Non-Hermitian gauge field, non-Hermitian flow, and quantum localization

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1. Imaginary gauge transformation
2. Anderson localization
3. Random quantum walk

Non-Hermiticity due to the vector potential

$$H_g = \frac{(p - ig)^2}{2m} + V(x)$$

g: imaginary vector potential

$V(x)$: real scalar potential

Imaginary gauge transformation

$$H_g = \frac{(p - ig)^2}{2m} + V(x)$$

eA

$$g = 0 : \; H_0 \psi_0 = E_0 \psi_0$$

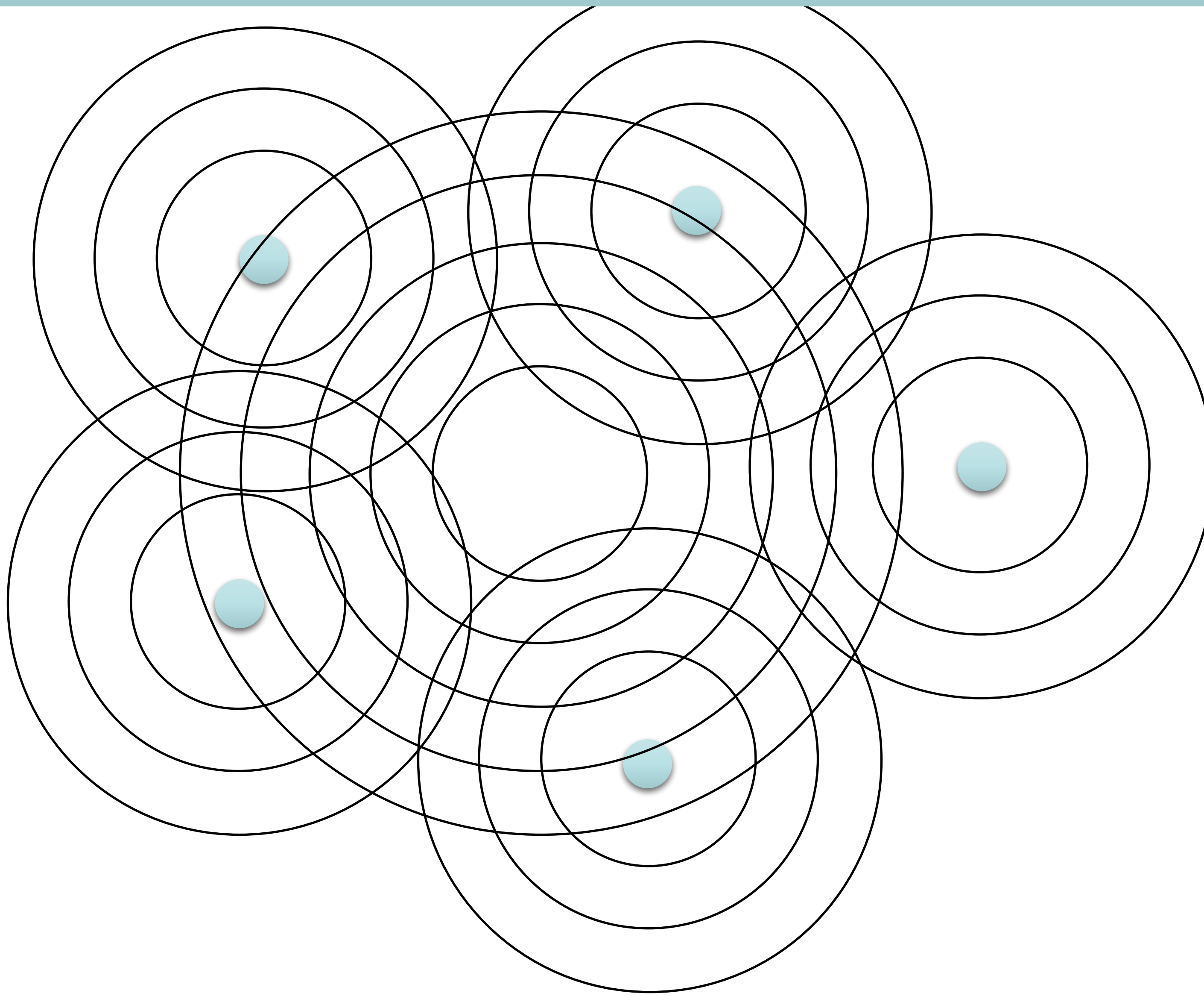
$$\Downarrow$$

$$g \neq 0 : \; H_g \psi_g = E_g \psi_g$$

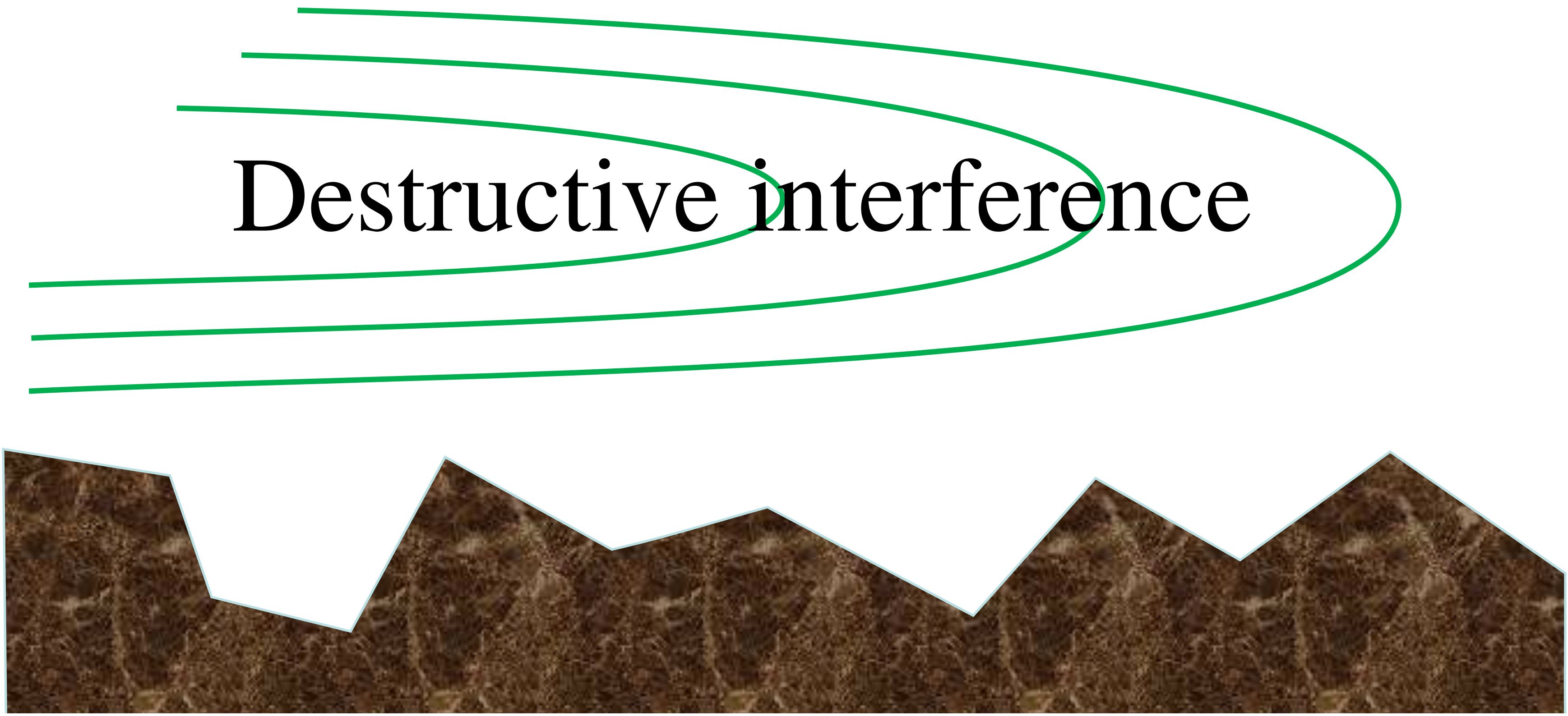
$$\psi_g = \psi_0 e^{\int g\,dx}; \; E_g = E_0$$

