Basics of quantum measurement with quantum light

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Quantum light – Fock states

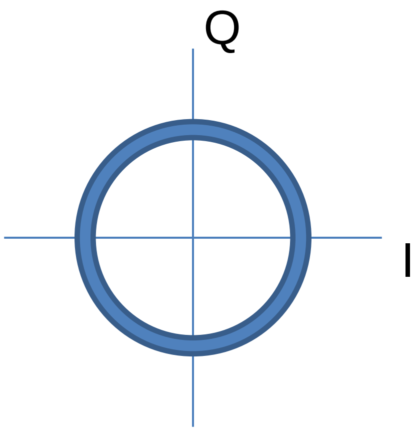
Quantum state with definite number of photons in it!

$$|n\rangle = 1,2,3\ldots$$

n is an eigenstate of number operator $\hat{n} = a^{\dagger}a$

$$\hat{n}|n\rangle = n$$

Let's also look at what a coherent state looks like in terms of its *quadratures* $(I,Q) = \frac{a \pm a^{\dagger}}{2}$ (equivalent to x and p for SHO)



Quantum light – coherent states

The quantum state of a driven harmonic oscillator

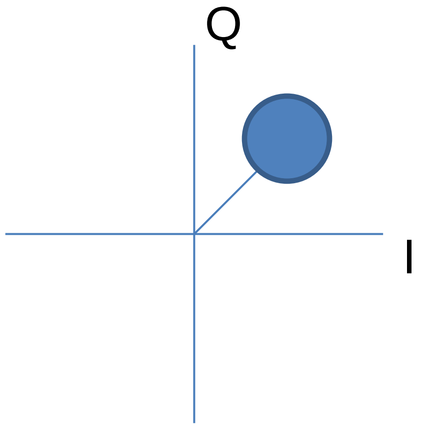
$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}$$

This state is not a eigenstate of photon number, but instead of lowering operator w/ complex eigenvalue α

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

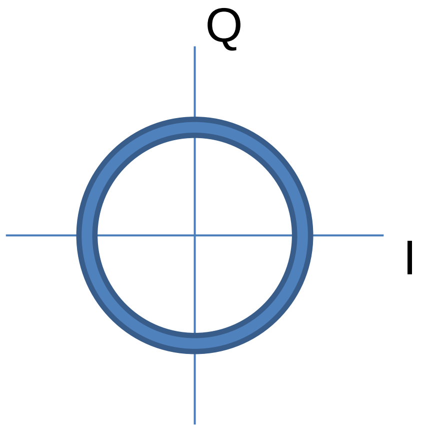
In I/Q space, has gaussian distribution centered on coordinates

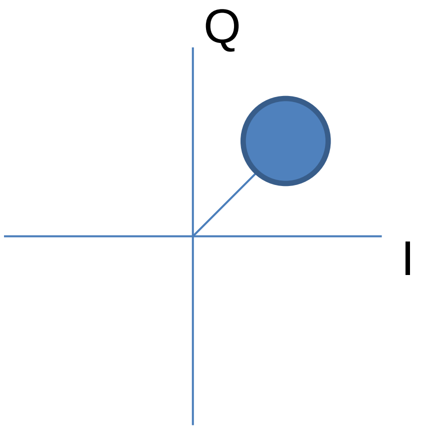
$$(\bar{I}, \bar{Q}) = (Re(\alpha), Im(\alpha))$$



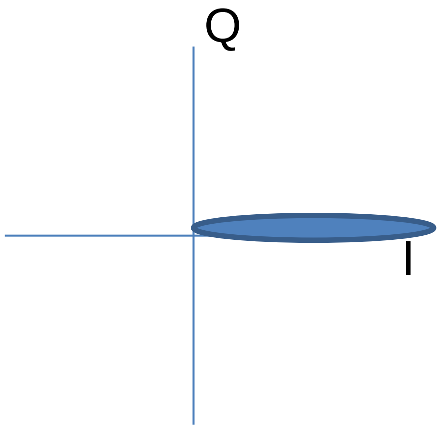
Quantum light – squeezed states

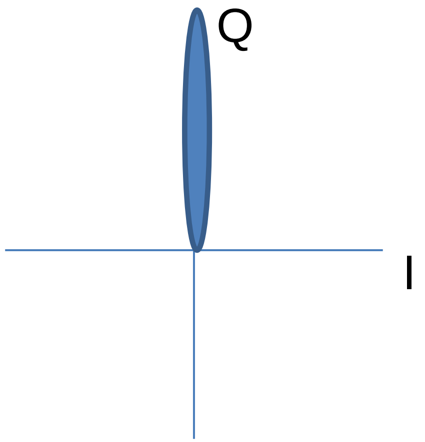
These basic quantum states have the same area in phase space





We can consider other, more complicated states which ‘squeeze’ the light in certain quadratures while leaving area unchanged





What to do with quantum light?

- Detect/measure it to prove quantum properties
  - Q-function, Wigner Tomography
  - time/space correlations
  - bunching/anti-bunching

- Use it as a tool/sensor
  - too many to count….

- Use it for quantum information
  - (flying) qubit
  - readout of another (stationary) qubit

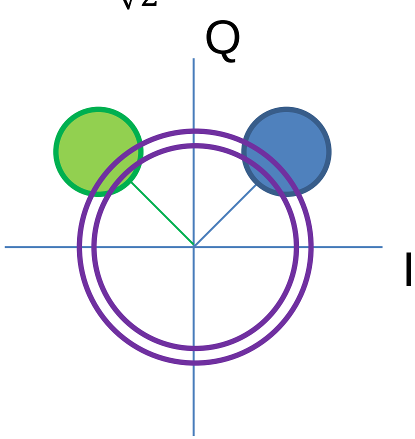
We must match the light to its detector

- ‘Click’ or photon counting detectors project flying light onto a Fock state
  - great for Fock states, not a good basis for discriminating coherent light

Output: ‘click’ or number w/ probability

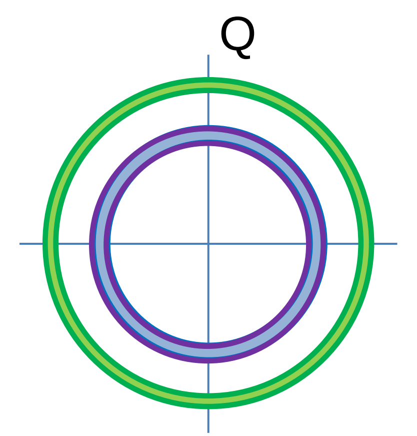
$$P_n = \left|\langle n|\psi_{light}\rangle\right|^2$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\alpha_1\rangle + |\alpha_2\rangle)$$

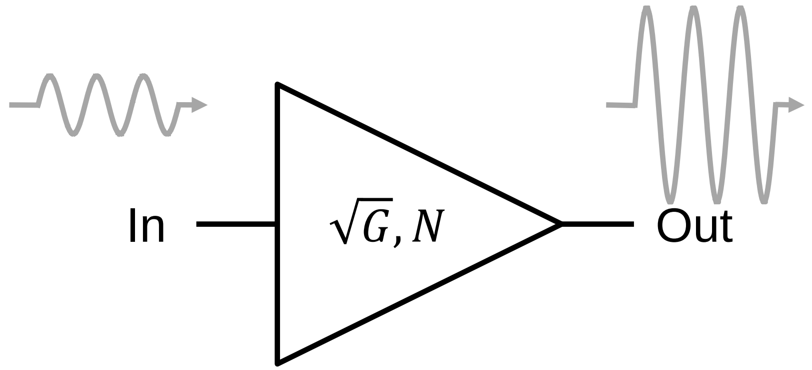


vs.

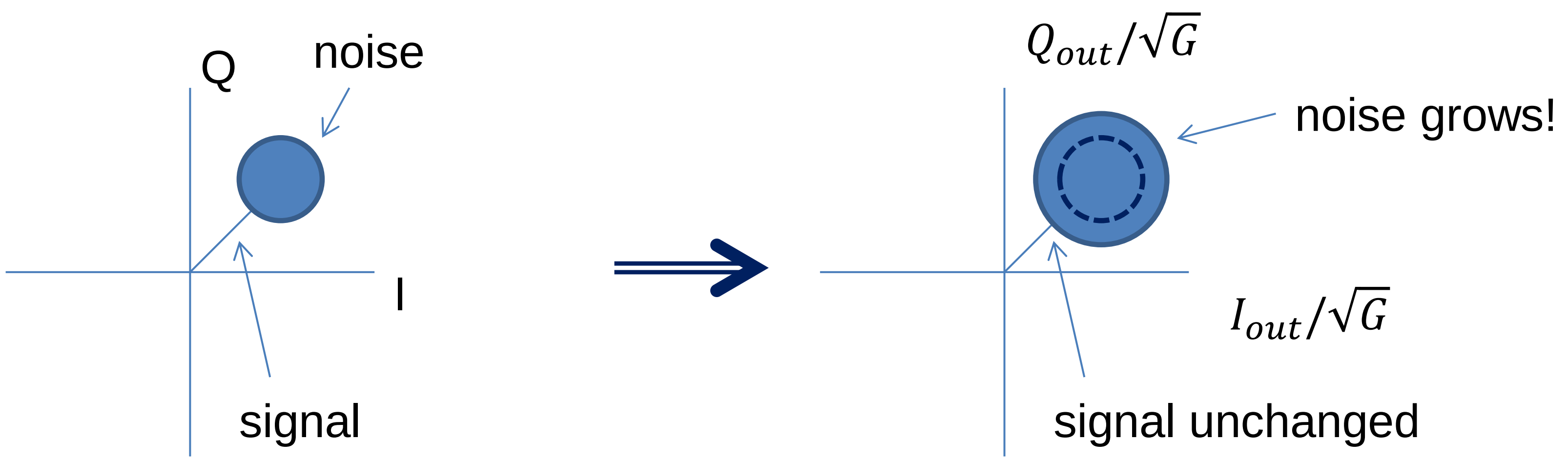
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|n\rangle + |m\rangle)$$



Classical (microwave) amplifier



Classical I/Q plot



- Amplifiers **DEGRADE** Signal-to-Noise Ratio (SNR)
- Their job is to swamp noise of later elements

Amplifiers have many properties

Electrical Specifications at 25°C

Parameter	Condition (MHz)	ZVA-183+ ▲ZVA-183X+			Units
		Min.	Typ.	Max.	
Frequency Range		700	—	18000	MHz
Gain	700 - 18000	24	26	—	dB
Gain Flatness	700 - 18000	—	±1.0	—	dB
Output Power at 1dB compression	700 - 18000	21	24	—	dBm
Noise Figure	700 - 18000	—	3.0	5.5	dB
Output third order intercept point	700 - 18000	—	+33	—	dBm
Input VSWR	700 - 18000	—	1.35	—	:1
Output VSWR	700 - 18000	—	1.25	—	:1
DC Supply Voltage		—	12*	—	V
Supply Current		—	—	400	mA

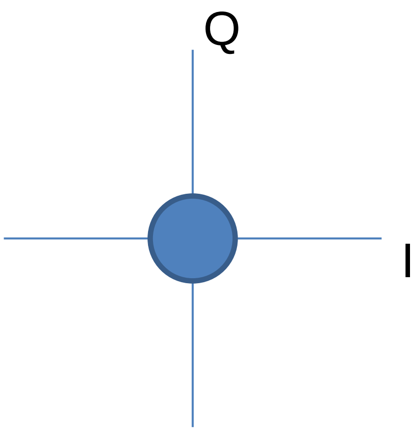
First and foremost: Noise added by amplifier

- Goes by many names: noise figure, noise factor, noise temperature
- Here we’ll focus on how many noise quanta the amplifier adds.
- In units of added quanta this amplifier adds 830 @ 7.5 GHz

Quantum limit on phase-preserving amplifier

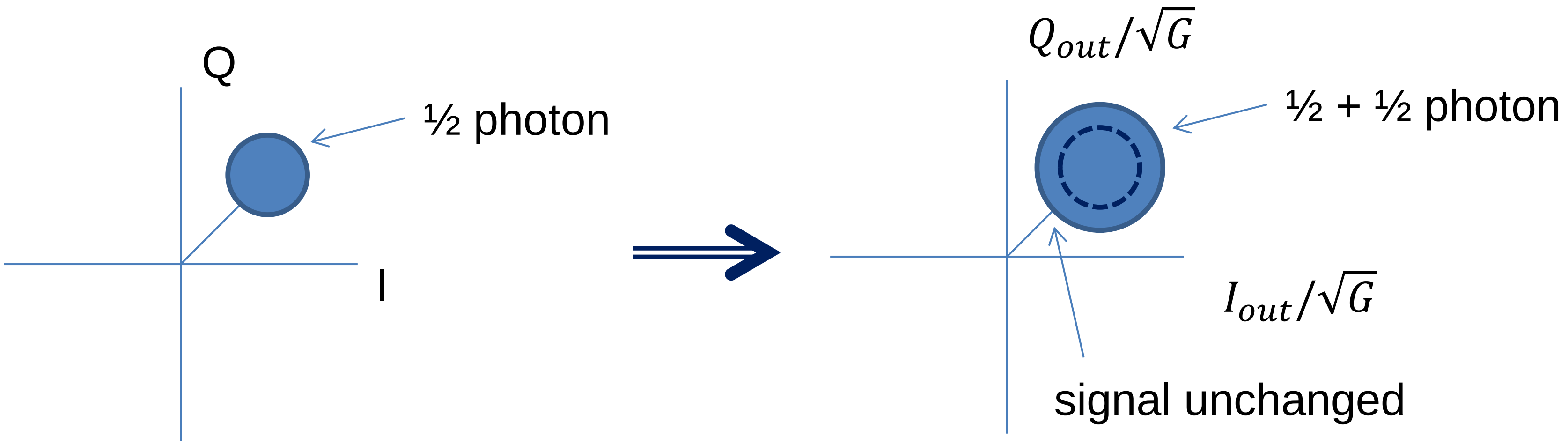
Caves 1984

Vacuum state ($|0\rangle$)



Operators (I/Q) or (x,p) do not commute, and so there is minimum area in phase space, associated w/ ½ photon energy.

Mininum added noise is ½ photon



Outline for the remainder of our tutorials

Tutorial 1: **Basics of quantum measurement with quantum light**

- Intro to parametric amplification
- Classification of amplifiers by type and interaction
- Limitations of parametric amplification
- An introduction to superconducting circuits

Tutorial 2: **Quantum measurement of coherent states and qubits with phase-sensitive amplifiers**

- Phase-sensitive amplification with superconducting microwave circuits
- Single-shot qubit readout
- Weak measurements and quantum back-action
- Entanglement via sequential measurement

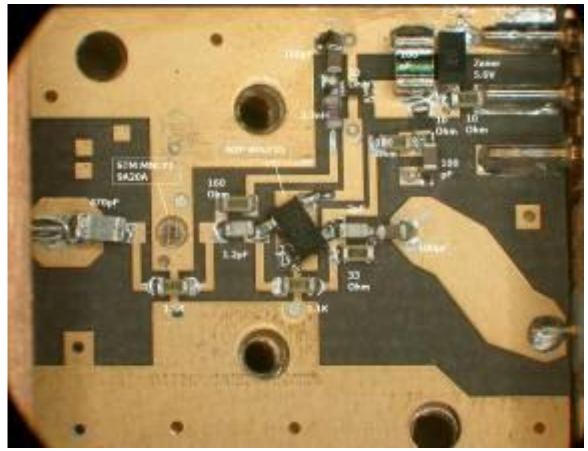
Tutorial 3: **Quantum measurement of coherent states and qubits with phase-preserving amplifiers**

- Phase-preserving amplification with superconducting microwave circuits
- Single-shot qubit readout
- Weak measurements and quantum back-action
- Entanglement via amplification

Introduction to parametric amplifiers

How to build a classical amplifier

- Inductors, capacitors, resistors, transistors + DV voltage for gain



How to build a quantum amplifier?

