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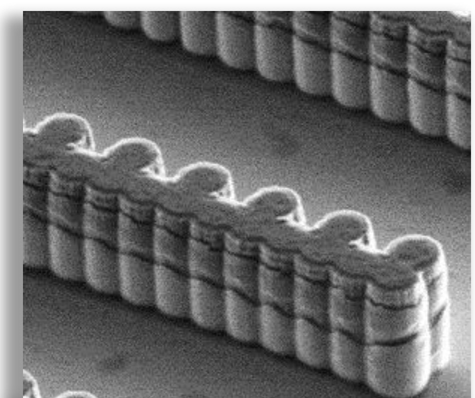
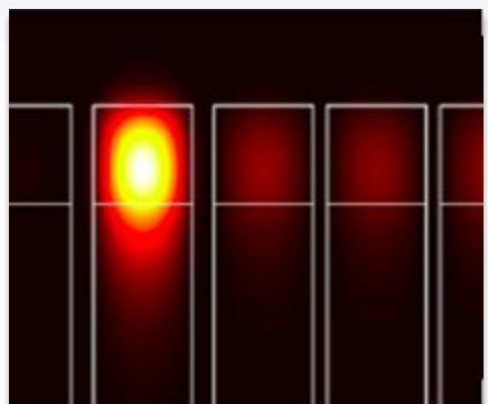
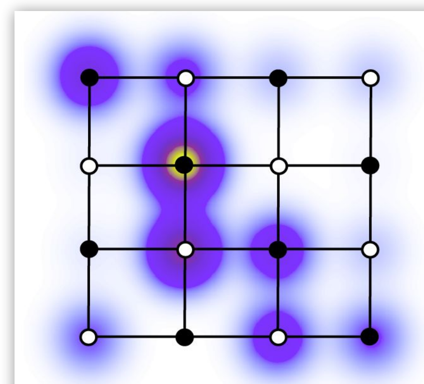
Non-Hermitian Parity-Hole Symmetry, Flat Band, and Linear Localization

Li Ge

City University of New York (CUNY)







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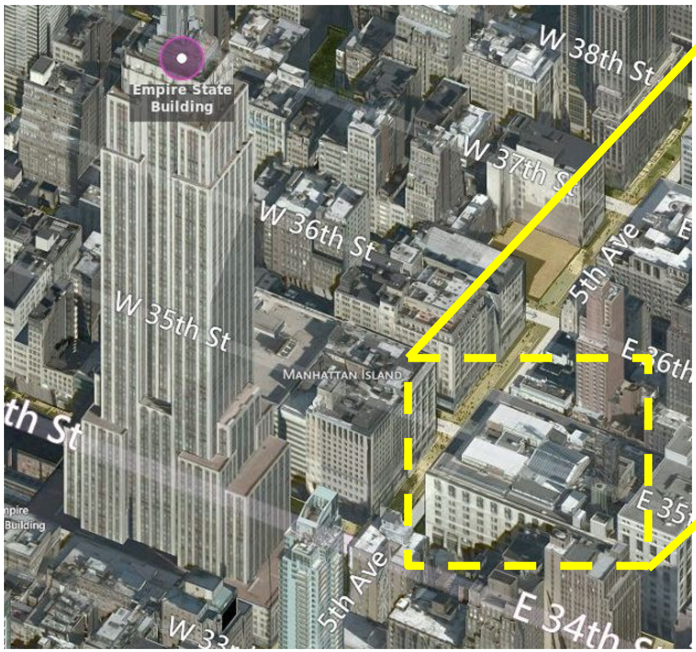


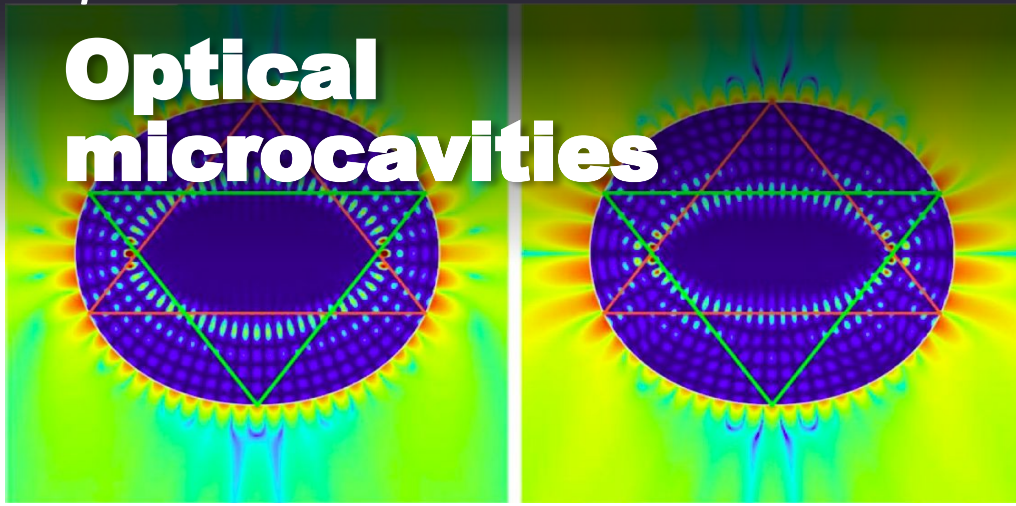
College of Staten Island Seal

Motto	Opportunity and Challenge
Type	Public
Established	1956
President	William J. Fritz, PhD.
Provost	Gary Reichard, PhD
Academic staff	1,239 (Fall 2015)[1]
Administrative staff	1,138 (Fall 2015)[1]
Students	13,798 (Fall 2015)[1]
Mascot	Dolphin
Affiliations	The City University of New York
Website	www.csi.cuny.edu

The City University of New York College of Staten Island

Special Issue on

Optical microcavities



PHOTONICS Research

2016 Impact Factor

4.679 9/92 Optics

Published in Vol 5, Iss 6, Dec. 2017

Guest Editors

Li Ge, *City University of New York, USA*

Liang Feng, *University of Pennsylvania, USA*

Harald G. L. Schwefel, *University of Otago, New Zealand*

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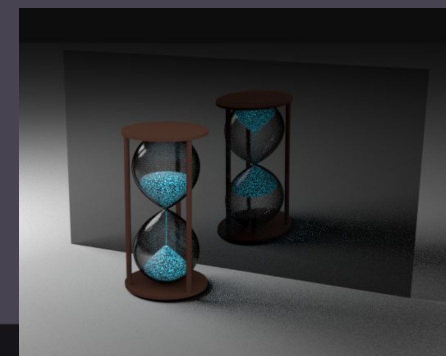
Focus | 30 November 2017

PT symmetry



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REVIEW ARTICLE | FOCUS

https://doi.org/10.1038/s41566-017-0031-1

nature photonics

Non-Hermitian photonics based on parity–time symmetry and beyond

Liang Feng[1]*, Ramy El-Ganainy[2,3] and Li Ge[4,5]*

Outline

- Non-Hermitian particle-hole symmetry and anti-PT symmetry
- Non-Hermitian zero modes
- Non-Hermitian flat bands and defect states
- Linear localization
- Summary

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What is anti-PT symmetry?

LG and Tureci, PRA 88, 053810 (2012)

PT symmetry

$$n(-\boldsymbol{x}) = n^*(\boldsymbol{x})$$

$$n(-\boldsymbol{x}) = -n^*(\boldsymbol{x})$$

Anti-PT symmetry

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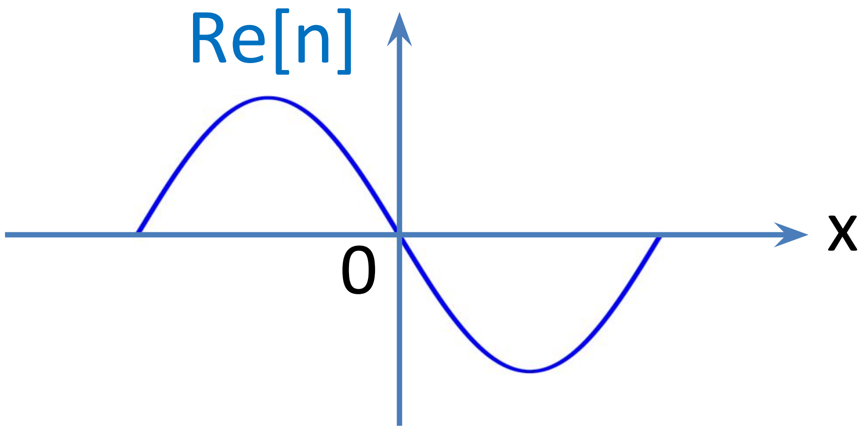
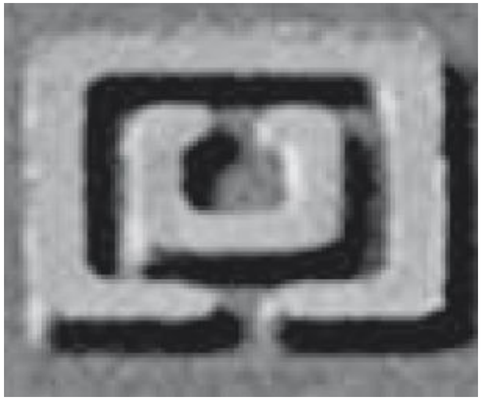
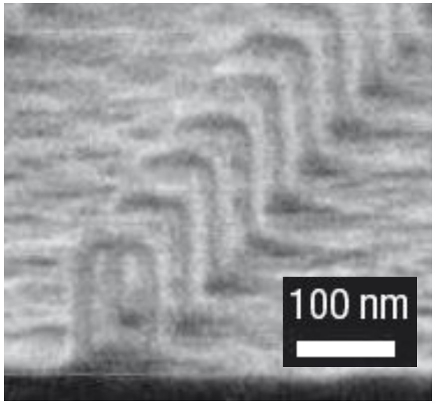
Anti-PT symmetry

$$\mu(-x) = -\mu(x)$$

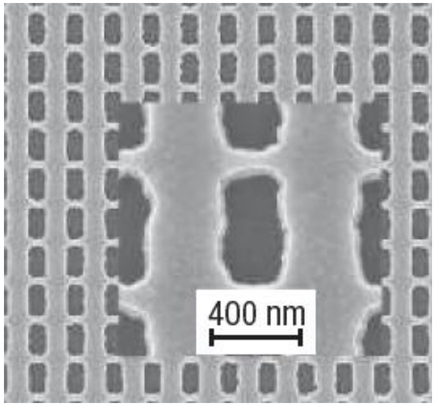




Yen et al Science 303, 1494 (04)



Zhang et al PRL 94, 37402 (05)



Dolling et al. Opt. Lett. 31, 1800 (06)

