Non-Hermitian Topological Phases

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Based on

MS, K. Hasebe, K. Esaki, M.Kohmoto, Time-reversal symmetry in non-Hermitian systems, Progress of Theoretical Physics 127, 937 (2011)

K.Esaki, MS, K.Hasebe, M. Kohmoto, Edge states and topological phases in non-Hermitian systems, Phys. Rev. B84, 205129 (2011)

$+\ \alpha$ (new)

- Kenta Esaki (ISSP)
- Kazuki Hasebe (Kagawa National College of Technology)
- Mahito Kohmoto (ISSP)

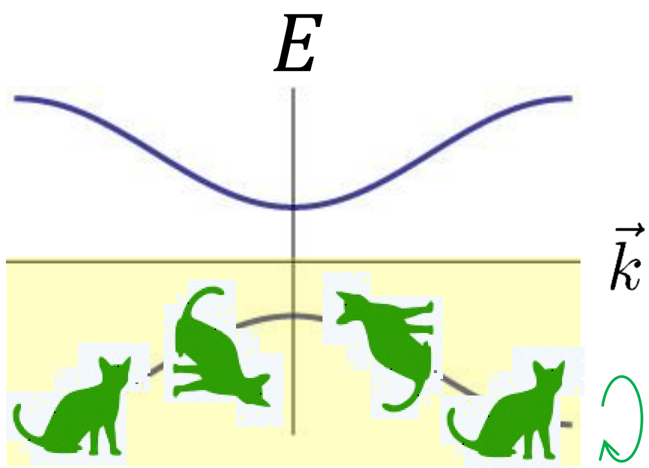
Outline

- Why non-hermitian topological phase?
- Basic symmetries in non-hermitian systems
- Topological phases intrinsic to non-Hermitian systems

What is topological phase

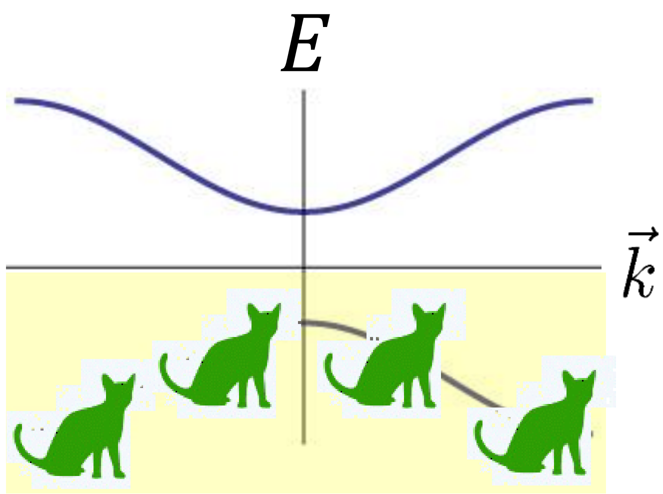
Top. Phase = bulk top # of occupied state or gapped Hamiltonian

TI/TSC



$N_{\text{top}} \neq 0$

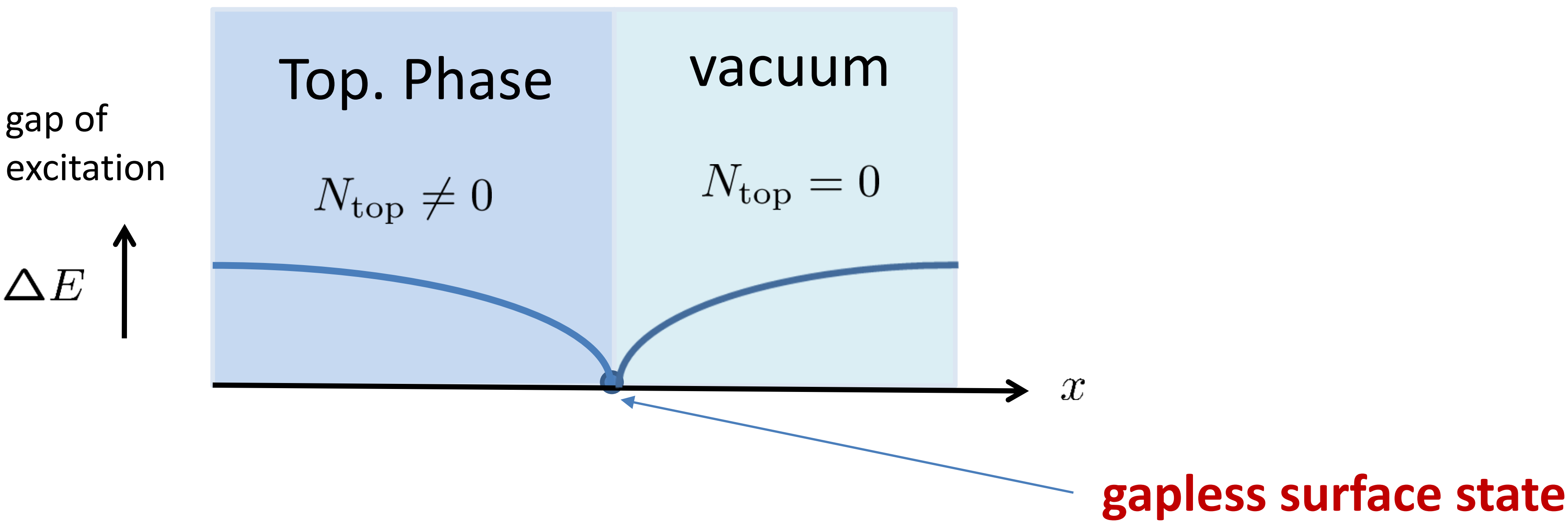
Ordinary insulator/SC



$N_{\text{top}} = 0$

The idea of topological phases (topological insulators and topological superconductors) has been successfully established with many experimental supports

Indeed, the non-trivial topological structure can be detected experimentally as the existence of gapless surface states

