Non-Hermitan Quantum Systems as Quantum Devices

Manas Kulkarni

International Centre for Theoretical Sciences

ICTS- TIFR, Bangalore

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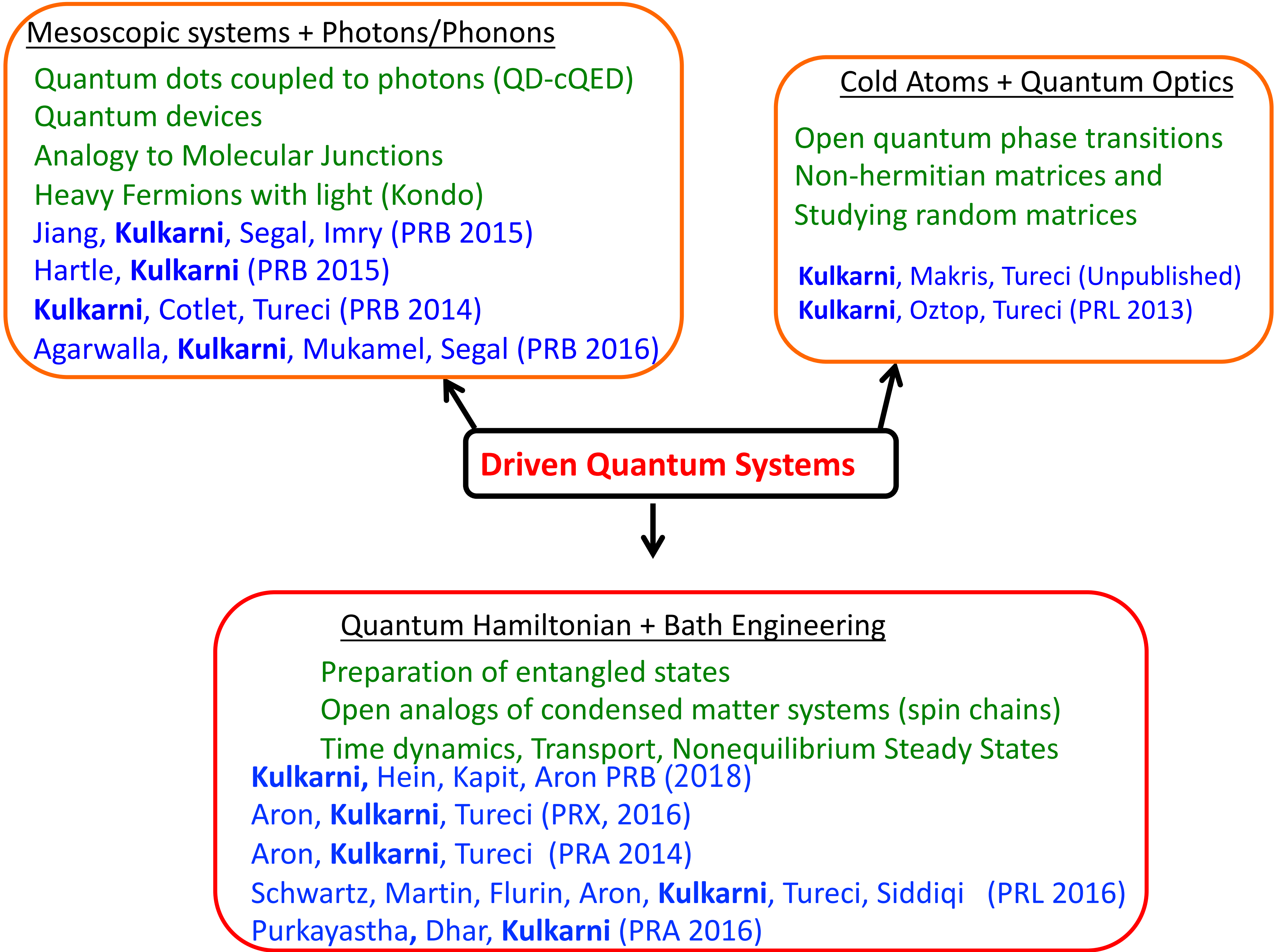


Mesoscopic systems + Photons/Phonons

Quantum dots coupled to photons (QD-cQED)
Quantum devices
Analogy to Molecular Junctions
Heavy Fermions with light (Kondo)
Jiang, **Kulkarni**, Segal, Imry (PRB 2015)
Hartle, **Kulkarni** (PRB 2015)
Kulkarni, Cotlet, Tureci (PRB 2014)
Agarwalla, **Kulkarni**, Mukamel, Segal (PRB 2016)

Cold Atoms + Quantum Optics

Open quantum phase transitions
Non-hermitian matrices and
Studying random matrices

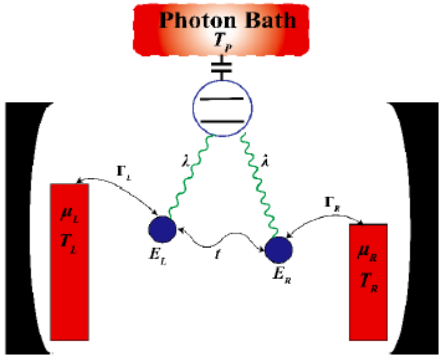
Kulkarni, Makris, Tureci (Unpublished)
Kulkarni, Oztop, Tureci (PRL 2013)

Driven Quantum Systems

Quantum Hamiltonian + Bath Engineering

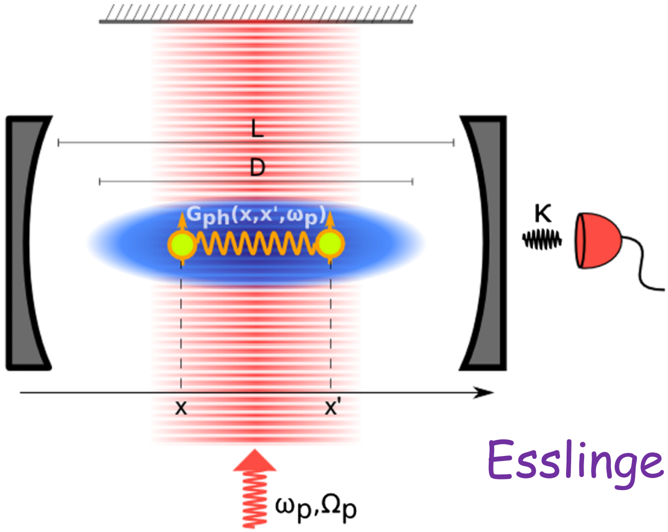
Preparation of entangled states
Open analogs of condensed matter systems (spin chains)
Time dynamics, Transport, Nonequilibrium Steady States
Kulkarni, Hein, Kapit, Aron PRB (2018)
Aron, **Kulkarni**, Tureci (PRX, 2016)
Aron, **Kulkarni**, Tureci (PRA 2014)
Schwartz, Martin, Flurin, Aron, **Kulkarni**, Tureci, Siddiqi (PRL 2016)
Purkayastha**,** Dhar, **Kulkarni** (PRA 2016)

Mesoscopic systems + Photons/Phonons



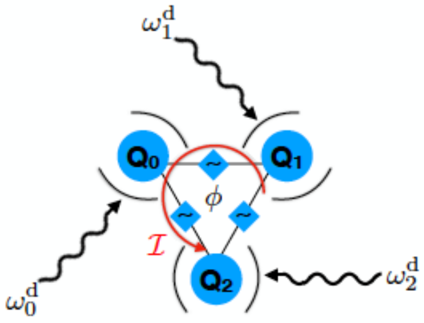
Petta (Princeton)
Kontos (ENS Paris)

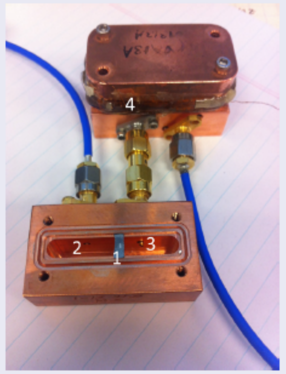
Cold Atoms + Quantum Optics



Esslinger (Zurich)

Driven Quantum systems





courtesy of Siddiqi's group
@ UC Berkeley

Sidiqqi (Berkeley)
Martinis (Google-Santa Barbara)
Vijay (TIFR)

Quantum Hamiltonian + Bath Engineering

Common Methods we use:

- Diagrammatic Keldysh
- Lindblad Master Eq
- Redfield Equation
- Exact brute-force numerics
- Quantum Langevin

Engineer /Design an interesting systems made of matter (atoms or artificial atoms) and bosons (photons, phonons)

<u>Study Fundamental Concepts in Physics and Mathematics</u>

- Entanglement
- Transport
- Correlations
- Non-Hermitian Matrices, Pseudo-Spectrum
- Phase Transitions
- Statistics

<u>Use this for technological Applications</u>

- Quantum Devices (diodes Rectifiers, Transistors)
- Photon Amplifers
- Lasers in microwave regime
- Target State Preparation

Non-Hermitan Quantum Systems as Quantum Devices

Part - A

Microwave Ampliers and Masers
[Agarwalla, Kulkarni, Segal, in preparation]

Agarwalla -- Dept of Physics, IISER Pune

Segal --Chemical Physics Theory Group,
Centre of Quantum Information & Quantum Control,
Univ of Toronto

Part - B

Diodes, Rectifier and Transistors
[Lu, Wang, Ren, Kulkarni, Jiang, in preparation]

Lu, Wang, Jiang --School of Physical Science and Technology
& Collaborative Innovation Center of Suzhou Nano Science
and Technology, Soochow University