- Inductors, capacitors, resistors, transistors + DV voltage for gain

Quantum harmonic oscillators

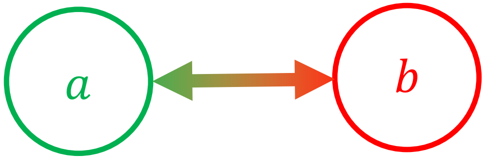
Ports (no internal losses)

Nonlinear Hamiltonian + parametric drive

Parametrically driven couplings

direct exchange

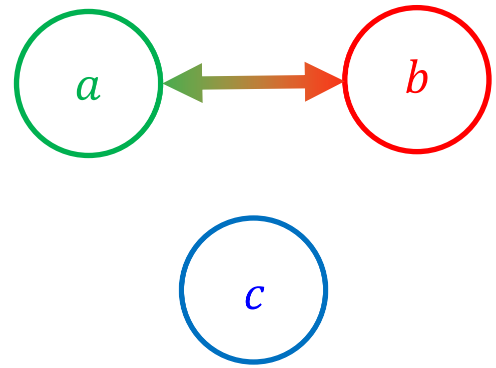
$$\frac{\mathcal{H}_{int}}{\hbar} = \omega_a a^\dagger a \qquad \frac{\mathcal{H}_{int}}{\hbar} = \omega_b b^\dagger b$$



$$\frac{\mathcal{H}_{int}}{\hbar} = g\left(a b^\dagger + a^\dagger b\right)$$

- if $\omega_a - \omega_b \gg \kappa_{a,b}$ this term dies due to energy conservation (RWA)
- interaction also turns off slowly vs. detuning, limiting the on/off ratio

parametrically driven exchange (Conv)



$$\frac{\mathcal{H}_{int}}{\hbar} = g\left(a b^\dagger c^\dagger + a^\dagger b c\right)$$

- if $\omega_c \neq \omega_a - \omega_b$ we can drive the c-mode ‘stiffly’

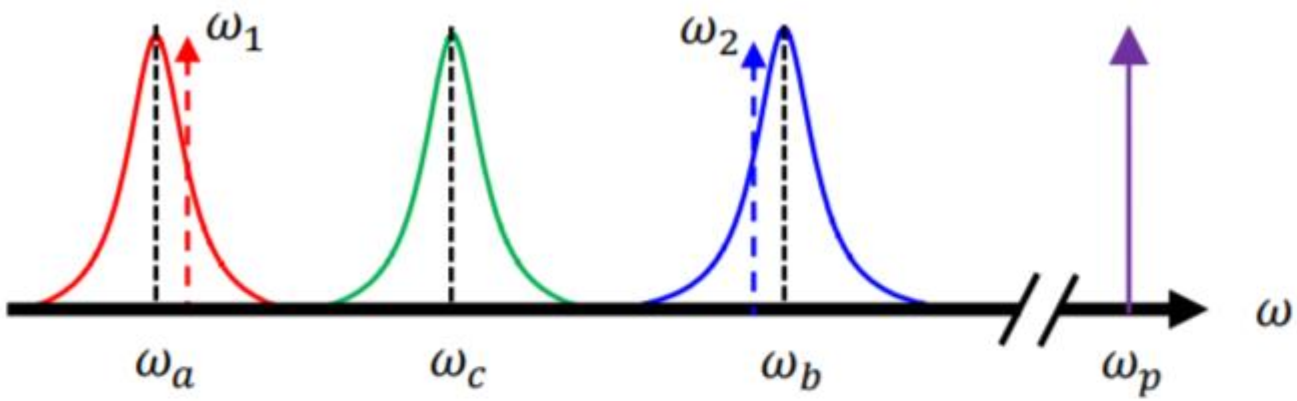
$$\frac{\mathcal{H}_{int}}{\hbar} = g a b^\dagger + g^* a^\dagger b$$

- Parametric drive fully controls strength and phase of interaction

System Dynamics – Phase preserving amplification

$$\frac{\mathcal{H}}{\hbar} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g\left(a^\dagger b^\dagger c + a b c^\dagger\right)$$

Pay careful attention to coupling, this form destroys one c ‘pump’ photon to create one photon each in a and b



- Modes we use for quantum signals should be driven near their resonance frequency
- We need the third ‘pump’ mode to be far away from the pump frequency so the pump can be ‘stiff’ and c becomes a number

System Dynamics continued

$$\frac{\mathcal{H}}{\hbar} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g\left(a^\dagger b^\dagger c + abc^\dagger\right)$$

- We'll see this explicitly later, but for now, consider the photon exchange process
- If $\omega_a + \omega_b = \omega_c$ the process works, else suppressed by energy non-conservation
- This feature will remain even after c-mode an (off-resonant) pump

Master Eqn

- In this example we will have our pump on, as well as a 'weak' signal in mode a
- Mode b will be left 'idle'

$$\frac{da}{dt} = i\left[\frac{\mathcal{H}}{\hbar}, a\right] - \frac{\kappa_a}{2} a + \sqrt{\kappa_a} a^{in}$$

$$\frac{da}{dt} = -i\omega_a a - \frac{\kappa_a}{2} a - igb^\dagger c + \sqrt{\kappa_a} a^{in}$$

$$\frac{db}{dt} = -i\omega_a b - \frac{\kappa_b}{2} b - iga^\dagger c + \sqrt{\kappa_b} b^{in} \qquad (b^{in} = 0)$$

Switch to Fourier Domain

$$a(t) \rightarrow a[\omega]$$

Taking Fourier transform to make clear which frequencies are linked:

- assuming we drive mode a at $\omega_1 = \omega_a + \Delta$
- signal in mode b will be at $\omega_2 = \omega_b - \Delta$

$$-i\omega_1 a[\omega_1] = -i\omega_a a[\omega_1] - \frac{\kappa_a}{2} a[\omega_1] - igb^\dagger[\omega_2]\langle c\rangle + \sqrt{\kappa_a} a^{in}[\omega_1]$$

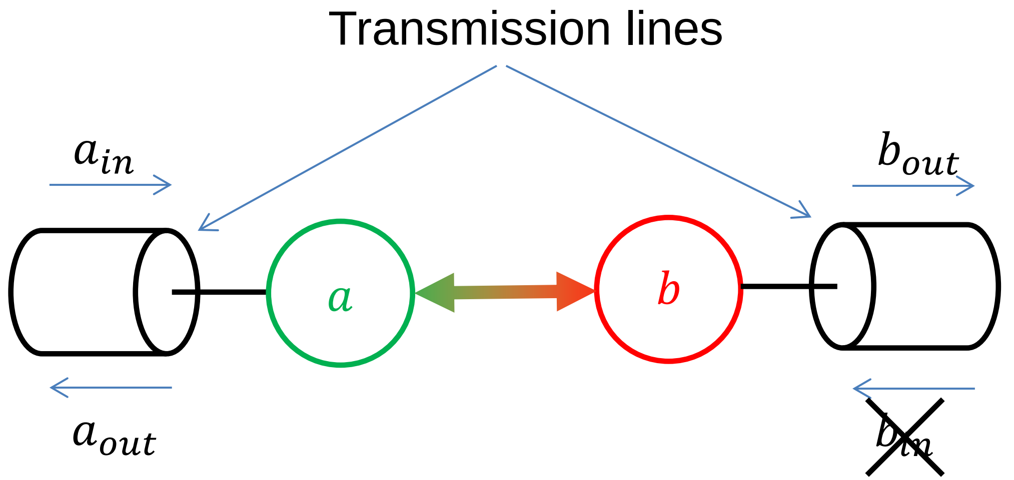
$$-i\omega_2 b[\omega_2] = -i\omega_b b[\omega_2] - \frac{\kappa_b}{2} b[\omega_2] - iga^\dagger[\omega_1]\langle c\rangle$$

where we have used the following relations:

$$\int_{-\infty}^{+\infty} a(t) e^{i\omega_1 t} dt = a[\omega_1] \qquad \int_{-\infty}^{+\infty} b(t) e^{i\omega_2 t} dt = b[\omega_2]$$

$$c = \langle c\rangle e^{-i\omega_c t} = \langle c\rangle e^{-i(\omega_1 + \omega_2)t}$$

Input-Output Theory



$$a_{in} + a_{out} = \sqrt{\kappa_a}\, a \qquad \cancel{b_{in}} + b_{out} = \sqrt{\kappa_b}\, b$$

Substituting back into our equations for a and b we find:

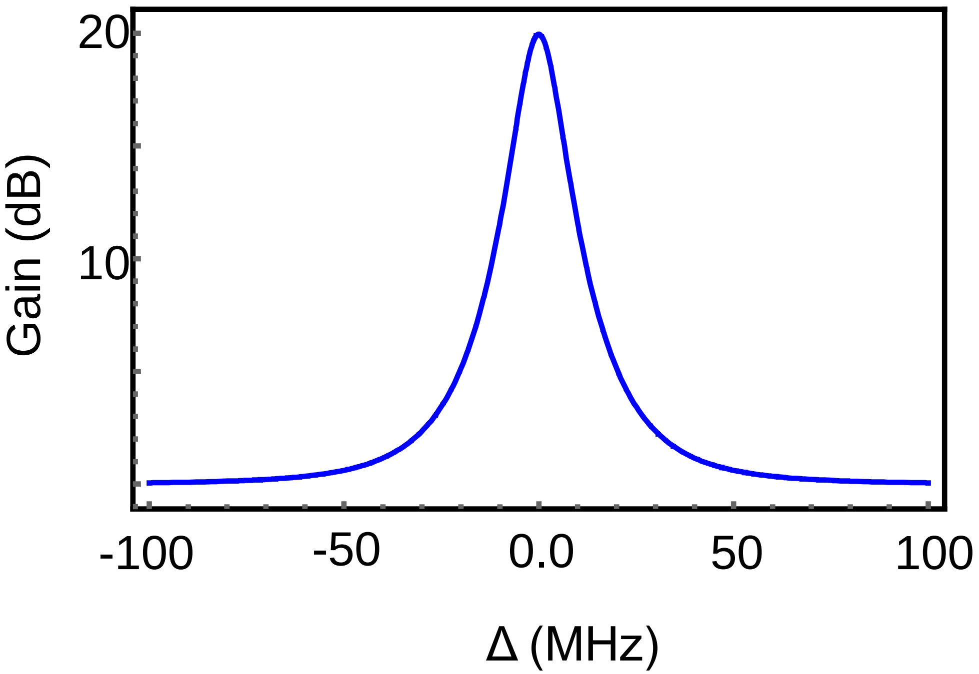
$$\left(\frac{\kappa_a}{2} - i\,\Delta\right)\frac{(1+\alpha)}{\sqrt{\kappa_a}} = -i\frac{g\langle c\rangle}{\sqrt{\kappa_b}}\beta + \sqrt{\kappa_a}$$

$$\left(\frac{\kappa_b}{2} + i\,\Delta\right)\frac{\beta^*}{\sqrt{\kappa_b}} = -i\frac{g\langle c\rangle}{\sqrt{\kappa_a}}(1+\alpha^*)$$

where $\alpha = \dfrac{a_{out}}{a_{in}}$ and $\beta = \dfrac{{b_{out}}^{\dagger}}{a_{in}}$

Closed form solution

$$|\alpha|^2 = \frac{16g^4\langle c\rangle^4 + 8g^2\langle c\rangle^2\left(4\Delta^2 + \kappa_a\kappa_b\right) + \left(4\Delta^2 + {\kappa_a}^2\right)\left(4\Delta^2 + {\kappa_b}^2\right)}{16g^4\langle c\rangle^4 + 8g^2\langle c\rangle^2(4\Delta^2 - \kappa_a\kappa_b) + (4\Delta^2 + {\kappa_a}^2)(4\Delta^2 + {\kappa_b}^2)}$$

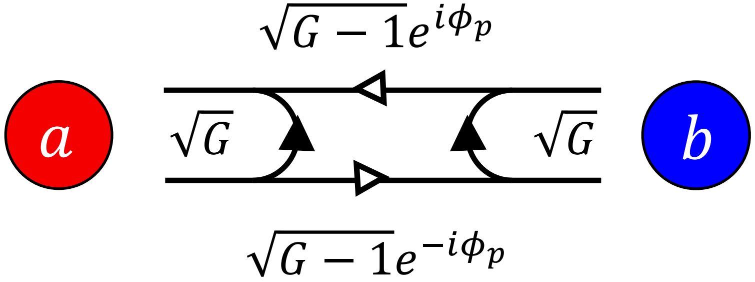


For example :

$\kappa_a = \kappa_b = 50\ MHz$

$g = 50\ MHz$

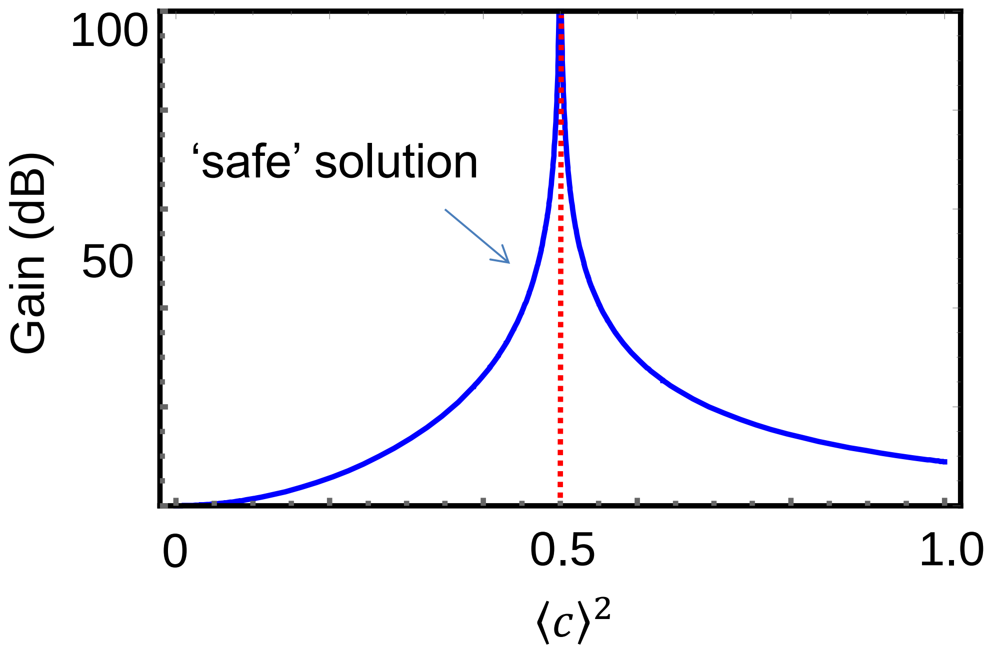
$\langle c\rangle^2 = 0.36$



Gain vs. pump power

When Δ = 0, gain simplifies to:

$$|\alpha|^2 = \left(\frac{1+|\rho|^2}{1-|\rho|^2}\right)^2 \quad \text{where} \quad |\rho|^2 = \frac{4g^2}{\kappa_a\kappa_b}\langle c\rangle^4$$

