Quantum Hamiltonian Engineering via Parametric Drives

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What a quantum machine looks like

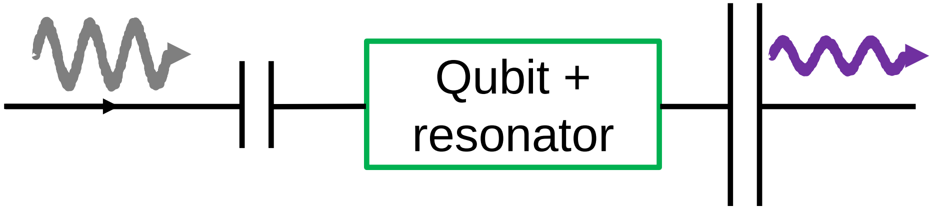
Classical control lines

Dilution unit (cryogenic cooling)

50 or so readout channels

50 or so quantum bits

Quantum signals from qubits



We might care to

- amplify this pulse
- route it to another qubit/cavity
- Interfere it with another pulse
- ….

Key limitation: quantum efficiency

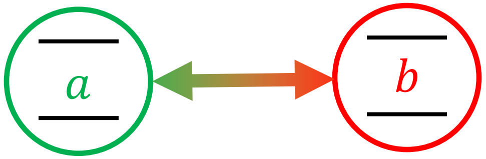
Solution: parametric amplification

Outline

- **Parametric amplification**
- **Effect of 4th order terms**
- **Multiple parametric drives**

Parametrically driven couplings

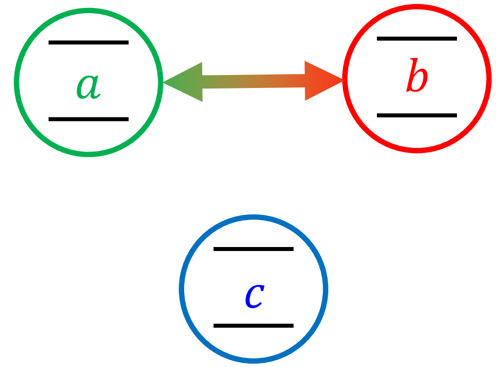
direct exchange



$$\frac{\mathcal{H}_{int}}{\hbar} = g\left(ab^\dagger + a^\dagger b\right)$$

- if $\omega_a - \omega_b \gg \kappa_{a,b}$ this term dies due to energy conservation (RWA)
- interaction also turns off slowly vs. detuning, limiting the on/off ratio

parametrically driven exchange (Conv)



$$\frac{\mathcal{H}_{int}}{\hbar} = g\left(ab^\dagger c^\dagger + a^\dagger bc\right)$$

- if $\omega_c \neq \omega_a - \omega_b$ we can drive the c-mode ‘stiffly’

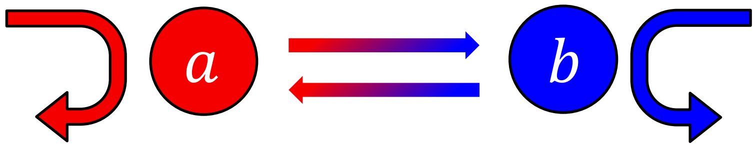
$$\frac{\mathcal{H}_{int}}{\hbar} = gab^\dagger + g^* a^\dagger b$$

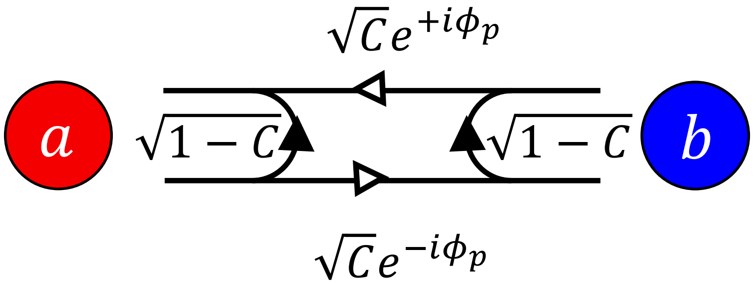
- Parametric drive fully controls strength and phase of interaction

Photon conversion (Conv)

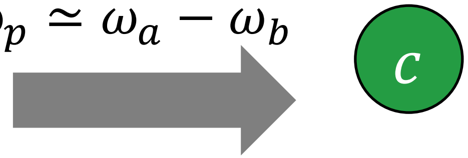
$$\omega_p = \omega_a - \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a^\dagger b e^{i\phi_p} + a b^\dagger e^{-i\phi_p}\right)$$

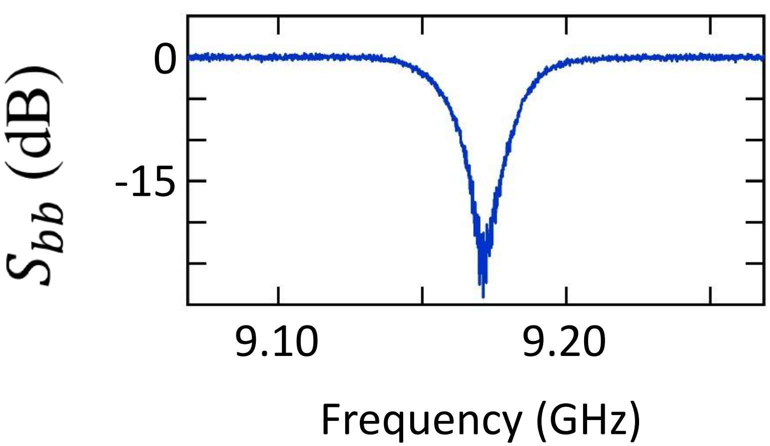


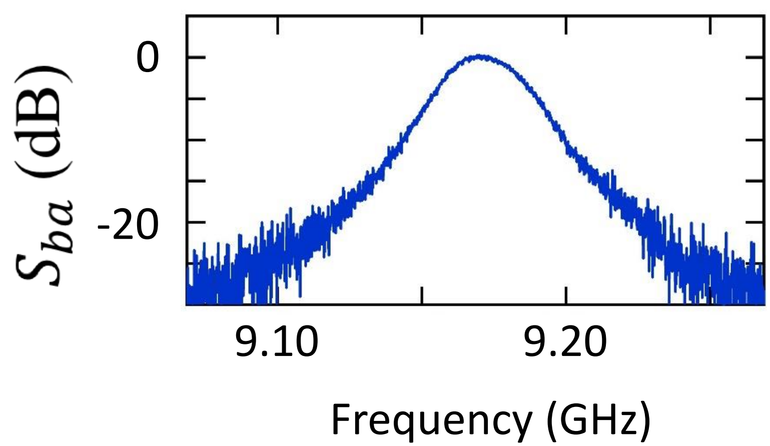


$$\omega_p \simeq \omega_a - \omega_b$$



$$C = \frac{2\,{}^{P_P}/_{P_C}}{\left(1 + {}^{P_P}/_{P_C}\right)^2}$$



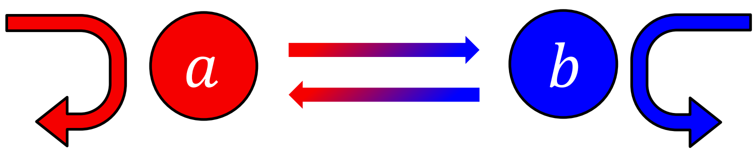


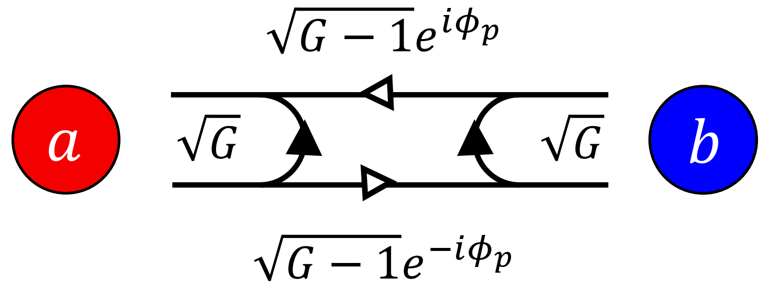
Sliwa *PRX* (2015)

Phase preserving gain (Gain)

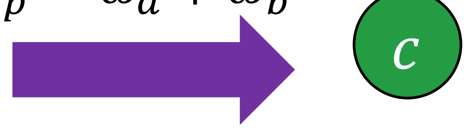
$$\omega_p = \omega_a + \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a^\dagger b^\dagger e^{i\phi_p} + abe^{-i\phi_p}\right)$$

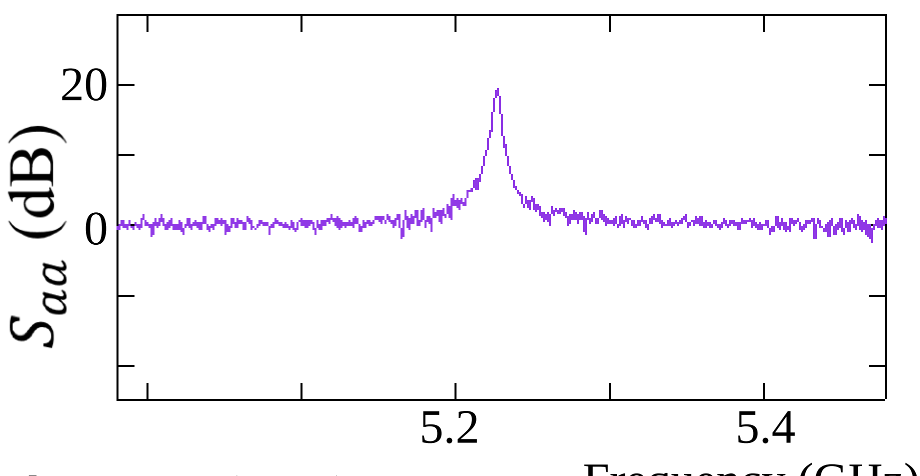


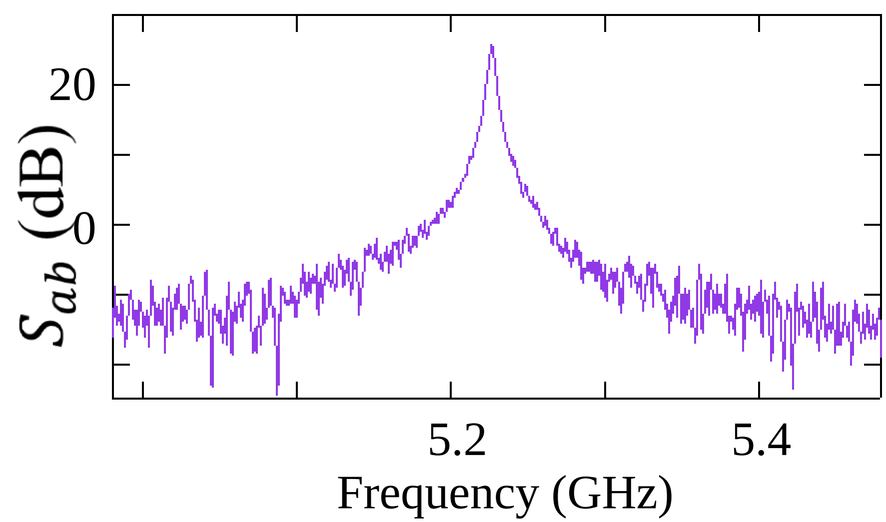


$$\omega_p \simeq \omega_a + \omega_b$$



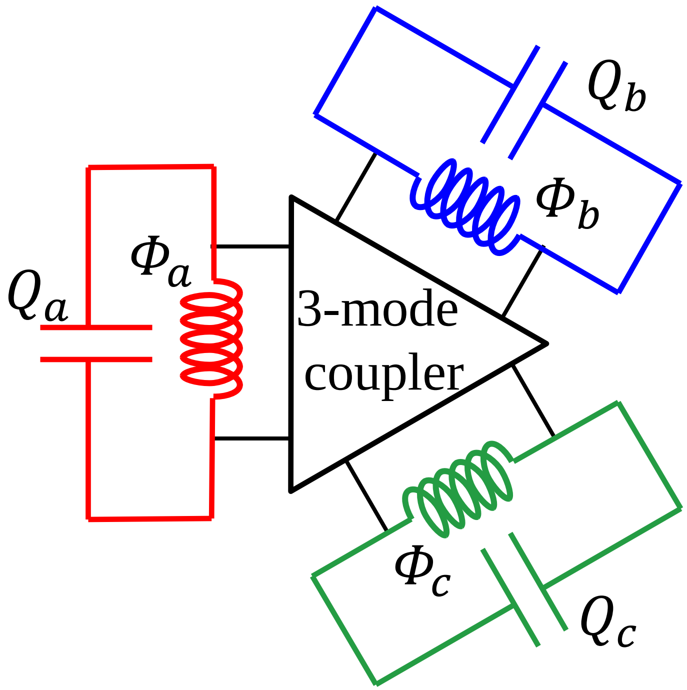
$$G = \frac{\left(1 + {P_P}/{P_C}\right)^2}{\left(1 - {P_P}/{P_C}\right)^2}$$





Sliwa *PRX* (2015)

Parametric coupling overview



$$H_{couple} \propto \Phi_a \Phi_b \Phi_c$$

Re-write in terms of a, b, c

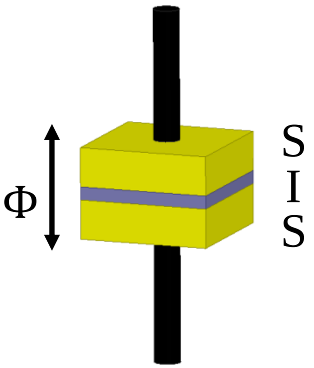
$$H_{couple} = \hbar g_3 (a + a^\dagger)(b + b^\dagger)(c + c^\dagger)$$
$$= \hbar g_3 (abc^\dagger + a^\dagger b^\dagger c + ab^\dagger c + a^\dagger bc^\dagger + \cdots)$$

- If frequencies $\omega_{a,b,c}$ all very different, all terms die in the rotating wave approx.
- Drive one mode (c) at $\omega_p = \omega_a + \omega_b \neq \omega_c$
- Stiff pump: $c \rightarrow \langle c \rangle = |c| e^{i\phi_p}$

$$H_G = \hbar g (a^\dagger b^\dagger e^{i\phi_p} + abe^{-i\phi_p})$$

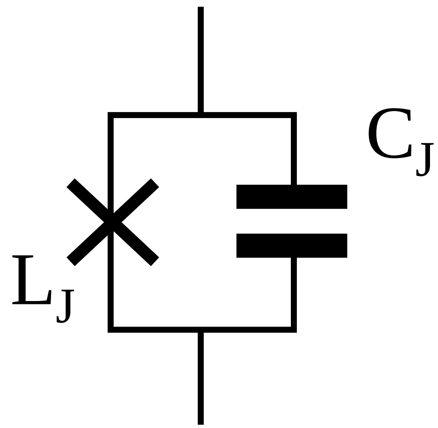
Physical Implementations: Josephson junctions, opto-mechanics, diodes, optical fibers….