Anderson Localization

Anderson Localization



In one dimension

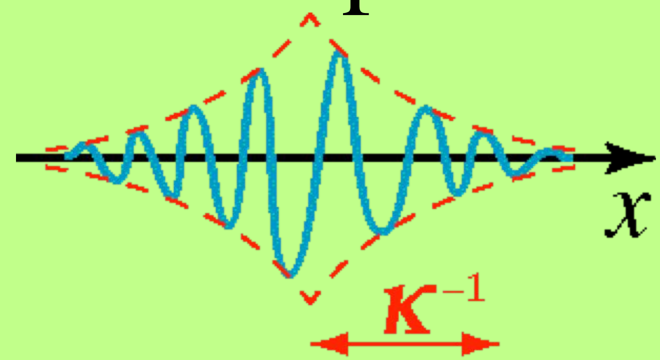


$V(x)$: (real) radom scalar potential

Anderson localization

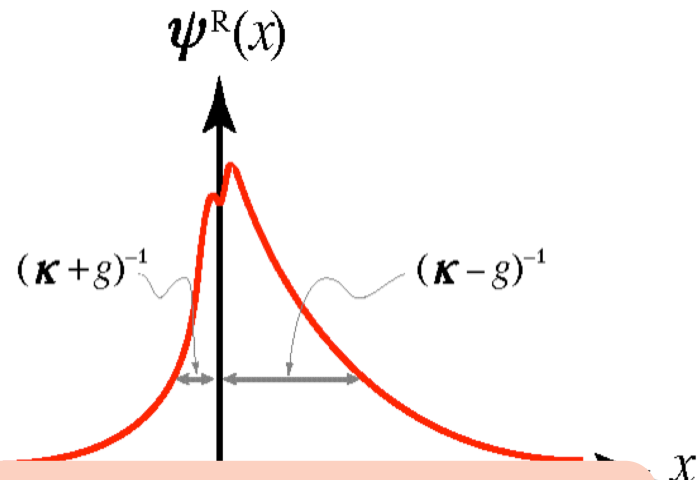
Localized states in a random potential

$$\psi_0 \sim e^{-\kappa|x|}$$



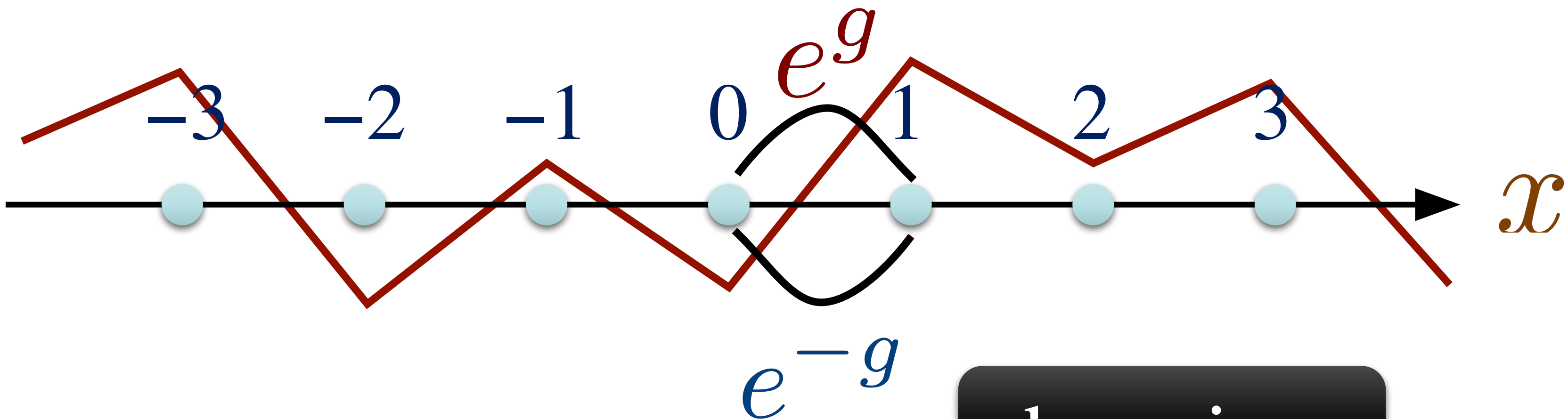
Imaginary gauge transformation

$$\psi_g \sim e^{-\kappa|x|+gx}$$



normalizable for $g < \kappa$ only

1d tight-binding model



hopping

$$H_g = -\frac{t}{2}\sum_x \left(e^{g} c^\dagger_{x+1} c_x + e^{-g} c^\dagger_x c_{x+1} \right)$$

$$+ \sum_x V_x c^\dagger_x c_x$$

real random potential

Non-Hermitian Anderson model

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$$H = \begin{pmatrix} \ddots & \ddots & & & & e^{g} \\ \ddots & V_{-1} & e^{-g} & & & \\ & e^{g} & V_0 & e^{-g} & & \\ & & e^{g} & V_1 & e^{-g} & \\ & & & e^{g} & V_2 & \ddots \\ e^{-g} & & & & \ddots & \ddots \end{pmatrix}$$