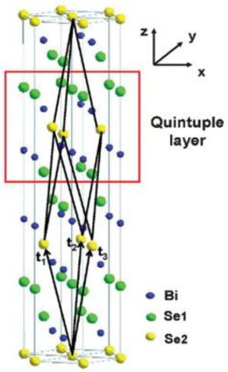


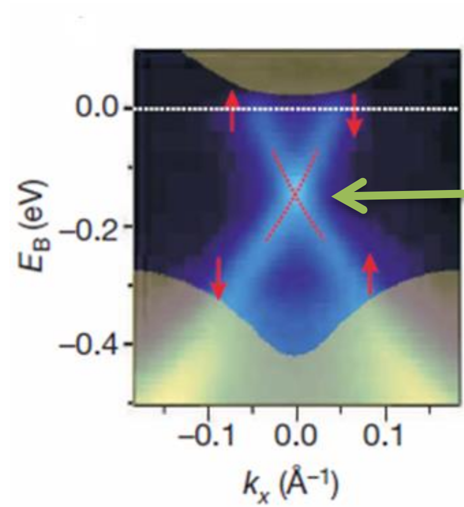
Materials in topological phases have gapless boundary states ensured by bulk topological numbers

bulk-boundary correspondence

Topological insulators

Bi_2Se_3





conduction band

surface Dirac fermions

valence band

[Hsieh et al (09)]

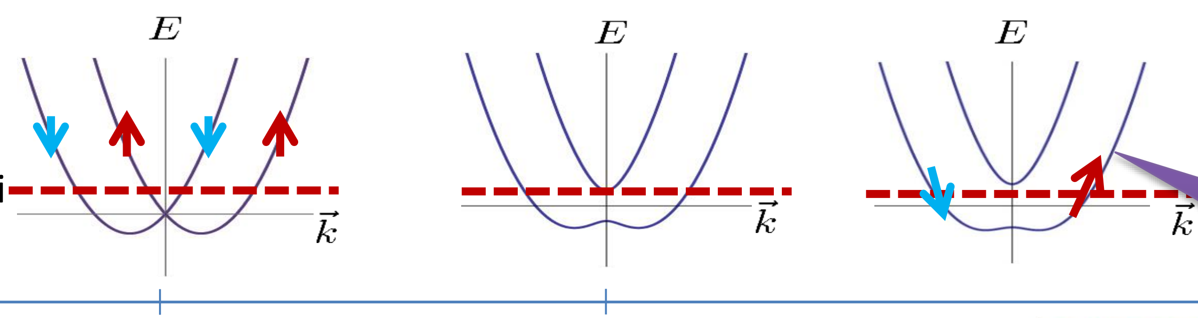
Topological superconductors

S-wave SC with Rashba SOC + Zeeman field

[MS-Takahashi-Fujimoto (09), J. Sau et al (10)]

Non spin-degenerate single Fermi surface

Fermi Level



Topological SC

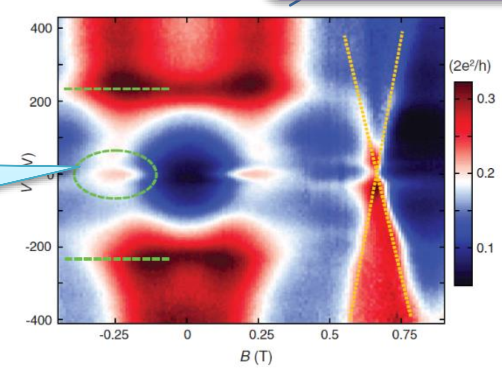
1D Nanowire

[Lutchyn et al (10), Y.O ...)]

nanowire

MF

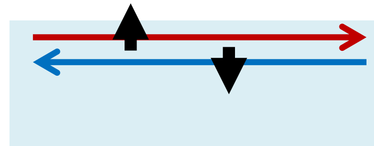
Majorana Fermion



Mourik *et al.*,(12)

Symmetry is very important to obtain top. phases

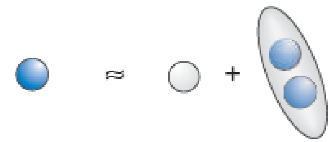
Time-reversal symmetry (TRS)



Kramers pair

- No back scattering
- topologically stable

Particle-hole symmetry (PHS)



Majorana fermion

[Schnyder-Ryu-Furusaki-Ludwig (12)]
[Avron-Seiler-Simon (83)]

	TRS	PHS	CS	d=1	d=2	d=3
A	0	0	0	0	**Z**	0
AIII	0	0	1	**Z**	0	**Z**
AI	1	0	0	0	0	0
BDI	1	1	1	**Z**	0	0
D	0	1	0	$\mathbf{Z_2}$	**Z**	0
DIII	-1	1	1	$\mathbf{Z_2}$	$\mathbf{Z_2}$	**Z**
AII	-1	0	0	0	$\mathbf{Z_2}$	$\mathbf{Z_2}$
CII	-1	-1	1	**2Z**	0	$\mathbf{Z_2}$
C	0	-1	0	0	**2Z**	0
CI	1	-1	1	0	0	**2Z**

IQHS

Majorana nanowire

p+ip chiral p
Sr_2RuO_4, ^3He-A

^3He-B

$Cu_xBi_2Se_3$

3D TI

QSH

But this is just a starting point …

Indeed by taking into account crystalline symmetry, we can obtain huge #s of new topological phases

SG	Short	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
1	$P1$	$\mathbb{Z}$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}$
2	$P\bar{1}$	$\mathbb{Z}^9$	0	$\mathbb{Z}^3$	$\mathbb{Z}_2$
3	$P2$	$\mathbb{Z}^5$	$\mathbb{Z}^5$	$\mathbb{Z}$	$\mathbb{Z}$
4	$P2_1$	$\mathbb{Z}$	$\mathbb{Z}+\mathbb{Z}_2^3$	$\mathbb{Z}$	$\mathbb{Z}$
5	$C2$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}$	$\mathbb{Z}$
6	Pm	$\mathbb{Z}^3$	$\mathbb{Z}^6$	$\mathbb{Z}^3$	0
7	Pc	$\mathbb{Z}$	$\mathbb{Z}^2+\mathbb{Z}_2$	$\mathbb{Z}+\mathbb{Z}_2$	0
8	Cm	$\mathbb{Z}^2$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	0
9	Cc	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}+\mathbb{Z}_2$	0