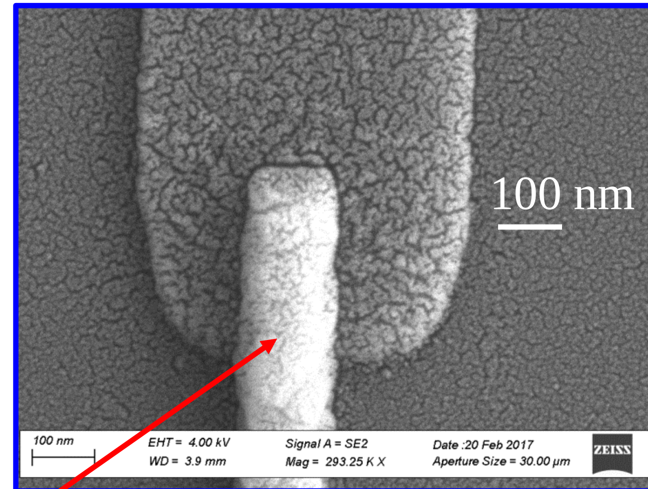
The Josephson tunnel junction



SUPERCONDUCTING
TUNNEL JUNCTION

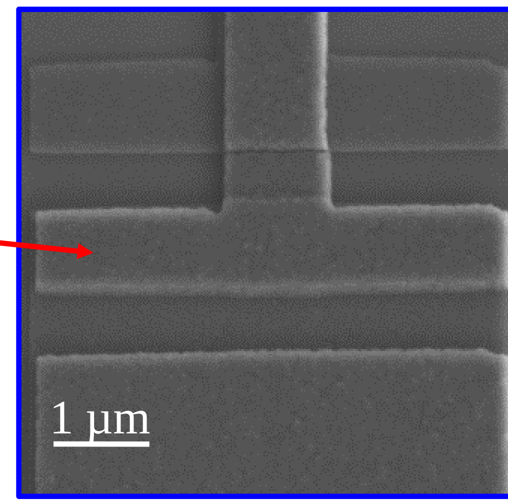
$$I = I_0 \sin\left(\frac{2\pi}{\Phi_0}\Phi\right)$$





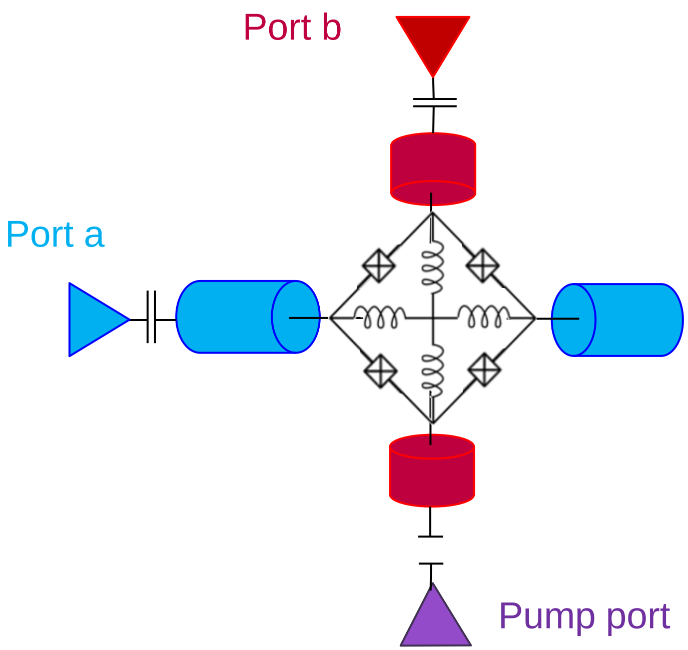
$Al/AlO_x/Al$
tunnel junction

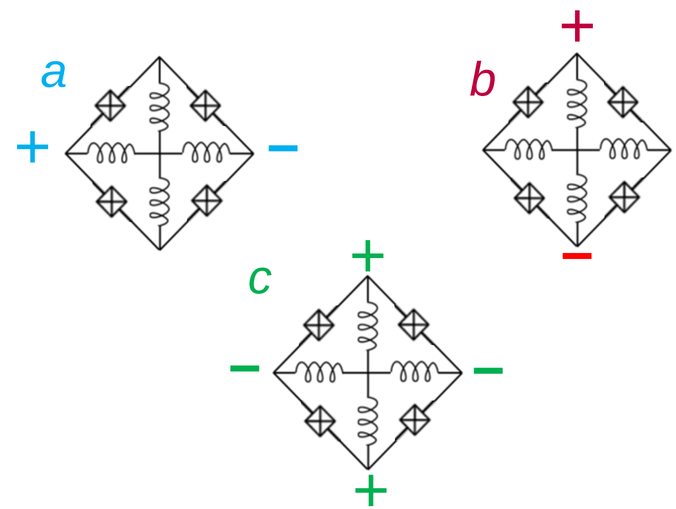
T ~ 20 mK



$$H = \frac{Q^2}{2C} - E_J \cos\left(\frac{2\pi}{\Phi_0}\Phi\right) = \hbar\omega_0 b^\dagger b - \lambda\left(b^\dagger b\right)^2 + \cdots$$

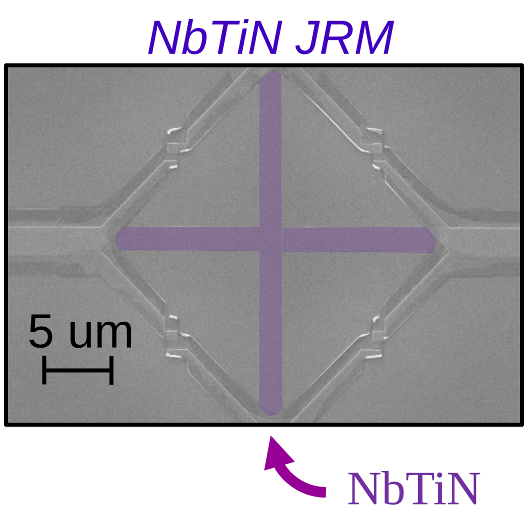
Josephson parametric converter (JPC)





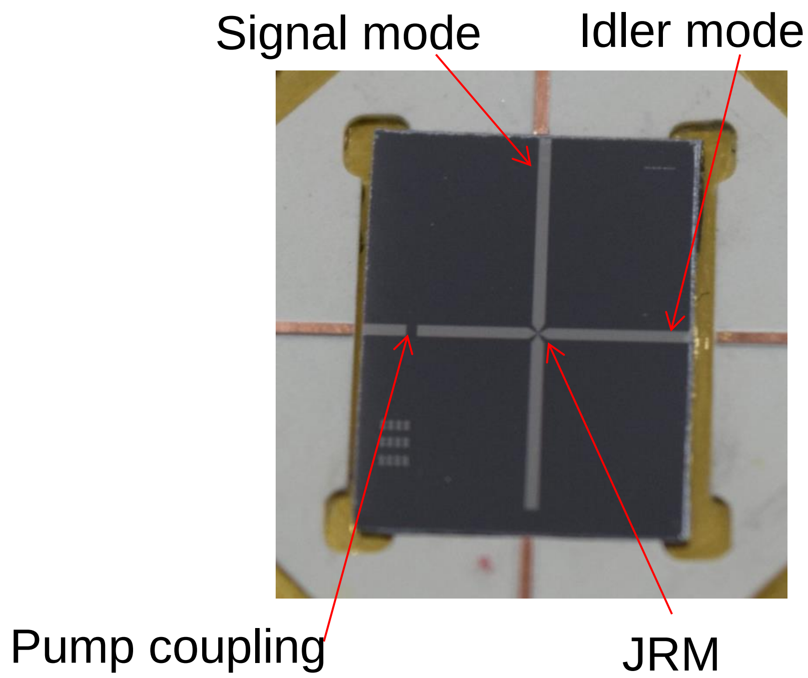
$$\frac{H_{Coupling}}{\hbar} = g\,(a + a^{\dagger})(b + b^{\dagger})(c + c^{\dagger})$$

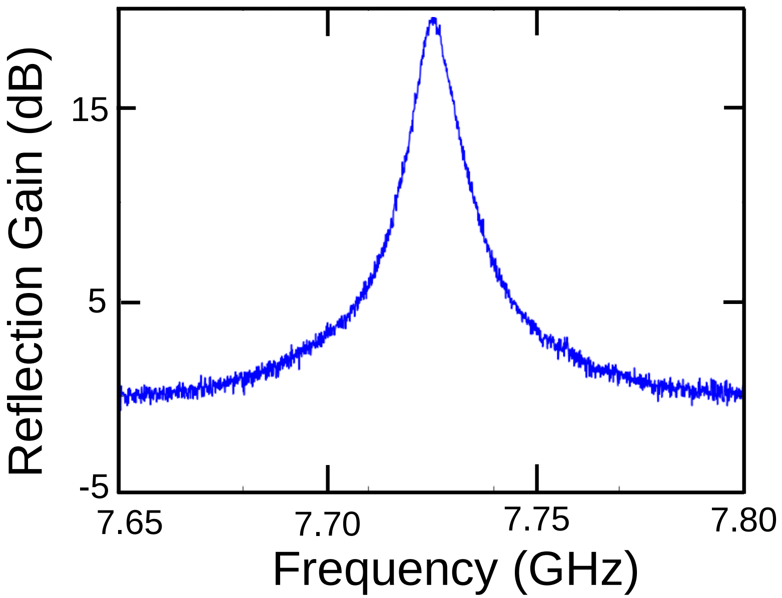
$$- \sum_{m=a}^{c} \sum_{n=m}^{c} K_{mn} a^{\dagger}{}_{m} a_{m} a^{\dagger}{}_{n} a_{n} + O\left(a_i^5\right) + \cdots$$

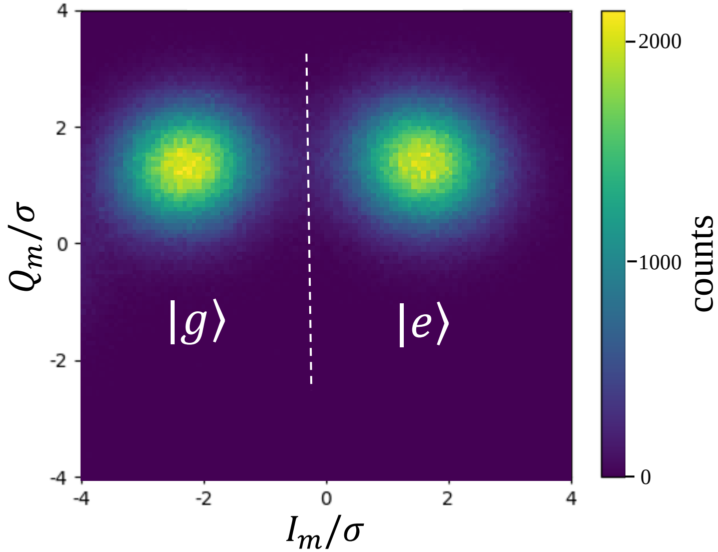


Bergeal *Nature Physics* (2010)

The 8-junction Josephson Parametric Converter



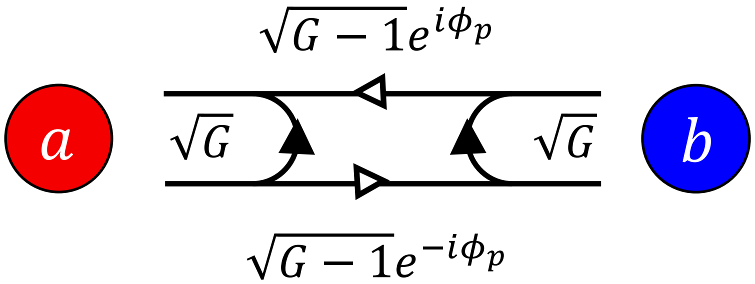




Nearly quantum limited ($\eta \sim 0.5 - 0.6$)!

Amplifier Limitations

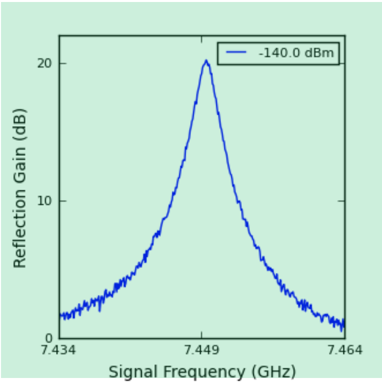
1. Operates in reflection



2. Has narrow bandwidth

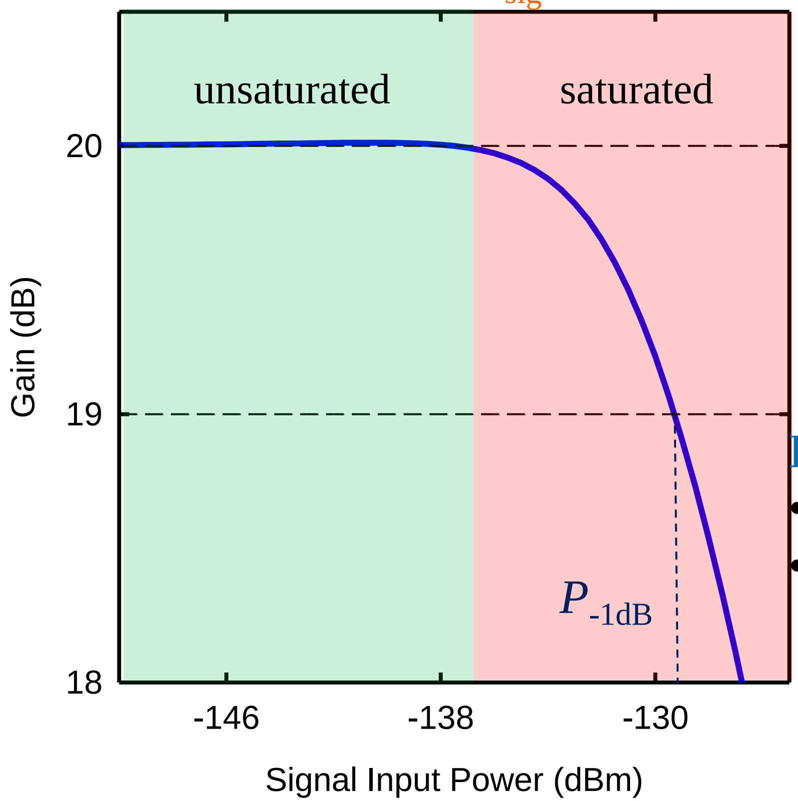
$$2\pi\, B \simeq \sqrt{\kappa_a \kappa_b} \big/ \sqrt{G}$$

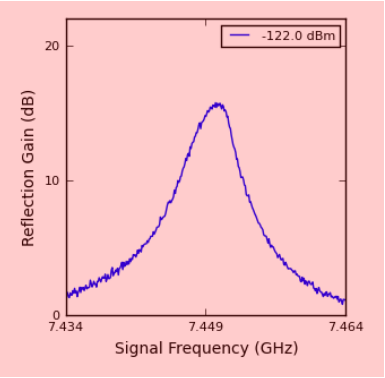
Limitation 3: Gain Saturation



$$G_0 = \left(\frac{1 + n_p/n_p^c}{1 - n_p/n_p^c} \right)^2$$

G vs P_{sig}





Pump depletion (PD) theory:

- P_{sig} ↗, n_p ↘, G ↘
- n_p ↗, P_{-1dB} ↗

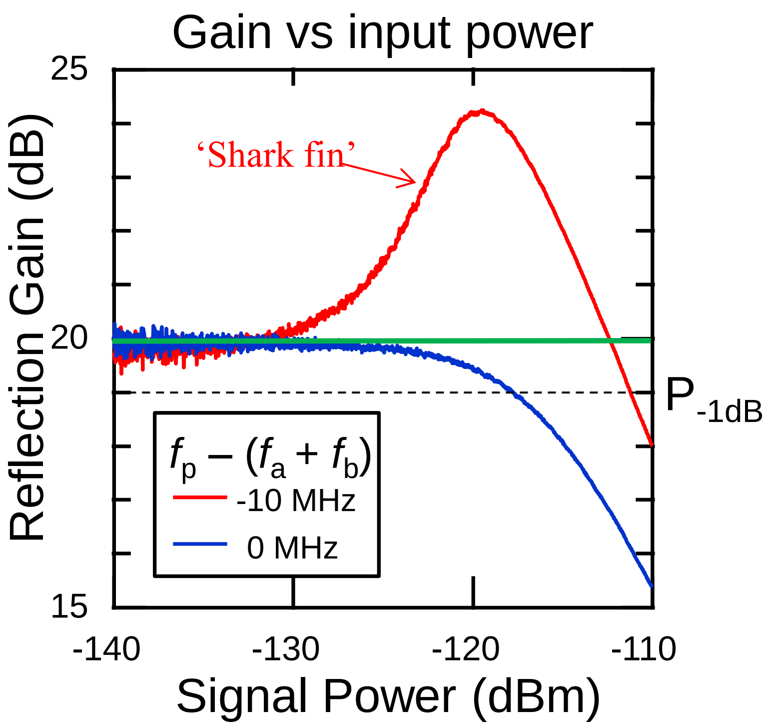
Problem: We don't understand well all causes

