Nonclassicality in a parity-time-symmetric optomechanical system

Anirban Pathak

Jaypee Institute of Information Technology

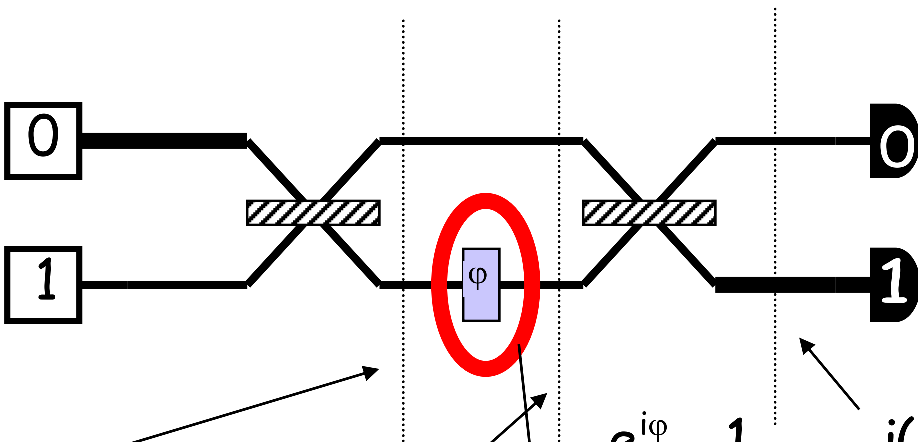
Disclaimer:

- Research activities of our group are not focused on $\mathcal{PT}$ symmetric systems in particular or non-Hermitian Physics in general.
- I represent quantum optics and quantum information group and we often work with non-Hermitian systems/operators/gates and use them in beneficial manner. **So, in this talk the "non-Hermitian physics" will be used in its literary meaning and will not be restricted to non-Hermitian operators.**
- We work on open quantum systems, non-Gaussianity inducing operators (which are often non-Hermitian) and optomechanical systems=> These operators/systems are closely connected to the non-Hermitian physics, and this talk will reveal the connection, but will not be focused completely on $\mathcal{PT}$ symmetric systems.

What you see is what you want to see.
If you wish you can see non-Hermitian physics everywhere

What is a quantum gate? Can I treat a glass plate as a non-Hermitian gate? What about the light coming out of this laser pointer?

Phase plate in Mach-Zehnder interferometer is non-Hermitian gate.



$\sin^2\left(\frac{\varphi}{2}\right)$ (output 0)

$\cos^2\left(\frac{\varphi}{2}\right)$ (output 1)

$$\frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{i}{\sqrt{2}}|0\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|1\rangle$$

$$\frac{e^{i\varphi}-1}{2}|0\rangle + \frac{i(e^{i\varphi}+1)}{2}|1\rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \exp(i\phi) \end{pmatrix} \Rightarrow \text{non} - \text{Hermitian unless}\, \phi = 0, \pi$$

Now you can show this type of experiemnts in class room using IBM quantum computers

Non-hermitian physics is present everywhere, but it’s often hidden and sometime it’s superimposed by intentional truncation of the universe

There is plenty of ~~room~~ non-Hermiticity in the ~~bottom~~ framework of standard quantum mechanics

A simple argument: Evolution under any Hermitian operator is unitary. So a non-unitary evolution ~~would essentially~~ may corresponds to an evolution under a non-Hermitian effective Hamiltonian. Consequently, open quantum system dynamics which corresponds to energy non-conservative systems is expected to be related to evolution under non-Hermitian effective Hamiltonian which is essentially a partial description of the complete system as the bath does not include entire universe and leads to energy loss.

What is my contribution to non-Hermitian physics?

Environment plays a crucial role in non-Hermitian physics:

Nonclassical states

Informal definition: A state, which does not have any classical analogue, is called nonclassical.

Formal definition used by quantum optics community: For which Glauber-Sudarshan P function is not a classical probability density function

LIGO uses nonclassical state as input

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha| d^2\alpha. \qquad (A)$$

For a pure state $|\psi\rangle : \rho = |\psi\rangle\langle\psi|$

For a mixed state $\{\mathrm{p_i}, |\psi_i\rangle\} : \rho = \sum_{\mathrm{i}} \mathrm{p_i} |\psi_i\rangle\langle\psi_i|,$

where $0 \le \mathrm{p_i} \le 1.$

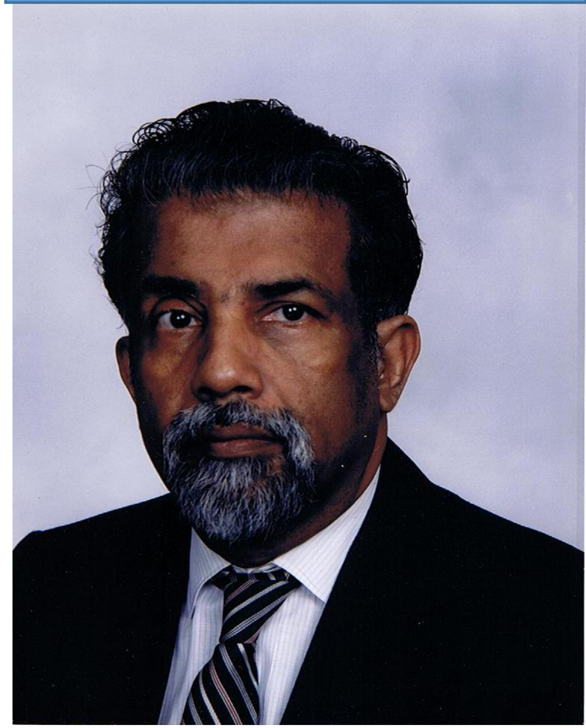
For a continuous variable mixed state created by mixture of coherent states with different values of α :

$$\rho = \int p(\alpha)|\alpha\rangle\langle\alpha| d^2\alpha, \qquad (B)$$

with $0 \le \mathrm{p}(\alpha) \le 1$ as $\mathrm{p}(\alpha)$ is the probability associated with $|\alpha\rangle$.

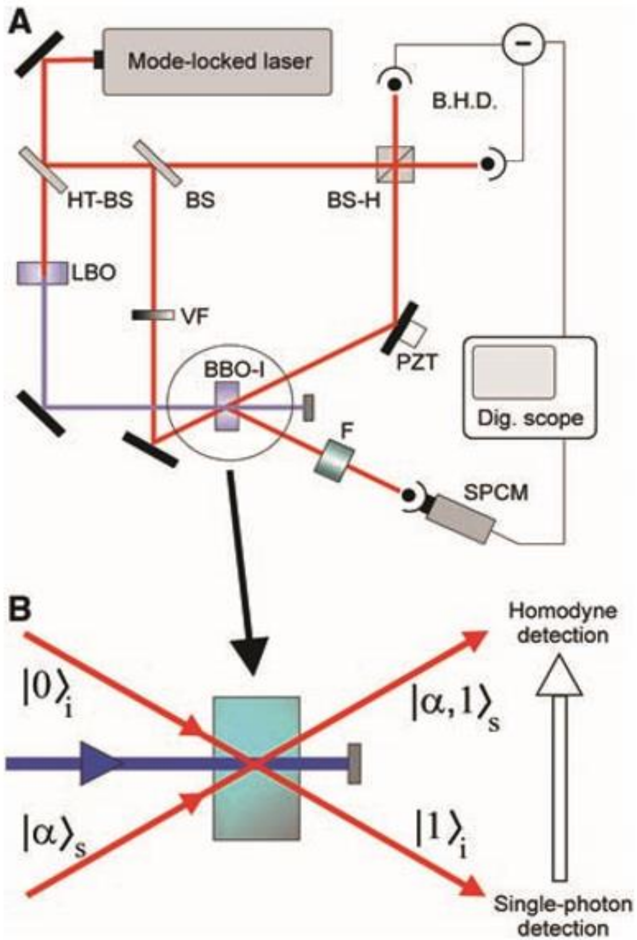
Compare (A) and (B), if $P(\alpha)$ is nonpositive then ρ in (A) is not a mixture of classical state and is called nonclassical.

Example, finite dimensional coherent states are nonclassical.



Father of nonclassicality

Photon addition is a non-Gauusianity inducing operation which is realized through a non-Hermitian operator



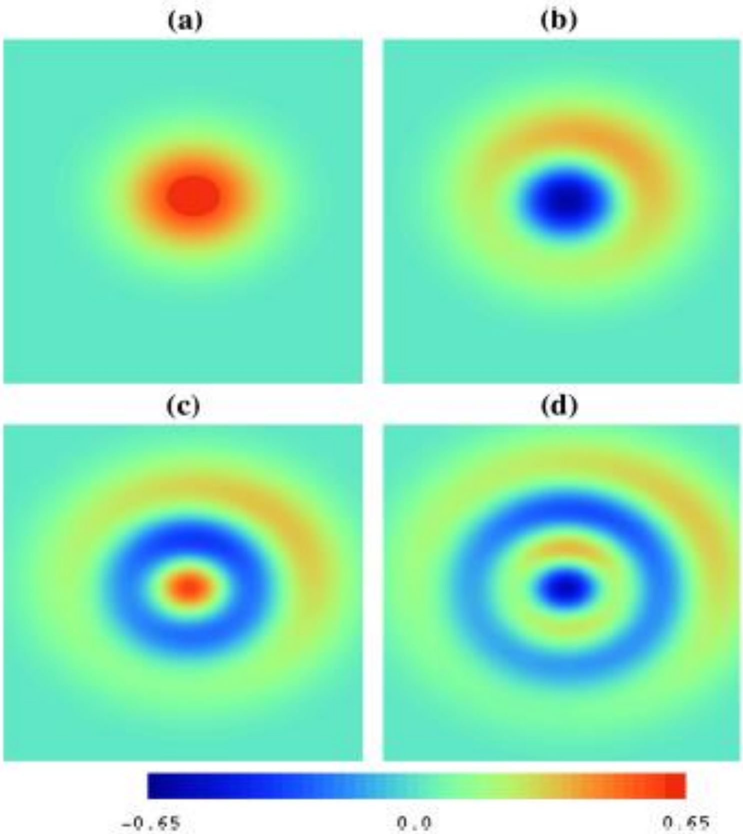
Alessandro Zavatta, et al. Science, **306,** 660 (2004)

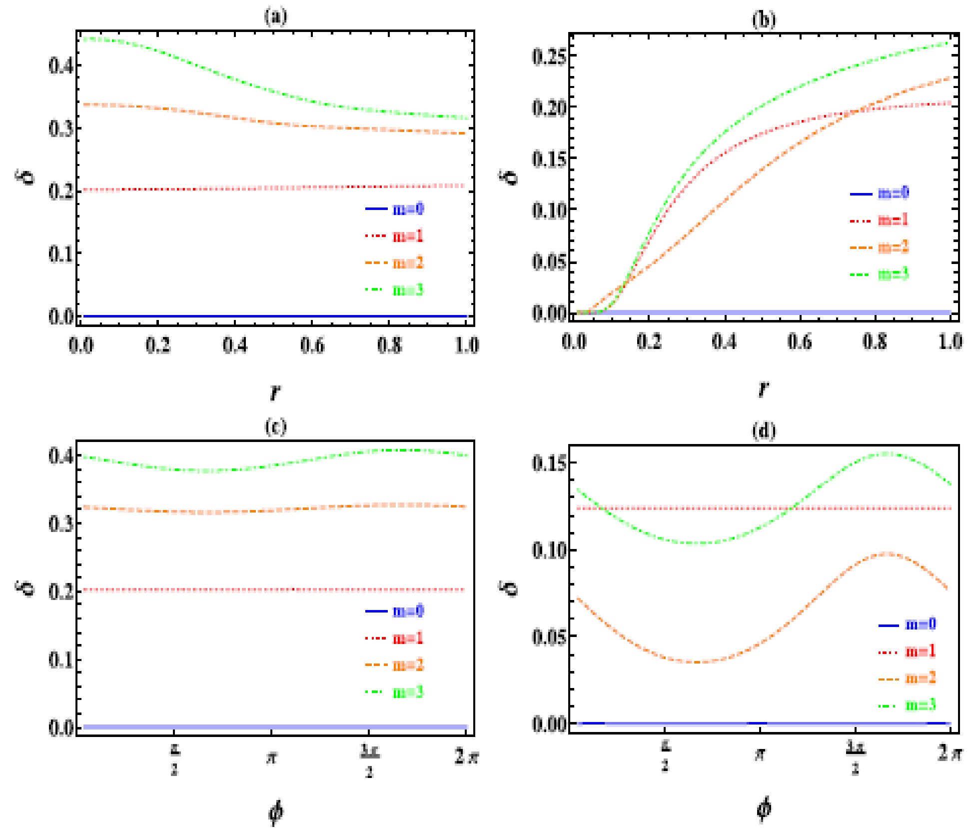
Effect of photon addition and subtraction: non-Hermitian operation can induce nonclassicality

Nonclassical volume is given by $\delta(|\psi\rangle) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |W_{\psi}(q,p)|\, dq\, dp - 1,$

where $W_{\psi}(q,p)$ is the Wigner function of the state $|\psi\rangle$.

Phys. Lett. A **381** (2017) 3178-3187





Why are we interested in nonclassical states? Classically impossible things may happen in the quantum world.

Is it related to non-Hermiticity?

We perform proof of principle experiment using IBM and fidelity was low: QINP 16 (2017) 292; PLA 381 (2017) 3860. QPT also show low gate fidelity arXiv:1805.07185

Cartoons used in this talk are from: Elements of Quantum Computation and Quantum Communication, **A Pathak**, CRC Press, Boca Raton, USA, (2013).

BCST: Fidelity under Amplitude damping and Phase damping noise is calculated as

$$
\begin{aligned}
F_{\mathrm{AD}} &= \frac{1}{16\left(2-4\eta_A+5\eta_A^2-4\eta_A^3+2\eta_A^4+\eta_A^2\cos2\theta_1\cos2\theta_2+\eta_A\left(2-3\eta_A+2\eta_A^2\right)(\cos2\theta_1+\cos2\theta_2)\right)} \\
&\times\Big[32-164\eta_A+57\eta_A^2-26\eta_A^3+10\eta_A^4+\eta_A\left(34-51\eta_A+30\eta_A^2\right)(\cos2\theta_1+\cos2\theta_2) \\
&+\eta_A^2\left(3-2\eta_A+2\eta_A^2\right)(\cos4\theta_1+\cos4\theta_2)+4\eta_A^3\left(3-2\eta_A+2\eta_A^2\right)(\cos2\theta_1\cos4\theta_2+\cos4\theta_1\cos2\theta_2) \\
&+16\eta_A^2\left(2-2\eta_A+\eta_A^2\right)\cos2\theta_1\cos2\theta_2+\eta_A^2\left(1-2\eta_A+2\eta_A^2\right)\cos4\theta_1\cos4\theta_2\Big].
\end{aligned}
$$

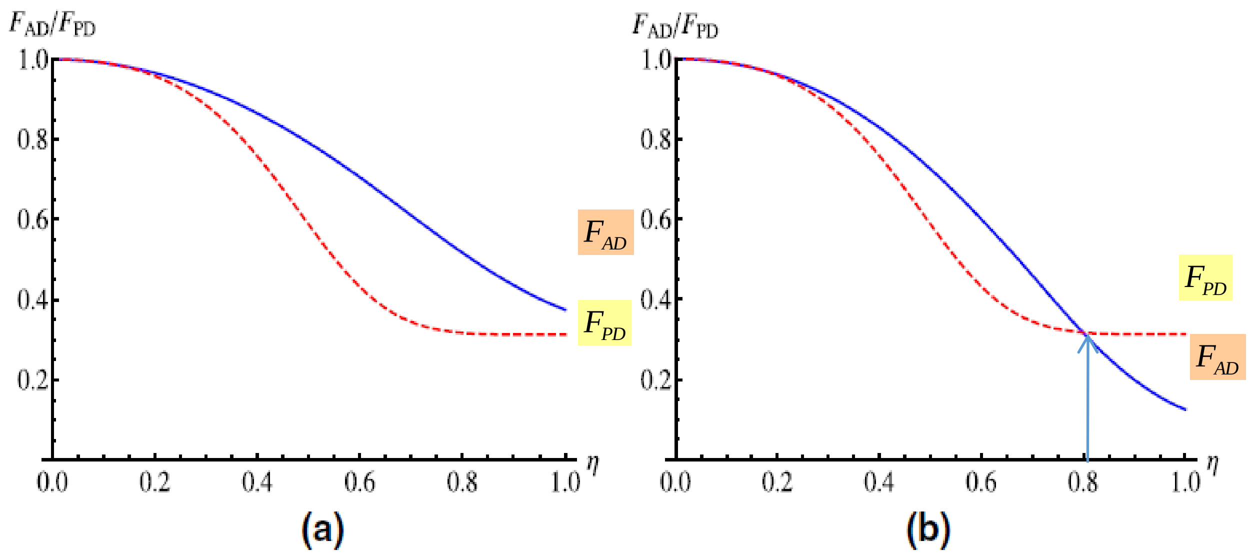
and

$$
F_{\mathrm{PD}} = \frac{32-128\eta_P+210\eta_P^2-164\eta_P^3+59\eta_P^4+\eta_P^2\left\{2-4\eta_P+3\eta_P^2\right\}(16\cos2\theta_1\cos2\theta_2+\cos4\theta_1\cos4\theta_2+3(\cos4\theta_1+\cos4\theta_2))}{16\left(2-8\eta_P+14\eta_P^2-12\eta_P^3+5\eta_P^4+\eta_P^2\left\{2-4\eta_P+3\eta_P^2\right\}\cos2\theta_1\cos2\theta_2\right)}.
$$

respectively. Here, for computational convenience, we have considered $a_i = Sin\theta_i,\ b_i = Cos\theta_i$, where $i \in \{1,2\}$.