SG	Short	$\epsilon(2_{001}, m_{001})$	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
10	$P2/m$	$+_{0,1/2}$	$\mathbb{Z}^{15}$	0	$\mathbb{Z}^3$	0
		$-$	$\mathbb{Z}$	$\mathbb{Z}^8$	$\mathbb{Z}$	0
11	$P2_1/m$	$+_{0,1/2}, -$	$\mathbb{Z}^6$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	0
12	$C2/m$	$+_{0,1/2}$	$\mathbb{Z}^{10}$	0	$\mathbb{Z}^2$	0
		$-$	$\mathbb{Z}^3$	$\mathbb{Z}^4$	$\mathbb{Z}$	0
13	$P2/c$	$+_{0,1/2}, -$	$\mathbb{Z}^7$	$\mathbb{Z}^2$	$\mathbb{Z}$	0
14	$P2_1/c$	$+_{0,1/2}, -$	$\mathbb{Z}^5$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
15	$C2/c$	$+_{0,1/2}, -$	$\mathbb{Z}^6$	$\mathbb{Z}$	$\mathbb{Z}$	0

SG	Short	$\epsilon(2_{100}, 2_{010})$	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
16	$P222$	$+_0$	$\mathbb{Z}^{13}$	$\mathbb{Z}_2$	0	$\mathbb{Z}$
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^{12}$	0	$\mathbb{Z}$
17	$P222_1$	$+_0, -_{1/2}$	$\mathbb{Z}^5$	$\mathbb{Z}^4+\mathbb{Z}_2$	0	$\mathbb{Z}$
18	$P2_12_12$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^2+\mathbb{Z}_2^3$	0	$\mathbb{Z}$
19	$P2_12_12_1$	$+_0, -_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}_4^3$	0	$\mathbb{Z}$
20	$C222_1$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^2+\mathbb{Z}_2^2$	0	$\mathbb{Z}$
21	$C222$	$+_0$	$\mathbb{Z}^8$	$\mathbb{Z}+\mathbb{Z}_2$	0	$\mathbb{Z}$
		$-_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^7$	0	$\mathbb{Z}$
22	$F222$	$+_0$	$\mathbb{Z}^7$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^6$	$\mathbb{Z}_2$	$\mathbb{Z}$
23	$I222$	$+_0$	$\mathbb{Z}^7$	$\mathbb{Z}_2^2$	0	$\mathbb{Z}$
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^6+\mathbb{Z}_2$	0	$\mathbb{Z}$
24	$I2_12_12_1$	$+_0, -_{1/2}$	$\mathbb{Z}^4$	$\mathbb{Z}^3+\mathbb{Z}_2$	0	$\mathbb{Z}$

SG	Short	$\epsilon(m_{100}, m_{010})$	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
25	$Pmm2$	$+_0$	$\mathbb{Z}^9$	$\mathbb{Z}^9$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^5$	$\mathbb{Z}^4$	0
26	$Pmc2_1$	$+_0$	$\mathbb{Z}^3$	$\mathbb{Z}^3+\mathbb{Z}_2^3$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^3$	$\mathbb{Z}^2+\mathbb{Z}_2$	0
27	$Pcc2$	$+_0$	$\mathbb{Z}^5$	$\mathbb{Z}^5$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2^4$	0
28	$Pma2$	$+_0, -_{1/2}$	$\mathbb{Z}^4$	$\mathbb{Z}^5$	$\mathbb{Z}$	0
29	$Pca2_1$	$+_0, -_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}+\mathbb{Z}_2^2$	$\mathbb{Z}_2$	0
30	$Pna2_1$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}_2$	0
31	$Pmn2_1$	$+_0, -_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^3+\mathbb{Z}_2$	$\mathbb{Z}$	0
32	$Pba2$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^3+\mathbb{Z}_2$	$\mathbb{Z}_2$	0
33	$Pna2_1$	$+_0, -_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}+\mathbb{Z}_4$	$\mathbb{Z}_2$	0
34	$Pnn2$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}_2$	0
35	$Cmm2$	$+_0$	$\mathbb{Z}^6$	$\mathbb{Z}^6$	0	0
		$-_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	0
36	$Cmc2_1$	$+_0$	$\mathbb{Z}^2$	$\mathbb{Z}^2+\mathbb{Z}_2^2$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^2+\mathbb{Z}_2$	$\mathbb{Z}$	0
37	$Ccc2$	$+_0$	$\mathbb{Z}^4$	$\mathbb{Z}^4$	0	0
		$-_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	$\mathbb{Z}_2^2$	0
38	$Amm2$	$+_0$	$\mathbb{Z}^6$	$\mathbb{Z}^6$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^4$	$\mathbb{Z}^3$	0
39	$Abm2$	$+_0$	$\mathbb{Z}^4$	$\mathbb{Z}^4+\mathbb{Z}_2$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}+\mathbb{Z}_2^2$	0
40	$Ama2$	$+_0, -_{1/2}$	$\mathbb{Z}^3$	$\mathbb{Z}^4$	$\mathbb{Z}$	0
41	$Aba2$	$+_0, -_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^2+\mathbb{Z}_2$	$\mathbb{Z}_2$	0
42	$Fmm2$	$+_0$	$\mathbb{Z}^5$	$\mathbb{Z}^5$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^3$	$\mathbb{Z}^2+\mathbb{Z}_2$	0
43	$Fdd2$	$+_0, -_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	$\mathbb{Z}_2$	0
44	$Imm2$	$+_0$	$\mathbb{Z}^5$	$\mathbb{Z}^5$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}^3+\mathbb{Z}_2$	$\mathbb{Z}^2$	0
45	$Iba2$	$+_0$	$\mathbb{Z}^3$	$\mathbb{Z}^3+\mathbb{Z}_2$	0	0
		$-_{1/2}$	$\mathbb{Z}$	$\mathbb{Z}+\mathbb{Z}_2$	$\mathbb{Z}_2^2$	0
46	$Ima2$	$+_0$	$\mathbb{Z}^3$	$\mathbb{Z}^3+\mathbb{Z}_2$	0	0
		$-_{1/2}$	$\mathbb{Z}^2$	$\mathbb{Z}^3$	$\mathbb{Z}$	0

List of top #s for materials with 230 space groups

[Shiozaki-MS-Gomi (18)]

(Cont.)