Complex-Eigenvalue Transition

Complex-eigenvalue transition

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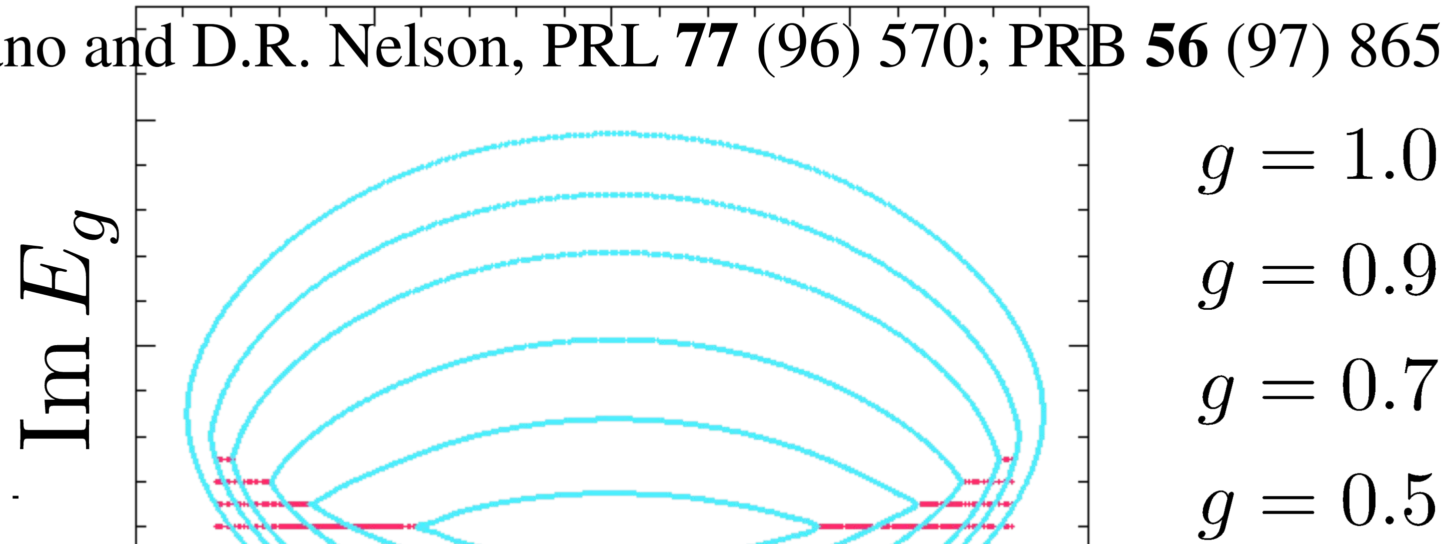

$V_x \in [-1, 1$

$$H_g = -\frac{t}{2}\sum_x \left(e^{ga} c^\dagger_{x+a} c_x + e^{-ga} c^\dagger_x c_{x+a} \right) + \sum_x V_x c^\dagger_x c_x$$

Imaginary gauge transformation

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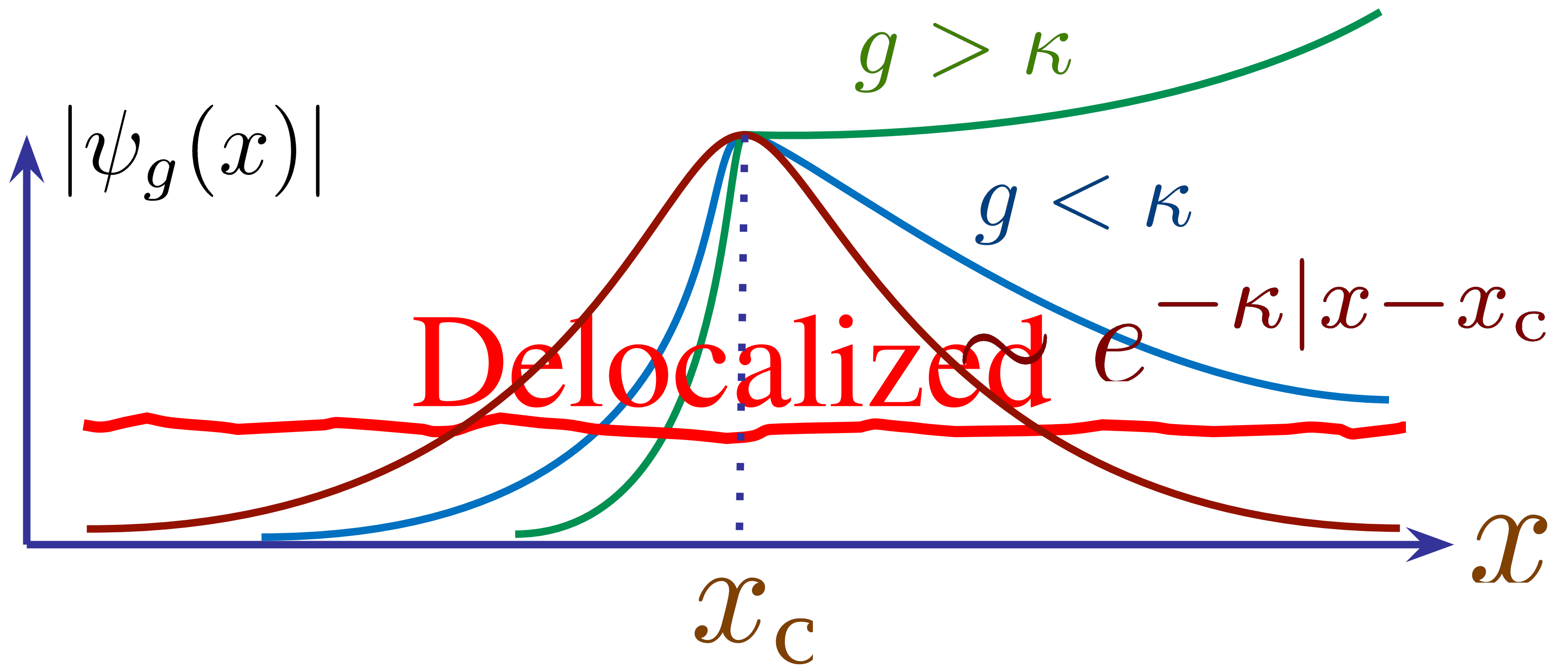
$$\psi_0(x) \longrightarrow \psi_g(x) = \psi_0(x)e^{gx}$$



Delocalization transition

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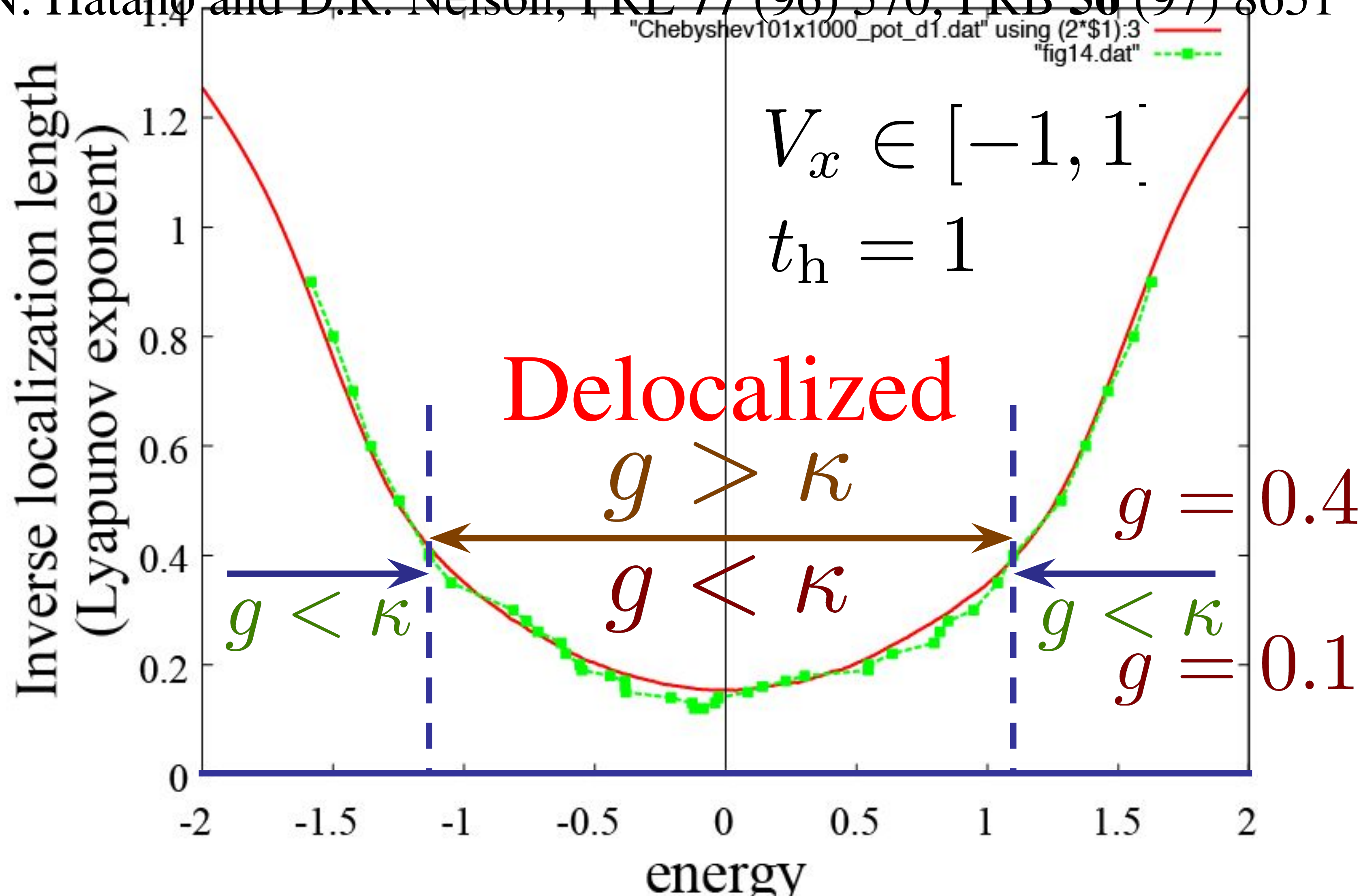