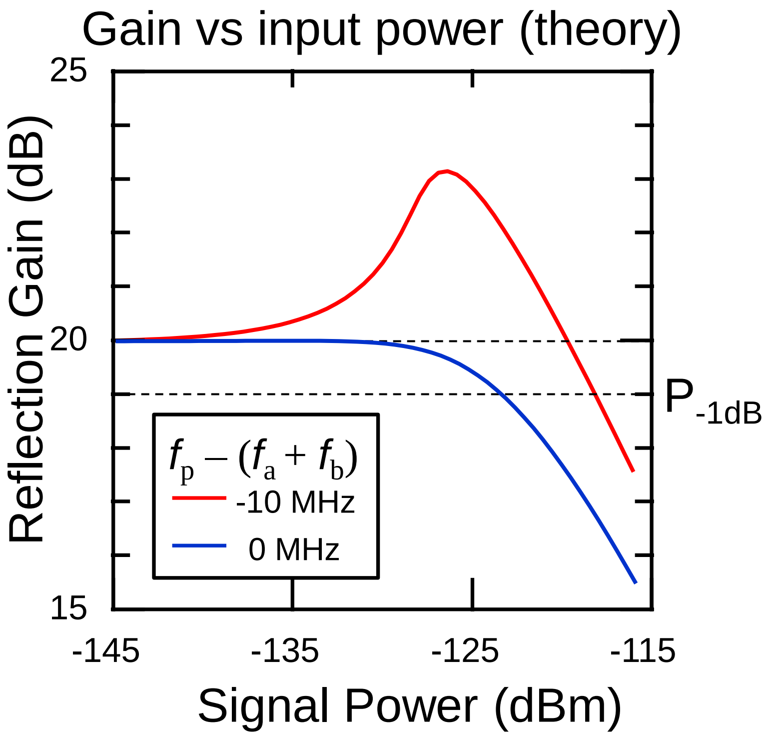
Pump depletion: Abdo Phys. Rev. B (2013); C. Eichler EPJ Quantum Technology (2014)

Cancelling Higher Order Effects

Kerr terms effect performance

$$H_{coupling} = g\left(ab + a^\dagger b^\dagger\right)$$
$$+\kappa_{aa}(a^\dagger a)^2 + \kappa_{bb}(b^\dagger b)^2 + \kappa_{ab}a^\dagger a b^\dagger b$$

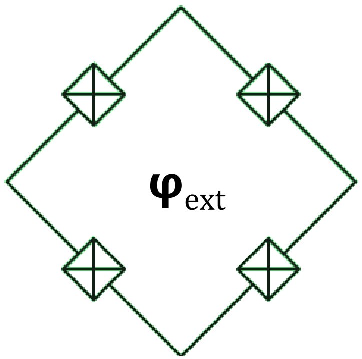




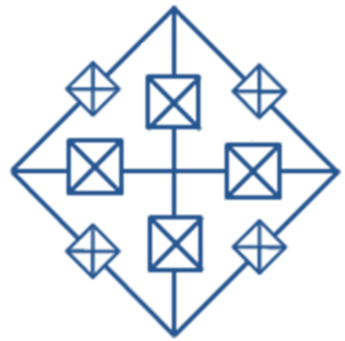
G. Liu et al., Appl. Phys. Lett. **111**, 202603 (2017)

Using ‘shunted’ JRMs to achieve cancellation

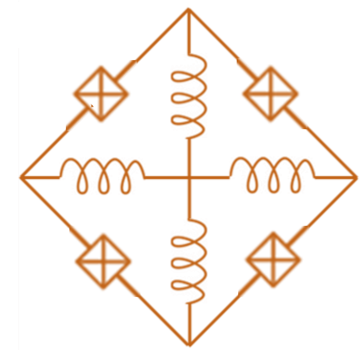
4-Junction JRM



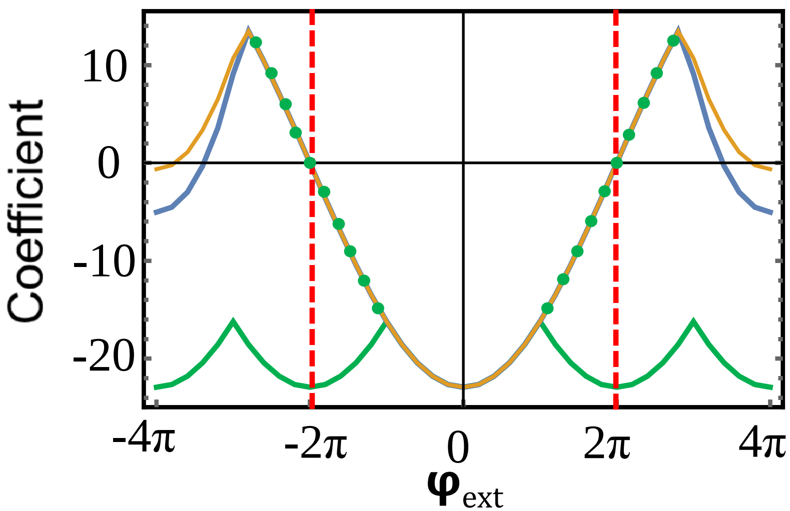
8-Junction JRM



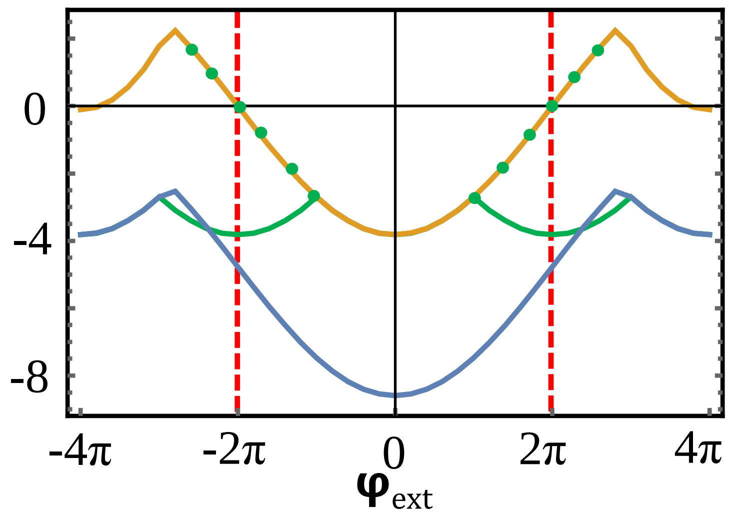
4-Junction JRM + Linear Inductance (WJRM)



Cross Kerr



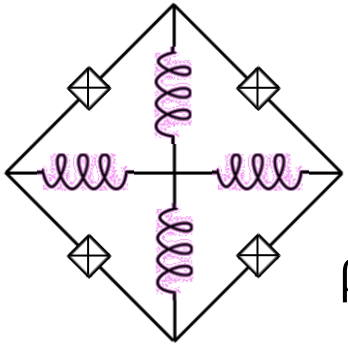
Self Kerr



Bergeal, N., et al. *Nature* 2010 Schackert, F. PhD thesis (Yale, 2013) Roch, N., et al PRL (2012)

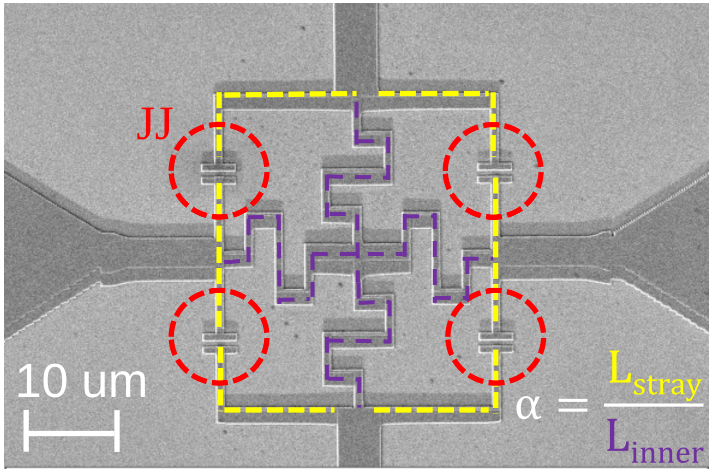
How strongly to shunt?

4 Junction JRM + Linear Inductances

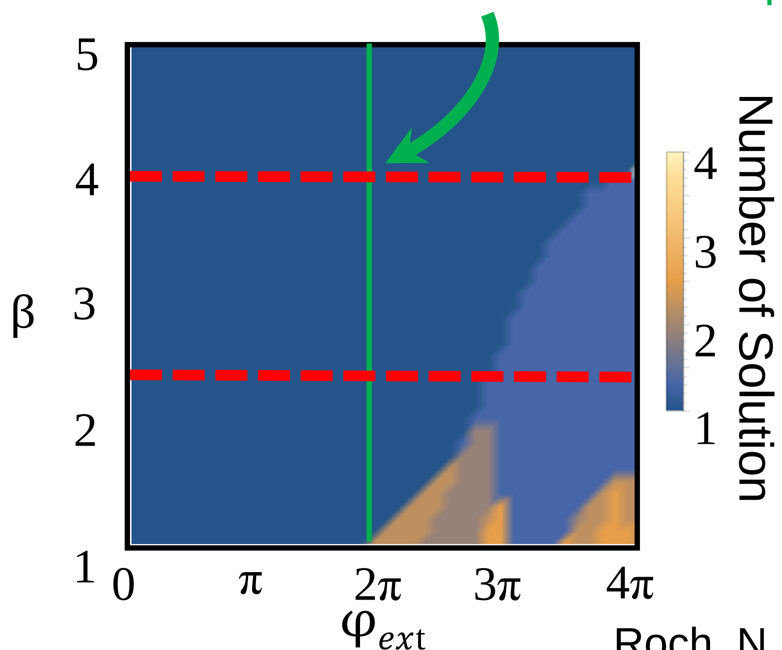


$$\beta = \frac{L_{inner}}{L_J}$$

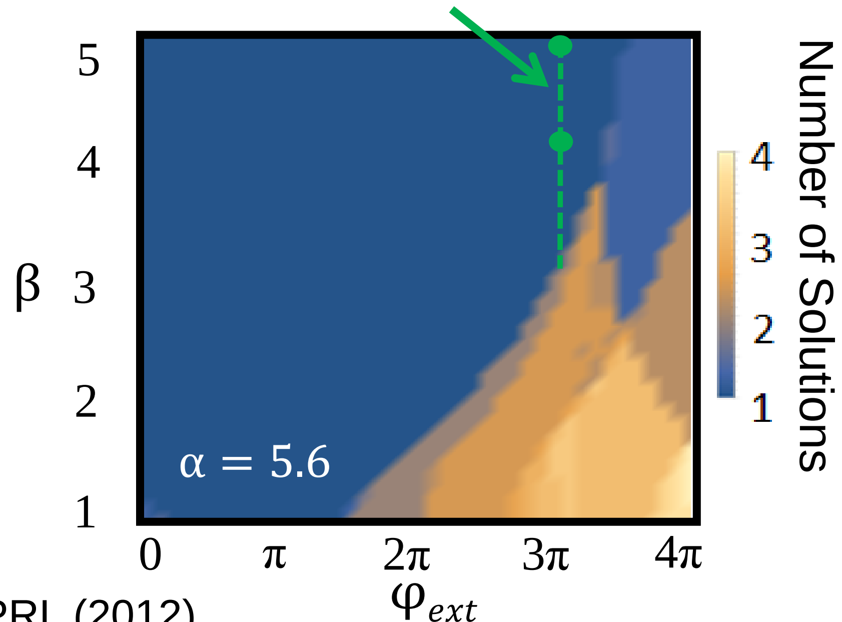
Stray Inductance (yellow)



4th order Cancellation points



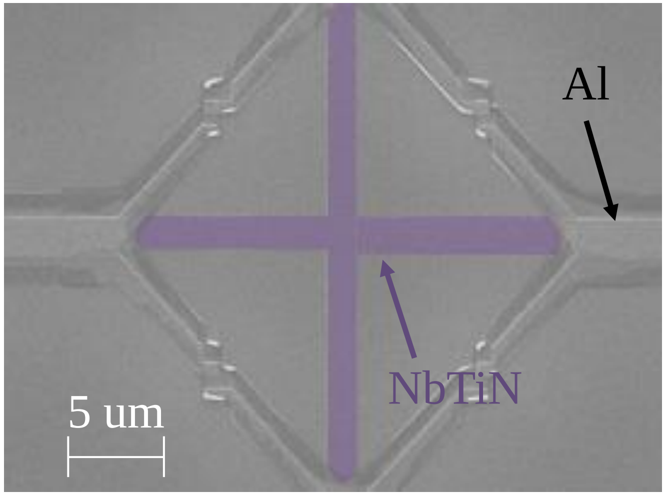
Cancellation Points

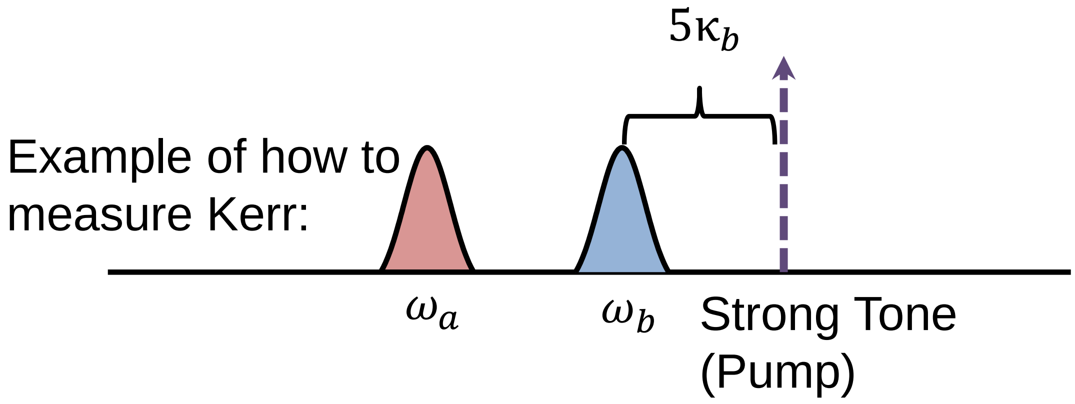


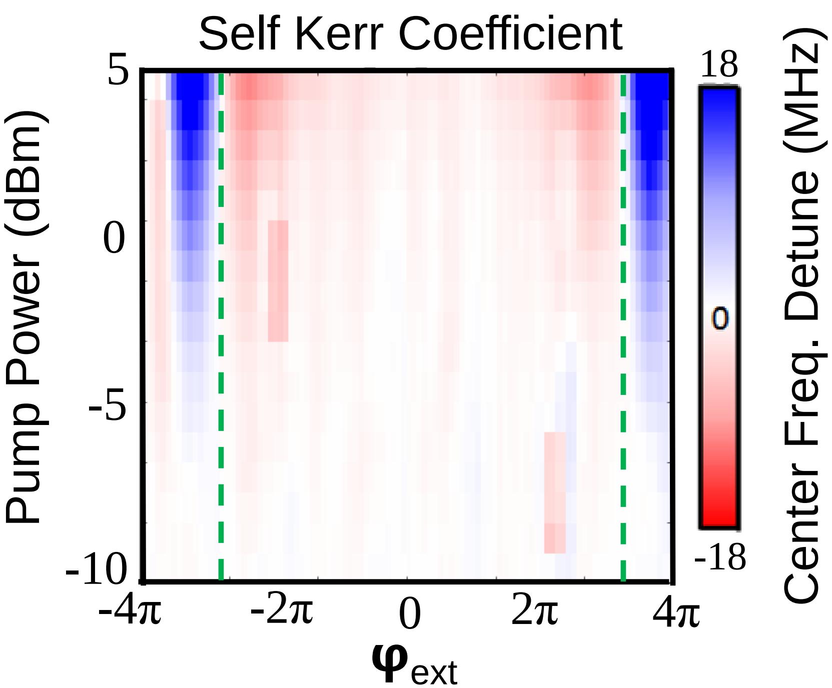
Roch, N., et al PRL (2012)

