Fall to the centre and fall to infinity

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Non-Hermitian Physics - PHHQP XVIII

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collaborators

- Cliff Burgess (McMaster / Perimeter Institute)
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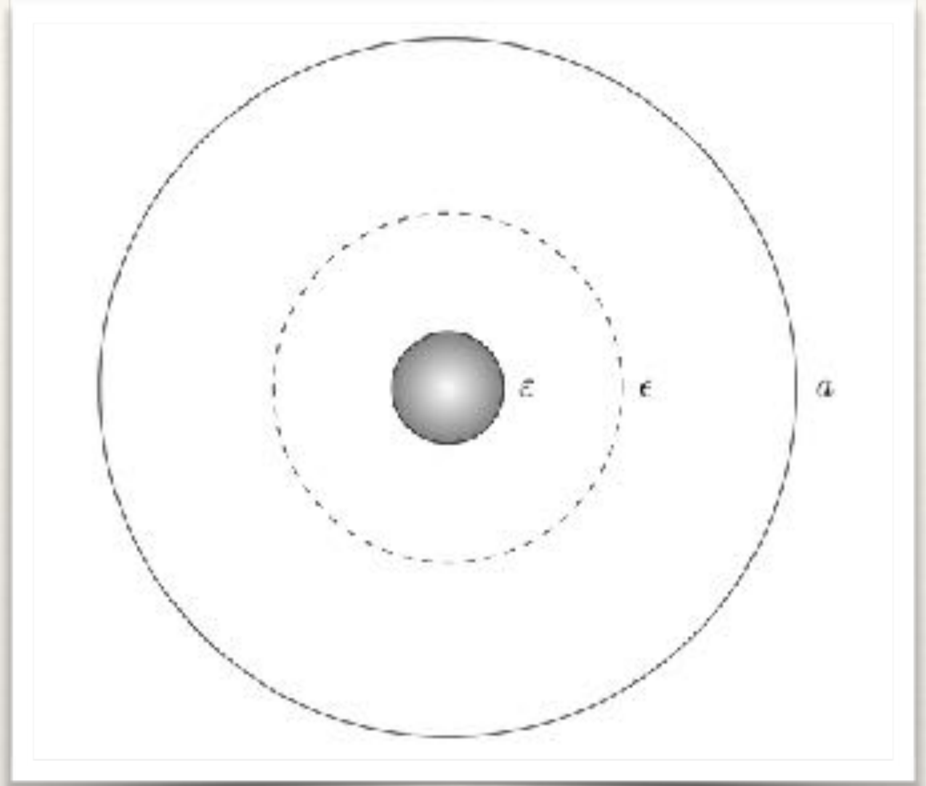
Mainly based on:

Fall to the Centre in Atom Traps and Point-Particle EFT for Absorptive Systems
R. Plestid, C.P. Burgess and D.H.J. O'Dell
arXiv:1804.103241 [hep-ph]

Plan

- Fall to the centre and point-particle EFT
- Experimental consequences for atoms interacting with a charged wire
- Fall to infinity (1 slide)
- Quantum catastrophes

Point-particle effective field theory



$$\varepsilon \ll \epsilon \ll a$$

- Physics should not depend on choice of gaussian pillbox surface ϵ
- Perform systematic expansion of source properties ("multipole expansion")

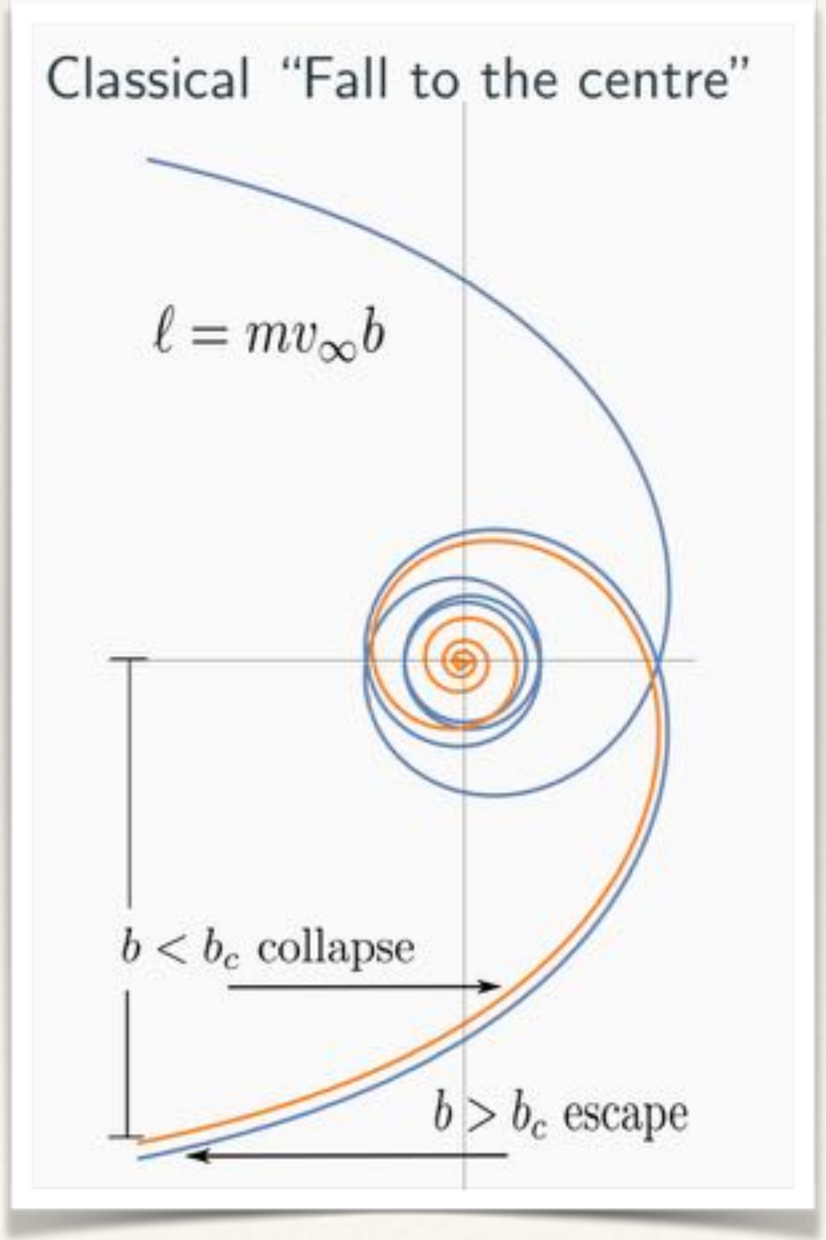
$$S_{\text{total}} = S_{\text{Bulk}} + S_{\text{particle}}$$

$$S_{\text{Bulk}} = \int \mathrm{d}t\ \mathrm{d}^d x\ \left[\frac{\mathrm{i}}{2}(\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*) - \frac{1}{2m}|\nabla\Psi|^2 - V(\mathbf{x})|\Psi|^2\right]$$

$$S_{\text{particle}} = -\int \mathrm{d}t\ \mathrm{d}^d x\ \ \delta^d(x)\ \left[\lambda|\Psi|^2 + \cdots\right]$$

"multipole expansion" describing physics of source at origin

Fall to the centre



central force

$$V(r) \quad \rightarrow J = \text{constant}$$

$$\frac{1}{2}m\dot{r}^2 + V(r) + \frac{J^2}{2mr^2} = E$$

To penetrate **angular momentum barrier:** $V(r) \rightarrow -\infty$

EITHER	as	$-\dfrac{g}{r^2}$	with	$g > \dfrac{J^2}{2m}$
OR	as	$-\dfrac{1}{r^n}$	with	$n > 2$

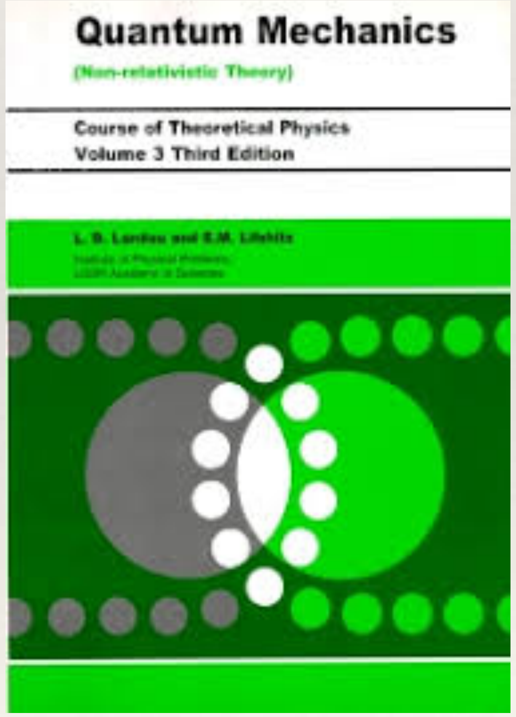
Quantum mechanics

$$-\frac{1}{2m}\left[\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}-\frac{l(l+1)}{r^2}R\right]-\frac{g}{r^n}R=ER$$

When $n \geq 2$

- Energy unbounded from below (!!)
- Bound energies become continuous (!!)

Non-self Adjoint problem



Fall to centre in QM

- Case, Phys. Rev. **80**, 797 (1950)
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- Alliluev, JETP **34**, 8 (1972)
- Gupta & Rajeev, Phys. Rev. D 48, 5940 (1993)
- Camblong et al., Phys. Rev. Lett. **85**, 1590 (2000)
- Beane et al., Phys. Rev. A **64**, 042103 (2001)
- Coon & Holstein, Am. J. Phys. **70**, 513 (2002)
- Camblong & Ordonez, Phys. Rev. D **68**, 125013 (2003)
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- Essin and Griffiths, Am. J. Phys **74**, 109 (2006)
- Hammer & Swingle, Annals Phys. 321, 306 (2006)
- Moroz & Schmidt, Annals Phys. **325**, 491 (2010)
- Khelashvili & Nadareishvili, Am. J. Phys. **79**, 668 (2011)
- Bouaziz & Bawin, Phys. Rev. A **89**, 022113 (2014)

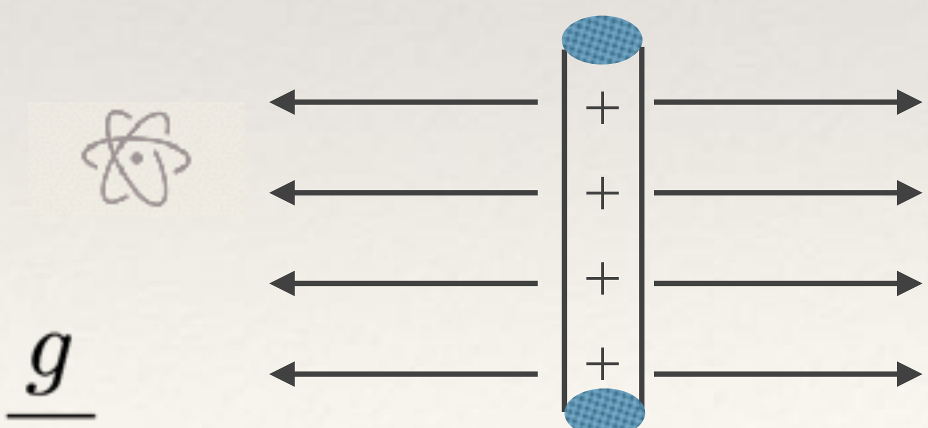
…and many more…

Inverse square potential

$$\boxed{V(r) = -\frac{g}{r^2}}$$

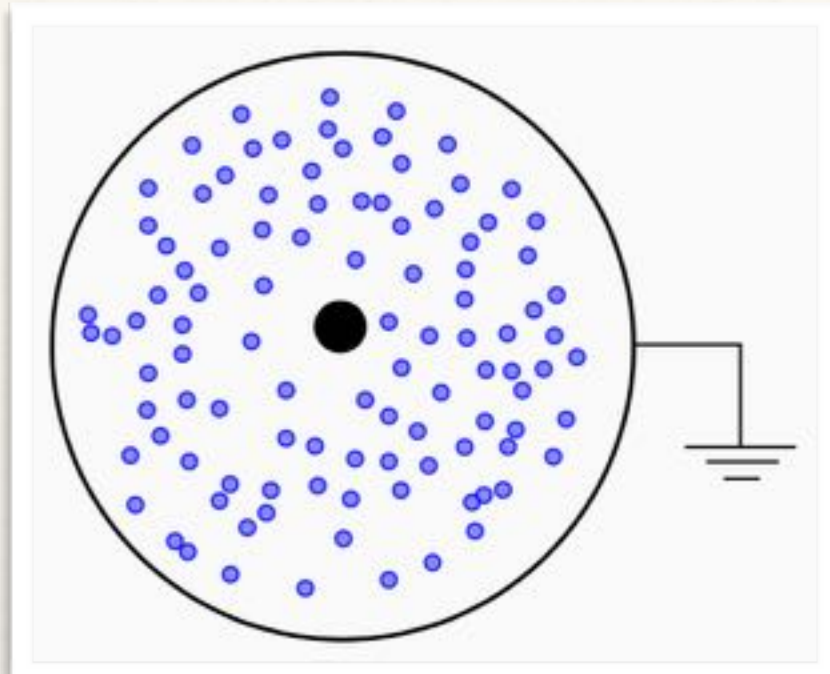
Physical example: neutral polarizable atoms in presence of a charged wire