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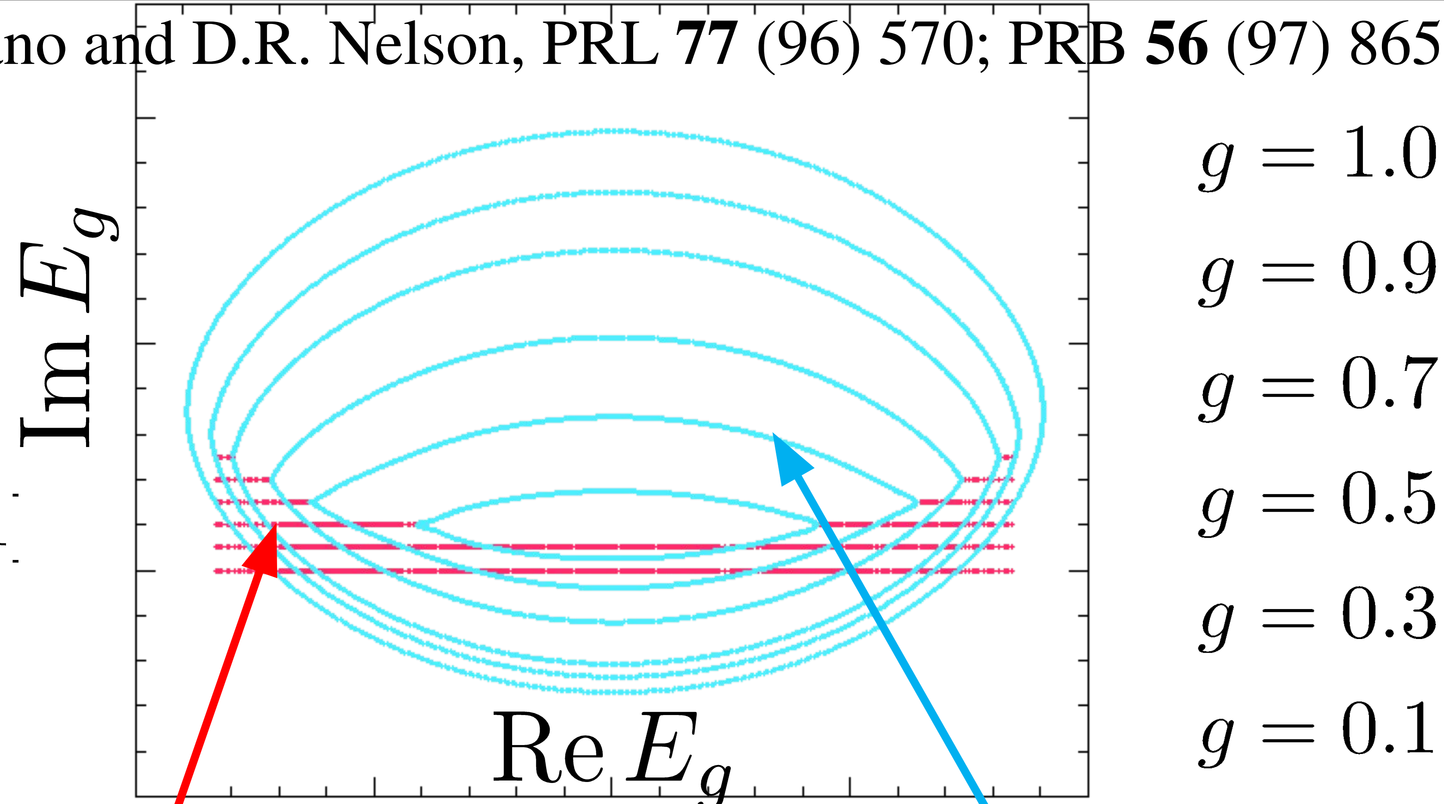

$V_x \in [-1, 1$

1000 sites, pe... b... c...

Complex Eigenvalue Transition $\Leftrightarrow$ Non-Hermitian delocalization transition

Non-Hermitian Anderson model

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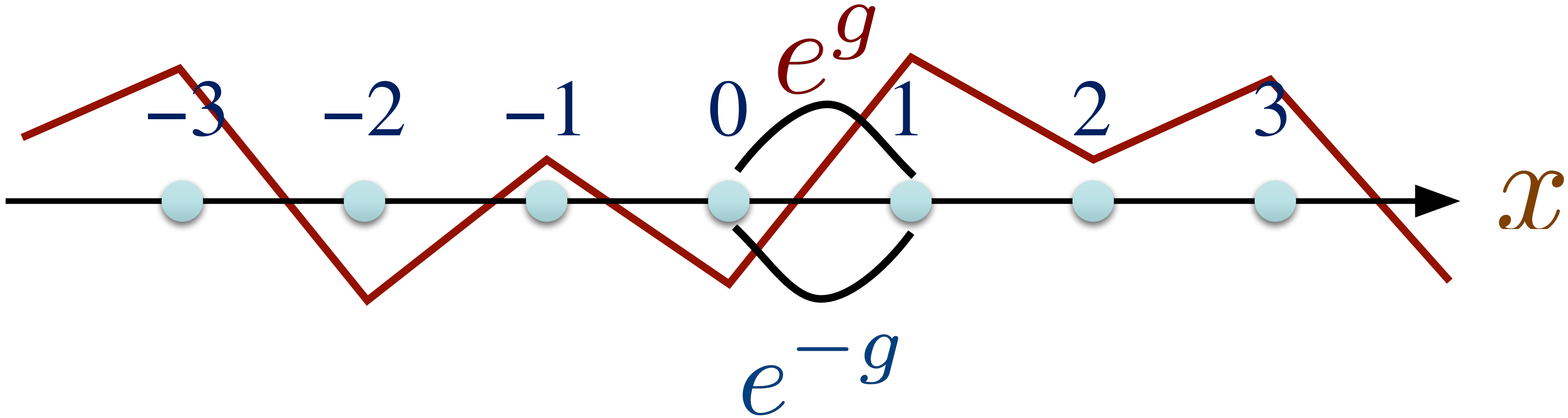