Key Lab of Advanced Optical Manufacturing Technologies
of Jiangsu Province & Key Lab of Modern Optical Technologies of
Education Ministry of China, Soochow University

Ren - Center for Phononics and Thermal Energy Science,
School of Physics Science and Engineering, Tongji University

Related older papers

Agarwalla, **Kulkarni**, Mukamel, Segal, (PRB 2016)
Kulkarni, Cotlet, Tureci (PRB 2014)

Microwave amplifiers,
Single photon sources

Purkayastha, Dhar, **Kulkarni**, (PRA 2016)
Jiang, **Kulkarni**, Segal, Imry (PRB 2015)

Rectifiers, Transistors

Acknowledgements

Petta Lab (Princeton)
Kontos Lab (ENS, Paris)
Siddiqi Lab (Berkeley)

Part - A

Microwave Ampliers and Masers

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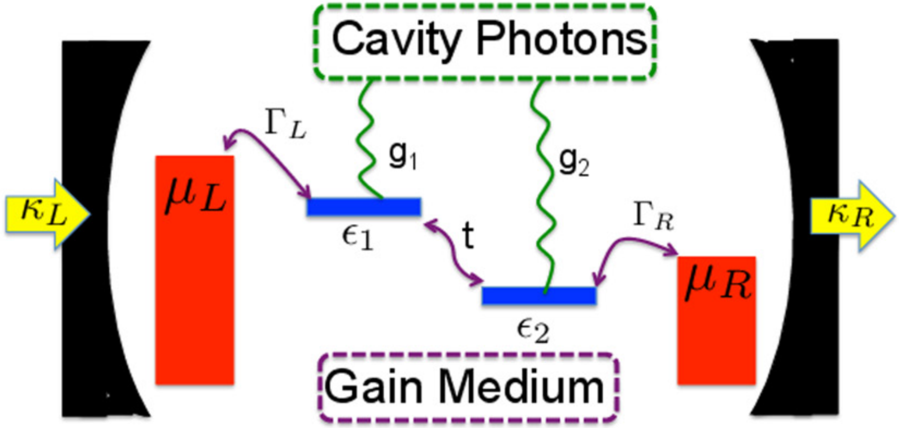

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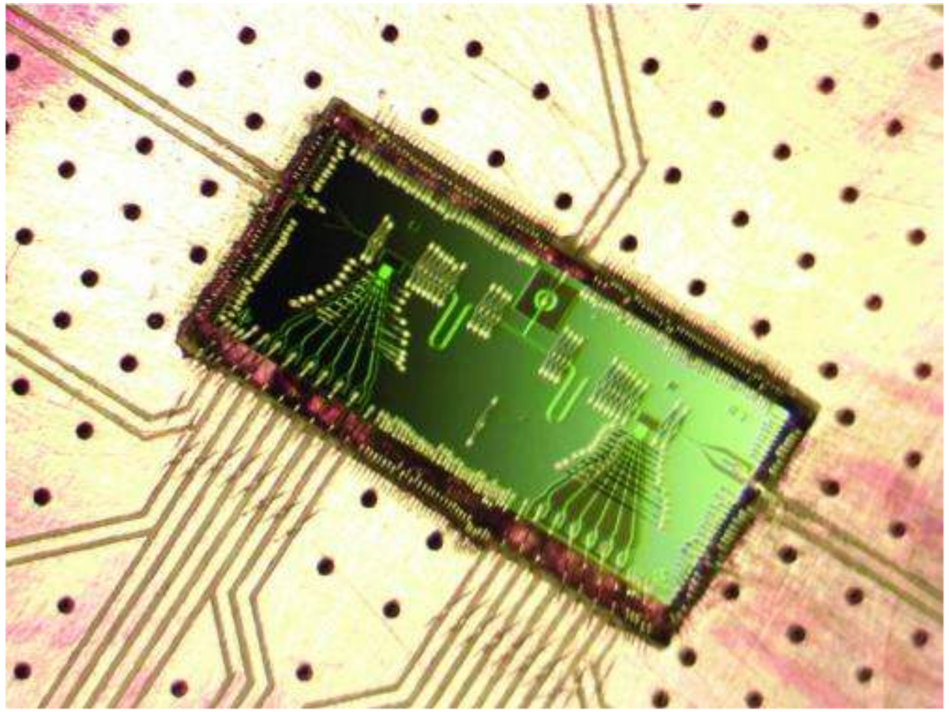
NEWS

Rice-sized laser, powered one electron at a time, bodes well for quantum computing

January 15, 2015



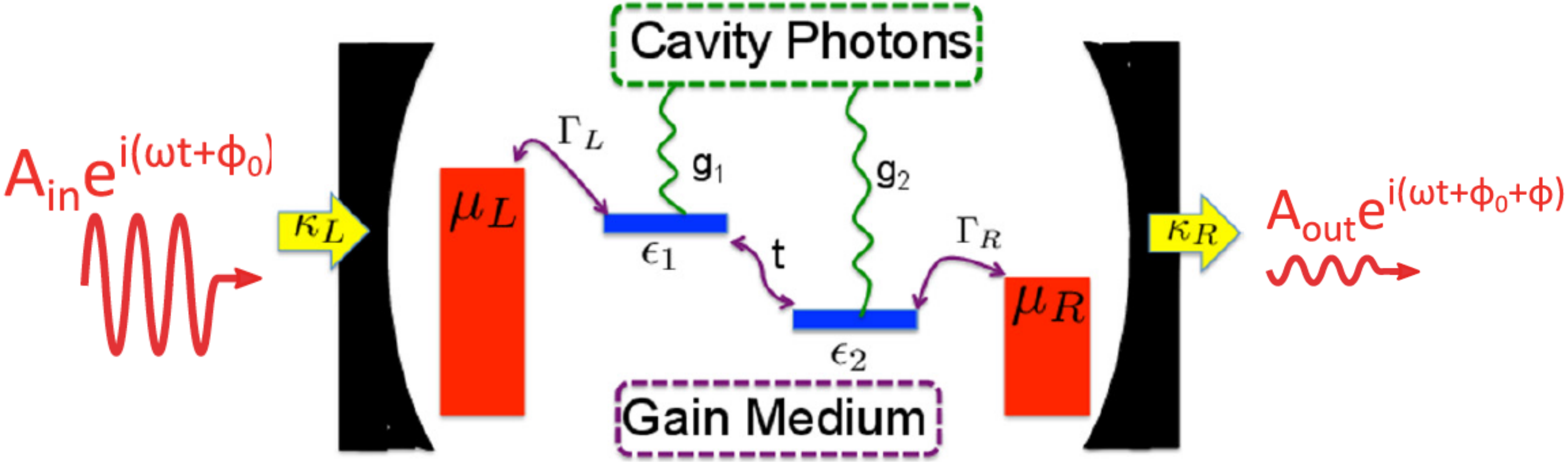
Princeton University researchers have built a rice grain-sized microwave laser. Credit: Jason Petta, Princeton University

Two Double-Quantum Dots

Science **347**, 285 (2015)
Petta Lab Princeton

Also, one double-quantum dot, Petta Group (PRL 2017)

DQD circuit-QED setup



Double Quantum Dot c-QED setup

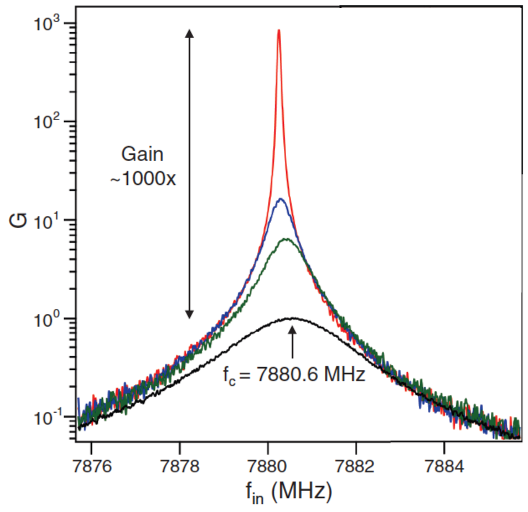
Agarwalla, **Kulkarni**, Mukamel, Segal (PRB 2016)
Kulkarni, Cotlet, Tureci (PRB 2014)

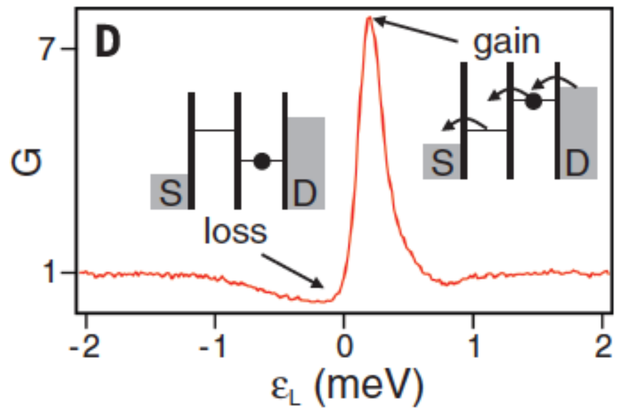
} Diagrammatic Keldysh Approach

Experimental observables

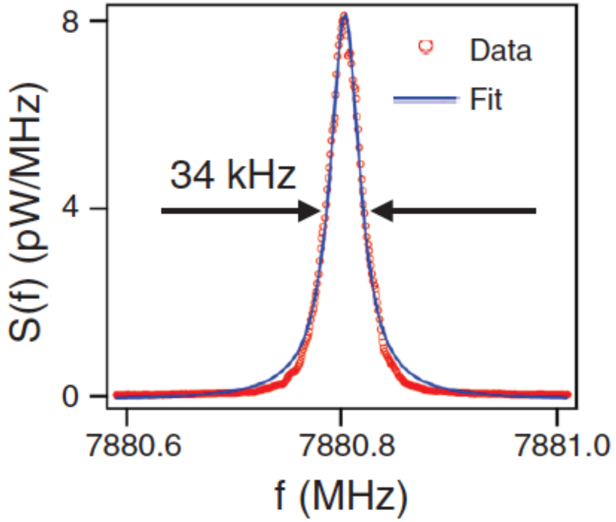
Photonic Measurements:

