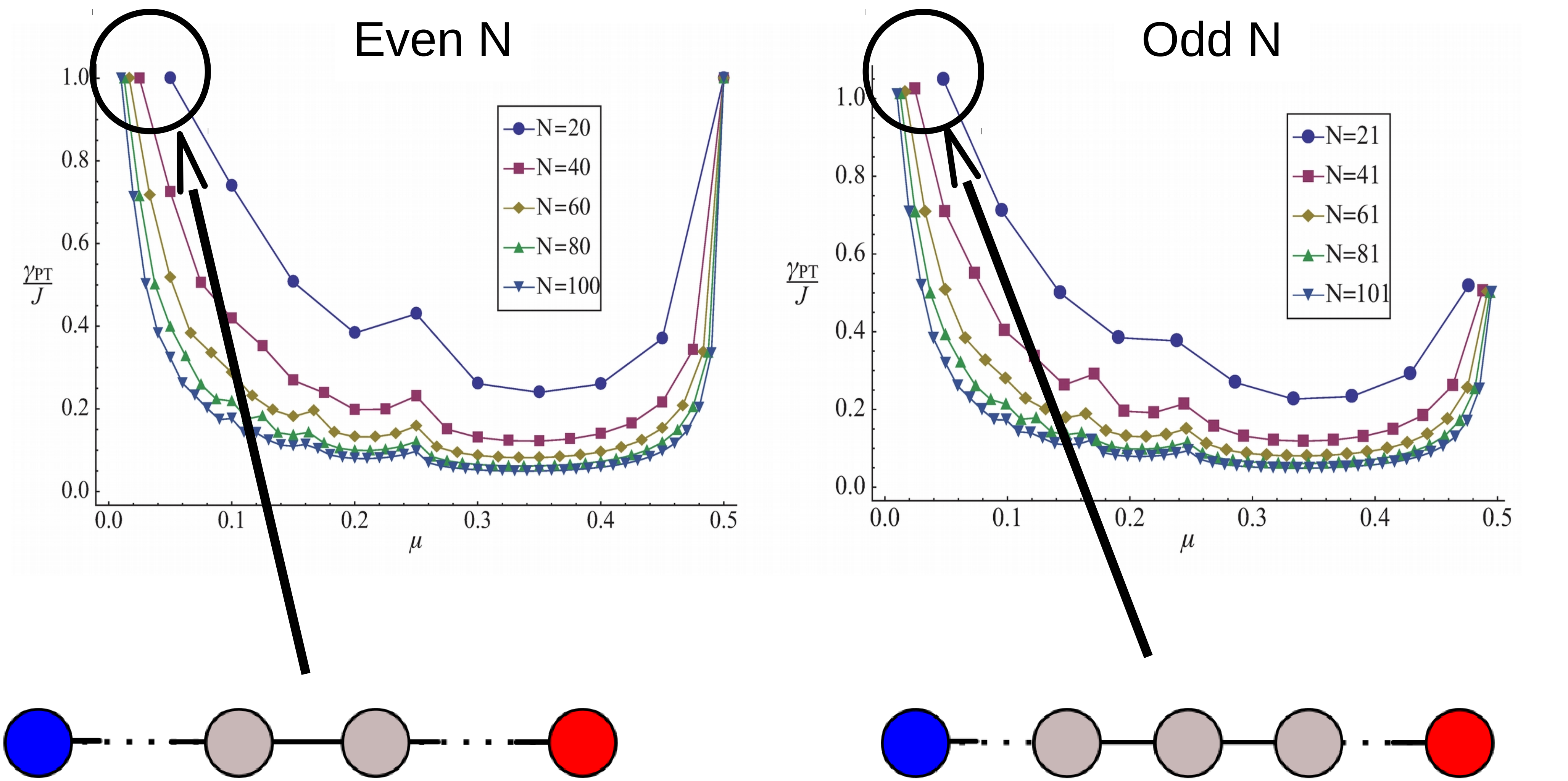

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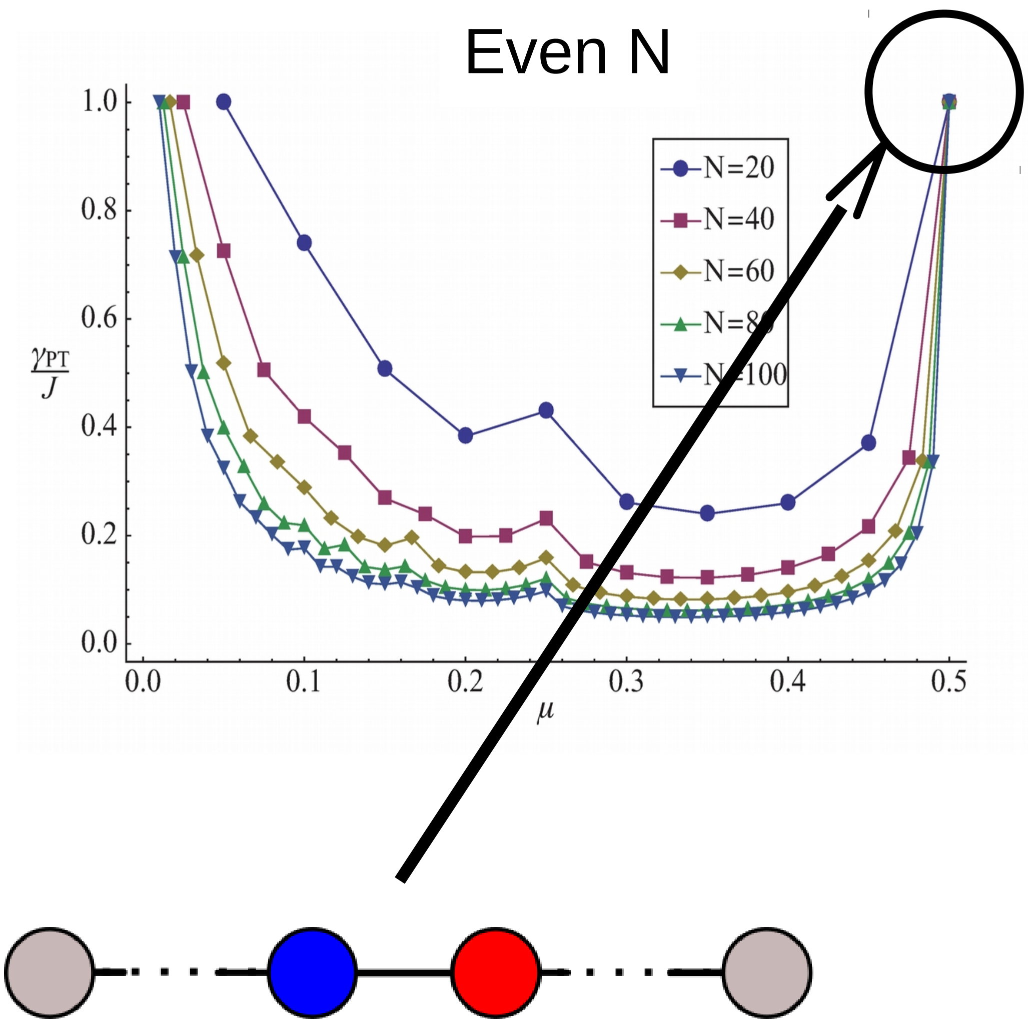