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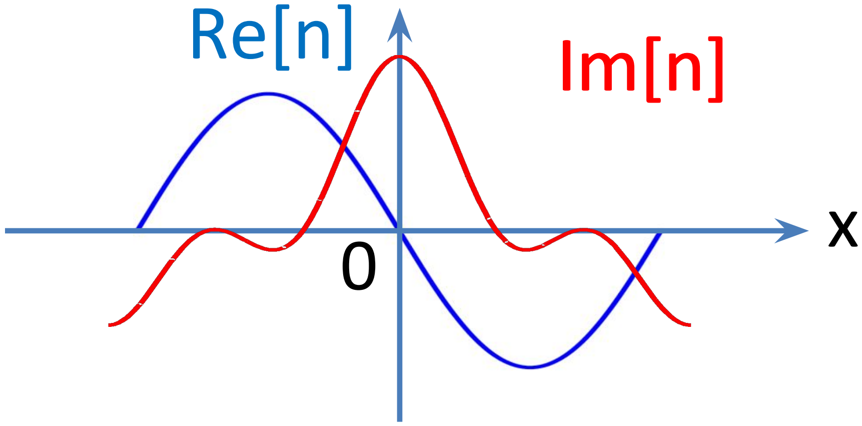
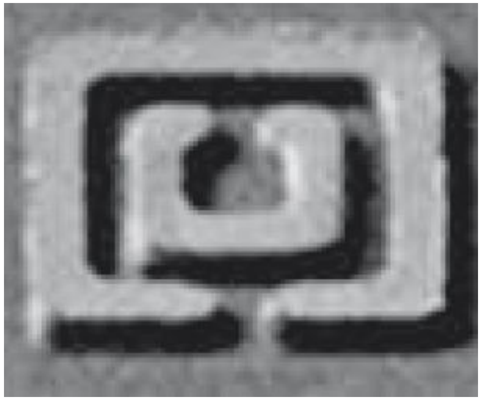
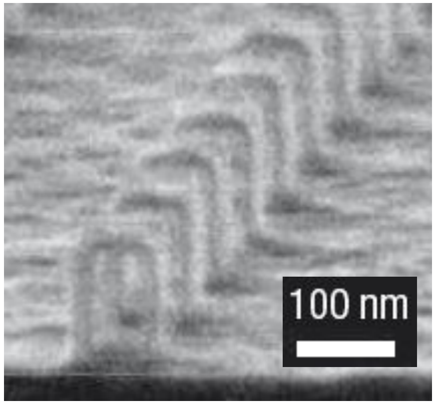
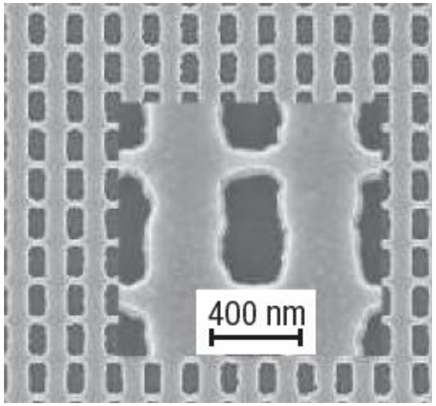




Yen et al Science 303, 1494 (04)



Zhang et al PRL 94, 37402 (05)



Dolling et al. Opt. Lett. 31, 1800 (06)

What is anti-PT symmetry?

- "Optical potential" is anti-PT:

$$\{PT, n(x)\} = PTn(x) + n(x)PT = 0$$

- Next step: Whole system is anti-PT

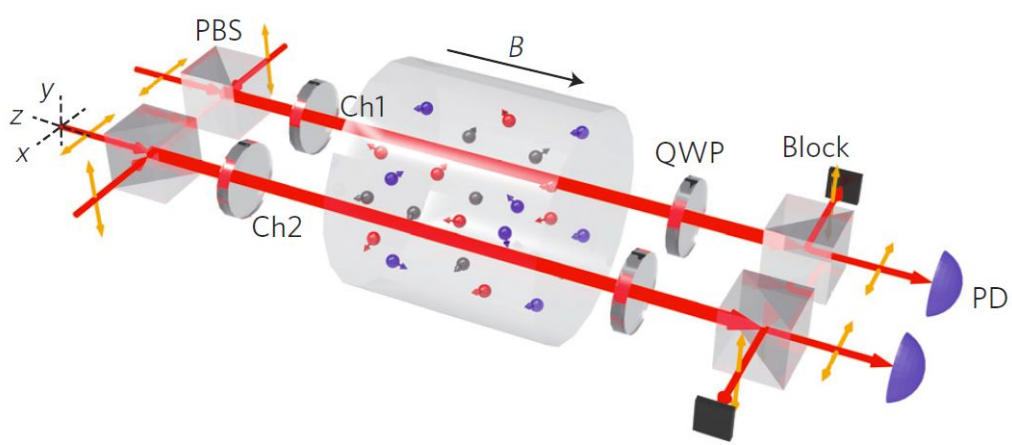
$$\{PT, H(x)\} = 0$$

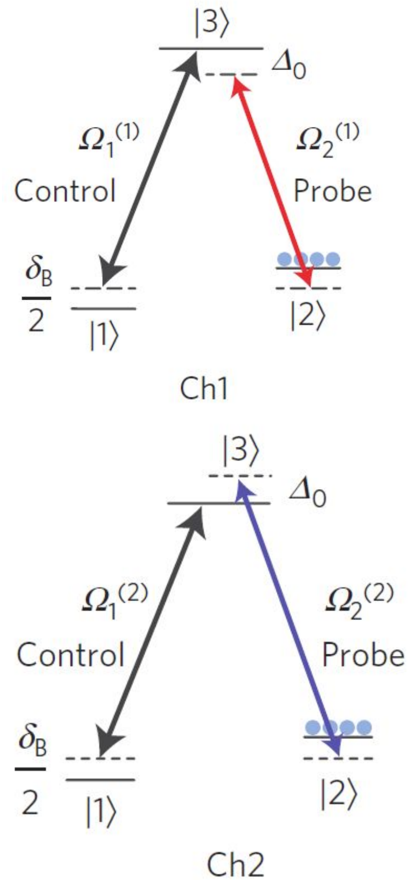
$$H = \begin{pmatrix} \Delta + i\gamma & g \\ -g^* & -\Delta + i\gamma \end{pmatrix} \quad (\Delta, \gamma \in \boldsymbol{R})$$

  – The additional step: nontrivial couplings

Realizing anti-PT symmetry

$$H = \begin{pmatrix} \Delta + i\gamma & g \\ -g^* & -\Delta + i\gamma \end{pmatrix}$$





$$H = \begin{pmatrix} \Delta + i\gamma & i\Gamma e^{-2i\Delta t} \\ i\Gamma e^{2i\Delta t} & -\Delta + i\gamma \end{pmatrix}$$

Peng et al., Nat. Phys. 12, 1139 (2016)

Why anti-PT symmetry is interesting?

- It has completely different physical consequences from PT
- An example of non-Hermitian particle-hole symmetry

$$\{XT, H\} = 0$$

the realization of a *fermionic* symmetry in a (non-Hermitian) *bosonic* system LG, PRA 95, 023812 (2017)

- It gives rise to non-Hermitian zero modes, analogous to Majorana zero modes in condensed matter physics

$$\mathrm{Re}[E] = 0$$

$$\{XT, H\} = 0 \begin{cases} \text{symmetric:} & E_i = -E_i^*, \quad \Psi_i \propto XT\Psi_i \\ \text{spontaneously broken:} & E_1 = -E_2^*, \quad \Psi_1 \propto XT\Psi_2 \end{cases}$$

Outline

- Non-Hermitian particle-hole symmetry and anti-PT symmetry
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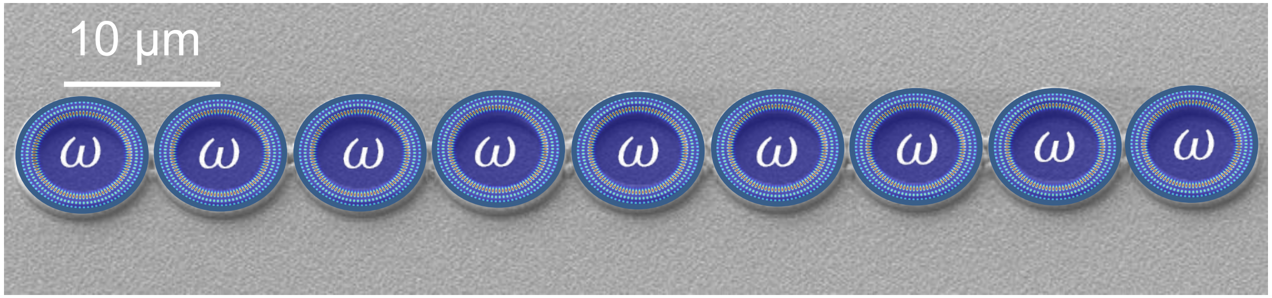
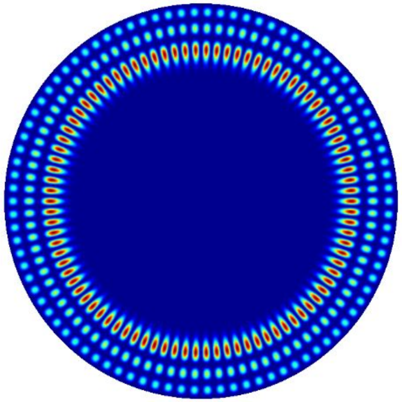
Zero modes in Hermitian systems

- Zero modes are eigenstates of “zero” energy respect to a well-defined energy level (e.g., Fermi level)
- In a coupled system with identical subsystems, the zero can be chosen as the energy of a particular interesting state of the subsystems.





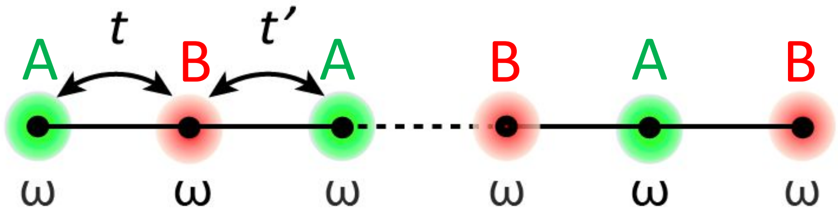
Symmetry-protected zero modes in Hermitian systems

- Chiral symmetry (aka. sublattice symmetry)

$$\{H, \mathcal{C}\} = 0 \rightarrow \begin{Bmatrix} \mathcal{C}\Psi_n = \Psi_m, \\ \omega_n = -\omega_m \end{Bmatrix}$$ (*C*: a linear operator)

Zero modes: $n = m \rightarrow \omega_n = 0$

- Easy to achieve in lattice systems, e.g., 1D SSH model



$$H\Psi = \begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix} \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} \quad \mathcal{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \{H, \mathcal{C}\} = 0$$