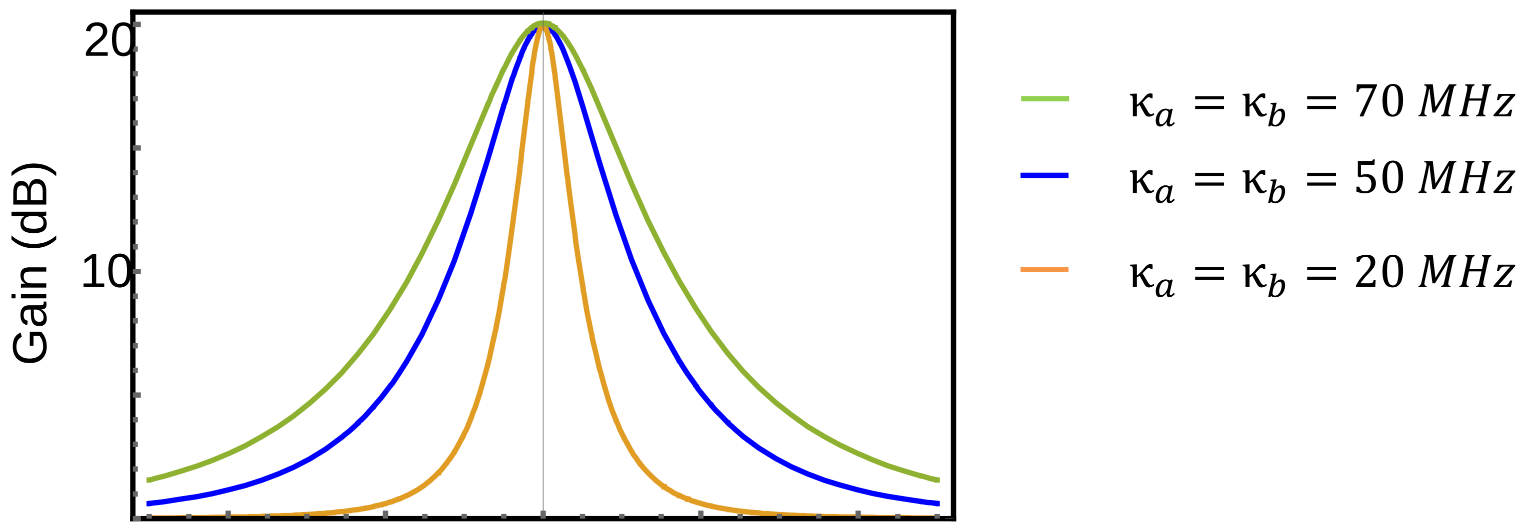


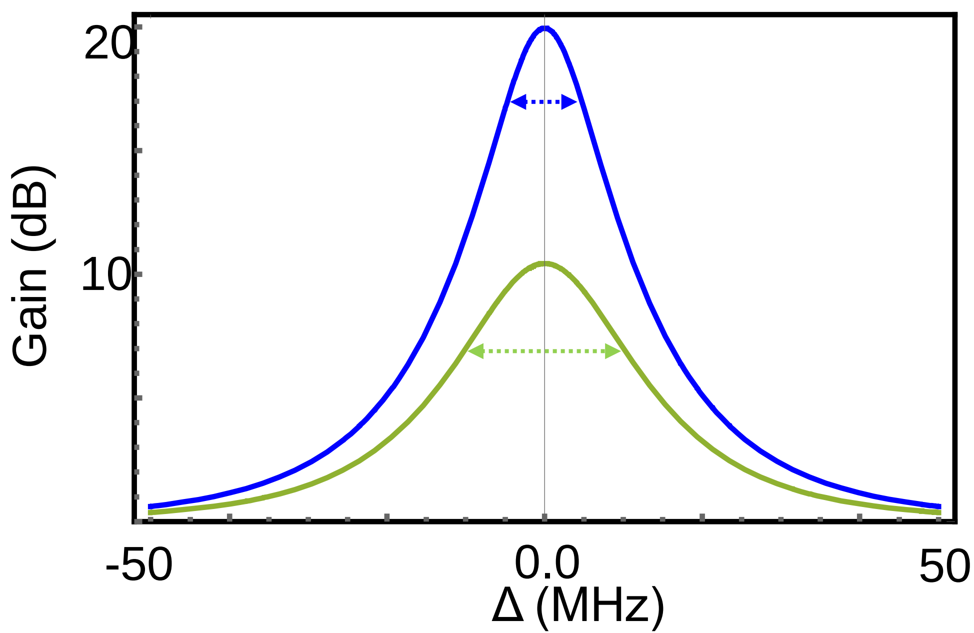
Where

$\kappa_a = \kappa_b = 50\ MHz$

$g = 50\ MHz$

Amplifier bandwidth vs. gain and mode bandwidth



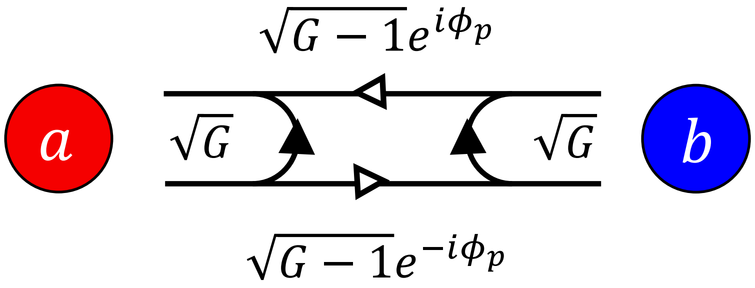


$\kappa_a = \kappa_b = 50\ MHz$

$$2\pi\ B \simeq \sqrt{G} \Big/ \sqrt{\kappa_a \kappa_b}$$

Amplifier Limitations

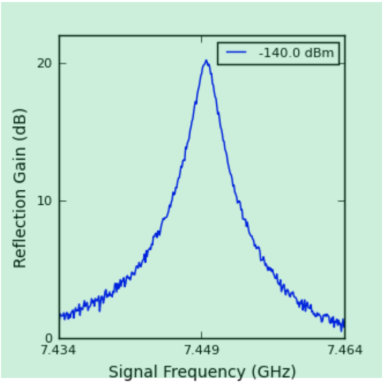
1. It operates in reflection



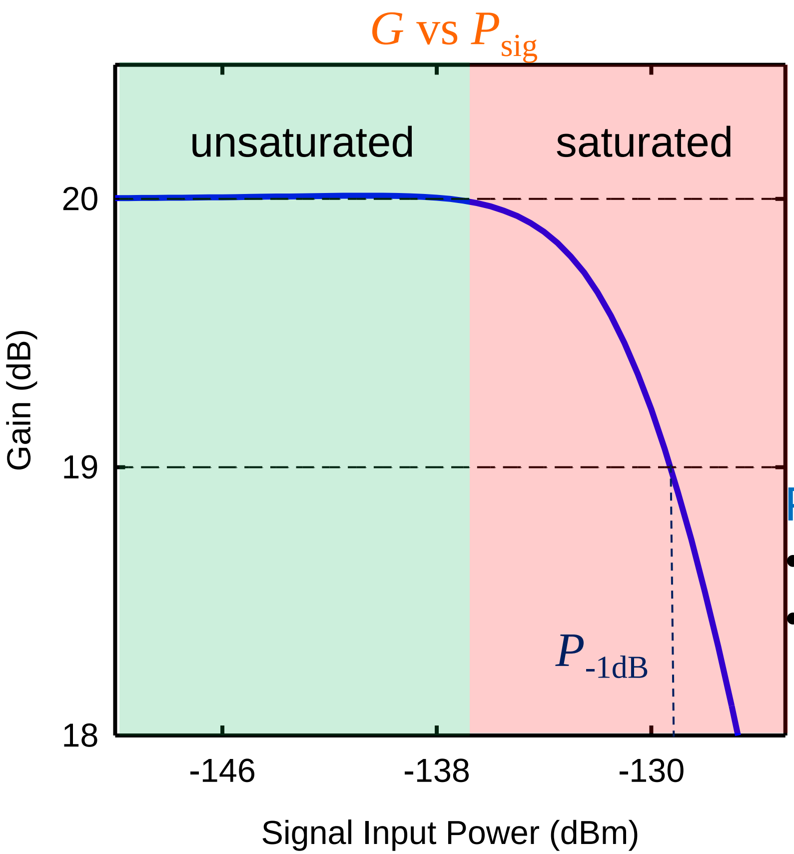
2. It has narrow bandwidth

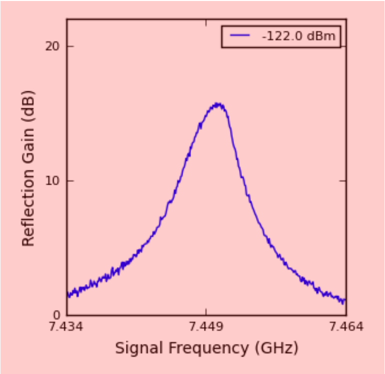
$$2\pi\, B \simeq \sqrt{\kappa_a \kappa_b} \Big/ \sqrt{G}$$

Limitation 3: Gain Saturation



$$G_0 = \left(\frac{1 + n_p/n_p^c}{1 - n_p/n_p^c}\right)^2$$





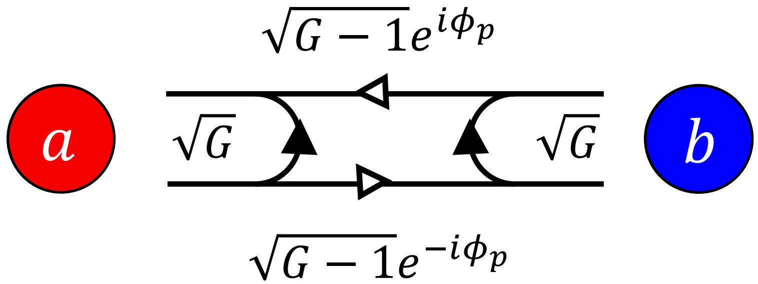
Pump depletion (PD) theory:

- P_{sig} ↗, n_p ↘, G ↘
- n_p ↗, $P_{\text{-1dB}}$ ↗

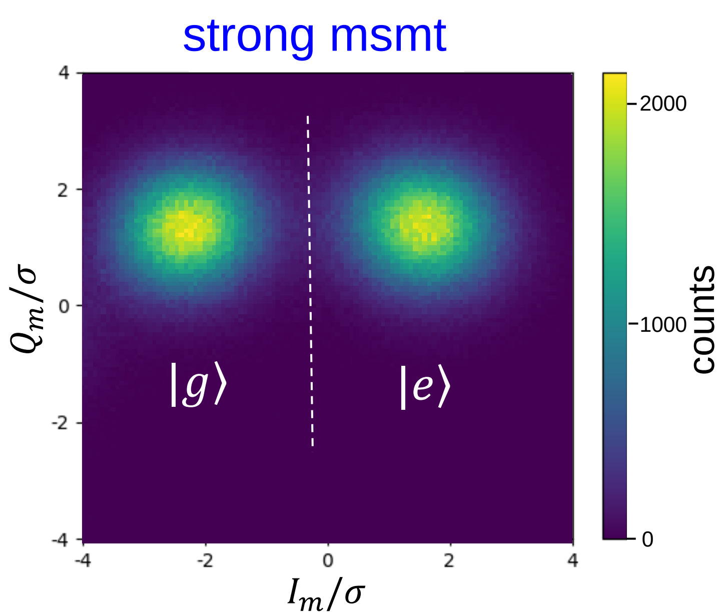
Problem: We don’t understand well all causes

Pump depletion: Abdo Phys. Rev. B (2013); C. Eichler EPJ Quantum Technology (2014)

But it is quantum limited!



- Consider how much noise comes out of mode a with no inputs signals
- The amplifier symmetrically handles both modes; the noise out of a is double its input due to the added noise from the 'idler' mode irrespective of what they are
- For quantum signals you will get 1 photon tota fluctuations (1/2 + 1/2); second 1/2 physicall sourced in idler mode
- We'll show how to test this with qubits in tutorial 3.



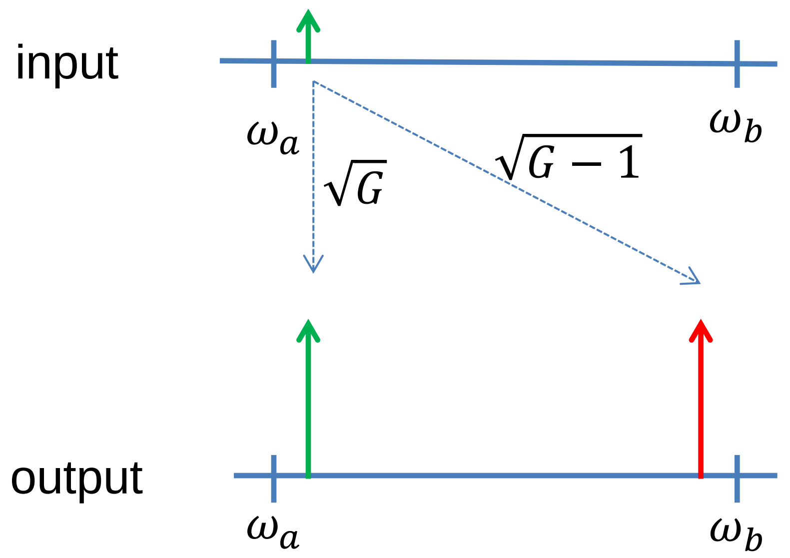
Phase-sensitive amplifier

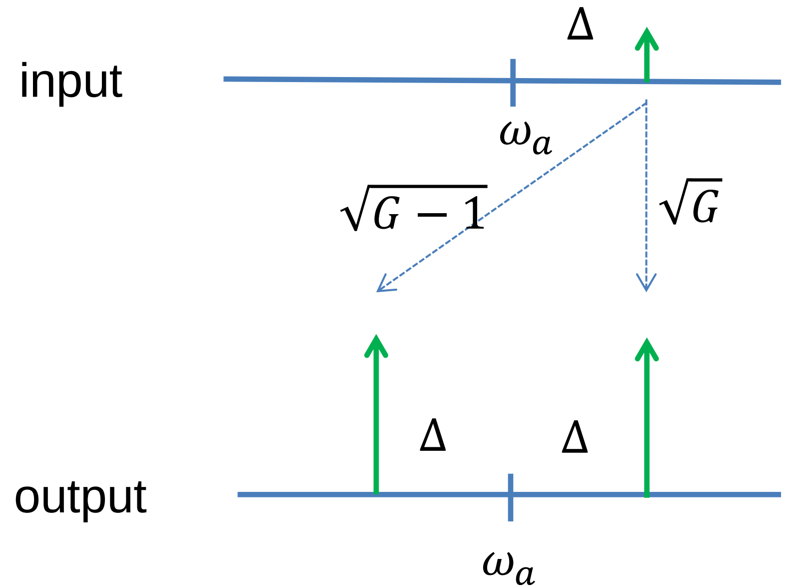
$$\frac{\mathcal{H}_{int}}{\hbar} = g\left(a^\dagger a^\dagger c + abc^\dagger\right)$$

- Note that this is not the coupling Vijay will present tomorrow (his has 4 a operators; 1 each for signal and idler and 2 for pump)

- To see the parallel, make the subsitution (a,b) - > (a_u , a_l). An signal to a_u will be both amplified and coupled to a_l

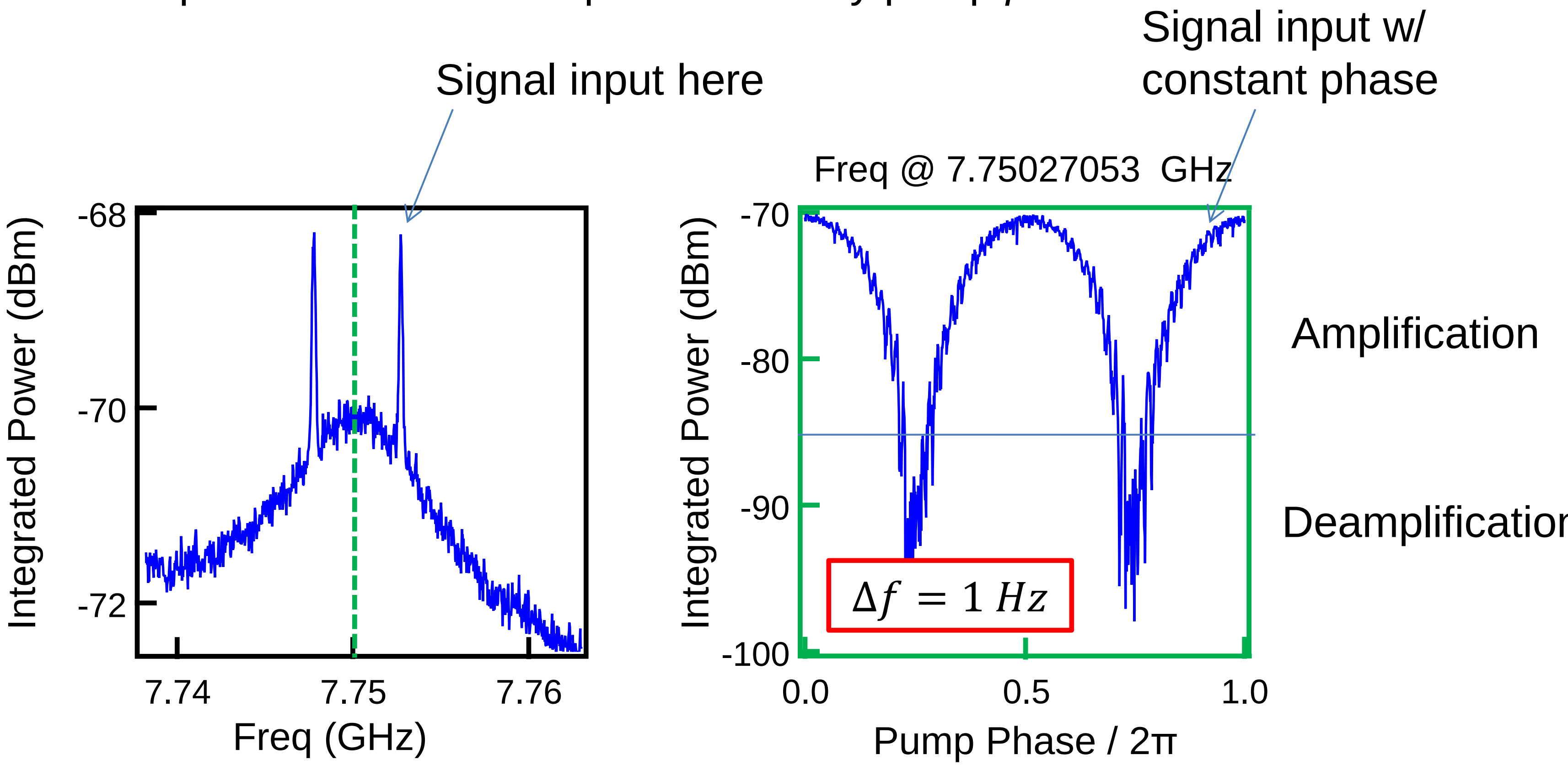
- Essentially one half of the amplifier acts as the ‘signal’ while the other half acts as the ‘idler’





Phase sensitive amplifier data

- At zero offset frequencies these signal and idler tones interfere
- One quadrature is amplified, the other deamplified
- Which quadrature will be amplified is set by pump *phase*



Classifying amplifiers*

	3-wave coupling	4-wave coupling
one mode	'Singly degenerate' phase-sensitive	'Doubly degenerate' phase-sensitive
two-mode	'Non-degenerate' phase-preserving	??