SG	Short	$(\epsilon(m_{100}, m_{010}), \epsilon(m_{100}, m_{001}), \epsilon(m_{010}, m_{001}))$	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
47	$Pmmm$	$(+,+,+)_0$	$\mathbb{Z}^{27}$	0	0	0
		$(-,-,-)_{1/2}$	$\mathbb{Z}^9$	0	$\mathbb{Z}^6$	0
		$(-,+,+),(+,-,+),(+,+,-)$	$\mathbb{Z}^3$	$\mathbb{Z}^{12}$	0	0
		$(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^5$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	0
48	$Pnnn$	$(+,+,+)_0,(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^9$	0	$\mathbb{Z}_2$	0
		$(-,-,-)_{1/2},(-,+,+),(+,-,+),(+,+,-)$	$\mathbb{Z}^3$	$\mathbb{Z}^6$	0	0
49	$Pccm$	$(+,+,+)_0,(+,-,-)$	$\mathbb{Z}^{14}$	0	$\mathbb{Z}$	0
		$(-,-,-)_{1/2},(-,+,+)$	$\mathbb{Z}^6$	$\mathbb{Z}^4$	$\mathbb{Z}$	0
		$(+,-,+),(+,+,-)$	$\mathbb{Z}$	$\mathbb{Z}^{10}$	0	0
		$(-,+,-),(-,-,+)$	$\mathbb{Z}^5$	$\mathbb{Z}^2$	$\mathbb{Z}_2$	0
50	$Pban$	$(+,+,+)_0,(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^9$	0	$\mathbb{Z}_2$	0
		$(-,-,-)_{1/2},(-,+,+),(+,-,+),(+,+,-)$	$\mathbb{Z}^3$	$\mathbb{Z}^6$	0	0
51	$Pmma$	$(+,+,+)_0,(+,-,+)$	$\mathbb{Z}^{12}$	$\mathbb{Z}^3$	0	0
		$(-,-,-)_{1/2},(-,+,-)$	$\mathbb{Z}^7$	$\mathbb{Z}$	$\mathbb{Z}^3$	0
		$(+,+,-),(+,-,-)$	$\mathbb{Z}^4$	$\mathbb{Z}^7$	0	0
		$(-,+,+),(-,-,+)$	$\mathbb{Z}$	$\mathbb{Z}^5+\mathbb{Z}_2$	$\mathbb{Z}$	0
52	$Pnna$	all	$\mathbb{Z}^5$	$\mathbb{Z}^2$	0	0
53	$Pmna$	$(+,+,+)_0,(-,-,-)_{1/2},(+,+,-),(-,-,+)$	$\mathbb{Z}^9$	$\mathbb{Z}$	$\mathbb{Z}$	0
		$(+,-,+),(-,+,+),(+,-,-),(-,+,-)$	$\mathbb{Z}^2$	$\mathbb{Z}^5$	0	0

(Cont.)

SG	Short	$(\epsilon(m_{100},m_{010}),\epsilon(m_{100},m_{001}),\epsilon(m_{010},m_{001}))$	$E_2^{0,0}$	$E_2^{1,0}$	$E_2^{2,0}$	$E_2^{3,0}$
54	$Pcca$	$(+,+,+)_0,(+,+,-),(+,-,+),(+,-,-)$	$\mathbb{Z}^6$	$\mathbb{Z}^3$	0	0
		$(-,-,-)_{1/2},(-,+,+),(-,+,-),(-,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}$	$\mathbb{Z}_2$	0
55	$Pbam$	$(+,+,+)_0,(-,+,+)$	$\mathbb{Z}^9$	$\mathbb{Z}_2^3$	0	0
		$(-,-,-)_{1/2},(+,-,-)$	$\mathbb{Z}^7$	0	$\mathbb{Z}^2+\mathbb{Z}_2$	0
		$(+,-,+),(+,+,-),(-,+,-),(-,-,+)$	$\mathbb{Z}$	$\mathbb{Z}^4+\mathbb{Z}_2$	0	0
56	$Pccn$	$(+,+,+)_0,(+,+,-),(+,-,+),(+,-,-)$	$\mathbb{Z}^5$	$\mathbb{Z}^2+\mathbb{Z}_2$	0	0
		$(-,-,-)_{1/2},(-,+,+),(-,+,-),(-,-,+)$	$\mathbb{Z}^3$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0
57	$Pbcm$	$(+,+,+)_0,(+,+,-),(-,+,+),(-,+,-)$	$\mathbb{Z}^5$	$\mathbb{Z}^2+\mathbb{Z}_2$	0	0
		$(-,-,-)_{1/2},(+,-,+),(+,-,-),(-,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	$\mathbb{Z}$	0
58	$Pnnm$	$(+,+,+)_0,(-,-,-)_{1/2},(-,+,+),(+,-,-)$	$\mathbb{Z}^8$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
		$(+,+,-),(+,-,+),(-,+,-),(-,-,+)$	$\mathbb{Z}$	$\mathbb{Z}^4+\mathbb{Z}_2$	0	0
59	$Pmmn$	$(+,+,+)_0,(+,+,-),(+,-,+),(+,-,-)$	$\mathbb{Z}^7$	$\mathbb{Z}^4$	0	0
		$(-,-,-)_{1/2},(-,+,+),(-,+,-),(-,-,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^2+\mathbb{Z}_2$	$\mathbb{Z}^2$	0
60	$Pbcn$	all	$\mathbb{Z}^4$	$\mathbb{Z}+\mathbb{Z}_2$	0	0
61	$Pbca$	all	$\mathbb{Z}^3$	$\mathbb{Z}_2^2$	0	0
62	$Pnma$	$(+,+,+)_0,(+,-,+),(-,+,+),(-,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}+\mathbb{Z}_2^2$	0	0
		$(-,-,-)_{1/2},(+,-,-),(-,+,-),(+,+,-)$	$\mathbb{Z}^3$	$\mathbb{Z}+\mathbb{Z}_2$	$\mathbb{Z}$	0
63	$Cmcm$	$(+,+,+)_0,(+,+,-)$	$\mathbb{Z}^8$	$\mathbb{Z}^2$	0	0
		$(-,-,-)_{1/2},(-,-,+)$	$\mathbb{Z}^5$	$\mathbb{Z}$	$\mathbb{Z}^2$	0
		$(+,-,+),(+,-,-)$	$\mathbb{Z}^2$	$\mathbb{Z}^3$	$\mathbb{Z}$	0
		$(-,+,+),(-,+,-)$	$\mathbb{Z}^4$	$\mathbb{Z}^4$	0	0
64	$Cmca$	$(+,+,+)_0,(+,+,-)$	$\mathbb{Z}^7$	$\mathbb{Z}+\mathbb{Z}_2$	0	0
		$(-,-,-)_{1/2},(-,-,+)$	$\mathbb{Z}^5$	0	$\mathbb{Z}+\mathbb{Z}_2$	0
		$(+,-,+),(+,-,-)$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	$\mathbb{Z}_2$	0
		$(-,+,+),(-,+,-)$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	0	0
65	$Cmmm$	$(+,+,+)_0$	$\mathbb{Z}^{18}$	0	0	0
		$(-,-,-)_{1/2}$	$\mathbb{Z}^8$	0	$\mathbb{Z}^4$	0
		$(+,+,-),(+,-,+)$	$\mathbb{Z}^2$	$\mathbb{Z}^8$	0	0
		$(-,+,+)$	$\mathbb{Z}^6$	$\mathbb{Z}^6$	0	0
		$(+,-,-)$	$\mathbb{Z}^6$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	0
		$(-,+,-),(-,-,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^4$	$\mathbb{Z}$	0
66	$Cccm$	$(+,+,+)_0,(+,-,-)$	$\mathbb{Z}^{11}$	0	$\mathbb{Z}$	0
		$(-,-,-)_{1/2},(-,+,+)$	$\mathbb{Z}^7$	$\mathbb{Z}^2$	$\mathbb{Z}$	0
		$(+,+,-),(+,-,+)$	$\mathbb{Z}$	$\mathbb{Z}^7$	0	0
		$(-,+,-),(-,-,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}_2$	0
67	$Cmma$	$(+,+,+)_0$	$\mathbb{Z}^{13}$	$\mathbb{Z}$	0	0
		$(-,-,-)_{1/2}$	$\mathbb{Z}^5$	$\mathbb{Z}$	$\mathbb{Z}^2+\mathbb{Z}_2$	0
		$(+,+,-),(+,-,+)$	$\mathbb{Z}^5$	$\mathbb{Z}^5$	0	0
		$(-,+,+)$	$\mathbb{Z}$	$\mathbb{Z}^7+\mathbb{Z}_2$	0	0
		$(+,-,-)$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}_2^2$	0
		$(-,+,-),(-,-,+)$	$\mathbb{Z}^6$	$\mathbb{Z}$	$\mathbb{Z}$	0
68	$Ccca$	$(+,+,+)_0,(+,-,-)$	$\mathbb{Z}^7$	$\mathbb{Z}$	$\mathbb{Z}_2$	0
		$(-,-,-)_{1/2},(-,+,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	$\mathbb{Z}_2$	0
		$(+,+,-),(+,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}^4$	0	0
		$(-,+,-),(-,-,+)$	$\mathbb{Z}^6$	0	$\mathbb{Z}_2$	0
69	$Fmmm$	$(+,+,+)_0$	$\mathbb{Z}^{15}$	0	0	0
		$(-,-,-)_{1/2}$	$\mathbb{Z}^6$	0	$\mathbb{Z}^3+\mathbb{Z}_2$	0
		$(+,+,-),(+,-,+),(-,+,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^6$	0	0
		$(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	$\mathbb{Z}$	0
70	$Fddd$	$(+,+,+)_0,(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^6$	0	$\mathbb{Z}_2$	0
		$(-,-,-)_{1/2},(+,+,-),(+,-,+),(-,+,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^3$	0	0
71	$Immm$	$(+,+,+)_0$	$\mathbb{Z}^{15}$	0	0	0
		$(-,-,-)_{1/2}$	$\mathbb{Z}^6$	$\mathbb{Z}_2$	$\mathbb{Z}^3$	0
		$(+,+,-),(+,-,+),(-,+,+)$	$\mathbb{Z}^3$	$\mathbb{Z}^6$	0	0
		$(+,-,-),(-,+,-),(-,-,+)$	$\mathbb{Z}^4$	$\mathbb{Z}^2$	$\mathbb{Z}$	0