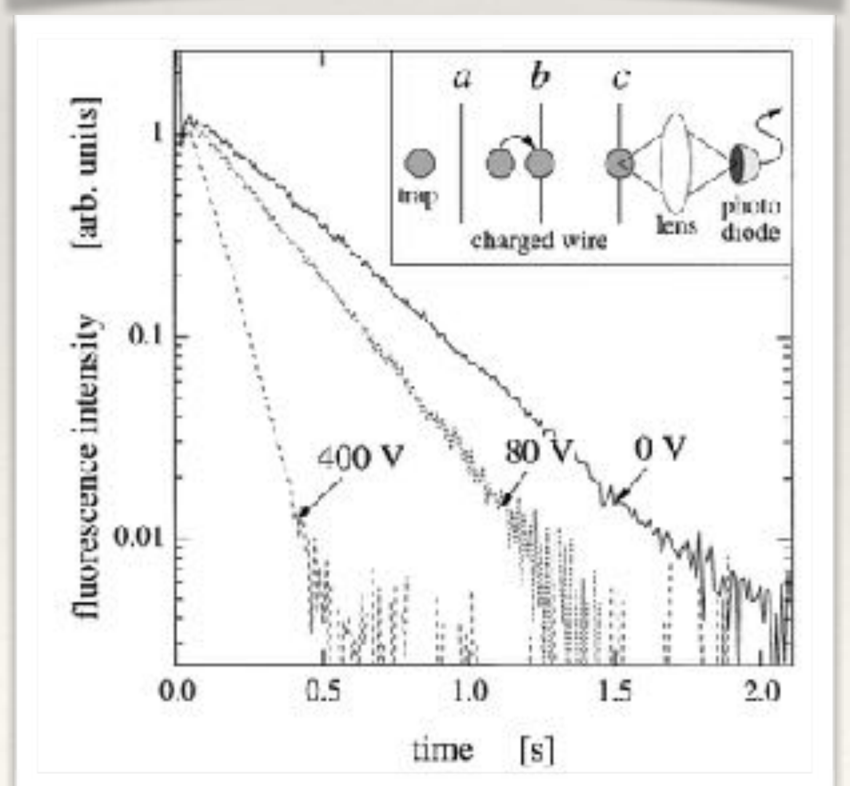
$$\mathbf{E} = \frac{\rho_{1D}}{2\pi\epsilon_0 r}\hat{\mathbf{r}} \qquad \mathbf{d} = \alpha\mathbf{E}$$



$$V = -\frac{1}{2}\mathbf{d}\cdot\mathbf{E} = -\frac{\alpha\rho_{1D}^2}{8(\pi\epsilon_0)^2}\frac{1}{r^2} \equiv -\frac{g}{r^2}$$

Experimental realization with gas of atoms





J. Denschlag, G. Umshaus and J. Schmiedmayer
PRL 81, 737 (1998)

$$V(r) = -\frac{g}{r^2}$$

- g tunable via electric potential on wire
- atoms touching wire are absorbed
- measure absorption by decay of fluorescence
- good agreement with classical theory:

critical impact parameter

$$\sigma^{(\mathrm{cl})}_{\mathrm{abs}} = \frac{2}{v}\sqrt{\frac{2g}{m}} = 2b_c$$

[cross section has units of length in 2D]

For comparison: Rutherford scattering has $\sigma \propto \frac{1}{v^4}$

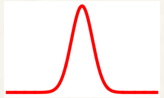
Scale invariance

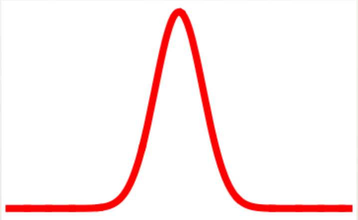
$$x \to \lambda x \qquad\qquad t \to \lambda^2 t$$

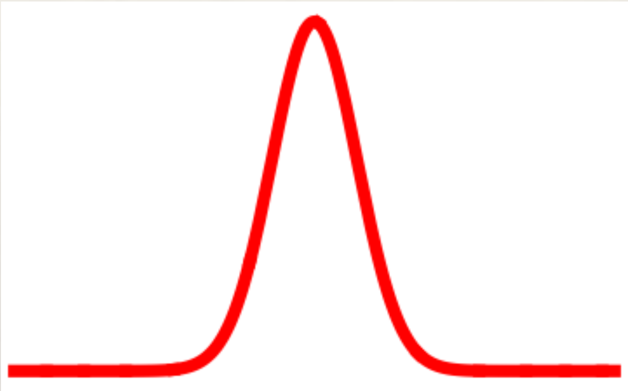
$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2\nabla^2}{2m} - \frac{g}{r^2}\right)\psi \quad \to \quad \frac{1}{\lambda^2}\, i\hbar\frac{\partial}{\partial t}\psi = \frac{1}{\lambda^2}\left(-\frac{\hbar^2\nabla^2}{2m} - \frac{g}{r^2}\right)\psi$$

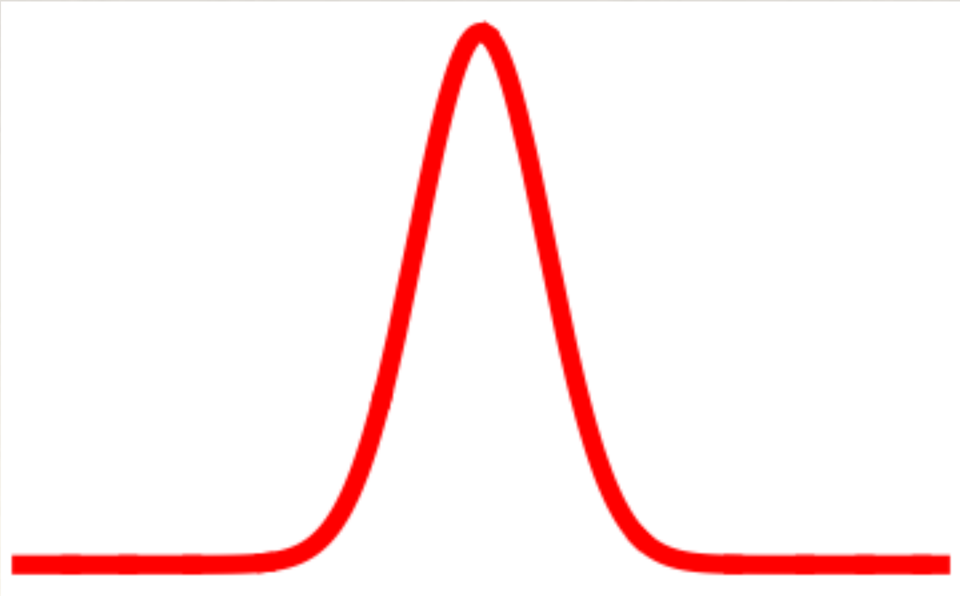
no intrinsic length scale in problem $\Rightarrow$ **"classical" scale invariance**

Problem: continuum of bound states







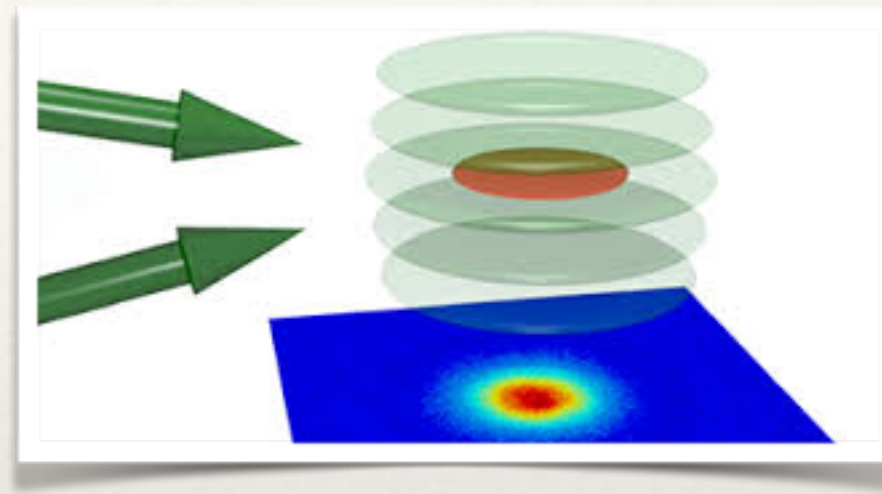


But…bound state spectrum should be discrete

Other examples of scale invariance

- **2D gas**

$$\hat{H} = \sum_i -\frac{\hbar^2}{2m}\nabla_i^2 + \frac{g}{2}\sum_{i,j}\delta^2(\mathbf{r}_i - \mathbf{r}_j) + \underbrace{\sum_i \frac{1}{2} m\omega_{\mathrm{trap}}^2 r_i^2}_{\text{harmonic trap breaks scale invariance}}$$



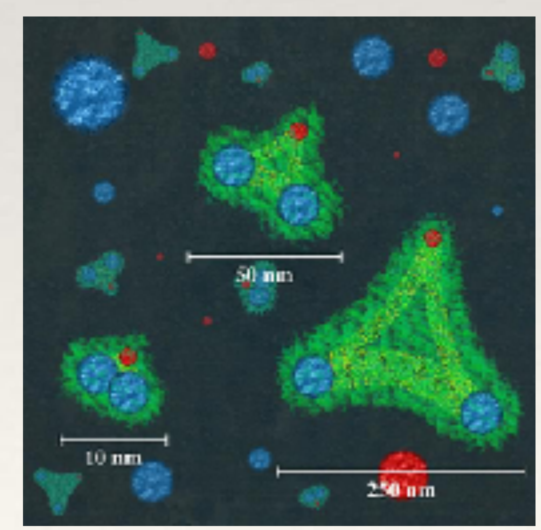
but: hidden SO(2,1) scaling symmetry [Pitaevskii & Rosch, PRA **55** R853 (1997)]
$\Rightarrow$ ladder of evenly spaced breathing modes $E_n = 2\, n\, \hbar\, \omega_{\mathrm{trap}}$

- **Efimov states**

Quantum 3-body problem at low energies reduces to:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(R) - \frac{g}{R^2}\psi(R) = E\psi(R) \qquad R = \sqrt{(r_{12}^2 + r_{23}^2 + r_{31}^2)/3}$$

Scale invariance becomes **discrete**: infinite geometric progression of bound states $E_N = E_0\lambda^{-2N}$ $\lambda = 22.69438\ldots$

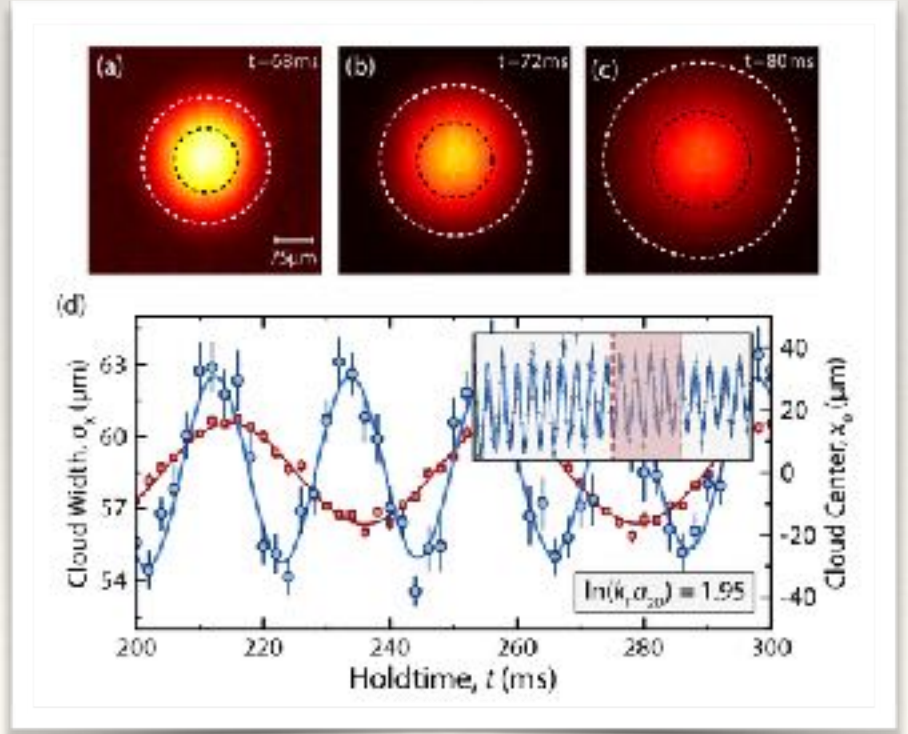


Schematic representation of Efimov trimers formed from two caesium and one lithium atom R. Pires et al, PRL 112, 250404 (2014)