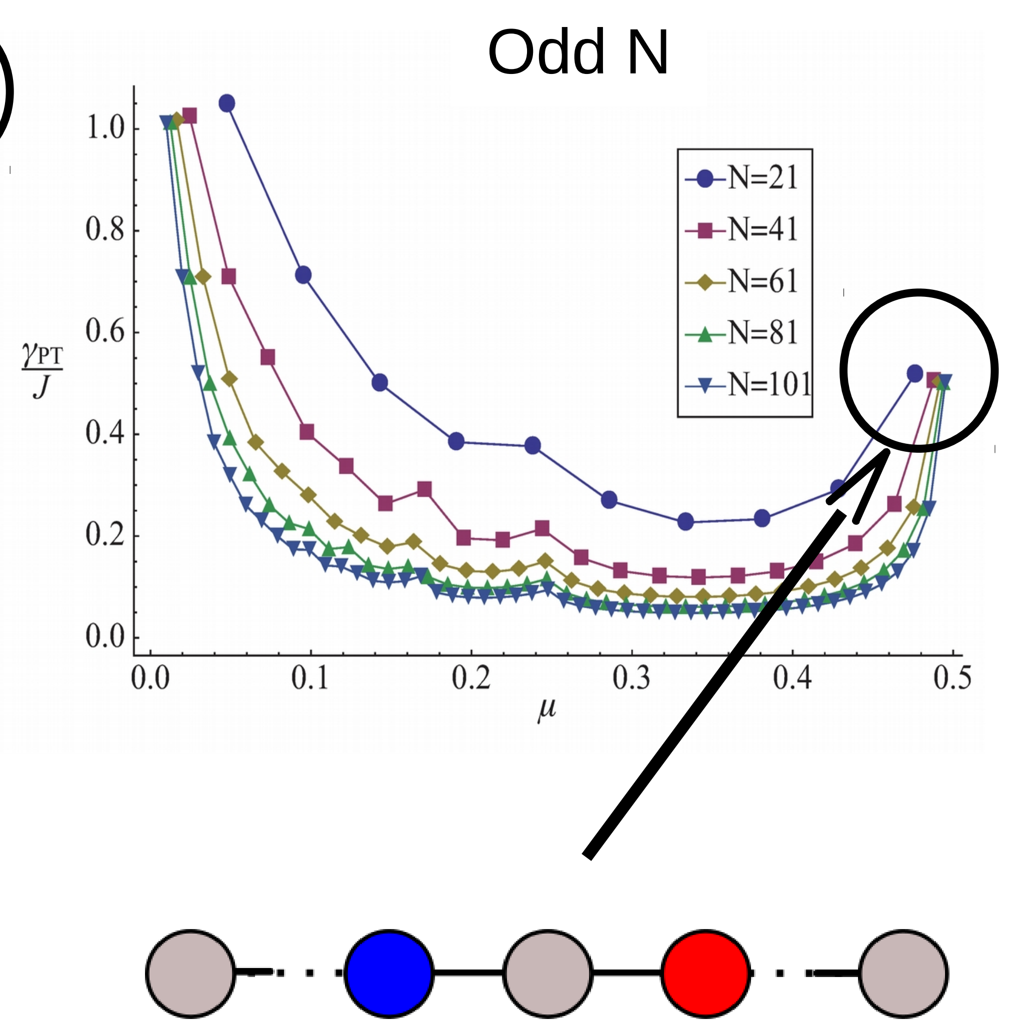

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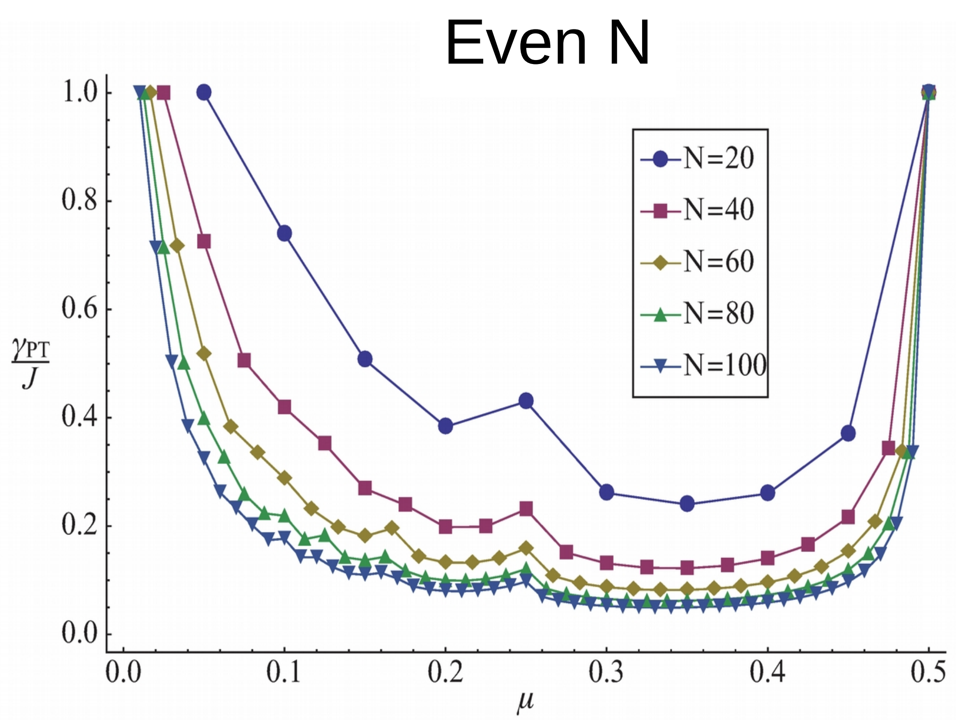
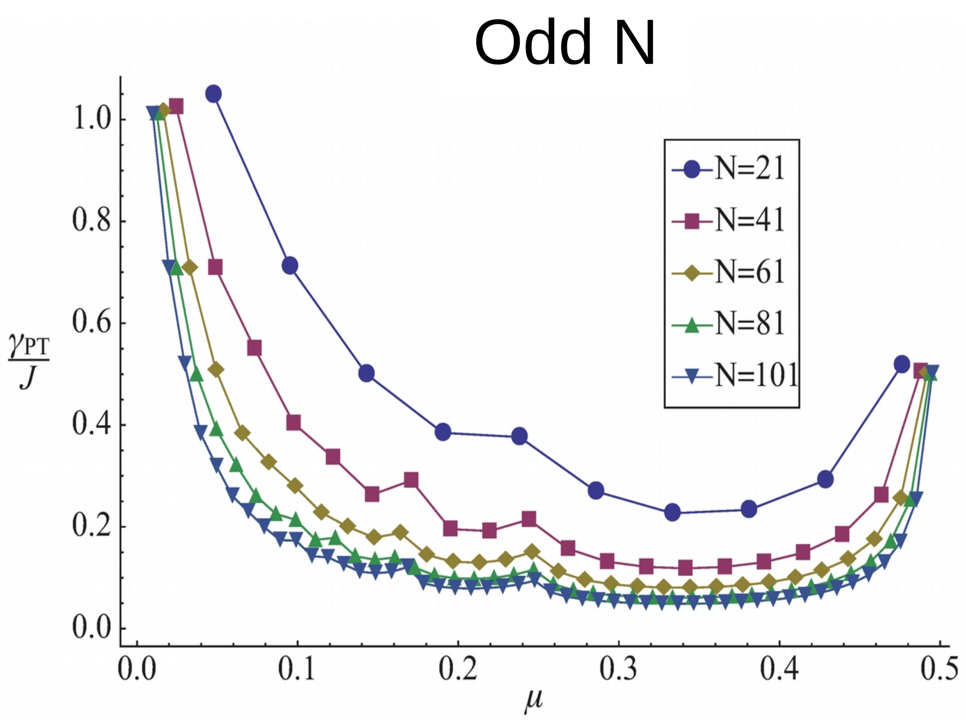
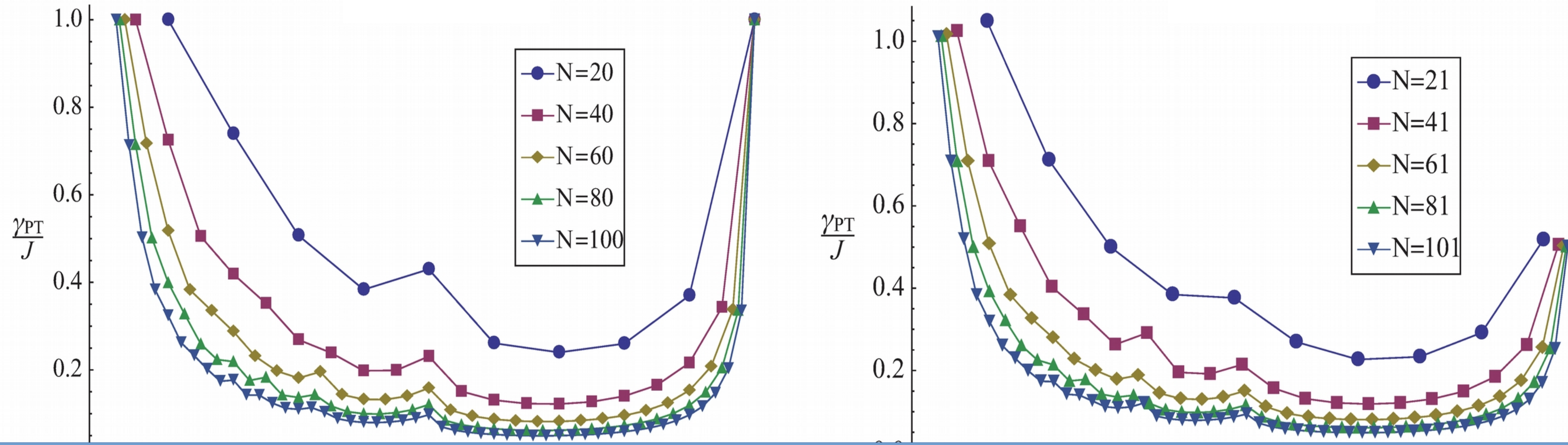




- The $\mathcal{PT}$ symmetry breaking threshold is enhanced with the gain-loss potentials are at the boundary
- $\max\{\gamma_{th}\} = J$

Y.N. Joglekar et al. PRA 83, 030103(R) (2010)

Even N   Odd N



Physical limitions prevent any futher increase of the threshold

- The $\mathcal{PT}$ symmetry breaking threshold is enhanced with the gain-loss potentials are at the boundary
- $\max\{\gamma_{th}\} = J$

Outline

- 1D $\mathcal{PT}$-symmetric discrete lattice model
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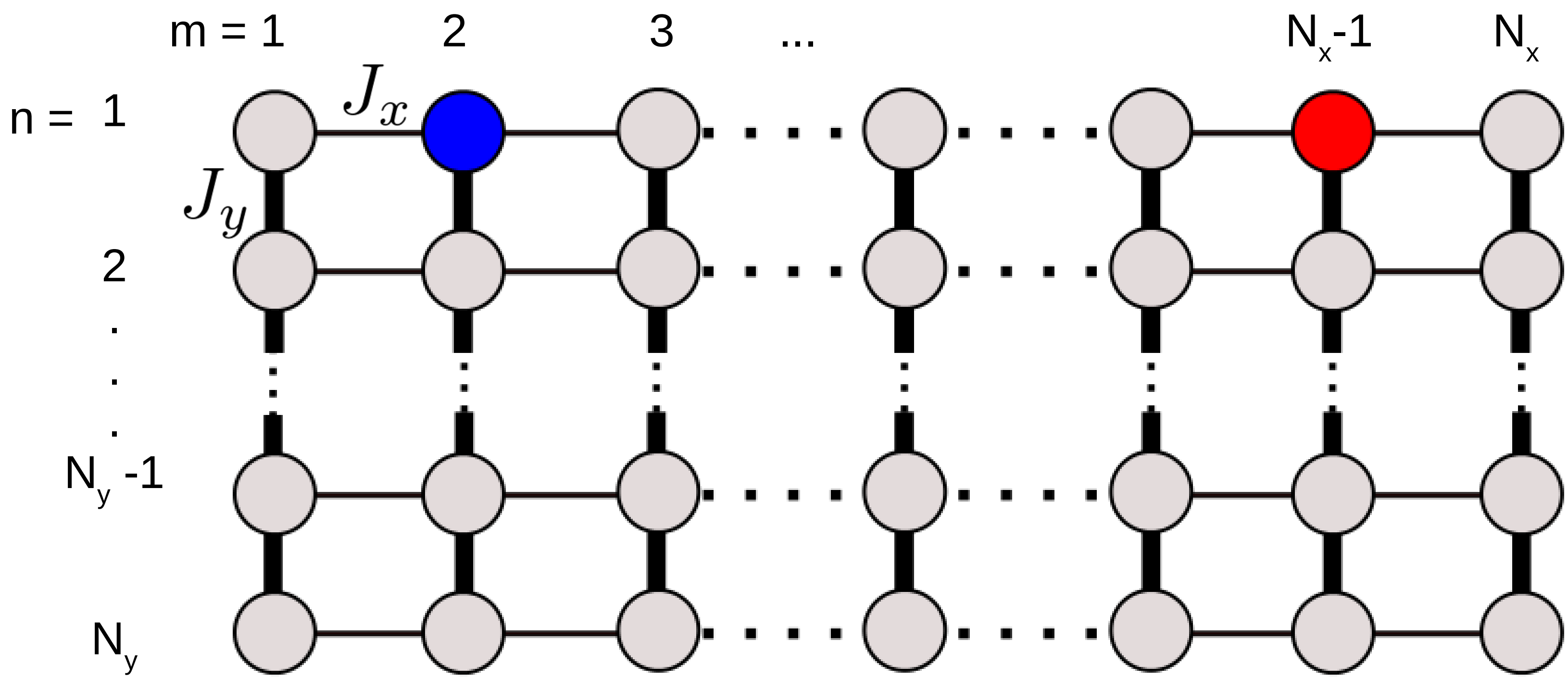
2D $\mathcal{PT}$ tight binding lattice model : Introducing neutral hermitian chains

Consider N_x sites on a single chain with 1 pair of gain-loss sites strongly coupled $N_y - 1$ chains of the same length



2D $\mathcal{PT}$ tight binding lattice model : Introducing neutral hermitian chains

Hermitian

$$H_0 = -J_x \sum_{m,n} |m,n\rangle \langle m+1,n| + \text{h.c.}$$
$$-J_y \sum_{m,n} |m,n\rangle \langle m,n+1| + \text{h.c.}$$