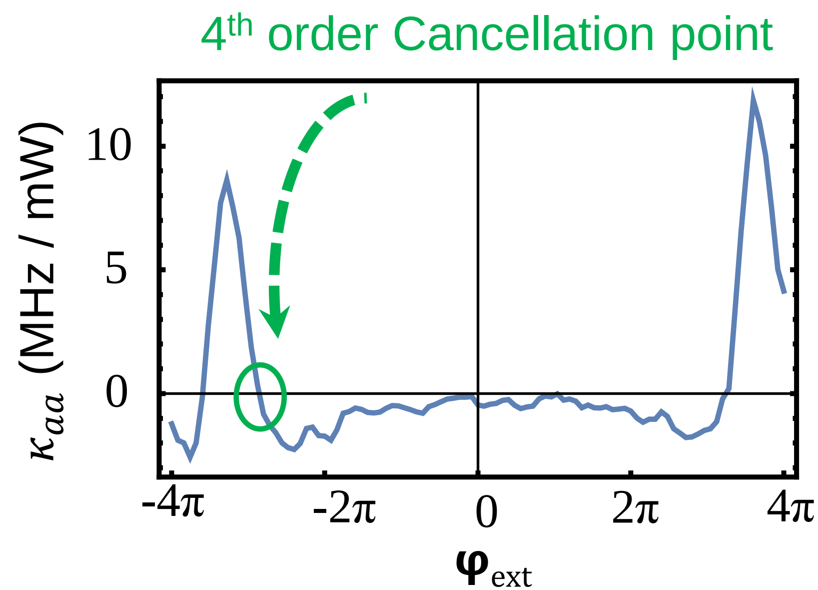
Kerr measurement of NbTiN shunted JRM

$\beta \sim 4$

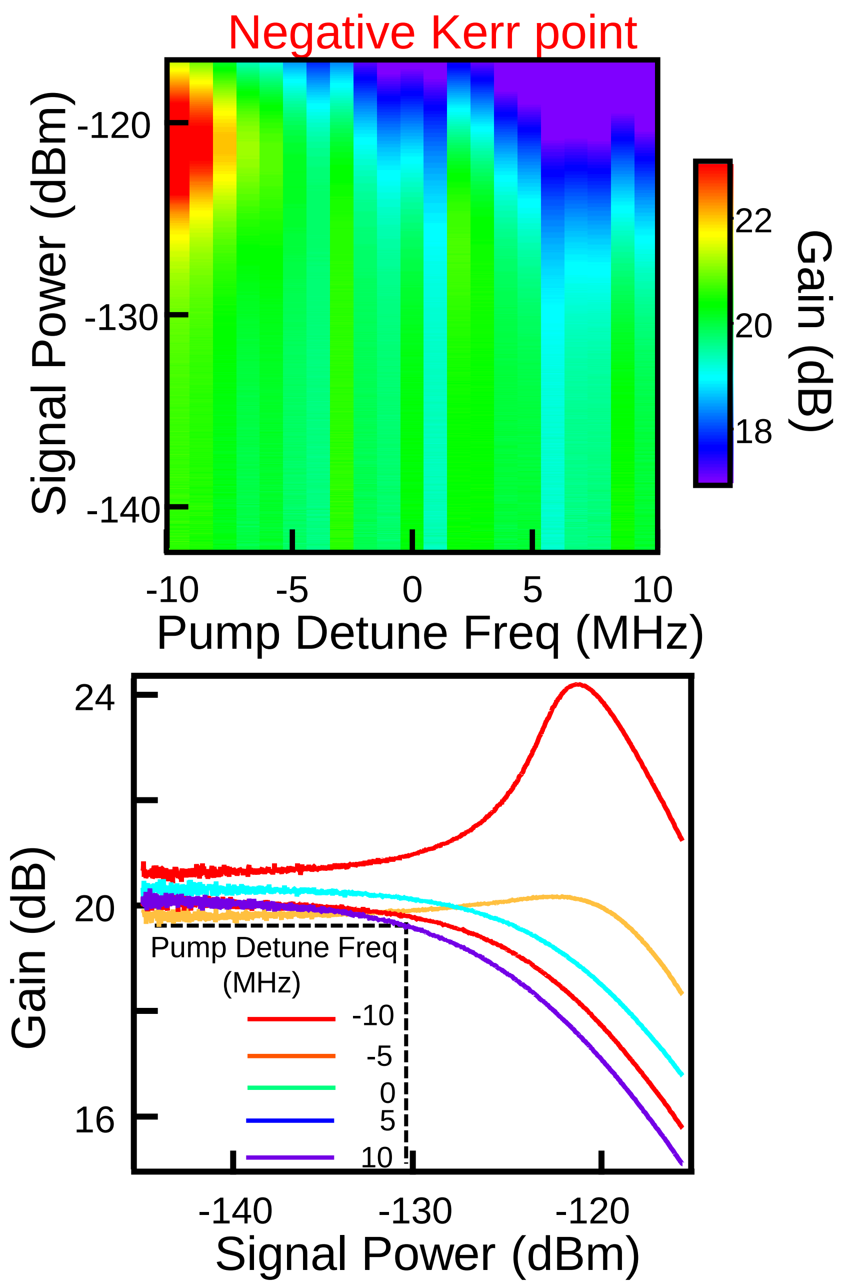


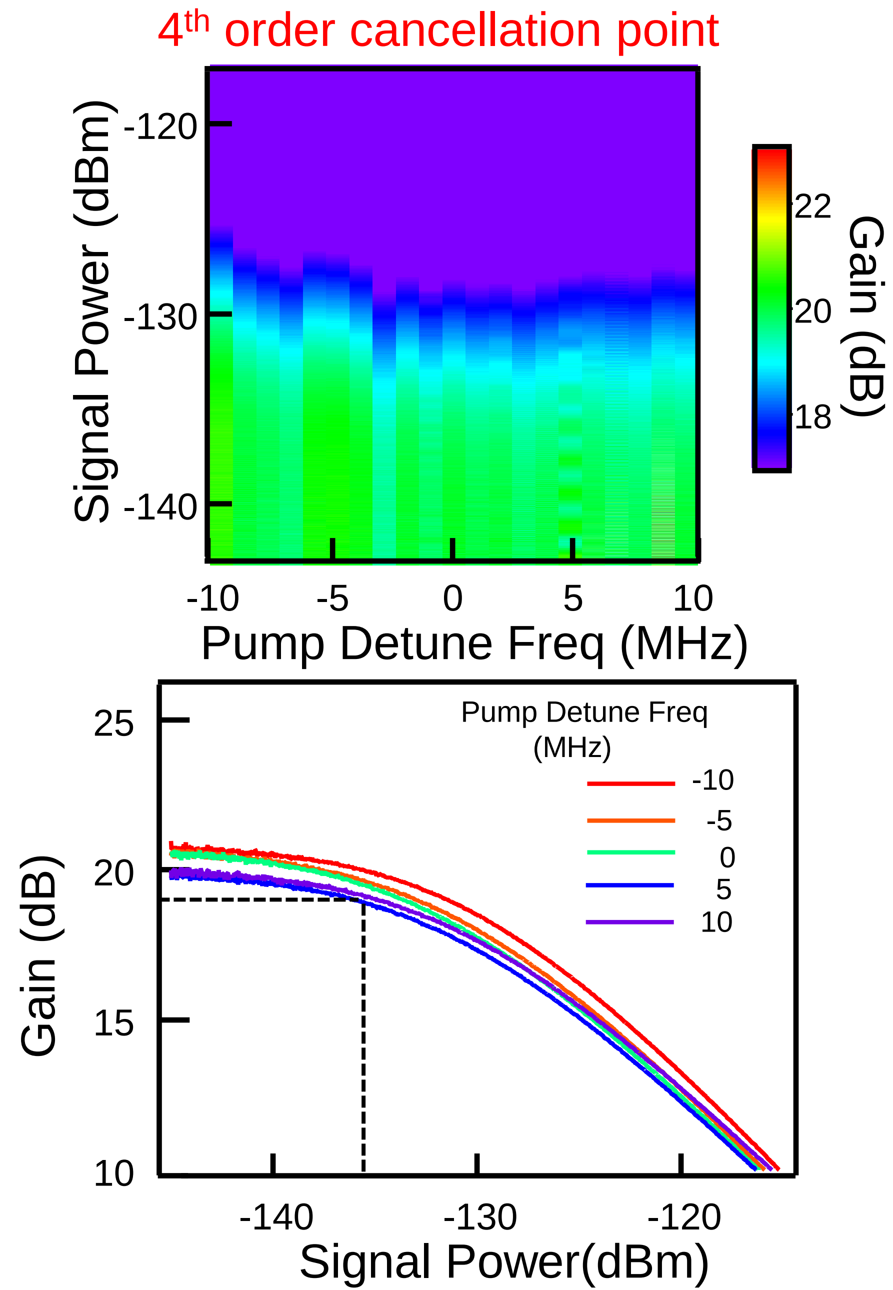






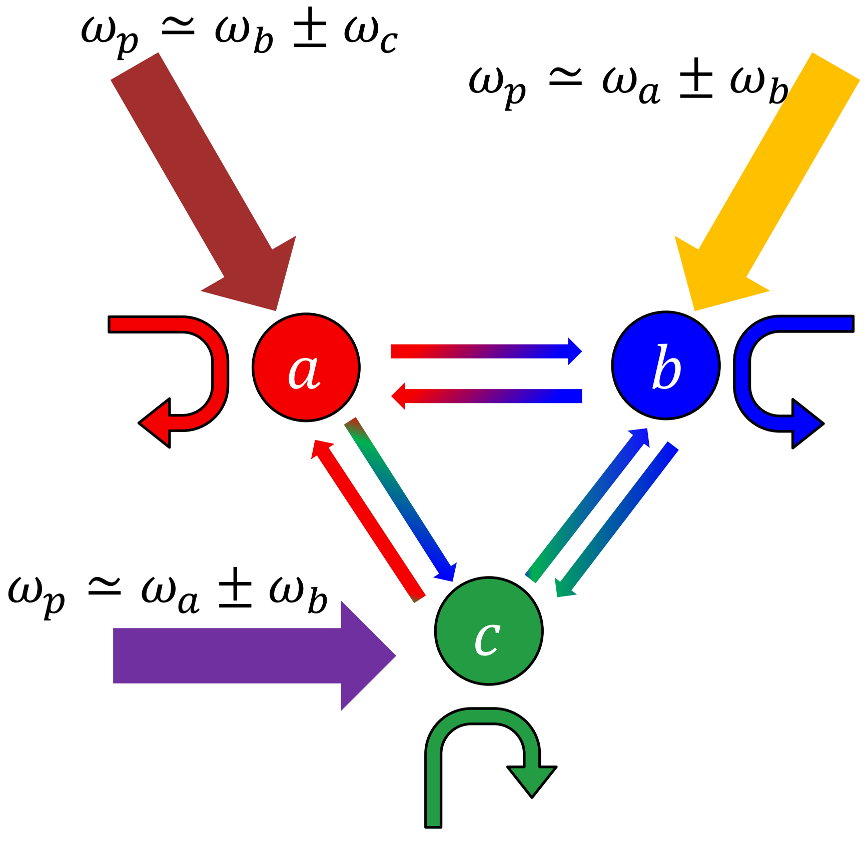
Cancellation Point Comparison





Multiple Parametric Couplings

Adding interference



- Pumps span frequencies and imprint phases on each leg

- Controls:
  - Coupling type (ω_p)
  - Coupling strength (P_p)
  - Coupling phase (ϕ_p)

- Frequency (non) conjugation

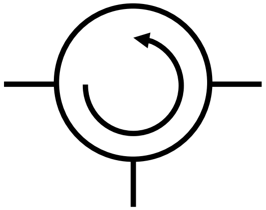
Gain: $(\omega_a + \Delta) \rightarrow (\omega_b - \Delta)$
Conv: $(\omega_a + \Delta) \rightarrow (\omega_b + \Delta)$

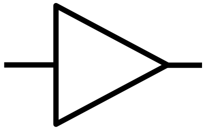
Ranzani and Aumentado *NJP* (2015)
Metelmann and Clerk *PRX* (2015)

Engineering quantum information processors

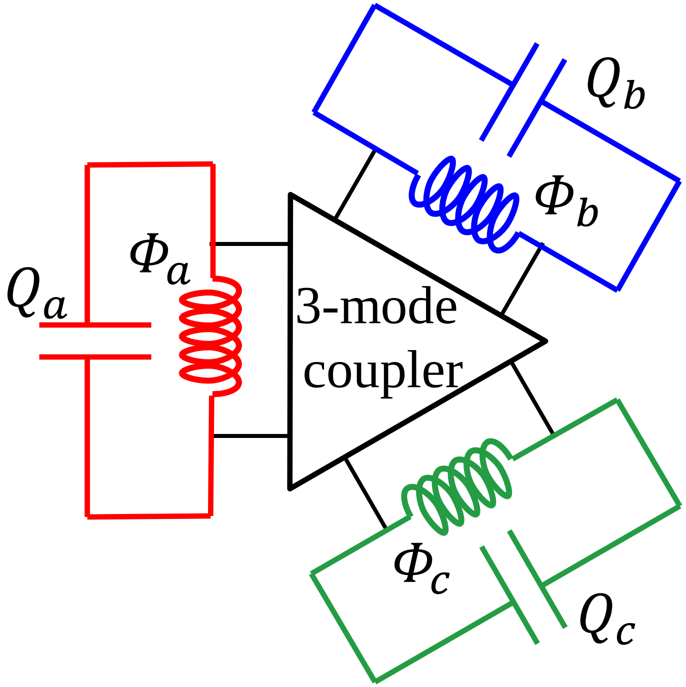
- Passive/reciprocal circuit elements (hybrids, directional couplers) are straightforward

- Gainful and non-reciprocal devices are much harder





- These *quantum* devices must be lossless, and operate on *propagating* states of microwave light

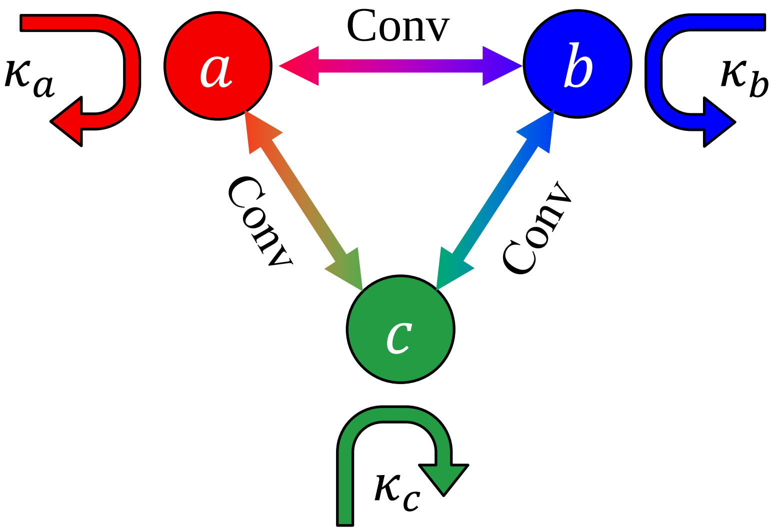


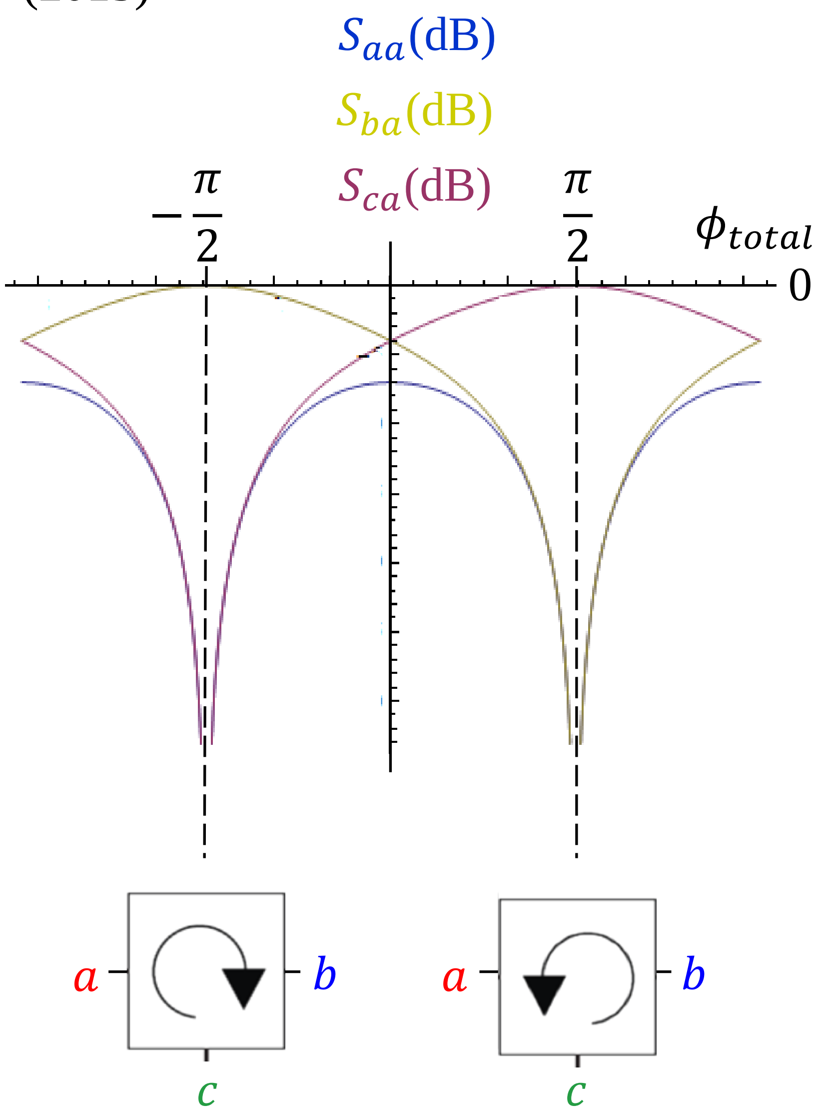
- idea is to engineer these devices from the Hamiltonian up using parametric drives