* At optical frequencies phase-preserving => heterodyne detection
phase-sensitive => homodyne detection

More useful: specify parametric interaction

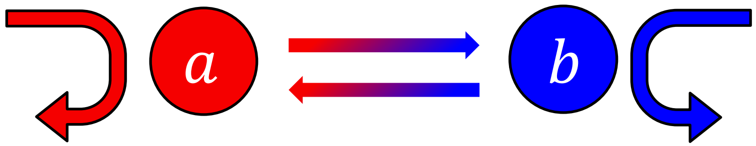
	3-wave coupling	4-wave coupling
one mode (a)	$g a^\dagger a^\dagger c$	$g a^\dagger a^\dagger a a$
two-mode (a,b)	$g a^\dagger b^\dagger c$	$g a^\dagger b^\dagger c c$ **

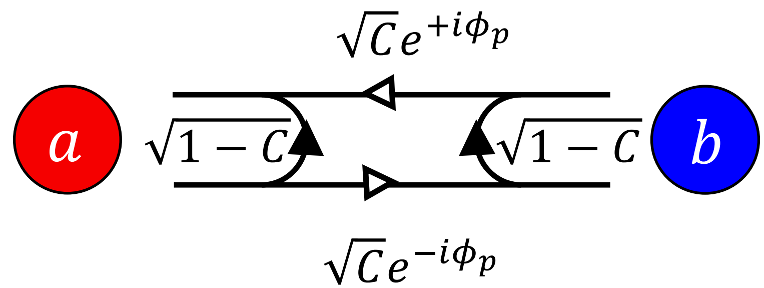
** I'm not aware of this one being built yet

Other Couplings: Photon conversion

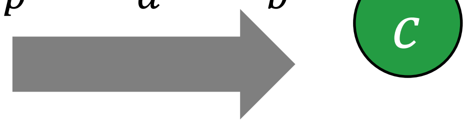
$$\omega_p = \omega_a - \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a^\dagger b e^{i\phi_p} + a b^\dagger e^{-i\phi_p}\right)$$

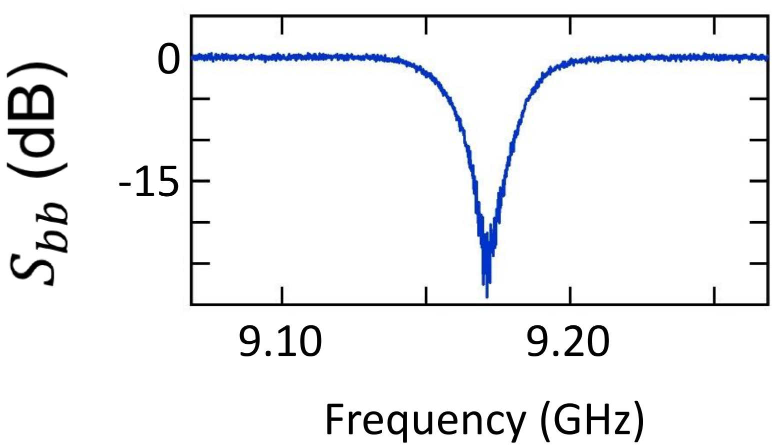


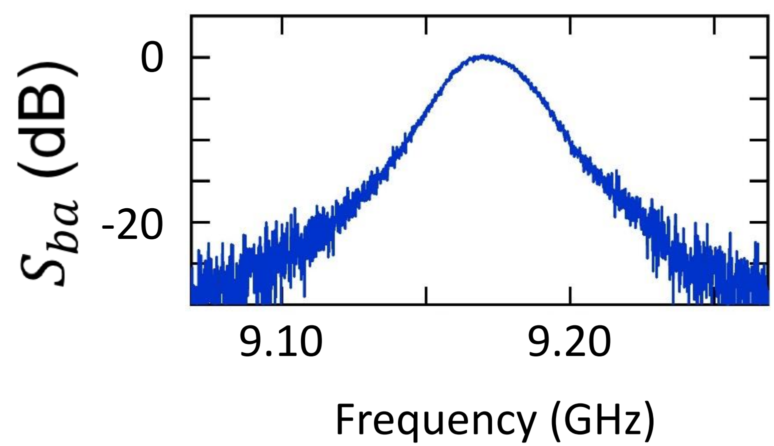


$\omega_p \simeq \omega_a - \omega_b$



$$C = \frac{2\, {}^{P_P}/_{P_C}}{\left(1 + {}^{P_P}/_{P_C}\right)^2}$$





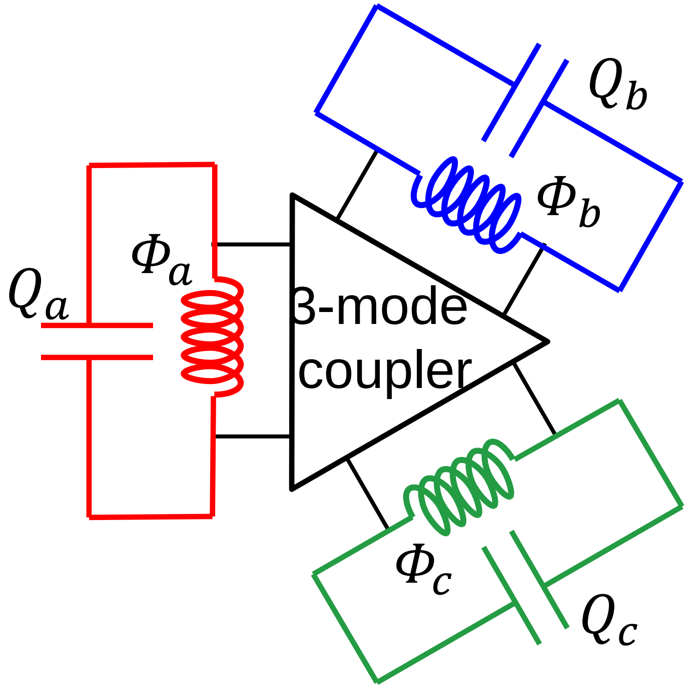
Sliwa *PRX* (2015)

What about still odder couplings?

Ex.: $g a^\dagger a^\dagger c c c$?

- This will be a phase-sensitive amplifier when driven at $2\omega_a = 3\omega_c$
- Such couplings may limit current device performance

More exciting: multiple, simultaneous couplings



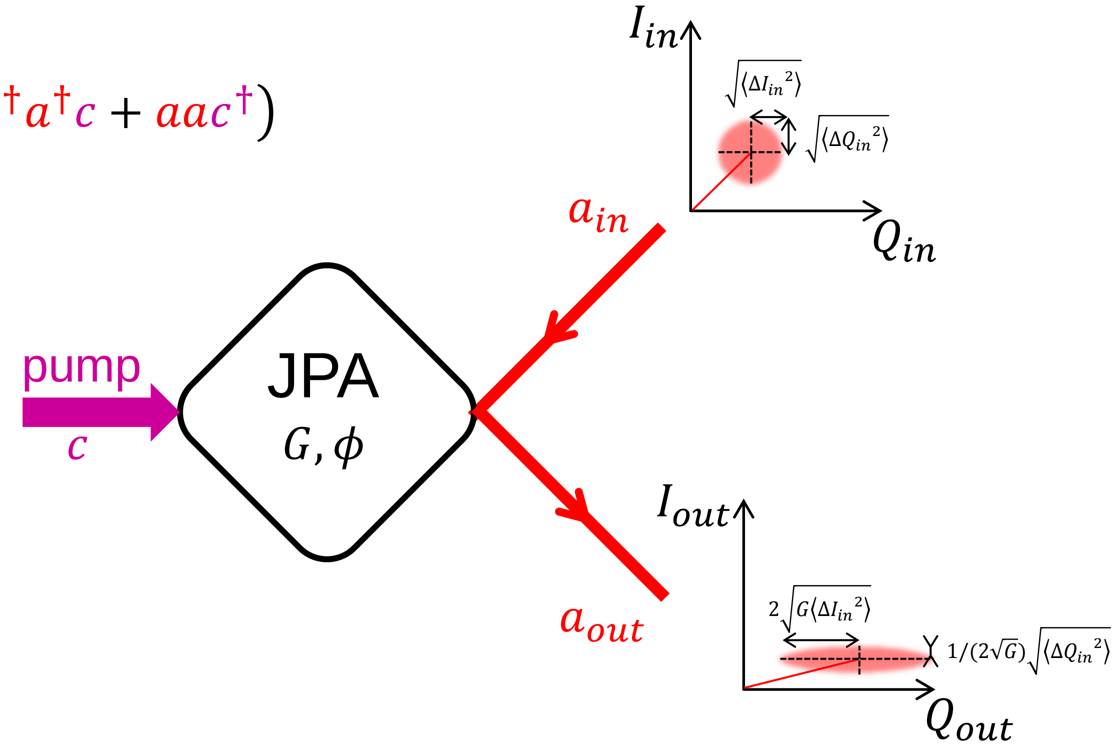
- Can make non-reciprocal devices: circulators, directional amplifiers
- Can also address other shortcomings
- **Your theory proposal here!**

Bergeal *et al*. *Nature Physics* (2010)
Ranzani and Aumentado *NJP* (2015),
Metelmann and Clerk *PRL* (2015)
Sliwa *et al. PRX* (2015)
….

Amplifier outputs are squeezed light!

Squeezed light from a phase-sensitive amplifier

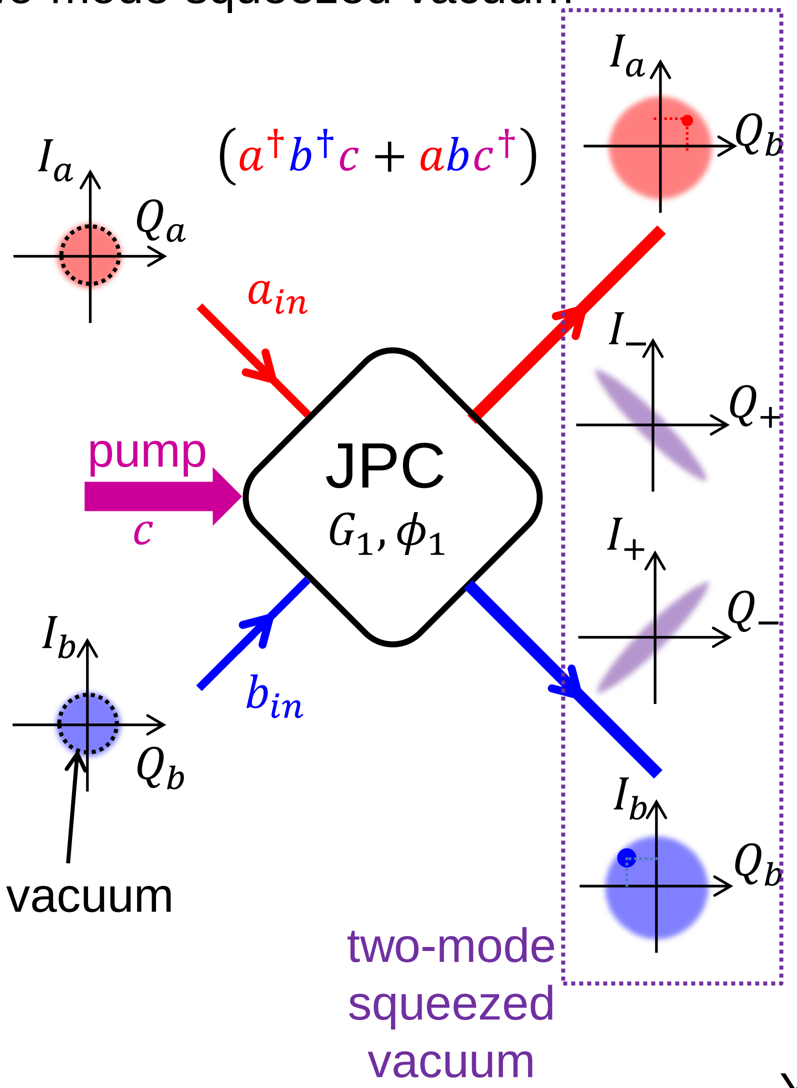
$\left(a^{\dagger}a^{\dagger}c + aac^{\dagger}\right)$



Castellanos-Beltran et al., *Nature Physics* (2008)

Two-mode Squeezing

Example: two-mode squeezed vacuum

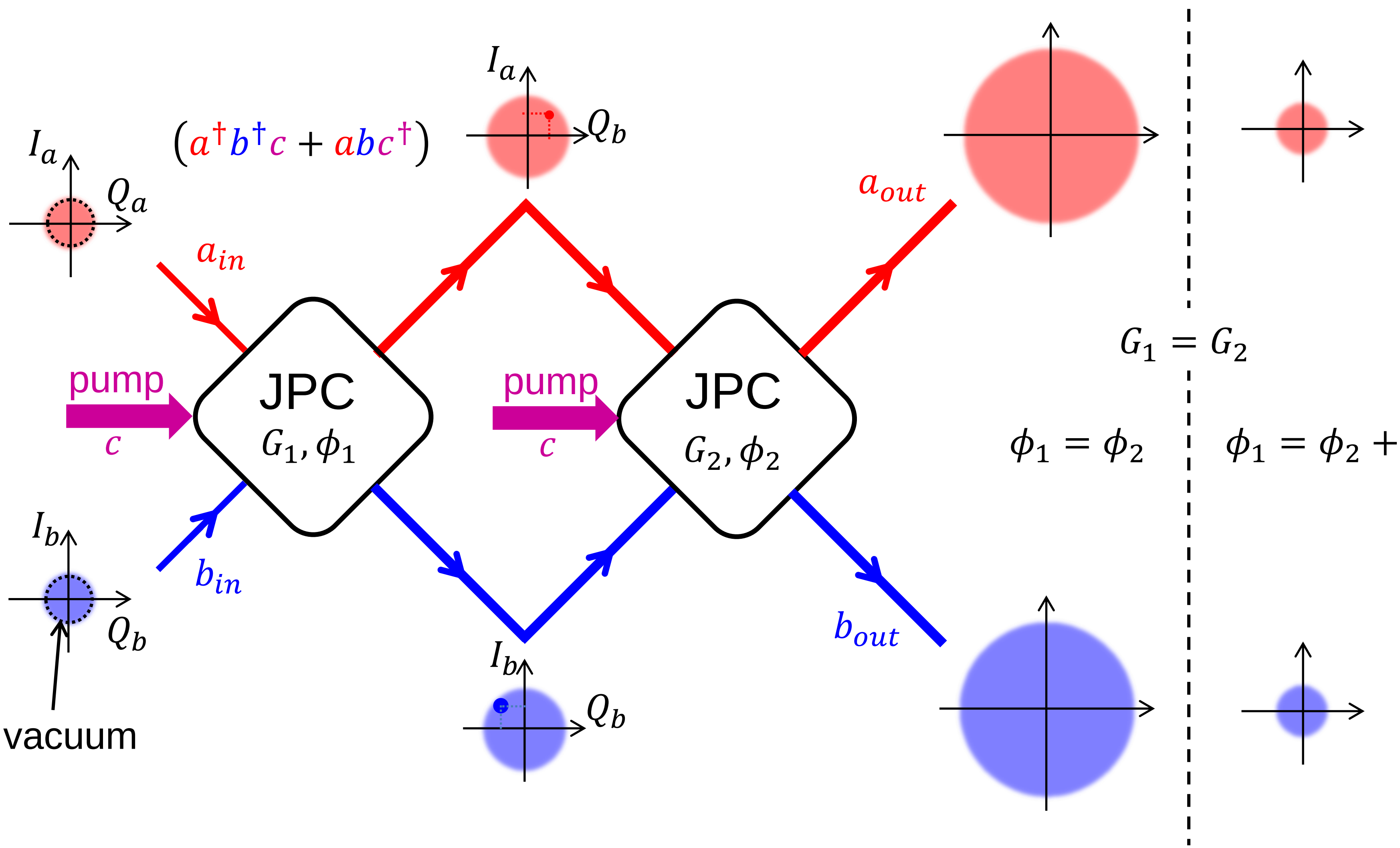


$$I_\pm = I_a \pm I_b$$

$$Q_\pm = Q_a \pm Q_b$$

Yurke *et al.*, *PRA* (1986)
Bergeal *et al.*, *Nature Physics* (2010)

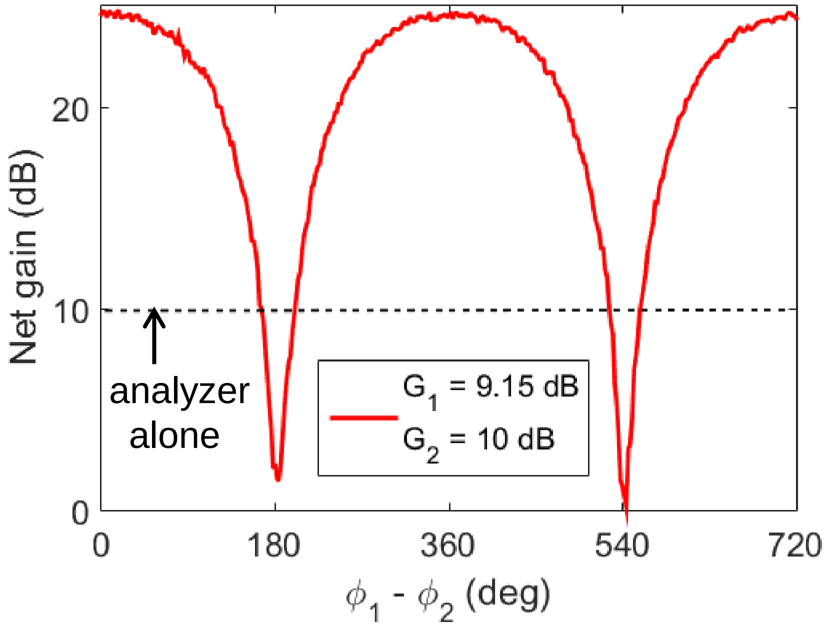
Amplification is a unitary transformation