(Cont.)

another 5 pages …

By imposing symmetry, we can obtain rich structures of top. phases

Question

Is there another way to obtain new top. phases?

Our answer

→ Yes.

Non-Hermiticity opens a new direction in topological phases

Why non-Hermitian systems?

Key point

For non-Hermitian Hamiltonians, complex conjugation and transpose are different

$$H^\dagger \neq H \quad \Rightarrow \quad H^* \neq H^t$$

This small change gives a crucial difference in symmetry consideration.

Basic symmetry in non-Hermitian Hamiltonian

- TRS and PHS (=AZ sym.) are fundamental symmetry for topology since they are robust against disorders.
- For Hermitian Hamiltonians, both can be given as anti-unitary symmetries, but for non-Hermitian ones, PHS cannot be.

BdG Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{ij}(c_i^\dagger, c_i)\underline{H_{ij}}\begin{pmatrix} c_j \\ c_j^\dagger \end{pmatrix}$$

$$(c_i^\dagger, c_i) = (c_i, c_i^\dagger)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv (c_i, c_i^\dagger)\tau_x$$

PHS $$\tau_x H \tau_x^{-1} = -H^t$$

non-Hermiticity $\neq -H^*$

PHS is not anti-unitary sym.

From this property, the basic sym. should change accordingly

[MS(18)]

[Bernard-LeClair (01)]

Hermitian case		→	non-Hermitian case	
TRS	$\mathcal{T}H\mathcal{T}^{-1}=H, \quad \mathcal{T}=U_T K$ (K: complex conj.) $U_T^{-1}HU_T=H^*$	→	$U_T^{-1}HU_T=H^*$	K-sym.
PHS	$\mathcal{C}H\mathcal{C}^{-1}=-H, \quad \mathcal{C}=U_C K$ $U_C^{-1}HU_C=-H^*$	→	$U_C^{-1}HU_C=-H^t$	C-sym.
CS	$\Gamma^{-1}H\Gamma=-H$	→	$\Gamma^{-1}H\Gamma=-H$	P-sym.
CS' (TRSxPHS)	$\Gamma'^{-1}H\Gamma'=-H$ $\Gamma'=U_C U_T^*$	→	$\Gamma'^{-1}H\Gamma'=-H^\dagger$	Q-sym.

New types of symmetry appear !!

