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Squeezed atom laser for non-Hermitian Bose-Einstein condensate

Non-Hermitian physics - PHHQP XVIII

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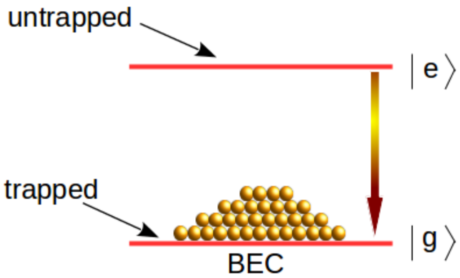
What is atom LASER!

- It is a coherent state of propagating atoms behaving like a wave
- It is created out of a Bose-Einstein condensate of atoms that are output coupled to the light
- Working principle: coupling between atom (in the BEC) and light

Why atom LASER?

- It can be used wherever usual LASER is used
- It is utilized to increase the accuracy in interferometry, etc.



W. Ketterle, Rev. Mod. Phys. **74**, 1131 (2002)

Squeezed atom LASER

Various ways to construct

1. Coupling a squeezed optical beam with the BEC

Squeezed atom LASER

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3. Topic of the discussion in this talk [S. Dey, V. Hussin, arXiv:1805.12363]

Why squeezed atom laser should be interesting?

Squeezed atom LASER

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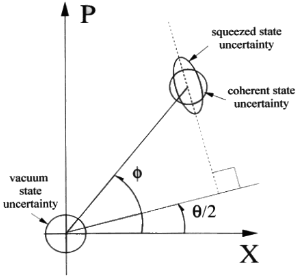

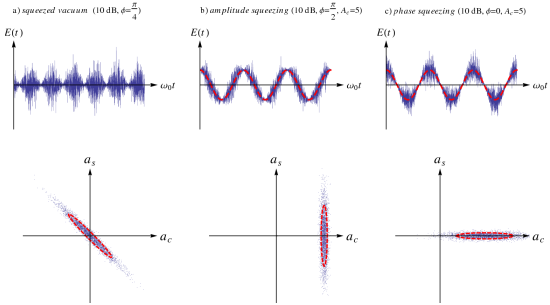

Technicalities for construction

- Hamiltonian for N identical two level atoms:

$$H = \omega_r a^\dagger a + \omega_0 b_e^\dagger b_e + \Omega\left(a b_e^\dagger b_g + a^\dagger b_g^\dagger b_e\right), \quad \Omega = \sqrt{\omega_r/2\epsilon_0 V}, \quad \hbar = 1$$

$b_g^\dagger(b_g), b_e^\dagger(b_e)$: Creation (annihilation) operators for $|g\rangle$ and $|e\rangle$
$a, a^\dagger$: Ladder operator of the optical radiation field
ω_0 : Level difference, V: Effective mode volume
ω_r : Frequency of the radiation field

S. Changpu et al., Commun. Theor. Phys. **29**, 161 (1998)

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- Bogoliubov approximation (Ignore the slow change of large number of atoms in $|g\rangle$, i.e. $b_g, b_g^\dagger \to \sqrt{N_c}$)

$$H = \omega \left(a^\dagger a + b^\dagger b \right) + \Omega \sqrt{N_c} \left(a b^\dagger + a^\dagger b \right), \qquad b_e \to b \text{ (notation)}$$

N. Bogoliubov, J. Phys. (USSR) **11**, 23 (1947)

Generalization of the radiation field

So called nonlinear generalization:

$$a \to A, \qquad a^\dagger \to A^\dagger$$

where

$$\begin{aligned} A &= af(\hat{n}) = f(\hat{n}+1)a \\ A^\dagger &= f(\hat{n})a^\dagger = a^\dagger f(\hat{n}+1) \end{aligned}$$

so that

$$A|n\rangle = \sqrt{n} f(n)|n-1\rangle, \quad A^\dagger|n\rangle = \sqrt{n+1} f(n+1)|n+1\rangle$$

If a Hamiltonian can be factorized as $A^\dagger A = a^\dagger a f^2(a^\dagger a)$, the eigenvalues can be expressed as $e_n = \hbar\omega n f^2(n)$