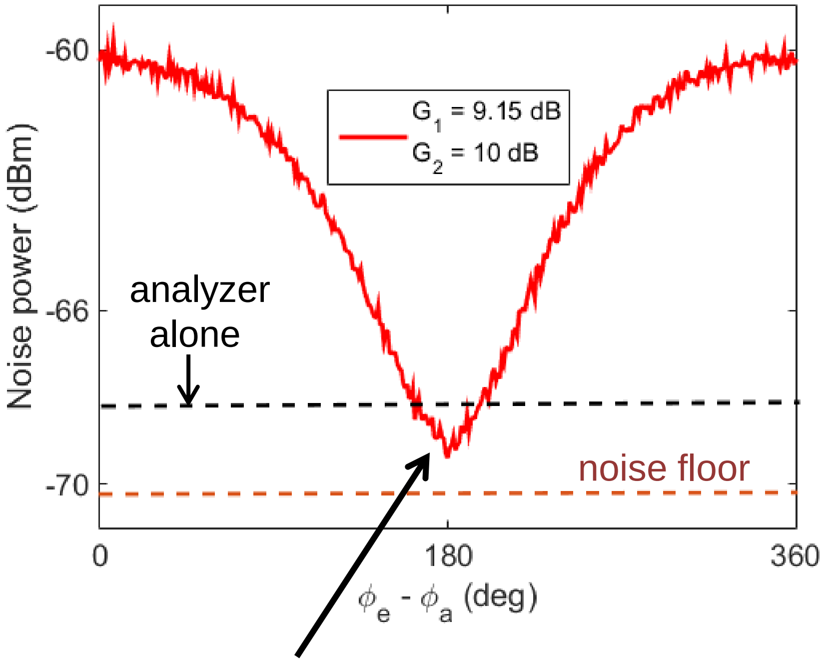
Yurke *et al.*, *PRA* (1986)
Flurin *et al.*, *PRL* (2012)

Experimental Evidence

Reflection gain vs $\phi_1 - \phi_2$, G_2= 10 dB

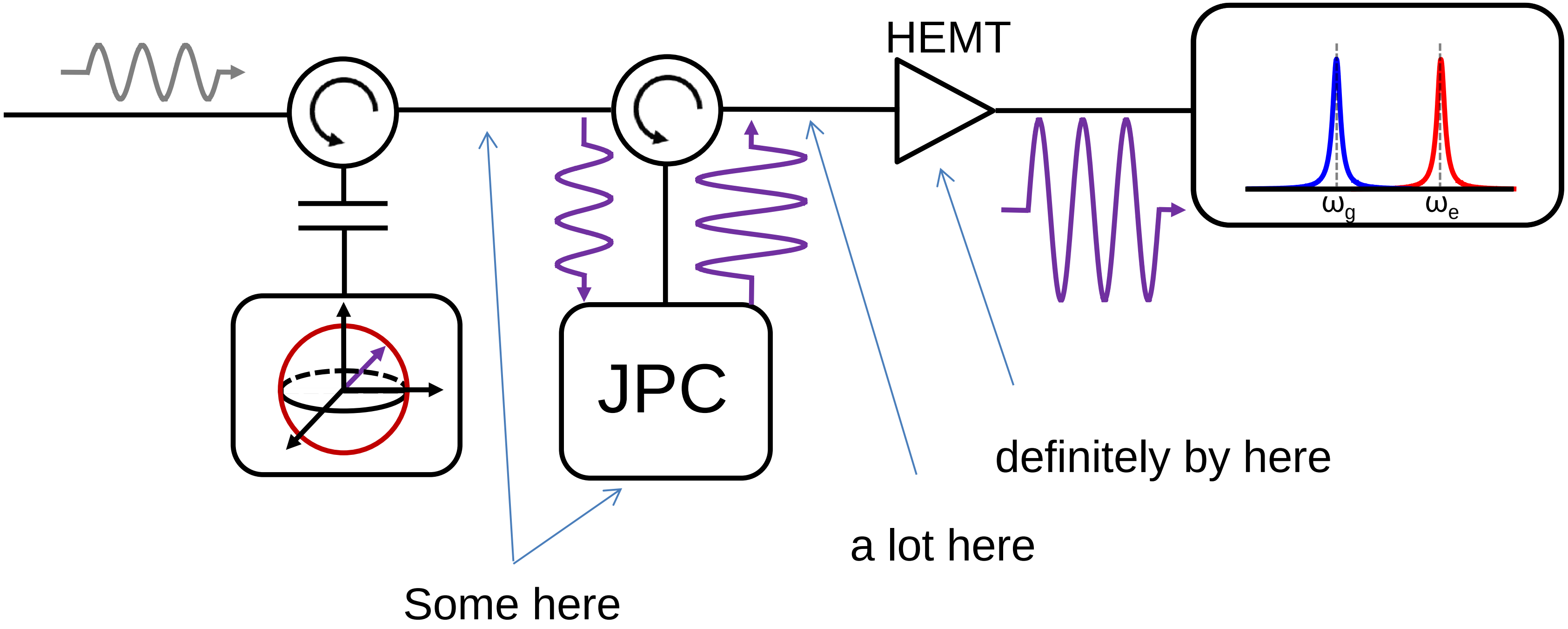


Two-mode squeezed vacuum fluctuation



~1 dB suppression of 'analyzer' noise output

So when do we measure?



We measure when we have lost 'enough' information to degrade our quantum coherence!

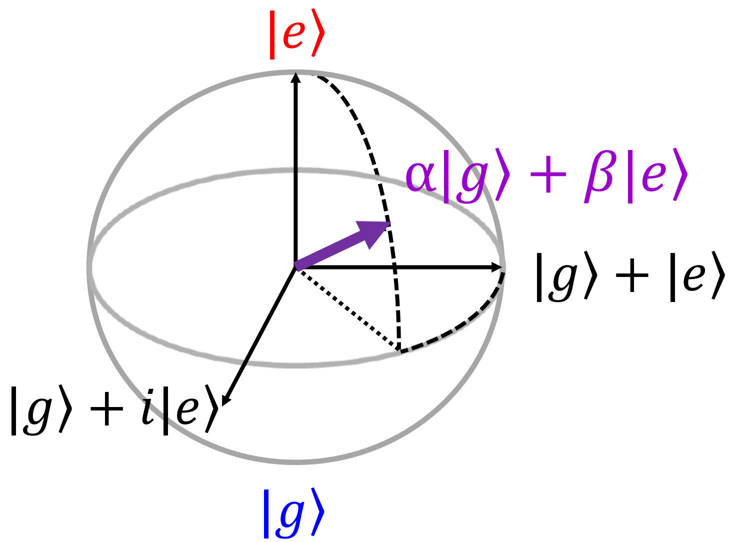
The Story So Far

- Parametric amplifiers are quantum-limited and well suited for amplifying coherent pulese of light
- The act of amplification is quantum, ‘measurement’ occurs when sufficient info lost
- The amplifiers’ outputs are interesting states of quantum light

Quantum Measurement in Superconducting Circuits

Qubits and entanglement

A Qubit



- Two level quantum system
- Can be prepared in well known initial state
- Can be placed in coherent superpositions states ($\alpha|g\rangle + \beta|e\rangle$)
- Characterized by energy relaxation (T_1) and dephasing (T_2) time-scales

Entanglement (e.g. Bell-states)

$|gg\rangle + |ee\rangle$

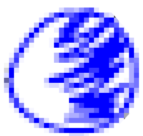
or

$|ge\rangle + |eg\rangle$

- Neither qubit contains information on its own (all measurements as random as a flipped coin)
- All information resides in relationship *between* qubits
- Building block for quantum communication, quantum computing
- One bit of entanglement referred to as an ‘e-bit’

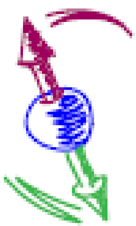
What is ‘Quantum Information’?

First, imagine that you have



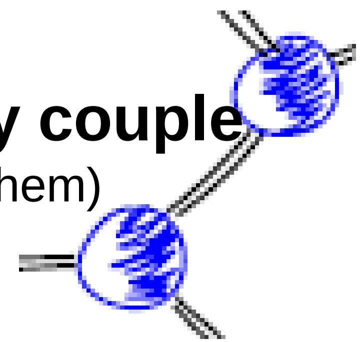
Well behaved quantum systems
(e.g. photon, electron, more generally “qubits”)

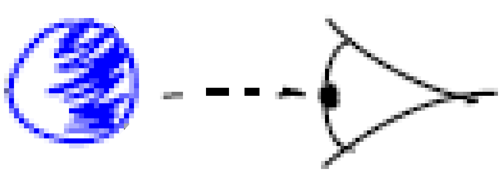
which you can



Individually control

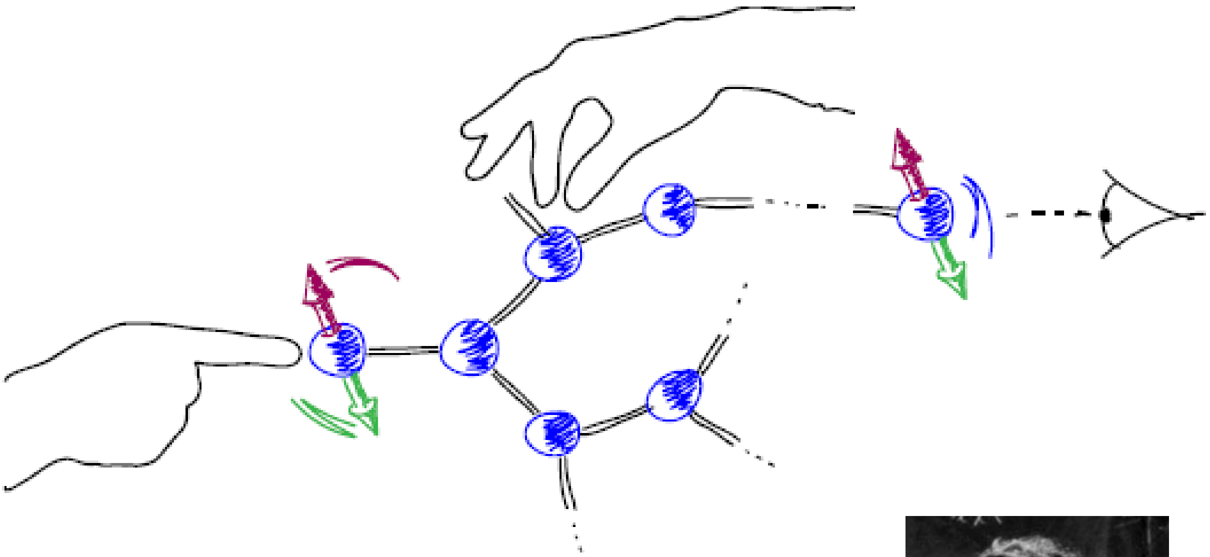
Controllably couple
(to entangle them)



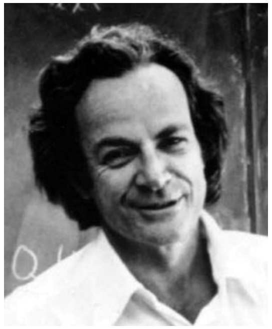


Efficiently and individually measure

Now put it all together

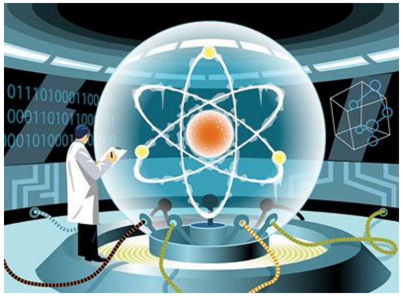


And use it to:



1. Simulate other quantum systems (Feynman 1982)

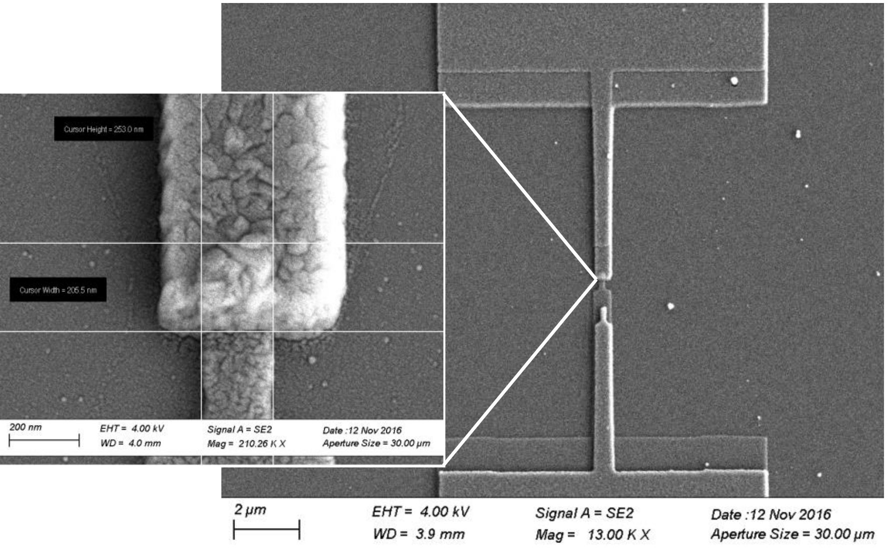
2. Perform algorithms which are hard for classical computers (Shor, Grover,…)



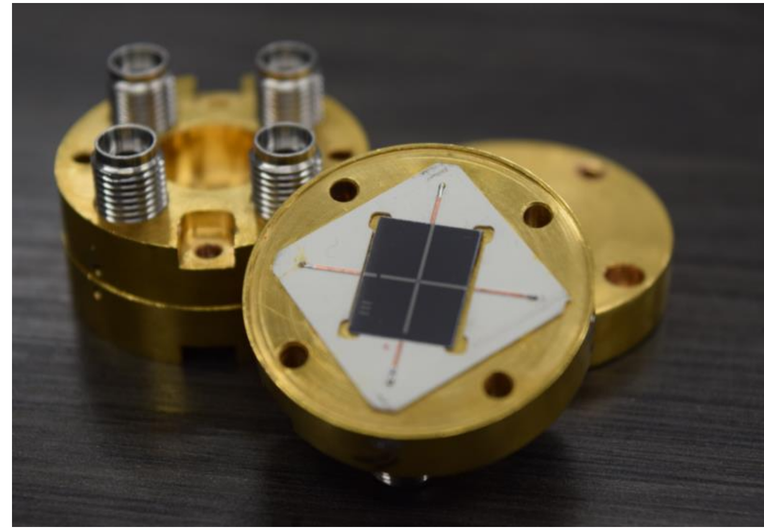
3. Do things we haven’t thought of/discovered yet!