Photon Transmission (Gain)

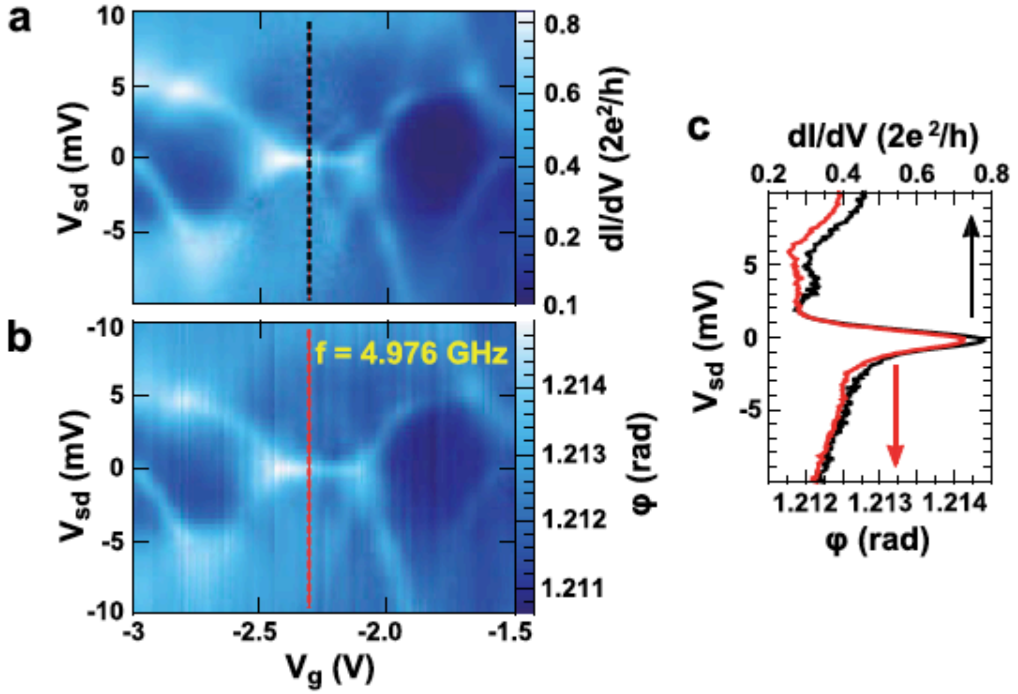




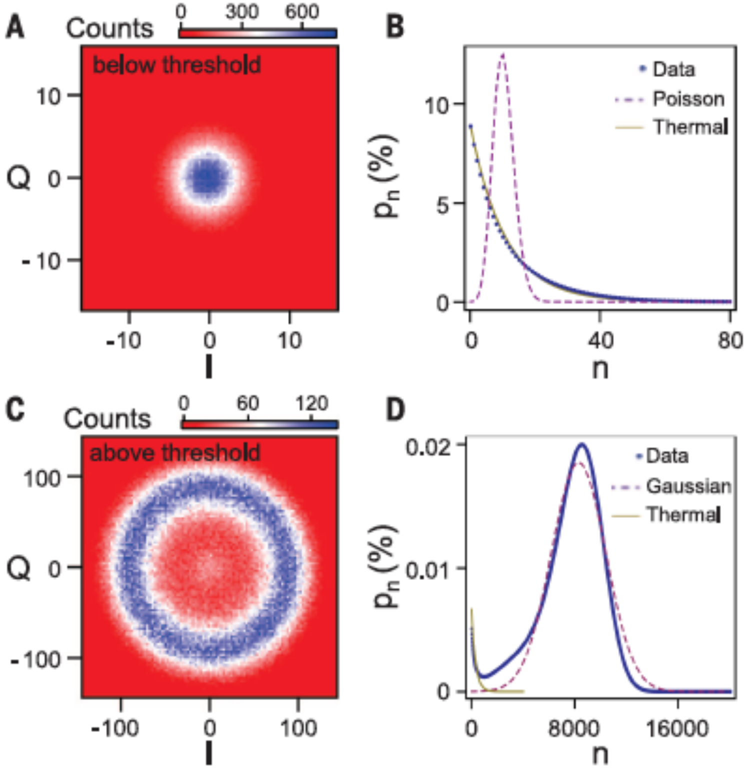
Emission Spectrum



phase spectroscopy

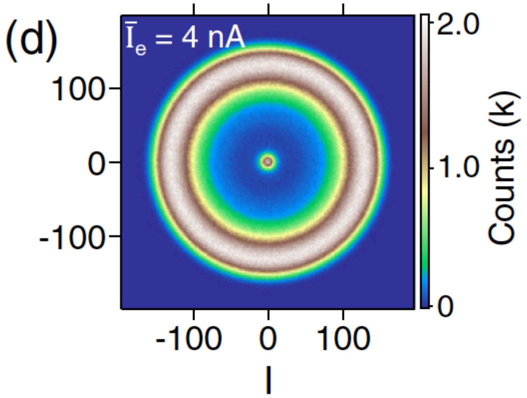


Mazing in double-double dot



Science **347**, 285 (2015)
Petta Lab Princeton

Mazing in single double dot



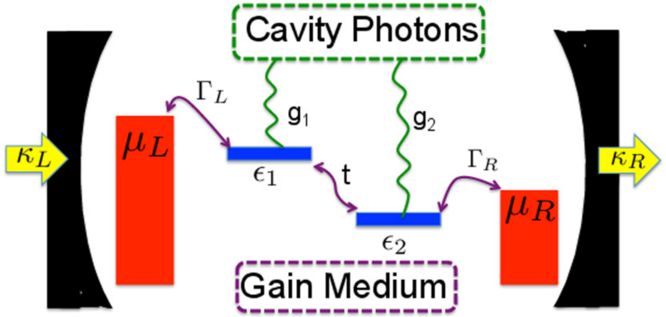
PRL 119, 097702 (2017)
Petta Lab Princeton

Photon Statistics:
No-Masing (Thermal)
Masing (Poisson)

Constructing the Hamiltonian and Bath

$$H = H_{matter} + H_{cavity} + H_{matter-cavity}$$

$$H_{matter} = H_{DQD} + H_{DQD-lead} + H_{DQD-phonon}$$



$$H_{DQD} = \frac{\epsilon}{2}\tau_z + t_c\tau_x$$

$$H_{DQD-lead} = \sum_{k,\alpha=L,R} \epsilon_{k\alpha} c^{\dagger}_{k\alpha} c_{k\alpha} + \sum_{k} \left[\lambda_{kL} c_{kL} |L\rangle\langle 0| + \lambda_{kR} c_{kR} |R\rangle\langle 0|\right] + h.c$$

$$H_{\mathrm{DQD-phonon}} = \sum_q \omega_q b^{\dagger}_q b_q + \tau_z \sum_q \lambda_q (b_q + b^{\dagger}_q)$$

Phonon bath

$$H_{cavity} = \omega_c a^{\dagger} a + \sum_{j\in K} \omega_{jK} a^{\dagger}_{jK} a_{jK} + \sum_{j\in K} \nu_j a^{\dagger}_{jK} a + h.c$$

Cavity Bath

$$H_{DQD-cavity} = g\tau_z (a + a^{\dagger})$$

Working with dot-Eigenstates

$$H_{DQD} = \frac{\Omega}{2}(d_e^\dagger d_e - d_g^\dagger d_g)$$

$$H_{DQD-cavity} = -g\sin\theta(d_e^\dagger d_g a + d_g^\dagger d_e a^\dagger)$$

$$\begin{aligned} H_{\mathrm{DQD-phonon}} = \sum_q \omega_q b_q^\dagger b_q + \sum_q \lambda_q \Big[\cos\theta\big(d_e^\dagger d_e - d_g^\dagger d_g\big) \\ - \sin\theta\big(d_e^\dagger d_g + d_g^\dagger d_e\big)\Big]\big(b_q + b_q^\dagger\big), \qquad (11) \end{aligned}$$

$$H_{DQD-lead} = \sum_k \begin{pmatrix} t_{kL} b_{kL} & t_{kR} b_{kR} \end{pmatrix} \begin{pmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix} \begin{pmatrix} d_e^\dagger \\ d_g^\dagger \end{pmatrix} + h.c.$$

We want to write the reduced density matrix of the dot and cavity mode

$$\dot{\rho} = -i[\underset{\substack{\uparrow \\ \text{Unitary Part}}}{H_0}, \rho] + \underbrace{\mathcal{L}_{elec}[\rho] + \mathcal{L}_{cavity}[\rho] + \mathcal{L}_{phon}[\rho]}_{\text{Non-Hermitian Part}}$$

Structure of non-Hermitian part

$$\dot{\rho} = -i[H_0, \rho] + \mathcal{L}_{elec}[\rho] + \mathcal{L}_{cavity}[\rho] + \mathcal{L}_{phon}[\rho]$$