Quantum anomalies

…when quantum effects break continuous scale invariance of classical Lagrangian

- Two photon decay of neutral pion [breaks axial SU(2) invariance]
- QED & QCD (coupling constants depend on scale)
- Efimov states (bound state breaks scale invariance)
- Scale invariance in 2D gas (shift of breathing frequency away from 2 ω_{trap})



Anomalous breaking of scale invariance in a two-dimensional Fermi gas

Holten et al.   arXiv:1803.08879

Boundary condition ambiguity in *1/r²* problem

radial equation $$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+\frac{\gamma}{r^2}+k^2\right)R=0$$

$$\gamma = 2mg/\hbar^2 - l(l+1)$$
$$l = 0, 1, 2\ldots$$

general solution at small *r* : $$R(r) = C_+ r^{s_+} + C_- r^{s_-}$$

$$s_+ = -\frac{1}{2}+\sqrt{\frac{1}{4}-\gamma}, \quad s_- = -\frac{1}{2}-\sqrt{\frac{1}{4}-\gamma}$$

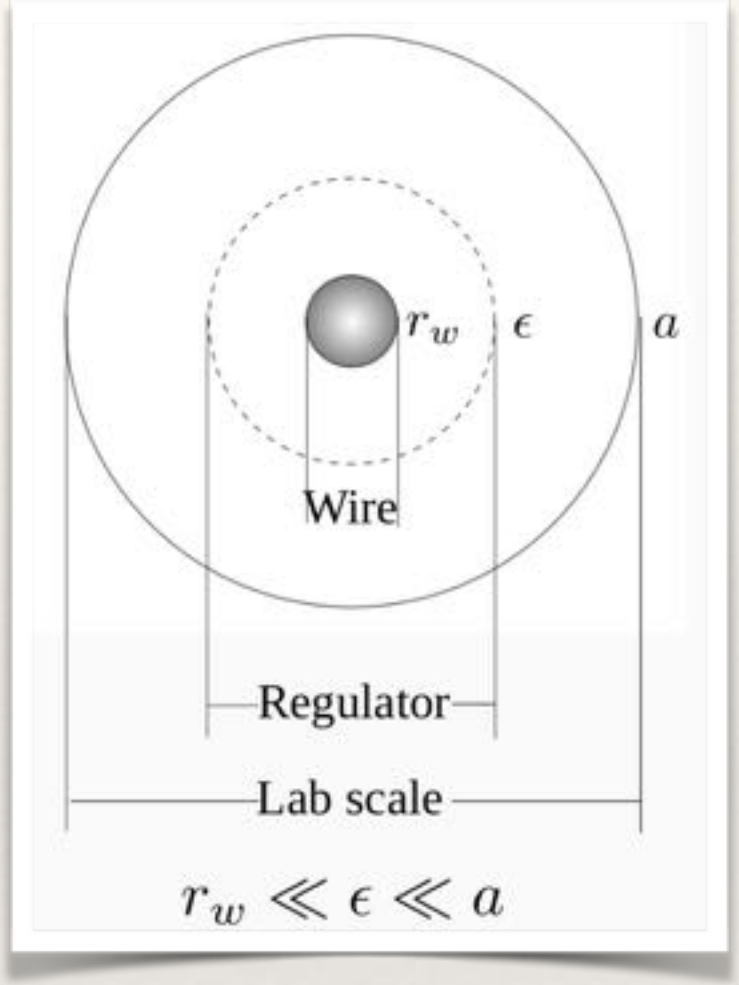
- **subcritical** $\gamma < 1/4$ $\quad s_1 \neq s_2$ ✓ (Typically choose less singular soln. $R = C_+ r^{s_+}$ $[C_- = 0]$)
- **supercritical** $\gamma > 1/4$ $\quad R = C_+\frac{1}{\sqrt{r}}r^{i\nu} + C_-\frac{1}{\sqrt{r}}r^{-i\nu}$ **Which to choose??**

where $\nu = \sqrt{\gamma - 1/4}$

Including atom-wire physics: PPEFT

Difficulty: $\lim_{r \to 0} R_{\pm}(r) = \infty$

- Need boundary condition to determine ratio C_+ / C_-



Delta-shell regularization

$$V_\epsilon(r) = -\frac{g}{r^2} + \frac{\lambda(\epsilon)}{2\pi\epsilon}\delta(r-\epsilon) \qquad \text{(2D)}$$

$$\frac{\pi}{m}\left(r\frac{\partial R}{\partial r}\right)_{r=\epsilon} = \lambda(\epsilon)R(\epsilon)$$

Point-particle effective field theory: $S_{\text{particle}} = -\int \mathrm{d}t\ \mathrm{d}^d x\ \ \delta^d(x)\ [\lambda|\Psi|^2 + \cdots]$ "multipole expansion"

Non-self adjoint extension

- Adding a boundary condition **breaks scale invariance** by fixing ratio C_+ / C_-
- Many ways to do it
- Most authors choose a "self adjoint extension"
- Point-particle EFT is algorithmic and allows non-self adjoint extensions

$$S_{\rm total} = S_{\rm Bulk} + S_{\rm particle}$$

$$S_{\rm Bulk} = \int {\rm d}t\ {\rm d}^d x\ \left[\frac{\rm i}{2}(\Psi^*\partial_t\Psi - \Psi\partial_t\Psi^*) - \frac{1}{2m}|\nabla\Psi|^2 + \frac{g}{\mathbf{x}^2}|\Psi|^2\right] \qquad S_{\rm particle} = -\int {\rm d}t\ {\rm d}^d x\ \ \delta^d(x)\left[\lambda|\Psi|^2 + \cdots\right]$$

- Absorption onto wire at can be captured by complex λ
- Radial probability flux at $r{=}\epsilon$:

$$J_r(\epsilon) = \frac{\rm i}{2m}\left(R\partial_r R^* - R^*\partial_r R\right)_{r=\epsilon} = {\rm i}(\lambda^* - \lambda)\frac{|R(\epsilon)|^2}{2\pi\epsilon}$$

- Net rate of probability flow out sphere of radius $r{=}\epsilon$:

$$\oint_{r=\epsilon} J_r\ {\rm d}\Omega = 2|R(\epsilon)|^2\ {\rm Im}\lambda$$

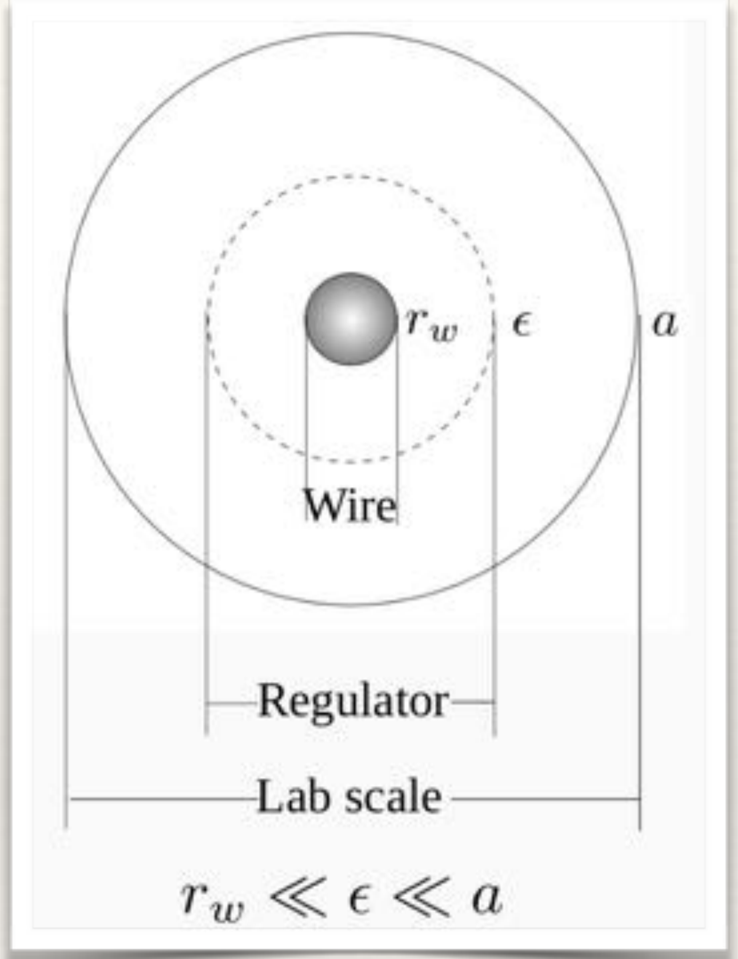
sink

or

source

Renormalization

- Physics should not depend on regulator scale ϵ
- Coupling $\lambda = \lambda(\epsilon)$ runs with the scale ϵ
- Running breaks scale invariance (except at fixed points)
- Ratio C_+ / C_- is physical observable (e.g. via scattering) and so must be independent of ϵ :



$$\frac{C_+}{C_-} = \frac{\zeta - \lambda(\epsilon)}{\zeta + \lambda(\epsilon)} (2ik\epsilon)^{\zeta}$$

$$\zeta = 2\sqrt{l^2 - 2mg/\hbar^2}$$

(2D)

AND $\quad \epsilon \frac{\mathrm{d}}{\mathrm{d}\epsilon} \frac{C_+}{C_-} = 0$

$$\Rightarrow \quad \boxed{\epsilon \frac{\mathrm{d}}{\mathrm{d}\epsilon} \lambda = \frac{1}{2} \left(\zeta^2 - \lambda^2 \right)}$$

RG flow of microscopic coupling λ

- Physics is equivalent for all $\lambda(\epsilon)$ along one trajectory
- ϵ increases in direction of arrows
- colour: red=slow, blue=fast

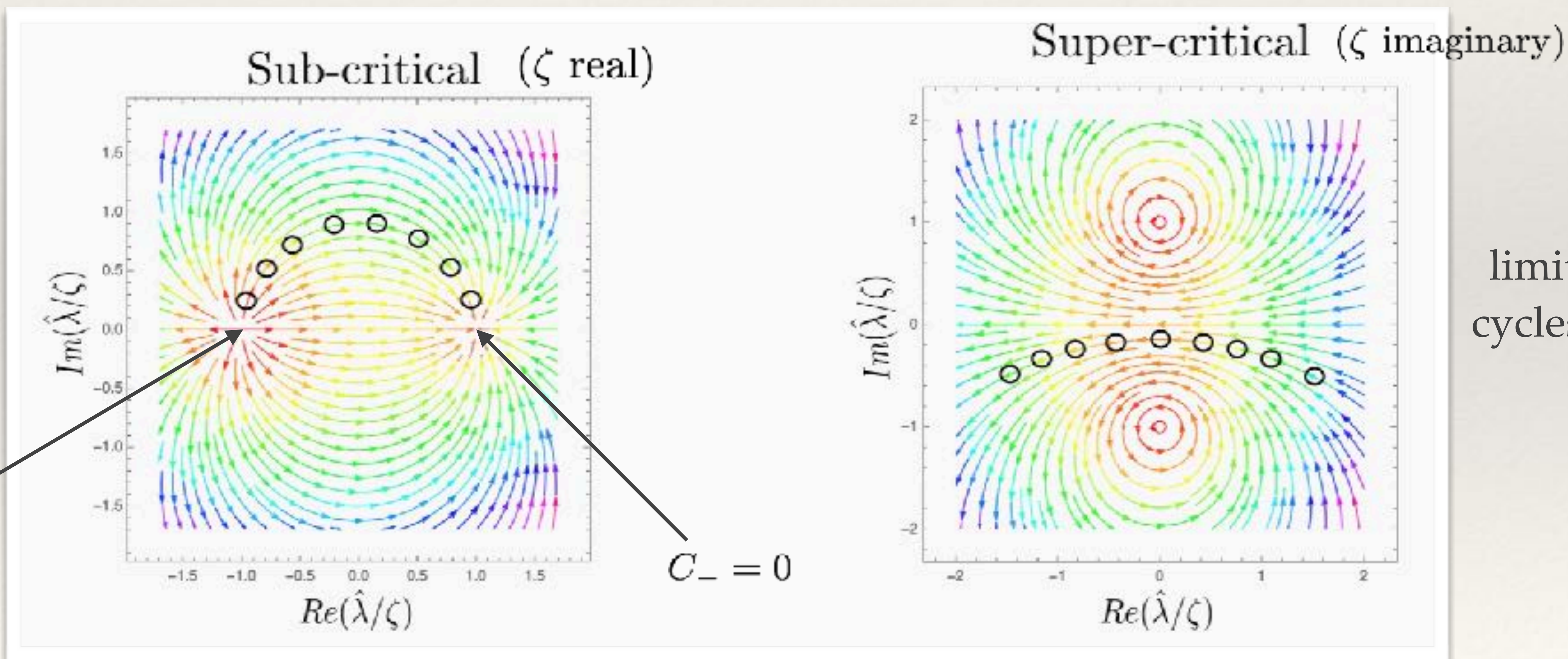
$$V_\epsilon(r) = -\frac{g}{r^2} + \frac{\lambda(\epsilon)}{2\pi\epsilon}\delta(r-\epsilon)$$