Art by W. Pfaff

Enter superconducting circuits

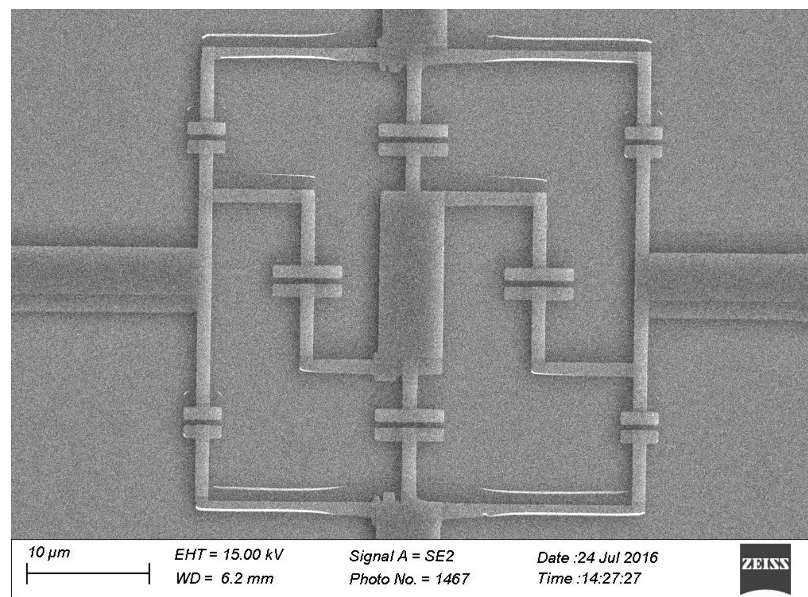


SEM image of qubit



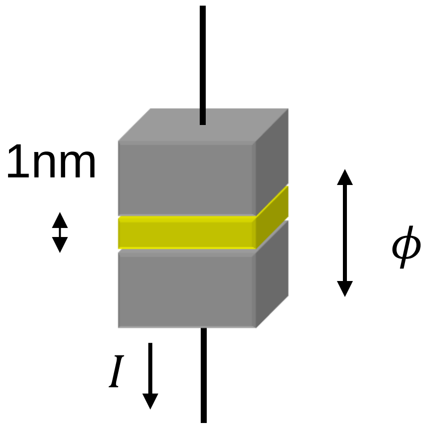
Josephson Parametric Converter

Qubit and cavity



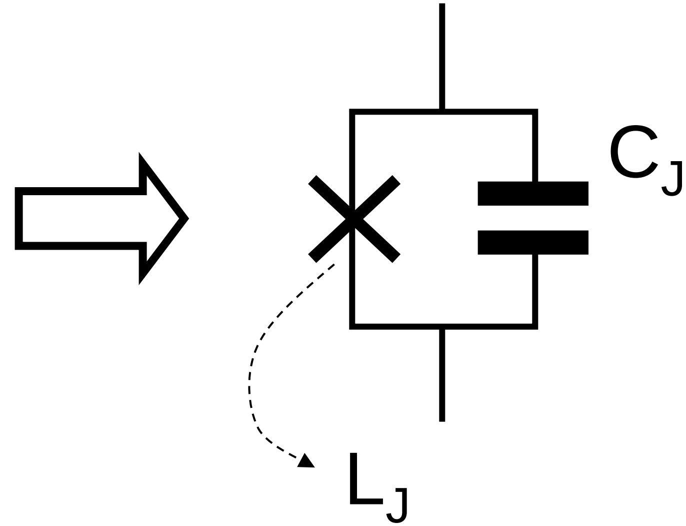
SEM image of Josephson Ring Modulator

The Josephson tunnel junction

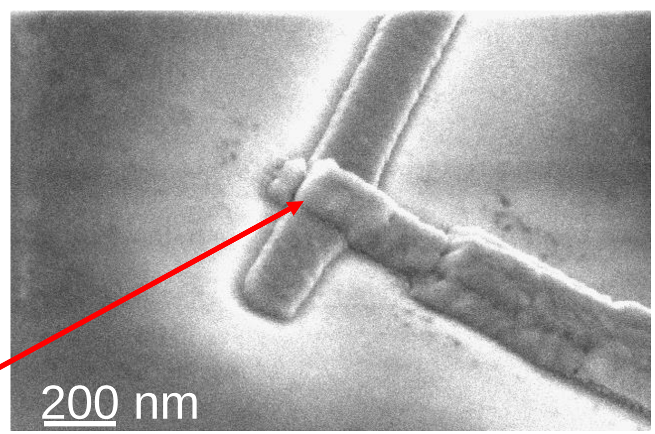


$$I = I_0 \sin\left(\phi/\varphi_0\right)$$

$$\varphi_0 = \frac{h}{2e}$$



SUPERCONDUCTING
TUNNEL JUNCTION

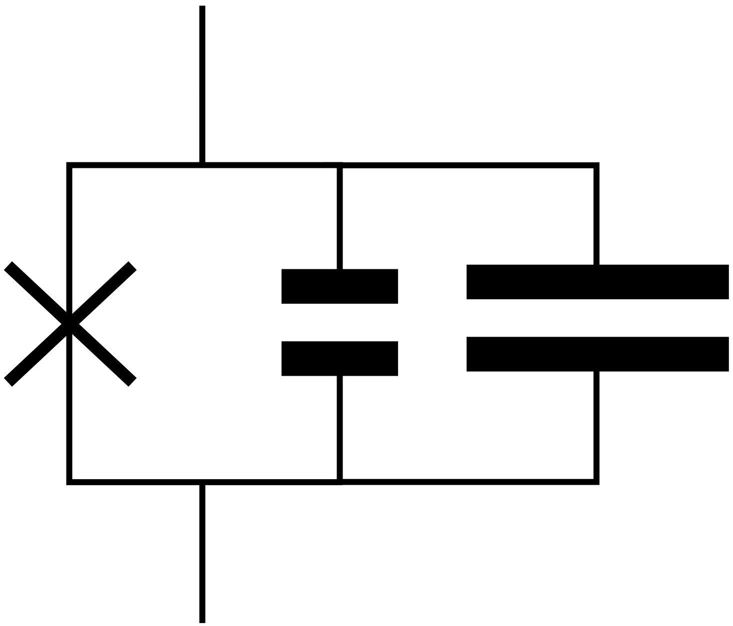


nonlinear inductor
shunted by capacitor

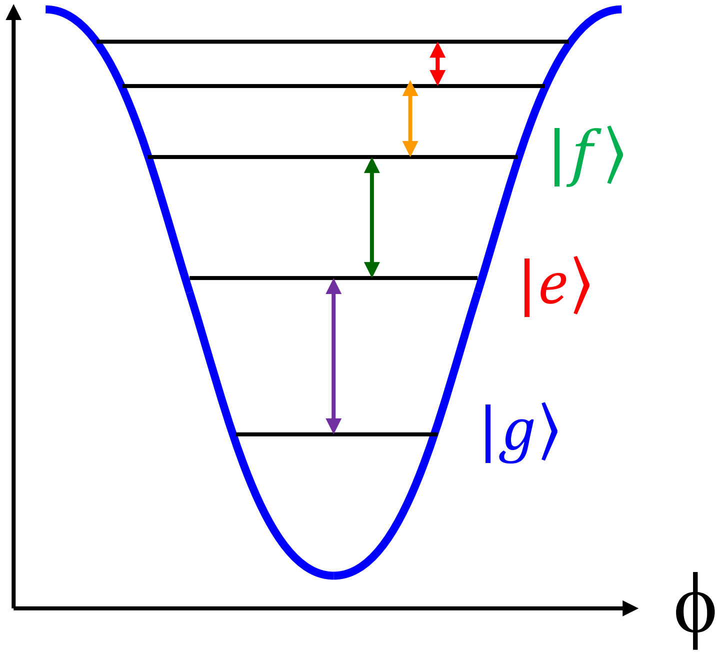
$Al/AlO_x/Al$
tunnel junction

Superconducting transmon qubit

Josephson junction with shunting capacitor ➔ anharmonic oscillator



Potential energy

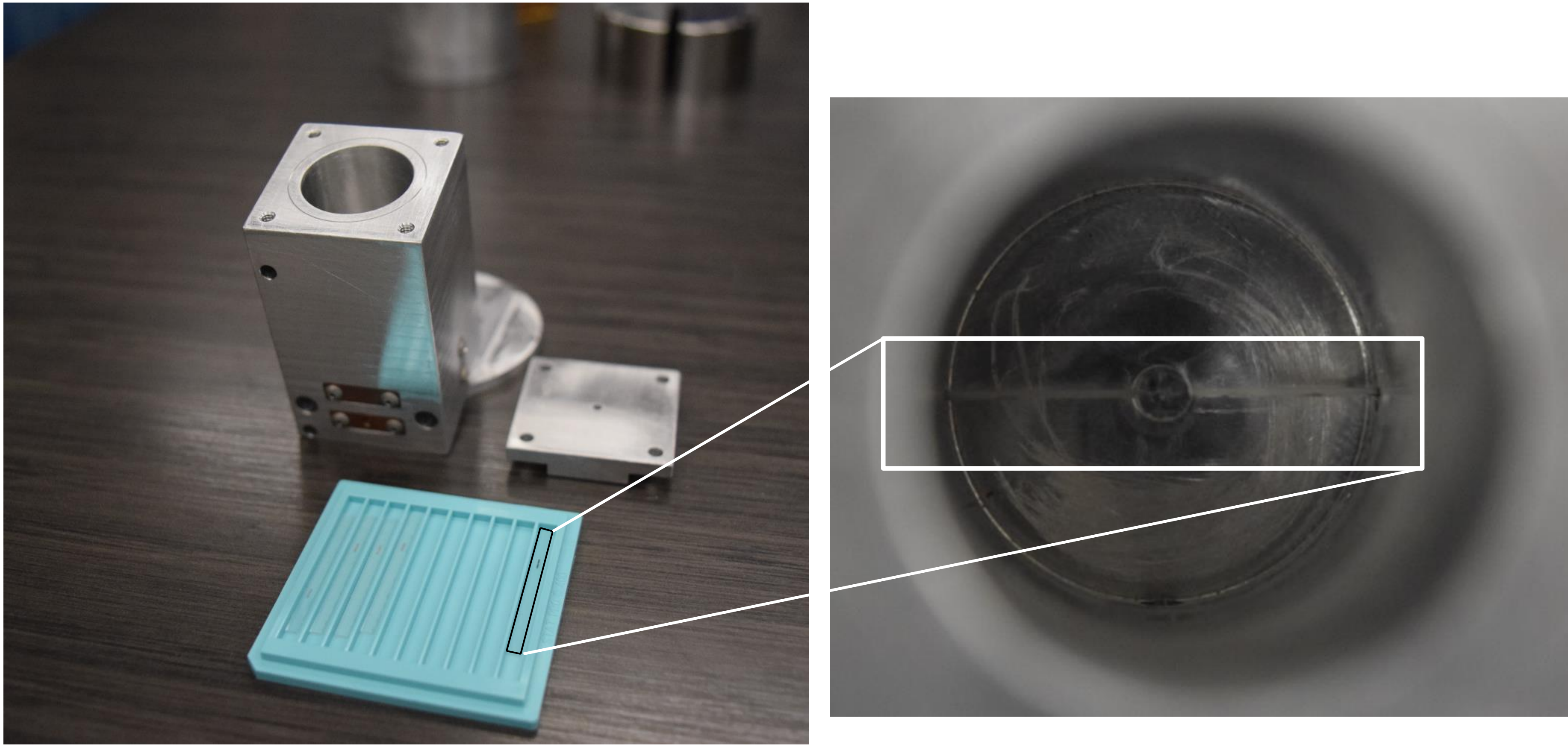


lowest two levels form qubit

f_{ge} ~ 5.025 GHz, f_{ef} ~ 4.805 GHz

Koch et al., Phys. Rev. A (2007)

Isolating the transmon from the environment



Cavity

$f_{c,g}$ = 7.4817 GHz

$1/\kappa$ = 30 ns

Qubit

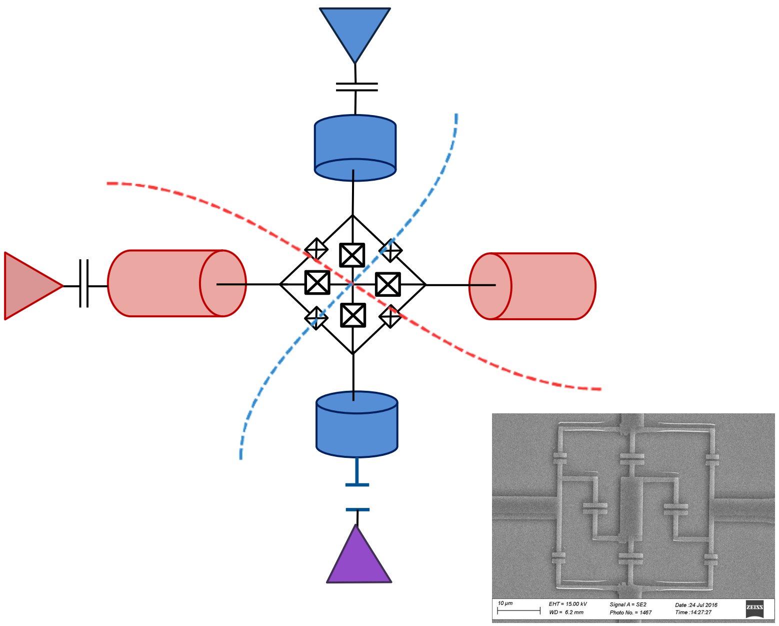
f_Q=5.0252 GHz

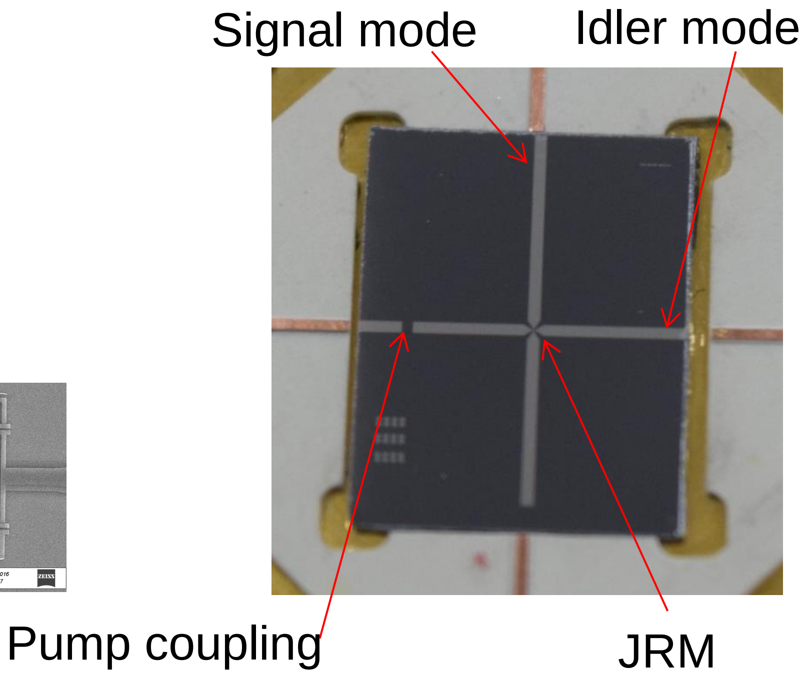
T_1 = 30 μs

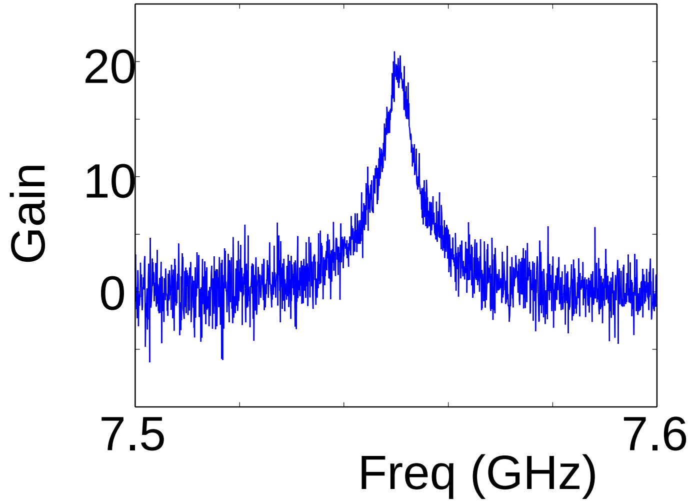
T_{2R} = 8 μs

The 8-junction Josephson Parametric Converter

(a)





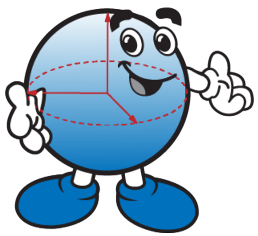


Bergeal *et al* Nature (2010)
See also Roch *et al* PRL (2012)