We can observe that $F_{AD/PD}$ depend on the decoherence rate $\eta_{A/P}$ and amplitude information a_i, b_i and are free from phase .

K. Thapliyal and **A. Pathak**, Quant. Info. Process **14** (2015) 2599–2616.

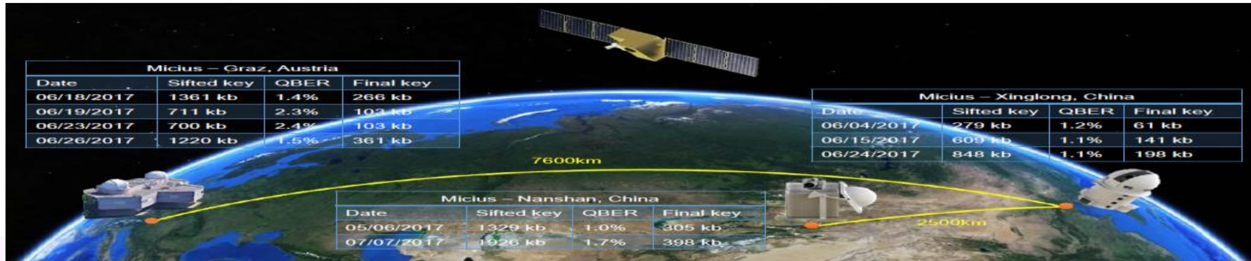


Comparison of the effect of amplitude-damping noise (solid line) with phase damping noise (dashed line) by assuming $\eta_A = \eta_P = \eta$ and (a) with $\theta_1 = \frac{\pi}{4}, \theta_2 = \frac{\pi}{6}$ (b) with $\theta_1 = \frac{\pi}{4}, \theta_2 = \frac{\pi}{3}$.

(a) $F_{AD} > F_{PD}$ for the same value of decoherence rate η. Whereas;

(b) $F_{AD} < F_{PD}$ for the same value of decoherence rate η after certain value of η, i.e., for $\eta > 0.8$

Why are we interested in nonclassical states? It can provide unconditional security.



Micius – Graz, Austria			
Date	Sifted key	QBER	Final key
06/18/2017	1361 kb	1.4%	266 kb
06/19/2017	711 kb	2.3%	103 kb
06/23/2017	700 kb	2.4%	103 kb
06/26/2017	1220 kb	1.5%	361 kb

Micius – Xinglong, China			
Date	Sifted key	QBER	Final key
06/04/2017	279 kb	1.2%	61 kb
06/15/2017	609 kb	1.1%	141 kb
06/24/2017	848 kb	1.1%	198 kb

Micius – Nanshan, China			
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05/06/2017	1329 kb	1.0%	305 kb
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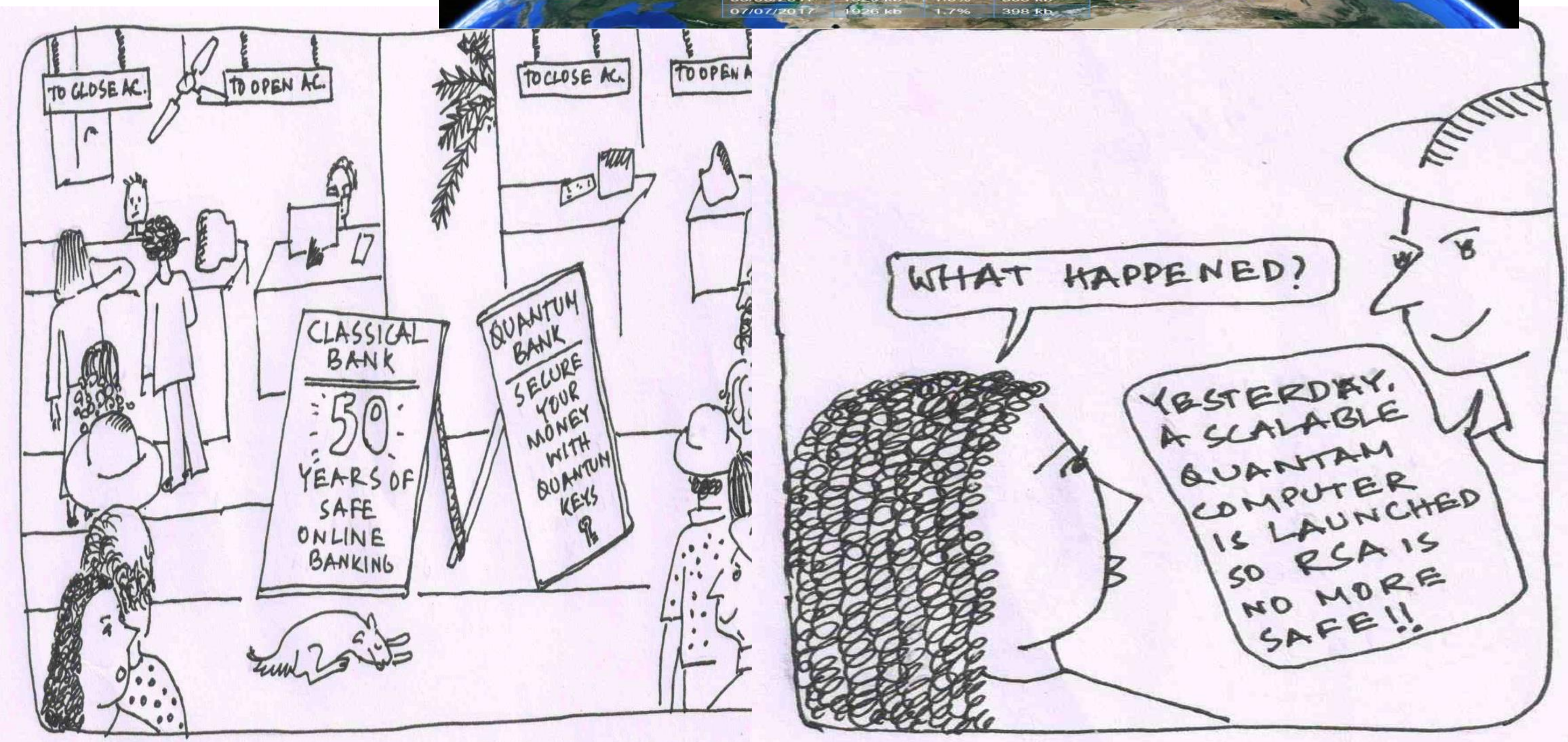


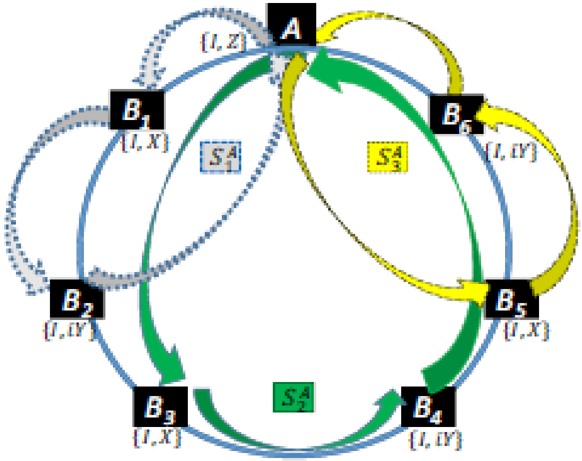
Why can’t we built a scalable quantum computer? Is there any connection with non-Hermitian physics? Investigations are done using Master equation & Kraus operators

Which concepts are connected to non-Hermiticity?

Teleportation QINP 16, 76 (2017) & QINP 16, 292 (2017) & Controlled telepor-tation QINP 14, 2599 (2015) & QINP 14, 4601 (2015)

Hierarchical quantum communication QINP 16, 205 (2017)

Direct secure quantum communication QINP 16, 115 (2017) & Asymmetric quantum dialogue QINP 16, 49 (2017)



Quantum voting IJQI 15, 1750007 (2017) & Decoy qubits QINP 15, 1703 (2016) & QINP 15, 4681 (2016)

Quantum key distribution arxiv:1609.07473v1 (2016) & Quantum conference arxiv:1702.00389v1 (2017) & Quantum e-commerce QINP 16, 295 (2017)

Controlled direct secure quantum communication QINP 16, 115 (2017)

Quantum sealed bid auc-tion QINP 16, 169 (2017)

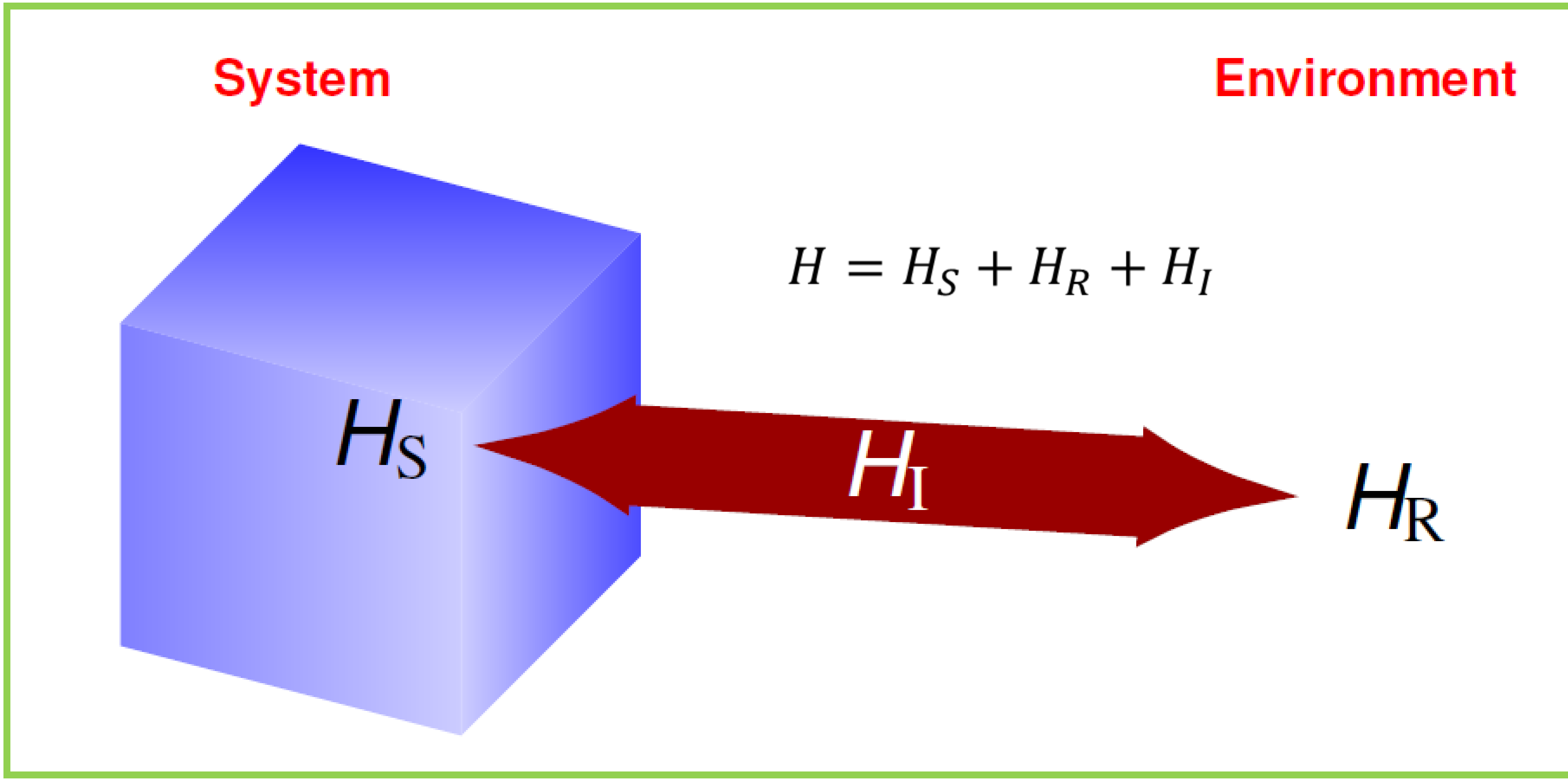
Quantum private compar-ison arxiv:1608.00101v1 (2016)

Optically implementable MDI-DSQC

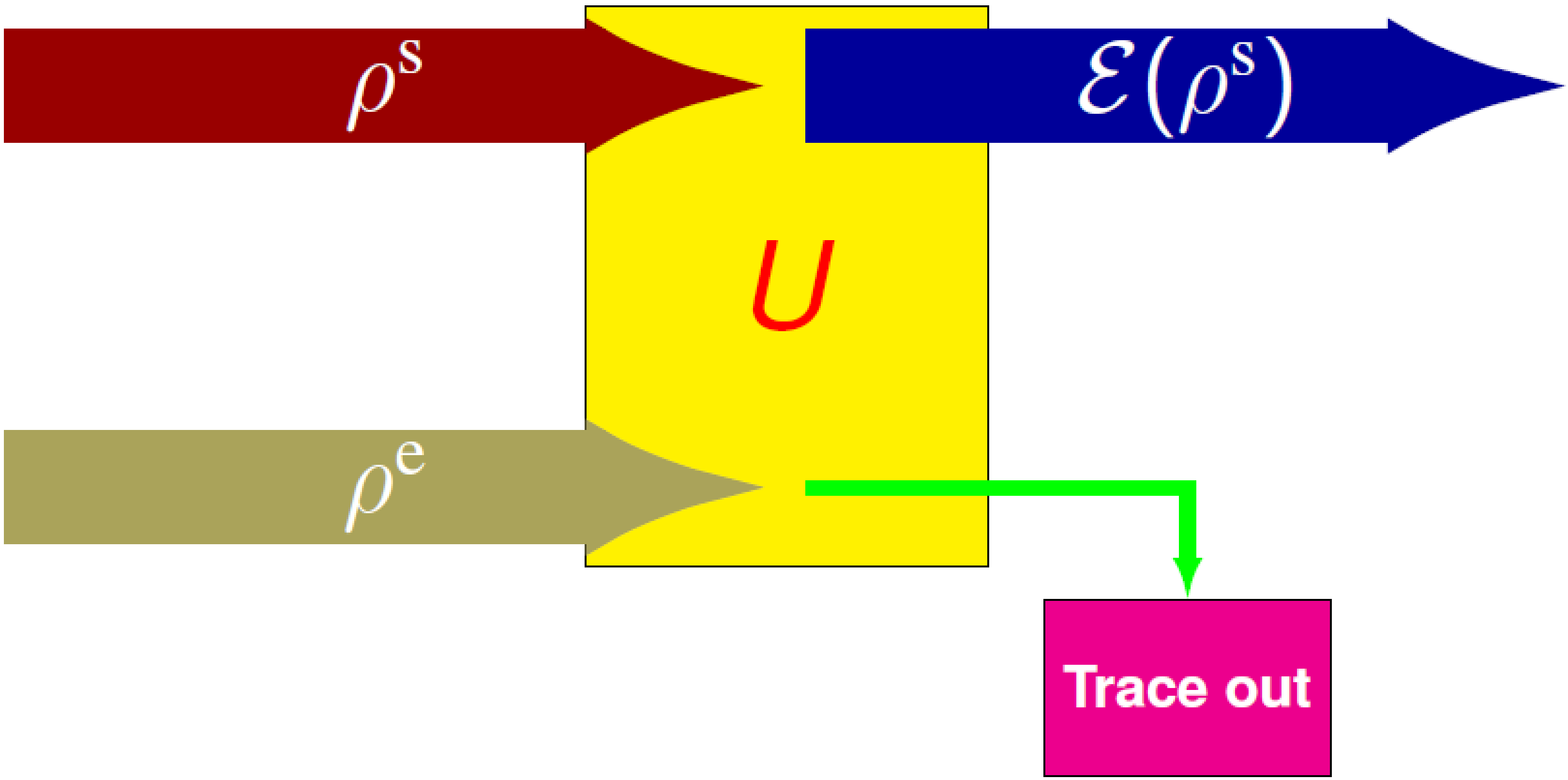
15 / 20

Entangled & nonclassical states, PRA, 93 (2016) 022107, 93 (2016) 012340, 91 (2015) 042309, 90 (2014) 013808, 89 (2014) 033812, 89 (2014) 033628, 87 (2013) 022325, Ann. Phys. 366 (2016) 148, 362 (2015) 261

Environment matters: How we model it?