$$\epsilon\frac{\mathrm{d}}{\mathrm{d}\epsilon}\lambda = \frac{1}{2}\left(\zeta^2 - \lambda^2\right)$$

$$\zeta = 2\sqrt{l^2 - 2mg/\hbar^2}$$

flow from UV to IR

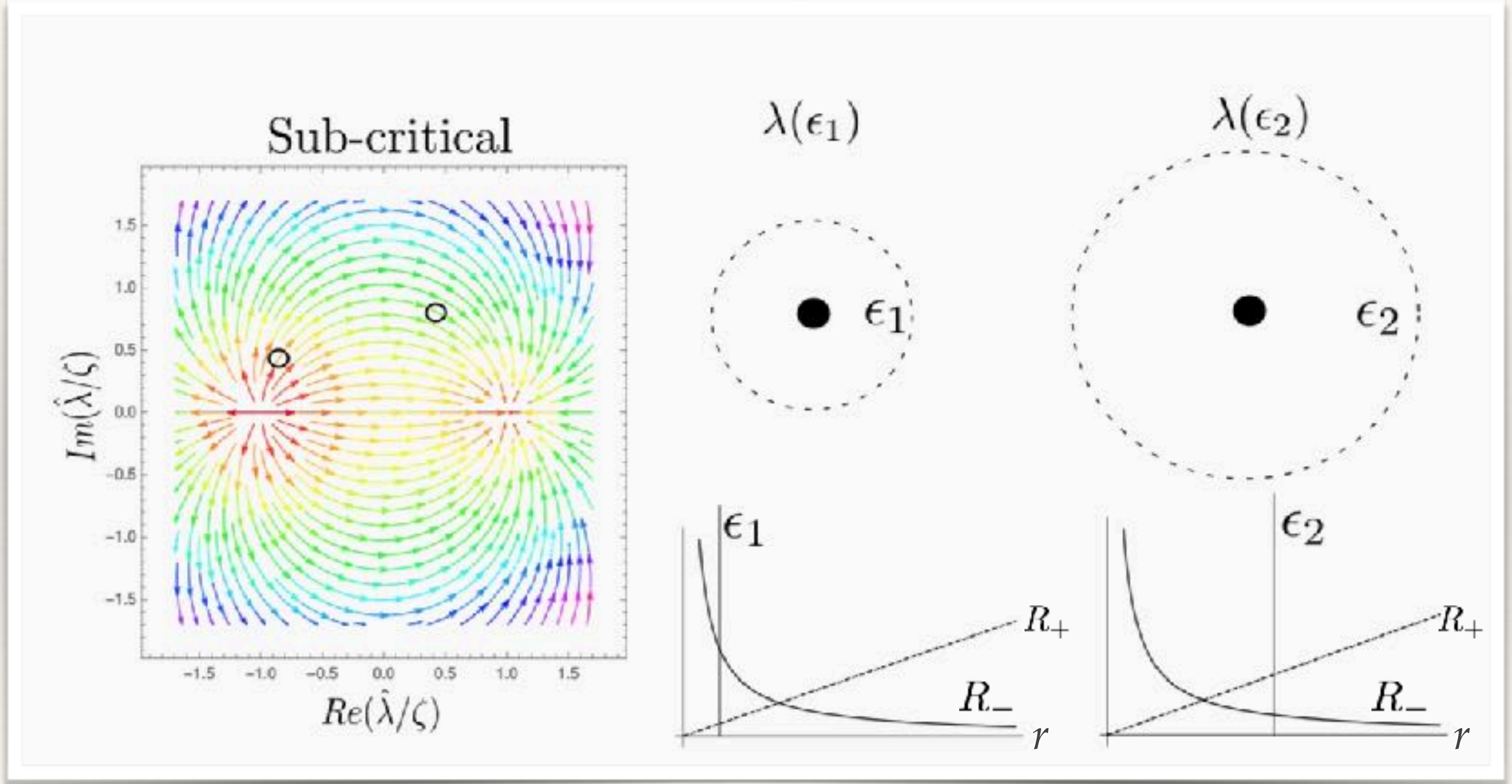
$C_+ = 0$



limit cycles!

see: Kaplan et al Phys. Rev. D **80**, 125005 (2009)

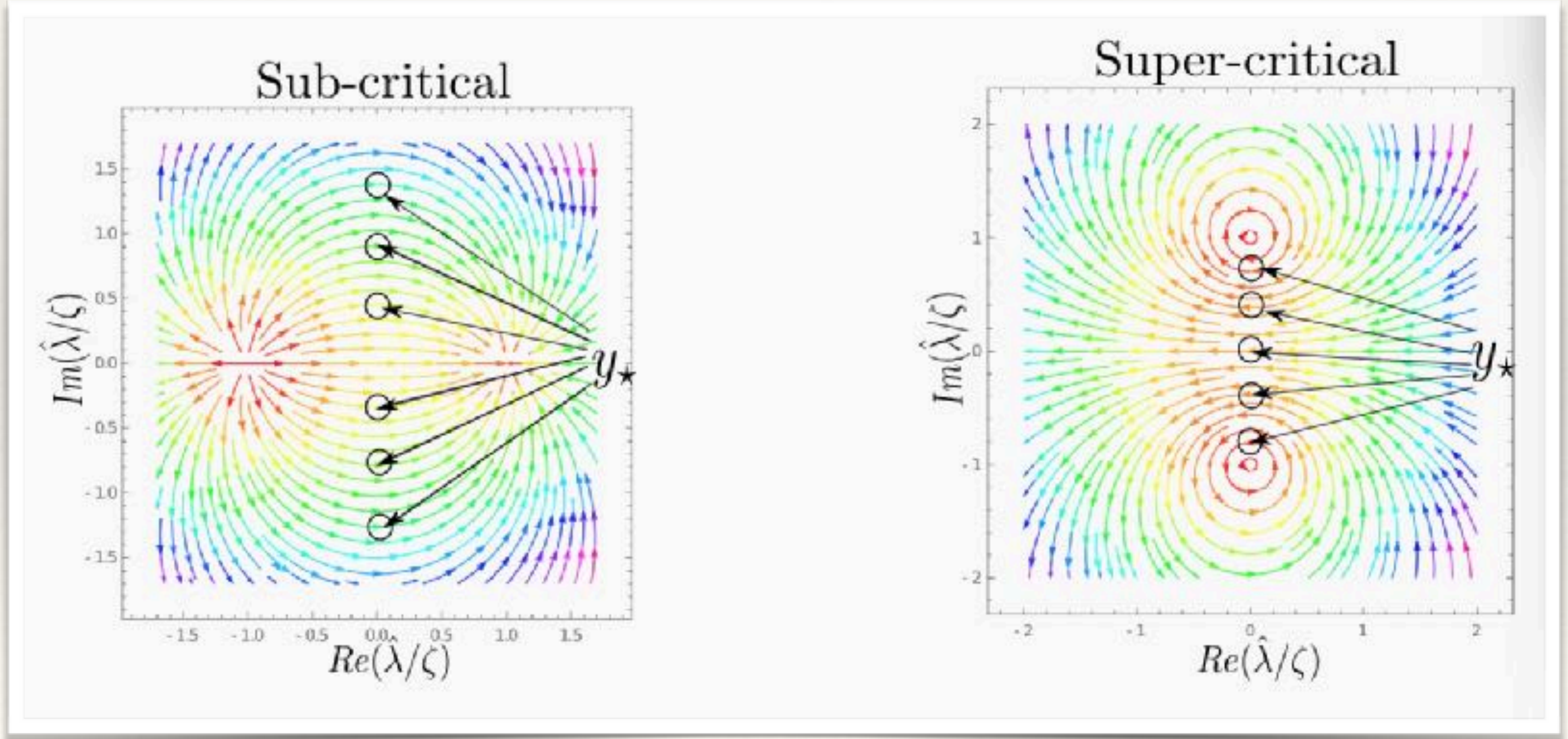
RG flow…



Sub-critical
$Im(\hat{\lambda}/\zeta)$
$Re(\hat{\lambda}/\zeta)$
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5
$\lambda(\epsilon_1)$
ϵ_1
$\lambda(\epsilon_2)$
ϵ_2
R_+
R_-
r

RG flow

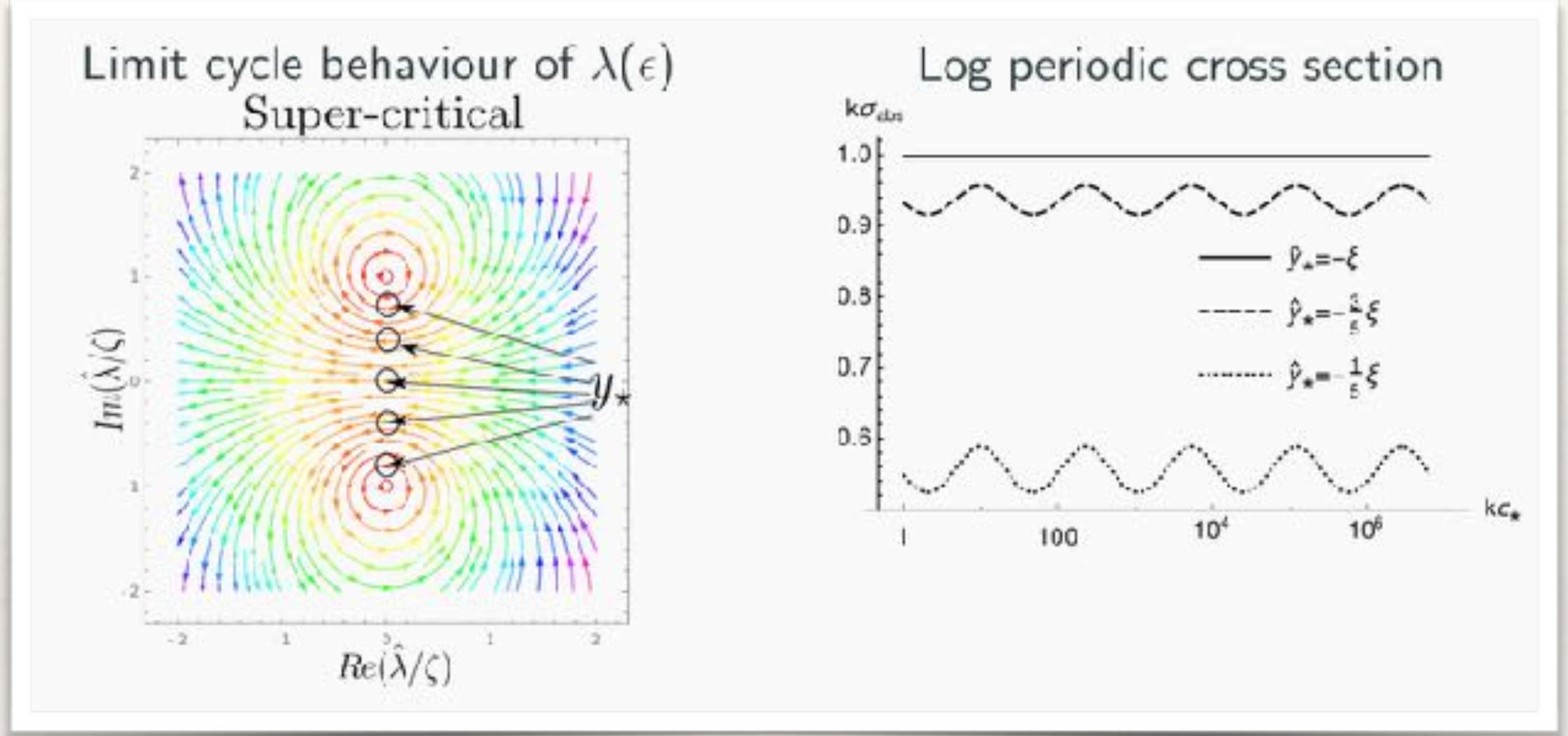
- Each trajectory is characterized by its values of $y_\star$ and $\epsilon_\star$
- $\epsilon_\star$ is the scale where Re $\lambda(\epsilon_\star)$ vanishes
- $y_\star$ is Im $\lambda(\epsilon_\star)$