Electron bath (leads)

$$\mathcal{L}_{elec}[\rho] = \sum_{\alpha=L,R,\nu=e,g} \mathcal{L}_{\alpha\nu}[\rho]$$

$$\mathcal{L}_{\alpha\nu}[\rho] = \frac{1}{2}\Gamma_{\alpha\nu}(\theta)\left[f_\alpha(\epsilon_\nu)D[d_\nu, \rho] + (1 - f_\alpha(\epsilon_\nu))D[d_\nu^\dagger, \rho]\right]$$

Fermi distribution

$$\Gamma_{Le(Rg)}(\theta) = \Gamma_{Le(Rg)} \cos^2(\tfrac{\theta}{2})$$

$$\Gamma_{Lg(Re)}(\theta) = \Gamma_{Lg(Re)} \sin^2(\tfrac{\theta}{2})$$

Photon bath (cavity leakage)

$$\mathcal{L}_{cavity}[\rho] = \frac{\kappa}{2}D[a^\dagger, \rho]$$

Structure of non-Hermitian part

Phonon bath

$$\mathcal{L}_{phon}[\rho] = \frac{\gamma_u(\epsilon)}{2} D[d_g^\dagger d_e, \rho] + \frac{\gamma_d(\epsilon)}{2} D[d_e^\dagger d_g, \rho] + \frac{\gamma_\phi(\epsilon)}{2} D[(d_e^\dagger d_e - d_g^\dagger d_g), \rho]$$

Bosonic distribution

Phonon bath spectral density

$$\gamma_u(\epsilon) = 2\sin^2(\theta)\, n_{th}(\Omega)\, J(\Omega)$$ Upward pump

$$\gamma_d(\epsilon) = 2\sin^2(\theta)\, (1 + n_{th}(\Omega))\, J(\Omega)$$ Downward pump

$$\gamma_\phi(\epsilon) = 2\cos^2(\theta)[1 + 2n_{th}(0)]J(0)$$ Dephasing

We will now write equations of motion along with a semi-classical approximation

$$\langle d_e^\dagger d_g a \rangle \approx \langle d_e^\dagger d_g \rangle \langle a \rangle$$

Defining $< d_e^\dagger d_g >= \rho_{eg}, \quad < d_e^\dagger d_e >= \rho_{ee}$ and so on...

Reduced density matrix equations

$$\dot{\rho}_{ee} = \left(\Gamma^{c}_{Le} + \Gamma^{s}_{Re}\right)\rho_{00} - \left(\bar{\Gamma}^{c}_{Le} + \bar{\Gamma}^{s}_{Re} + \gamma_d\right)\rho_{ee} + \gamma_u \rho_{gg} - ig\sin\theta\left(\rho_{ge}\langle a\rangle - h.c\right)$$

$$\dot{\rho}_{gg} = \left(\Gamma^{s}_{Lg} + \Gamma^{c}_{Rg}\right)\rho_{00} - \left(\bar{\Gamma}^{s}_{Lg} + \bar{\Gamma}^{c}_{Rg} + \gamma_u\right)\rho_{gg} + \gamma_d \rho_{ee} + ig\sin\theta\left(\rho_{ge}\langle a\rangle - h.c\right)$$

$$\dot{\rho}_{eg} = -i\Omega\rho_{eg} + ig\sin\theta\left(\langle n_e\rangle - \langle n_g\rangle\right)\langle a\rangle - \left(\frac{1}{2}\Gamma_{eff} + 2\gamma_\phi\right)\rho_{eg}$$

$$\dot{\langle a\rangle} = -i\omega_c\langle a\rangle - \frac{1}{2}\left(\kappa\langle a\rangle + 2ig\sin\theta\rho_{eg}\right) + \sqrt{\frac{\kappa}{2}}E\cos(\omega_d t)$$

Experimentally, one can measure the absolute value and phase of $t(\omega_d) \equiv \frac{\sqrt{2\kappa}\langle a\rangle}{E}$

$$t(\omega_d) \equiv \frac{\sqrt{2\kappa}\langle a\rangle_{ss}}{E} = \frac{i\kappa/2}{(\omega_d - \omega_c) + i\kappa/2 - \chi_{\mathrm{el}}(\omega_d)}$$

$$\chi_{el}(\omega_d) = \frac{g^2\sin^2(\theta)}{(\omega_d - \Omega) + i(\frac{1}{2}\Gamma_{eff} + 2\gamma_\phi)}(\rho_{gg} - \rho_{ee})|_{g=0}$$

- bias voltage and the temperatures of the leads
- Coupling between the DQD and fermionic leads as
- Coupling between the DQD and the phononic bath.

Transmission and Phase Response

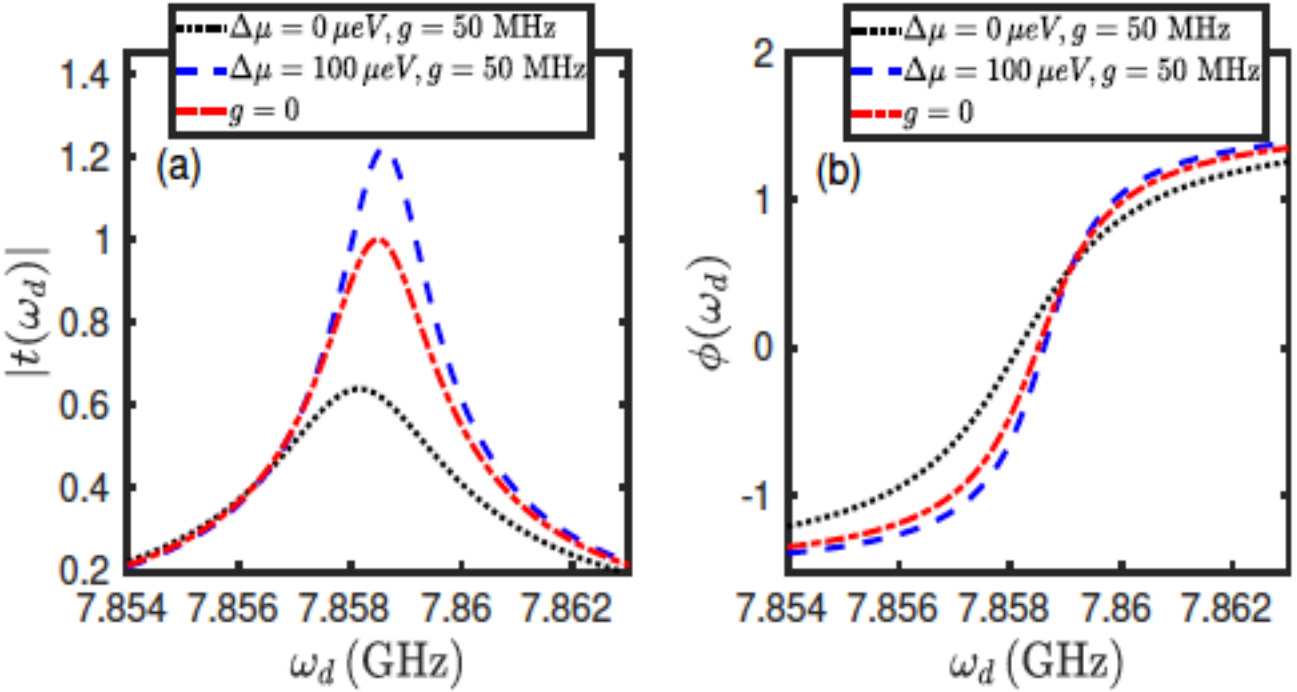
$$t(\omega_d) = \frac{i\kappa/2}{(\omega_d - \omega_c) + i\kappa/2 - \chi_{el}(\omega_d)}$$

$$|t(\omega_d\!=\!\omega_c)| = \frac{\kappa/2}{\left[(\chi'_{el})^2 + (\kappa/2 - \chi''_{el})^2\right]^{1/2}}$$

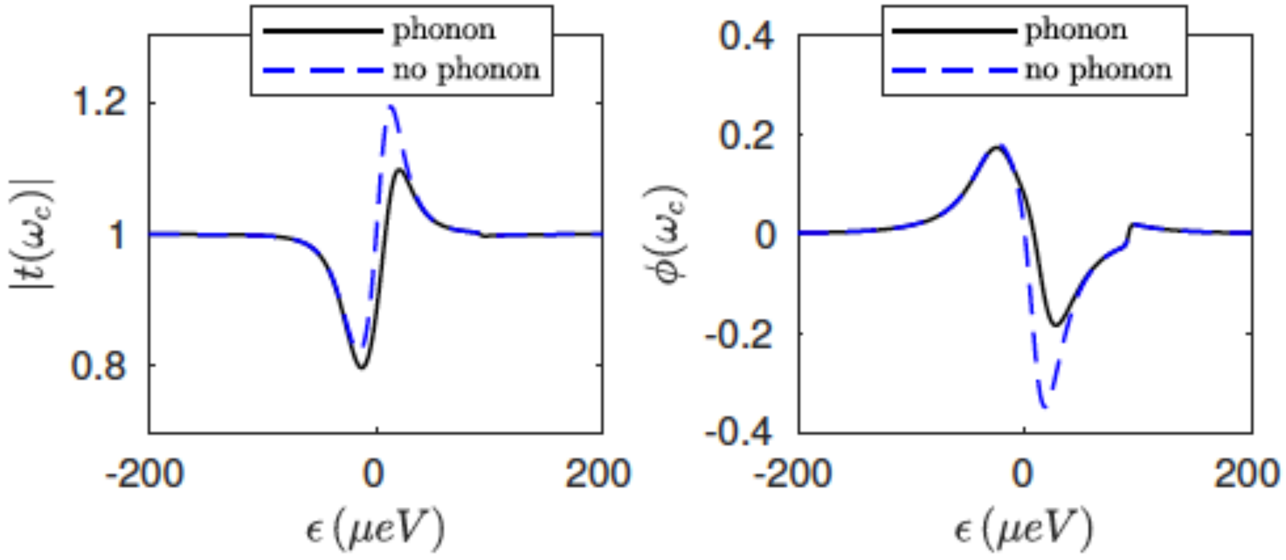
$$\tan\phi(\omega_d\!=\!\omega_c) = \frac{\chi'}{\chi'' - \kappa/2}$$