Here H_S is the system Hamiltonian, H_I is the system-reservoir interaction Hamiltonian and H_R is the reservoir Hamiltonian.



Evolution of the system-bath combination is unitary and is given by Liouville-von Neumann equation as $\dot{\rho}(t) = -i[H, \rho(t)]$, where $\rho = \rho^S \otimes \rho^{\mathrm{e}}$ is the quantum state in combined Hilbert space $H^S \otimes \mathrm{H}^{\mathrm{e}}$.

Tracing over the environment degrees of freedom, one can obtain $\dot{\rho}^S(t) = \mathcal{L}[\rho^S(t)]$, where $\mathcal{L}$ is the superoperator acting on the system state.

In operator-sum (or Kraus representation), a superoperator $\mathcal{E}$ acting on a system due to interaction with ambient environment is given by $\rho \rightarrow \mathcal{E}(\rho) = \sum_k \langle e_k | U(\rho \otimes |f_0\rangle\langle f_0|) U^\dagger | e_k \rangle = \sum_j E_j \rho E_j^\dagger$, where U is the unitary operator for free evolution of system, reservoir and interaction between them. Here, $|f_0\rangle$ is the environment's initial state, and $\{|e_k\rangle\}$ is a basis of enviroment.
This gives $E_j = \langle e_k | U | f_0 \rangle$ are the Kraus operators satisfying completeness condition $E_j^\dagger E_j = \mathbb{I}$.

The construction of most general form of generator $\mathcal{L}$ leads to the Lindblad equation.

Writing Lindblad form of master equation following assumptions are involved:

1. Born approximation: Weak coupling between system (S) and reservoir (R).
2. Markov approximation: Memoryless (when the time scale associated with the reservoir correlations is much smaller than the time scale over which the state varies appreciably, which is easily justified for weak S − R coupling and high T).

3. Rotating wave approximation: Fast system dynamics compared to the relaxation time.

Non-Markovian channels

Typically, this is due to the fact that the relevant environmental correlation times are not small compared to the

system's relaxation or decoherence time, rendering the

standard Markov approximation impossible.

The violation of this separation of time scales can occur, for example, in the cases of strong system-environment couplings, structured or finite reservoirs, low temperatures, or large initial system environment correlations.

Markovian channels

Examples of Kraus operators

Type of noise model	Kraus operators
Amplitude damping	
$H_I = a^\dagger b + a b^\dagger$	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix}.$
Phase damping	
$H_I = \chi a^\dagger a (b + b^\dagger)$	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix}.$

Here, *a* is system mode and *b* is reservoir mode.

Turchette et al., Phys. Rev. A **62**, 053807 (2000); M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (2008)

Markovian channels

Generalized amplitude damping (GAD)

$$E_0 = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta} \end{bmatrix}, \quad E_1 = \sqrt{p}\begin{bmatrix} 0 & \sqrt{\eta} \\ 0 & 0 \end{bmatrix},$$

$$E_2 = \sqrt{1-p}\begin{bmatrix} \sqrt{1-\eta} & 0 \\ 0 & 1 \end{bmatrix}, \quad E_3 = \sqrt{1-p}\begin{bmatrix} 0 & 0 \\ \sqrt{\eta} & 0 \end{bmatrix}.$$

These are generalization of AD to thermal and squeezed thermal reservoir.

Squeezed generalized amplitude damping (SGAD)

$$E_0 = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta} \end{bmatrix}, \quad E_1 = \sqrt{p}\begin{bmatrix} 0 & \sqrt{\eta} \\ 0 & 0 \end{bmatrix},$$

$$E_2 = \sqrt{1-p}\begin{bmatrix} \sqrt{1-\nu} & 0 \\ 0 & \sqrt{1-\mu} \end{bmatrix},$$

$$E_3 = \sqrt{1-p}\begin{bmatrix} 0 & \sqrt{\mu}e^{-i\xi} \\ \sqrt{\nu} & 0 \end{bmatrix}.$$

Markovian channels

Bit flip

$$E_0 = \sqrt{1-p}\,I_2, \quad E_1 = \sqrt{p}\,X.$$

Phase flip

$$E_0 = \sqrt{1-p}\,I_2, \quad E_1 = \sqrt{p}\,Z.$$

Depolarizing channel

$$E_0 = \sqrt{1-p}\,I_2, \quad E_1 = \sqrt{\frac{p}{3}}X,$$

$$E_1 = \sqrt{\frac{p}{3}}Y, \quad E_1 = \sqrt{\frac{p}{3}}Z.$$

Some collective noises

Collective rotation

$$U_r = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Collective dephasing

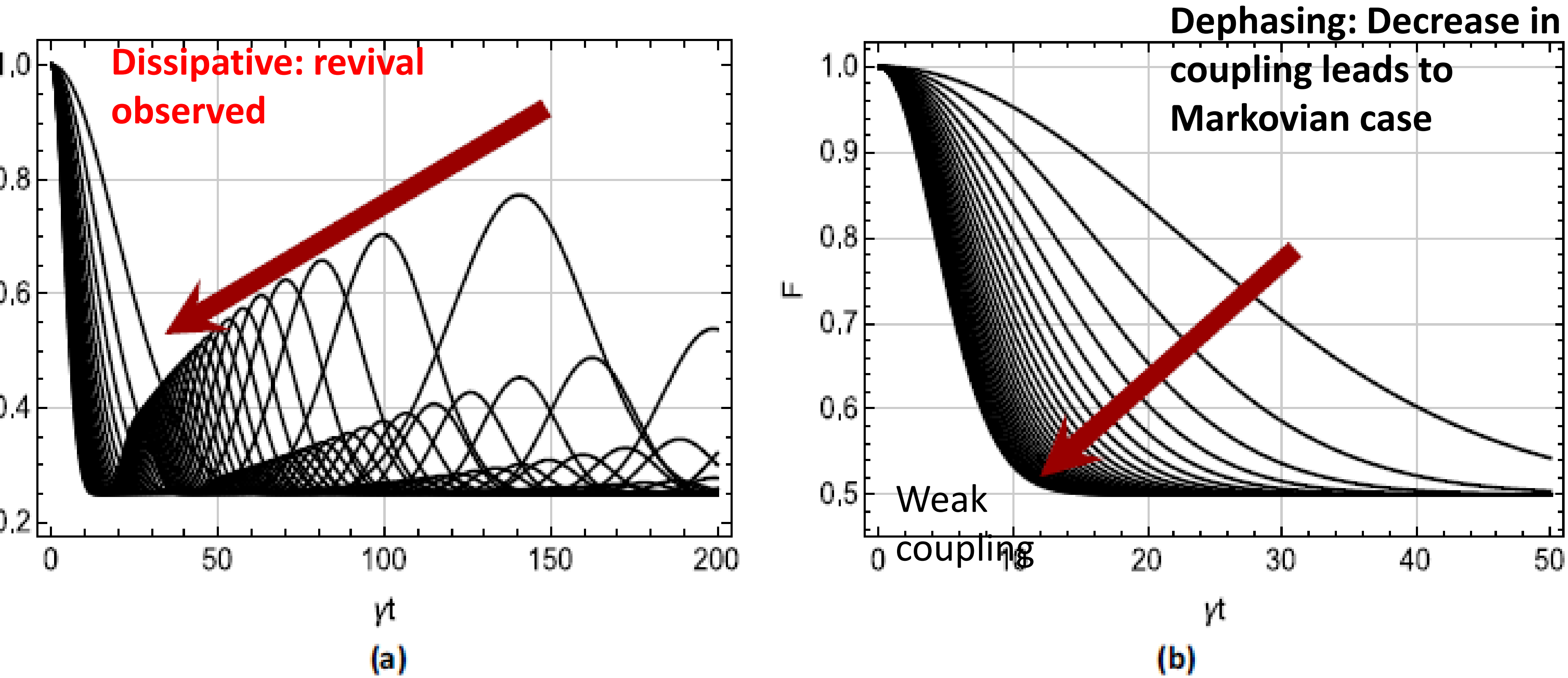
$$U_p = \begin{bmatrix} 1 & 0 \\ 0 & \exp(i\phi) \end{bmatrix}.$$

AD vs PD channels

Consider an arbitrary density matrix $\rho = \begin{bmatrix} a & b \\ b^* & c \end{bmatrix}$ evolving under AD channel becomes

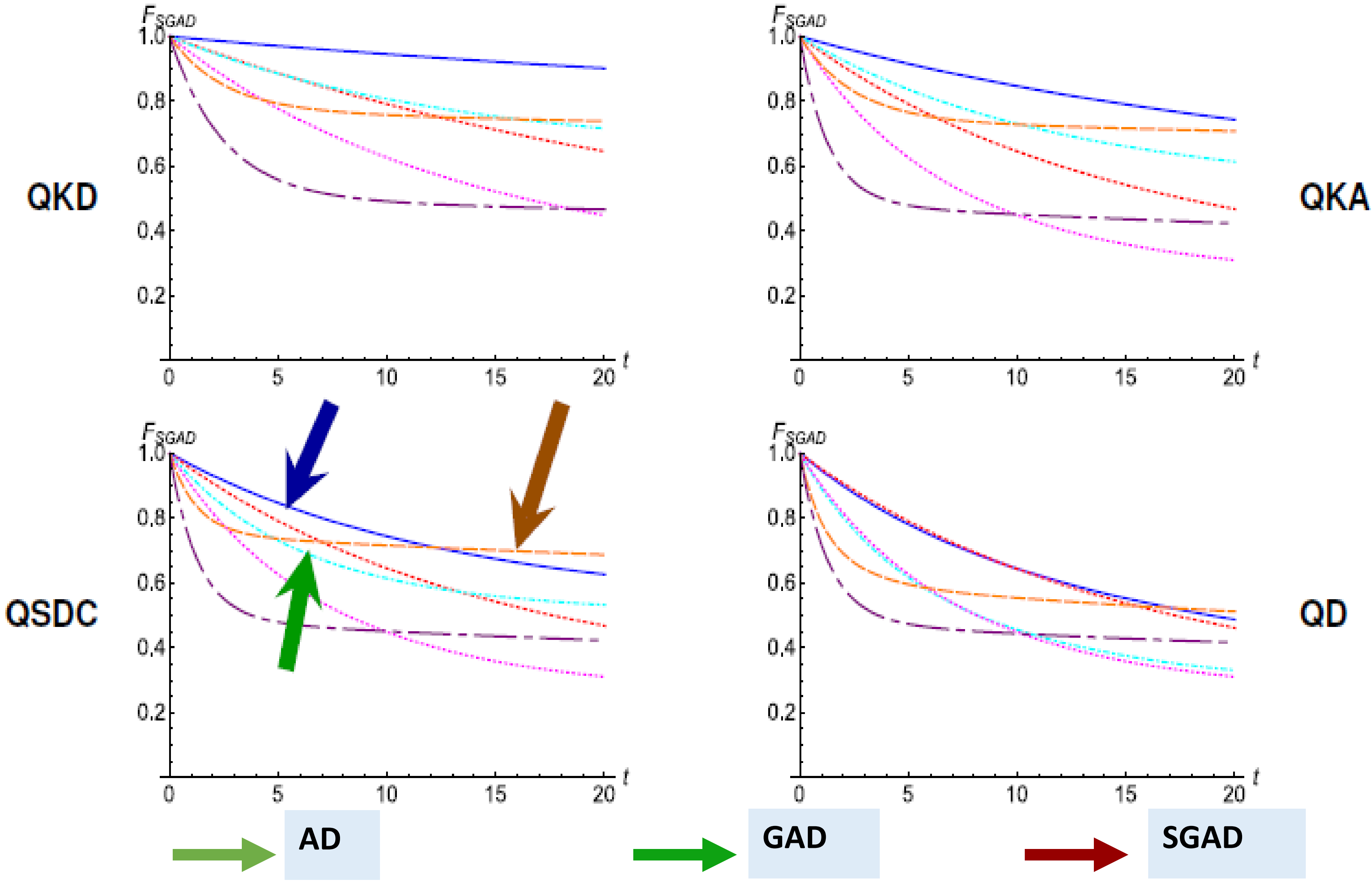
$$\rho' = \begin{bmatrix} a + pc & b\sqrt{1-p} \\ b^*\sqrt{1-p} & (1-p)c \end{bmatrix}.$$

Similarly, the state evolving under PD noise becomes $\rho' = \begin{bmatrix} a & b\sqrt{1-p} \\ b^*\sqrt{1-p} & c \end{bmatrix}.$



The effect of a change in the coupling strength on the fidelity is illustrated here with a set of plots for damping and dephasing non-Markovian noise in (a) and (b), respectively. Specifically, the parameter of the coupling strength Γ/γ varies from 0.001 to 0.03 in steps of 0.001 in both the plots.

K. Thapliyal, **A. Pathak,** S. Banerjee, Quant. Infor. Process. 16, 115 (2017)



V. Sharma, K. Thapliyal, **A. Pathak,** S. Banerjee, Quant. Infor. Process. 15 (2016) 4681.

Non-Hermitian Physics

Pre-Bender-paper era:

1. There was no confusion. Hardly any attention.
2. Gamow on α-decay: The tunnelling rate with which a particle can escape the nucleus can be effectively described through a complex energy eigenvalue.
3. complex (non-Hermitian) potentials were introduced by Feshbach, Porter and Weisskopf, to model scattering interactions between neutrons and nuclei2.
4. Nuclear Physics
5. Quantum Optics

Post-Bender-paper era:

1. Much attention has been received, and much confusion has been created.
2. Complex potentials have been studied
3. Non-Hermitian physics has been studied in the context of optics, optomechnics, thermodynamics and quantum mechanics

Bender paper: Bender and Boettcher. "Real spectra in non-Hermitian Hamiltonians having P T symmetry." PRL **80** (1998): 5243.

My first exposure to non-Hermitian Physics was in Pre-Bender era

- I was doing my MSc. Dissertation and from my BSc. quantum mechanics class I learned & believed that IN QUANTUM MECHANICS EACH PHYSICAL OBSERVABLE IS REPRESENTED BY A HERMITIAN OPERATOR.
- Somehow I learned about the quantum phase problem and I thought others have not tried it hard and if I try hard I'll be able to write a Hermitian phase operator (in the infinite dimensional Hilbert space). I tried hard, but failed!
- Do you know why? Eigen energies of a Harmonic oscillator is bounded from below! This is physical, this questions the validity of the idea that EACH PHYSICAL OBSERVABLE IS REPRESENTED BY A HERMITIAN OPERATOR.

Quantum phase problem in brief

P. A. M. Dirac in 1927 first time introduces quantum phase operator, with the assumption that the annihilation operator a can be factored out into a Hermitian function $f(n)$ of the number operator n and an operator U which define a Hermitian phase operator.

$$a = e^{i\emptyset}\sqrt{N} \quad a^\dagger = \sqrt{N}e^{-i\emptyset}$$

$$aa^\dagger = e^{i\emptyset}Ne^{-i\emptyset}$$

$$a^\dagger a = \sqrt{N}e^{-i\emptyset}e^{i\emptyset}\sqrt{N} = N$$

$$\begin{aligned}[a, a^\dagger] &= aa^\dagger - a^\dagger a \\ &= e^{i\emptyset}Ne^{-i\emptyset} - N \\ &= \left(e^{i\emptyset}N - Ne^{i\emptyset}\right)e^{-i\emptyset}\end{aligned}$$

$$U = e^{i\emptyset}$$

Quantum phase problem in brief

In Dirac formalism the annihilation and creation operator satisfy the usual Commutation relation

$$[\boldsymbol{a}, \boldsymbol{a}^{\dagger}] = \mathbf{1}$$

only if $\boldsymbol{e^{i\emptyset}N - Ne^{i\emptyset} = e^{i\emptyset}}$ i.e. the commutation relation $[n, \emptyset] = i$ is satisfied. Immediately it was shown that there is problem with uncertainty relation

$$\Delta \boldsymbol{n} \Delta \emptyset \geq \frac{\mathbf{1}}{\mathbf{2}}$$

For example, uncertainty in $\emptyset$ to be greater than 2π.

Non-Hermiticity and quantum phase problem: A paper that I read in pre-Bender era

ANNALS OF PHYSICS 101, 319–341 (1976)

Who is Afraid of Nonhermitian Operators? A Quantum Description of Angle and Phase

JEAN-MARC LÉVY-LEBLOND*

Laboratoire de Physique Théorique et Hautes Energies, Université Paris VII

Received March 16, 1976

The physical characteristics of a quantum property consist essentially of its eigenvalues and eigenstates. As a consequence, hermitian operators are shown to define an unduly restricted framework for the theoretical description of quantum properties; **nonhermitian operators, for instance unitary, but also nonnormal ones, may be acceptable as well if the projectors onto their eigenstates allow for a resolution of the identity operator, so as to preserve the probabilistic interpretation of the Hilbert space formalism**.