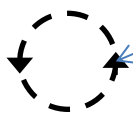
What a quantum machine looks like

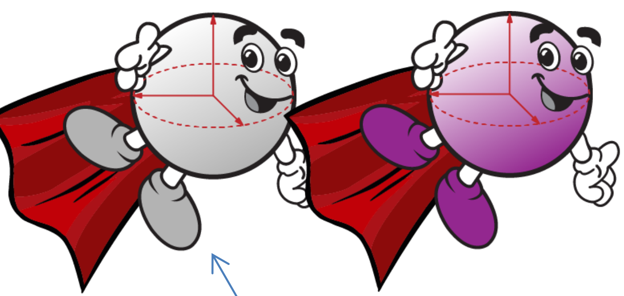
Measurement via flying qubits

This qubit is well protected, with long coherence times

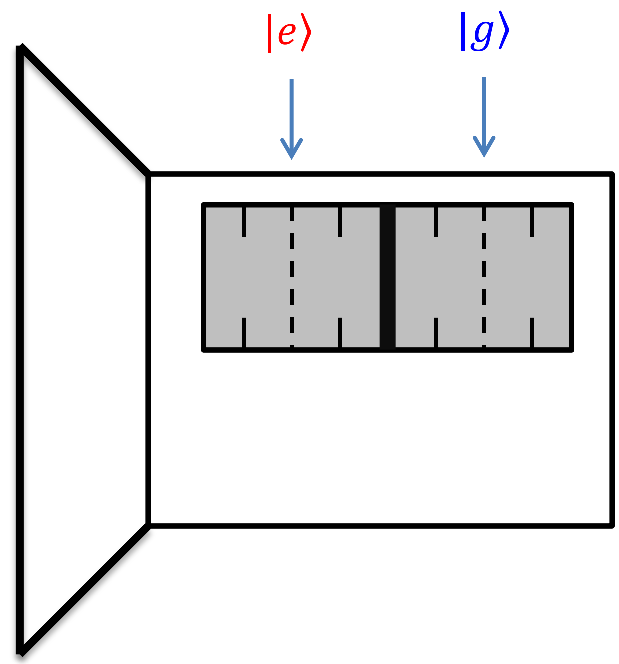


dispersive interaction





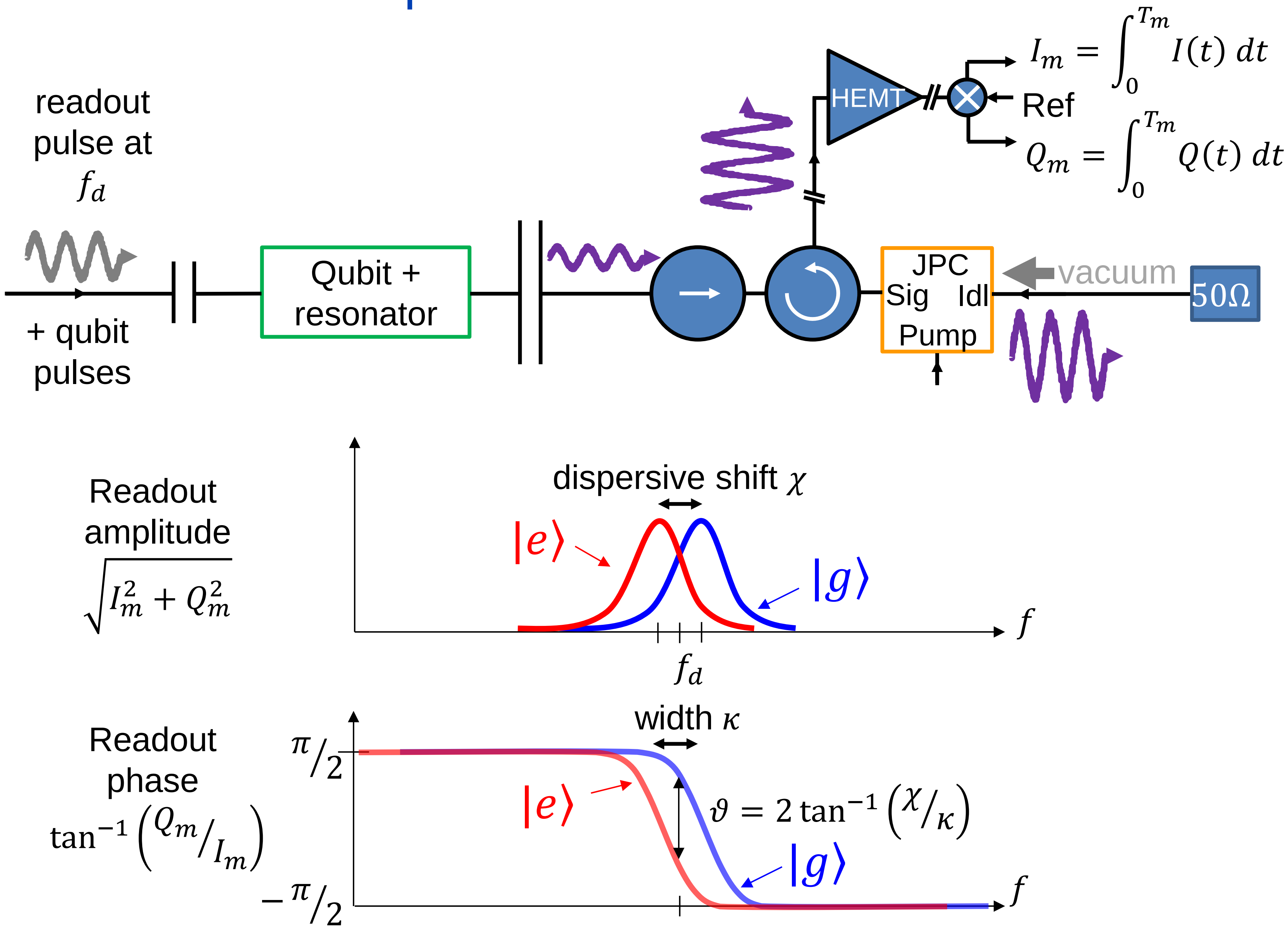
This flying "ancillary" qubit suffers the slings and arrows of outrageous fortune on its way to the detector



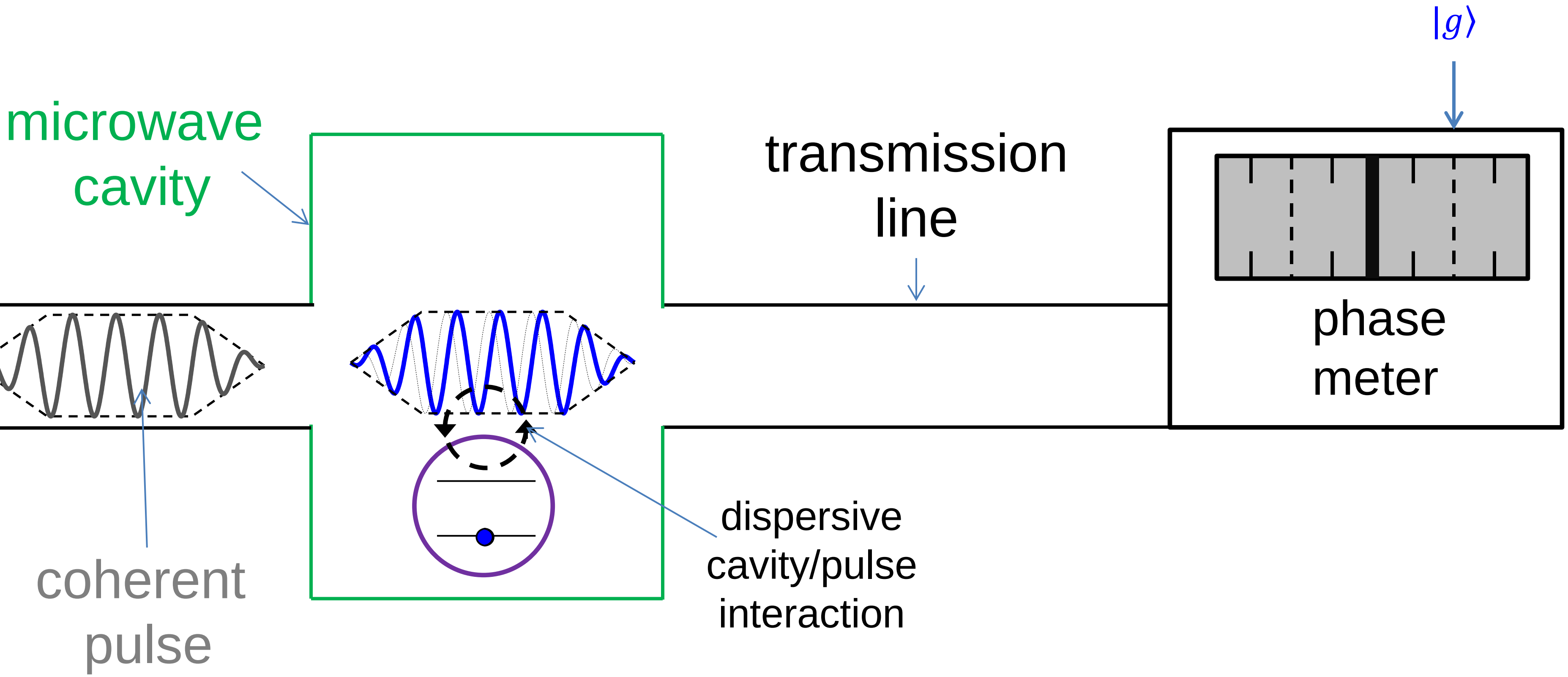
Art by W. Pfaff

If you are quiet and/or I talk too fast…

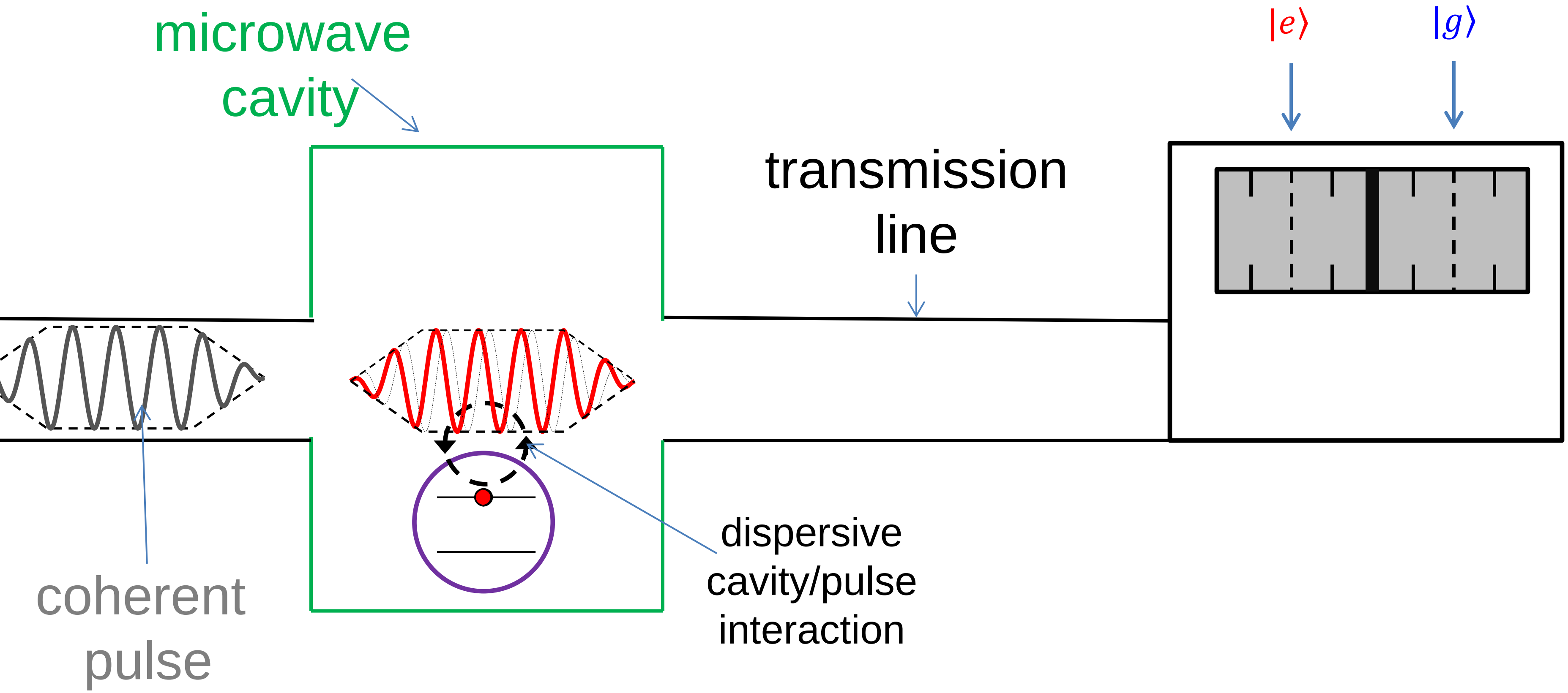
Dispersive Measurement



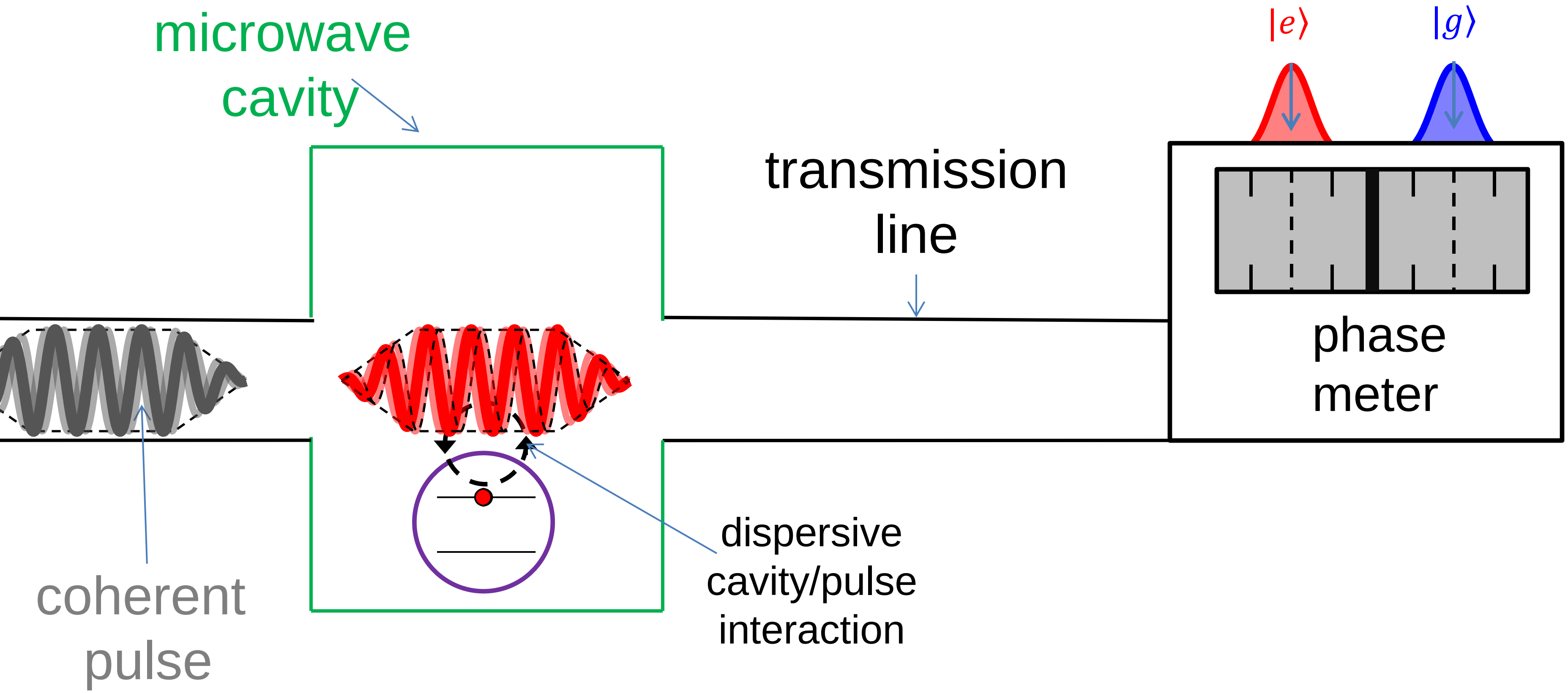
Dispersive measurement: classical version



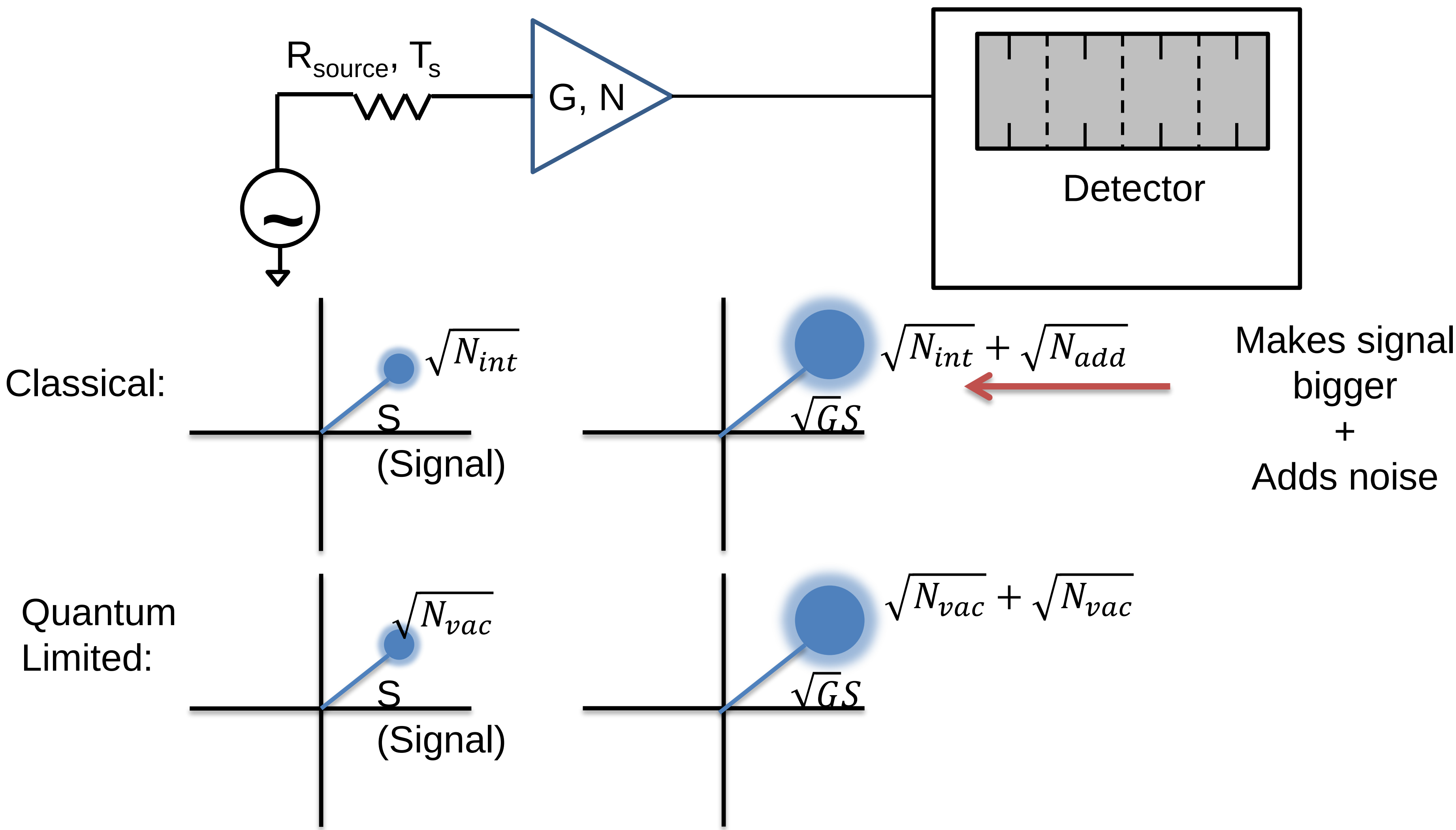
Dispersive measurement: classical version