Non-Hermiticity
$\rightarrow$ Delocalizing

Randomness
$\rightarrow$ Localizing

Complex current

$$H_g = -\frac{t}{2}\sum_x \left(e^g c_{x+1}^\dagger c_x + e^{-g} c_x^\dagger c_{x+1}\right)$$

Eigenfn. $\psi_g(x) = e^{ikx}$

Eigenv. $E_g = \cos(k - ig)$

$$= \cos k \cosh g + i \sin k \sinh g$$

$$J_A = \frac{\partial E_A}{\partial A} \quad \Rightarrow \quad J_g = \frac{\partial E_g}{\partial (ig)} = \sin(k - ig)$$

Imaginary-time path integral

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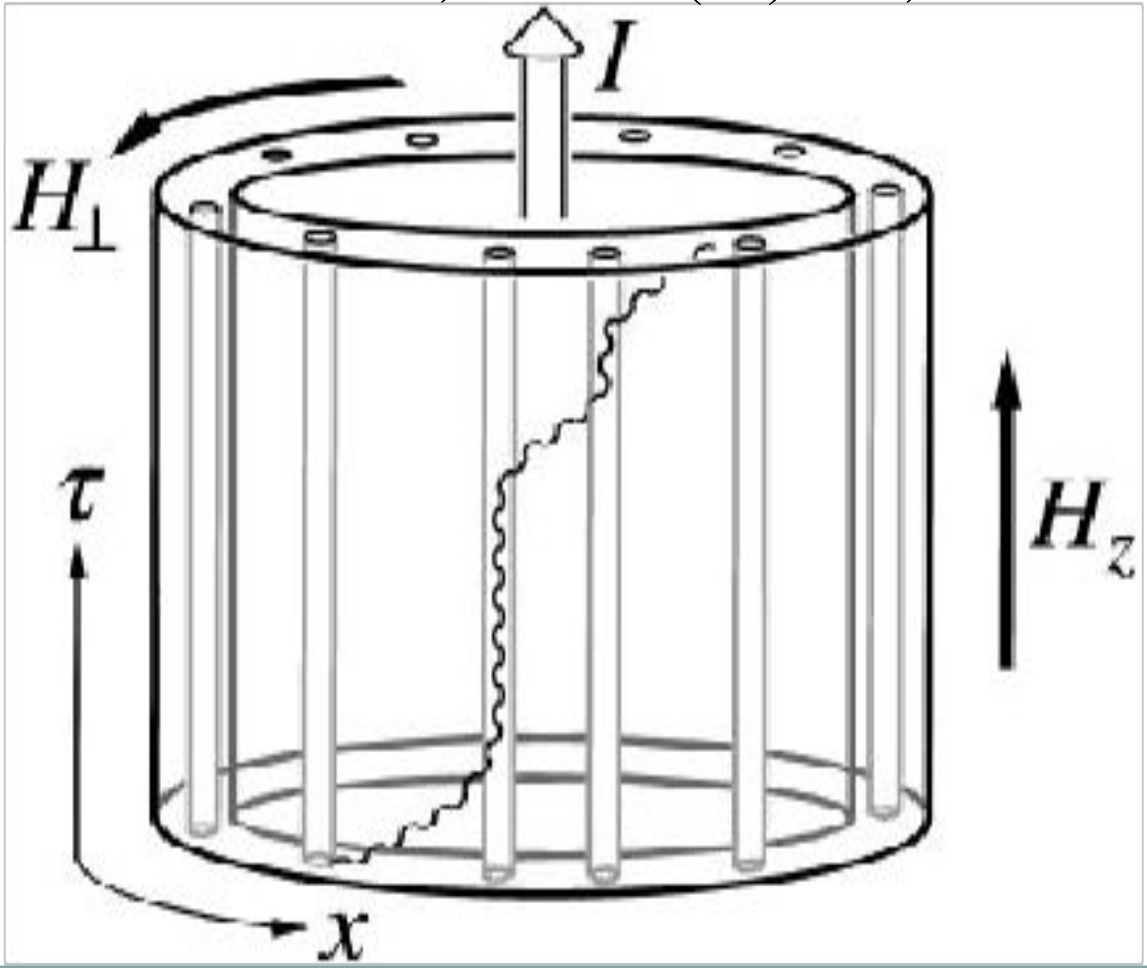
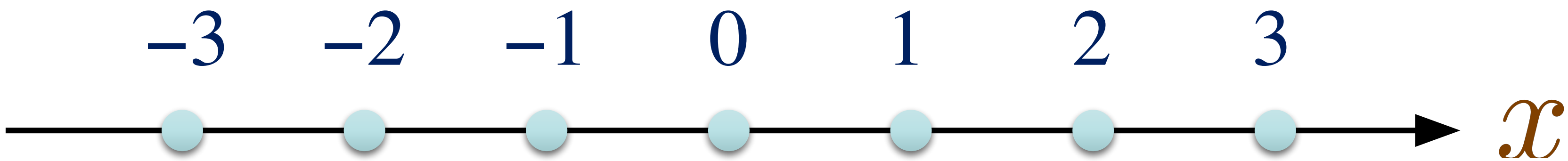


Magnetic flux in a superconducting sheet

Discrete-time quantum walk



$$\begin{array}{cccccc} \cdots & |-1L\rangle & |0L\rangle & |1L\rangle & |2L\rangle & \cdots \\ \cdots & |-1R\rangle & |0R\rangle & |1R\rangle & |2R\rangle & \cdots \end{array}$$

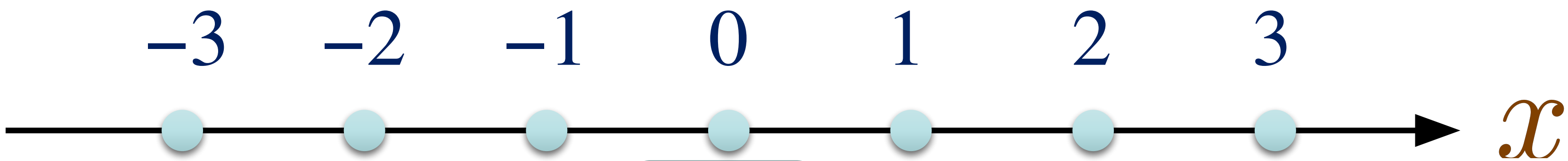
Unitary operators

Coin operator *C*: $C\begin{pmatrix}|0L\rangle \\ |0R\rangle\end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix}|0L\rangle \\ |0R\rangle\end{pmatrix}$