2nd Merger of Quantum Optics and Non-Hermitian Physics happened in last 2-3 years

How to look at nonclassicality: moment based criteria used by us

Squeezing : $(\Delta X_i)^2 < \frac{1}{4}$, where $X_1 = \frac{1}{2}\left[\left(a + a^\dagger\right)\right]$, $X_2 = \frac{1}{2i}\left[\left(a - a^\dagger\right)\right]$

Antibunching : $d_a^{(n-1)} = \left\langle a^{\dagger n} a^n \right\rangle - \left\langle a^\dagger a \right\rangle^n < 0.$

Amplitude powered Squeezing : $A_{i,a} = \left\langle \left(\Delta Y_{i,a}\right)^2 \right\rangle - \frac{1}{2}\left|\left\langle \left[Y_{1,a}, Y_{2,a}\right]\right\rangle\right| < 0,$

where $Y_{1,a} = \frac{1}{2}\left[\left(a^k + a^{\dagger k}\right)\right]$, $Y_{2,a} = \frac{1}{2i}\left[\left(a^k - a^{\dagger k}\right)\right]$

Duan criteria of entanglement : $\left\langle (\Delta u)^2 \right\rangle + \left\langle (\Delta v)^2 \right\rangle - 2 < 0,$

where $\Delta u = \frac{1}{\sqrt{2}}\left[\left(a + a^\dagger\right) + \left(b + b^\dagger\right)\right]$,

$\Delta v = \frac{1}{i\sqrt{2}}\left[\left(a - a^\dagger\right) + \left(b - b^\dagger\right)\right]$

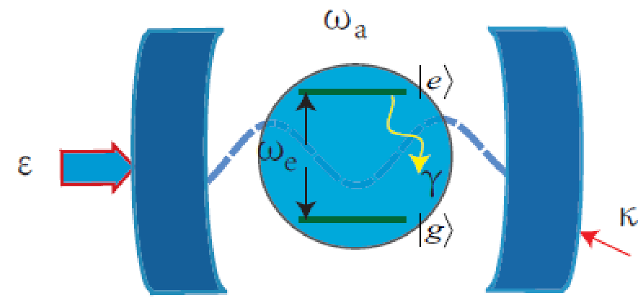
Entanglement : $E_{ab}^{m,n} = \left\langle a^{\dagger m} a^m b^{\dagger n} b^n \right\rangle - \left|\left\langle a^m b_1^{\dagger n} \right\rangle\right|^2 < 0,$

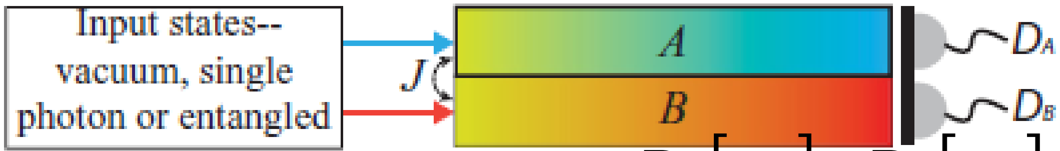
$$E_{ab}^{'m,n} = \left\langle a^{\dagger m} a^m \right\rangle \left\langle b^{\dagger n} b^n \right\rangle - \left|\left\langle a^m b_1^n \right\rangle\right|^2 < 0.$$

Usually we use Sen-Mandal approach to obtain time evolution of field operators

We also use negative values of Wigner function, zeroes of Q and characteristic function based criteria of Perina.

Some recent results connecting nonclassicality or quantum optics and non-Hermitian Hamiltonian





A (B) is a gain (absorbing) medium

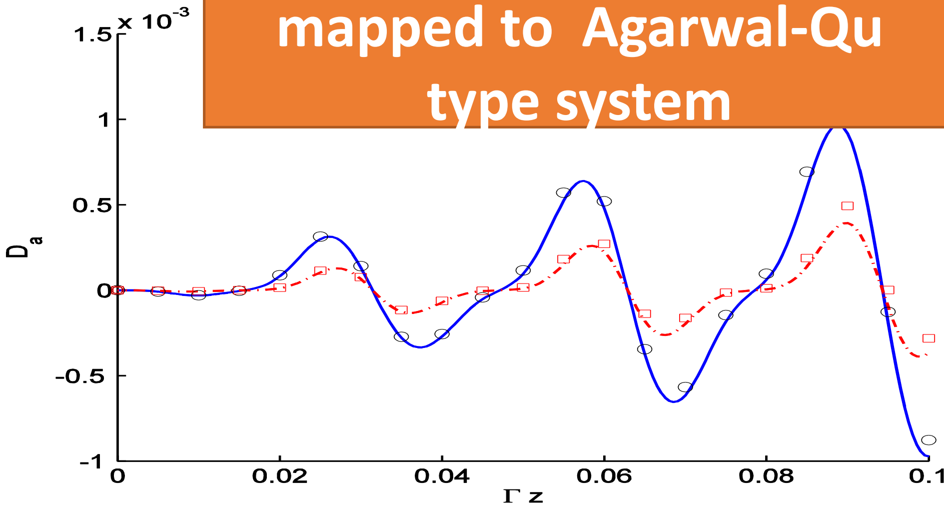
$$\mathrm{Re}[\chi_A]=\mathrm{Re}[\chi_B], \mathrm{Im}[\chi_A]=-\mathrm{Im}[\chi_B]; n^2=1+4\pi\chi$$

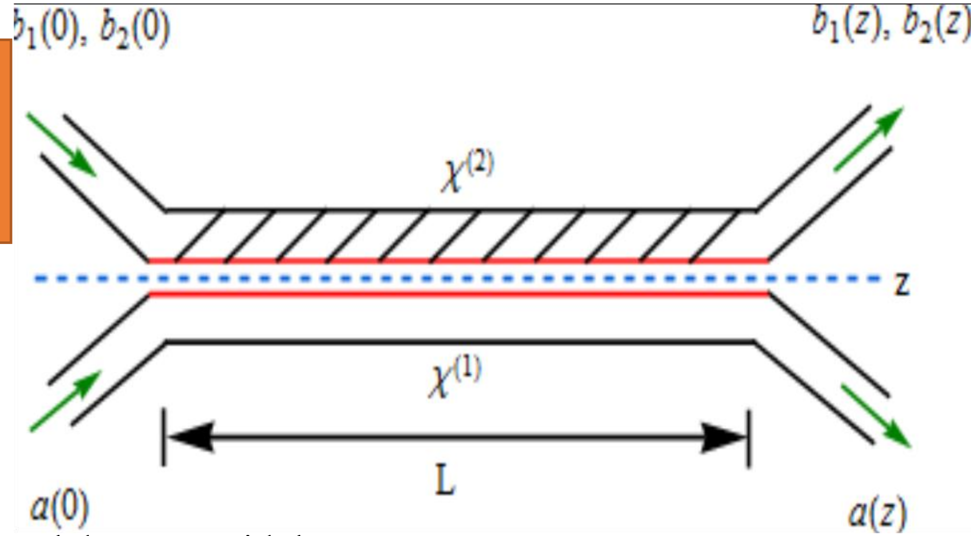
- Recently various bosonic Haniltonian has been investigated using the procedures of conventional quantum optics.
    - Full Hamiltonian is written in rotating frame
    - It's Langevin equation is written and transformed to semiclassical Langevin equation by neglecting quantum noise.
    - An effective Hamiltonian is found whose Heisenberg's equation is the same as the Langevin equation obtained above.
- Antibunching (Zhou et al., PRA **97**, 043819 (2018)) photon blockade induced by a non-Hermitian Hamiltonian with a gain cavity and intermodal antibunching in χ_1-χ_1 waveguide (Agarwal and Qu, PRA **85**, 031802(R) (2012)) are reported
- Entanglement and other quantum correlations are reported for two qubits trapped in two spatially separated cavities connected by an optical fiber (Mohamed, QINP **17** (2018) 96)

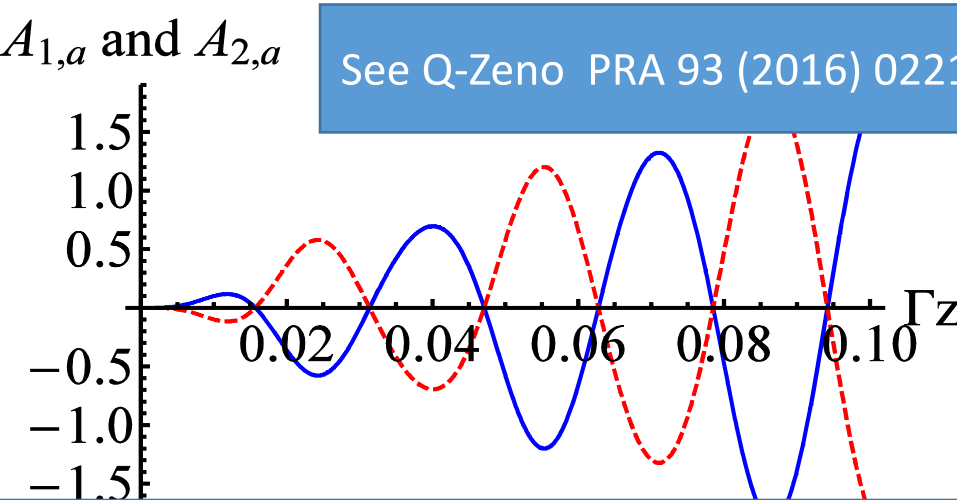
K. Thapliyal, A. Pathak, B. Sen, and J. Perina, PRA 90 (2014) 013808.

$$G_{\text{int}} = -\hbar k a b_1^{\dagger} - \hbar \Gamma b_1^2 b_2^{\dagger} \exp(i\Delta k z) + \text{H.c.}$$

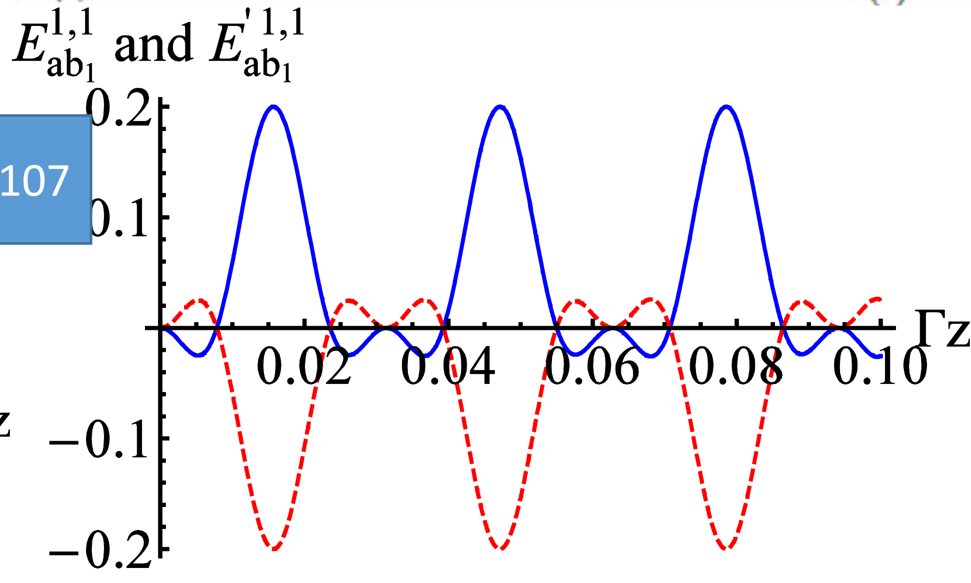
These results can be mapped to Agarwal-Qu type system







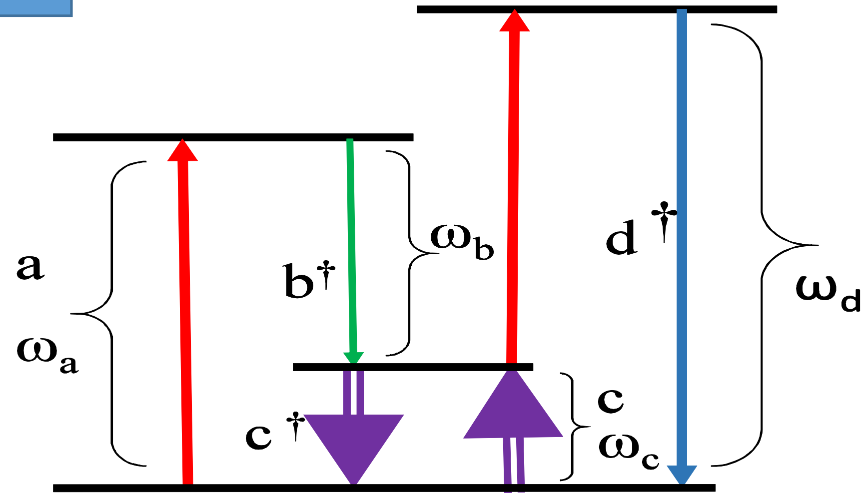
See Q-Zeno PRA 93 (2016) 022107

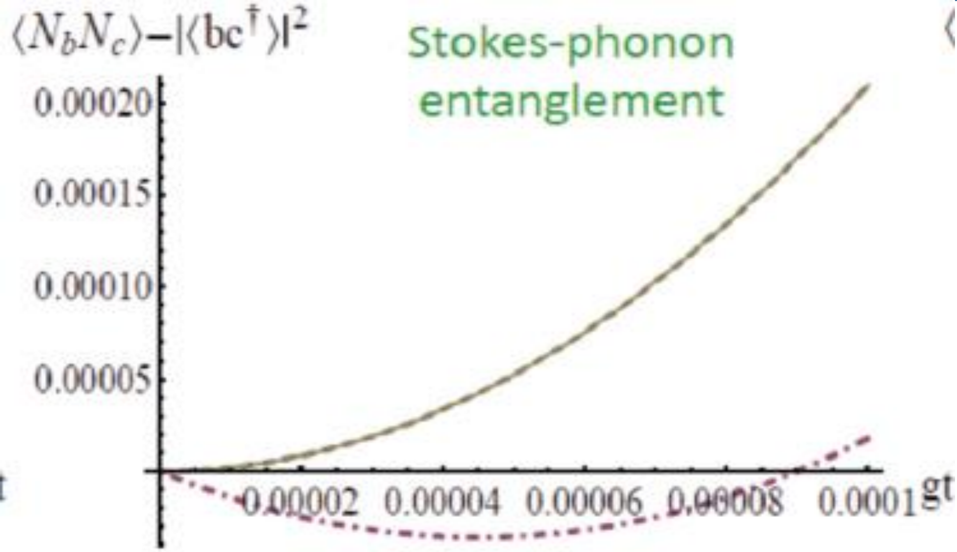


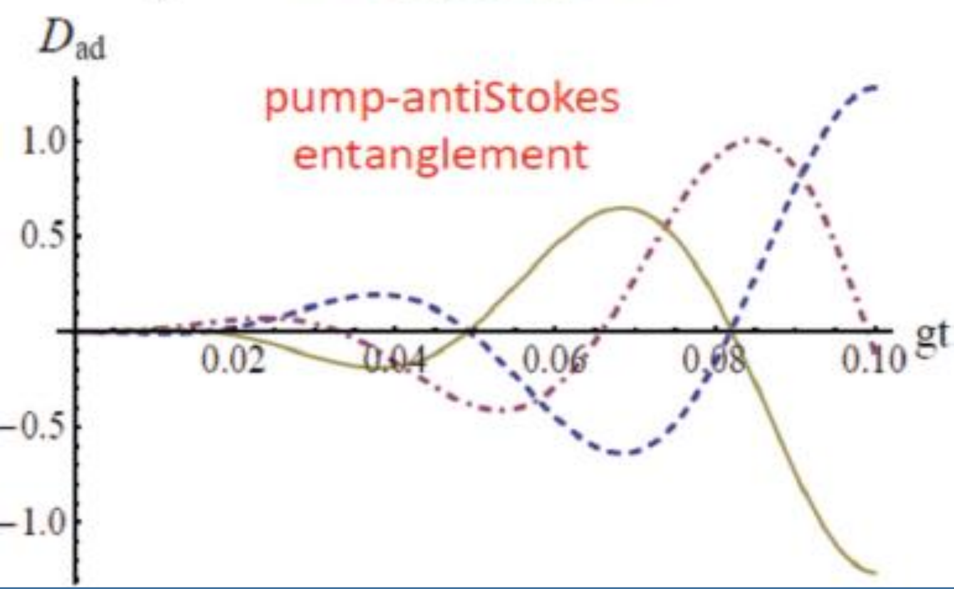
Similar result for contradirectional case in: K. Thapliyal, A. Pathak, B. Sen, and J. Perina, PLA 378 (2014) 3431.