Symmetry-protected zero modes in Hermitian systems

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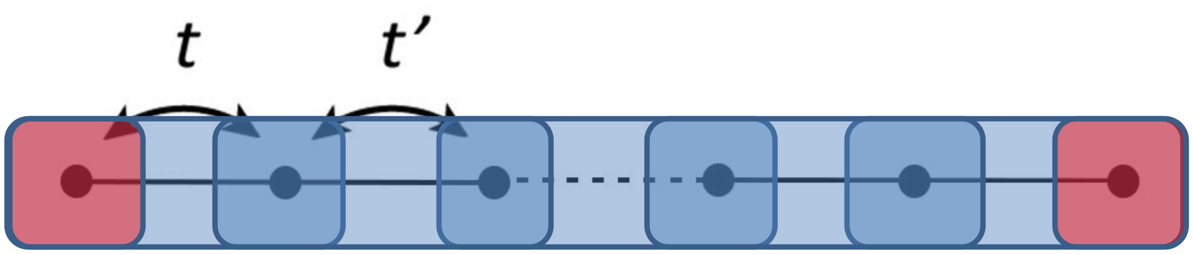
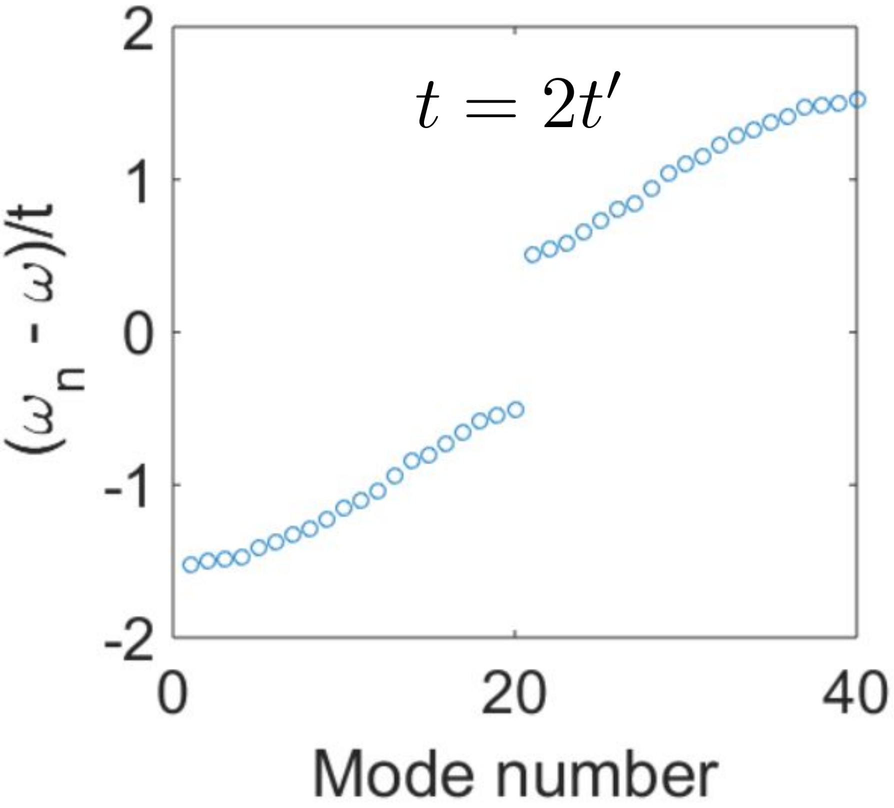


– When t>t', no zero modes ("dimerized" chain)
– When t<t', two zero modes

Example of zero modes

- 1D Su-Schrieffer-Heeger (SSH) model



Symmetry-protected zero modes in Hermitian systems

- Particle-hole symmetry (with *C* and complex conjugation *K)*:

$$\{H, \mathcal{C}\mathcal{K}\} = 0 \rightarrow \begin{Bmatrix} \mathcal{C}\mathcal{K}\Psi_n = \Psi_m, \\ \omega_n = -\omega_m \end{Bmatrix}$$

Zero modes: $n = m \rightarrow \omega_n = 0$

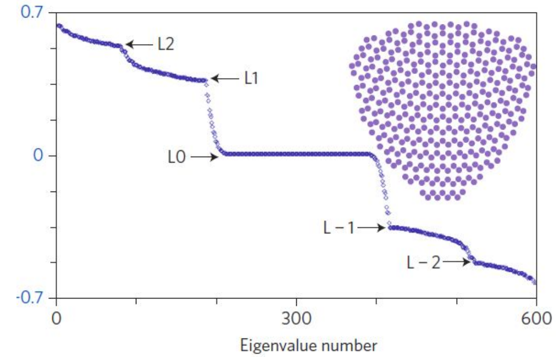
  - More restrictive (e.g., superconductors)

Zero modes in non-Hermitian systems (e.g., photonics)

nature photonics | ARTICLES
PUBLISHED ONLINE: 9 DECEMBER 2012 | DOI: 10.1038/NPHOTON.2012.302

Strain-induced pseudomagnetic field and photonic Landau levels in dielectric structures

Mikael C. Rechtsman[1†*], Julia M. Zeuner[2†], Andreas Tünnermann[2], Stefan Nolte[2], Mordechai Segev[1] and Alexander Szameit[2]



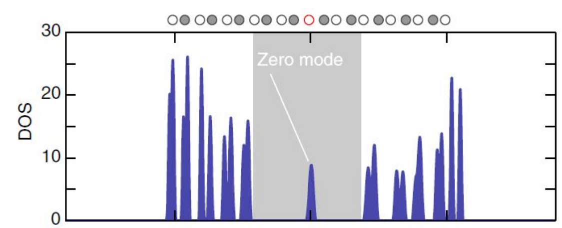
All originate from the chiral symmetry of the underlying Hermitian systems!

ARTICLE

Received 29 Jul 2014 | Accepted 19 Feb 2015 | Published 2 Apr 2015
DOI: 10.1038/ncomms7710 OPEN

Selective enhancement of topologically induced interface states in a dielectric resonator chain

Charles Poli[1], Matthieu Bellec[2], Ulrich Kuhl[2], Fabrice Mortessagne[2] & Henning Schomerus[1]



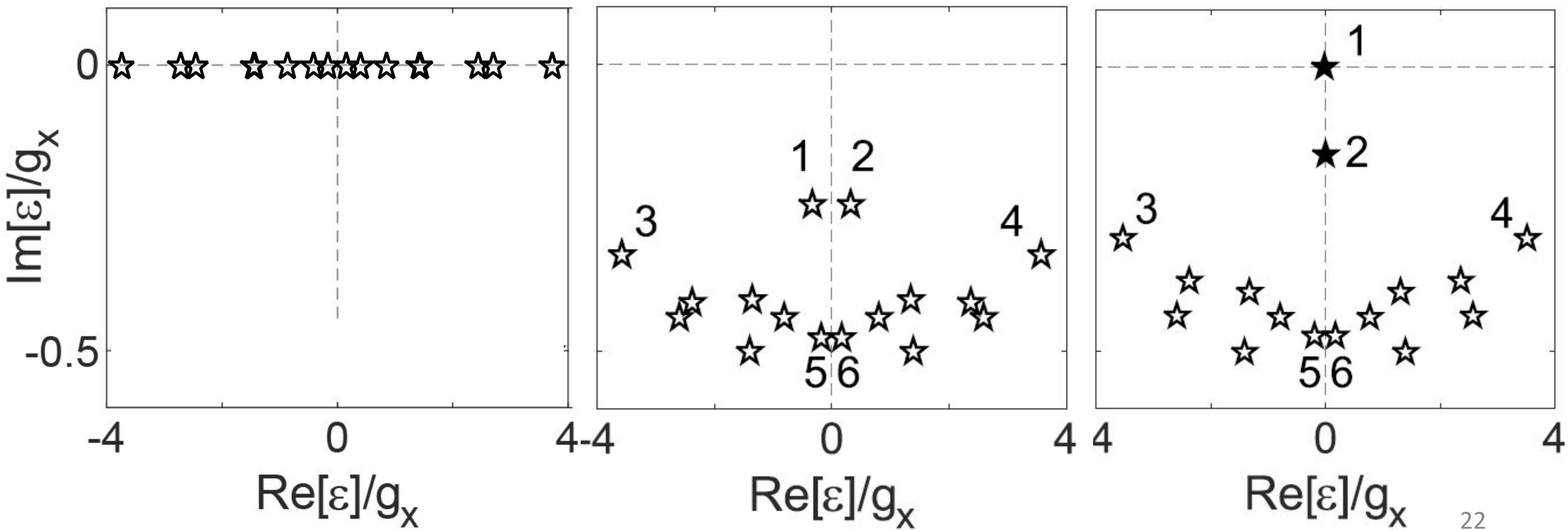
Symmetry-protected zero modes unique to non-Hermitian system

- Particle-hole symmetry:

Zero mode: $\omega_n = -\omega_m$ (Hermitian case)

$\omega_n = -\omega_m^*$ (non-Hermitian case)

$$n = m \rightarrow \mathrm{Re}[\omega_n] = 0$$

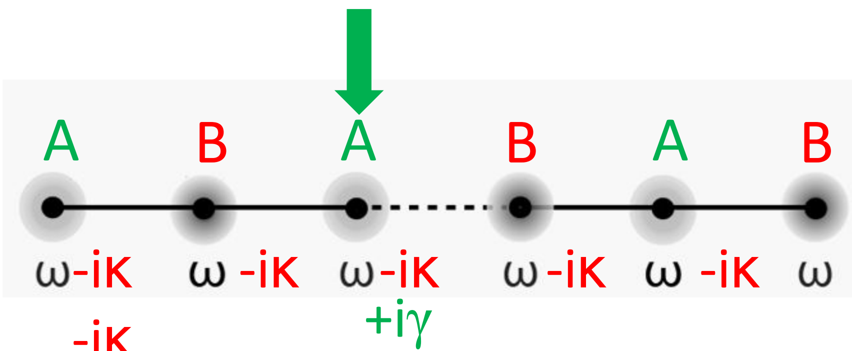


LG, PRA 95, 023812 (2017)

Physical realization of non-Hermitian particle-hole (NHPH) symmetry

- Use existing photonic platform
- Should be straightforward to implement non-Hermitian perturbations experimentally
- Candidate: loss and localized pump on a lattice that forms two sublattices

LG, PRA 95, 023812 (2017)



$$H\Psi = \begin{pmatrix} iD_A & T \\ T^\dagger & iD_B \end{pmatrix} \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix}$$

[redacted]

Key questions

Q: Can photonics zero modes be used for topological quantum computing as their condensed matter counterpart (e.g., Majoranas)?

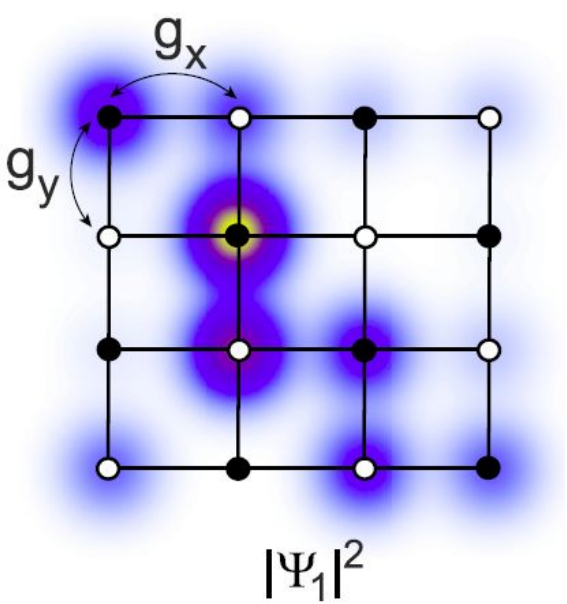
A: Too early to tell …

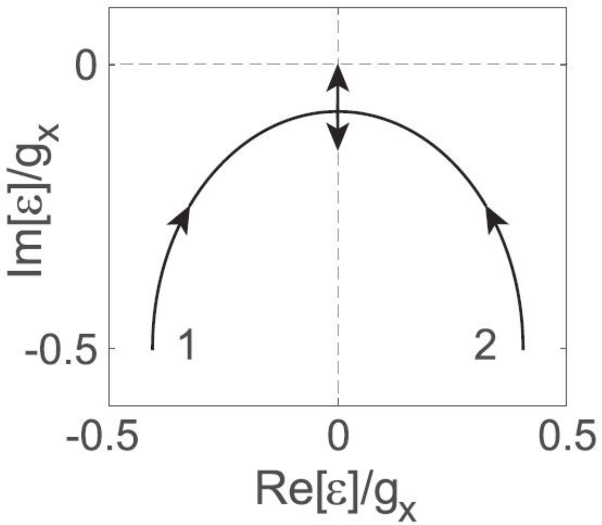
Q: Can photonic zero modes inspire a better design and enable new functionalities of photonic systems?

A: Yes!