$*$ All one needs to compute is the $f(n)$

Minimal length model

Define a new commutation relation:

$$[X, P] = i\hbar \left(1 + \check{\tau} P^2\right), \qquad X = \left(1 + \check{\tau} p^2\right) x, \quad P = p, \quad [x, p] = i\hbar$$

Harmonic oscillator:

$$H = \frac{P^2}{2m} + \frac{m\omega^2}{2} X^2 - \hbar\omega \left(\frac{1}{2} + \frac{\tau}{4}\right), \qquad H^\dagger \neq H$$

Reality of spectrum: $h = \eta H \eta^{-1}$, with $\eta = (1 + \check{\tau} p^2)^{-1/2}$

$$h = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \frac{\omega\tau}{4\hbar}(x^2 p^2 + p^2 x^2 + 2xp^2 x) - \hbar\omega \left(\frac{1}{2} + \frac{\tau}{4}\right) + \mathcal{O}(\tau^2)$$

Eigenvalues and eigenfunctions:

$$E_n = \hbar\omega n f^2(n) = \hbar\omega \left(An + Bn^2\right) + \mathcal{O}(\tau^2), \quad A = 1 + \frac{\tau}{2}, B = \frac{\tau}{2}$$

$$|\phi_n\rangle = |n\rangle - \frac{\tau}{16}\sqrt{(n-3)_4}\,|n-4\rangle + \frac{\tau}{16}\sqrt{(n+1)_4}\,|n+4\rangle + \mathcal{O}(\tau^2)$$

S. Dey, A. Fring, PRD **86**, 064038 (2012)

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$$H = \frac{P^2}{2m} + \frac{m\omega^2}{2}X^2 - \hbar\omega\left(\frac{1}{2}+\frac{\tau}{4}\right), \qquad H^\dagger \neq H$$

$$f(n) = \sqrt{A+Bn}$$

Reality of spectrum

$$h = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \frac{\omega\tau}{4\hbar}\left(x^2p^2+p^2x^2+2xp^2x\right) - \hbar\omega\left(\frac{1}{2}+\frac{\tau}{4}\right) + \mathcal{O}(\tau^2)$$

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Towards the solution

$$H_d = \omega\left(A^\dagger A + b^\dagger b\right) + \Omega\sqrt{N_c}\left(Ab^\dagger + A^\dagger b\right)$$
$$= \omega\left[\left(C + Da^\dagger a\right)a^\dagger a + b^\dagger b\right] + \Omega\sqrt{N_c}\left[ab^\dagger\sqrt{C + Da^\dagger a} + \sqrt{C + Da^\dagger a}\,a^\dagger b\right]$$

Introduce slowly varying polariton operators:

$$M(t) = \frac{1}{\sqrt{2}}e^{-2i\Omega\sqrt{N_c}t}[a(t) + b(t)], \qquad N(t) = \frac{1}{\sqrt{2}}e^{2i\Omega\sqrt{N_c}t}[a(t) - b(t)]$$

with $[M, M^\dagger] = [N, N^\dagger] = 1$ and invert them

$$a(t) = \frac{1}{\sqrt{2}}\left[e^{2i\Omega\sqrt{N_c}t}M(t) + e^{-2i\Omega\sqrt{N_c}t}N(t)\right]$$
$$b(t) = \frac{1}{\sqrt{2}}\left[e^{2i\Omega\sqrt{N_c}t}M(t) - e^{-2i\Omega\sqrt{N_c}t}N(t)\right]$$

$a(t), b(t) \rightarrow H_d$ and ignore the oscillating terms like $M^\dagger N, N^\dagger M$, etc.