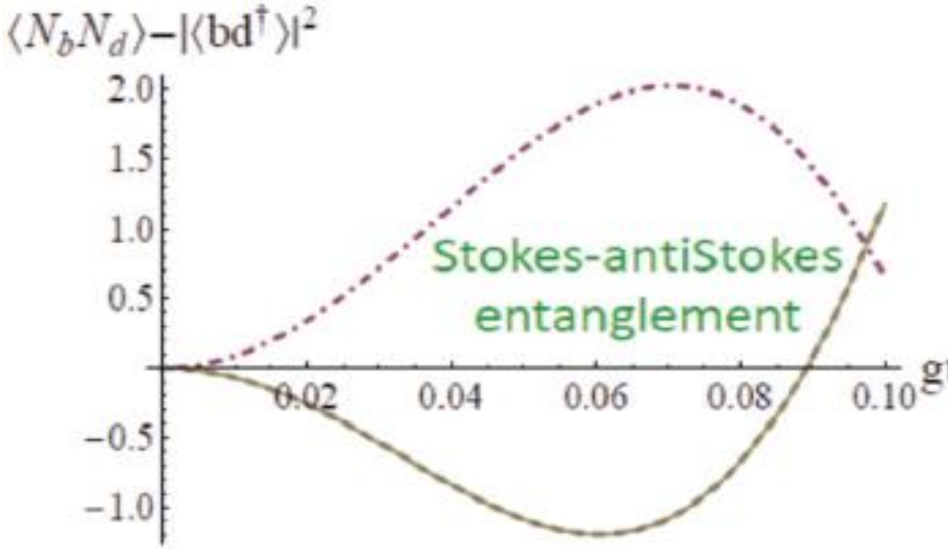
B. Sen, S. K. Giri, S. Mandal, C. H. R. Ooi, and A. Pathak, PRA **87** (2013) 022325.

$$H=\omega_a a^\dagger a+\omega_b b^\dagger b+\omega_c c^\dagger c+\omega_d d^\dagger d+g\left(ab^\dagger c^\dagger+\text{H.c.}\right)+\chi\left(acd^\dagger+\text{H.c.}\right)$$





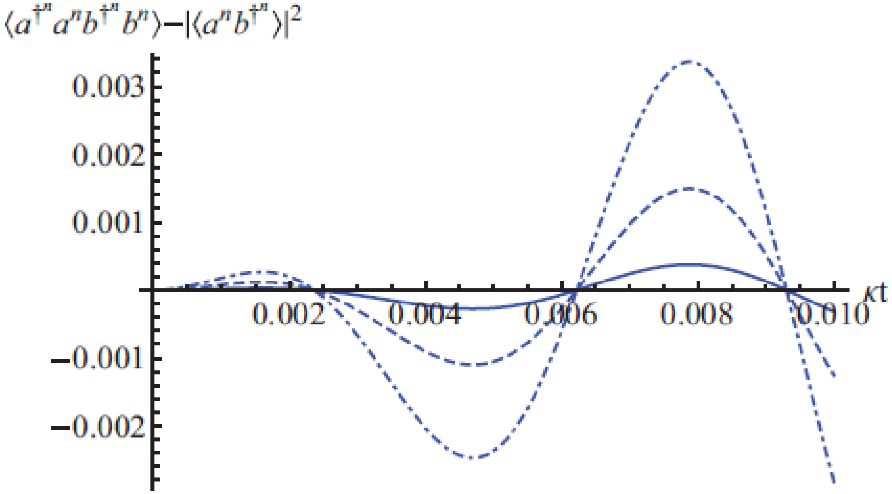


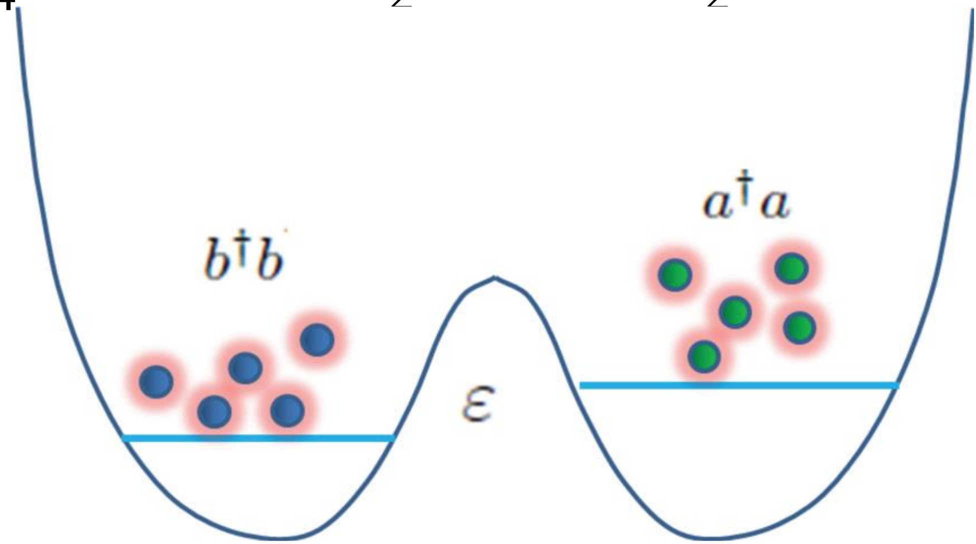


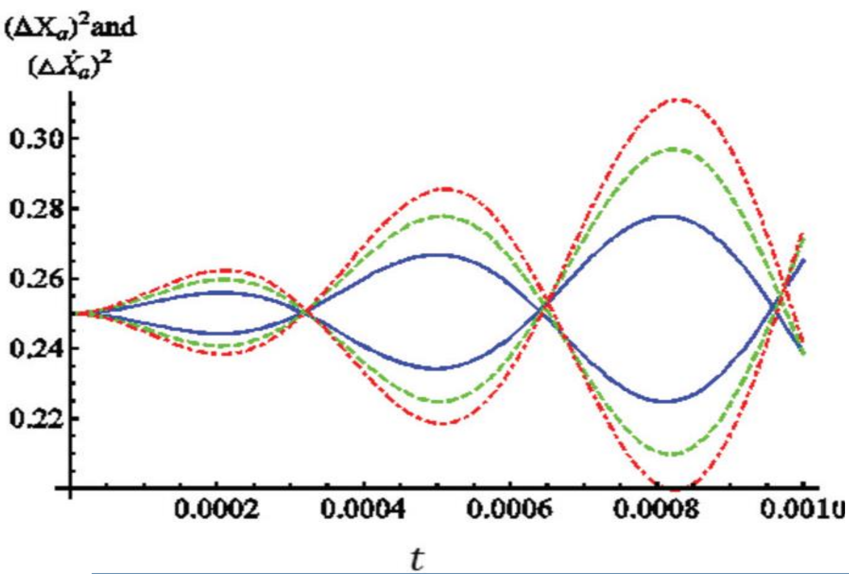
Similar result in: A. Pathak, J. Krepelka and J. Perina, Phys. Lett. A 377 (2013) 2692

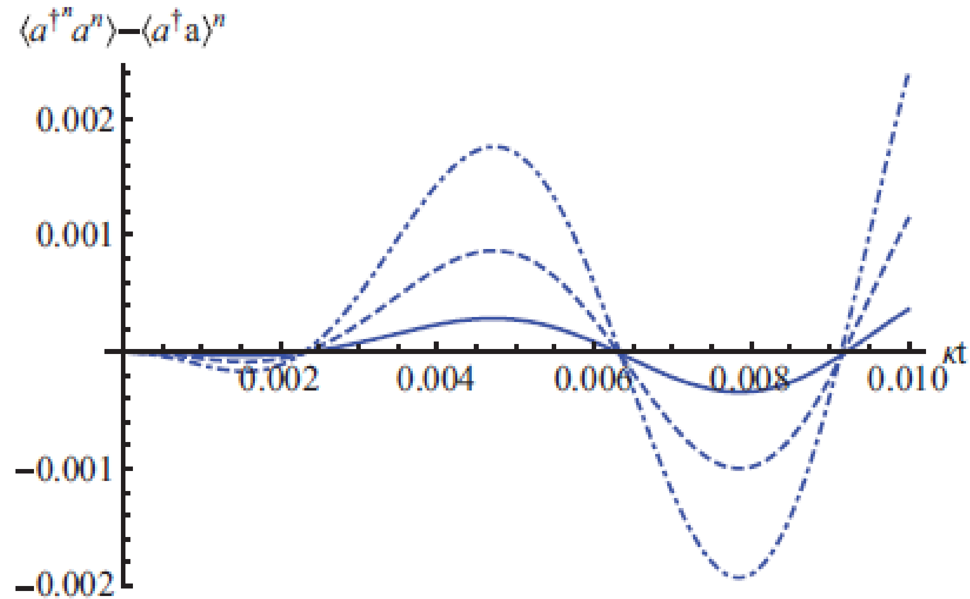
S. K. Giri, B. Sen, C. H. R. Ooi, and A. Pathak, PRA **89** (2014) 033628.

$$H = \frac{\kappa}{4}\left(a^{\dagger 2}a^{2} + b^{\dagger 2}b^{2}\right) + \frac{\Delta\mu}{2}\left(a^{\dagger}a - b^{\dagger}b\right) - \frac{\varepsilon}{2}\left(a^{\dagger}b - b^{\dagger}a\right).$$









This results are for atom-atom BEC. Similar result for atom-molecule BEC in: S. K. Giri, K. Thapliyal, B. Sen, and A. Pathak arXiv:1407

$\mathcal{PT}$ symmetry and bosonic operators

The notion of "parity-time symmetry" in optical and quantum systems starts with the action of parity ($\mathcal{P}$) and the time reversal operator ($\mathcal{T}$):

$$\mathcal{P}: \ (\boldsymbol{i}, \boldsymbol{x}, \boldsymbol{p}) \longrightarrow (\boldsymbol{i}, -\boldsymbol{x}, -\boldsymbol{p})$$
$$\mathcal{T}: (\boldsymbol{i}, \boldsymbol{x}, \boldsymbol{p}) \longrightarrow (-\boldsymbol{i}, \boldsymbol{x}, -\boldsymbol{p})$$

where x and p represent position and momentum operators, respectively and i is the imaginary unit.

- The operator $\mathcal{P}$ is *linear* and the operator $\mathcal{T}$ is *antilinear* [1].
- The operator $\mathcal{T}$ changes the sign of i to preserve the fundamental commutation relation $[x, p] = i$ of the dynamical variables in quantum mechanics.

$\mathcal{PT}$ operations for single mode bosonic fields operators

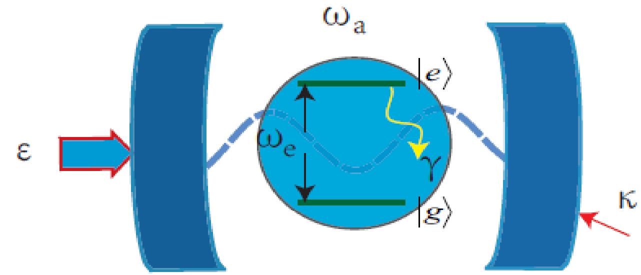
$\mathcal{P}$: $\quad \boldsymbol{a} \leftrightarrow -\boldsymbol{a}, \boldsymbol{a}^{\dagger} \leftrightarrow -\boldsymbol{a}^{\dagger}$

$\mathcal{T}$: $\quad \boldsymbol{a} \leftrightarrow \boldsymbol{a}, \boldsymbol{a}^{\dagger} \leftrightarrow \boldsymbol{a}^{\dagger}$

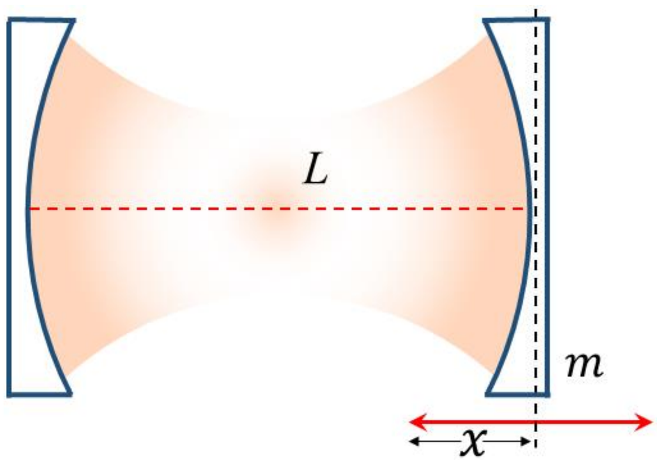
$$\boldsymbol{a}^{\dagger} = \sqrt{\mathbf{1}/\mathbf{2}\boldsymbol{m}\hbar\omega}(-\boldsymbol{i}\boldsymbol{p} + \boldsymbol{m}\omega\boldsymbol{x})$$
$$\boldsymbol{a} = \sqrt{\mathbf{1}/\mathbf{2}\boldsymbol{m}\hbar\omega}(\boldsymbol{i}\boldsymbol{p} + \boldsymbol{m}\omega\boldsymbol{x})$$

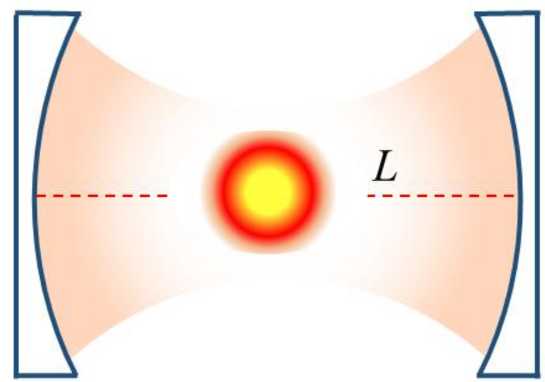
1. Carl M. Bender, arXiv: 0501052v1

The optomechanical system



Compare Zhou et al., and our sustem





The system Hamiltonian of optomechanical system with movable mirror

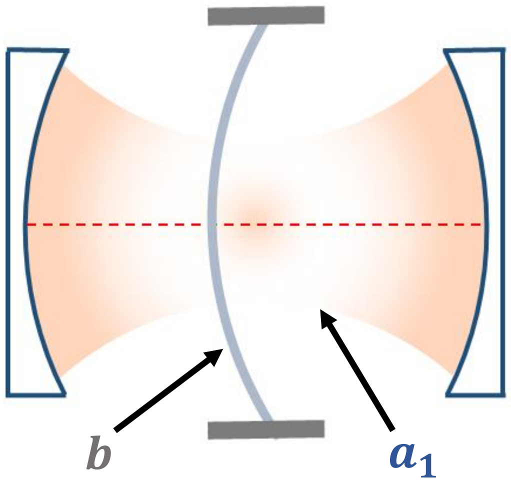
$$H_{sys} = \omega_0 a^\dagger a + \beta a^{\dagger 2} a^2 + \omega_M b^\dagger b + g(b^\dagger + b)a^\dagger a$$

The system Hamiltonian a BEC trapped optomechanical-like system

$H_{sys} = \omega_0 a^\dagger a +$ ~~$\beta a^{\dagger 2} a^2$~~ $+ \omega_M b^\dagger b + g(b^\dagger + b)a^\dagger a$

N. Alam, et al., arXiv: 1708.03967v1

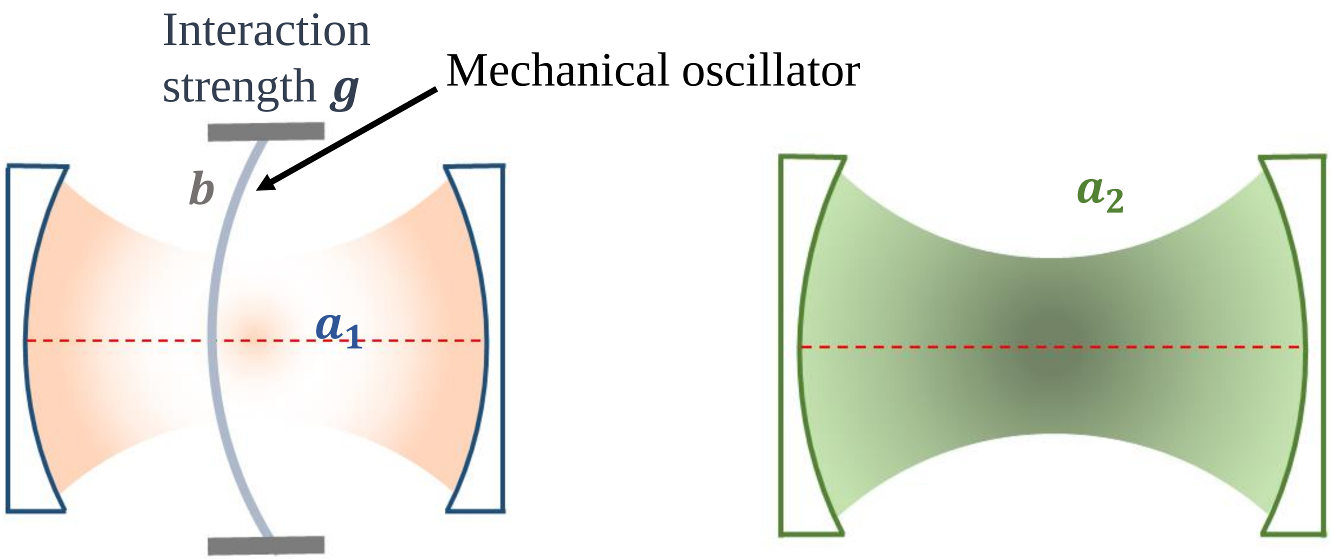
Let's construct a $\mathcal{PT}$ symmetric optomechanical system 1: start with a cavity and replace BEC by a mechanical membrane