Shift operator *S*: $S|1L\rangle = |0L\rangle$

$S|1R\rangle = |2R\rangle$

Discrete-time quantum walk



$$\cdots \quad |-1L\rangle \quad \boxed{|0L\rangle} \quad |1L\rangle \quad |2L\rangle \quad \cdots$$

$$\cdots \quad |-1R\rangle \quad \boxed{|0R\rangle} \quad |1R\rangle \quad |2R\rangle \quad \cdots$$

$$|\psi(0)\rangle = |0L\rangle$$

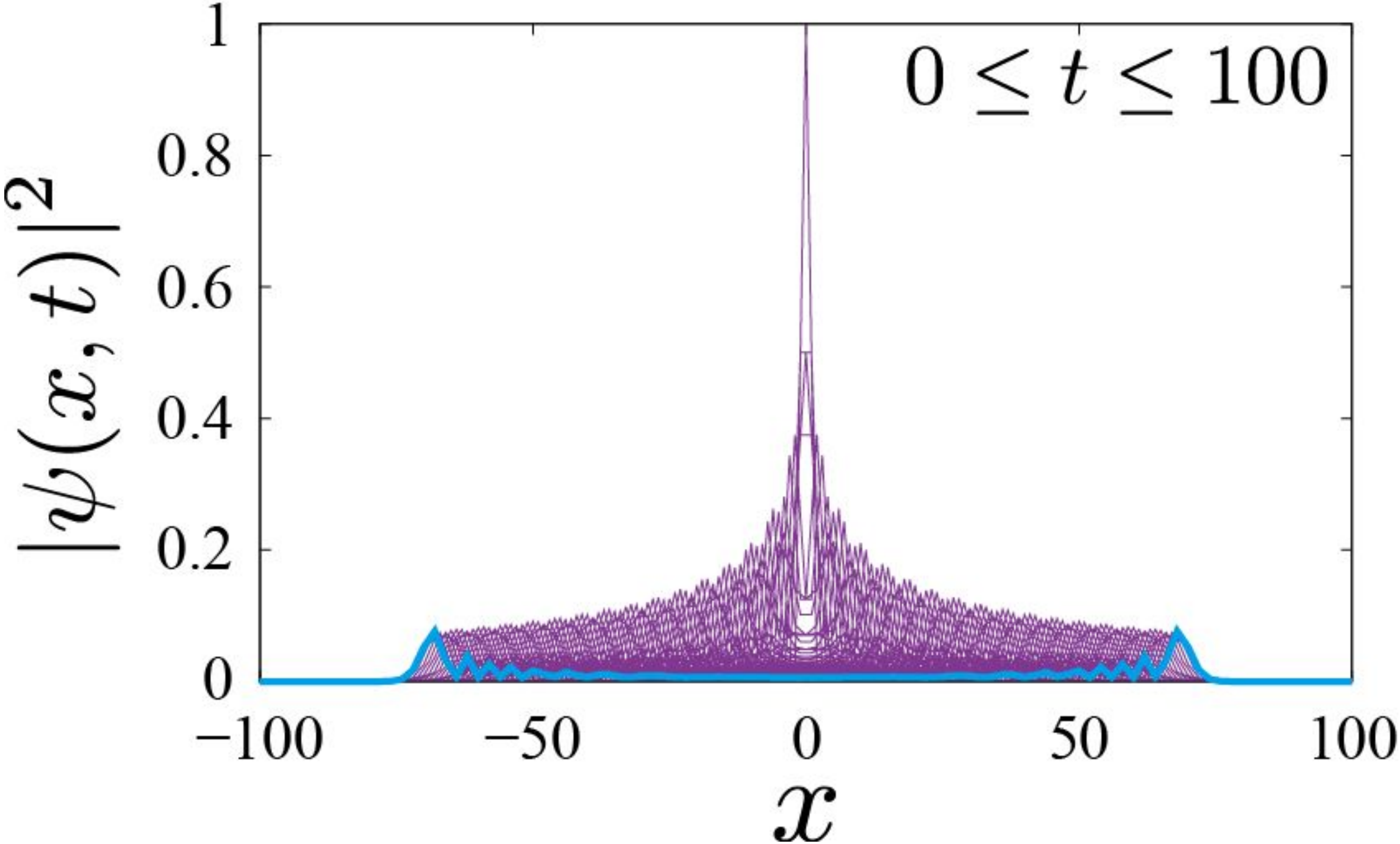
$$C|\psi(0)\rangle = a|0L\rangle + b|0R\rangle$$

$$SC|\psi(0)\rangle = a|-1L\rangle + b|1R\rangle = |\psi(1)\rangle$$

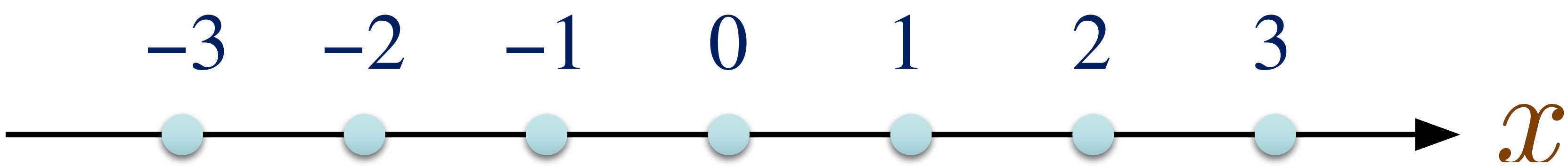
$$U = SC \qquad |\psi(t)\rangle = U^t|\psi(0)\rangle$$

$$H = i\log U \qquad = e^{-iHt}|\psi(0)\rangle$$

Discrete-time quantum walk



Random quantum walk



$$\cdots \quad |-1L\rangle \quad |0L\rangle \quad |1L\rangle \quad |2L\rangle \quad \cdots$$

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Random unitary

Unitary operators

Coin operator C: $\boxed{C}\begin{pmatrix} |0L\rangle \\ |0R\rangle \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} |0L\rangle \\ |0R\rangle \end{pmatrix}$