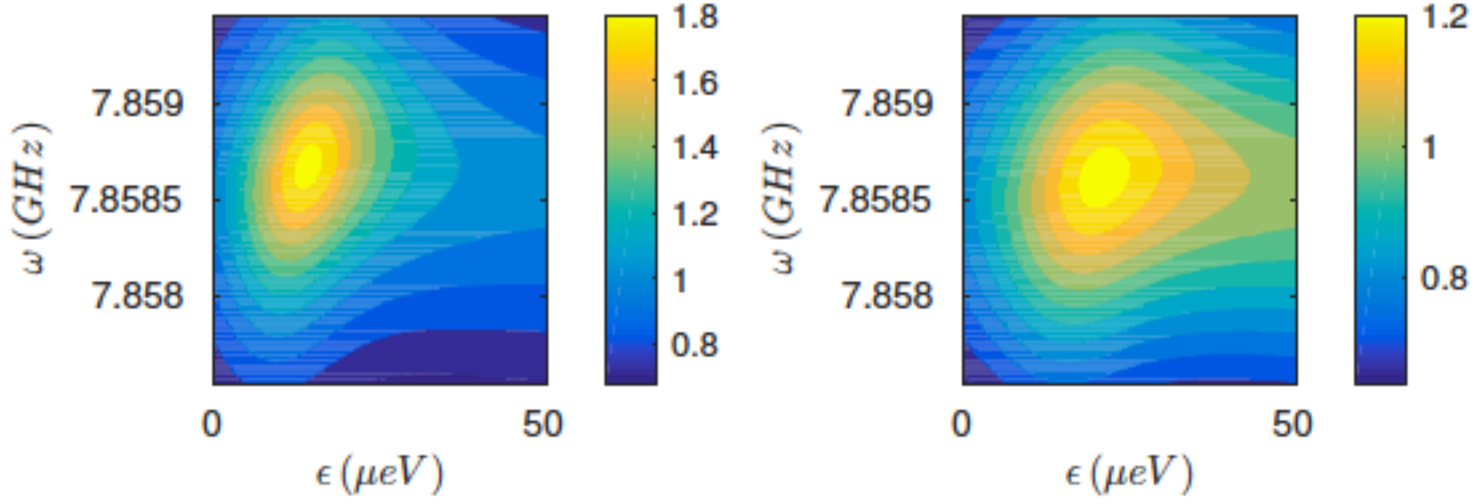
Threshold Condition

$$\frac{2g^2\sin^2(\theta)}{(\frac{1}{2}\Gamma_{eff} + 2\gamma_\phi)}(\rho_{ee} - \rho_{gg})|_{g=0} \quad = \kappa$$

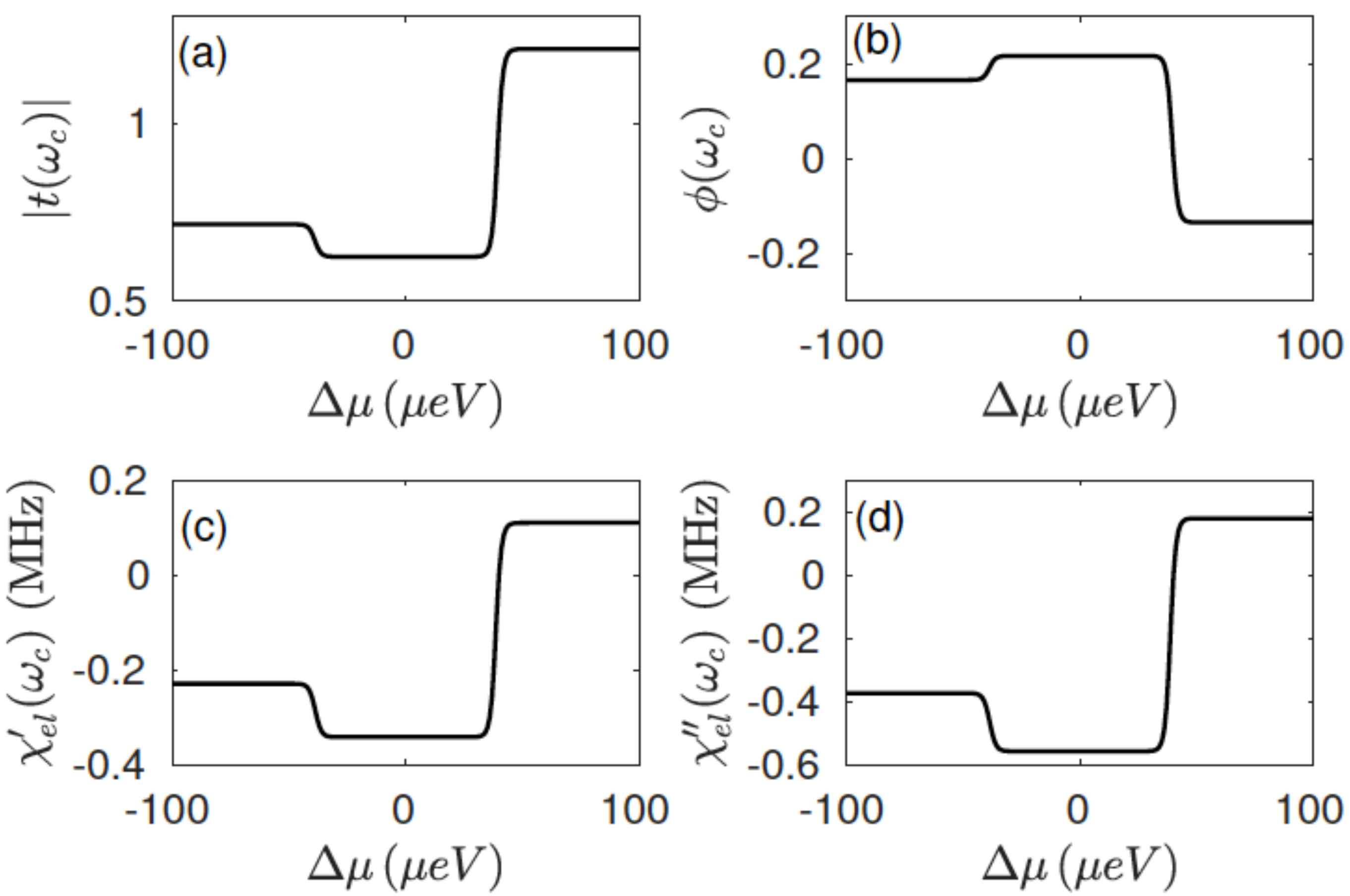


Transmission and Phase Response





Transmission, Phase Response, Charge Susceptibility



Photon Statistics

$$\rho_{phot}(t) = Tr_{elec+phon}[\rho(t)]$$

$$p_m(t) = \langle m|\rho_{ph}(t)|m\rangle$$

$$\begin{aligned}\frac{d}{dt}p_m &= -ig\sin\theta\Big[\sqrt{m+1}\big(\rho_{ge;m+1m} - \rho_{eg;mm+1}\big) + \sqrt{m}\big(\rho_{eg;m-1m} - \rho_{ge;mm-1}\big)\Big] \\ &+ \quad \kappa(1+\bar{n})\big[(m+1)p_{m+1} - mp_m\big] + \kappa\bar{n}\big[mp_{m-1} - \kappa\bar{n}(m+1)p_m\big]\end{aligned}$$

$$\begin{pmatrix} \dot{\rho}_{gg;m,n} \\ \dot{\rho}_{ee;m-1,n-1} \\ \dot{\rho}_{eg;m-1,n} \\ \dot{\rho}_{ge;m,n-1} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & -ig\sin\theta\sqrt{m} & ig\sin\theta\sqrt{n} \\ 0 & a_{22} & ig\sin\theta\sqrt{n} & -ig\sin\theta\sqrt{m} \\ -ig\sin\theta\sqrt{m} & ig\sin\theta\sqrt{n} & a_{33} - i(\Omega-\omega_c) & 0 \\ ig\sin\theta\sqrt{n} & -ig\sin\theta\sqrt{m} & 0 & a_{33} + i(\Omega-\omega_c) \end{pmatrix} \begin{pmatrix} \rho_{gg;m,n} \\ \rho_{ee;m-1,n-1} \\ \rho_{eg;m-1,n} \\ \rho_{ge;m,n-1} \end{pmatrix} + \begin{pmatrix} b_1 p_m \\ b_2 p_{m-1,n-1} \\ 0 \\ 0 \end{pmatrix} \tag{39}$$

Plugging back solution of above we get,

$$\frac{d}{dt}p_m = \frac{m\big[Ap_{m-1} - A_b p_m\big]}{1 + mC + \frac{(\Omega-\omega_c)^2}{a_{33}^2}} - \frac{(m+1)\big[Ap_m - A_b p_{m+1}\big]}{1 + (m+1)C + \frac{(\Omega-\omega_c)^2}{a_{33}^2}} + \kappa\big[(m+1)p_{m+1} - mp_m\big]$$