Non-Hermitian

$$\Gamma = +i\gamma(|m_0,n_0\rangle \langle m_0,n_0| - |\bar{m}_0,n_0\rangle \langle \bar{m}_0,n_0|)$$

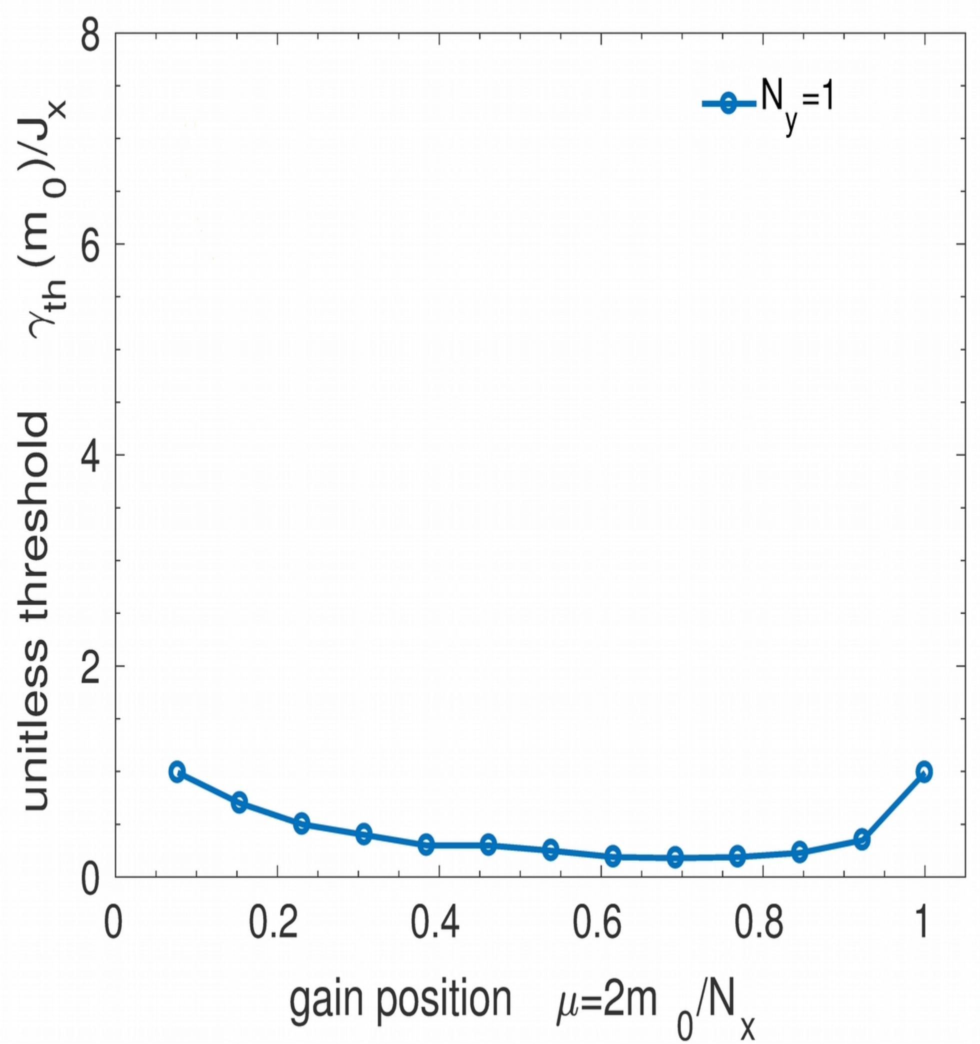
$\mathcal{PT}$ symmetry :

$\mathcal{P}$: $(m_0,n_0) \to (\bar{m}_0,n_0)$, where $\bar{m}_0 = (N_x+1-m_0)$

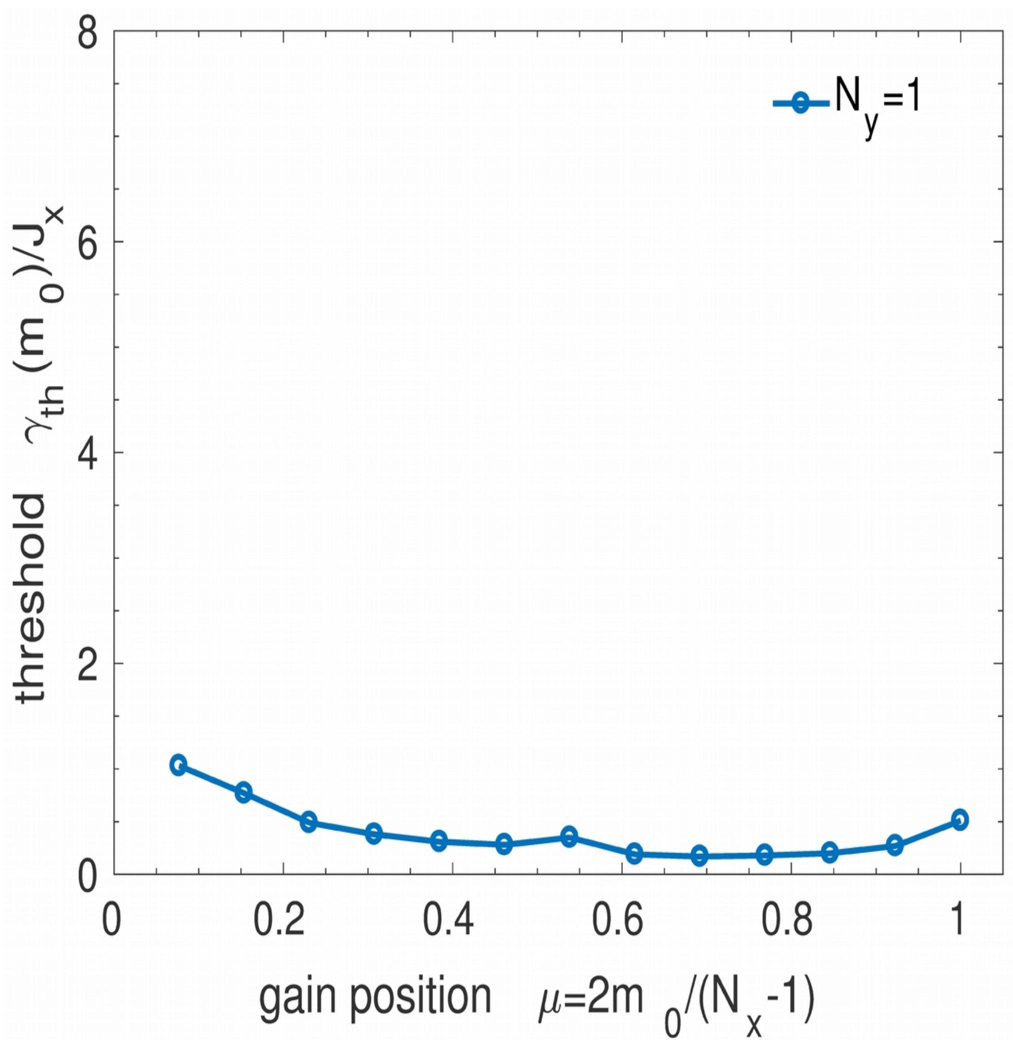
$\mathcal{T}$: $i \to -i$

2D $\mathcal{PT}$ chains (Numerical results)