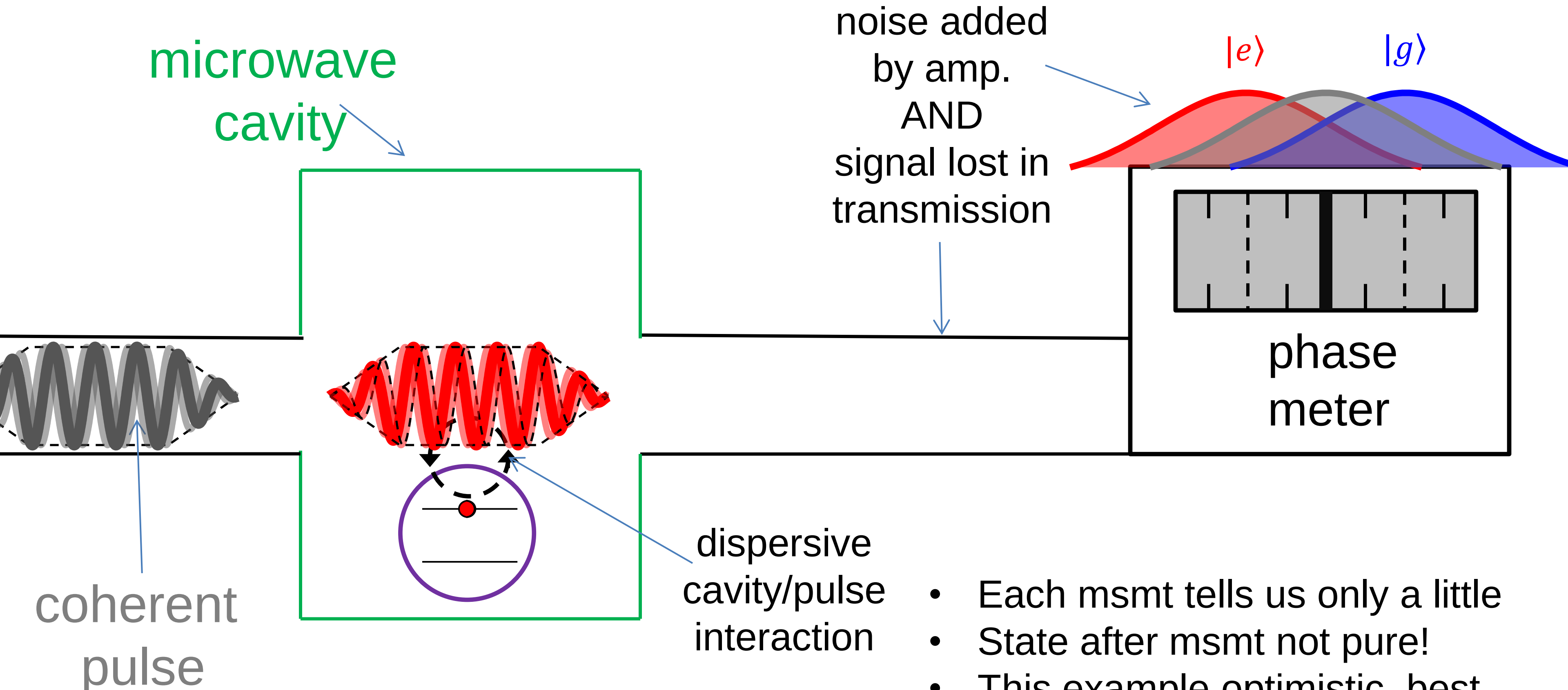
Now a wrinkle: finite phase uncertainty



Amplifier Theory

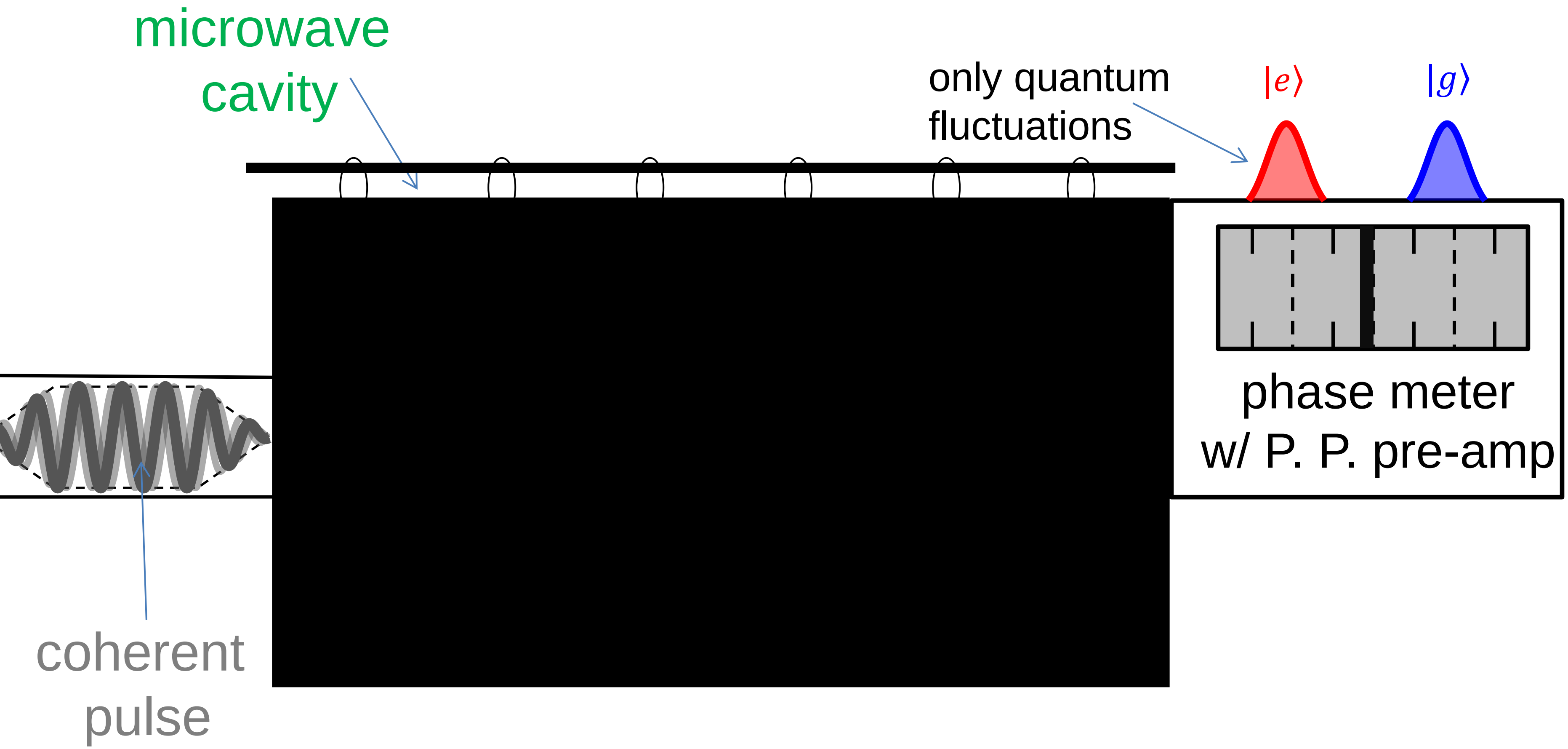


Measurement with bad meter (still classical)



- Each msmt tells us only a little
- State after msmt not pure!
- This example optimistic, best commercial amp adds 20-30x noise
- We fix this with quantum-limited amplification

Quantum-limited amplification: projective msmt

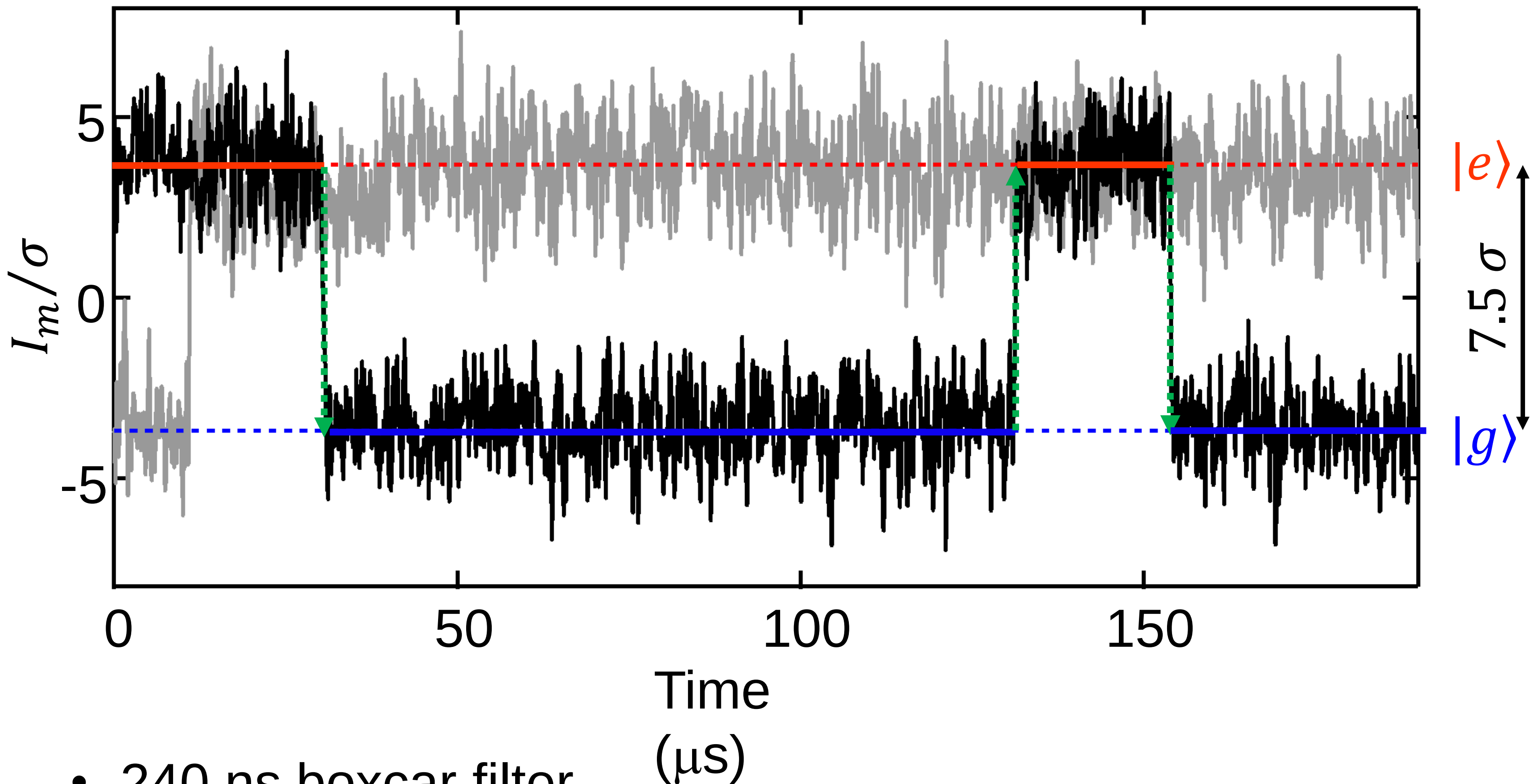


- state of qubit pure after each msmt
- For unknown initial state

$c_g|g\rangle + c_e|e\rangle$, repeat

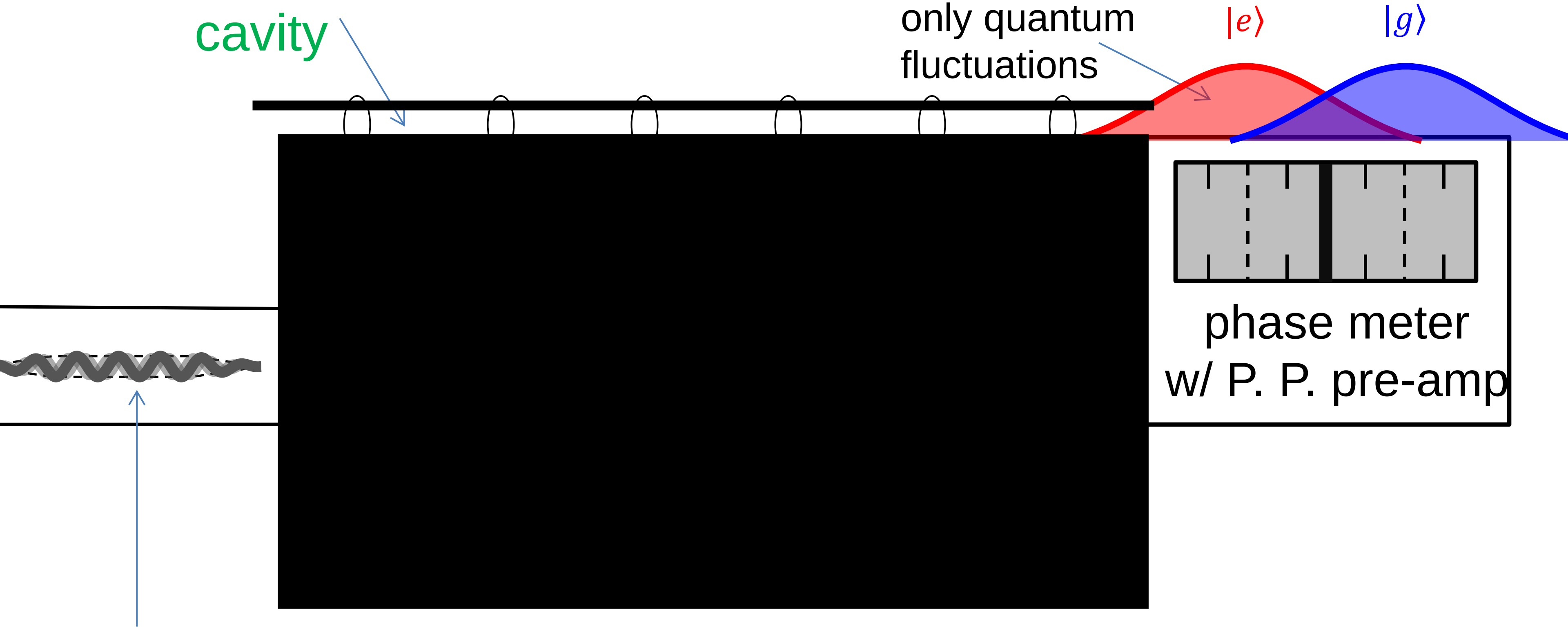
many times to estimate $|c_g|^2$, $|c_e|^2$

Quantum jumps



- 240 ns boxcar filter
- $T_1(\bar{n} = 10) \cong 50\mu s$
- Fully linear (can see $|f\rangle, |h\rangle$… in IQ plane)

Quantum-limited amplification: ‘partial’ msmt



- state of qubit pure after each msmt
- counter-intuitive, but is achievable in the laboratory

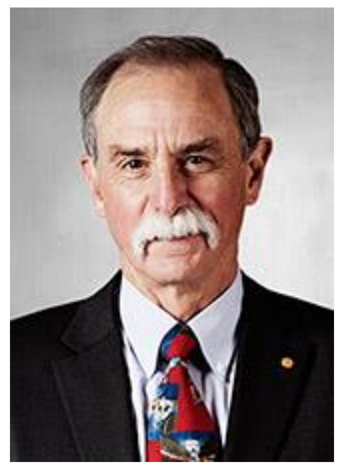
Extra Slides

Can we build such systems?

"we never experiment with just one electron or atom or (small) molecule. In thought experiments we sometimes assume we do; this invariably entails ridiculous consequences […]

*it is fair to state that we are not experimenting with single particles, any more that we can raise Ichtyosauria in the zoo."**

*Schroedinger, *Brit. J. Phil.* Sci **3**, 233 (1952)



The Nobel Prize in Physics 2012 was awarded jointly to Serge Haroche and David J. Wineland *"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"*