Below threshold

$$\frac{d}{dt}p_m = m\left[Ap_{m-1} - A_b p_m\right] - (m+1)\left[Ap_m - A_b p_{m+1}\right] + \kappa\left[(m+1)p_{m+1} - mp_m\right]$$

Solution:

$$p_m = \left(1 - \frac{A}{A_b+\kappa}\right)\left(\frac{A}{A_b+\kappa}\right)^m \qquad T_{\text{eff}} = \frac{\hbar\omega_c}{k_B \ln\left(\frac{A_b+\kappa}{A}\right)}$$

Thermal with effective temperate $A - A_b \leq \kappa$

All regimes

$$p_m = \frac{p_0 y! x^m}{(m+y)!}$$

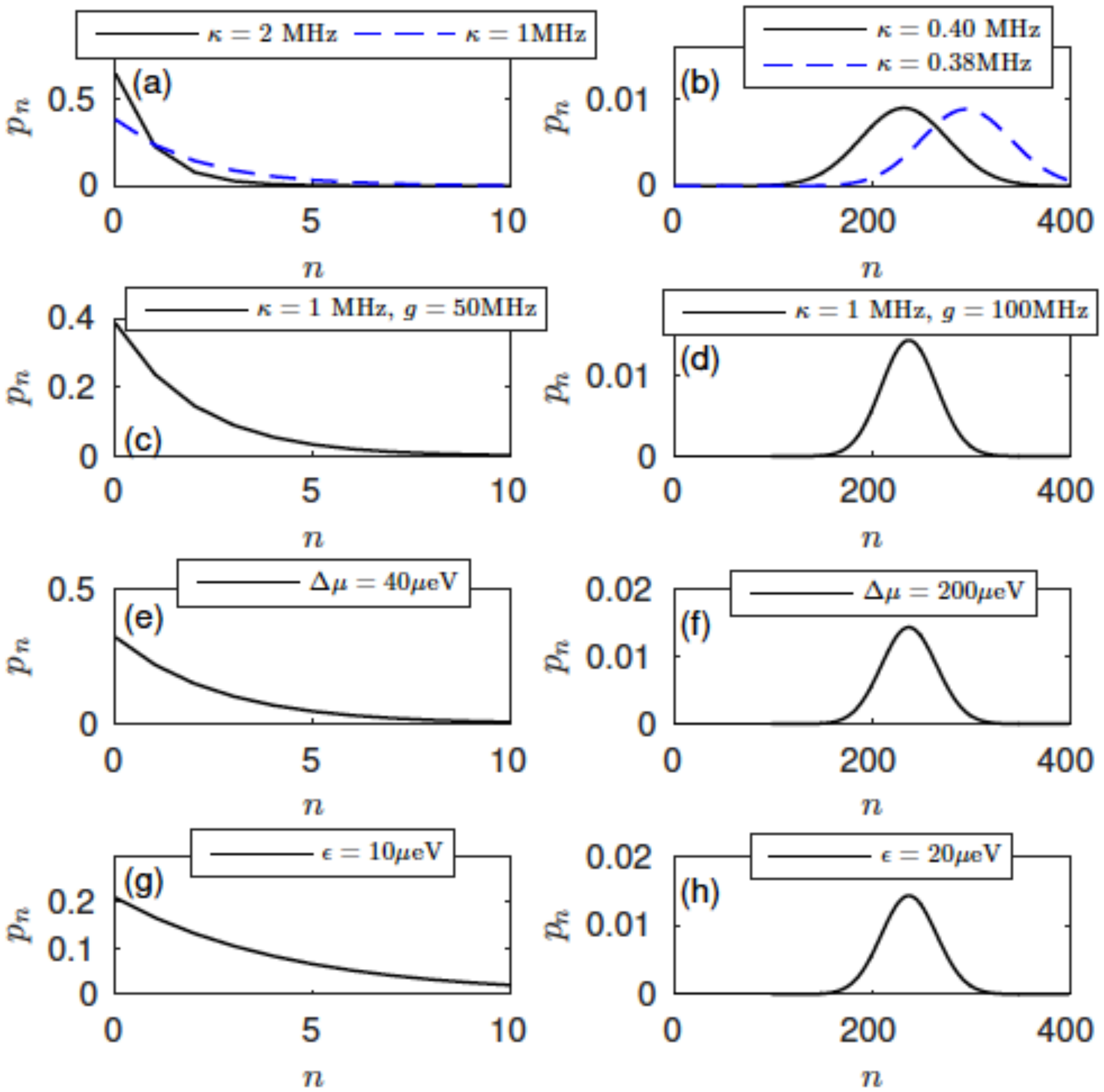
where

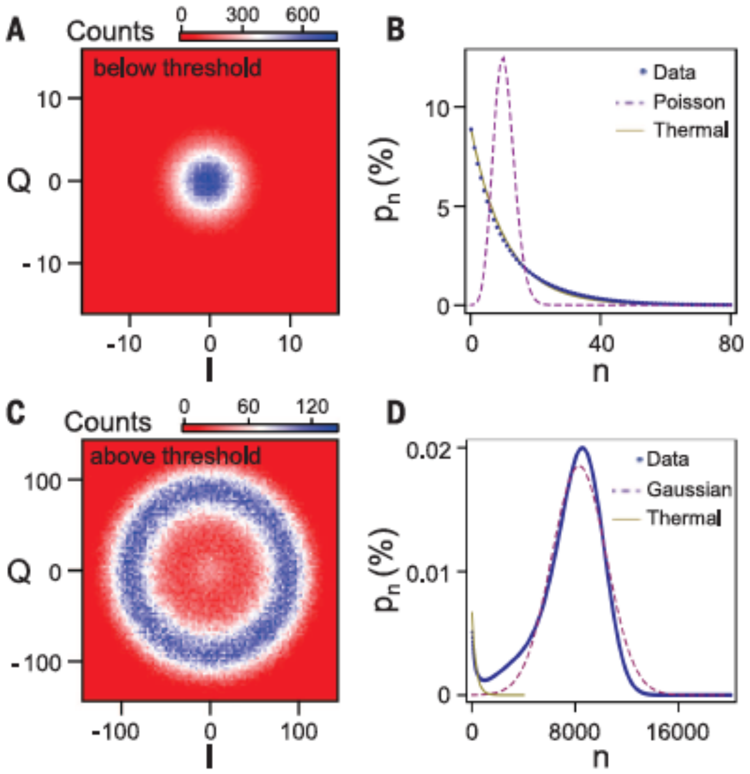
$$x = \frac{A}{\kappa c}, \qquad y = \frac{1}{\kappa c}(\kappa + A_b)$$

Poissonian with effective temperate

Agarwalla, Kulkarni, Segal (2018, in preparation)

Thermal to Poissonian Statistics





We will also compute Fano factors

Agarwalla, Kulkarni, Segal (2018, in preparation)

Part - B

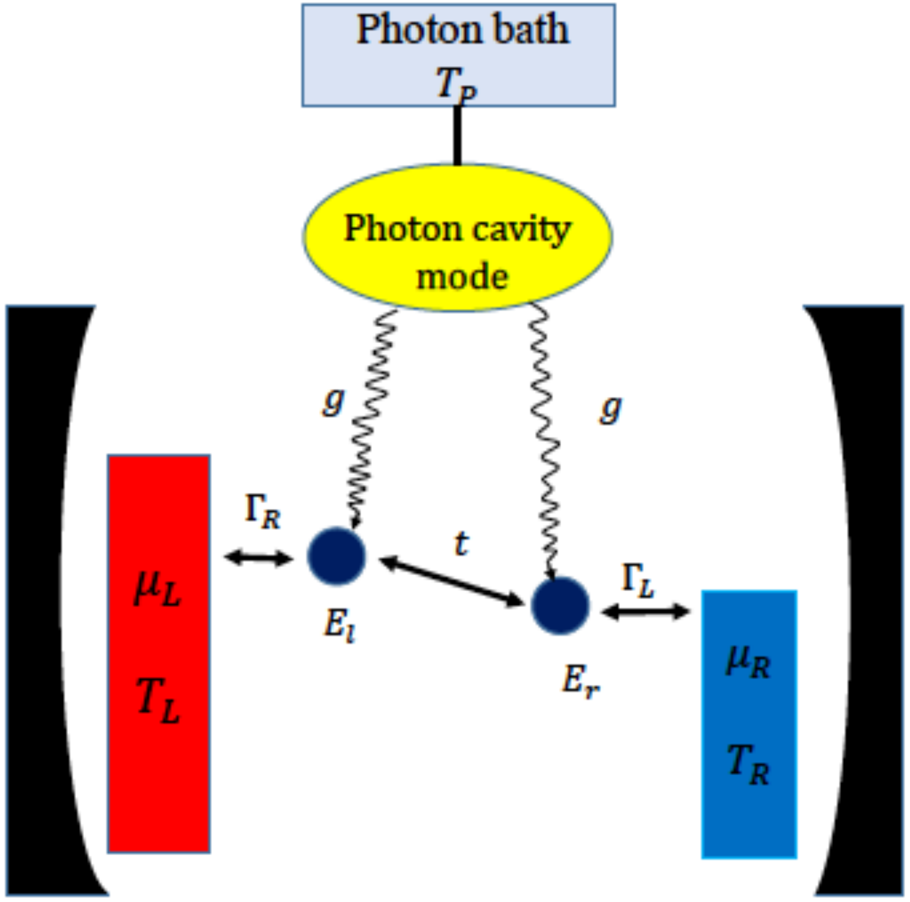
Diodes, Rectifier and Transistors

[Lu, Wang, Ren, Kulkarni, Jiang, in preparation]

Lu, Wang, Jiang --School of Physical Science and Technology & Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University