Shift operator S:
$$S|1L\rangle = |0L\rangle$$
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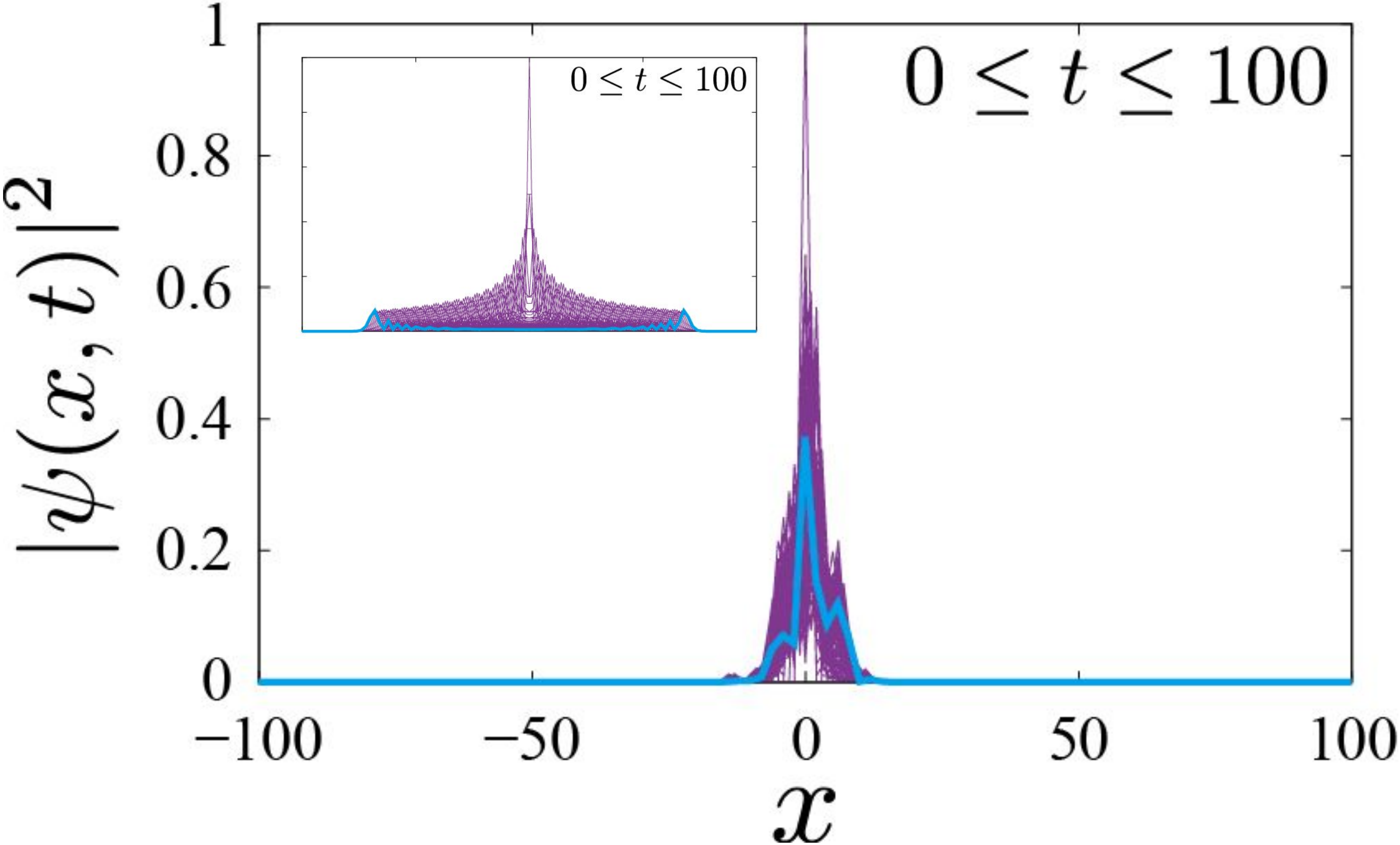
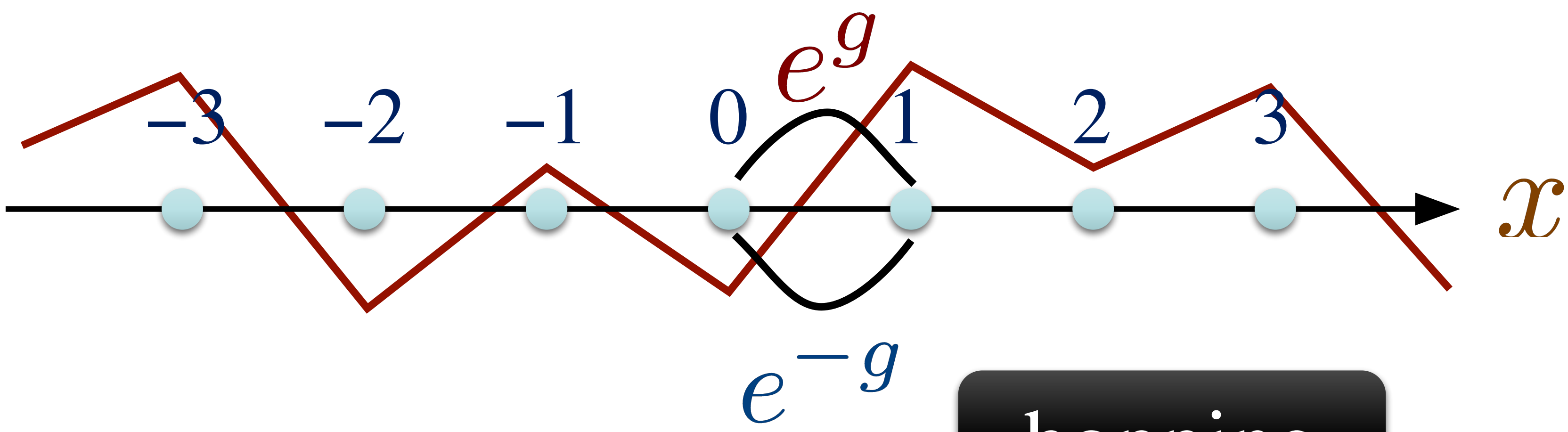
Random quantum walk



1d tight-binding model



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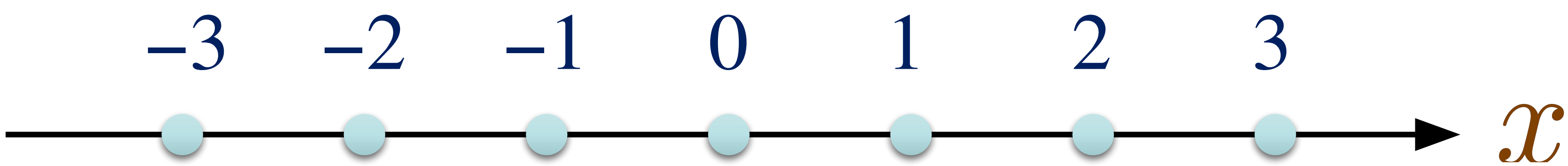
$$+ \sum_x V_x c^\dagger_x c_x$$