Zero-mode lasers with tunable spatial profiles

- Building block: optical microcavities (micro-disk, micro-ring, micro-toroid, etc.)

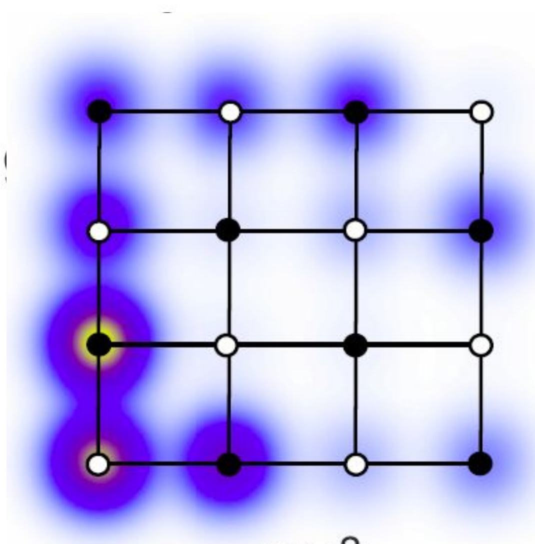




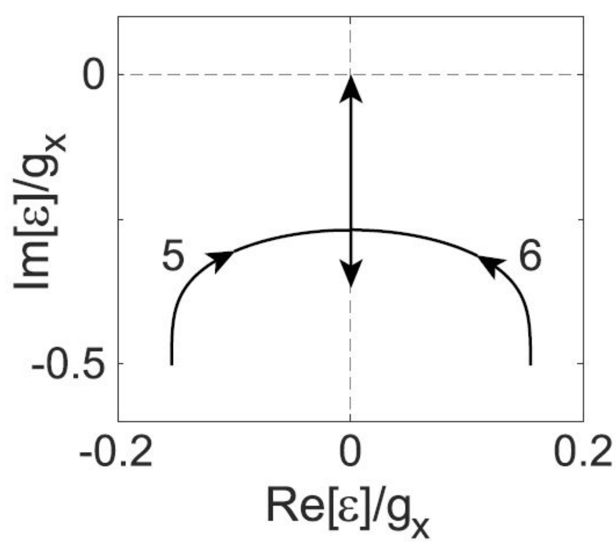
LG, PRA 95, 023812 (2017)

Zero-mode lasers with tunable spatial profiles

- Building block: optical microcavities (micro-disk, micro-ring, micro-toroid, etc.)
- Different pump configurations lead to zero modes lasing with different spatial profiles



$|\Psi_5|^2$



LG, PRA 95, 023812 (2017)

Zero-mode lasers with tunable spatial profiles

Advantages:

- May be used for spatial encoding in telecommunication
- Symmetry protection against position and coupling disorders

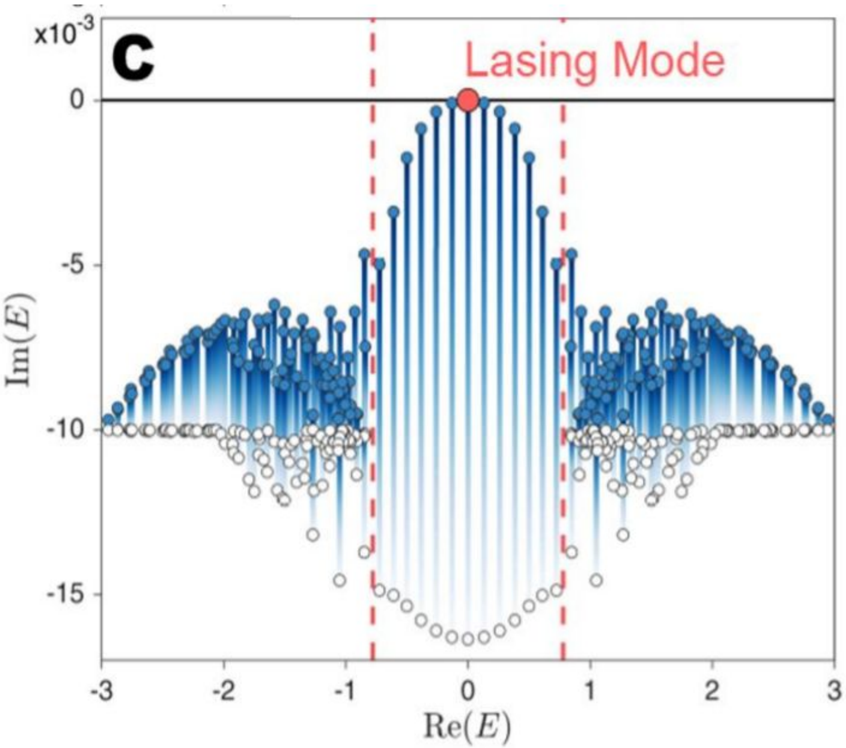
$$H\Psi = \begin{pmatrix} iD_A & T \\ T^\dagger & iD_B \end{pmatrix} \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix}$$

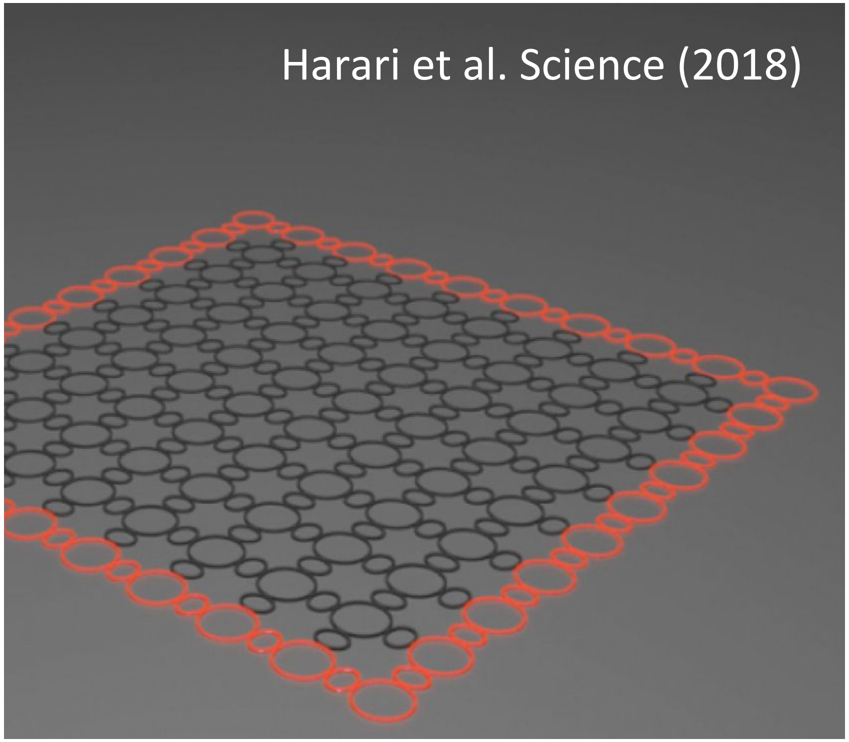
[redacted]

- Robust single-mode performance
- Robust against cavity detuning and higher-order couplings

LG, PRA 95, 023812 (2017)

Topological insulator laser





NHPH symmetry provides the symmetry protection of the chiral edge mode.

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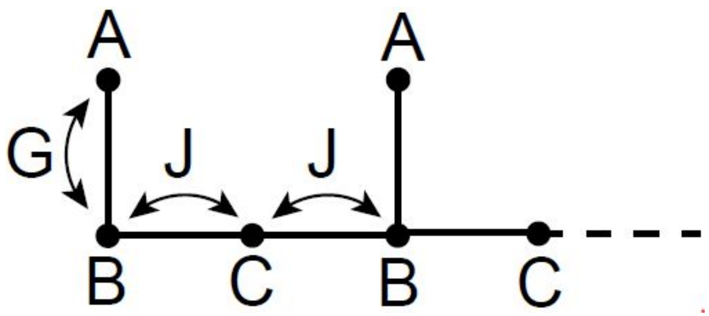
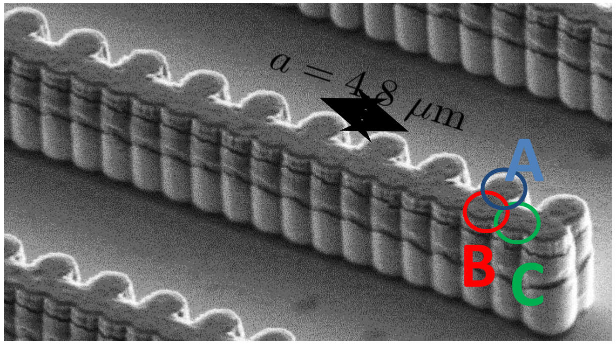
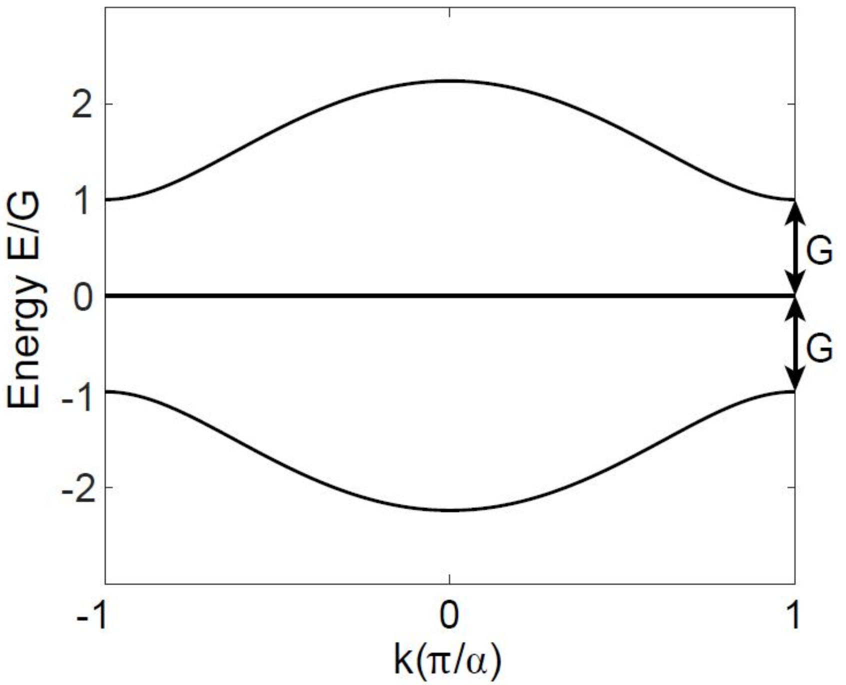
What is a flat band?