Self-adjoint case: $y_\star = 0$ and $y_\star \to \pm\infty$

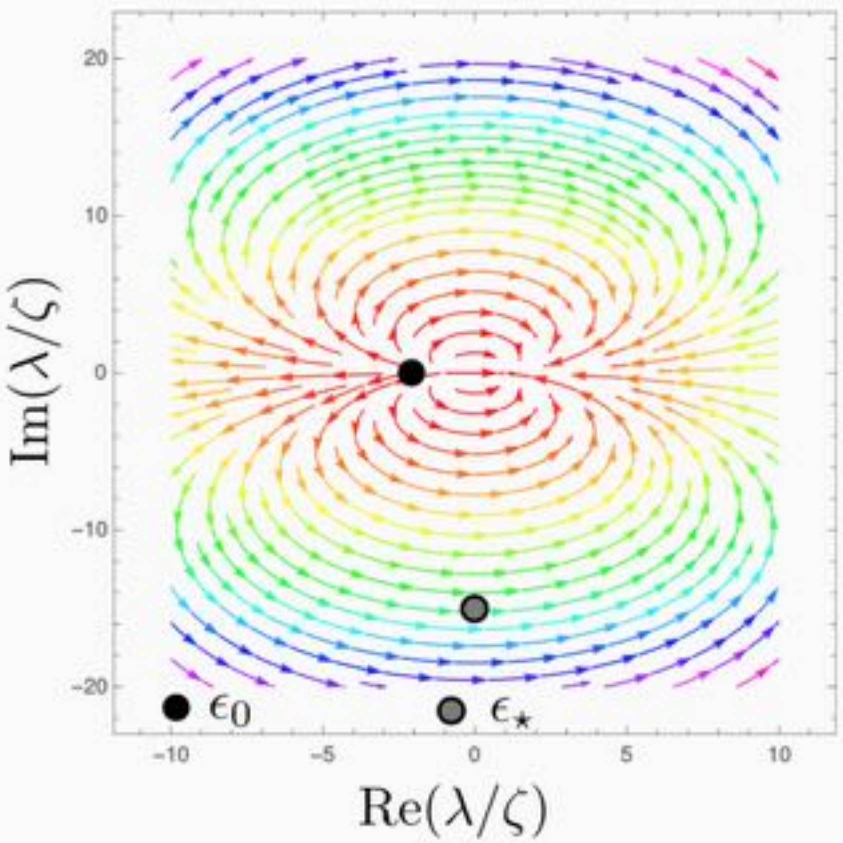
Experimental consequences

Partial-wave absorptive cross sections (super-critical)



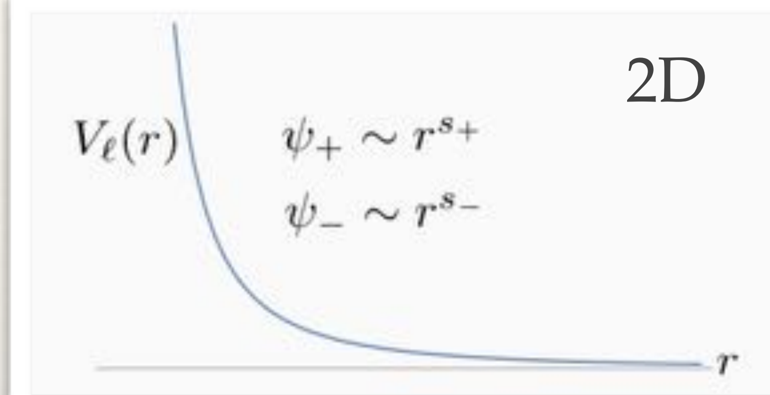
- Limit cycle leads to log-periodic absorption cross section in the super-critical case
- Continuous scale invariance broken to a discrete sub-group
- Two fixed points at $\lambda = \pm\, i\, \zeta$
- $y_\star = -i\, \zeta$ is a perfect absorber

Exotic bound state in sub-critical regime



$$\epsilon_\star = \epsilon_0 \left[\frac{(\lambda_0 + \zeta) - 2}{(\lambda_0 + \zeta)} \right]^{1/\zeta}$$

ζ real ($\zeta = 2\sqrt{l^2 - 2mg/\hbar^2}$)



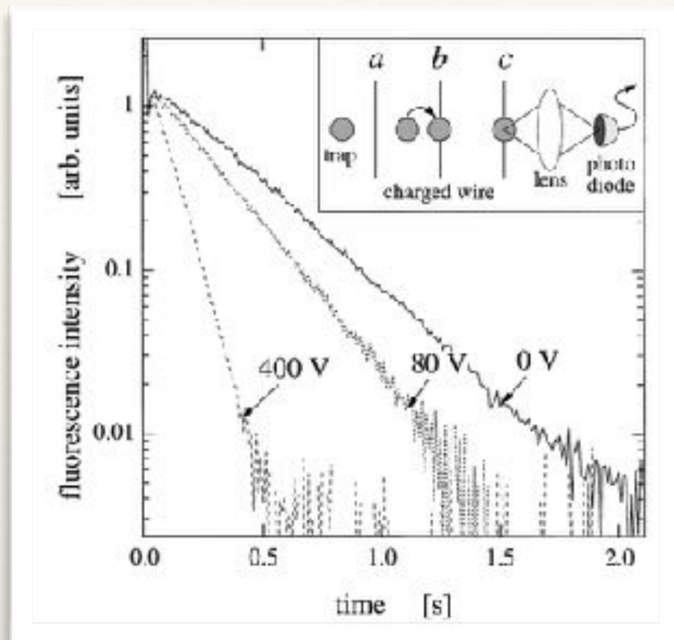
- repulsive effective potential!
- bound state supported by δ-function

ζ can be tuned via applied voltage:

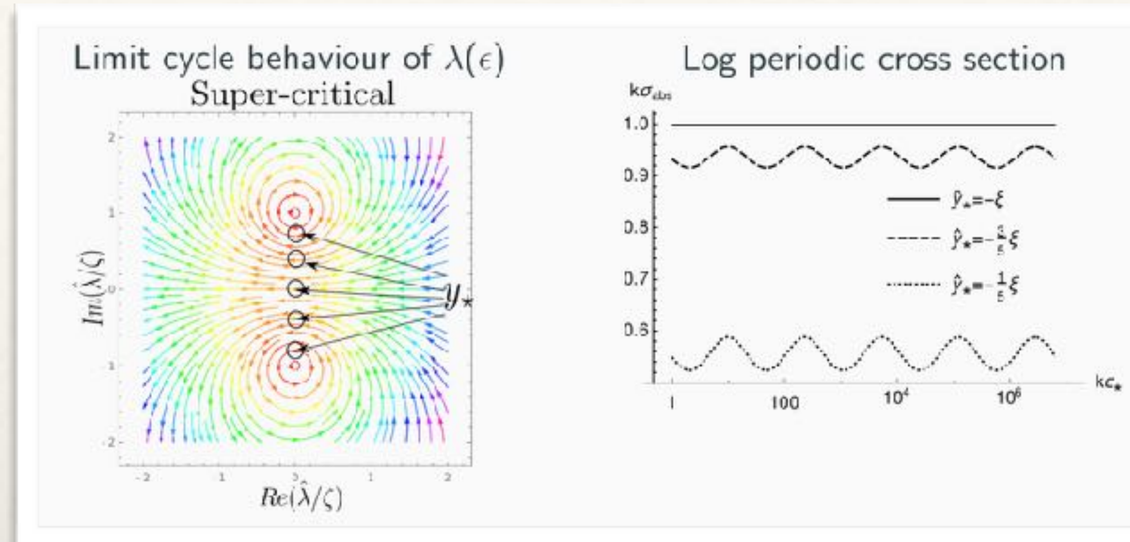
$$\zeta = 2\sqrt{l^2 - (V/V_c)^2}$$

see: Bouaziz & Bawin, Phys. Rev. A **89**, 022113 (2014)

Charged wire as perfect absorber



J. Denschlag, G. Umshaus and J. Schmiedmayer
PRL 81, 737 (1998)

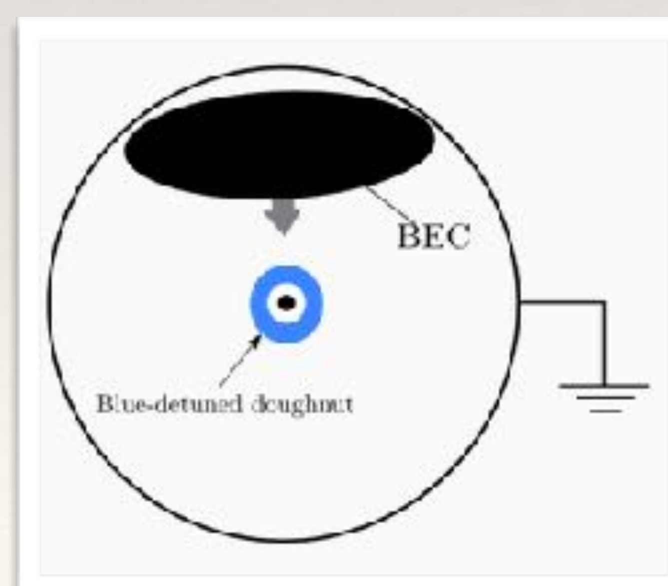


But there is no anomaly for a perfect absorber!

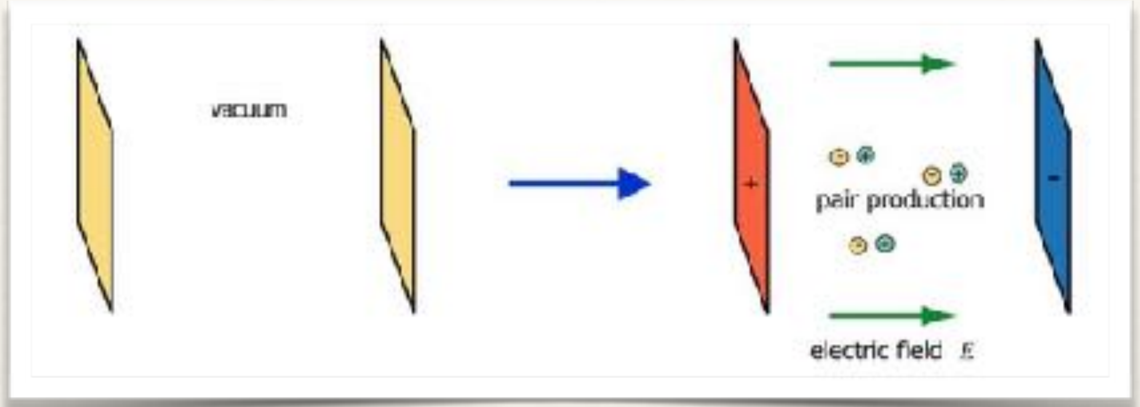
$$\sigma_{\text{abs}}^{(\text{cl})} = \frac{2}{v}\sqrt{\frac{2g}{m}} = 2b_c$$

solution??

…use blue-detuned laser to shield wire and control absorption



Fall to infinity (poster)- Sriram Sundaram



Schwinger pair creation (1951)

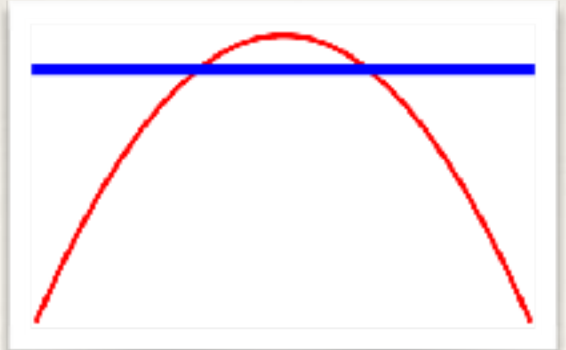
$$\left[\hbar^2 \frac{\partial^2}{\partial x^2} - \left(\frac{\hbar}{c}\frac{\partial}{\partial t} - \mathrm{i}\frac{qEx}{c}\right)^2\right]\phi(x,t) = m^2c^2\phi(x,t)$$

Klein-Gordon equation for particle of charge q in a constant electric field E

Put: $\phi(x,t) = e^{-\mathrm{i}\omega t}\phi(x)$

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{m}{2}\left(\frac{qE}{mc}\right)^2\left(x + \frac{\hbar\omega}{qE}\right)^2\right]\phi(x) = -\frac{mc^2}{2}\phi(x)$$