N_x = Even

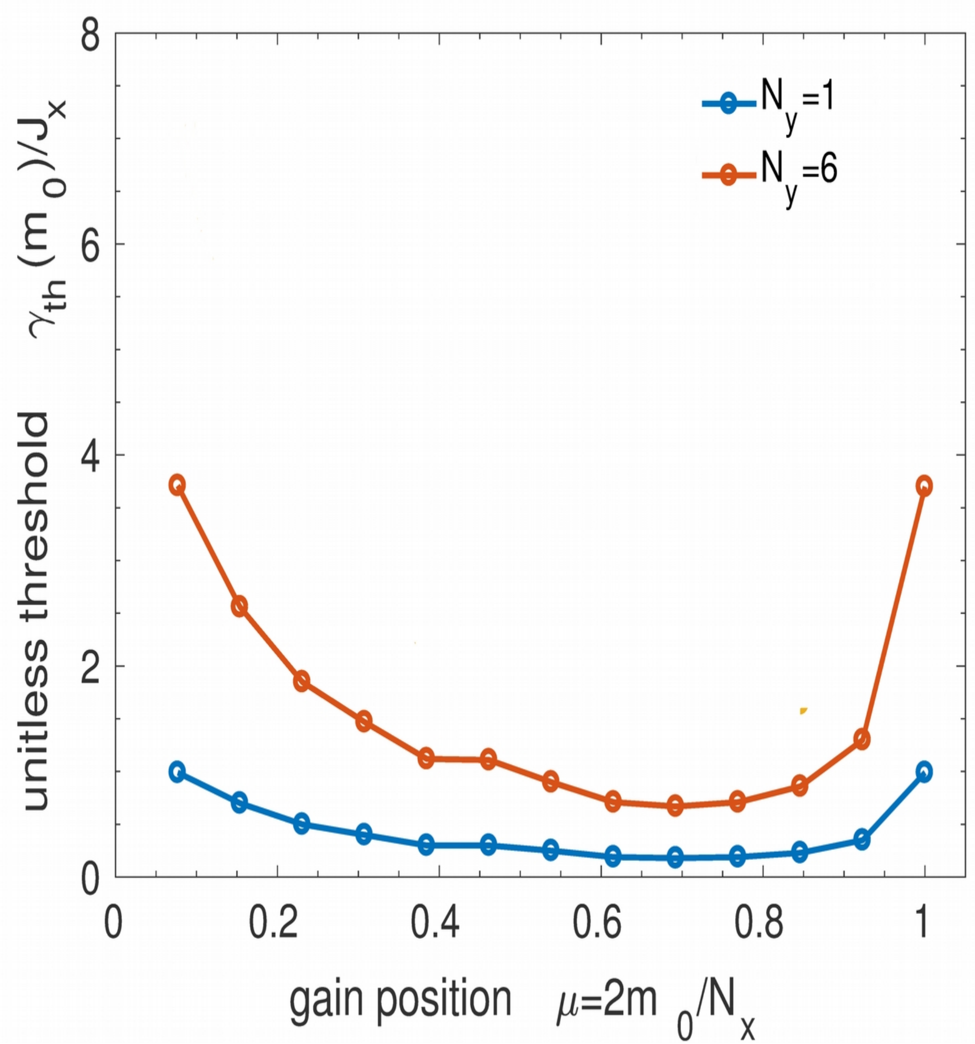


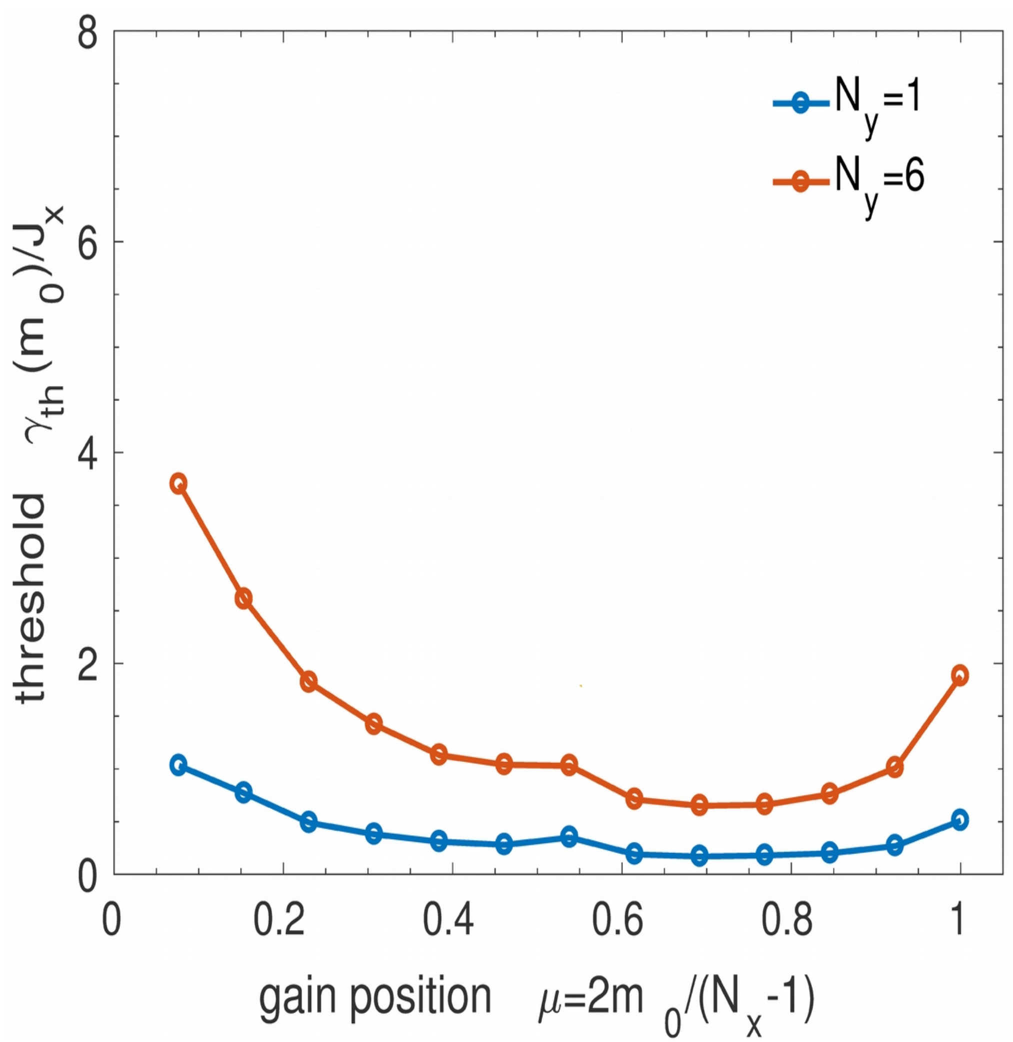
N_x = Odd



2D $\mathcal{PT}$ chains (Numerical results)

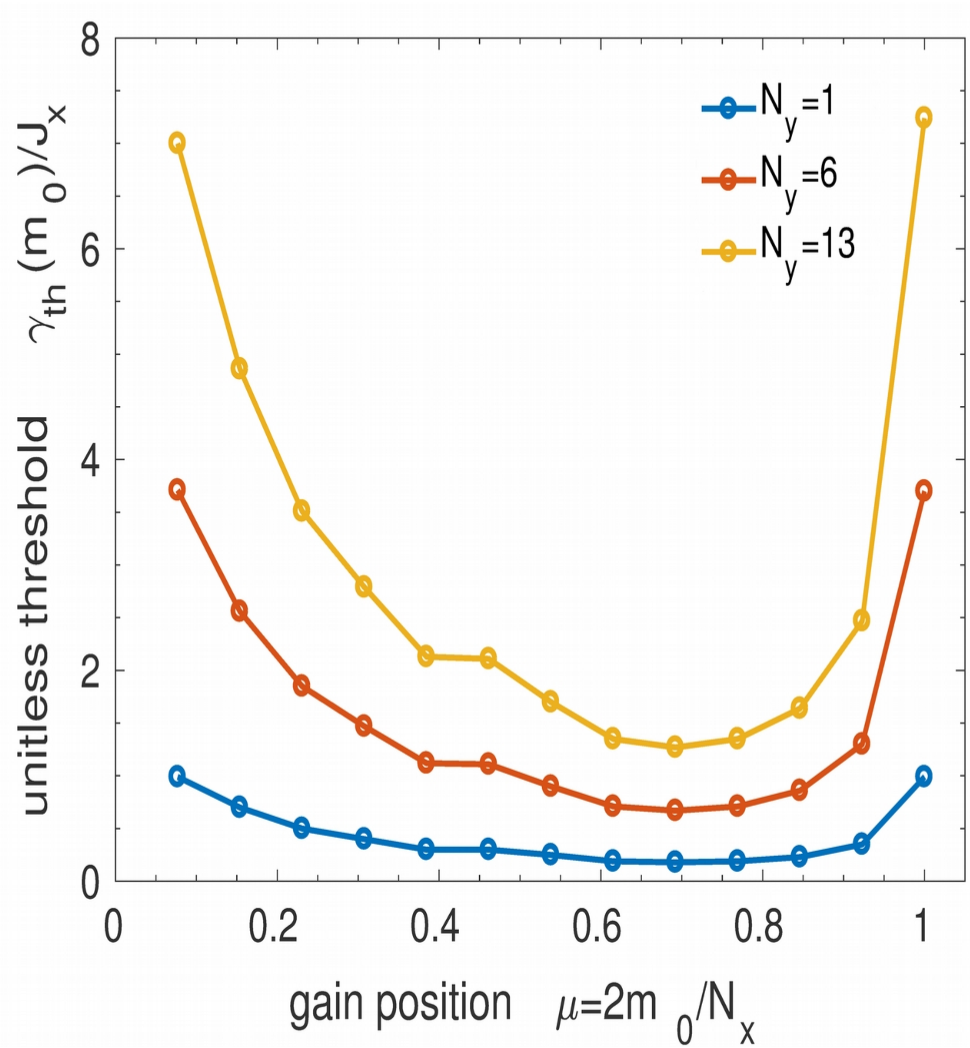
N_x = Even **J_y/J_x = 20** **N_x = Odd**