Solution

Solve the Heisenberg equations of motion:

$$\frac{dM(t)}{dt} = i[H, M(t)], \qquad \frac{dN(t)}{dt} = i[H, N(t)]$$

so that

$$M(t) = M(0)e^{-i\left\{\frac{\omega}{2}(C+D+1)+\frac{2\alpha_1}{D}\left(C+\frac{D}{2}\right)+\frac{D}{2}\left(\omega+\frac{2\alpha_1}{D}M^\dagger M\right)+\omega DN^\dagger N\right\}t}$$

$$N(t) = N(0)e^{-i\left\{\frac{\omega}{2}(C+D+1)-\frac{2\alpha_1}{D}\left(C+\frac{D}{2}\right)+\frac{D}{2}\left(\omega-\frac{2\alpha_1}{D}M^\dagger M\right)+\omega DN^\dagger N\right\}t}$$

Finally, the light field operator turns out to be

$$a(t) = \frac{1}{\sqrt{2}}\left[M(0)e^{-i\Theta_M t} + N(0)e^{-i\Theta_N t}\right]$$

with

$$\Theta_M = \frac{\omega}{2}(C+D+1) + \alpha_3 + (\alpha_1+\alpha_2)M^\dagger M + 2\alpha_2 N^\dagger N,$$

$$\Theta_N = \frac{\omega}{2}(C+D+1) - \alpha_3 + (\alpha_2-\alpha_1)M^\dagger M + 2\alpha_2 N^\dagger N,$$

Analysis of squeezing / nonclassicality

- Quadrature squeezing: $(\Delta y)^2 < 1/2$ or $(\Delta z)^2 < 1/2$ has to hold, where

$$y = \frac{1}{\sqrt{2}}\left[a^\dagger(t) + a(t)\right], \qquad z = \frac{i}{\sqrt{2}}\left[a^\dagger(t) - a(t)\right].$$

  Equivalently $S_1 < 0$ or $S_2 < 0$

$$S_1 = 2(\Delta y)^2 - 1 = 2\left\{\langle a^\dagger(t)a(t)\rangle + \mathrm{Re}[\langle a(t)^2\rangle] - \mathrm{Re}[\langle a(t)\rangle^2] - |\langle a(t)\rangle|^2\right\}$$
$$S_2 = 2(\Delta z)^2 - 1 = 2\left\{\langle a^\dagger(t)a(t)\rangle - \mathrm{Re}[\langle a(t)^2\rangle] + \mathrm{Re}[\langle a(t)\rangle^2] - |\langle a(t)\rangle|^2\right\}$$

- Photon antibunching: Mandel parameter $Q < 0$

$$Q = \frac{\langle\{a^\dagger(t)a(t)\}^2\rangle - \langle a^\dagger(t)a(t)\rangle^2}{\langle a^\dagger(t)a(t)\rangle} - 1,$$

- We require: $\langle a(t)\rangle, \langle a(t)^2\rangle, \langle a^\dagger(t)a(t)\rangle, \langle a^\dagger(t)a(t)a^\dagger(t)a(t)\rangle$

$\langle a(t)^2 \rangle$

$$\langle a(t)^2 \rangle = \frac{\alpha^2}{4} e^{-|\alpha|^2} \left(\gamma_+ e^{-i\epsilon_+} + \gamma_- e^{-i\epsilon_-} + \xi e^{-i\lambda} \right),$$

where

$$\gamma_\pm = e^{\frac{|\alpha|^2}{2}[\cos(4\alpha_2 t)+\cos\{2t(\alpha_2 \mp \alpha_1)\}]}$$

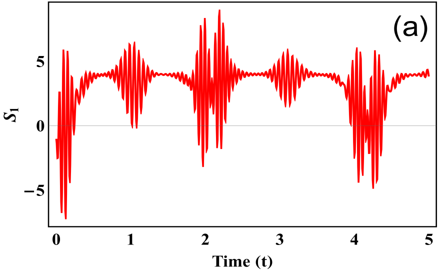
$$\epsilon_\pm = \frac{1}{2}\left[4t\left\{\frac{\omega}{2}(C+D+1) \mp \alpha_3\right\} + |\alpha|^2\{\sin(4t\alpha_2) + \sin[2t(\alpha_2 \mp \alpha_1)]\}\right]$$