Schrödinger equation for particle in inverted harmonic oscillator



SU(1,1) symmetry

Quantum catastrophes

General solution at small *r* : $$R(r) = C_+ r^{s_+} + C_- r^{s_-}$$

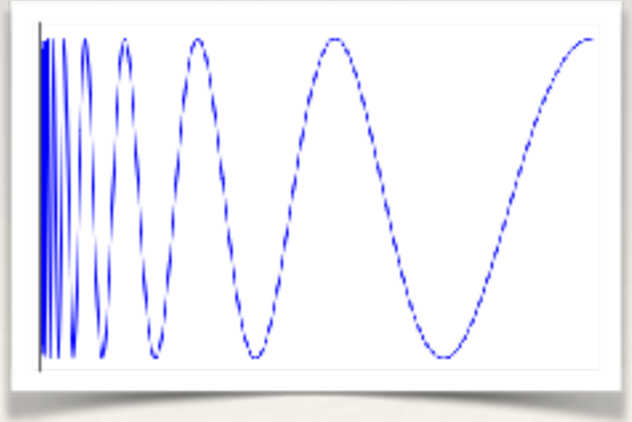
where $$s_\pm = \frac{1}{2}(2 - d \pm \zeta)$$ d = dimension

$$\zeta = \sqrt{(d-2)^2 + 4[\omega_d(l) - 2mg/\hbar^2]}$$

$$\begin{cases} \omega_3(l) = l(l+1) \\ \omega_2(l) = l^2 \end{cases}$$

For ζ imaginary: $\zeta = i\xi$ solutions are oscillatory:

$$\left(\frac{r}{r_0}\right)^{s_\pm} = \left(\frac{r}{r_0}\right)^{(d-2)/2} \mathrm{e}^{\pm \mathrm{i} \frac{\xi}{2} \log(r/r_0)}$$

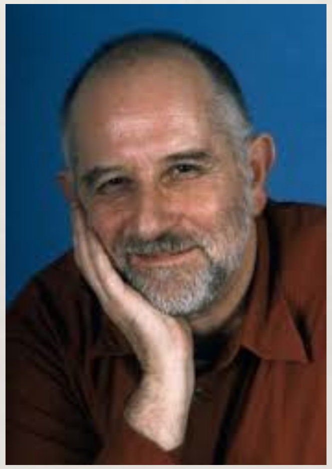


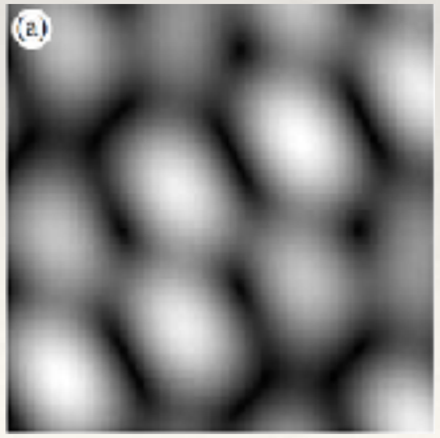
Logarithmic Phase singularity: phase takes all possible values

The big idea

When do we need to 2nd-quantize waves in order to avoid singularities?

M. V. Berry and M. R. Dennis, J. Opt. A: Pure Appl. Opt. **6**, S178 (2004), "**Quantum cores of optical phase singularities**"





$2\,\lambda$

Superposition of 4 plane waves gives nodal points where nodal lines pierce the plane.

Quantum catastrophe: Hawking radiation?

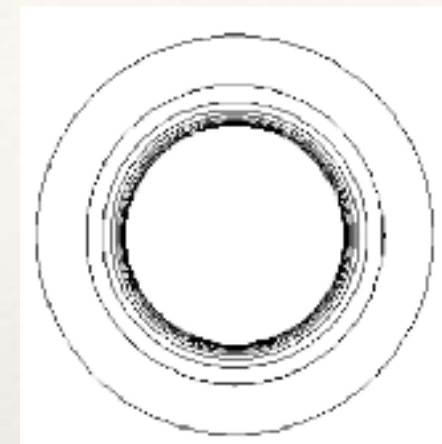
U. Leonhardt, Nature **415**, 406 (2002)

A laboratory analogue of the event horizon using slow light in an atomic medium

Ulf Leonhardt

School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, Fife KY16 9SS, UK

Singularities underlie many optical phenomena[1]. The rainbow, for example, involves a particular type of singularity—a ray catastrophe—in which light rays become infinitely intense. In practice, the wave nature of light resolves these infinities, producing interference patterns. At the event horizon of a black hole[2], time stands still and waves oscillate with infinitely small wavelengths. However, the quantum nature of light results in evasion of the catastrophe and the emission of Hawking radiation[3]. Here I report a theoretical laboratory analogue of an event horizon: a parabolic profile of the group velocity[7] of light brought to a standstill in an atomic medium[4–6] can cause a wave singularity similar to that associated with black holes. In turn, the quantum vacuum is forced to create photon pairs with a characteristic spectrum, a phenomenon related to Hawking radiation[3]. The idea may initiate a theory of 'quantum' catastrophes, extending classical catastrophe theory[8,9].



Wavelength appears to shrink linearly as event horizon is approached to compensate red shift:

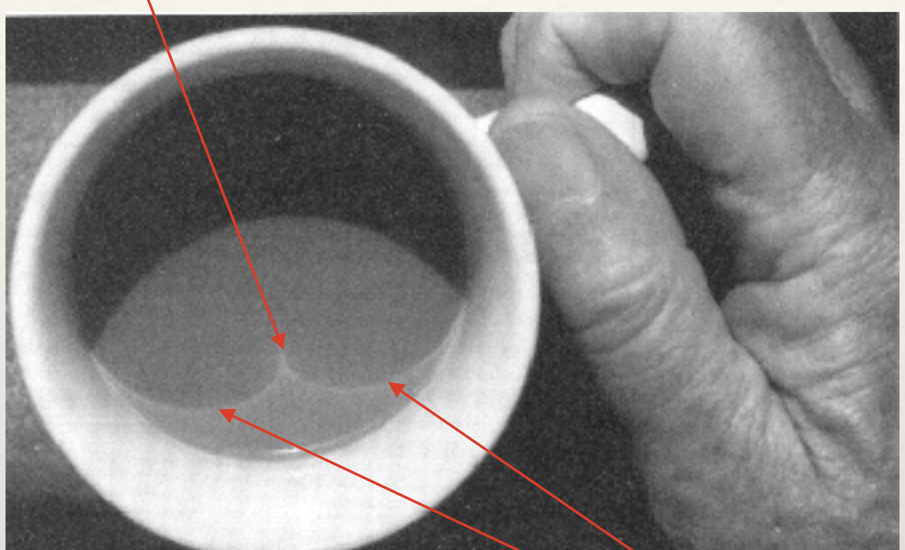
$$k \sim \frac{\nu}{r - r_{\text{eh}}}$$

$$\psi = e^{i[\int k(z)dz - \omega t]} = e^{i[\nu \log(r - r_{\text{eh}}) - \omega t]}$$
$$= (r - r_{\text{eh}})^{i\nu} e^{i\omega t}$$

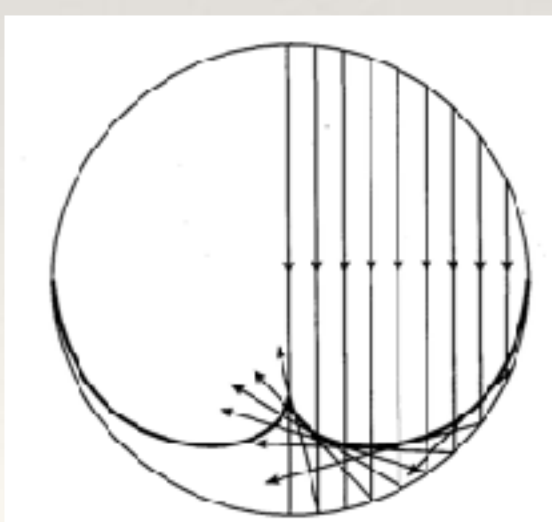
where: $\Re(\nu) = \omega \frac{c}{g_{\text{eh}}}$

Ordinary ray caustics

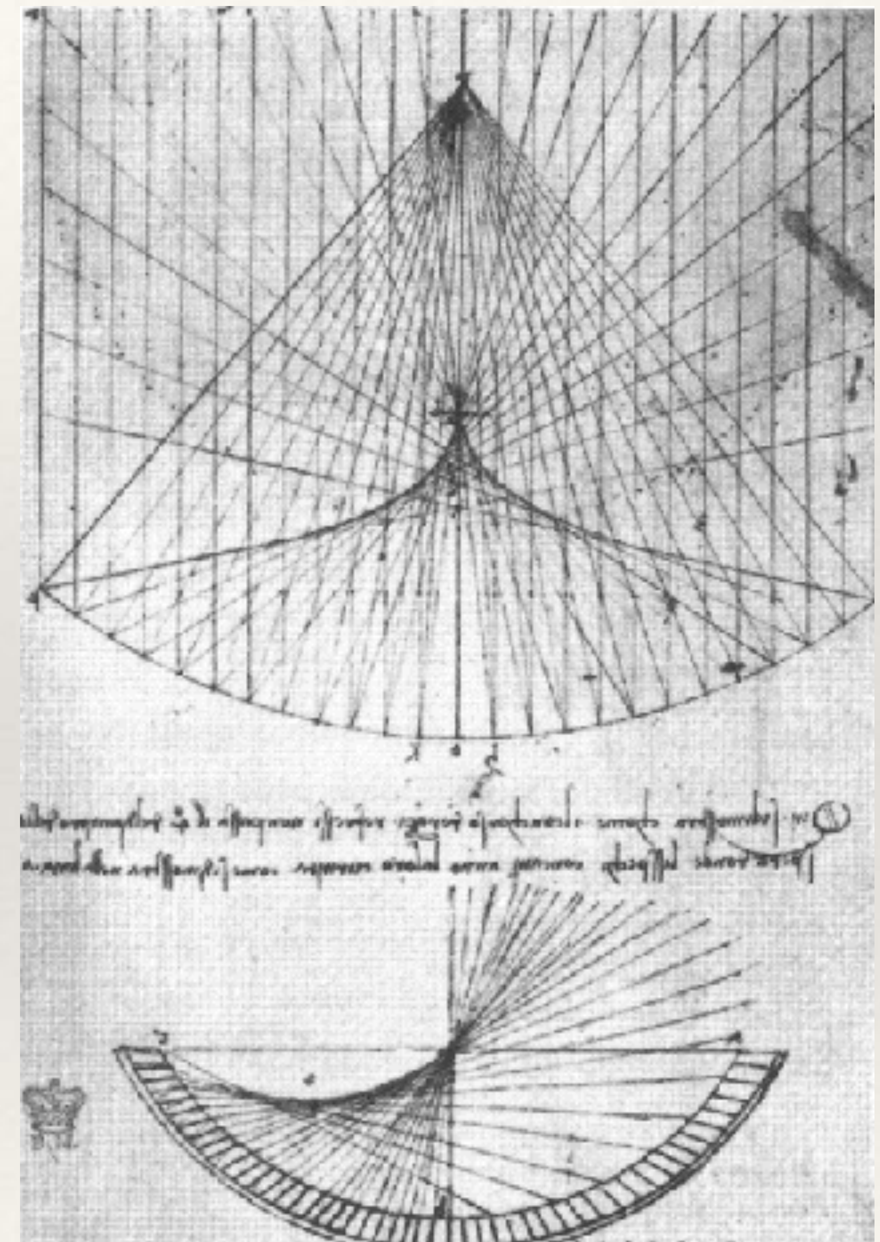
cusp point



fold lines



- Structurally stable
- Generic
- Singular: intensity diverges (in ray theory)



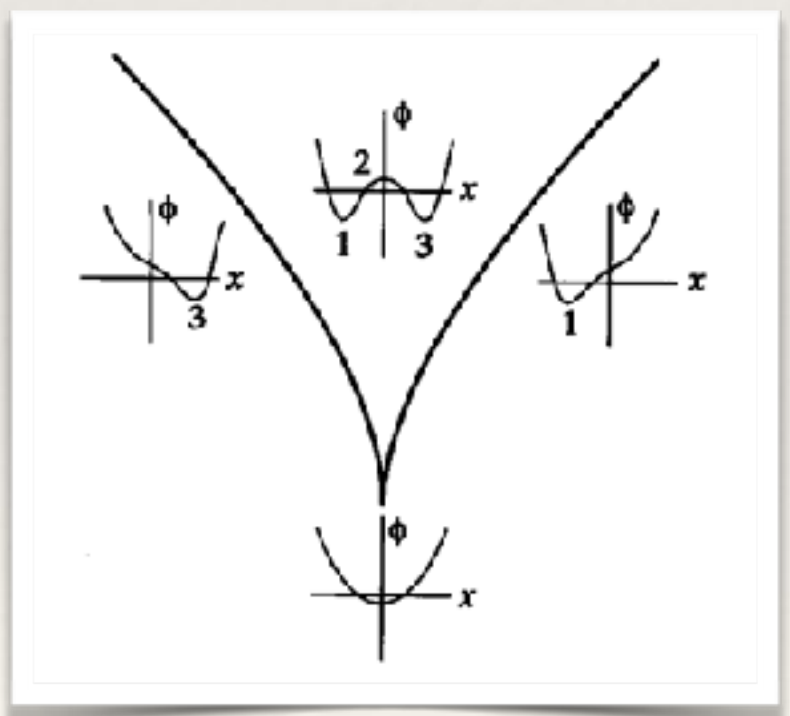
Leonardo de Vinci c. 1508

The Pearcey function

"Wave catastrophe"