- key element non-linear Josephson junctions

3 port circulator

Sliwa *Phys. Rev. X* (2015)





- balance three hopping term amplitudes with dissipation rates

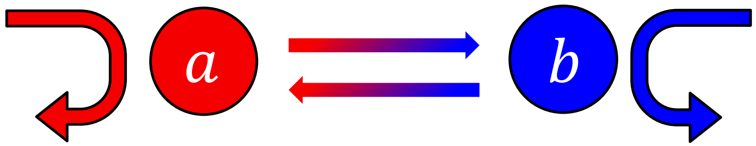
- balance phases so that loop phase $\phi_{total} = \phi_{ab} + \phi_{bc} - \phi_{ca} = \pm\pi/2$

Koch *Phys. Rev.* A (2010), Sliwa *PRX* (2015), Kerckhoff *Phys. Rev. A* (2015), Lecocq *Phys. Rev. A* (2016), …

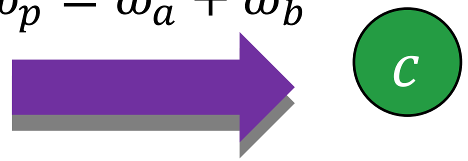
Bi-direction phase sensitive gain (XX)

$$\omega_{p1,p2} = \omega_a \pm \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a + a^\dagger\right)\left(b + b^\dagger\right)$$



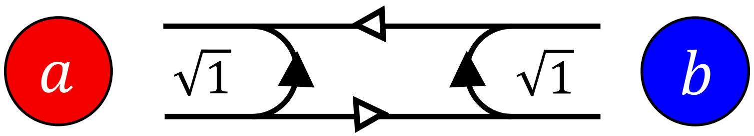
$\omega_p \simeq \omega_a + \omega_b$



$\omega_p \simeq \omega_a - \omega_b$

$P_a \rightarrow P_b = 1/\sqrt{G}$

$X_b \rightarrow X_a = \sqrt{G}$



$P_a \rightarrow P_b = \sqrt{G}$

$X_a \rightarrow X_b = 1/\sqrt{G}$

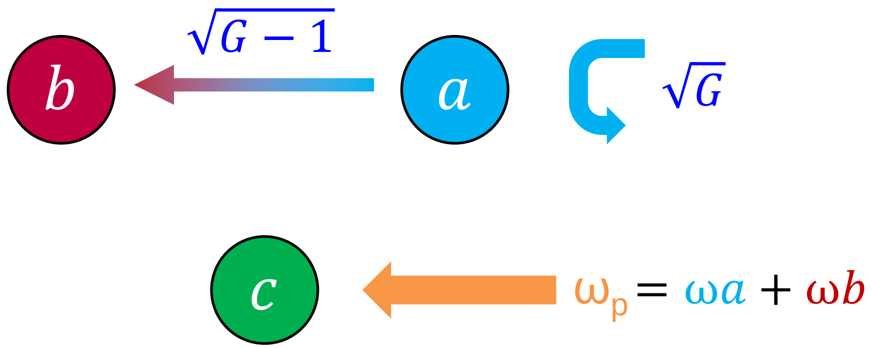
$$G = {}^{P_P}/_{P_C}$$

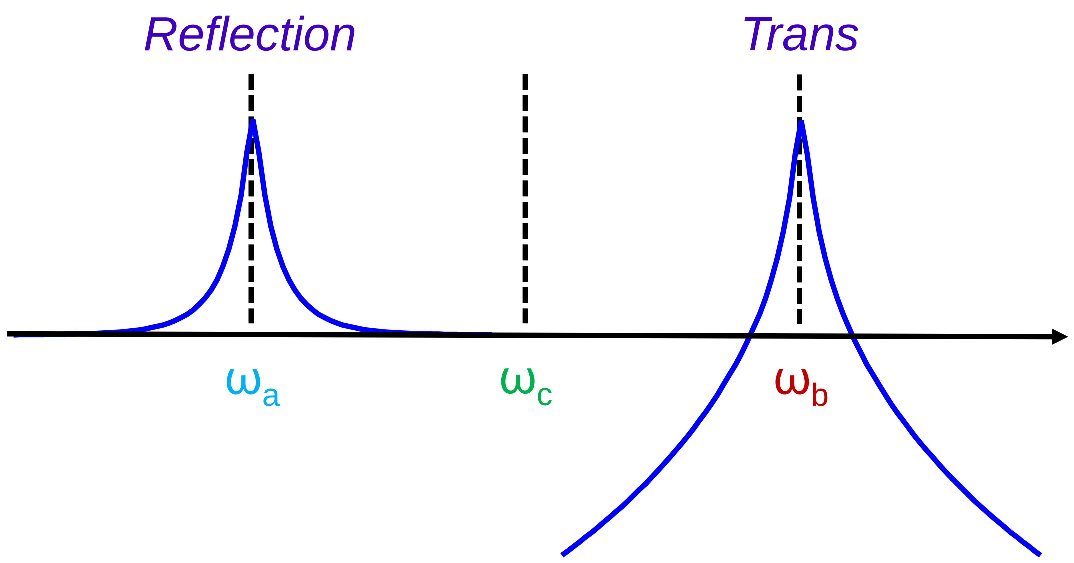
- **Enhanced bandwidth $BW \cong \frac{1}{2}\kappa/2\pi$ and saturation power!**
- Have to match mode bandwidths
- Have to pump HARD!

Metelmann *PRX* (2015)

Sliwa *APS March Meeting* (2016)

Gain / Conversion

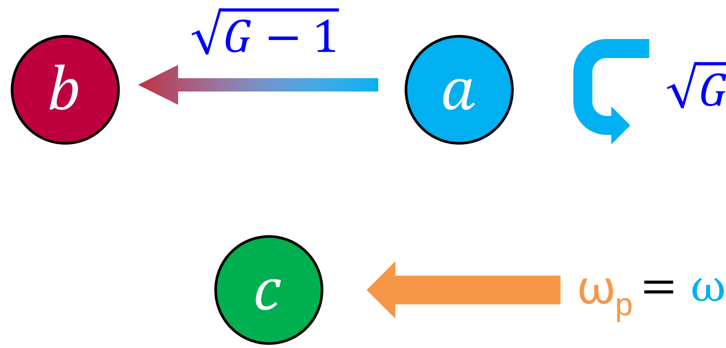




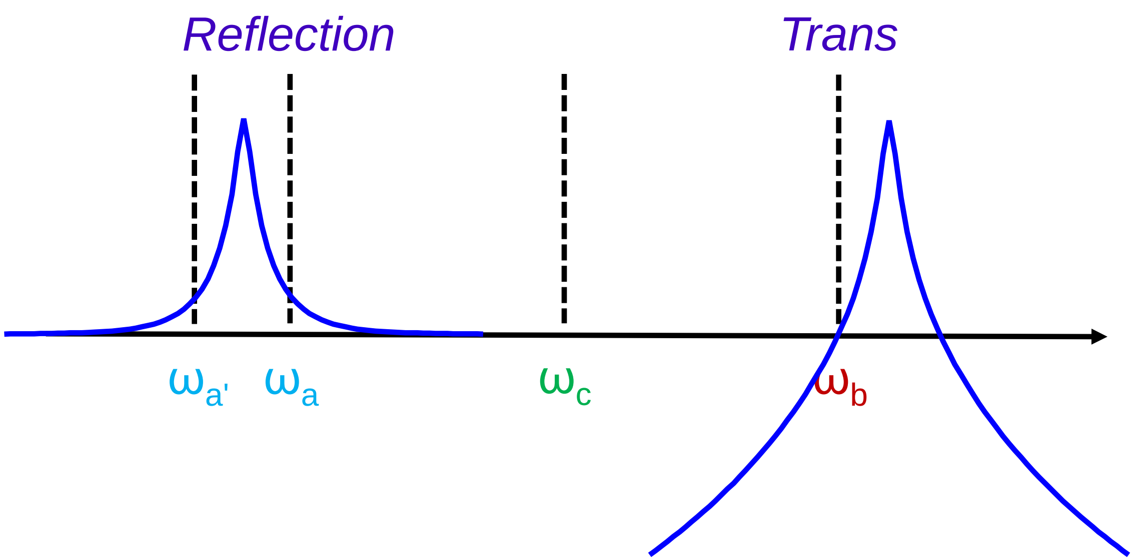
$$\frac{H_{Coupling}}{\hbar} = g_G(a^\dagger\, b^\dagger + a\, b)$$

Bergeal et al. *Nature Physics* (2010)

Gain / Conversion



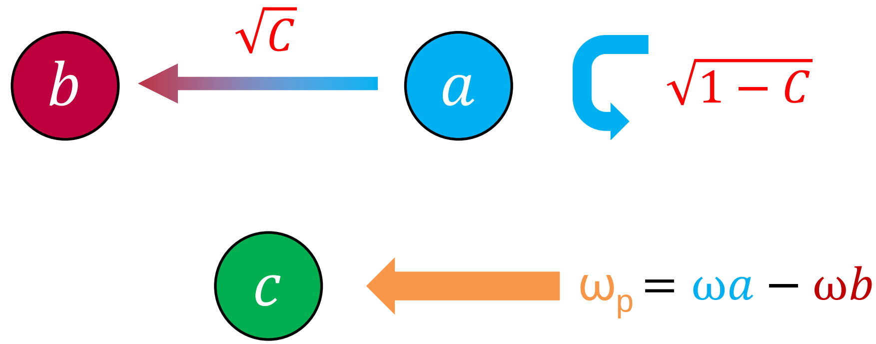
$$\omega_p = \omega a' + \omega b + K_{ac}\langle c^\dagger c\rangle$$

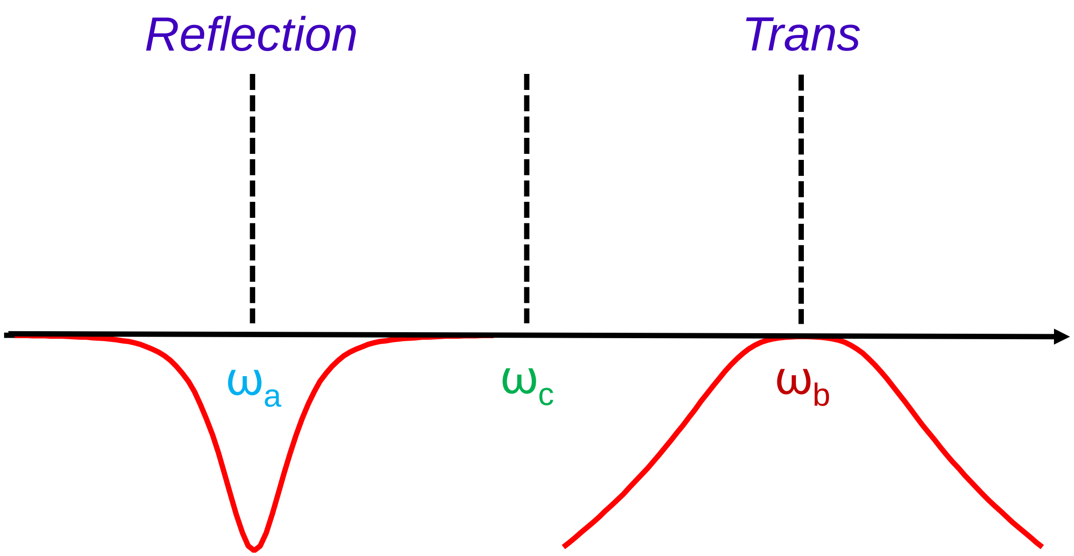


$$\frac{H_{Coupling}}{\hbar} = g_G\left(a^\dagger\, b^\dagger + a\, b\right) - K_{ac} a^\dagger a\,\langle c^\dagger c\rangle$$

Bergeal et al. *Nature Physics* (2010)

Gain / Conversion

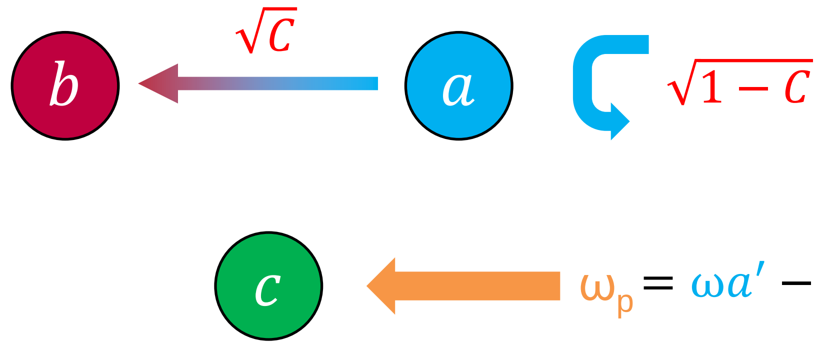




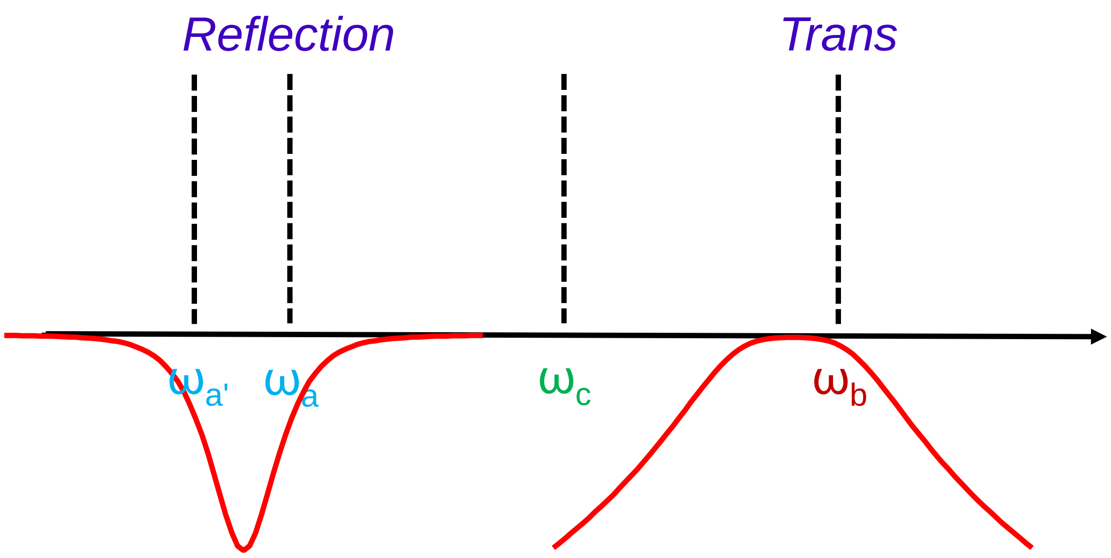
$$\frac{H_{Coupling}}{\hbar} = g_C\left(a^{\dagger}b + a\, b^{\dagger}\right)$$