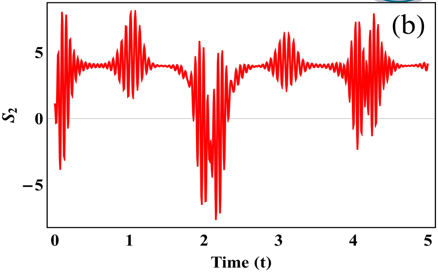
$$\xi = e^{\frac{|\alpha|^2}{2}[\cos\{2t(\alpha_1 - 3\alpha_2\} + \cos\{2t(\alpha_1 + 3\alpha_2)\}]}$$

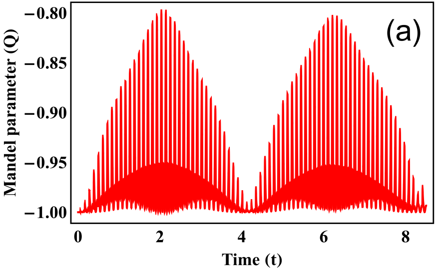
$$\lambda = 4t\left\{\frac{\omega}{2}(C+D+1) \mp \alpha_3\right\} - \frac{|\alpha|^2}{2}\{\sin[2t(\alpha_1 - 3\alpha_2)] - \sin[2t(\alpha_1 + 3\alpha_2)]\}$$

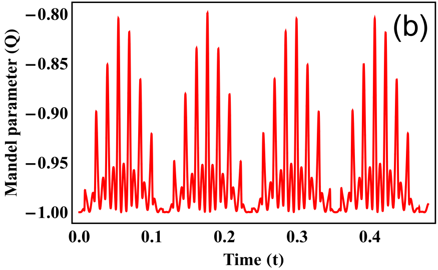
- We require: $\langle a(t) \rangle, \langle a(t)^2 \rangle, \langle a^\dagger(t) a(t) \rangle, \langle a^\dagger(t) a(t) a^\dagger(t) a(t) \rangle$

Numerical results









More details on the model used

Overview of quantum gravity

- Usual quantum theory fails to renormalize the QFT in UV regime.
- Recent approach: search for a quantum theory which is consistent with the quantum gravity models.

Noncommutative space-time: An essential theory for QG

- Version 1: [H. S. Snyder, Phys. Rev. **71**, 38 (1947)]

$$\begin{aligned} [x^\mu, x^\nu] &= i\theta\,(x^\mu p^\nu - x^\nu p^\mu) \\ [x^\mu, p_\nu] &= i\hbar\,(\delta^\mu_\nu + \theta p^\mu p_\nu) \\ [p_\mu, p_\nu] &= 0 \end{aligned}$$

- Version 2 [N. Seiberg, E. Witten, JHEP **1999**, 032 (1999)]

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad [x^\mu, p_\nu] = i\hbar\delta^\mu_\nu \quad \text{and} \quad [p_\mu, p_\nu] = 0$$

Motivations of the study continued

- Version 3 (Noncommutative phase space)

$$[x_i, x_j] = i\theta_{ij}, \quad [p_i, p_j] = i\Omega_{ij}, \quad [x_i, p_j] = i\hbar\delta_{ij}\left(1 + \frac{\theta_{ik}\Omega_{kj}}{4\hbar^2}\right)$$

- Version 4 (Motivated by generalized uncertainty principle) [our model]:

$$[X, P] = i\hbar(1 + \check{\tau} P^2)$$

MInimal length $\sim \hbar\sqrt{\check{\tau}}$ [B. Bagchi, A. Fring, PLA **373**, 4307-4310 (2009)]

More information on quantum optics and information theory based on version 4

S. Dey, A. Fring, V. Hussin, arXiv:1801.01139

Conclusions

- We have explored an alternative method to construct a squeezed atom laser
- Analysed the nonclassical / squeezing properties both analytically and numerically
- Our model being originated from QG theories, could provide an insight of a consistent quantum theory compatible with gravity
- Nonlinearity being the key feature, in near future, it may be possible to perform experiments.

Thank you for your attention

Based on: S. Dey & V. Hussin, arXiv:1805.12363