Key Lab of Advanced Optical Manufacturing Technologies of Jiangsu Province & Key Lab of Modern Optical Technologies of Education Ministry of China, Soochow University

Ren - Center for Phononics and Thermal Energy Science, School of Physics Science and Engineering, Tongji University



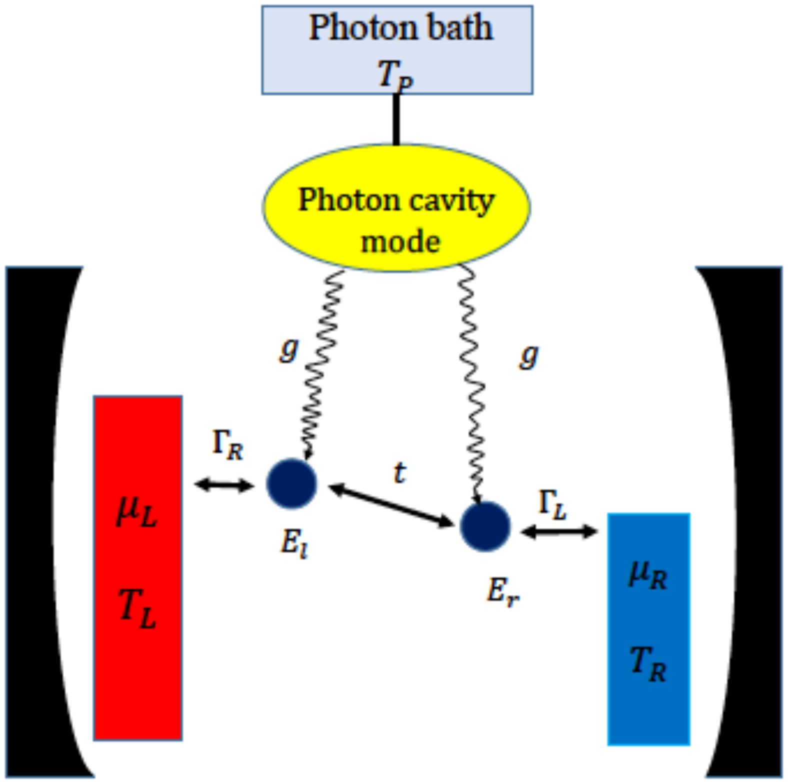
$$\hat{H} = \hat{H}_{c-DQD} + \hat{H}_{lead} + \hat{H}_{dot-lead}$$

$$\hat{H}_{c-DQD} = \hat{H}_{DQD} + \hat{H}_{p} + \hat{H}_{e-p}$$

$$\hat{H}_{DQD} = \sum_{i=\ell,r} E_i \hat{d}_i^\dagger \hat{d}_i + (t\hat{d}_\ell^\dagger \hat{d}_r + \text{H.c.})$$

$$\hat{H}_{e-p} = g\omega_c \hat{(} d_\ell^\dagger \hat{d}_\ell + d_r^\dagger \hat{d}_r)(\hat{a}_q + \hat{a}_q^\dagger),$$

$$\hat{H}_p = \omega_c \hat{a}_q^\dagger \hat{a}_q.$$



$$\hat{H}_{lead} = \sum_{j=L,R} \sum_{k} \varepsilon_{j,k} \hat{c}_{j,k}^\dagger \hat{c}_{j,k}.$$

$$\hat{H}_{dot-lead} = \sum_{k} V_{L,k} \hat{c}_\ell^\dagger \hat{c}_{L,k} + \sum_{k} V_{R,k} \hat{c}_r^\dagger \hat{c}_{R,k} + \text{H.c.}$$

Quantities of Interest

$$I_e^L|_{el} = e\int \frac{d\omega}{2\pi}\mathrm{Tr}(\hat{\Gamma}^L_{rot}(\omega)\hat{G}^r_{tot}(\omega)[\hat{\Sigma}_l^<(\omega) + 2f_L(\omega)\hat{\Sigma}_l^r(\omega)]\hat{G}^a_{tot}(\omega)),$$

$$I_e^L|_{inel} = e\int \frac{d\omega}{2\pi}\mathrm{Tr}(\hat{\Gamma}^L_{rot}(\omega)\hat{G}^r_1(\omega)[\hat{\Sigma}_P^<(\omega) + 2f_L(\omega)\hat{\Sigma}_P^r(\omega)]\hat{G}^a_1(\omega)).$$

$$I_Q^L|_{el} = \int \frac{d\omega}{2\pi}(\omega-\mu_L)\mathrm{Tr}(\hat{\Gamma}^L_{rot}(\omega)\hat{G}^r_{tot}(\omega)[\hat{\Sigma}_l^<(\omega) + 2f_L(\omega)\hat{\Sigma}_l^r(\omega)]\hat{G}^a_{tot}(\omega)),$$

$$I_Q^L|_{inel} = \int \frac{d\omega}{2\pi}(\omega-\mu_L)\mathrm{Tr}(\hat{\Gamma}^L_{rot}(\omega)\hat{G}^r_1(\omega)[\hat{\Sigma}_P^<(\omega) + 2f_L(\omega)\hat{\Sigma}_P^r(\omega)]\hat{G}^a_1(\omega)).$$

-- Treat the Green’s function of c-DQD (polaron) part non-pertubatively

-- After that, follow the standard procedure, of polaron – lead system, which is weak in system-bath coupling.

Non-Perturbative Hybrodized Dots Greens Function

$$G^{r(a)}_{0D}(\omega) = \frac{1}{Z}\sum_{n,m=0}^{\infty}\left[\frac{e^{-\beta m\omega_c} + e^{-\beta(n\omega_c+\tilde{E}_D)}}{\omega - \Delta^{(1)}_{mn} \pm i0^+} + \right.$$
$$\left. \frac{e^{-\beta(m\omega_c+\tilde{E}_d)} + e^{-\beta(n\omega_c+\tilde{E}_d)}}{\omega - \Delta^{(2)}_{mn} \pm i0^+}\right] D^2_{nm}(g_D),$$

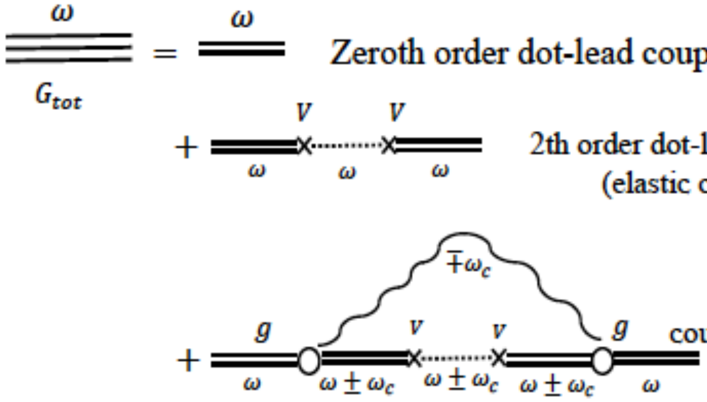
where

$$\Delta^{(1)}_{nm} = (n-m)\omega_c + \tilde{E}_D,$$
$$\Delta^{(2)}_{nm} = (n-m)\omega_c + (\tilde{E}_D - 2\omega_c g_D g_d),$$
$$D_{nm}(g_D) = e^{-g_D^2/2}\sum_{k=0}^{\min\{n,m\}}\frac{(-1)^k\sqrt{n!m!}g_D^{n+m-2k}}{(n-k)!(m-k)!k!},$$
$$Z = (1+N_P)(1+e^{-\beta\tilde{E}_D} + e^{-\beta\tilde{E}_d} + e^{-\beta\tilde{E}_{Dd}})$$

Similarly compute Keldysh G^K

And then calculate currents

Symbol	Meaning
—	bare dot
═	c-DQD (dressed electron)
〰	photon
┈	leads
×	lead-dot coupling, coupling strength V
○	Electron-photon coupling, coupling strength g
≡	G_{tot}