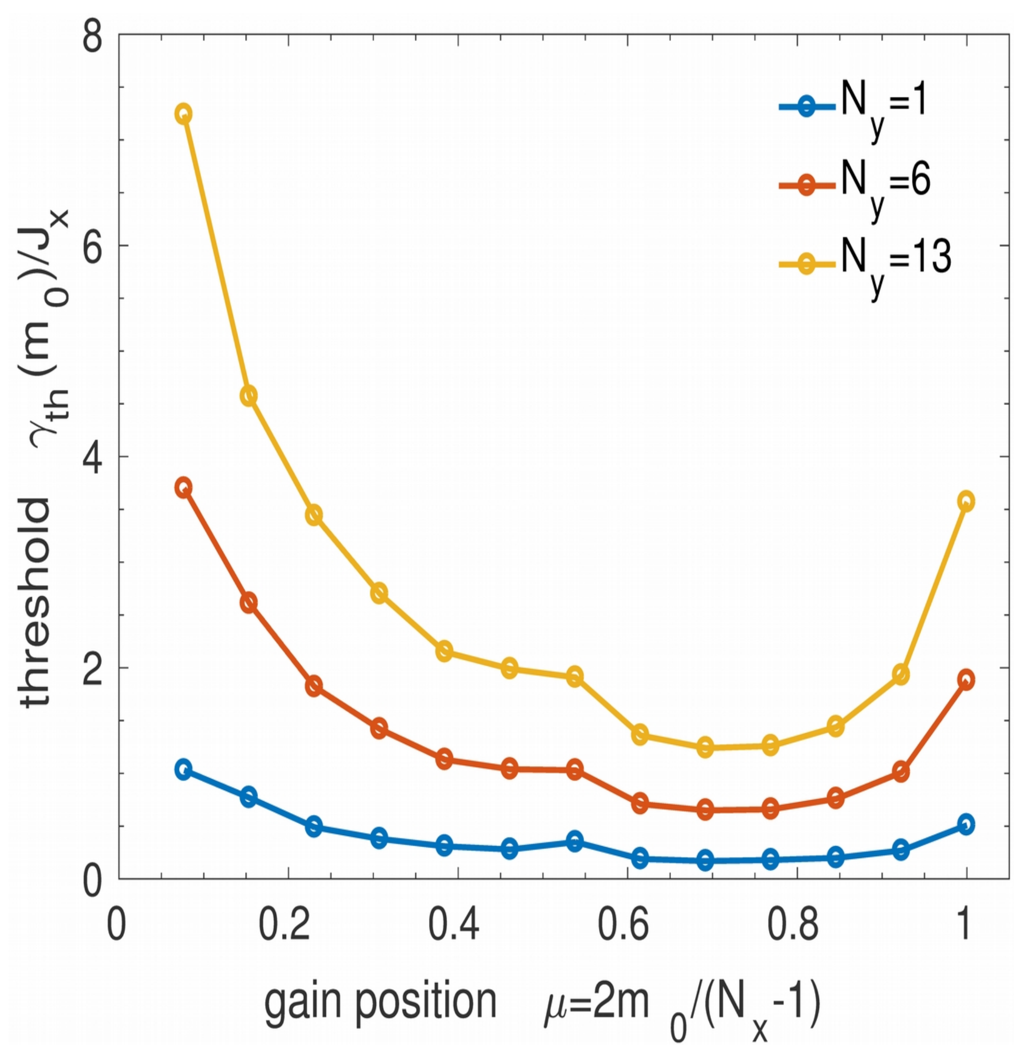




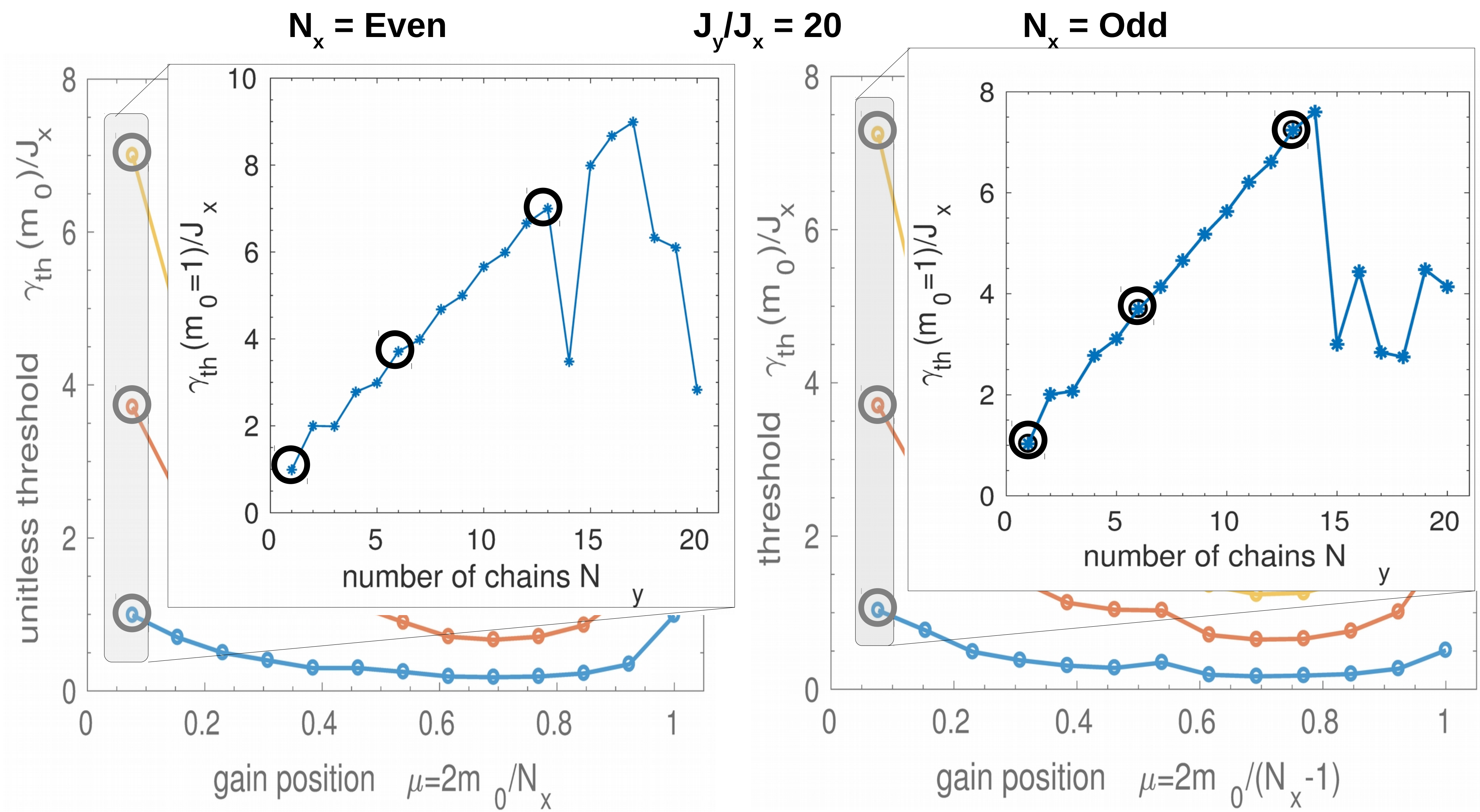
2D $\mathcal{PT}$ chains (Numerical results)

N_x = Even **$J_y/J_x = 20$** **N_x = Odd**



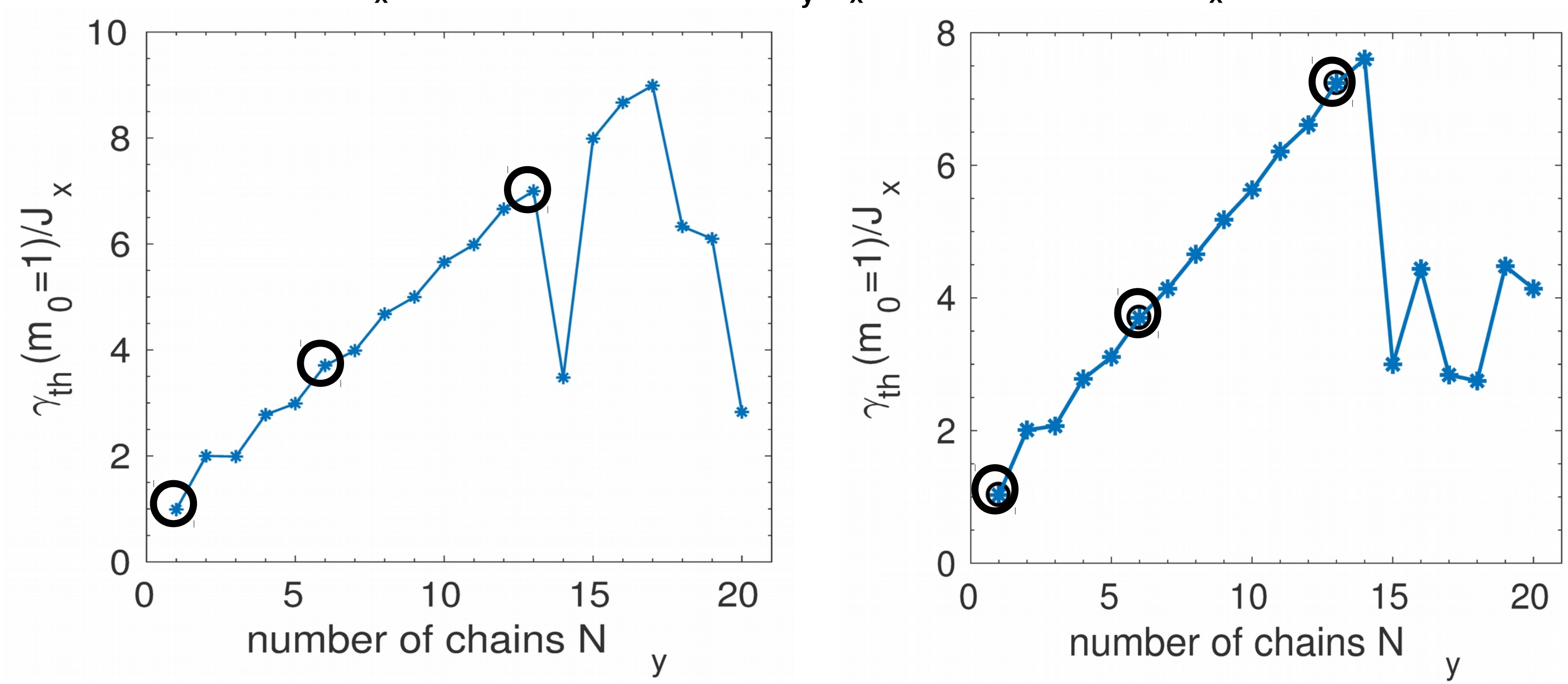


2D $\mathcal{PT}$ chains (Numerical results)



2D $\mathcal{PT}$ chains (Numerical results)

N_x = Even $J_y/J_x = 20$ N_x = Odd



We now can find a scaling law for the PT-threshold !!!

2D $\mathcal{PT}$ chains (Analytical results)

Is this analytically tractable?

2D $\mathcal{PT}$ chains (Analytical results)

Is this analytically tractable? ... Yes !

2D $\mathcal{PT}$ chains (Analytical results)

Is this analytically tractable? ... Yes !

- For the <u>strong coupling limit</u> we impose that the energy bands are well separated i.e.

$$E_{N_x,1} < E_{1,2}$$

Energy **<u>level</u>** index Energy **<u>band</u>** index

$$E_{p,q} = -2J_x \cos\left(\frac{p\pi}{N_x+1}\right) - 2J_y \cos\left(\frac{q\pi}{N_y+1}\right)$$

$$\frac{J_y}{J_x} > \left[\frac{\cos\left(\frac{\pi}{N_x+1}\right)-\cos\left(\frac{N_x\pi}{N_x+1}\right)}{\cos\left(\frac{\pi}{N_y+1}\right)-\cos\left(\frac{2\pi}{N_y+1}\right)}\right]$$

- Reducing the Hamiltonian into an effective 2-level system.

2D $\mathcal{PT}$ chains (Analytical results)

Here is the math...

$$H_{\text{eff}}(m_0, n_0) = (E_{p,q} - E_{p+1,q})\frac{\sigma_z}{2} + i\Delta_{p,q}(m_0, n_0)\sigma_x$$

2D $\mathcal{PT}$ chains (Analytical results)

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$$\langle p, q|\,\Gamma\,|p+1, q\rangle$$

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$$\langle p, q| \, \Gamma \, |p + 1, q\rangle$$

$$2\mathrm{i}\gamma A^2 \sin(k_p m_0) \sin(k_{p+1} m_0) \sin^2(k_q n_0)$$

2D $\mathcal{PT}$ chains (Analytical results)

Here is the math...

$$H_{\text{eff}}(m_0, n_0) = (E_{p,q} - E_{p+1,q})\frac{\sigma_z}{2} + i\Delta_{p,q}(m_0, n_0)\sigma_x$$

$$\sqrt{\frac{4}{(N_X+1)(N_y+1)}}$$

$$\langle p, q| \,\Gamma\, |p+1, q\rangle$$

$$2\mathrm{i}\gamma A^2 \sin(k_p m_0) \sin(k_{p+1} m_0) \sin^2(k_q n_0)$$

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$$\implies |\Delta_{p,q}\gamma_{var}| = \frac{1}{2}|(E_{p,q} - E_{p+1,q})|$$

2D $\mathcal{PT}$ chains (Analytical results)

Here is the math...

$$H_{\text{eff}}(m_0, n_0) = (E_{p,q} - E_{p+1,q})\frac{\sigma_z}{2} + i\Delta_{p,q}(m_0, n_0)\sigma_x$$

$$\implies |\Delta_{p,q}\gamma_{var}| = \tfrac{1}{2}|(E_{p,q} - E_{p+1,q})|$$

$$\boxed{\frac{\gamma(N_y)}{\gamma(N_y = 1)} = \frac{(N_y + 1)}{2}\text{cosec}^2\left(\frac{q\pi n_0}{N_y + 1}\right)}$$