- Dispersionless: $\omega(k) = const$
- One example: A Lieb lattice







Baboux, LG et al, PRL 116, 066402 (2015)

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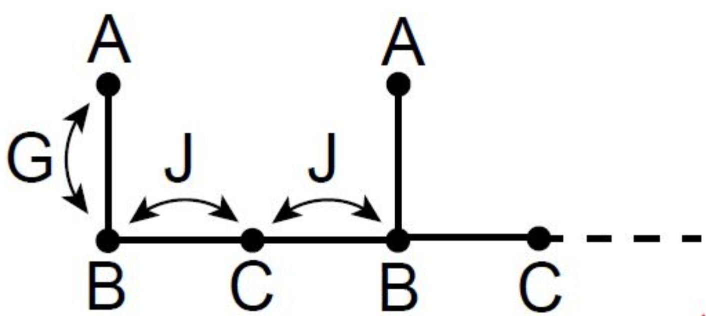
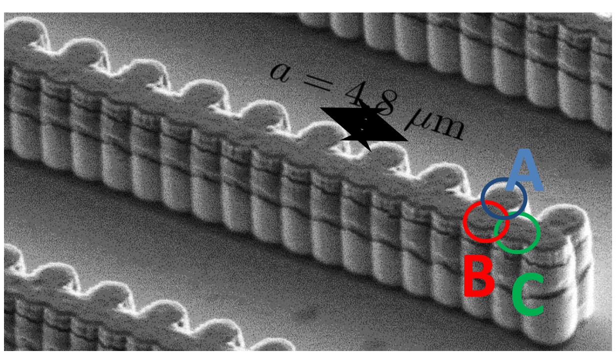
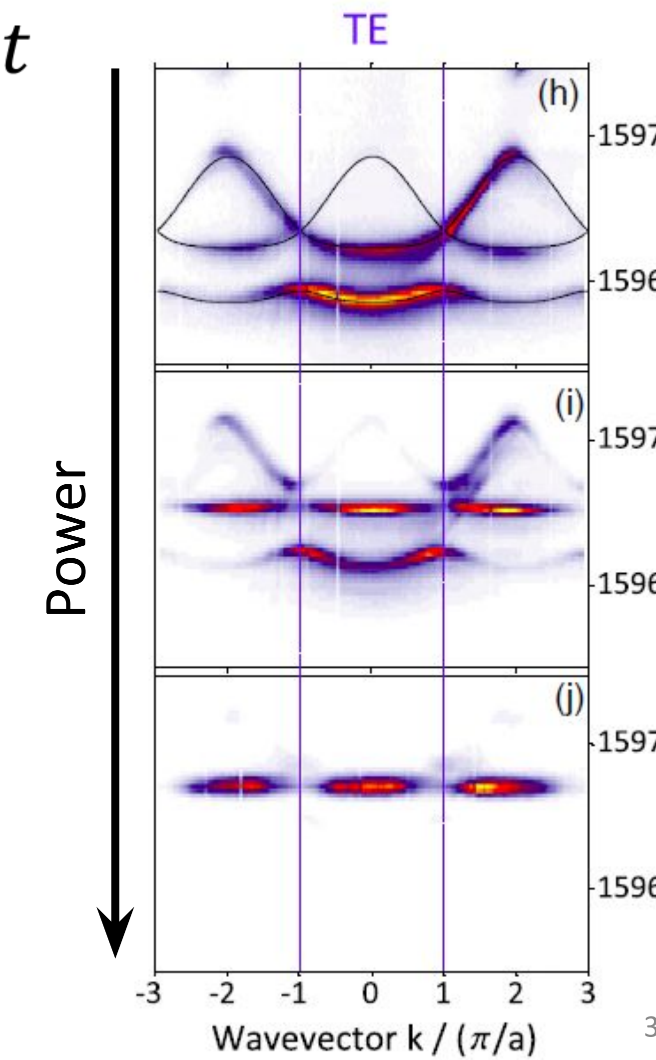


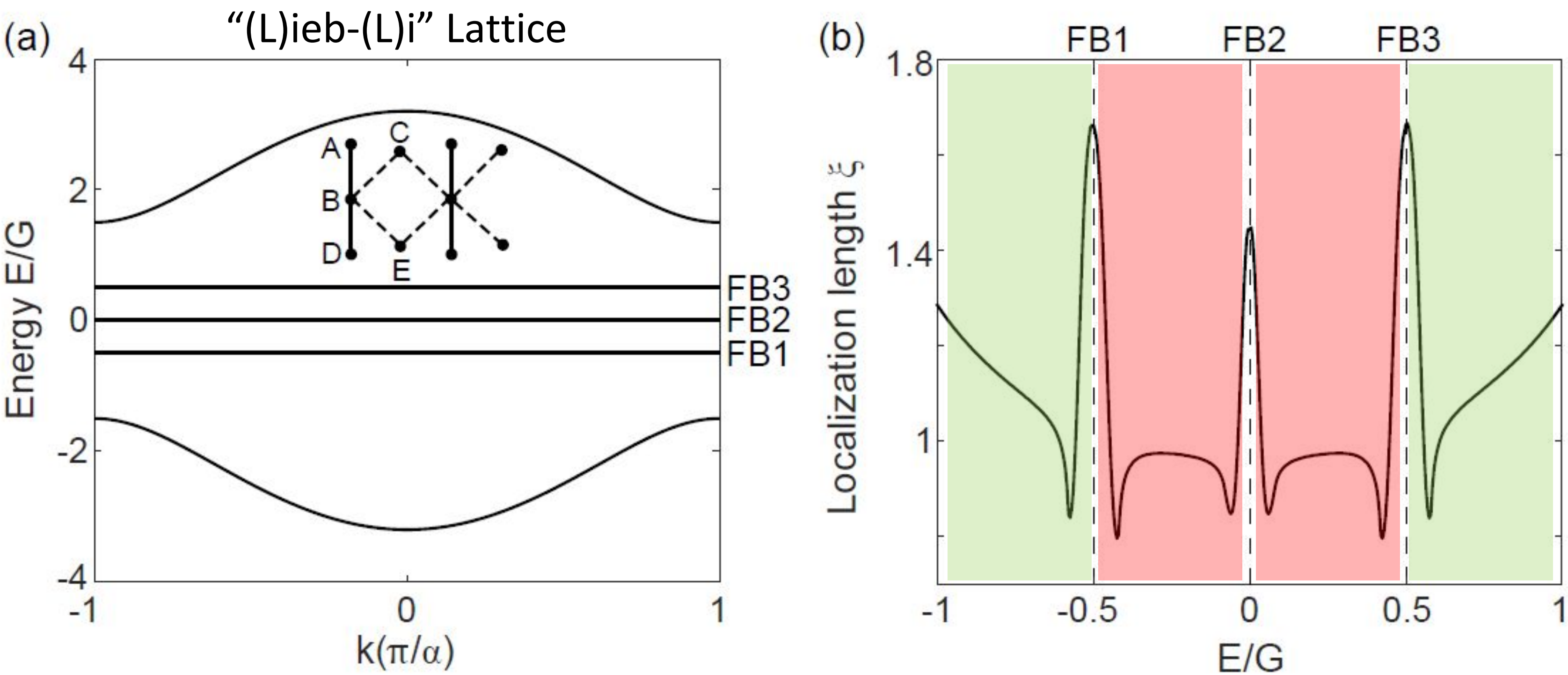




Baboux, LG et al, PRL 116, 066402 (2015)

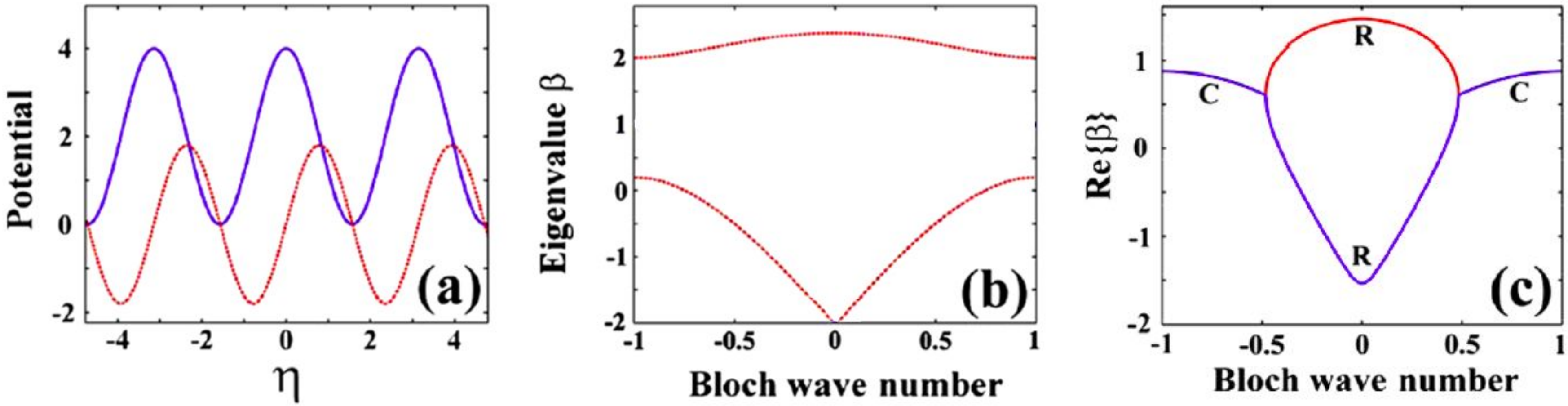
Localization in a flat-band system

- Typical: one localization minimum in a band gap
- Anomalous: more than one such minimum



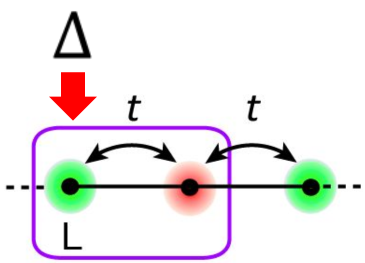
LG, Ann. Phys. 527, 201600182 (2017)

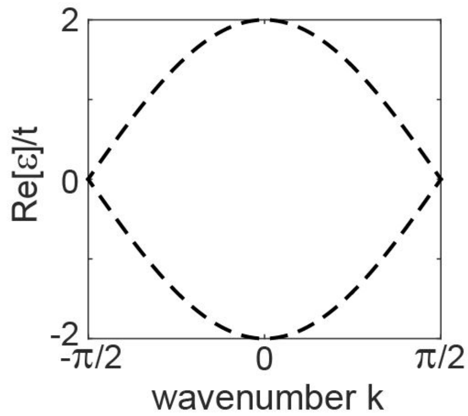
NHPH symmetry induced flat band



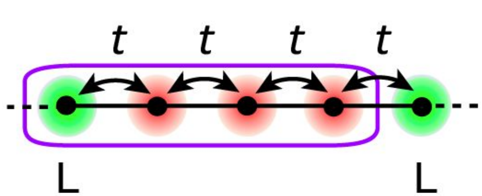
Makris, El-Ganainy, and Christodoulides, PRL 100, 103904 (2008)