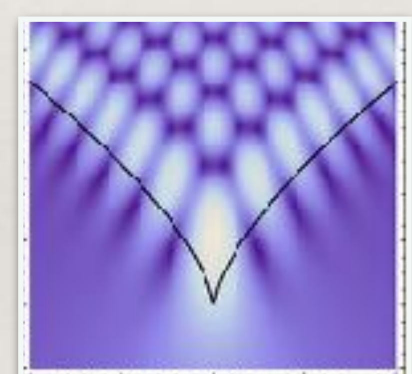
$$\Psi_{\text{cusp}}(C_1, C_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^4/4 + C_2 s^2/2 + C_1 s)} ds$$

T. Pearcey, Phil. Mag. **37**, 311 (1946)

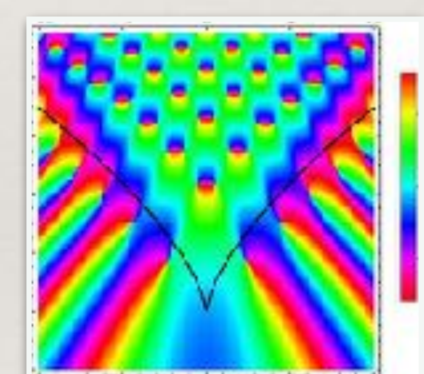


C_2

C_1



amplitude



phase

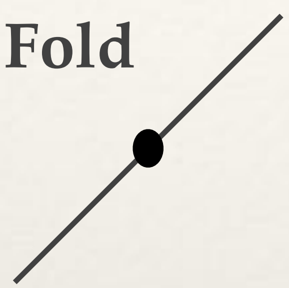
scaling property:

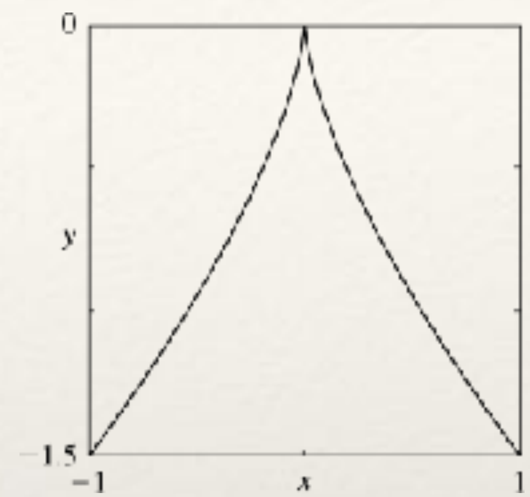
$$\Psi(C_1, C_2; k) = \left(\frac{k}{k_0}\right)^{1/4} \Psi[(k/k_0)^{3/4} C_1, (k/k_0)^{1/2} C_2, k_0]$$

There are three classical solutions inside the cusp and one outside

Elementary catastrophes (structurally stable singularities)

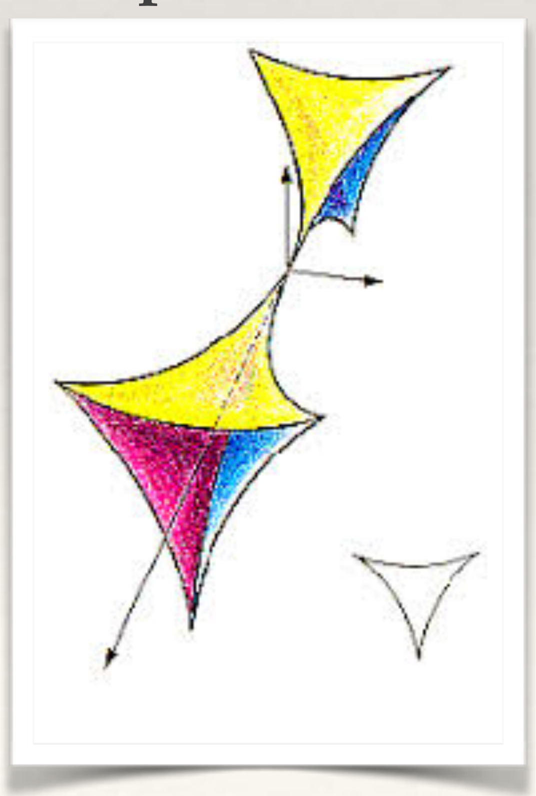
1D: Fold



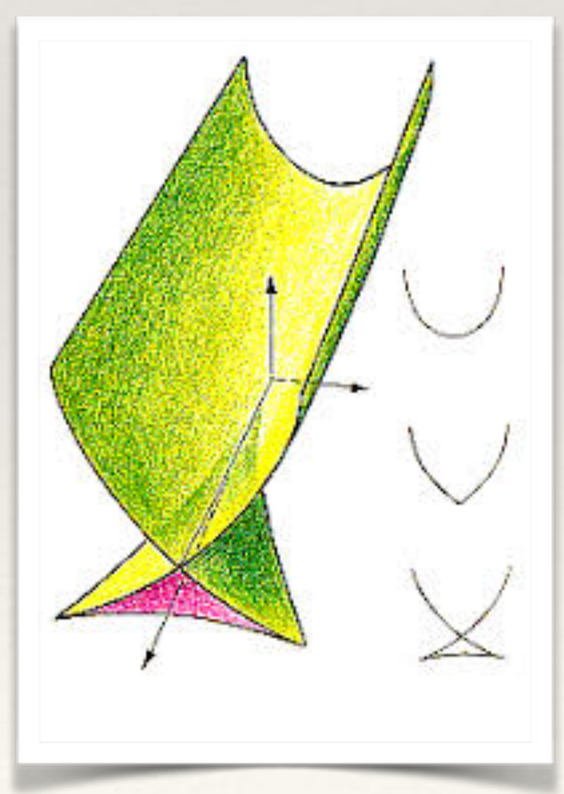


2D: Cusp

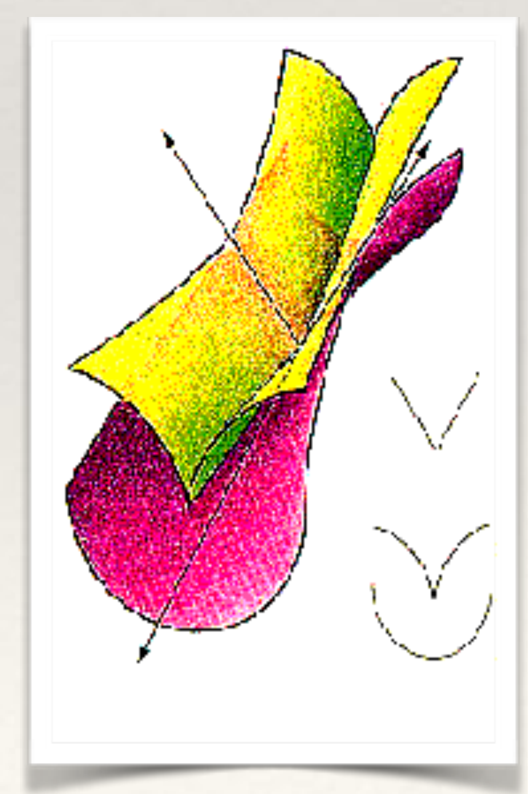
3D: **elliptic umbilic**



swallowtail

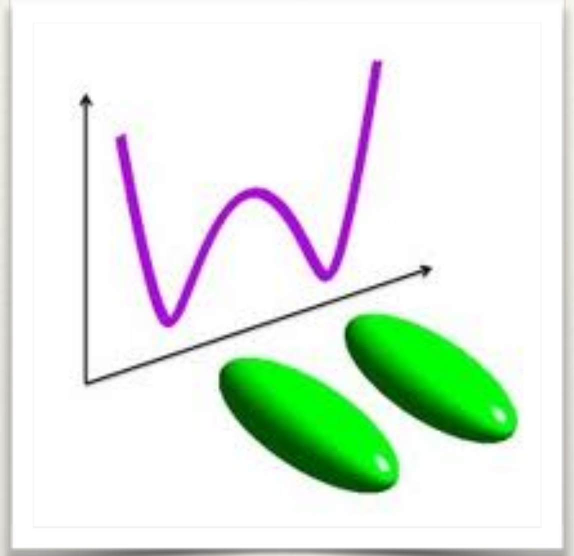


hyperbolic umbilic

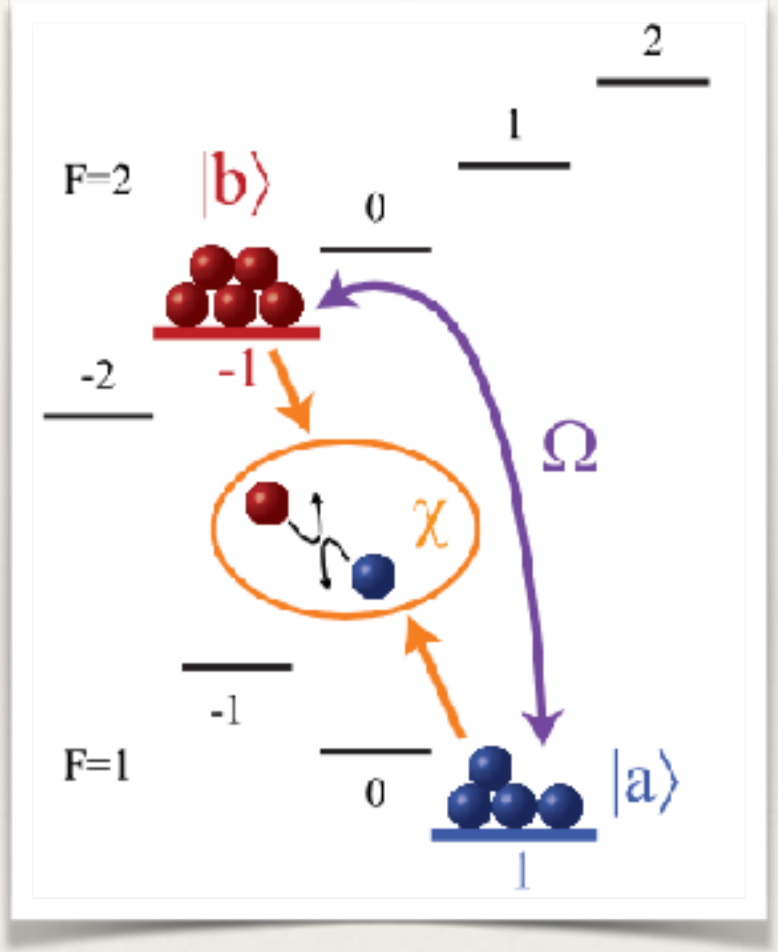


Another example: Bose-Einstein condensate in a double well potential

Two external states: double well trap OR **Two internal states**



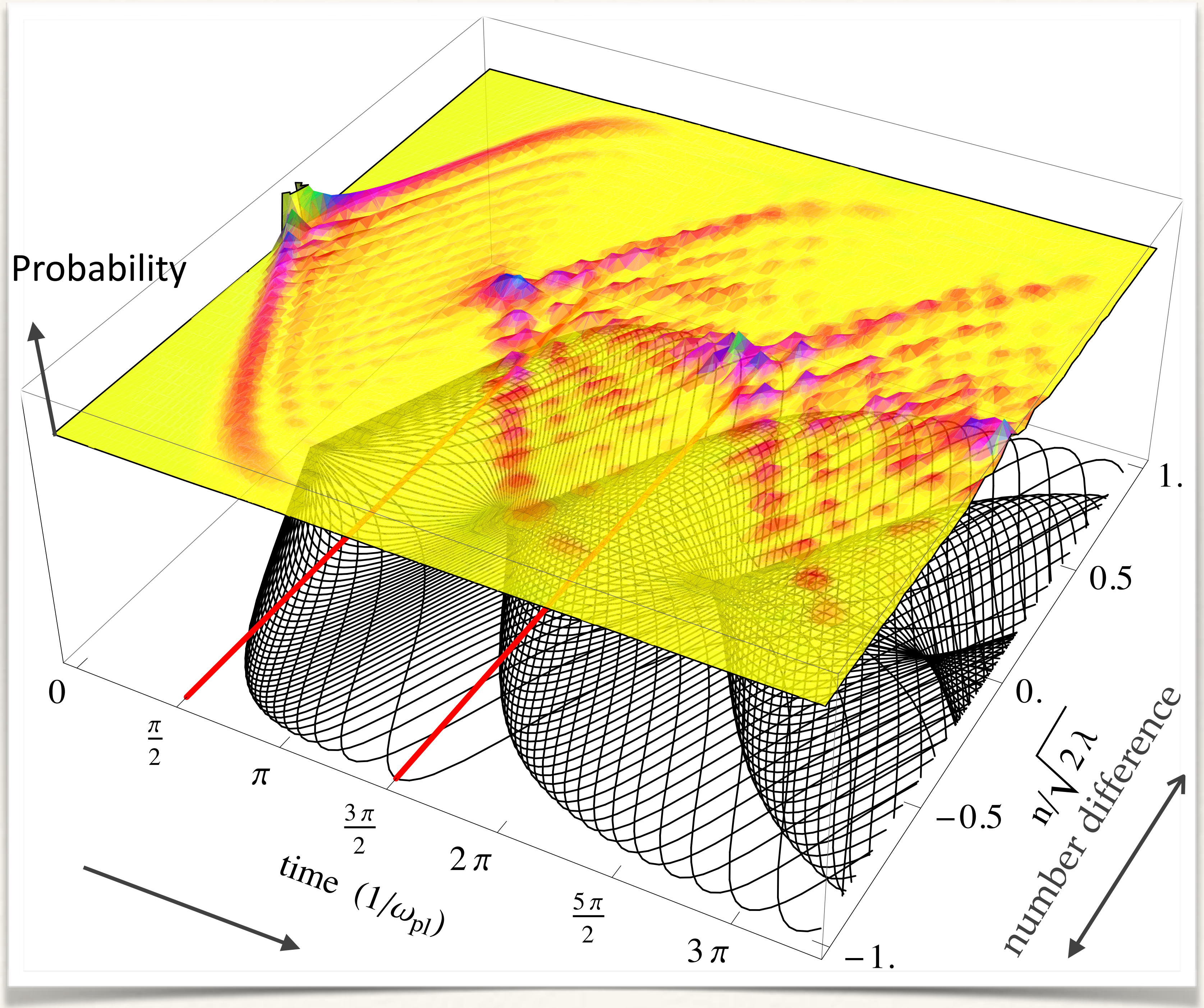
Oberthaler (Heidelberg) 2005
Steinhauer (Technion) 2007
Schmiedmayer (Vienna) 2007
Hinds (Imperial College) 2010
Thywissen (Toronto) 2011
Roati (Florence) 2015
Fattori (Florence) 2016