The Hamiltonian of the cavity with mechanical membrane

$$H_{sys} = -[\Delta_1] a_1^\dagger a_1 + (\omega_M) b^\dagger b + \frac{g x_{zpf}}{\sqrt{2}} (b^\dagger + b) a_1^\dagger a_1$$

Let's construct a $\mathcal{PT}$ symmetric optomechanical system 2: Add another cavity



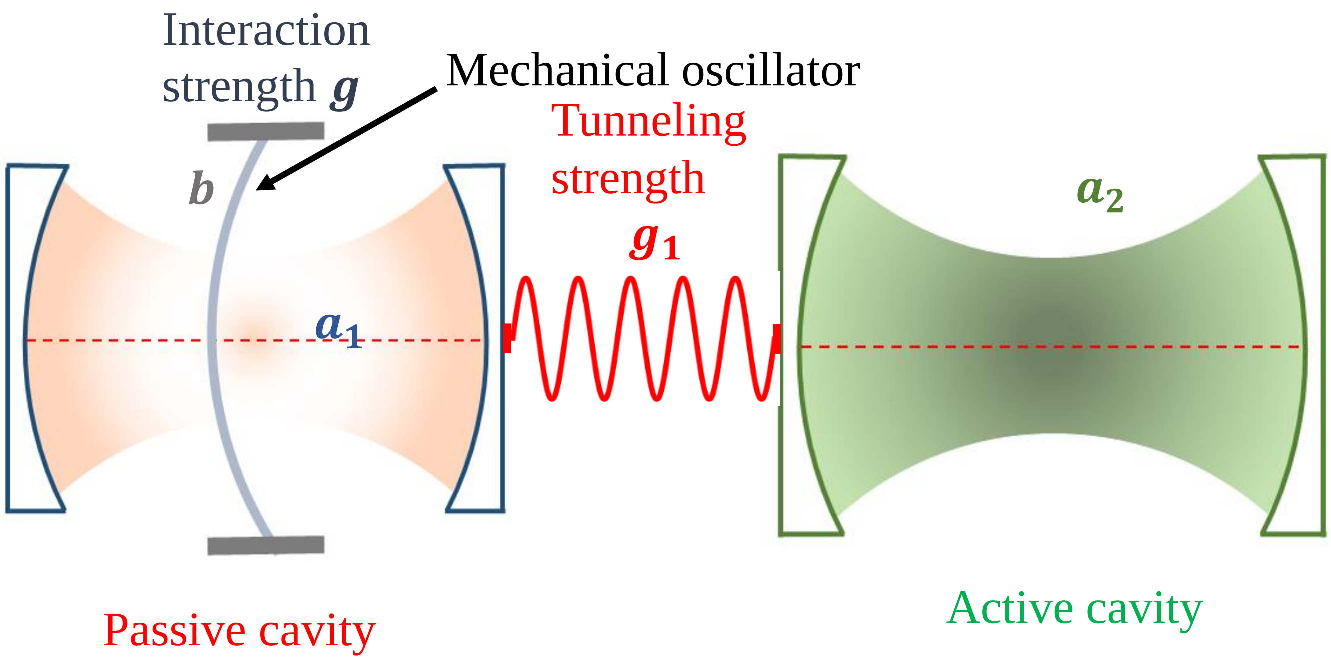
Passive cavity

Active cavity

The Hamiltonian of the 2-cavity system is

$$H_{sys} = -[\Delta_1]a_1^\dagger a_1 - [\Delta_2]a_2^\dagger a_2 + (\omega_M)b^\dagger b + \frac{g x_{zpf}}{\sqrt{2}}(b^\dagger + b)a_1^\dagger a_1$$

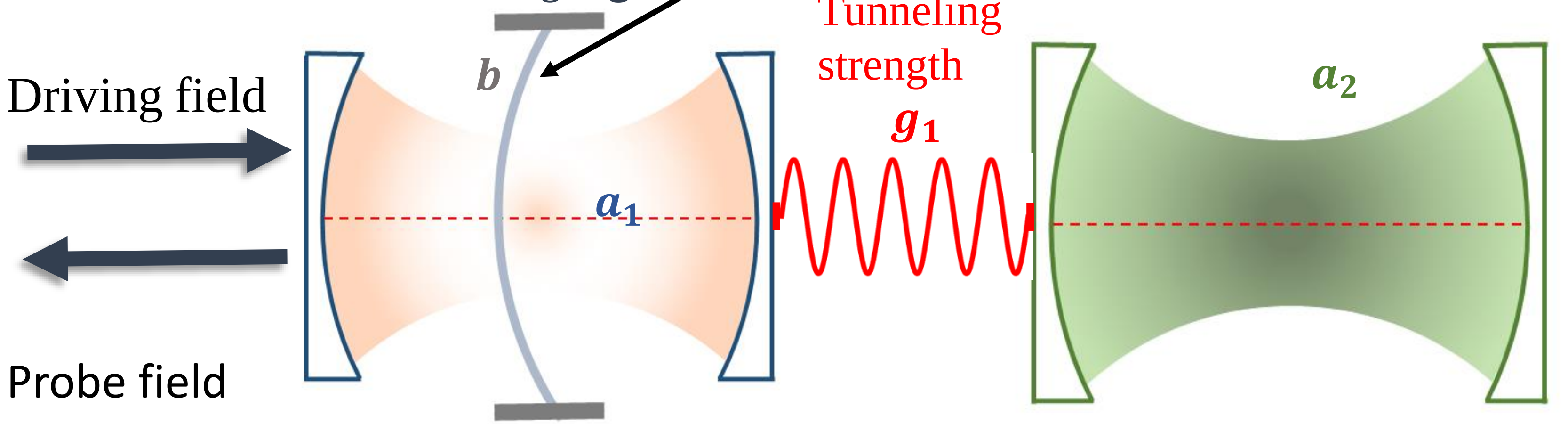
Let's construct a $\mathcal{PT}$ symmetric optomechanical system 3: Allow inter-cavity interaction



The Hamiltonian of the optomechanical system

$$H_{sys} = -[\Delta_1]a_1^\dagger a_1 - [\Delta_2]a_2^\dagger a_2 + (\omega_M)b^\dagger b + \frac{g x_{zpf}}{\sqrt{2}}(b^\dagger + b)a_1^\dagger a_1 - g_1(a_1^\dagger a_2 + a_1 a_2^\dagger)$$

Let's construct a PT symmetric optomechanical system 4: Introduce a driving field and a probe field (treat them classically)



The Hamiltonian of the optomechanical system

$$H_{sys} = -[\Delta_1]a_1^\dagger a_1 - [\Delta_2]a_2^\dagger a_2 + (\omega_M)b^\dagger b + \frac{g x_{zpf}}{\sqrt{2}}(b^\dagger + b)a_1^\dagger a_1 - g_1(a_1^\dagger a_2 + a_1 a_2^\dagger) + i(\epsilon_d a_1^\dagger - \epsilon_d^* a_1) + i\epsilon_p(a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t})$$

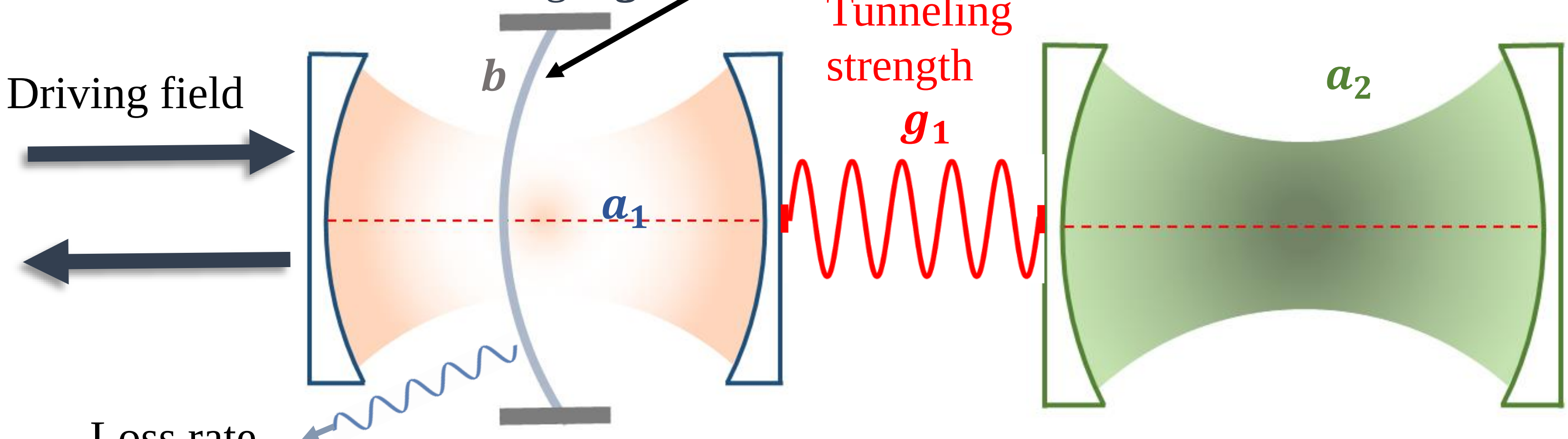
Amplitude of driving field

Driving field

Amplitude of probe field

Probe field

Let's construct a PT symmetric optomechanical system 5: Introduce loss from passive cavity and gain by active cavity, and thus non-Hermiticity



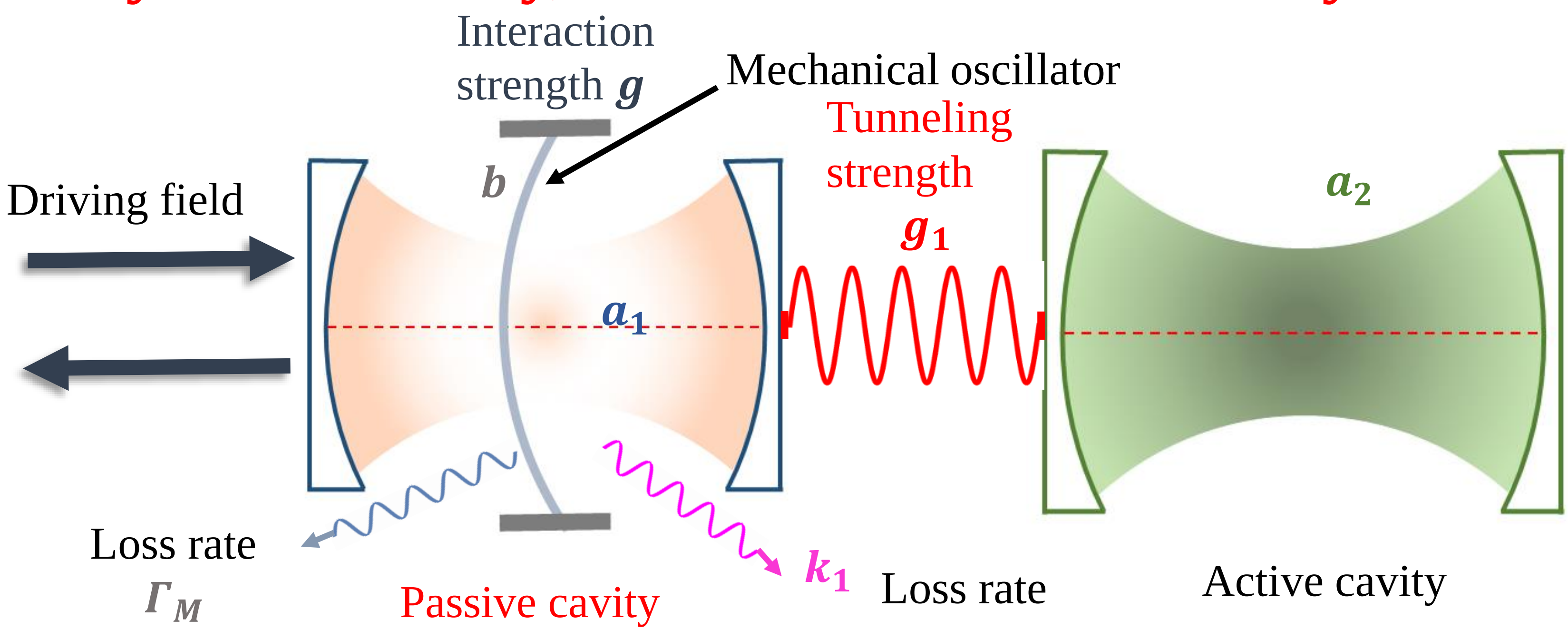
The effective non-Hermitian Hamiltonian of the optomechanical system

$$H_{eff} = -[\Delta_1]a_1^\dagger a_1 + [\Delta_2]a_2^\dagger a_2 + \left(\omega_M - i\frac{\Gamma_M}{2}\right)b^\dagger b$$
$$+\frac{g x_{zpf}}{\sqrt{2}}(b^\dagger + b)a_1^\dagger a_1 - g_1(a_1^\dagger a_2 + a_1 a_2^\dagger) + i(\epsilon_d a_1^\dagger - \epsilon_d^* a_1)$$
$$+i\epsilon_p(a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t})$$

Driving field

Probe field

Let's construct a PT symmetric optomechanical system 5: Introduce loss from passive cavity and gain by active cavity, and thus non-Hermiticity



The effective non-Hermitian Hamiltonian of the optomechanical system

$$H_{eff} = -\left[i\frac{k_1}{2} + \Delta_1\right] a_1^\dagger a_1 - [\Delta_2] a_2^\dagger a_2 + \left(\omega_M - i\frac{\Gamma_M}{2}\right) b^\dagger b$$

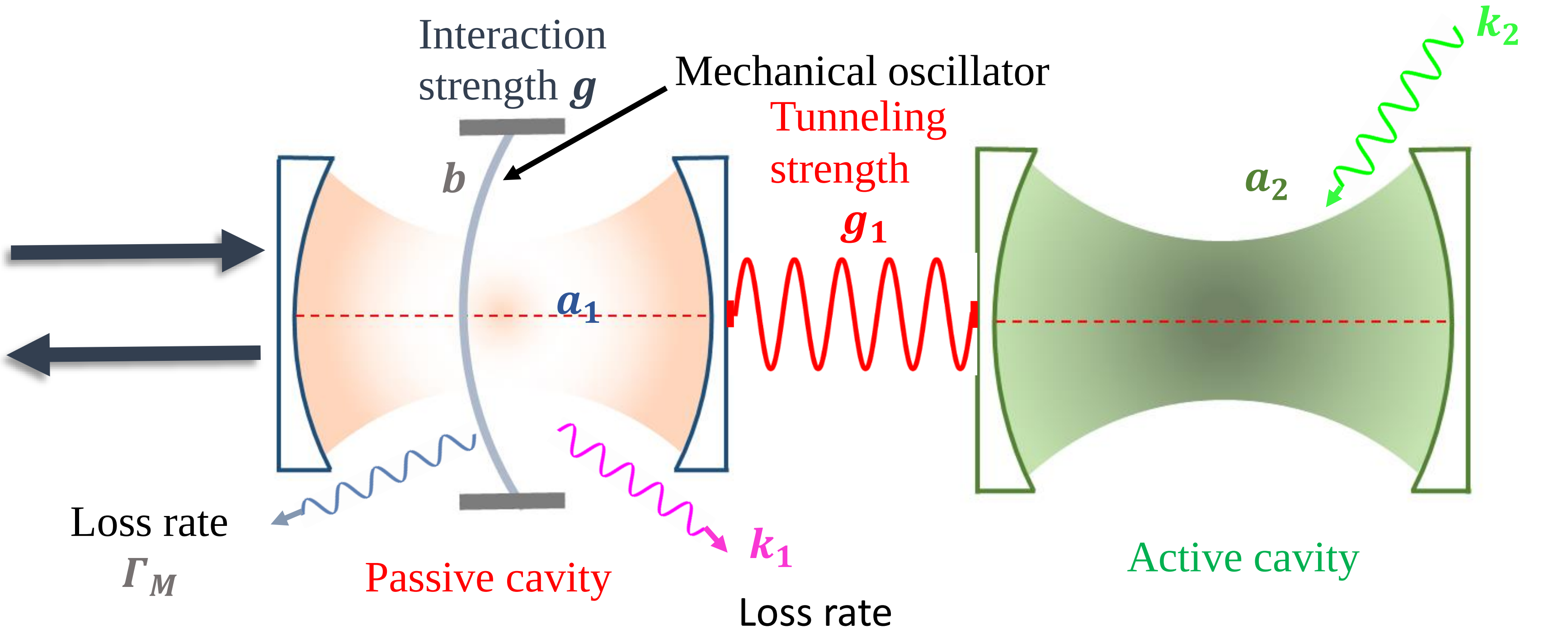
$$+ \frac{g x_{zpf}}{\sqrt{2}} (b^\dagger + b) a_1^\dagger a_1 - g_1 (a_1^\dagger a_2 + a_1 a_2^\dagger) + i(\epsilon_d a_1^\dagger - \epsilon_d^* a_1)$$

$$+ i\epsilon_p (a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t})$$

Driving field

Probe field

The optomechanical system



The effective non-Hermitian Hamiltonian of the optomechanical system

$$H_{eff} = -\left[i\frac{k_1}{2}+\Delta_1\right]a_1^\dagger a_1 + \left[i\frac{k_2}{2}-\Delta_2\right]a_2^\dagger a_2 + \left(\omega_M - i\frac{\Gamma_M}{2}\right)b^\dagger b$$