Taking into account these new types of sym., we can obtain extended fundamental symmetry classes in non-Hermitian systems

Hermitian case

	TRS	PHS	CS
A	0	0	0
AIII	0	0	1
AI	1	0	0
BDI	1	1	1
D	0	1	0
DIII	-1	1	1
AII	-1	0	0
CII	-1	-1	1
C	0	-1	0
CI	1	-1	1

10 classes (AZ classes)

Non-Hermitian case **43 classes**

No sym + P-sym	2 classes
C sym + P-sym	8 classes
Q sym +P-sym	4 classes
K sym+P-sym	5 classes
C sym +**Q-sym**+P-sym	24 classes

New families of classes intrinsic to non-Hermitian Hamiltonians

In the remaining time, I will show that these new families of symmetry classes indeed may host new topological phases intrinsic to non-Hermitian Hamiltonians

1) Q sym protected topological phase

- Graphene with non-Hermitian onsite potential

2) C sym +**Q-sym** protected topological phase

- SO(3,2) Luttinger model

K.Esaki, MS, K.Hasebe, M. Kohmoto, PRB(11)

1) Q-sym protected topological phase

$$\Gamma^{-1} H \Gamma = -H^\dagger \qquad \Gamma^2 = 1 \qquad \Gamma^\dagger = \Gamma$$ Q-sym

From this symmetry, the spectrum of the system has a unique feature, which enables us to obtain topological stability

$$H|u_n\rangle = E_n|u_n\rangle \qquad H^\dagger|u_n\rangle\rangle = E_n^*|u_n\rangle\rangle$$

$$\Rightarrow \quad H\Gamma|u_n\rangle\rangle = -E_n^*\Gamma|u_n\rangle\rangle$$

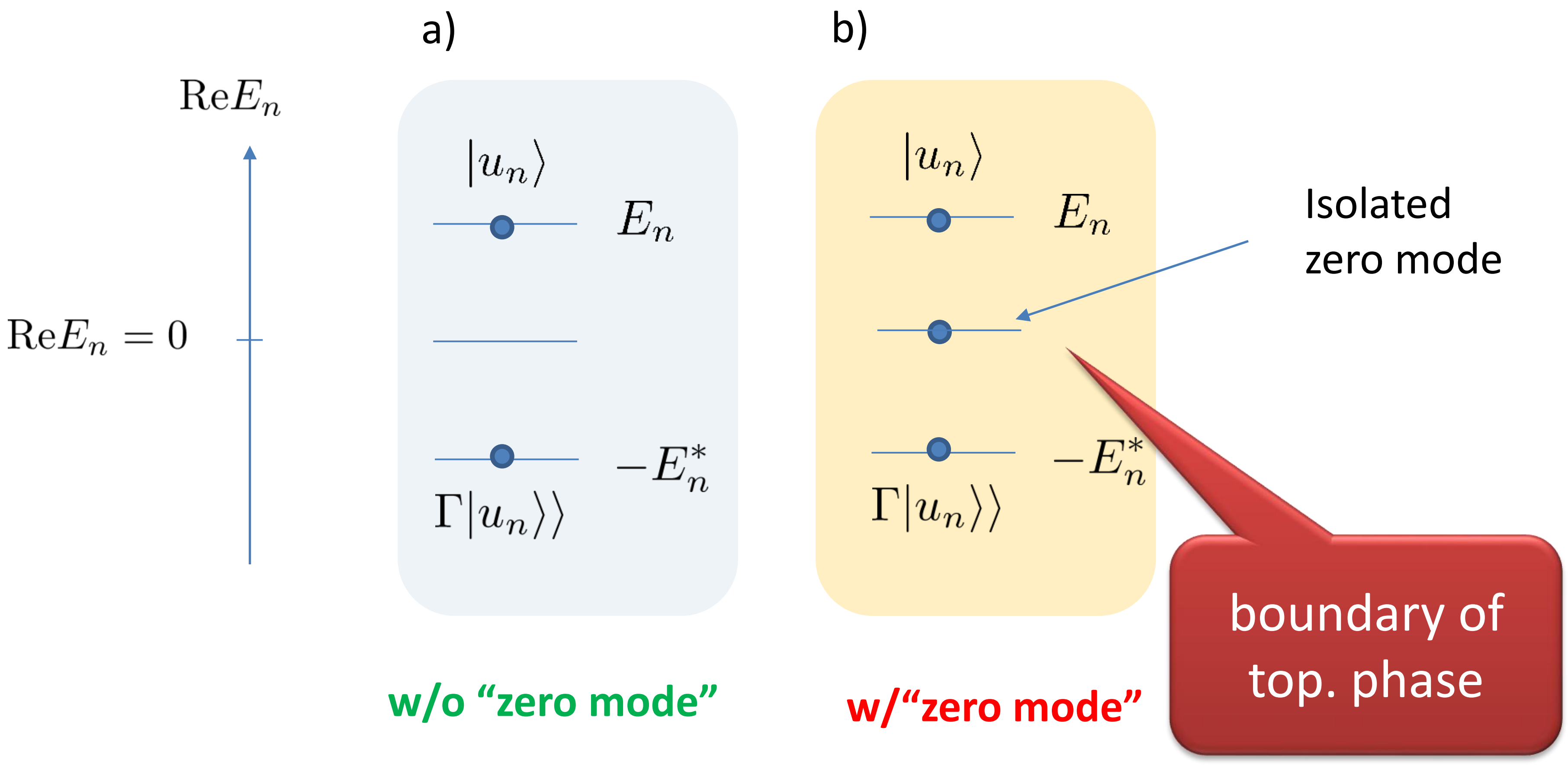
If Re $E_n \neq 0$, it holds that $E_n \neq -E_n^*$, so $|u_n\rangle$ and $\Gamma|u_n\rangle\rangle$ are independent

$$\Gamma|u_n\rangle\rangle \longleftrightarrow |u_n\rangle$$ Pair of "gapped" state

On the other hand, for Re $E_n = 0$, they are not independent with the same energy

$$\Gamma|u_n\rangle\rangle \sim |u_n\rangle$$ Isolated "zero" mode

From this property, we have two different patterns of spectrum

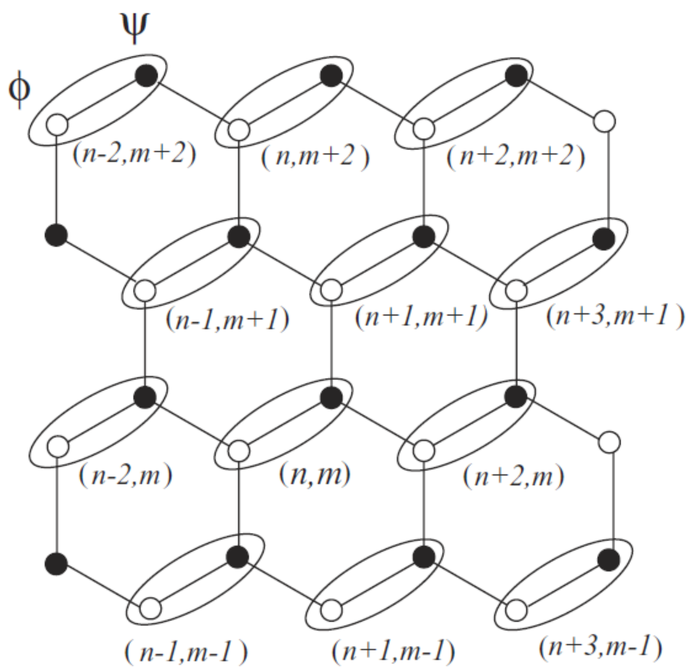


These two patterns of spectrum are not adiabatically connected to each other, so the latter can keep stable zero mode