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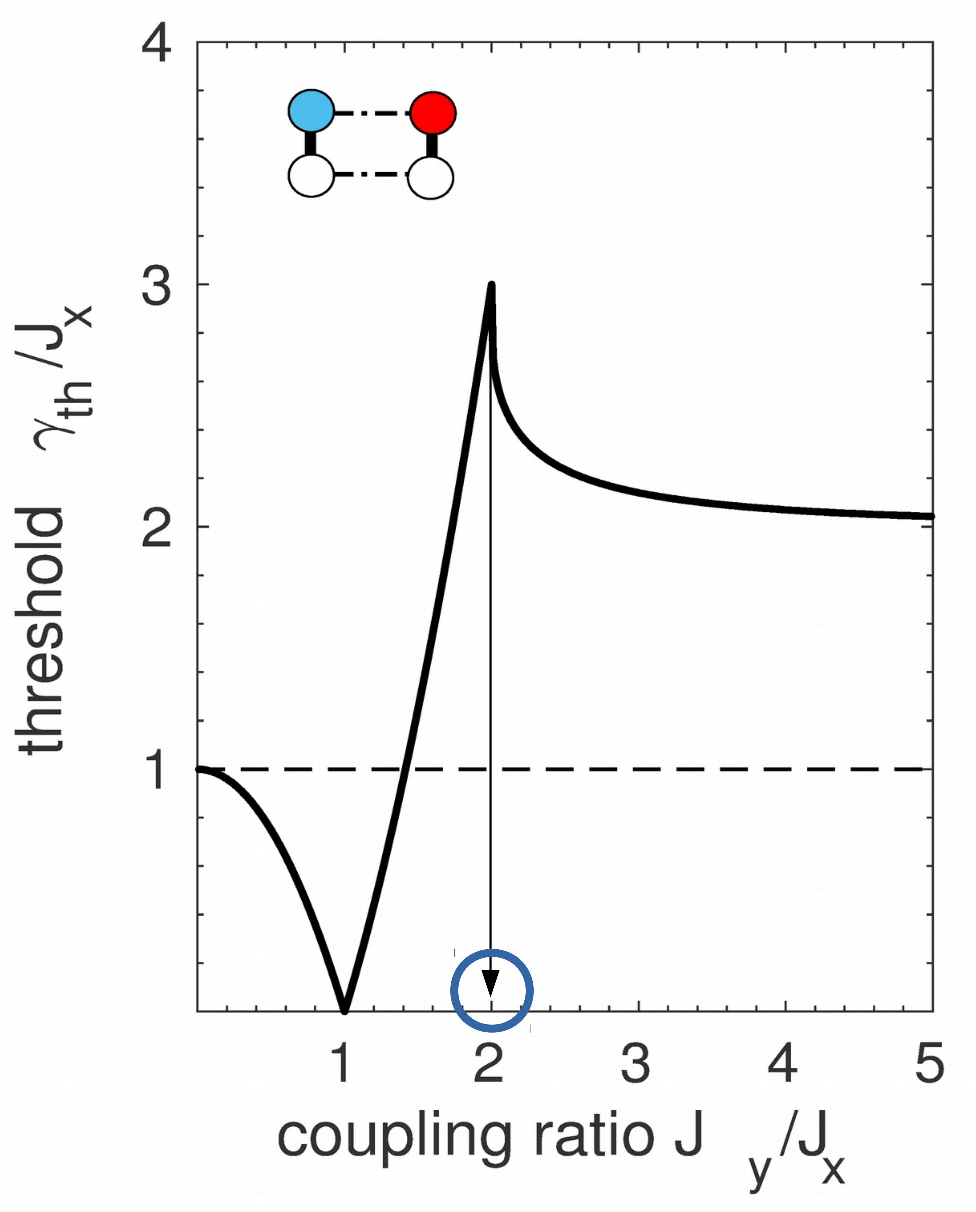
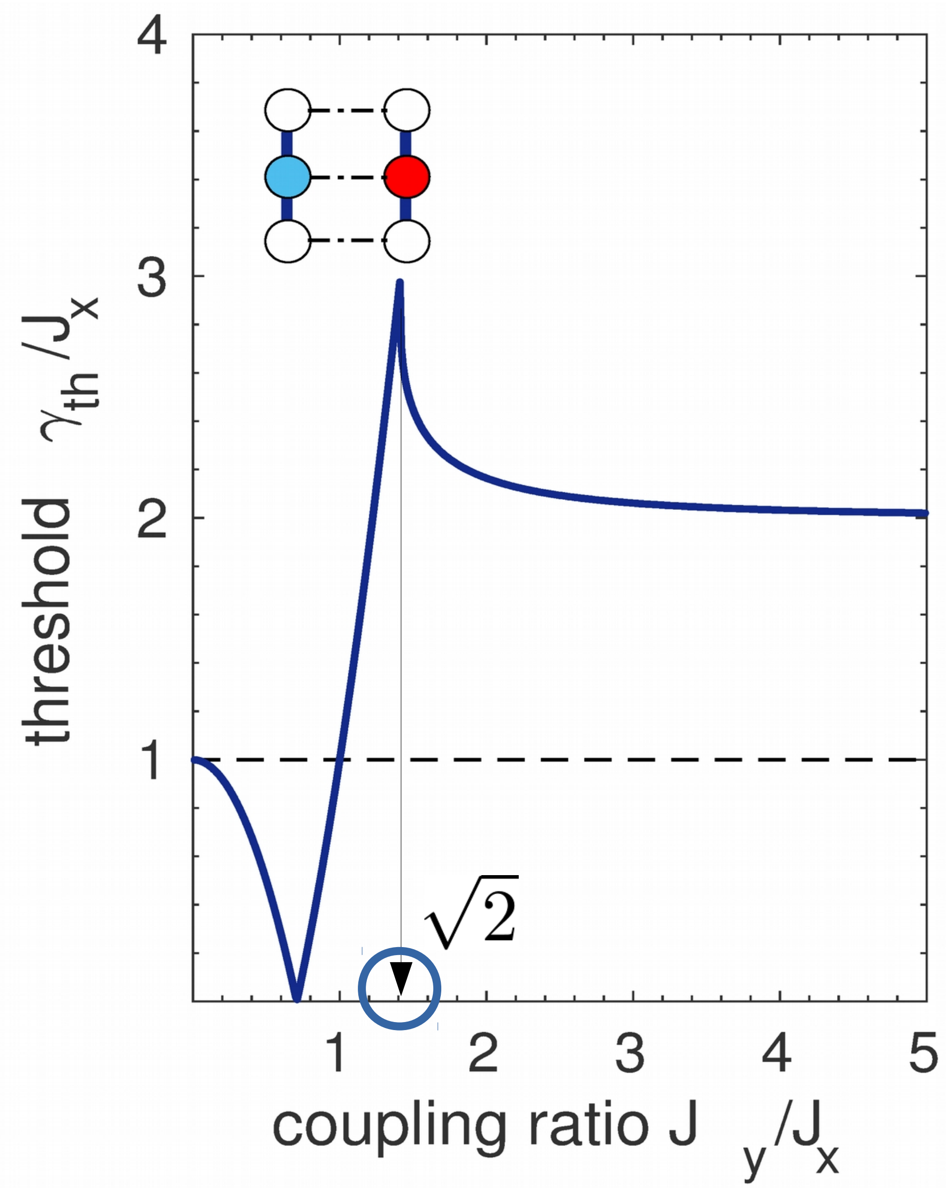
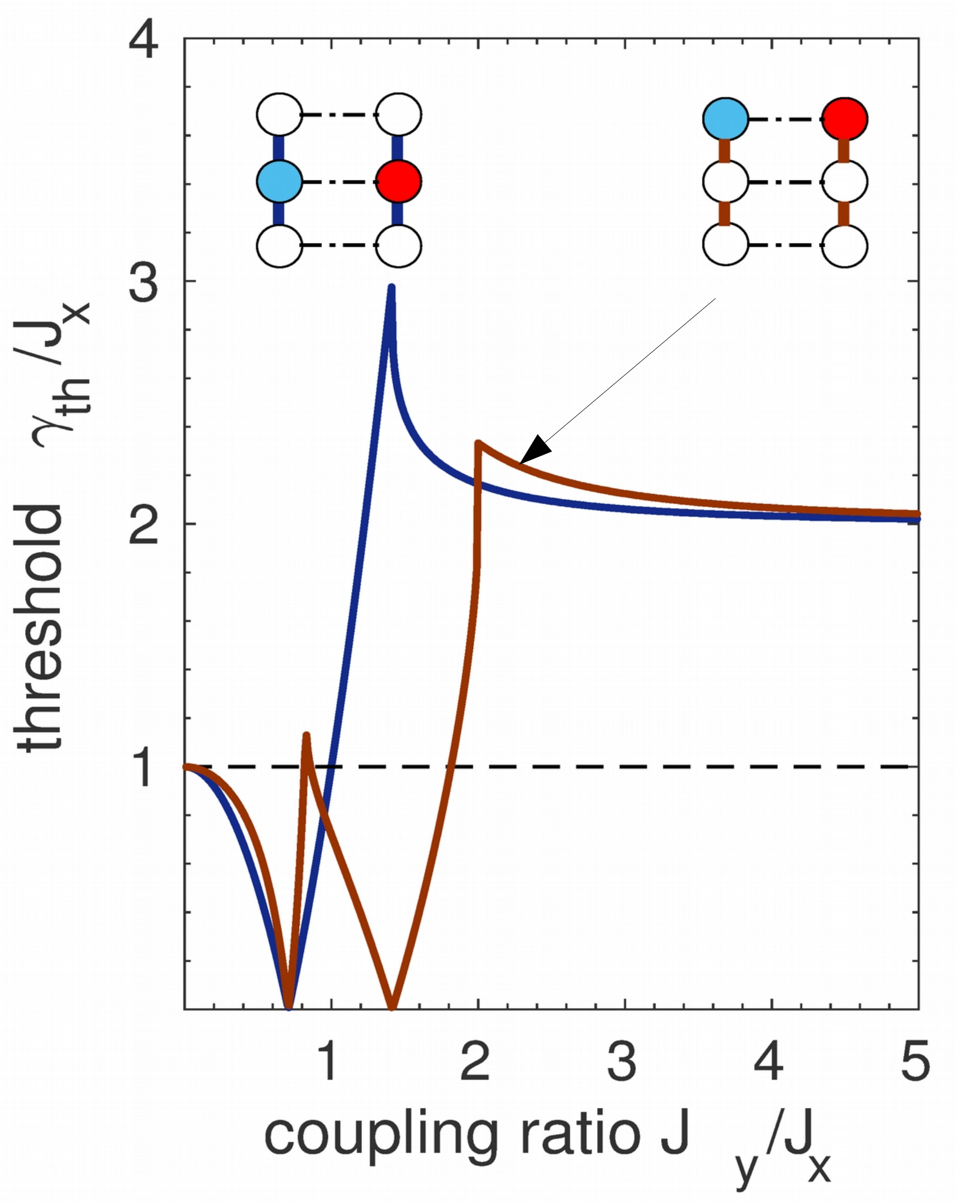
$\mathcal{PT}$-dimer chains