real random potential

NH Random quantum walk



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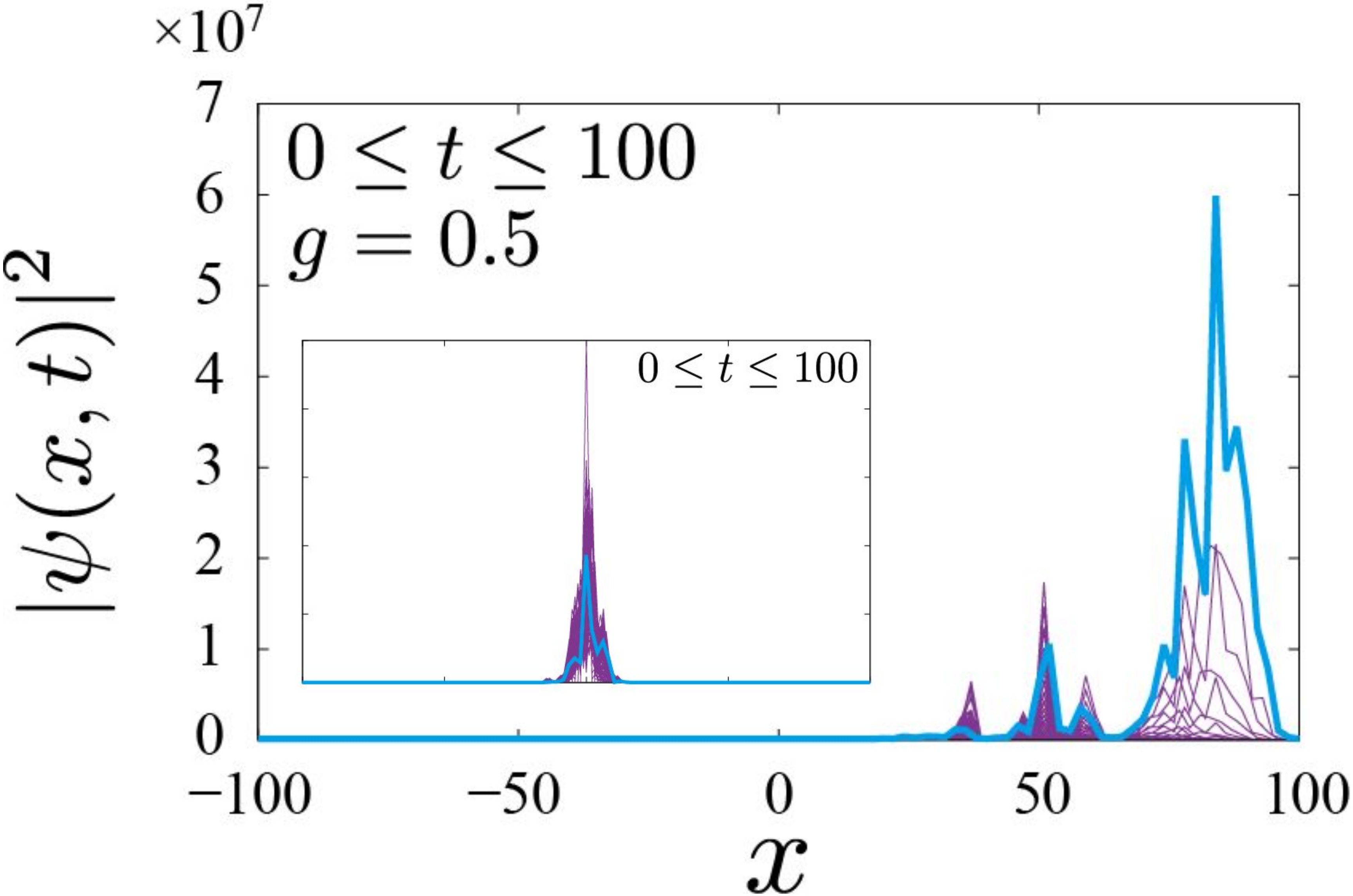
Random unitary

Unitary operators

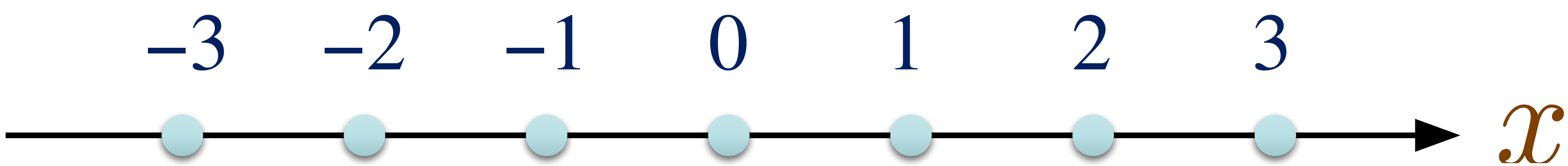
Coin operator C: $$C\begin{pmatrix} |0L\rangle \\ |0R\rangle \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} |0L\rangle \\ |0R\rangle \end{pmatrix}$$

Shift operator S: $$S|1L\rangle = |0L\rangle \times e^{-g}$$
$$S|1R\rangle = |2R\rangle \times e^{g}$$

NH Random quantum walk



Discrete-time quantum walk



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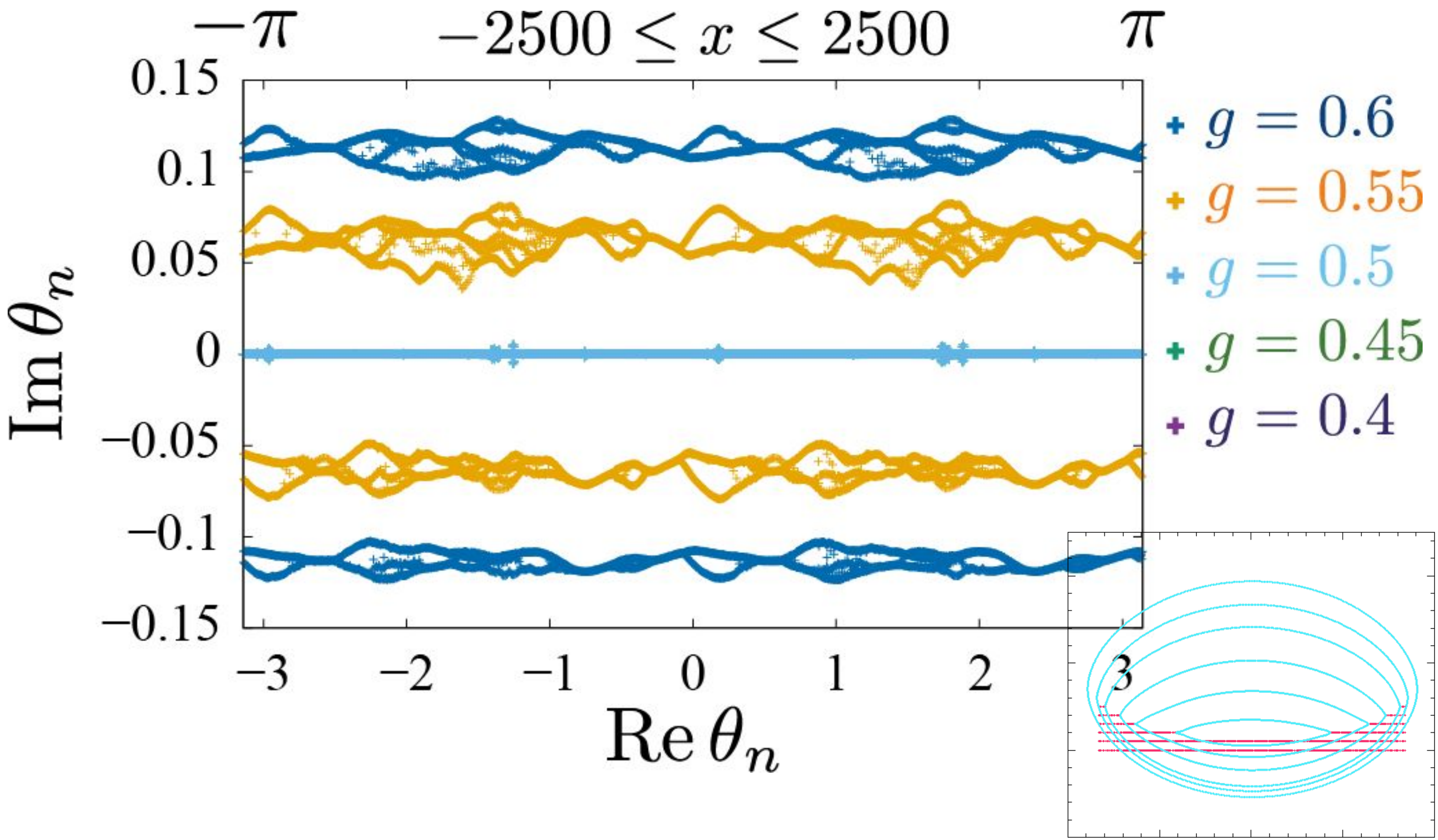
NH Random quantum walk



Summary

Imaginary vector potential

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Anderson localization
Random quantum walk

Non-Hermiticity
→ Delocalizing

Randomness
→ Localizing