Graphene with non-Hermitian on-site potential

[Esaki-MS-Hasebe-Kohmoto (11)]

honeycomb lattice



$$H = t\sum_{\langle i,j\rangle}\left(c_i^\dagger c_j + H.c\right) + i\lambda_{\mathrm{v}}\sum_i \xi_i c_i^\dagger c_i$$

non-Hermitian on-site potential

In momentum space, we have

$$H(\boldsymbol{k}) = \begin{pmatrix} i\lambda_{\mathrm{V}} & D(\boldsymbol{k}) \\ D^\dagger(\boldsymbol{k}) & -i\lambda_{\mathrm{V}} \end{pmatrix}$$

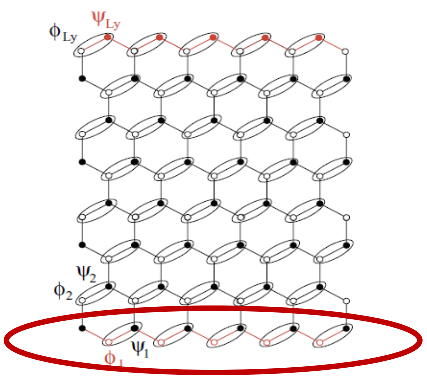
$$D(\boldsymbol{k}) = 2t\cos(k_x/2) + te^{i\sqrt{3}k_y/2}$$

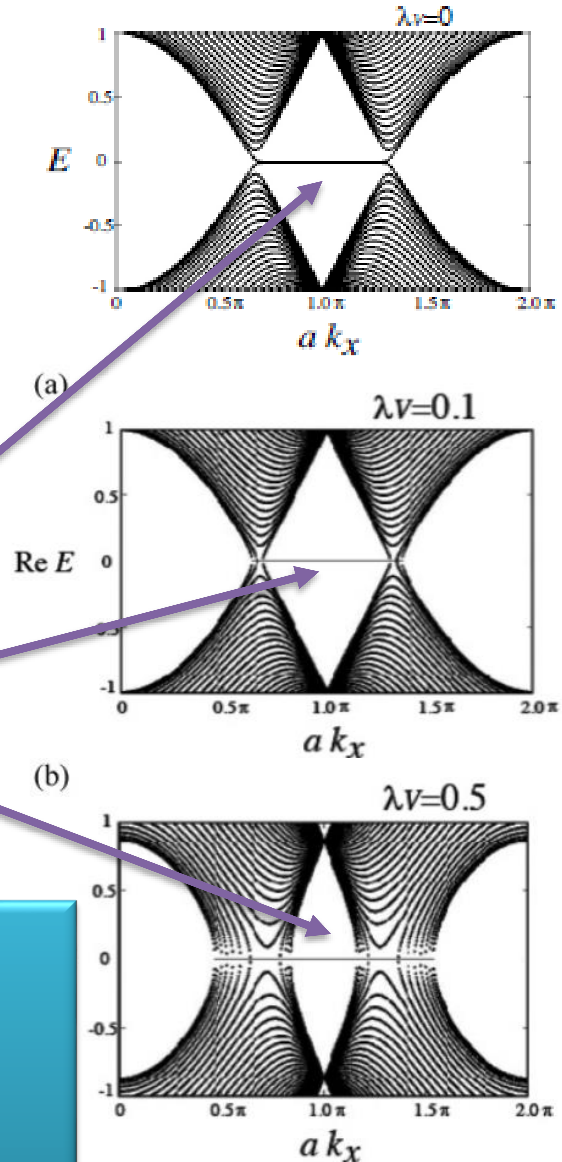
$$\Gamma^{-1}H(\boldsymbol{k})\Gamma = -H^\dagger(\boldsymbol{k}) \qquad \Gamma = \sigma_z$$

Q-symmetry

Spectrum with boundary

Hermiticity





$\lambda_V = 0$

$\lambda_V = 0.1$

$\lambda_V = 0.5$

non-Hermiticity

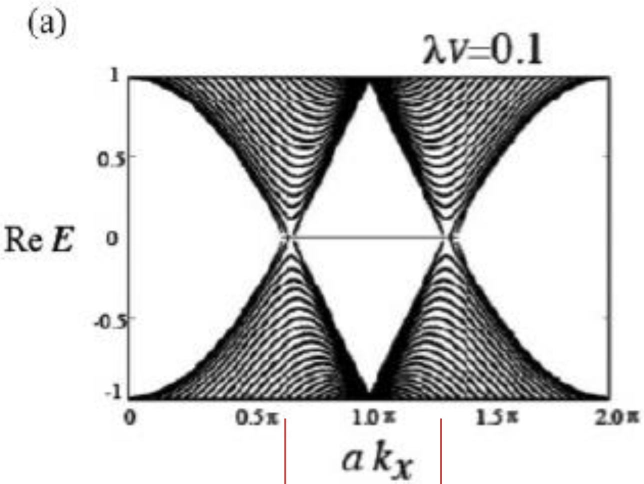
Zero energy state localized on boundary

Topological phase

In contrast to ordinary topological phase, non-hermitian topological phase has an edge state with an imaginary part of the spectrum.