Bergeal et al. *Nature Physics* (2010), Abdo et al. *Phys. Rev. B* (2013)

Gain / Conversion



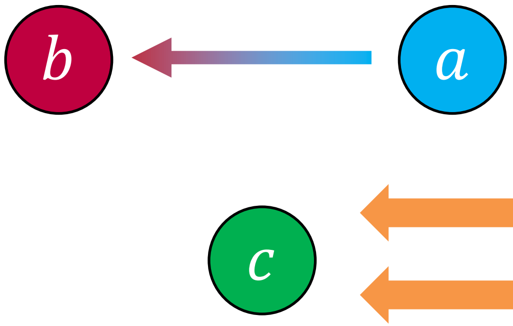
$\omega_p = \omega a' - \omega b + K_{ac}\langle c^\dagger c \rangle$



$$\frac{H_{Coupling}}{\hbar} = g_C\left(a^\dagger b + a\, b^\dagger\right) - K_{ac} a^\dagger a \left\langle c^\dagger c \right\rangle$$

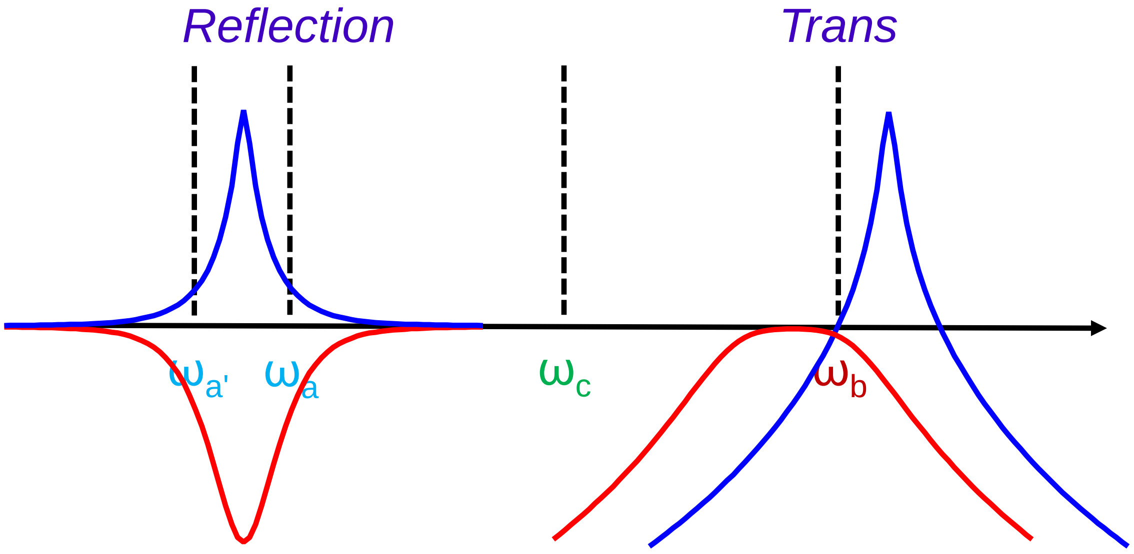
Bergeal et al. *Nature Physics* (2010), Abdo et al. *Phys. Rev. B* (2013)

G-C amplification w/ 4th order



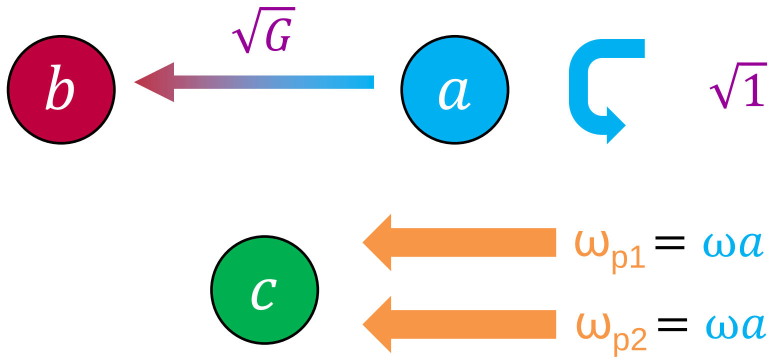
$$\omega_{p1} = \omega a' + \omega b + K_{ac}\langle c^\dagger c\rangle$$

$$\omega_{p2} = \omega a' - \omega b + K_{ac}\langle c^\dagger c\rangle$$



$$\frac{H_{Coupling}}{\hbar} = g_G\left(a^\dagger\ b^\dagger + a\ b\right) + g_C\left(a^\dagger b + a\ b^\dagger\right) - K_{ac}a^\dagger a\ \langle c^\dagger c\rangle$$

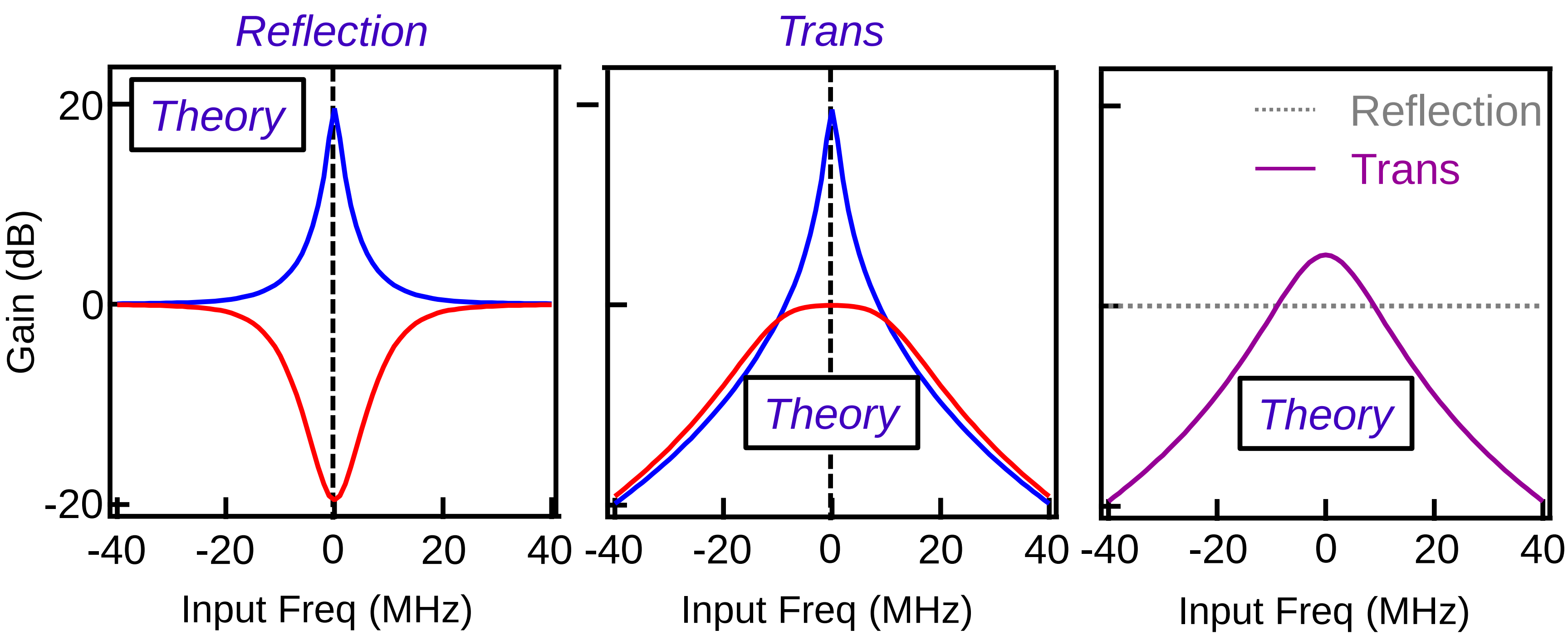
G-C amplification w/o 4th order - Theory



$\omega_{p1} = \omega a + \omega b$

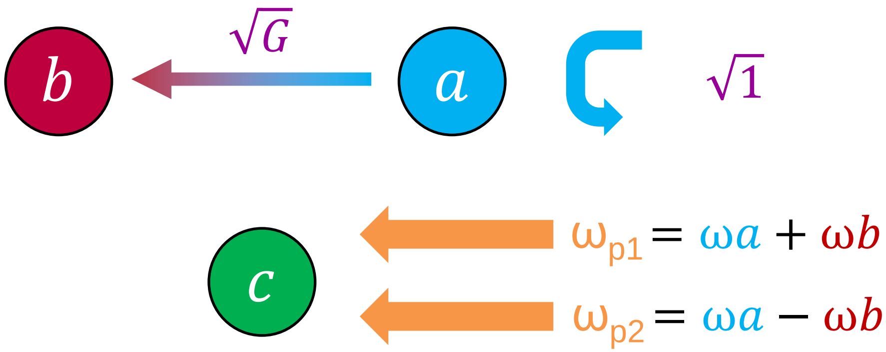
$\omega_{p2} = \omega a - \omega b$

$$\frac{H_{Coupling}}{\hbar} = g_G\left(a^\dagger\ b^\dagger + a\ b\right) + g_C\left(a^\dagger b + a\ b^\dagger\right)$$

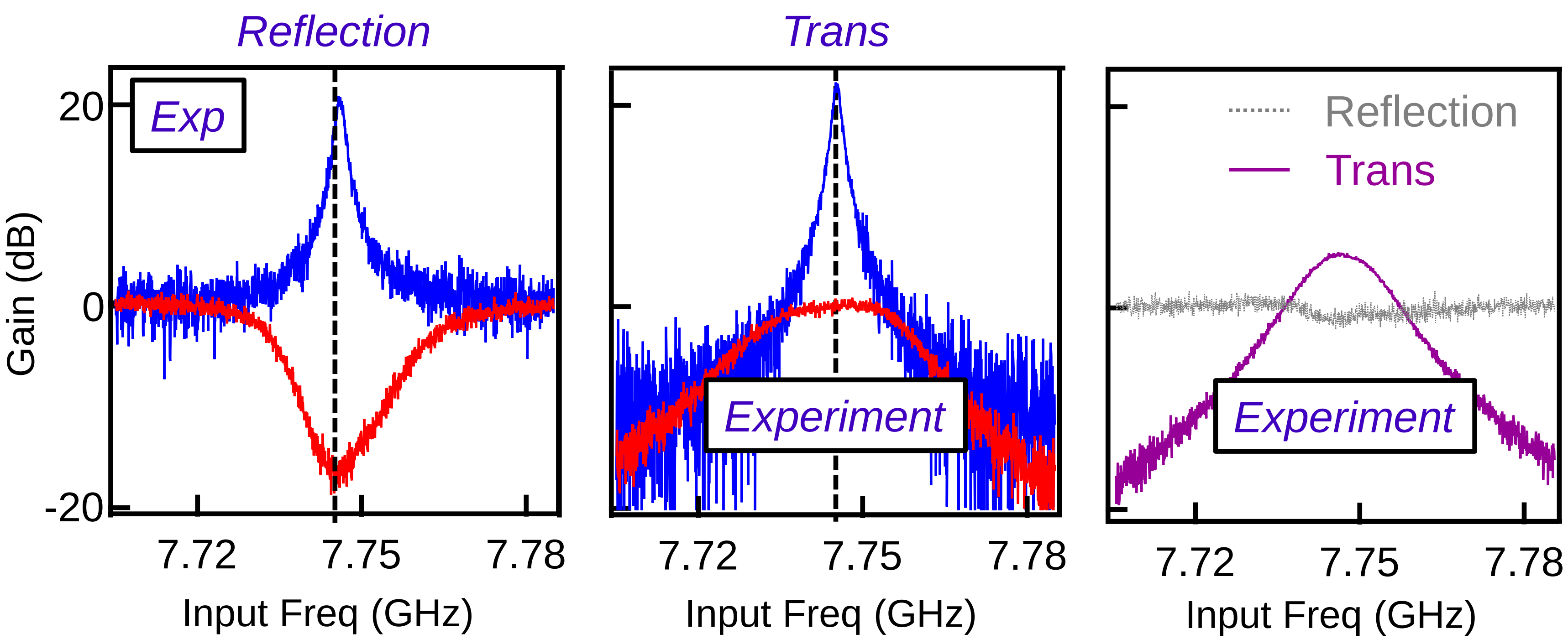


Metelmann et al. PRX (2015)

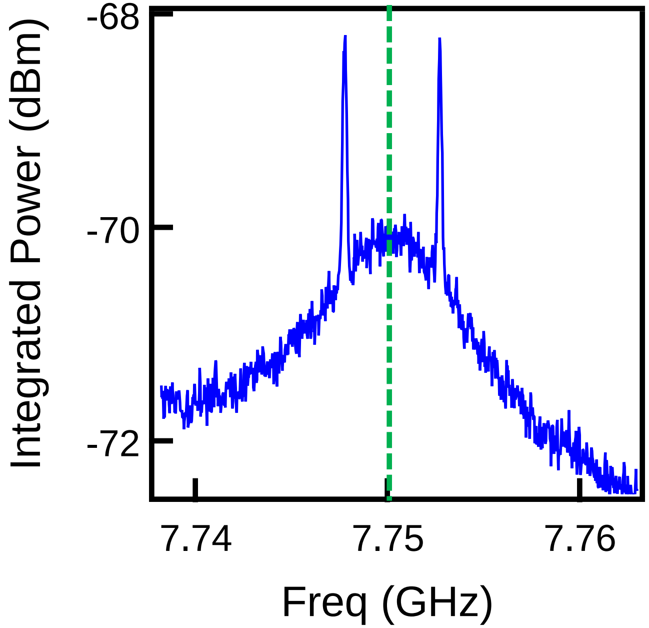
G-C amplification w/o 4th order - Experiment

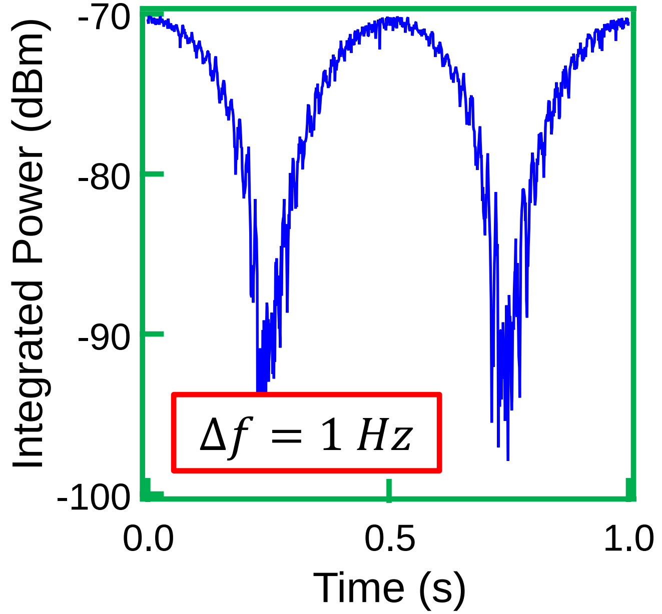


$$\frac{H_{Coupling}}{\hbar} = g_G(a^\dagger\ b^\dagger + a\ b) + g_C(a^\dagger b + a\ b^\dagger)$$