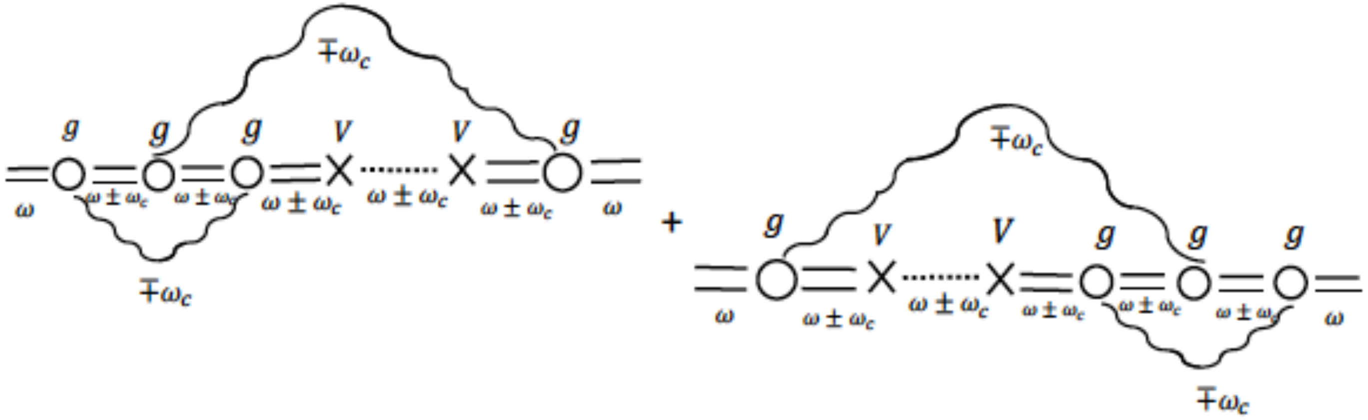


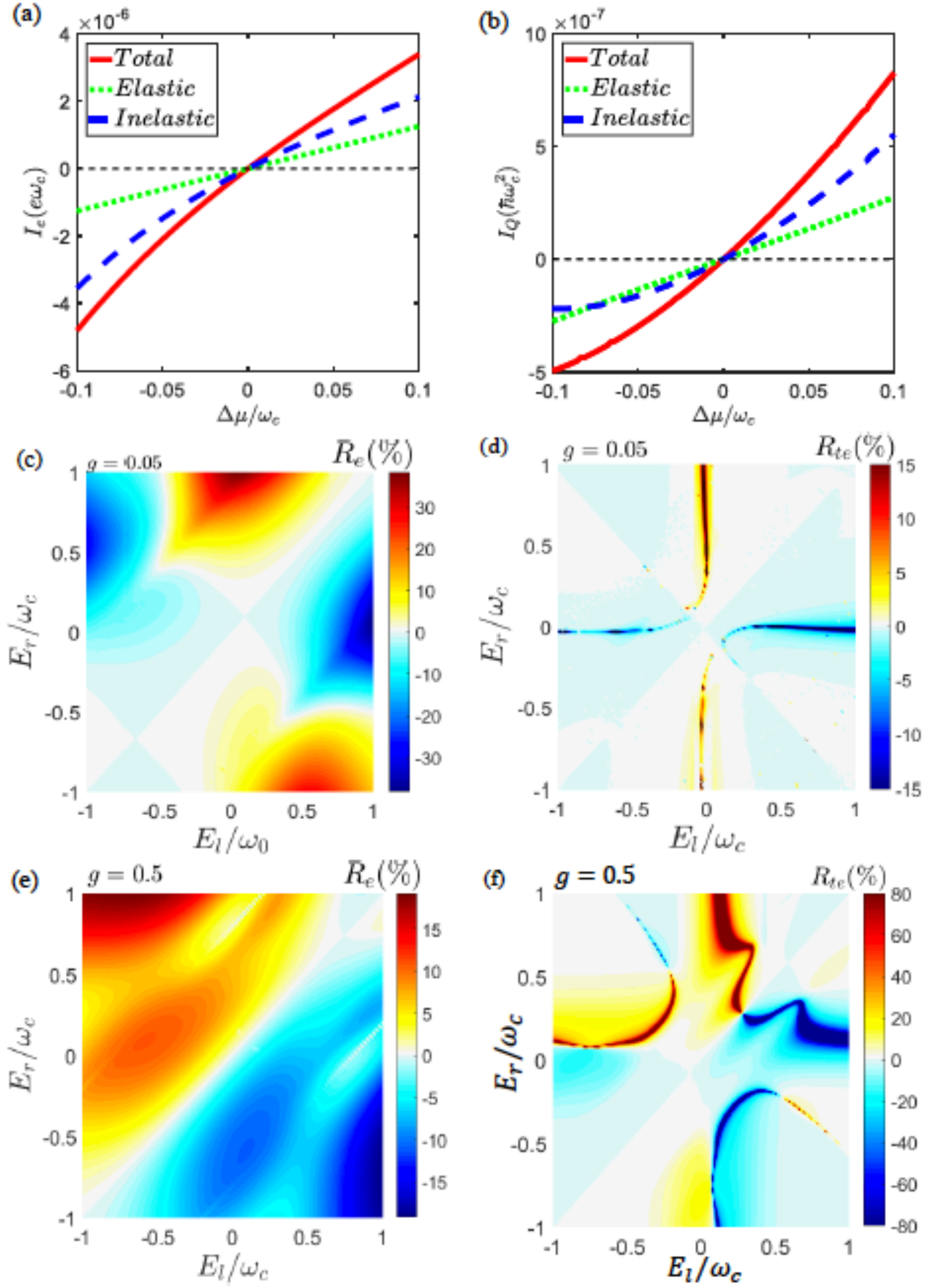
Zeroth order dot-lead coupling

2th order dot-lead coupling
(elastic current)

2th order dot-lead
coupling (inelastic current)

Neglected Diagrams Below





Rectification effects in QD-cQED systems

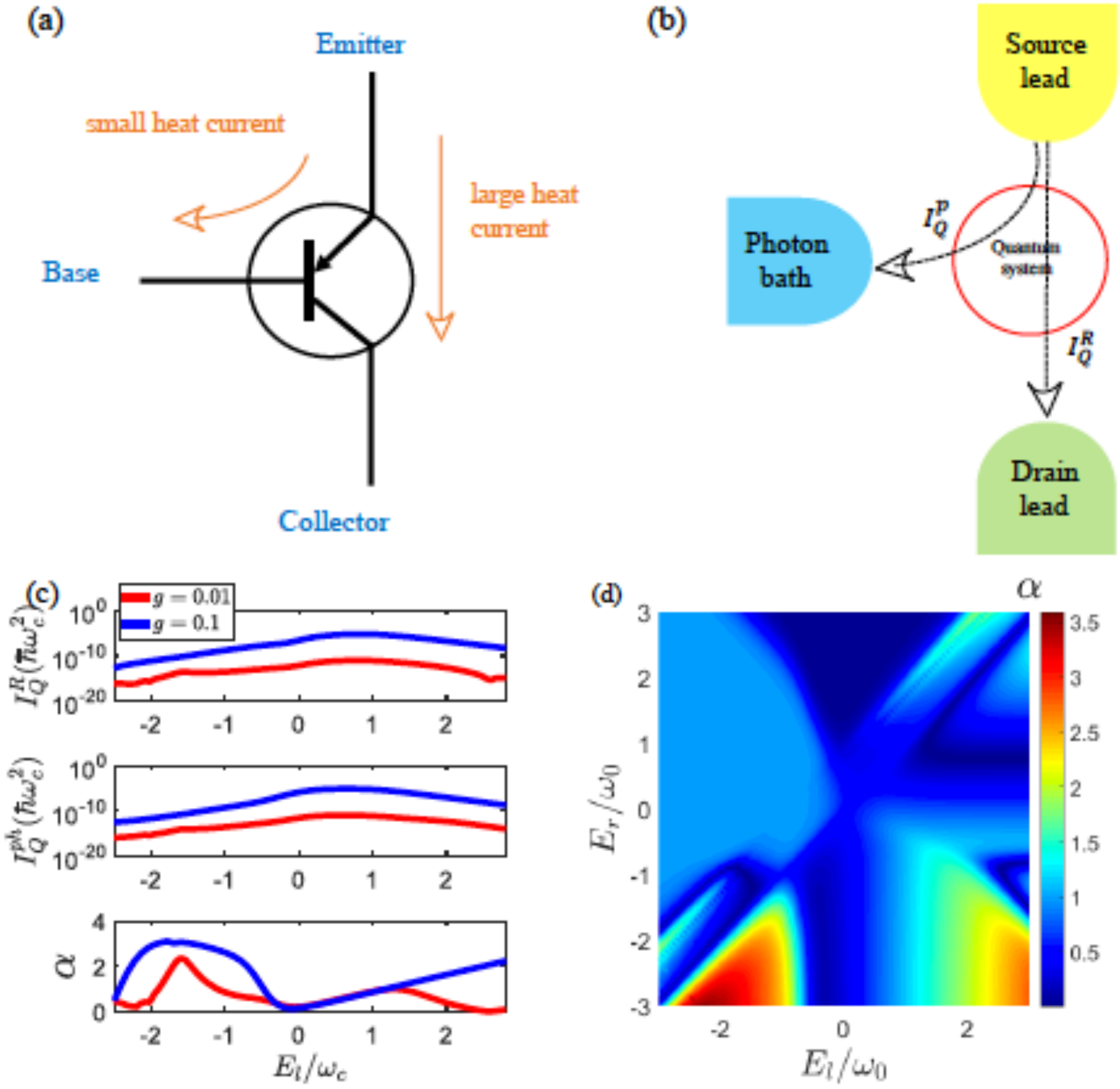
Charge Rectification

$$R_e = \frac{I_e(\Delta\mu) + I_e(-\Delta\mu)}{|I_e(\Delta\mu) + I_e(-\Delta\mu)|}$$

Cross-Rectification

$$R_{te} = \frac{I_Q(\Delta\mu) + I_Q(-\Delta\mu)}{|I_Q(\Delta\mu) + I_Q(-\Delta\mu)|}$$

Transistor Effects



$$\begin{pmatrix} I_Q^R \\ I_Q^P \end{pmatrix} = \begin{pmatrix} K_{PP} & K_{PR} \\ K_{RP} & K_{RR} \end{pmatrix} \begin{pmatrix} T_P - T_L \\ T_R - T_L \end{pmatrix}$$

$$\alpha = \left| \frac{\partial_{T_P} I_Q^R}{\partial_{T_P} I_Q^P} \right| = \frac{K_{RP}}{K_{PP}}$$

Conclusions & Outlook

Part - A

-- QD-cQED systems can be good microwave amplifiers, photon sources and masers
-- tuning parameters can help us change photonic statistics

-- Scalibility and exotic many body phenomena ?
-- Non-pertubative in system lead coupling ?

Part - B

-- QD-cQED systems can be good rectifiers and transistors
-- Transistor effect exist even in the linear regime
-- Non-reciprocity

-- Including coulomb energy ?
-- Non-perturbative in system lead coupling ?
-- Spin rectification effects ?
--- Chain of dots ? [realized already in Princeton, 9 dots without cavity yet]