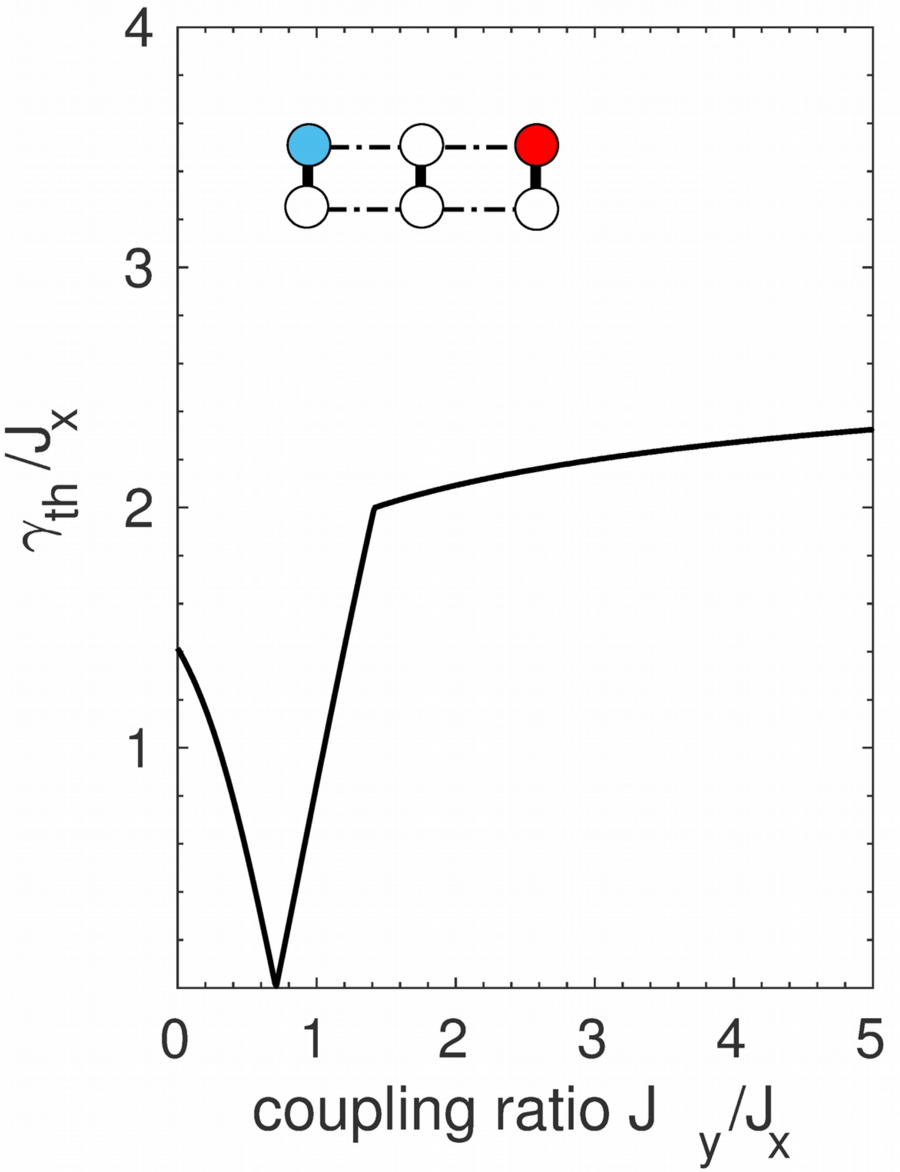
$\mathcal{PT}$-dimer chains

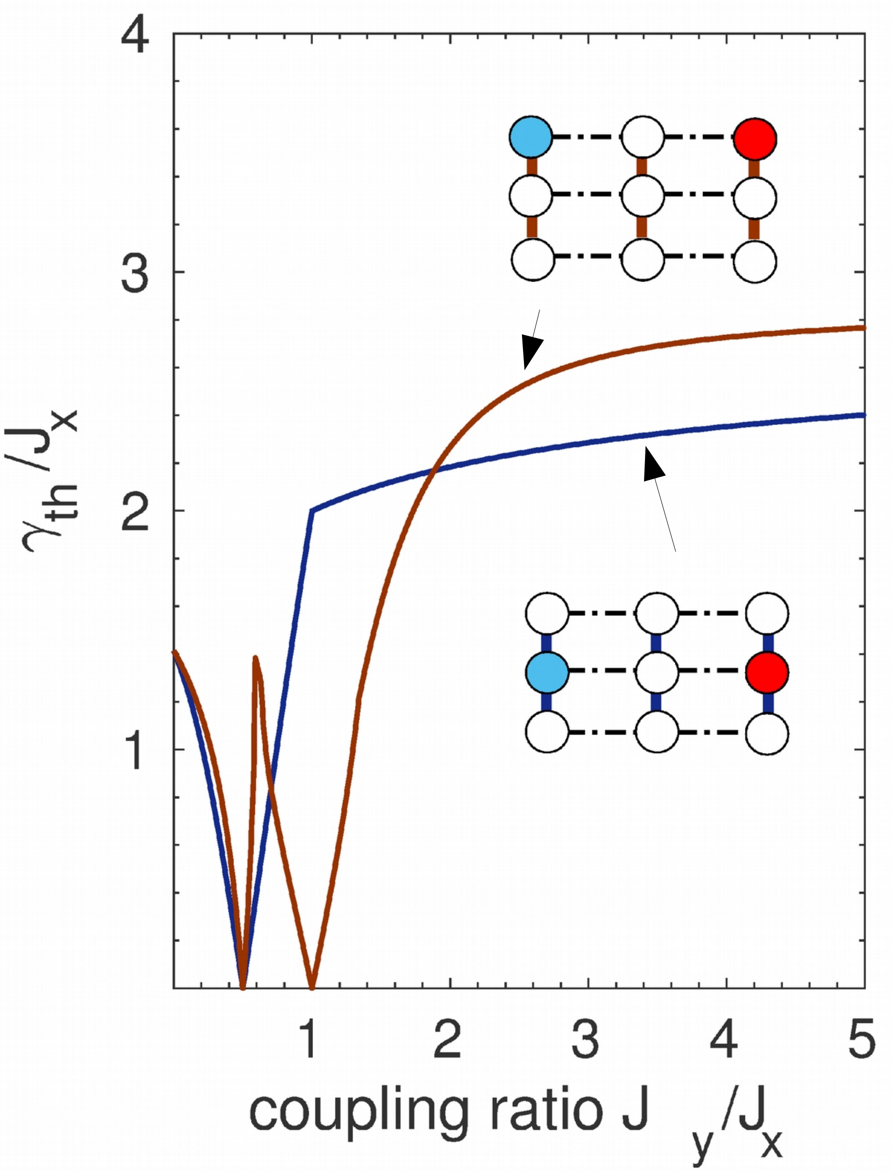


$\mathcal{PT}$-dimer chains



$\mathcal{PT}$-trimer chains





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Summary

- High asymmetry in 2D lattice models with a few gain-loss sites give rise to a scaling of the PT-threshold.
- PT-symmetric dimer and trimer case the threshold is doubled or even tripled depending on the coupling ratio.

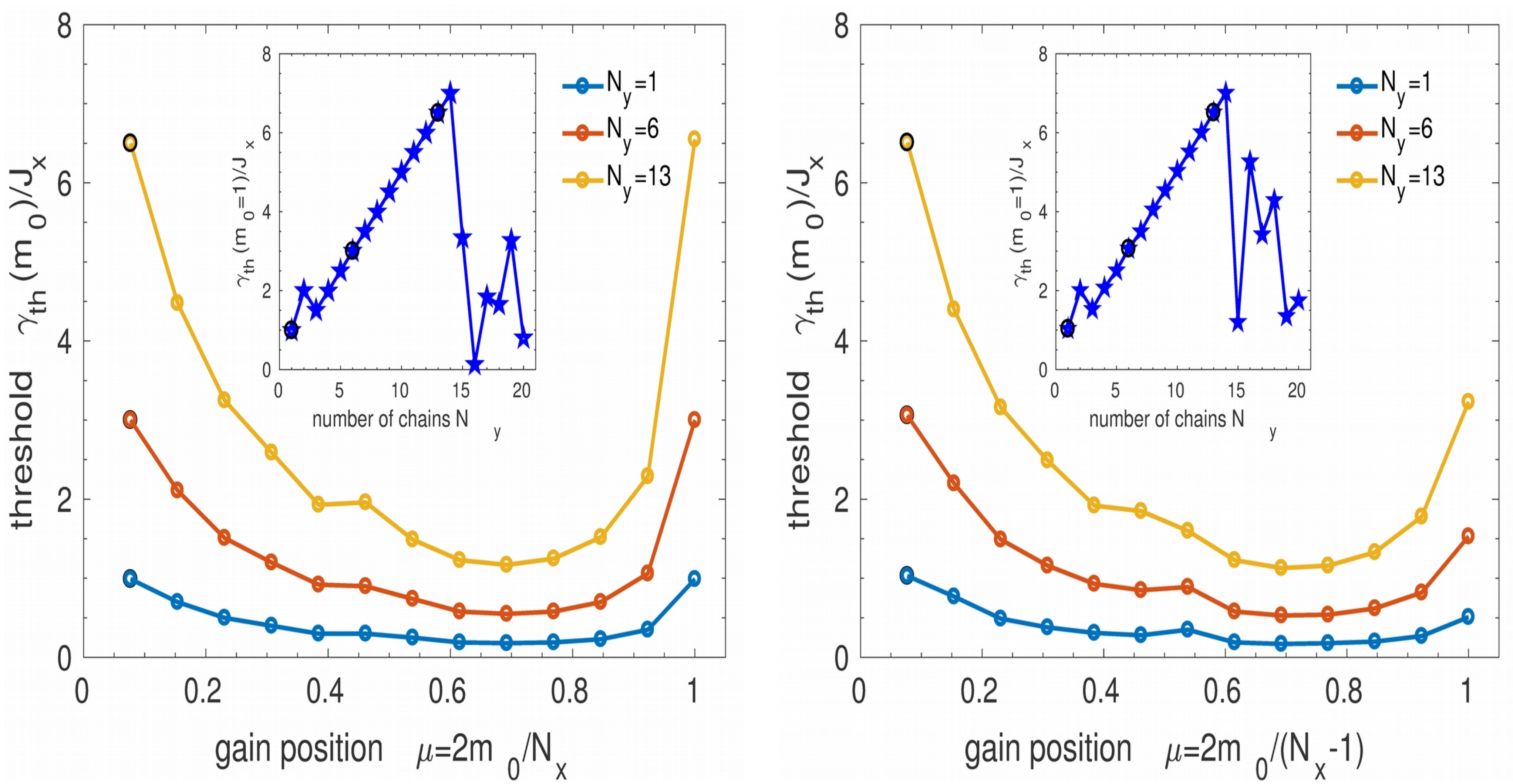
- This provides tunability of the $\mathcal{PT}$-threshold in experimental samples.

Investigation of these results in other $\mathcal{PT}$-symmetric systems would provide deeper insight.