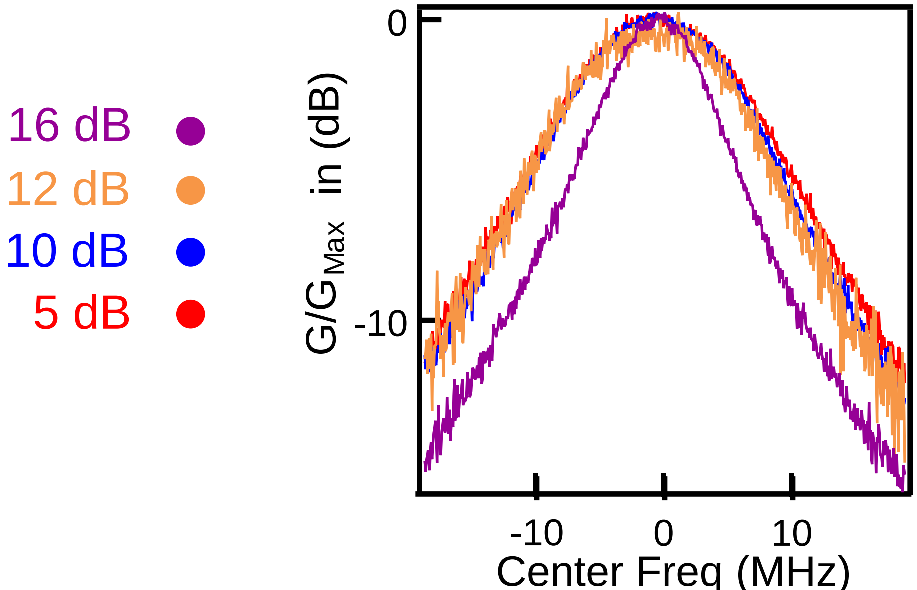


Phase sensitive amplifier

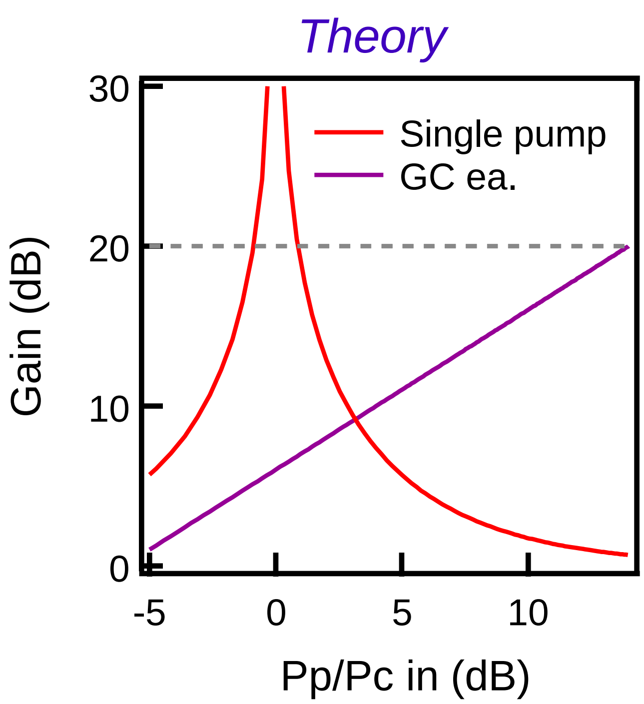


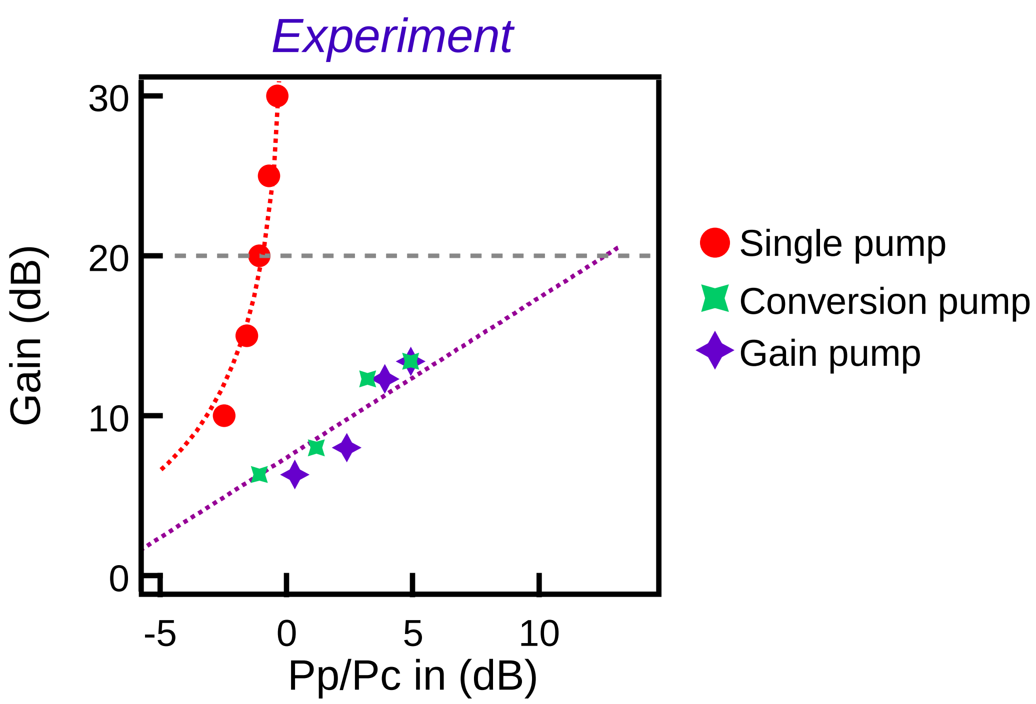


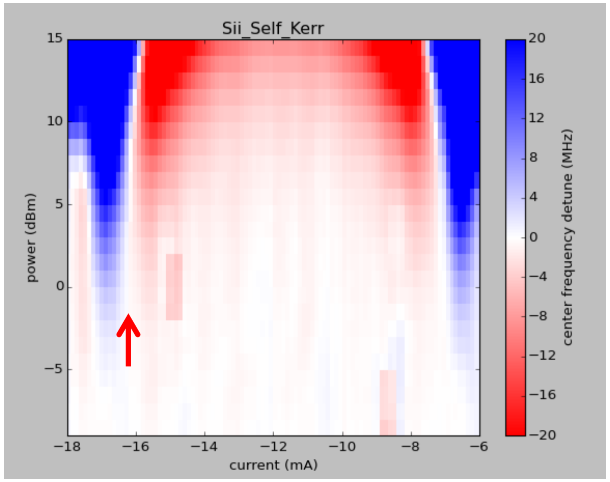
Fixed-bandwidth



Why not 30 dB ??


Theory
30
20
10
0
-5
0
5
10
Gain (dB)
Pp/Pc in (dB)
Single pump
GC ea.


Experiment
30
20
10
0
-5
0
5
10
Gain (dB)
Pp/Pc in (dB)
Single pump
Conversion pump
Gain pump


Sii_Self_Kerr
power (dBm)
current (mA)
center frequency detune (MHz)

Future work

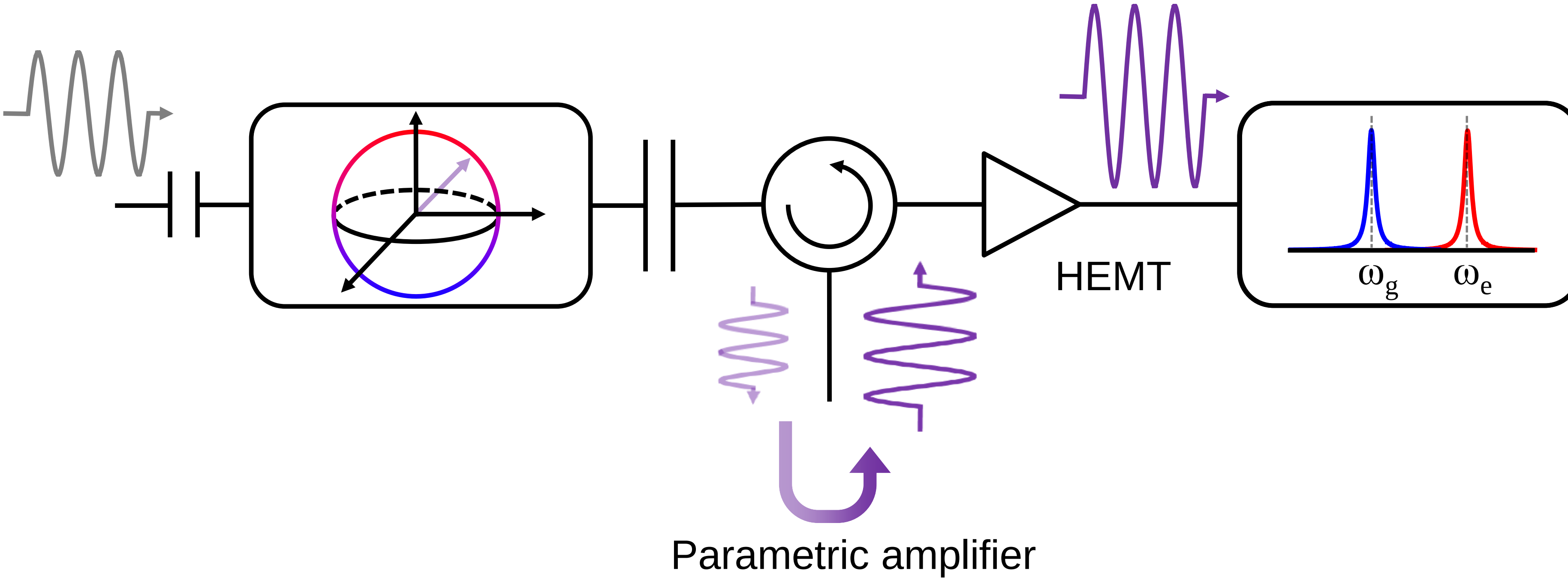
- More modes, more drives, better devices!
- New theory tools (is expansion of modes inadequate?)
- Better Hamiltonian engineering via improved fabrication
- Medium term: broadband, high-bandwidth directional amplifier

D. Pekker

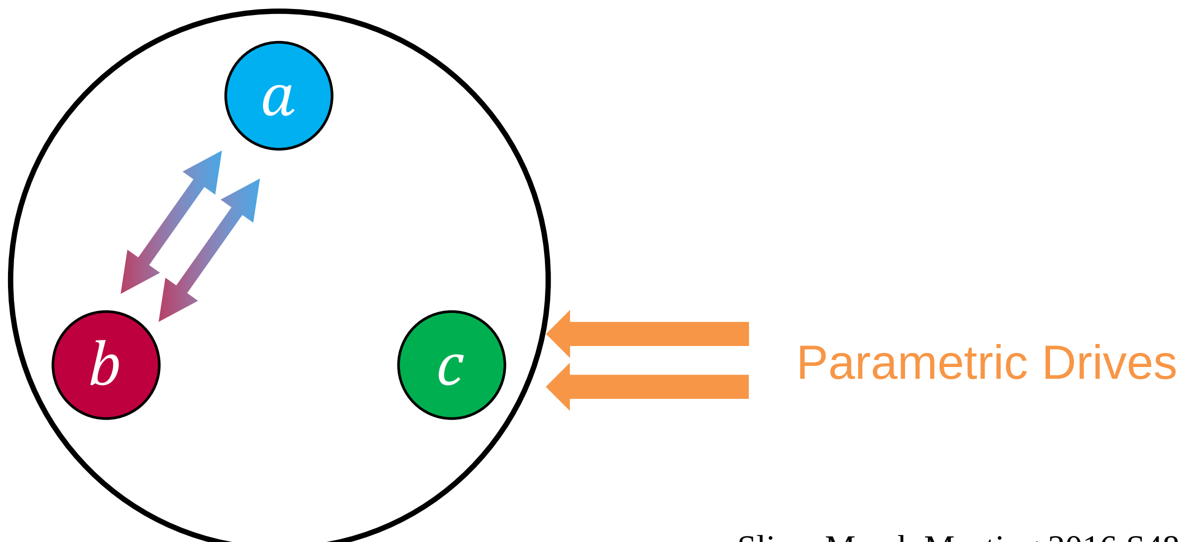
http://hatlab.pitt.edu/

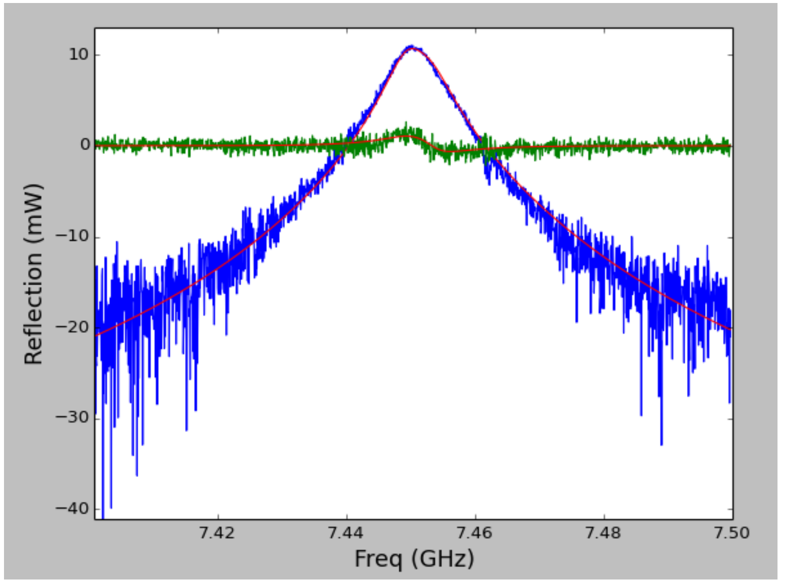
Back up

The ubiquitous parametric amplifier

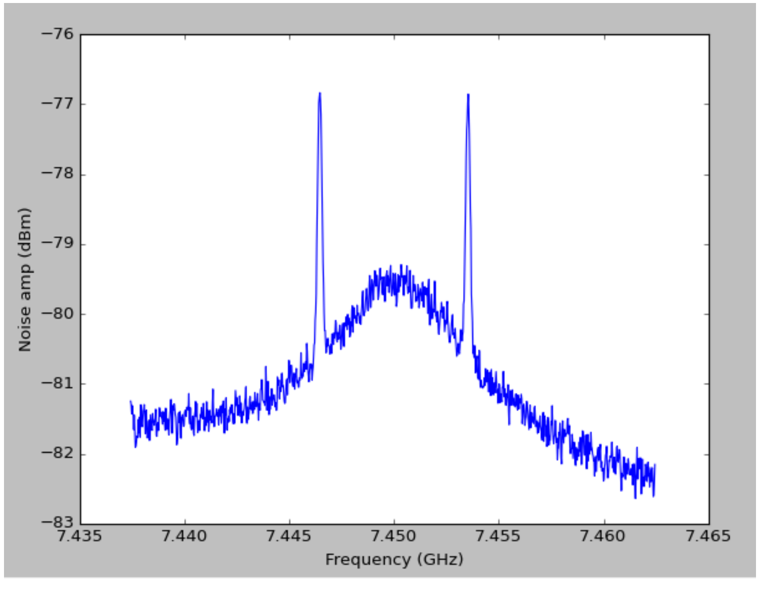


- Phase sensitive amplifier
- Fixed bandwidth
- Gain in transmission only
- Unity reflection gain

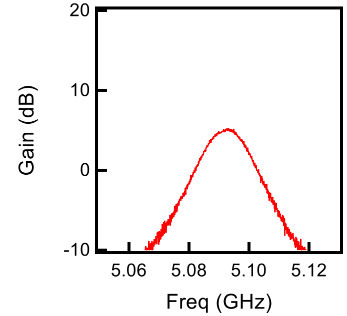




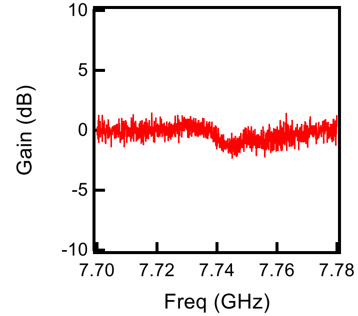
Reflection (mW)
Freq (GHz)
10
0
−10
−20
−30
−40
7.42
7.44
7.46
7.48
7.50



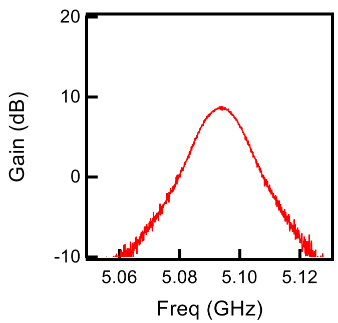
Noise amp (dBm)
Frequency (GHz)
−76
−77
−78
−79
−80
−81
−82
−83
7.435
7.440
7.445
7.450
7.455
7.460
7.465



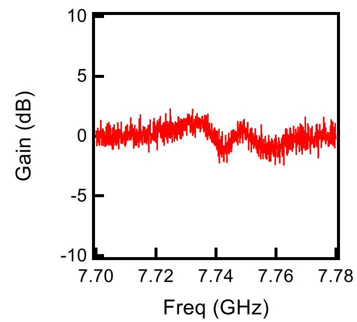
20
10
0
-10
Gain (dB)
5.06
5.08
5.10
5.12
Freq (GHz)