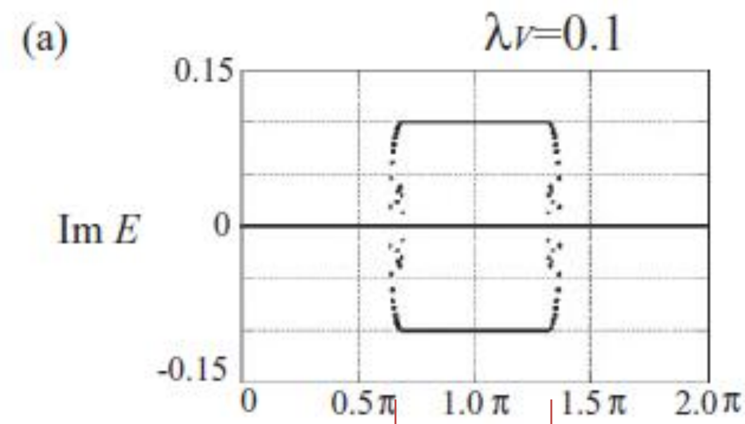
$\lambda_V = 0.1$

Real part of E



edge mode

Imaginary part of E



edge mode

Imaginary part of E for edge mode

2) C-sym+Q-sym protected topological phase

C-sym $U_C^{-1} H U_C = H^t$ $U_C U_C^* = -1$

Q-sym $\Gamma^{-1} H \Gamma = -H^\dagger$ $\Gamma^2 = 1$ $\Gamma U_C = -U_C \Gamma^*$

$H \to iH \quad U_T = \Gamma U_C$

K-sym $U_T^{-1} H U_T = H^*$ $U_T U_T^* = 1$

Q'-sym $\Gamma^{-1} H \Gamma = H^\dagger$ $\Gamma^2 = 1$ $\Gamma U_T = -U_T \Gamma^*$

$T = U_T K$

T^2=1 TRS $T H T^{-1} = H$ $T^2 = 1$

Psude-hermicity $\Gamma^{-1} H \Gamma = H^\dagger$ $\{\Gamma, T\} = 0$

Interestingly, due to the additional pseudo-hemiticity, T^2=1 TRS gives a kind of Kramers pairs.

$$H|\phi_n\rangle = E_n|\phi_n\rangle, \quad H^\dagger|\phi_n\rangle\rangle = E_n^*|\phi_n\rangle\rangle$$

From psuedo-hermiticity

$$\Gamma^{-1}H\Gamma = H^\dagger$$

$$H\Gamma|\phi_n\rangle\rangle = E_n^*\Gamma|\phi_n\rangle\rangle$$

Apply T from the left

$$HT\Gamma|\phi_n\rangle\rangle = E_n T\Gamma|\phi_n\rangle\rangle$$

So $|\phi_n\rangle$ and $T\Gamma|\phi_n\rangle\rangle$ have the same energy. We also

$T^2 = 1$ $\quad$ $\Gamma^\dagger = \Gamma$

$$\langle\langle\phi_n|T\Gamma|\phi_n\rangle\rangle = \langle\langle T^2\Gamma\phi_n|T\phi_n\rangle\rangle = \langle\langle\phi_n|\Gamma^\dagger T\phi_n\rangle\rangle = -$$

Anti-unitarity of T $\quad$ Ajoint of Γ $\quad$ $\{T,\Gamma\} = 0$

Possible new mechanism of topological stability

$|\phi_n\rangle$ $\xleftrightarrow{\text{independent}}$ $T\Gamma|\phi_n\rangle\rangle$

Kramers pair for T^2=1 TRS

2dim SO(3,2) Luttinger model

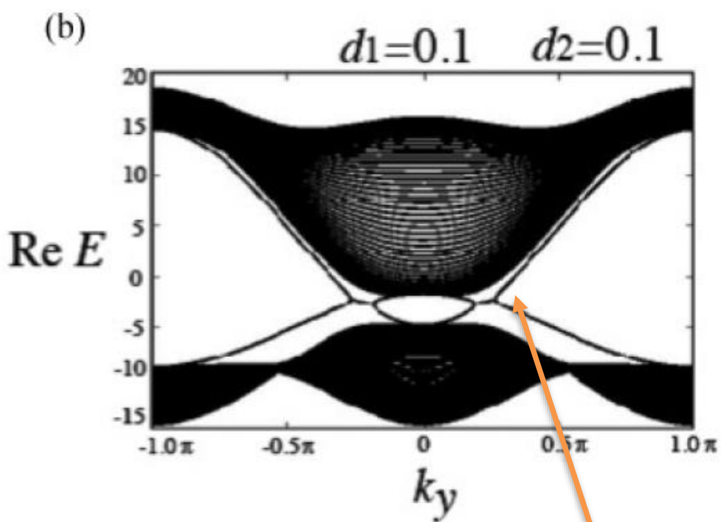
[Esaki-MS-Hasebe-Kohmoto (11)]

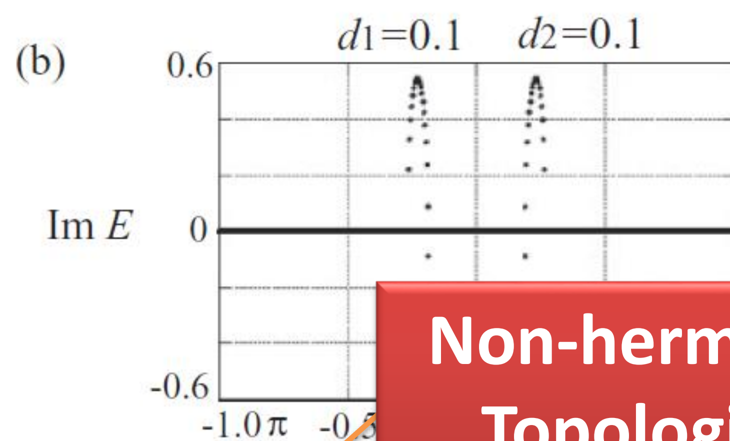
$$H(\boldsymbol{k}) = \epsilon(\boldsymbol{k}) + V \sum_{a=3,4,5} d_a(\boldsymbol{k})\Gamma_a + \underline{iV \sum_{a=1,2} d_a \Gamma_a}$$

TRS +pseudo hermiticity

non-Hermitian term

Spectrum with boundary





Non-hermitian Topological phase

Gapless boundary state with imaginary part of the spectrum

Summary

- To obtain topological phases, symmetry is very important.

- PHS is not anti-unitary for non-Hermitian Hamiltonians, which changes the basic set of symmetries relevant to topology

- The change of the basic set of symmetries makes it possible to obtain new topological phases intrinsic to non-Hermitian systems

For recent progress, see also poster by Kawabata