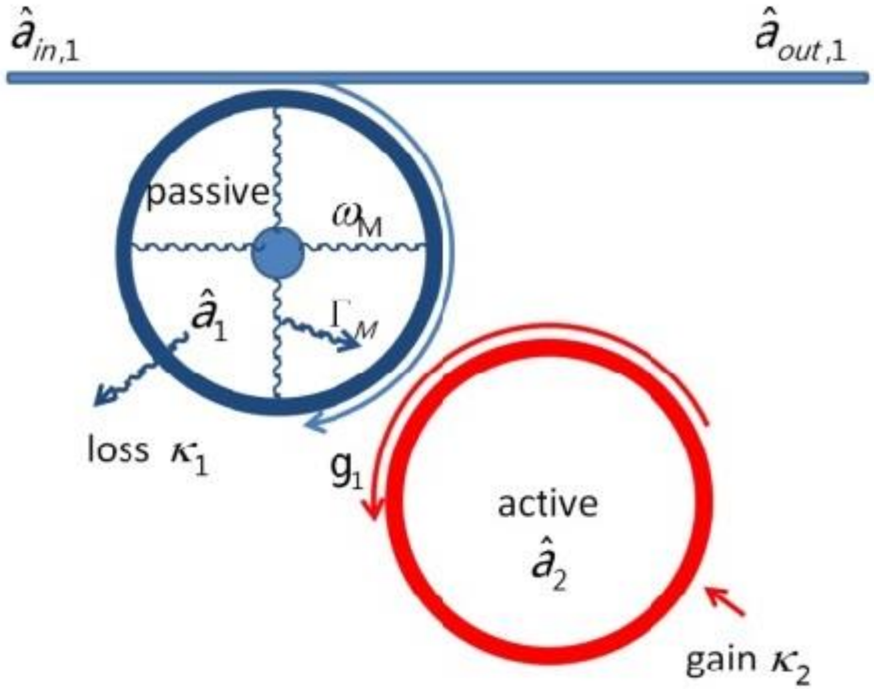
$$+\frac{g x_{zpf}}{\sqrt{2}}(b^\dagger + b)a_1^\dagger a_1 - g_1\left(a_1^\dagger a_2 + a_1 a_2^\dagger\right) + i\left(\epsilon_d a_1^\dagger - \epsilon_d^* a_1\right)$$

$$+i\epsilon_p\left(a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t}\right)$$

Driving field

Probe field

Equivalent optomechanical system



Xin-You Lü, et al., Phys. Rev. Lett., **114**, 253601 (2015)

$\mathcal{PT}$ symmetry in optomechanical systems has many similarity with our day-to-day life

- $\mathcal{PT}$ symmetry business specially, when it involves optics is similar to the business in real life as it solely depends on loss and gain. In our case, there are two cavities – one is active (which is like a business house and gains) and the other one is passive (which is like a common man who looses)!

The model Hamiltonian

The total Hamiltonian of the optomechanical system

$$H_{Tot} = -\Delta_1 a_1^\dagger a_1 - \Delta_2 a_2^\dagger a_2 + \frac{P^2}{2m_M} + \frac{1}{2} m_M \omega_M^2 x^2 + g a_1^\dagger a_1 x - g_1 \left(a_1^\dagger a_2 + a_1 a_2^\dagger\right) + i\left(\epsilon_d a_1^\dagger - \epsilon_d^* a_1\right) + i\epsilon_p \left(a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t}\right)$$

- a_1 (a_2) is the annihilation operator of the first (second) cavity.
- $\Delta_1 = \omega_d - \omega_{cl}$ ($\Delta_1 = \omega_d - \omega_{c2}$) is the detuning between driven field and passive(active) cavity. g_1 is the tunneling strength.
- g is the interaction strength, $\epsilon_d = \sqrt{\frac{2k_1 P_d}{\hbar\omega_d}}$ and $\epsilon_p = \sqrt{\frac{2k_1 P_p}{\hbar\omega_p}}$ are the amplitude of the driving and the probe fields related to powers P_p and P_d.
- $\delta = \omega_p - \omega_d$ is the detuning between the probe field and the driving field.
- m_M and ω_M are the effective mass and frequency of the mechanical oscillator.

Langevin equations

Langevin equations of the system are

$$\dot{\hat{a}}_1 = -\Big[\frac{\kappa_1}{2} - i\Delta_1\Big]\hat{a}_1 - ig\hat{a}_1\hat{x} + ig_1\hat{a}_2 + \epsilon_d + \epsilon_p e^{-i\delta t}$$
$$- \sqrt{\eta_c\kappa_1}\hat{a}_{\text{int},1} - \sqrt{(1-\eta_c)\kappa_1}\hat{a}_{\text{ext},1},$$
$$\dot{\hat{a}}_2 = \Big[\frac{\kappa_2}{2} + i\Delta_2\Big]\hat{a}_2 + ig_1\hat{a}_1 - \sqrt{\kappa_2}\hat{a}_{\text{int},2},$$
$$\dot{\hat{x}} = \frac{\hat{p}}{m_M},$$
$$\dot{\hat{p}} = -m_M\omega_m^2\hat{x} - g\hat{a}_1^\dagger\hat{a}_1 - \Gamma_M\hat{p} + \delta\hat{F}_{th},$$

The semiclassical Langevin equations of the system=> Take the average over the quantum Langevin equations (neglect quantum fluctuations in probe field)

$$\dot{a}_1 = -\Big[\frac{\kappa_1}{2} - i\Delta_1\Big]a_1 - igxa_1 + ig_1a_2 + \epsilon_d + \epsilon_p e^{-i\delta t},$$
$$\dot{a}_2 = \Big[\frac{\kappa_2}{2} + i\Delta_2\Big]a_2 + ig_1a_1,$$
$$\dot{x} = \frac{p}{m_M},$$
$$\dot{p} = -m_M\omega_m^2x - g|a_1|^2 - \Gamma_M p.$$

The effective Hamiltonian

The effective non-Hermitian Hamiltonian of the optomechanical system

$$\boldsymbol{H_{eff}} = -\left[\boldsymbol{i}\frac{\boldsymbol{k_1}}{\boldsymbol{2}} + \Delta_1\right]\boldsymbol{a_1^\dagger a_1} + \left[\boldsymbol{i}\frac{\boldsymbol{k_2}}{\boldsymbol{2}} - \Delta_2\right]\boldsymbol{a_2^\dagger a_2} + \left(\boldsymbol{\omega_M} - \boldsymbol{i}\frac{\boldsymbol{\Gamma_M}}{\boldsymbol{2}}\right)\boldsymbol{b^\dagger b}$$
$$+\frac{\boldsymbol{g x_{zpf}}}{\sqrt{\boldsymbol{2}}}(\boldsymbol{b^\dagger} + \boldsymbol{b})\boldsymbol{a_1^\dagger a_1} - \boldsymbol{g_1}(\boldsymbol{a_1^\dagger a_2} + \boldsymbol{a_1 a_2^\dagger}) + \boldsymbol{i}(\boldsymbol{\epsilon_d a_1^\dagger} - \boldsymbol{\epsilon_d^* a_1})$$
$$+\boldsymbol{i\epsilon_p}(\boldsymbol{a_1^\dagger e^{-i\delta t}} - \boldsymbol{a_1 e^{i\delta t}})$$

- a_1 (a_2) is the annihilation operator of the first (second) cavity.
- $\Delta_1 = \omega_d - \omega_{cl}$ ($\Delta_1 = \omega_d - \omega_{c2}$) is the detuning between driven field and passive(active) cavity. g_1 is the tunneling strength.
- g, is the interaction strength, $\epsilon_d = \sqrt{\frac{2k_1 P_d}{\hbar\omega_d}}$ and $\epsilon_p = \sqrt{\frac{2k_1 P_p}{\hbar\omega_p}}$ are the amplitude of the driving and the probe fields related to powers P_p and P_d.
- $\delta = \omega_p - \omega_d$ is the detuning between the probe field and the driving field.
- $k_1(\Gamma_M)$ and k_2 are the optical (mechanical) loss rate and gain rate.
- $\boldsymbol{x_{zpf}} = \sqrt{\frac{\hbar}{2m_M\omega_M}}$, m_M and ω_M are the effective mass and frequency of the mechanical oscillator.

$\mathcal{PT}$ operation in two-mode case

Putting $g = 0$ and turning off the driving and probe fields in H_{eff}, we can obtain the Hamiltonian H only for the two cavities, i.e.,

$$H = -\left[i\frac{k_1}{2} + \Delta_1\right] a_1^\dagger a_1 + \left[i\frac{k_2}{2} - \Delta_2\right] a_2^\dagger a_2 - g_1\left(a_1^\dagger a_2 + a_1 a_2^\dagger\right)$$

The Hamiltonian H is $\mathcal{PT}$ symmetric, where the action of the parity ($\mathcal{P}$) and the time reversal ($\mathcal{T}$) operation are to interchange the loss and gain oscillators [1], i.e.,

$$\mathcal{P}: \quad a_1 \leftrightarrow -a_2, \quad a_1^\dagger \leftrightarrow -a_2^\dagger$$
$$\mathcal{T}: \quad a_1 \leftrightarrow a_1, \; a_2 \leftrightarrow a_2, \; a_1^\dagger \leftrightarrow a_1^\dagger, \; a_2^\dagger \leftrightarrow a_2^\dagger$$

After the $\mathcal{PT}$ operation the Hamiltonian takes the form,

$$H^{\mathcal{PT}} = \mathcal{PT}H\mathcal{PT} = \left[i\frac{k_1}{2} - \Delta_1\right] a_2^\dagger a_2 + \left[-i\frac{k_2}{2} - \Delta_2\right] a_1^\dagger a_1 - g_1\left(a_1^\dagger a_2 + a_1 a_2^\dagger\right)$$

[1] C. M. Bender, et al., Phys. Rev. A **88**, 062111 (2013)

$\mathcal{PT}$ operation

The Hamiltonian H after the □□ operation takes the form,

$$H^{\mathcal{PT}} = \mathcal{PT}H\mathcal{PT}$$
$$= \left[i\frac{k_1}{2} - \Delta_1\right]a_2^\dagger a_2 + \left[-i\frac{k_2}{2} - \Delta_2\right]a_1^\dagger a_1 - g_1(a_1^\dagger a_2 + a_1 a_2^\dagger)$$

The above Hamiltonian is $\mathcal{PT}$ symmetric when $k_1 = k_2, \Delta_1 = \Delta_2$ and $4g_1 > k_1 + k_2$ therefore, with that condition

$$H = \left[i\frac{k}{2} - \Delta\right]a_2^\dagger a_2 + \left[-i\frac{k}{2} - \Delta\right]a_1^\dagger a_1 - g_1(a_1^\dagger a_2 + a_1 a_2^\dagger)$$

which is the $\mathcal{PT}$ symmetric Hamiltonian.

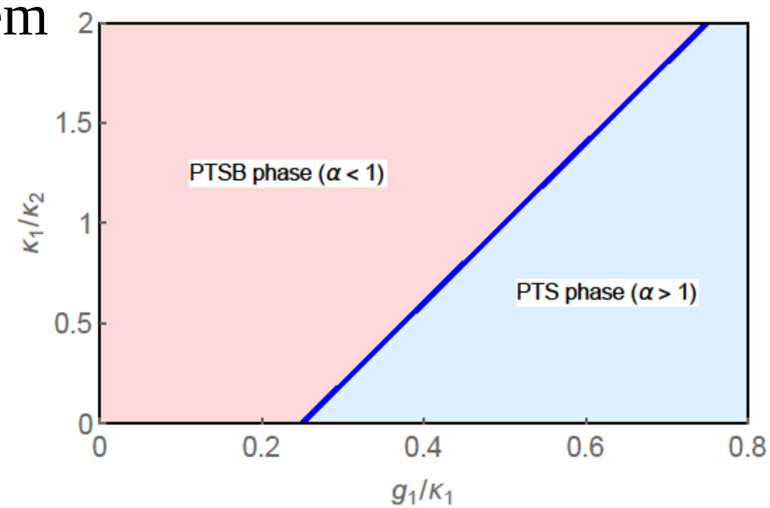
$\mathcal{PT}$ symmetry breaking

The coupling of the above Hamiltonian leads to the super modes $\boldsymbol{a}_{\pm} = \frac{1}{\sqrt{2}}(\boldsymbol{a_1} \pm \boldsymbol{a_2})$, with the eigenfrequencies $\omega_{\pm}$, and are given by

$$\omega_{\pm} = \Delta - \frac{i}{2}(k_1 - k_2) \pm \sqrt{(4g_1)^2 - (k_1 + k_2)^2}$$

where $\Delta_1 = \Delta_2 = \Delta$

- When $4g_1 > k_1 + k_2$ (i.e., $\alpha = \frac{4g_1}{k_1 + k_2} > 1$) the eigenfrequencies are real, and Hamiltonian of the double cavity system is $\mathcal{PT}$ symmetric.
- When $4g_1 < k_1 + k_2$ (i.e., $\alpha<1$), the eigen values are complex so the $\mathcal{PT}$ symmetry is broken.



Method: Example

Mandel parameter is defined by

$$Q_M \approx \frac{\left\langle \left(A^\dagger A\right)^2 \right\rangle - \left\langle A^\dagger A \right\rangle^2 - \left\langle A^\dagger A \right\rangle}{\left\langle A^\dagger A \right\rangle}$$

This quantity contains term $\left\langle \left(A^\dagger A\right)^2 \right\rangle$ which is a fourth order term that can be decomposed after writing it in normal-order form $\left\langle \left(A^\dagger A\right)^2 \right\rangle = \left\langle A^\dagger A^\dagger A A \right\rangle + \left\langle A^\dagger A \right\rangle$ and subsequently to use the decoupling relation

$$\langle ABCD \rangle \approx \langle AB \rangle\langle CD \rangle + \langle AC \rangle\langle BD \rangle + \langle AD \rangle\langle BC \rangle - 2\langle A \rangle\langle B \rangle\langle C \rangle\langle D \rangle.$$

Using this decoupling relation we can write

$$\begin{aligned}\left\langle A^\dagger A^\dagger A A \right\rangle &\approx \left\langle A^\dagger A^\dagger \right\rangle\langle AA \rangle + \left\langle A^\dagger A \right\rangle\left\langle A^\dagger A \right\rangle \\ &+ \left\langle A^\dagger A \right\rangle\left\langle A^\dagger A \right\rangle - 2\left\langle A^\dagger \right\rangle\left\langle A^\dagger \right\rangle\langle A \rangle\langle A \rangle, \\ &= \left\langle \left(A^\dagger\right)^2 \right\rangle\langle A^2 \rangle + 2\left\langle A^\dagger A \right\rangle^2 - 2\left\langle A^\dagger \right\rangle^2\langle A \rangle^2\end{aligned}$$

Thus, Mandel parameter can be written in terms of second order terms as

$$Q_M \approx \frac{\left\langle \left(A^\dagger\right)^2 \right\rangle\langle A^2 \rangle + \left\langle A^\dagger A \right\rangle^2 - 2\left\langle A^\dagger \right\rangle^2\langle A \rangle^2}{\left\langle A^\dagger A \right\rangle}$$

Method: Example

- We begin our solution scheme by taking an average of each terms appearing in Langevin equations for all the first and second order terms with respect to the initial state.
- This step yields a differential equation of the average of an operator in terms of averages of the remaining operators.
- Note that this step transforms the operator differential equation into a c-number differential equation which is much easier to handle.
- Assuming each reservoir to be in thermal equilibrium at temperature T, we can average over the system and reservoir degrees of freedom and using the fact that the reservoir average of the noise operator vanishes $\langle \widehat{F}_c \rangle_R = 0$, we end up with the following equation of motion for the average cavity photon number:

$$\frac{d\langle C\rangle}{dt} = -i\Delta_c \langle C\rangle - iG_A\langle A\rangle - iG_B\langle B\rangle - \frac{\Gamma c}{2}\langle C\rangle$$

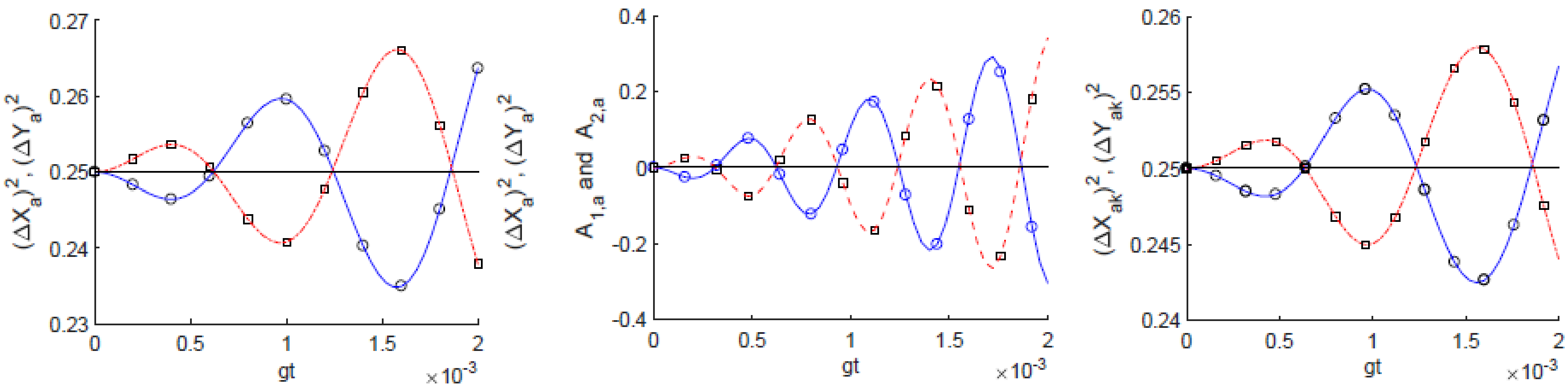
1. J. R. Anglin and A. Vardi, Physical Review A 64, 013605 (2001).
2. C. H. R. Ooi, Q. Sun, M. S. Zubairy, and M. O. Scully, Physical Review A 75, 013820 (2007).
3. S. K. Singh and C. H. R. Ooi, JOSA B 31, 2390 (2014).