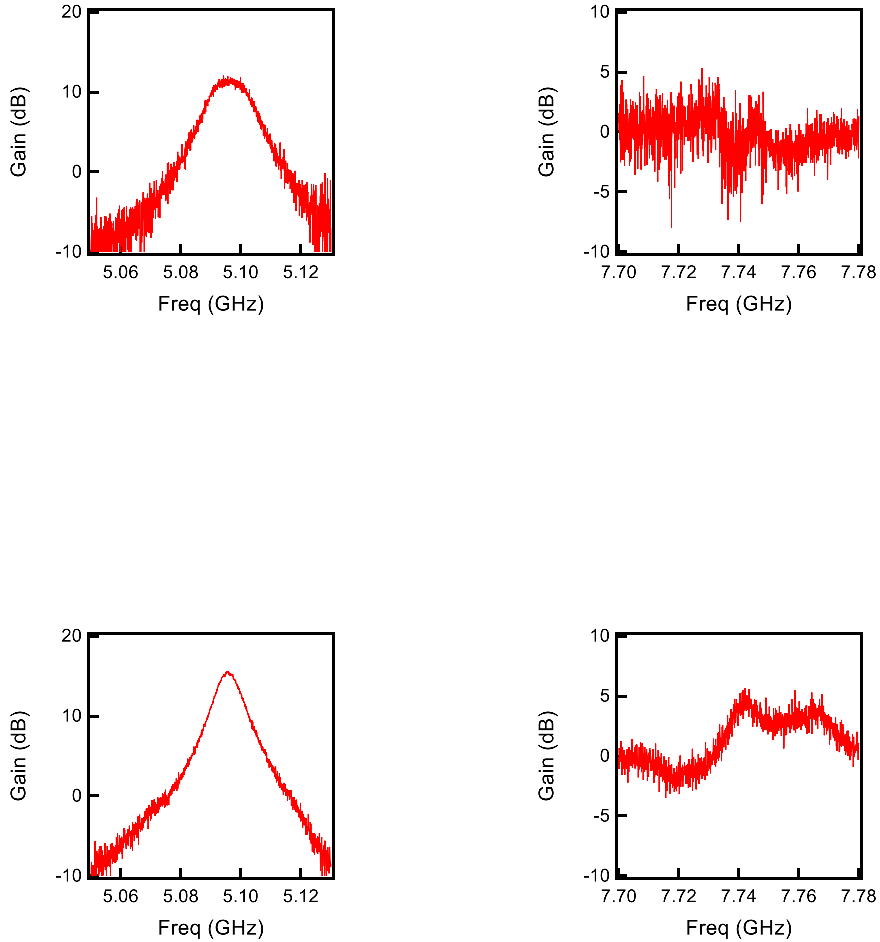
10
5
0
-5
-10
Gain (dB)
7.70
7.72
7.74
7.76
7.78
Freq (GHz)



20
10
0
-10
Gain (dB)
5.06
5.08
5.10
5.12
Freq (GHz)

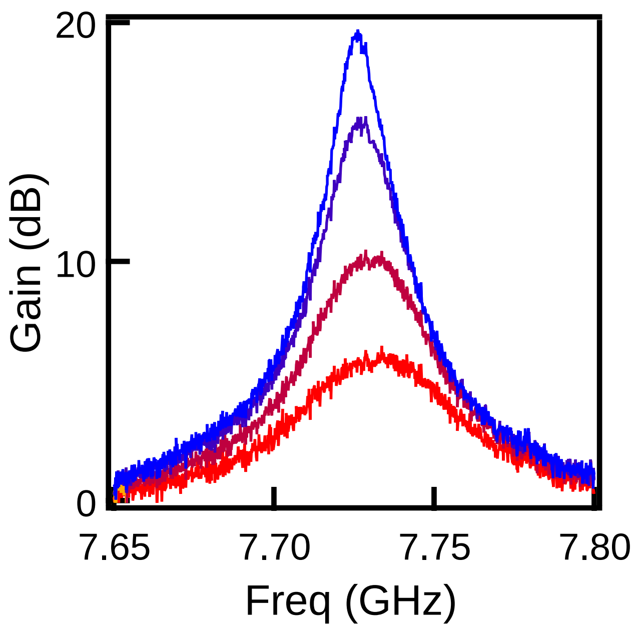


10
5
0
-5
-10
Gain (dB)
7.70
7.72
7.74
7.76
7.78
Freq (GHz)

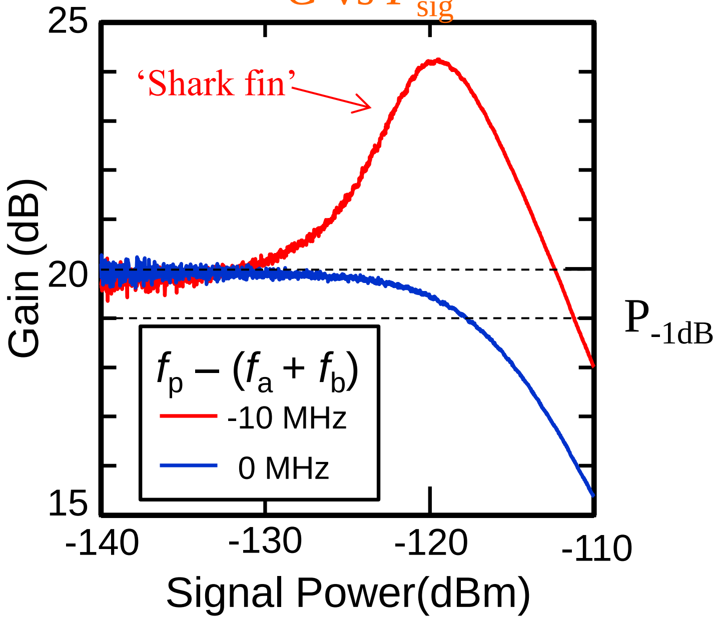

Gain (dB)
20
10
0
-10
5.06
5.08
5.10
5.12
Freq (GHz)
10
5
0
-5
-10
7.70
7.72
7.74
7.76
7.78

Higher order

Fixed Gain Peak



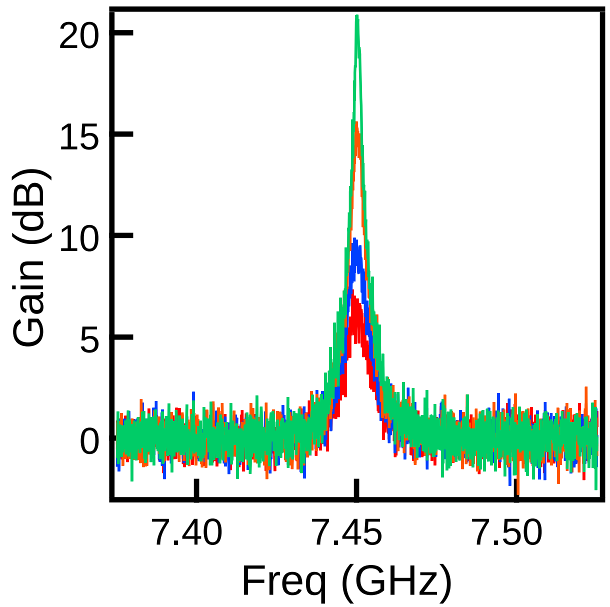
G vs P_{sig}



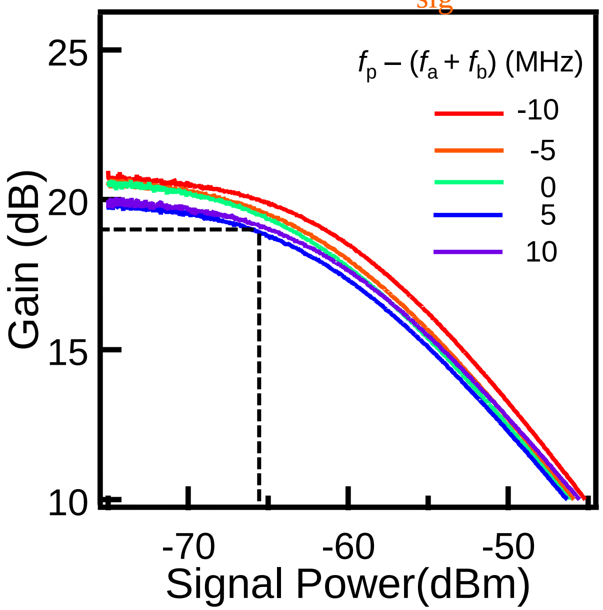
- 4th order terms shift mode freqs
- Shark-fin dynamic range

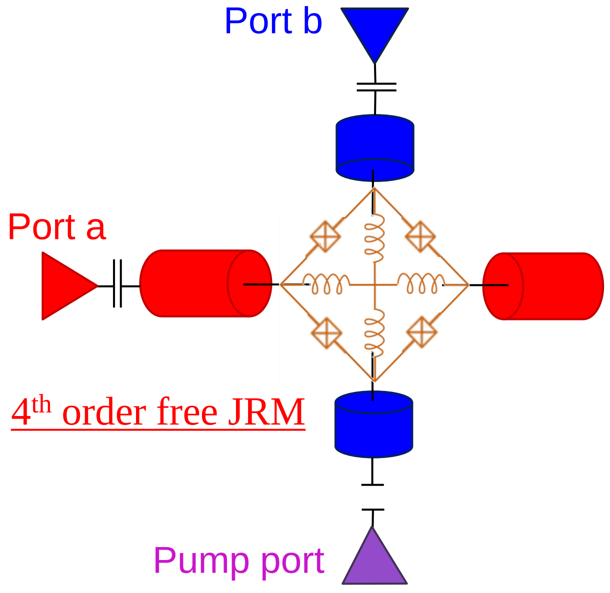
Higher order

Fixed Gain Peak



G vs P_{sig}

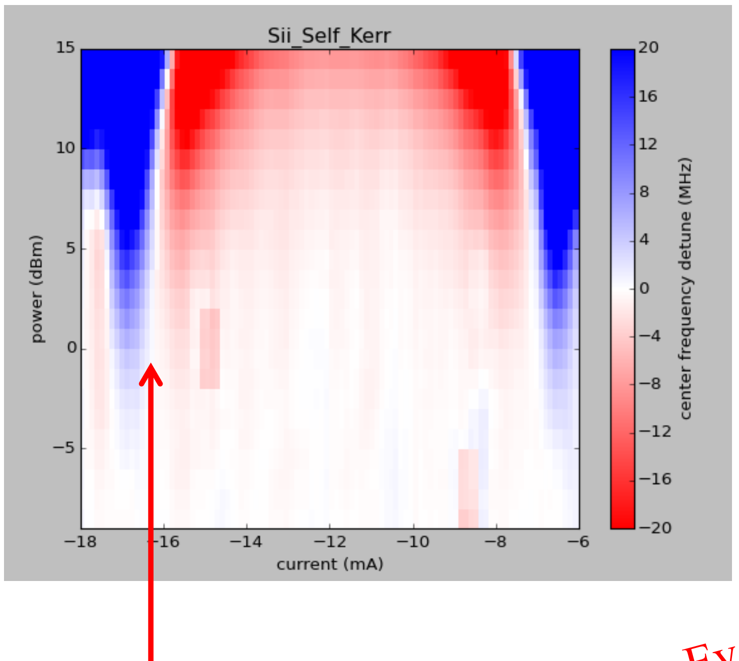


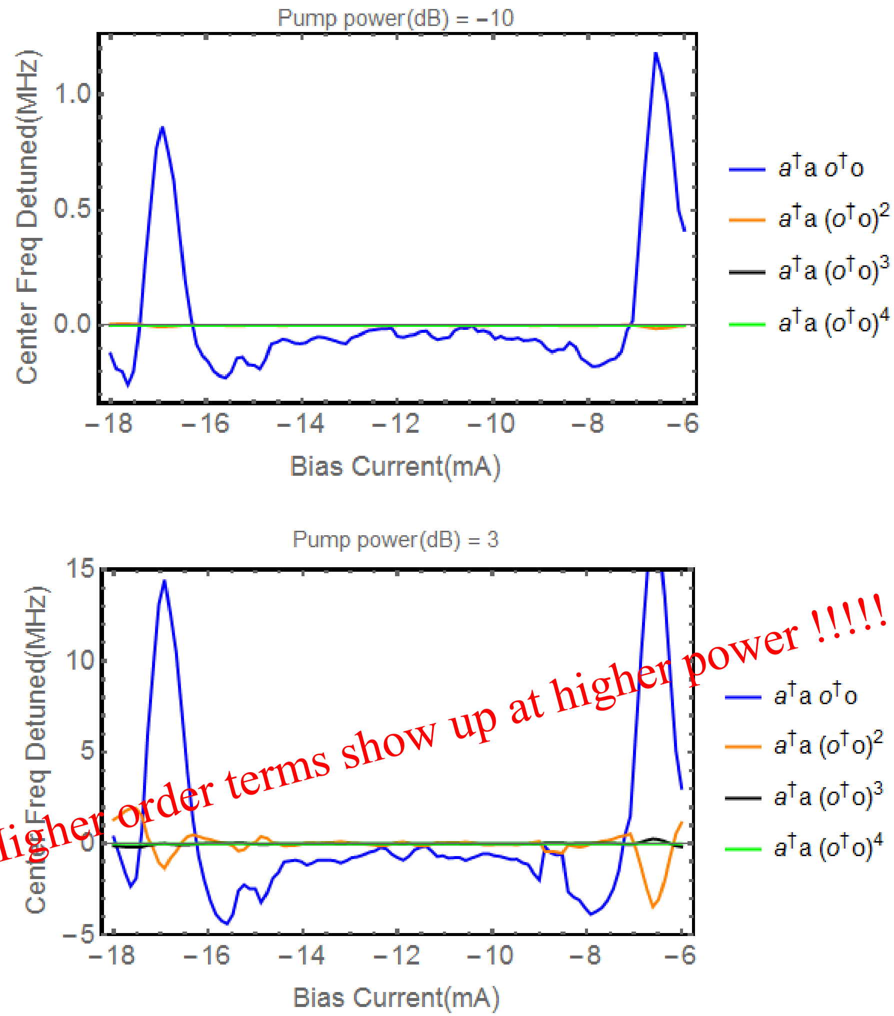


- 4th order terms shift mode freqs
- Shark-fin dynamic range
- 4th order free device

  (advertise Xi's talk)

Why not 30 dB Trans-Gain ?



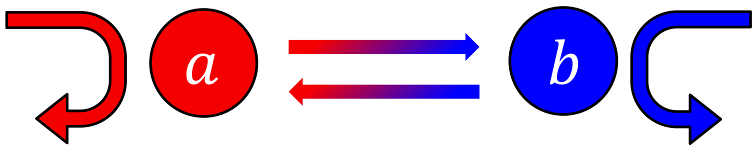


Even Higher order terms show up at higher power !!!!!

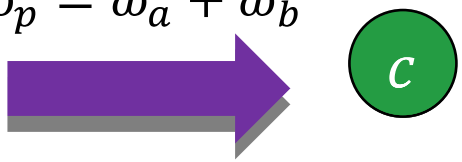
Bi-direction phase sensitive gain (XX)

$$\omega_{p1,p2} = \omega_a \pm \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a + a^\dagger\right)\left(b + b^\dagger\right)$$



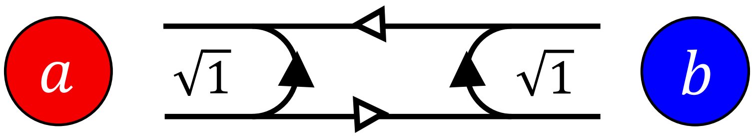
$$\omega_p \simeq \omega_a + \omega_b$$



$$\omega_p \simeq \omega_a - \omega_b$$

$$P_a \rightarrow P_b = 1/\sqrt{G}$$

$$X_b \rightarrow X_a = \sqrt{G}$$



$$P_a \rightarrow P_b = \sqrt{G}$$

$$X_a \rightarrow X_b = 1/\sqrt{G}$$

$$G = {}^{P_P}/_{P_C}$$

- **Enhanced bandwidth $BW \cong \frac{1}{2}\kappa/2\pi$ and saturation power!**
- Have to match mode bandwidths
- Have to pump HARD!

Metelmann *PRX* (2015)

Sliwa *APS March Meeting* (2016)