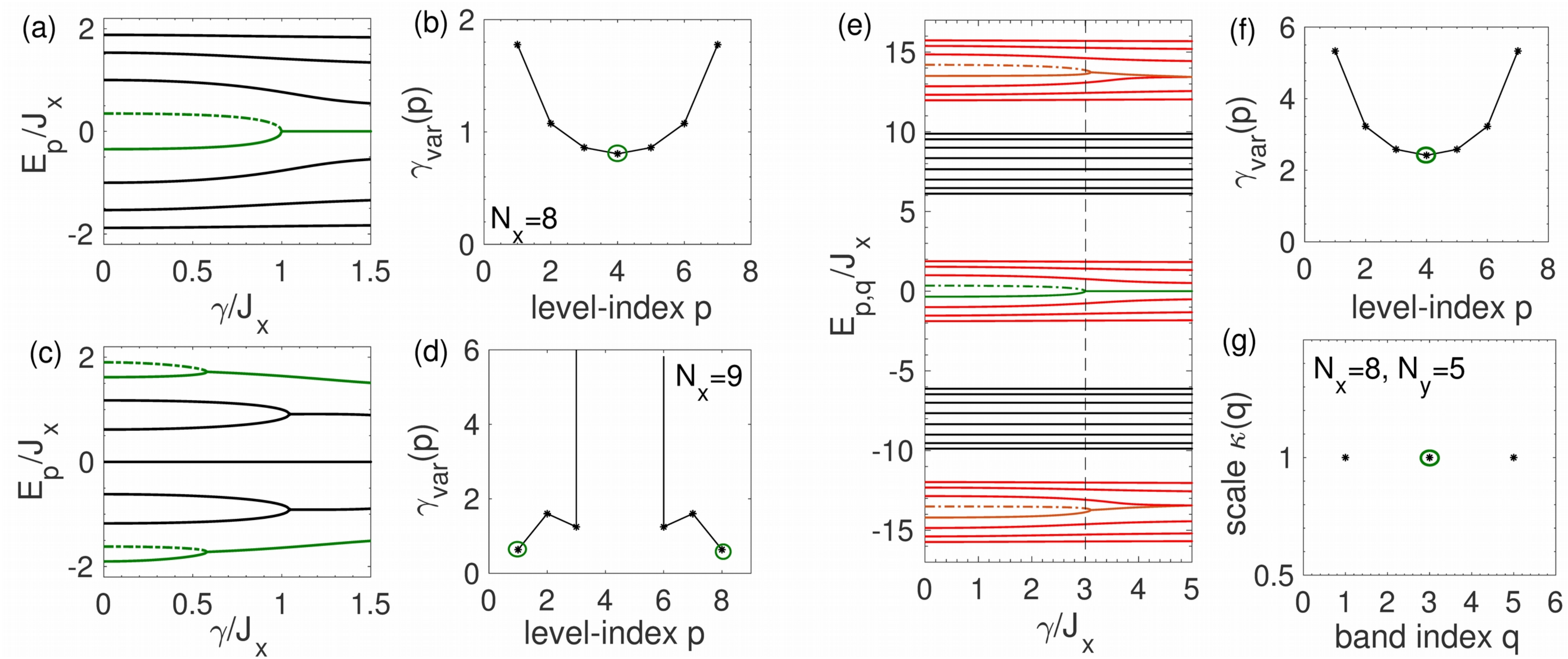
Thanks for listening...

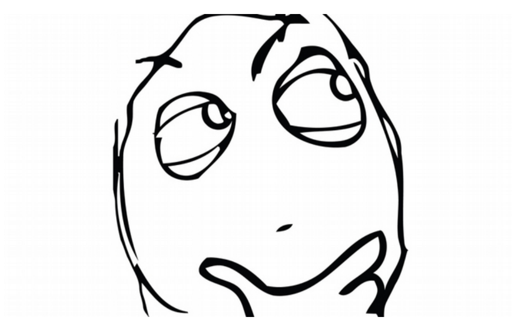
Any Questions ???

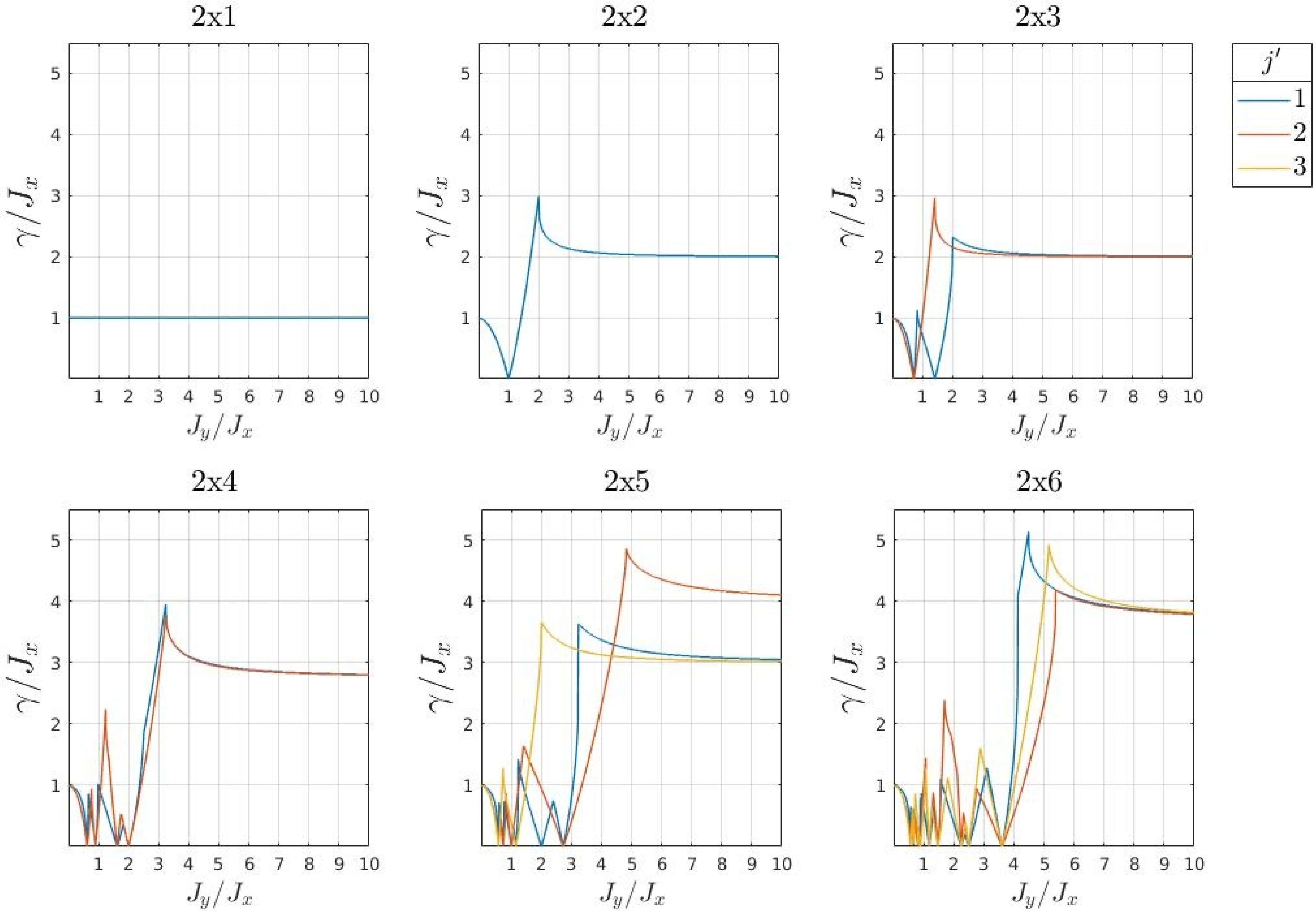
2D PT chains (periodic B.C.)

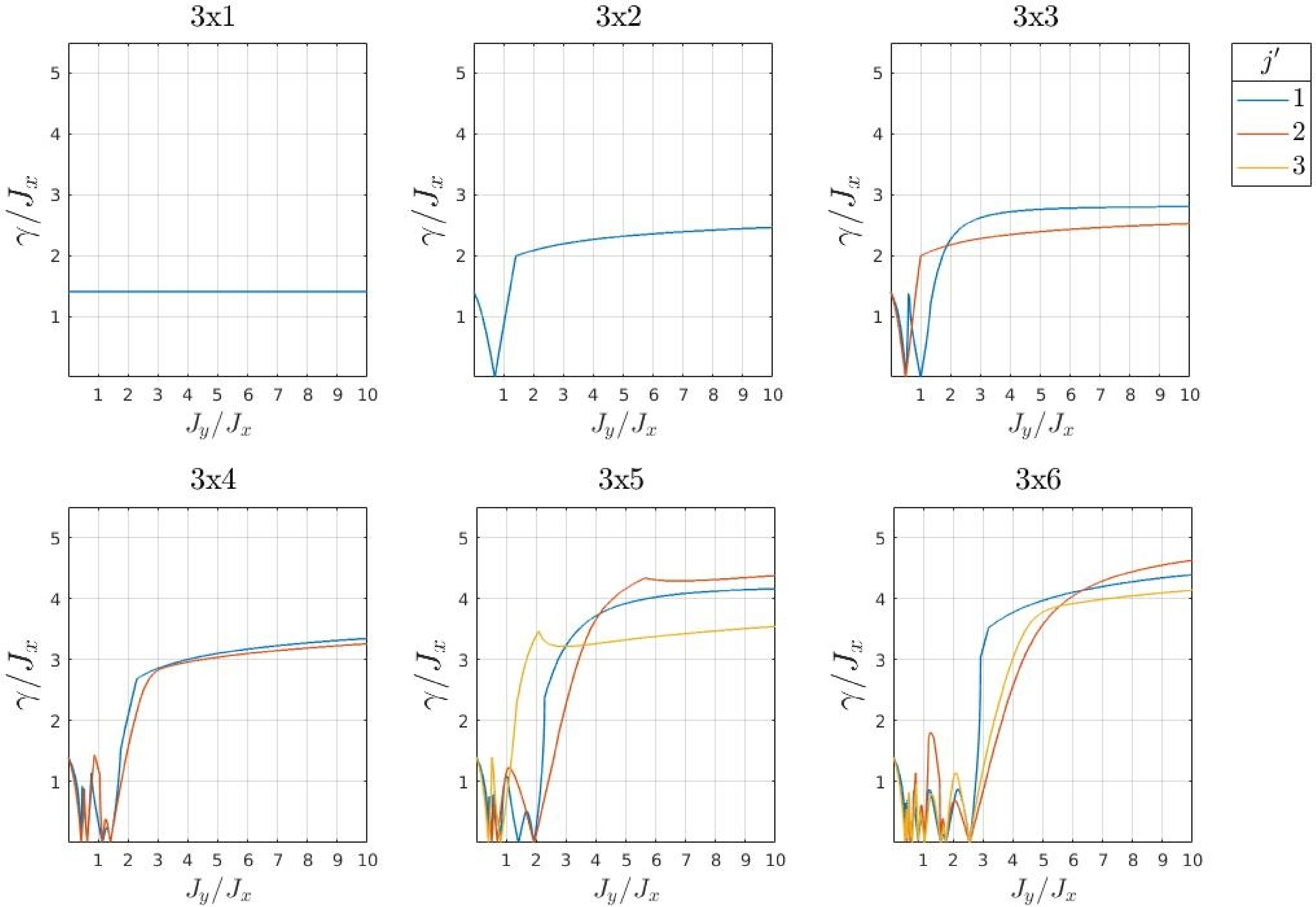


Variational analysis






2x1
2x2
2x3
2x4
2x5
2x6
γ/J_x
J_y/J_x
j'
1
2
3


3x1
3x2
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γ/J_x
J_y/J_x
j'
1
2
3

Enhancement of $\mathcal{PT}$-transition threshold by strong coupling to neutral chains

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