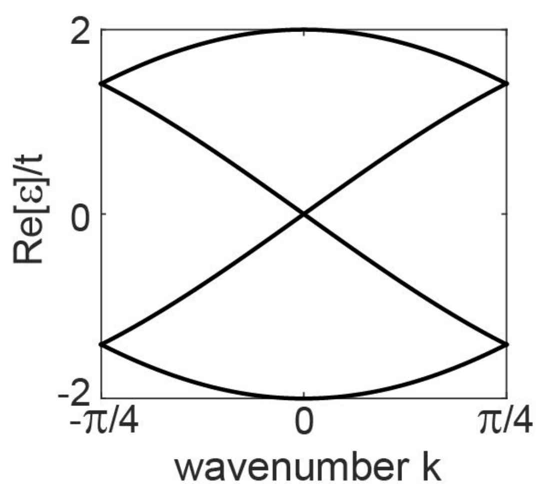
NHPH symmetry induced flat band





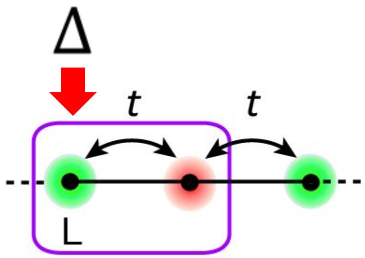
Increasing loss on one site

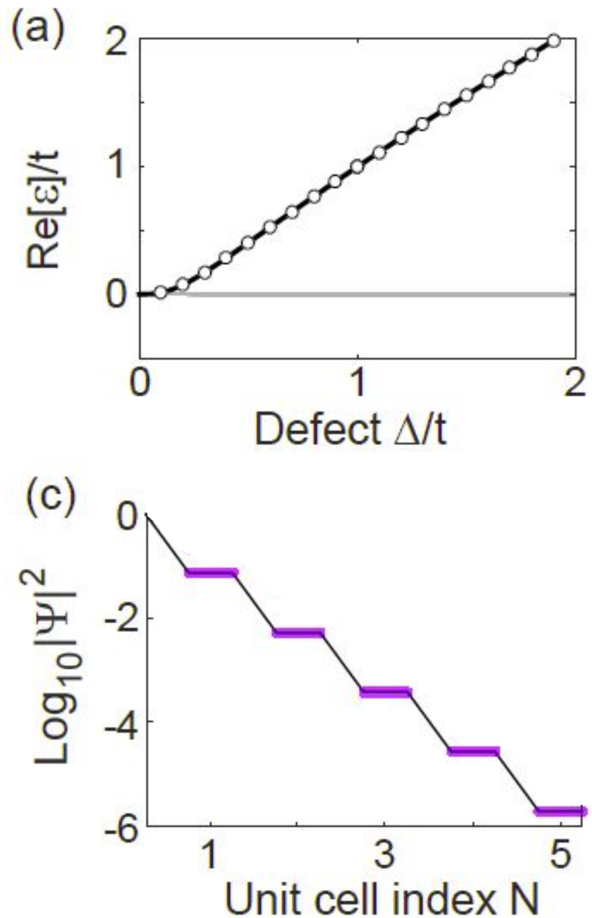




LG et al. PRL 120, 093901 (2018)

Defect states in a NHPH symmetry induced flat band

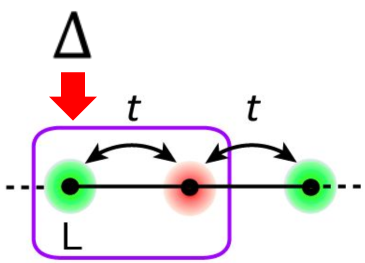




"Stretchable chain" with a soft link and a rigid link

LG et al. PRL 120, 093901 (2018)

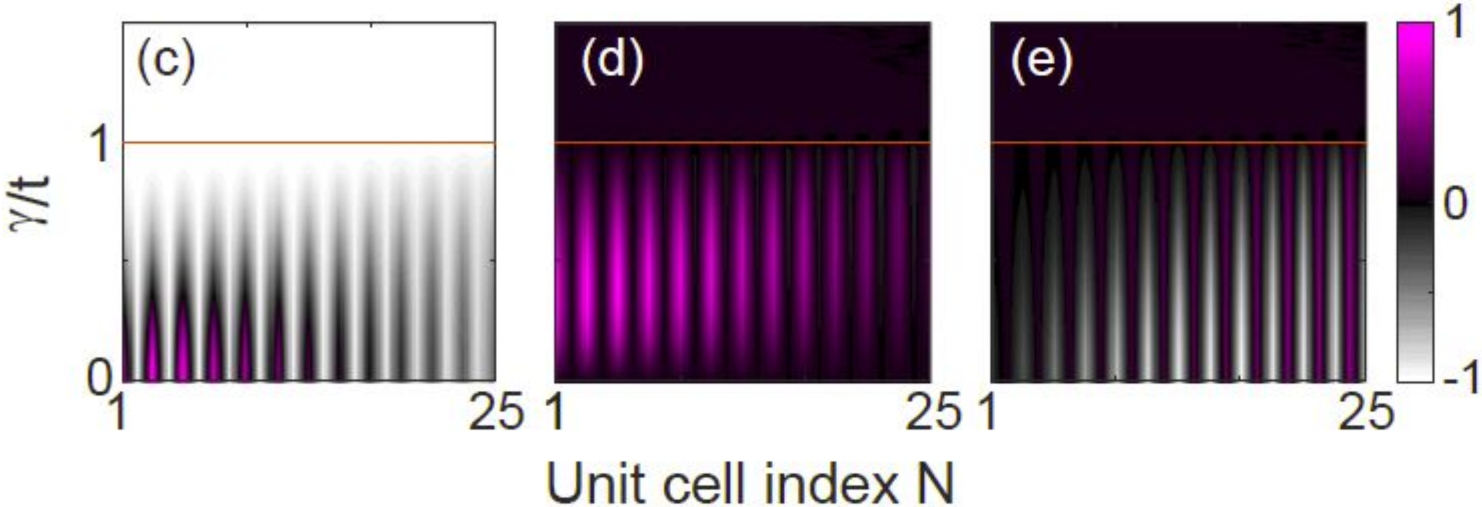
Defect states in a NHPH symmetry induced flat band



Anomalous spin alignment

$$H_2 = t(1+\cos ka)\,\sigma_x - t\sin ka\,\sigma_y + i\gamma\,\sigma_z \equiv -\boldsymbol{h}\cdot\boldsymbol{\sigma}$$

$$\boldsymbol{h}(\gamma) = [-t(1+\cos ka),\, t\sin ka,\, -i\gamma]$$



<u>LG</u> et al. PRL 120, 093901 (2018)

Non-Hermitian flatband

- What if we require $Im[\omega(k)]$ is also flat?
  - Trivial case: uniform loss (or gain) on a Hermitian lattice with a flat band: $\omega(k) \to \omega(k) \pm i\gamma$
  - Interesting case: with a Wannier function $W_n(x)$ that is also an eigenstate of the whole H [not $H_{Bloch}(k)$]

$$\Psi_n(x;k) = \sum_j e^{ikaj} W_n(x - ja)$$

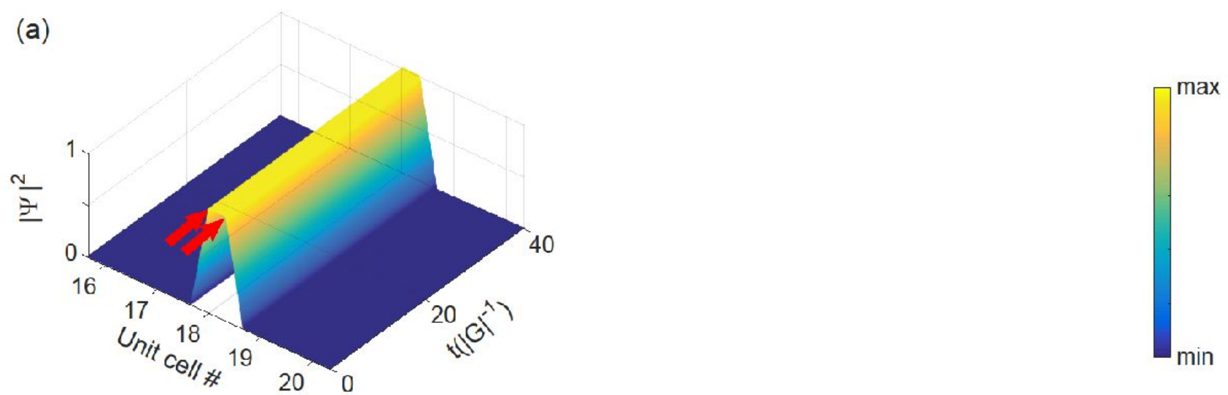
k: wave vector
j: lattice site index
n: band index

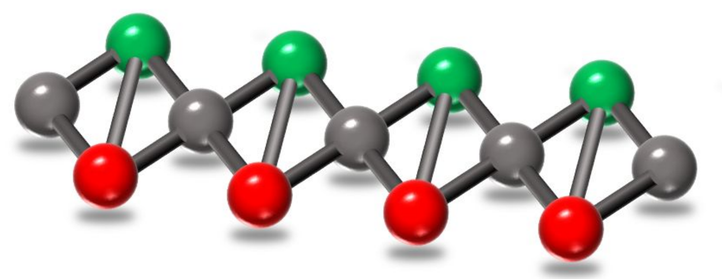
$$H_0 \Psi_n(x;k) = \omega_n(k) \Psi_n(x;k)$$

<u>LG</u>, Photon. Res. 6, A10 (2018)

Non-Hermitian flatband

- Power increase can be polynomial in time: as a result of exceptional points





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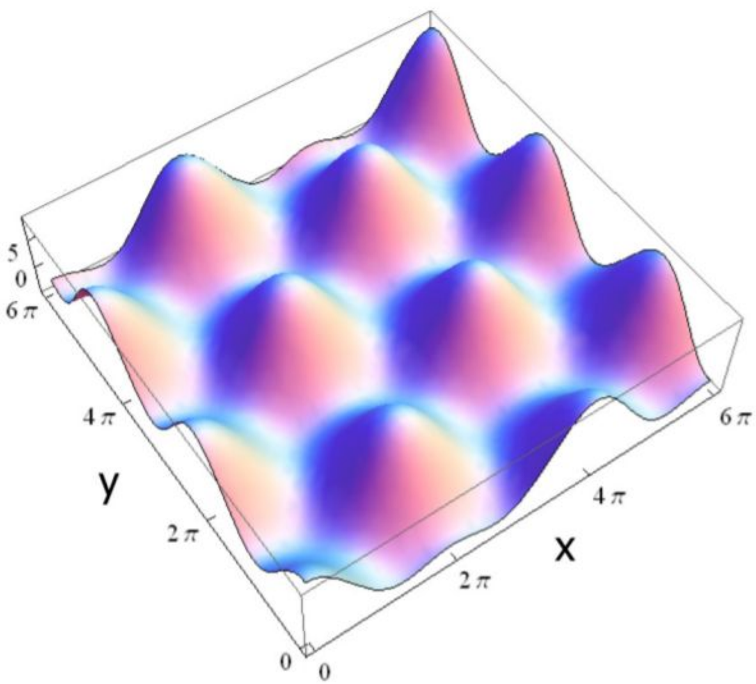
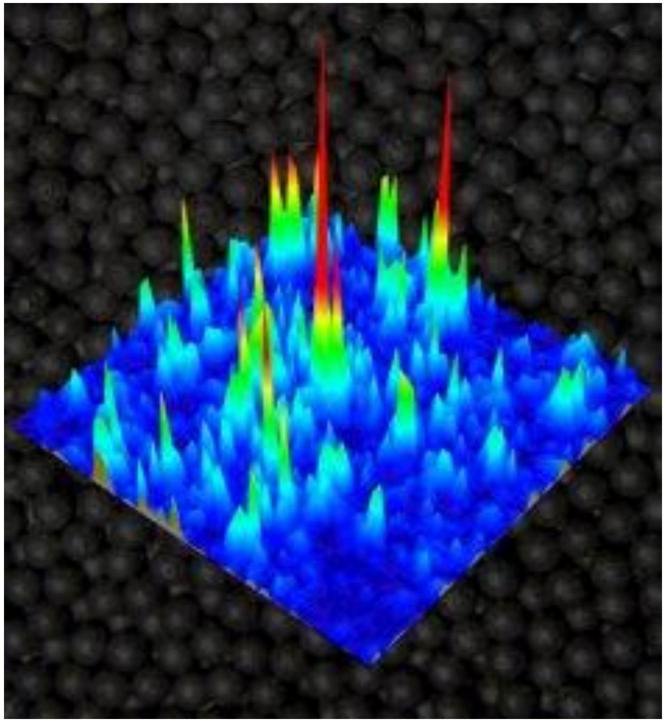
Outline

- Non-Hermitian particle-hole symmetry and anti-PT symmetry
- Non-Hermitian zero modes
- Non-Hermitian flat bands and defect states
- Linear localization
- Summary

Anderson localization



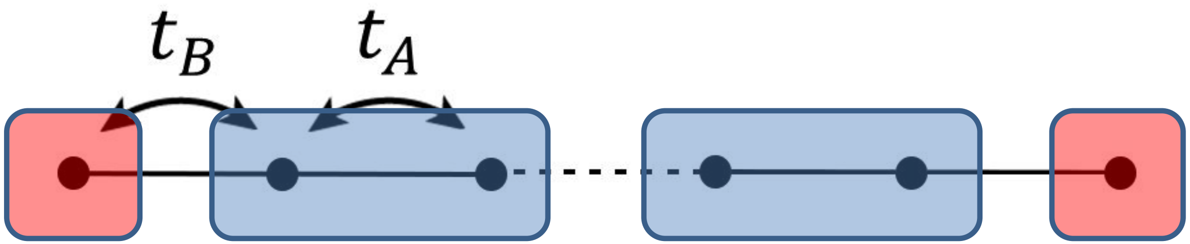
?