Squeezing

$$X_a = \frac{1}{2}(a + a^\dagger),\ Y_a = -\frac{i}{2}(a - a^\dagger) \qquad (\Delta X_a)^2 < \frac{1}{4},\ (\Delta Y_a)^2 < \frac{1}{4}$$

$$X_{ak} = \frac{1}{2\sqrt{2}}\{(a + a^\dagger) + (k + k^\dagger)\},$$
$$Y_{ak} = -\frac{i}{2\sqrt{2}}\{(a - a^\dagger) + (k - k^\dagger)\}$$
$$(\Delta X_{ak})^2 < \frac{1}{4},\ (\Delta Y_{ak})^2 < \frac{1}{4}$$

$$Y_{1,a} = \frac{a^l + a^{\dagger l}}{2},$$
$$Y_{2,a} = -i\frac{a^l - a^{\dagger l}}{2}$$
$$A_{i,a} = (\Delta Y_{i,a})^2 - \frac{1}{2}|\langle[Y_{1,a}, Y_{2,a}]\rangle| < 0,$$
$$A_{i,a} = \langle(\Delta Y_{i,a})^2\rangle - \langle N_a + \frac{1}{2}\rangle < 0,$$

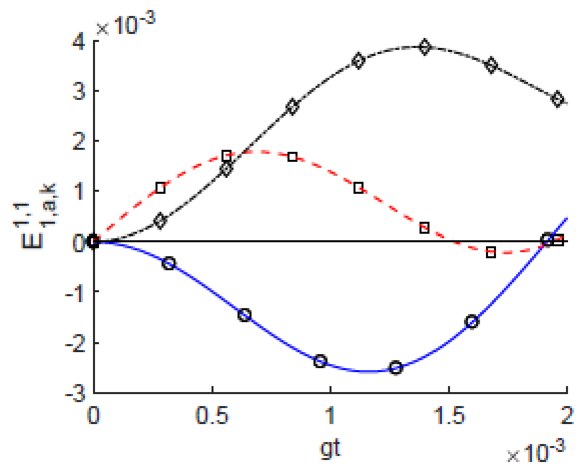


N. Alam, et al., arXiv: 1708.03967v1

Entanglement

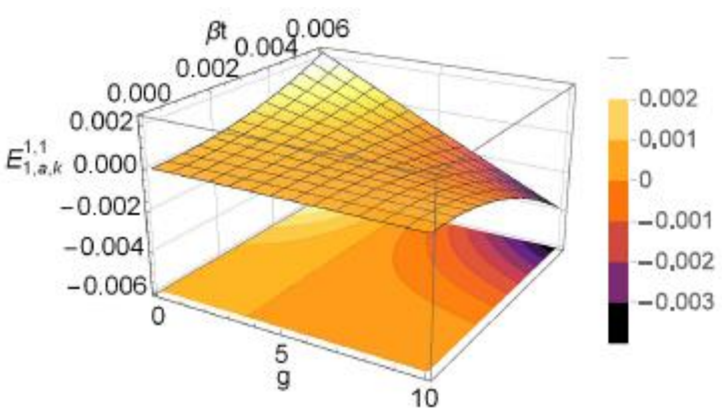
HZ-1 criterion

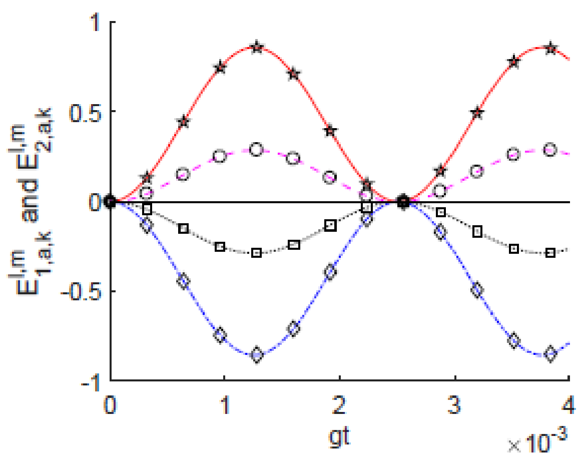
$$E^{1,1}_{1,a,k} = \langle a^\dagger(t)a(t)k^\dagger(t)k(t)\rangle - |\langle a(t)k^\dagger(t)\rangle|^2 < 0.$$

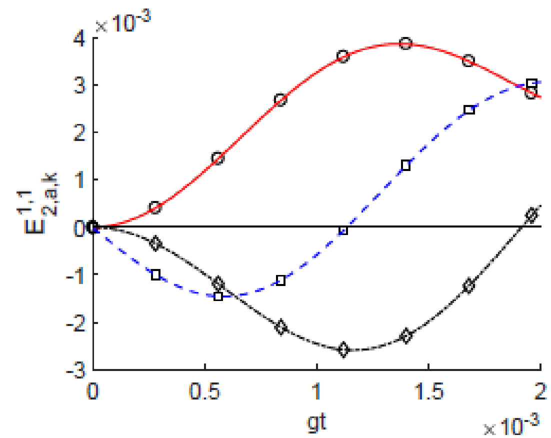


HZ-2 criterion

$$E^{1,1}_{2,a,k} = \langle a^\dagger(t)a(t)\rangle\langle k^\dagger(t)k(t)\rangle - |\langle a(t)k(t)\rangle|^2 < 0.$$







N. Alam, et al., arXiv: 1708.03967v1

Antibunching

Second order correlation function

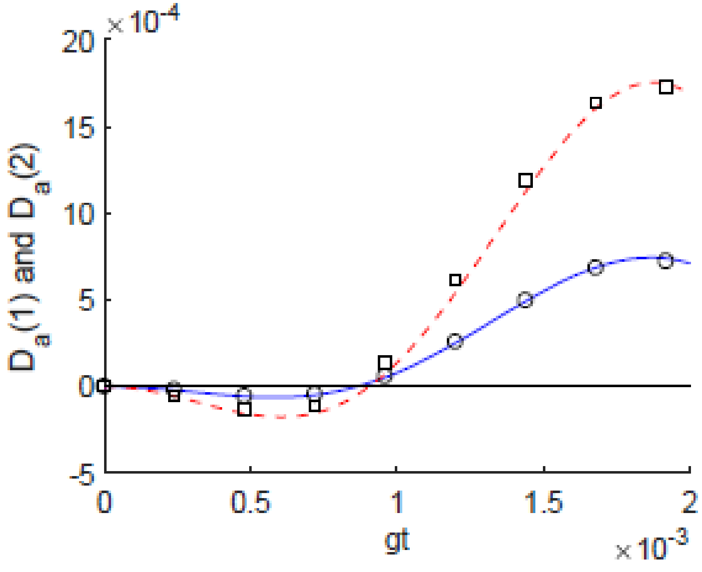
$$g^2(0) = \frac{\langle a^\dagger(t)a^\dagger(t)a(t)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t)a(t)\rangle}$$

$$g^2(0) - 1 = \frac{(\Delta N(t))^2 - \langle N(t)\rangle}{\langle N(t)\rangle^2} = \frac{D_a(1)}{\langle N(t)\rangle^2}$$

Higher-order antibunching

$$D(l-1) = \langle a^{\dagger l} a^l\rangle - \langle a^\dagger a\rangle^l < 0$$

$l = 2$, the above equation is reduce to normal order antibunching. For Higher-order antibunching $l > 2$.

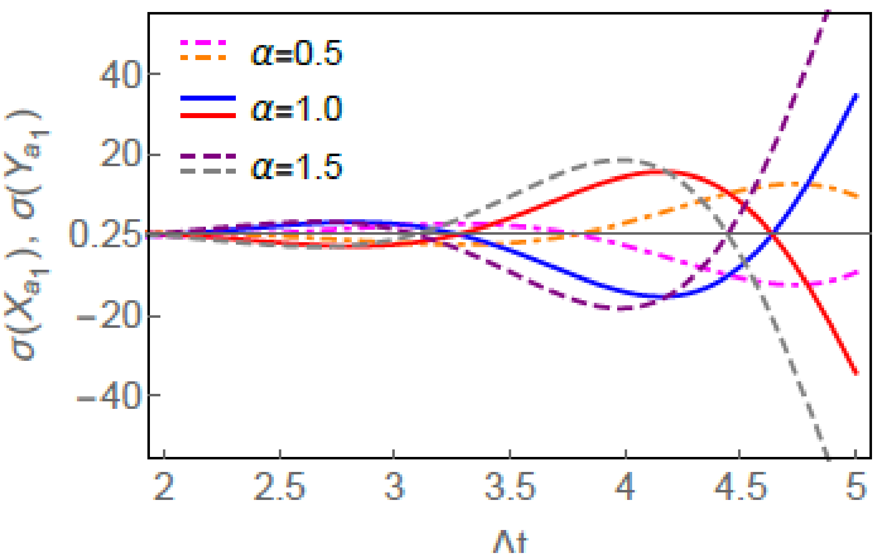


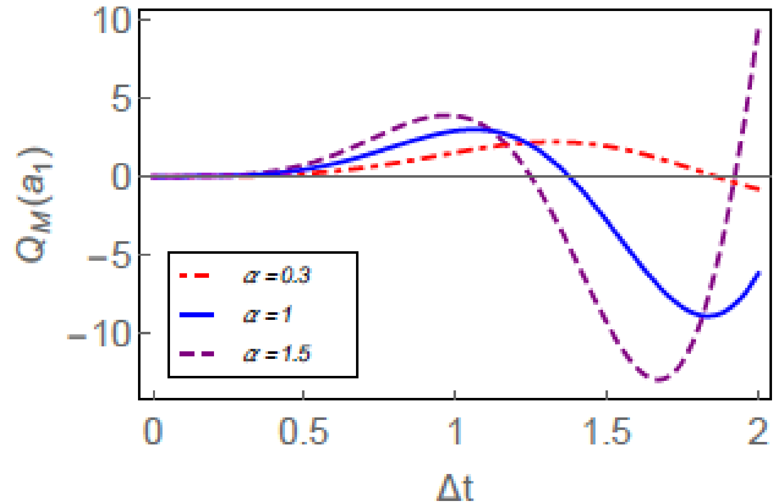
N. Alam, et al., arXiv: 1708.03967v1

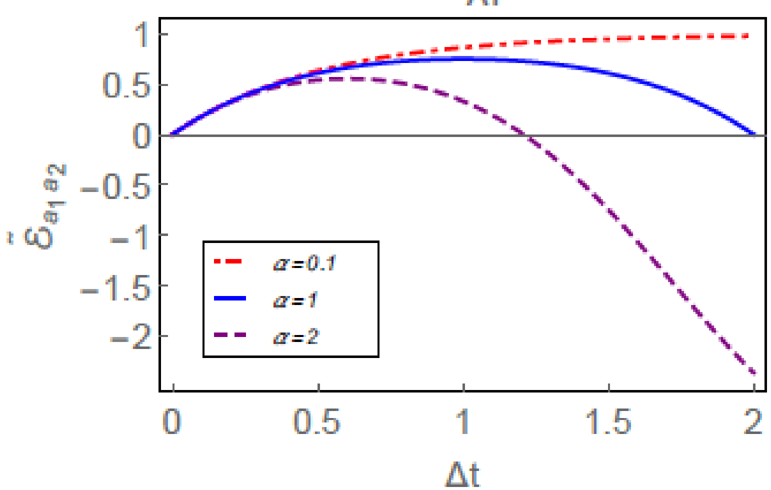
Nonclassicality in the $\mathcal{PT}$ symmetric optomechanical system

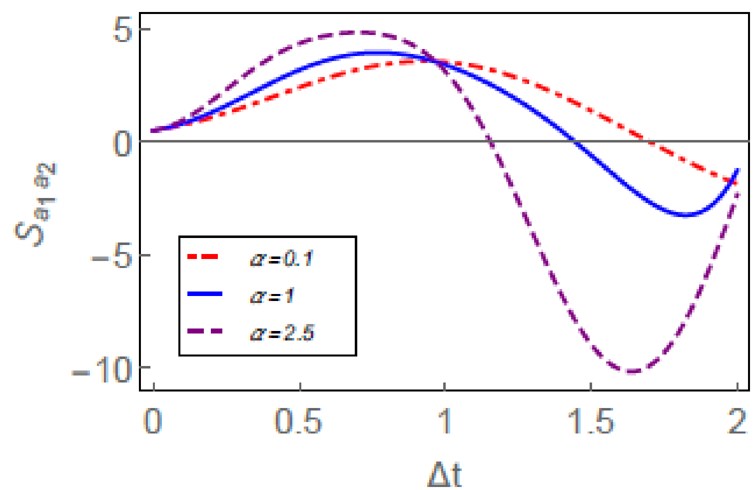
$$PTS: \alpha > 1$$

$$PTSB: \alpha < 1$$









Conclusion

- Non-Hermitian physics is now really getting connected to quantum optics (cf. recent review of El-Ganainy et al., Nature Physics **14** (2018) 11) and partially to quantum information.
- Quantum thermodynamics and non-Hermitian heat engines are getting connected & growing fast.
- New experiments and signatures of nonclassicalities in various non-Hermitian ($\mathcal{PT}$ symmetric system) would follow and some real devices based on those calculation are expected to appear.

THANK YOU

Supports Received from: DRDO; SERB, DST; CSIR

$\mathcal{PT}$ symmetry

Dirac Hermiticity:

$$H = H^{\dagger}$$ († means transpose + complex conjugate)

Hermiticity condition guarantees *real energy* and *probability-conserving* time evolution but it is a **mathematical** axiom and not a **physical** axiom of quantum mechanics.

- D. Bessis conjectured [1] on the basis of numerical studies that the spectrum of the Hamiltonian $H = p^2 + x^2 + ix^3$ is *real and positive.*
- Bender and Boettcher [1] showed that *wide class of* non-Hermitian Hamiltonians can actually possess *entirely real spectra* as long as they respect parity-time symmetry

1. C. Bender and S. Boettcher, ”Real spectra in non-Hermitian Hamiltonians having PT symmetry”, Phys. Rev. Lett. **80**, 5243 (1998).

$\mathcal{PT}$ symmetry

Dirac Hermiticity can be replaced by *physical* and *weaker* [1] condition of *PT* symmetry

The $\mathcal{PT}$ symmetry Hamiltonian means which commutes with $\mathcal{PT}$ i.e., $[H, \mathcal{PT}] = 0.$

Examples:

$\mathcal{PT}$ symmetric non-Hermitian Hamiltonian:

$$H = p^2 + ix^3 + ix$$

Entire spectrum is positive

Non- $\mathcal{PT}$ symmetric non-Hermitian Hamiltonian:

$$H = p^2 + ix^3 + x$$

Entire spectrum is complex

First Hamiltonian is non-Hermitian but its eigenvalues spectrum is real and this is due to the presence of symmetry and does possess $\mathcal{PT}$ symmetry. Therefore, Hermiticity is not an essential condition for the real eigenvalue in quantum mechanics.

1. Carl M. Bender, arXive:9809072v1

$\mathcal{PT}$ symmetry

The experimental observation of the predicted properties of the $\mathcal{PT}$-symmetric Hamiltonians have been observed at the classical level.

Examples:

(a) Superconductivity (b) optics, (c) microwave cavities, (d) atomic diffusion, (e) nuclear magnetic resonance, (f) coupled electronic and (g) mechanical oscillators.

Recently, a new approach has been proposed: simultaneously using gain and loss as a way of achieving optical behavior that is at present unattainable with standard arrangements.