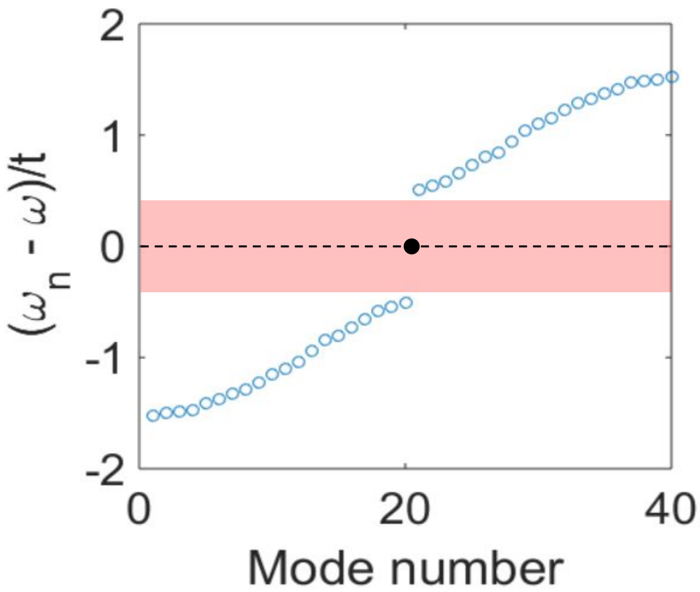


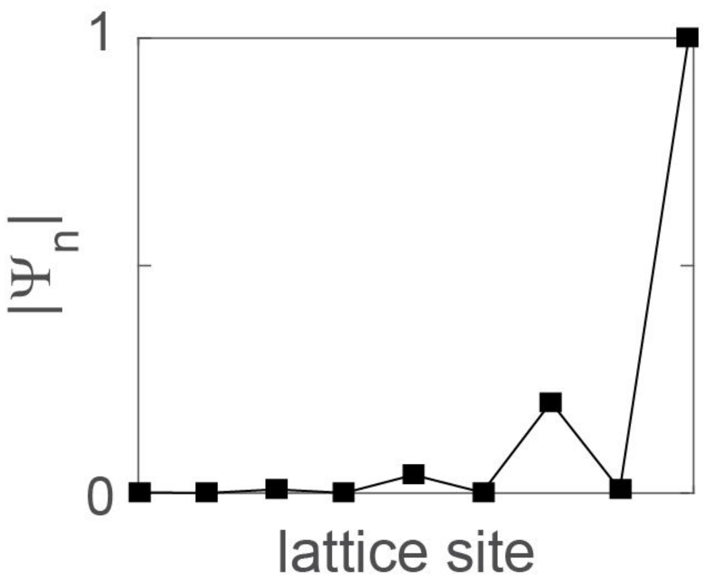
Introduce disorder

Localized zero mode

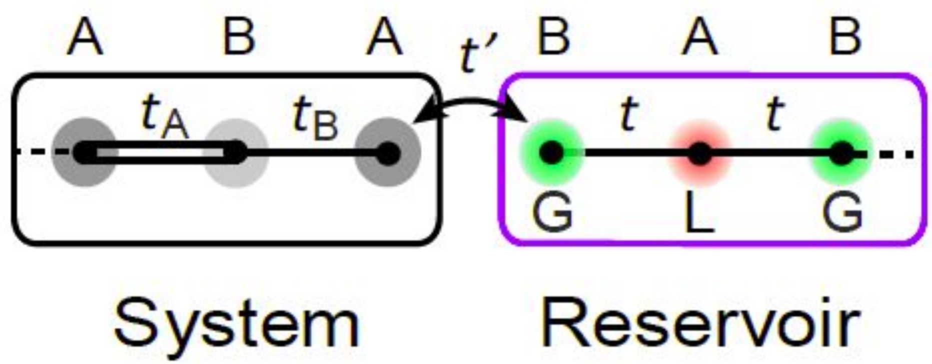
- SSH model: When $t_B<t_A$, two zero modes



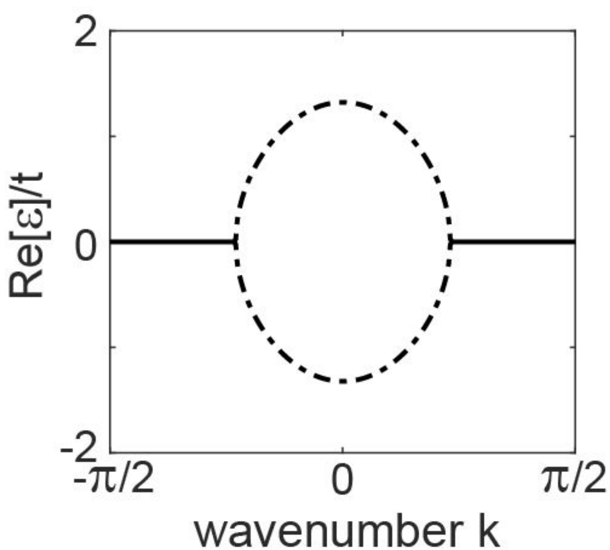


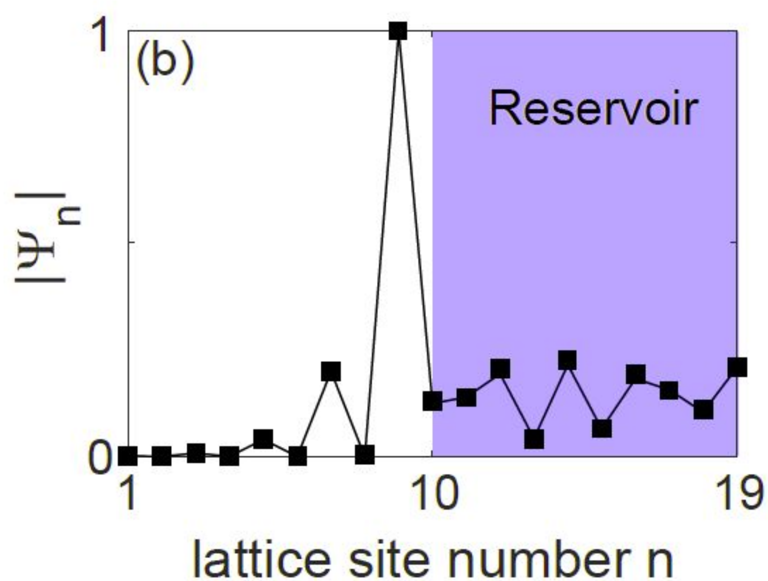


Localized zero mode coupled to a non-Hermitian reservoir

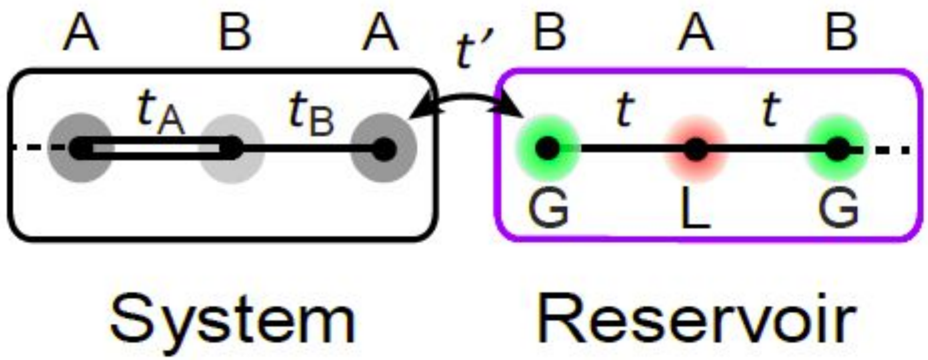


arXiv:1804.00579

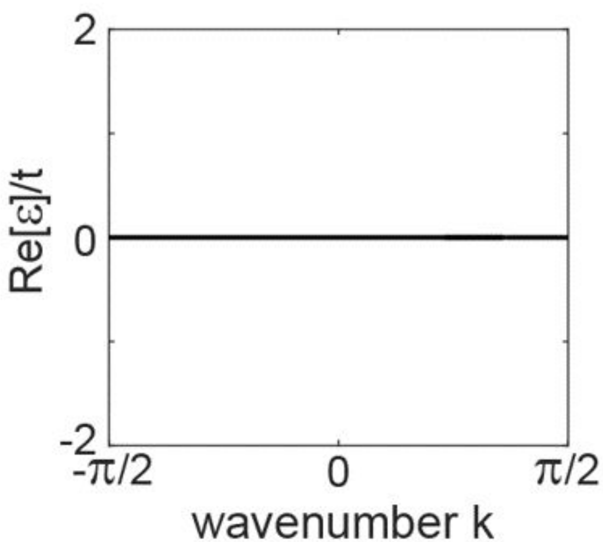


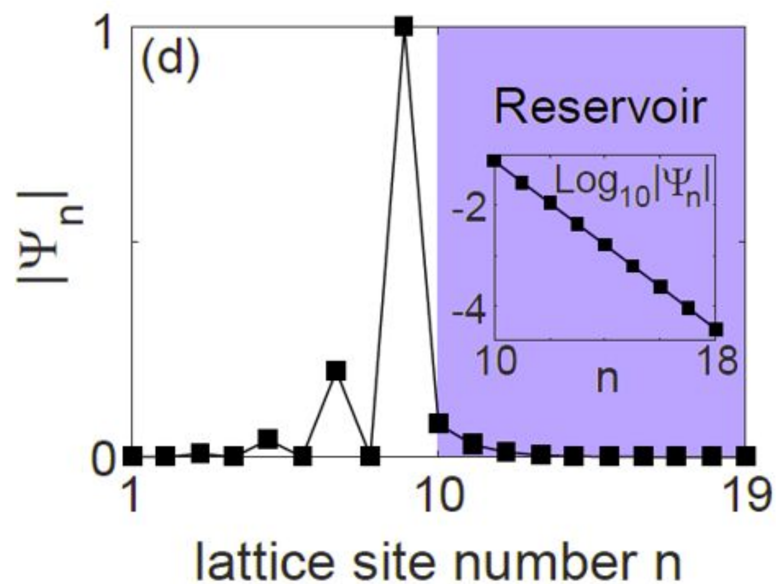


Localized zero mode coupled to a non-Hermitian reservoir

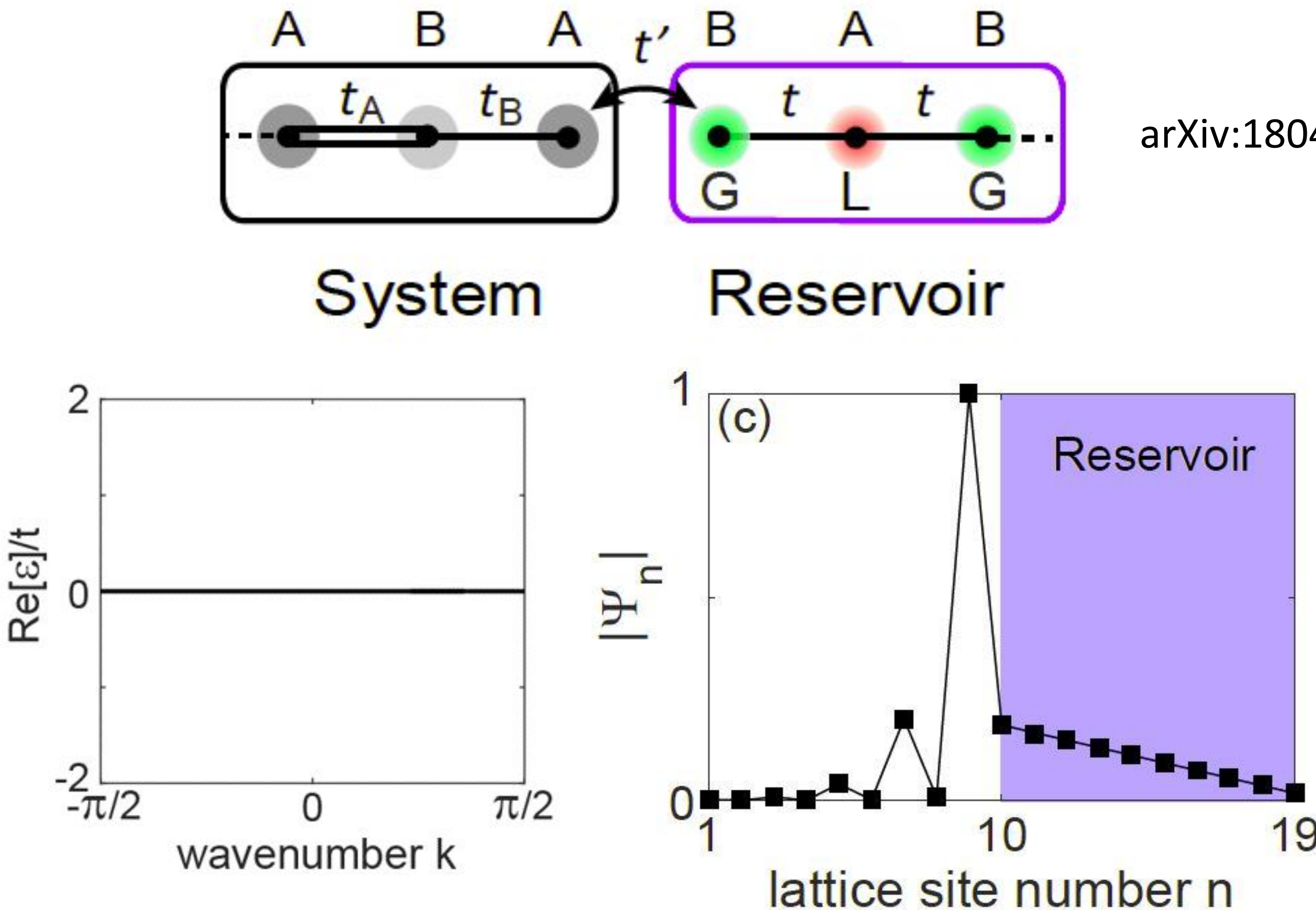


arXiv:1804.00579



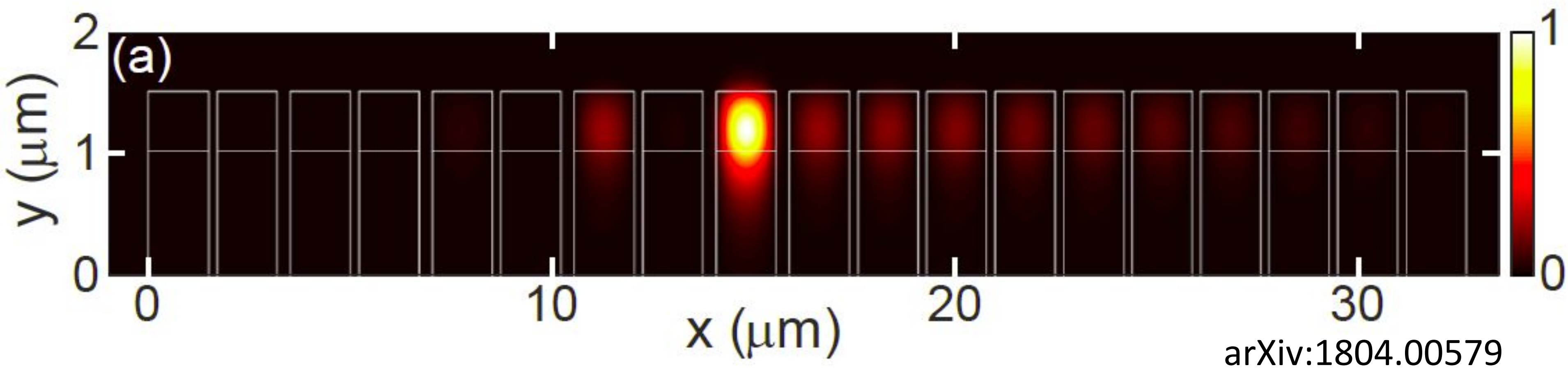


Localized zero mode coupled to a non-Hermitian reservoir



arXiv:1804.00579

Localized zero mode coupled to a non-Hermitian reservoir



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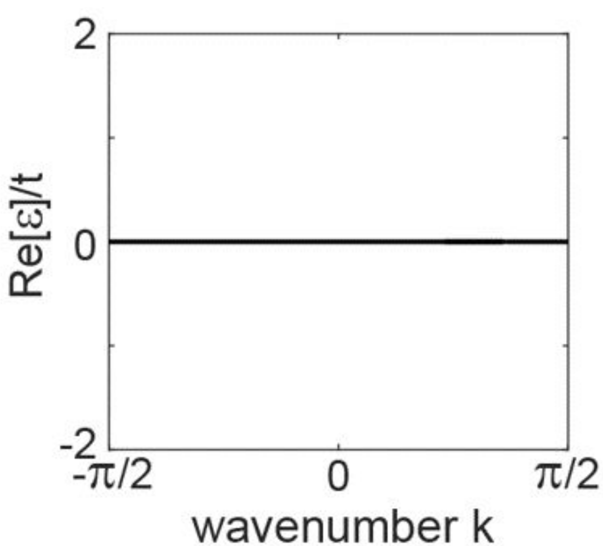
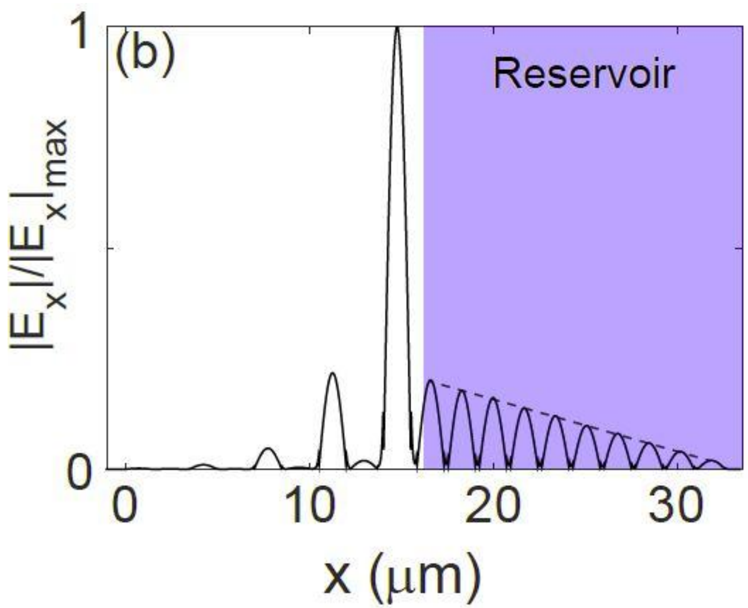
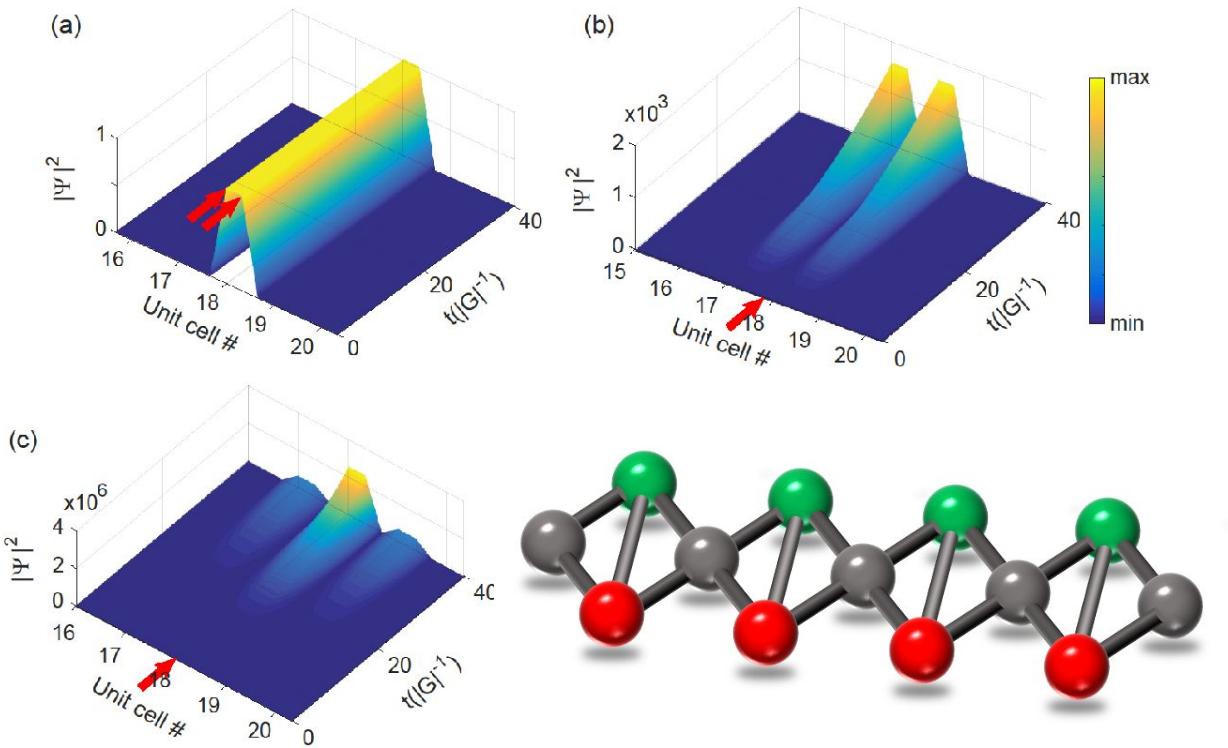




Localized zero mode coupled to a non-Hermitian reservoir

- Linear localization is NOT given by an exceptional point!



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Summary

- Non-Hermitian particle-hole symmetry and anti-PT symmetry
- Non-Hermitian zero modes
- Non-Hermitian flat bands and defect states
- Linear localization

National Science Foundation
WHERE DISCOVERIES BEGIN