Zibold et al (2010)

Catastrophes in Fock space following the sudden coupling of two independent superfluids

D. O'D., Phys. Rev. Lett. **109**, 150406 (2012)

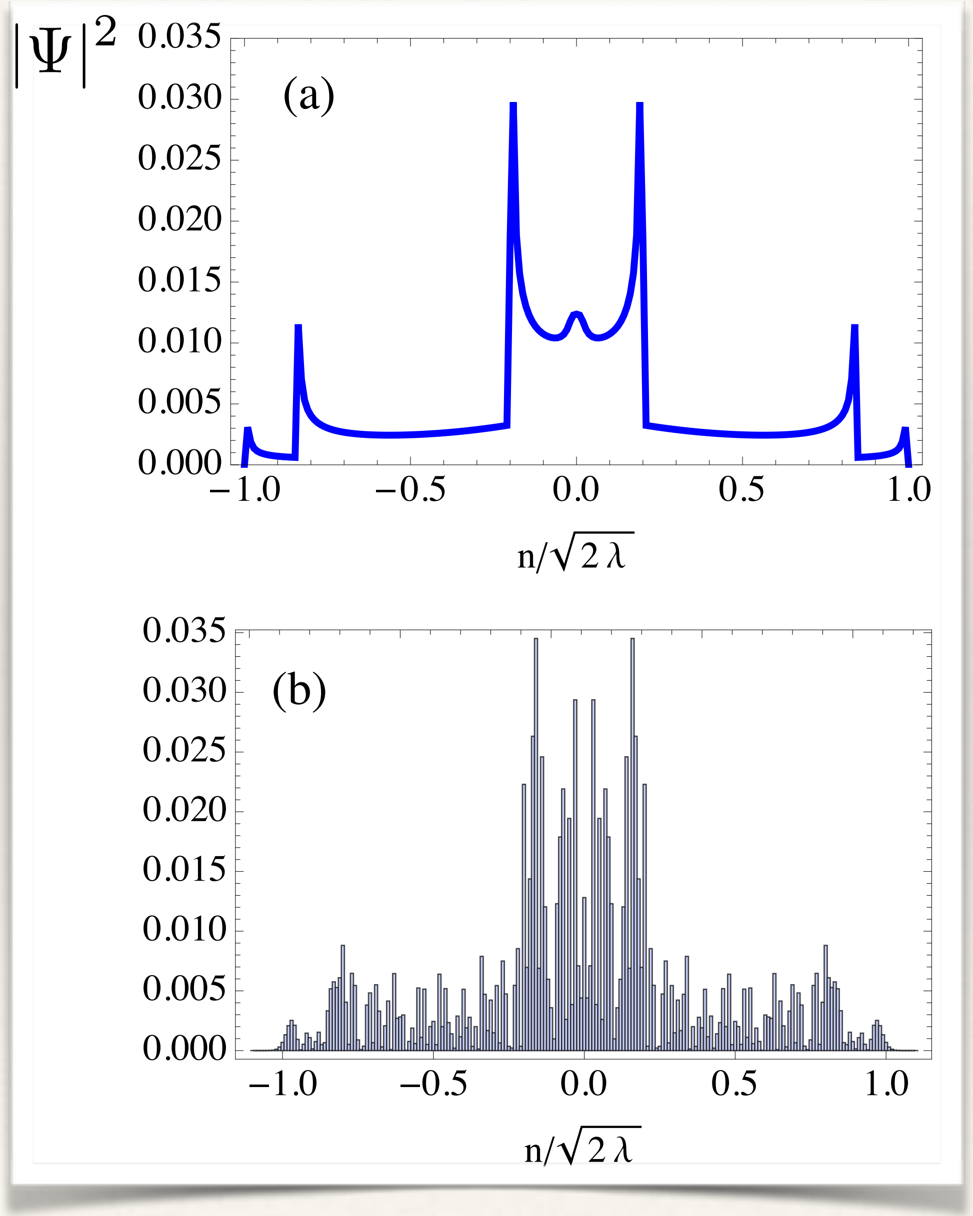


$$\Delta n \Delta \phi \geq 1$$

$$\lambda \equiv \frac{2E_J}{E_c} = 200$$

Quantum vs classical

$$\tau = 3.3\pi$$



Classical field
(Gross-Pitaevskii theory)

Quantum field
(Bose-Hubbard theory)

$$\lambda \equiv \frac{2E_J}{E_c} = 5000$$

Caustic singularities are removed by second-quantizing the field

Conclusions

- Inverse square potential has a quantum anomaly
- Can (and has) been realized in the laboratory
- Can have non-self adjoint extensions, e.g. describing absorption
- Still to be seen: log-periodic absorption, exotic bound state
- Wave function provides an example of a quantum catastrophe

See poster by Sriram Sundaram: "Schwinger pair production and fall to infinity"

Elementary catastrophes and their generating functions for K≤4

name	codimension K	$\phi(s;C)$ [generating function]
Fold	1	$s^3/3+Cs$
Cusp	2	$s^4/4+C_2s^2/2+C_1s$
Swallowtail	3	$s^5/5+C_3s^3/3+C_2s^2/2+C_1s$
Elliptic umbilic	3	$s_1^3-3s_1s_2^2- C_3(s_1^2+s_2^2)-C_2s_2-C_1s_1$
Hyperbolic umbilic	3	$s_1^3+s_2^3- C_3s_1s_2-C_2s_2-C_1s_1$
Butterfly	4	$s^6/6+C_4s^4/4+C_3s^3/3+C_2s^2/2+C_1s$
Parabolic umbilic	4	$s_1^4+s_1s_2^2+C_4s_2^2+C_3s_1^2+C_2s_2+C_1s_1$

R. Thom *Structural Stability and Morphogenesis* (Benjamin, 1975); **V.I. Arnol'd**, Russ. Math. Survs. *30 (5) (1975) p.1*

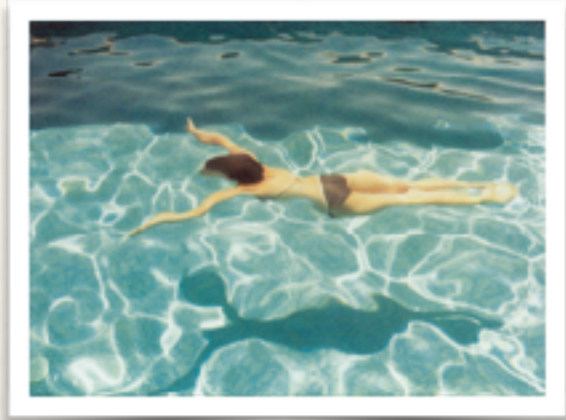
Catastrophe theory describes the **singularities of gradient maps** $\frac{\partial \phi}{\partial s_i} = 0$

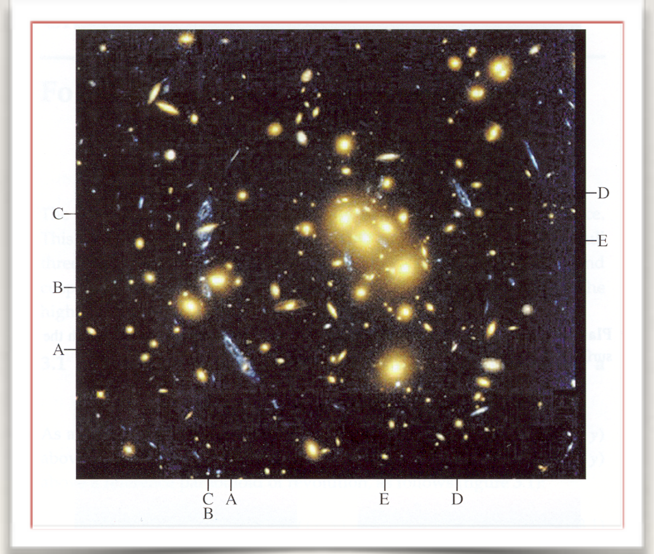
Scaling exponents

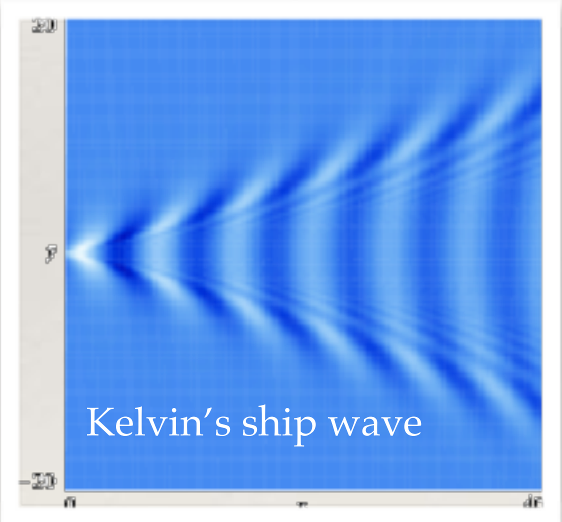
Catastrophe	Arnold Index β	Berry Indices σ_j	Berry Index γ
Fold	1/6	*2/3*	2/3
Cusp	1/4	*3/4, 1/2*	5/4
Swallowtail	3/10	*4/5, 3/5, 2/5*	9/5
Elliptic umbilic	1/3	*2/3, 2/3, 1/3*	5/3
Hyperbolic umbilic	1/3	*2/3, 2/3, 1/3*	5/3
Butterfly	1/3	*5/6, 2/3, 1/2, 1/3*	7/3
Parabolic umbilic	3/8	*5/8, 3/4, 1/2, 1/4*	17/8

$$\psi(C_j;k) = \left(\frac{k}{k_0}\right)^{\beta} \psi\left[(k/k_0)^{\sigma_j} C_j; k_0\right]$$

Natural focusing: caustics







+large scale structure in the universe (Zeldovich 1982)

+wave packet dynamics at the Anderson transition (Lemarie 2010)…..

Old non-hermitian work…

J. Phys. A: Math. Gen. **31** (1998) 2093–2101. Printed in the UK   PII: S0305-4470(98)89200-7

Diffraction by volume gratings with imaginary potentials

M V Berry and D H J O'Dell
H H Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK

$$\nabla^2\psi(x,z) + \left(k^2 + \mathrm{i}Q\cos^2(Kx)\right)\psi(x,z) = 0$$

Abstract. A medium described by an imaginary potential (or imaginary (refractive index)^2), that varies sinusoidally in one direction, acts as a volume grating for plane waves incident on it obliquely or normally. Two peculiar features are identified. First, if the potential is weak, so that there are only two significant diffracted beams near the Bragg angle, and three for normal incidence, diffraction is strongly affected by degeneracies of the non-Hermitian matrix generating the 'Bloch waves' in the grating; the effect of these degeneracies is very different from that of the Hermitian degeneracies for transparent gratings. Second, if the potential is strong and the grating thick, the asymptotic distribution of intensities among the diffracted beams (momentum distribution) is a rather narrow Gaussian, and dominated by a single set of complex rays; this is very different from the semiclassical limit for transparent gratings, where the rays form families of caustics proliferating with thickness, and with a wider momentum distribution.

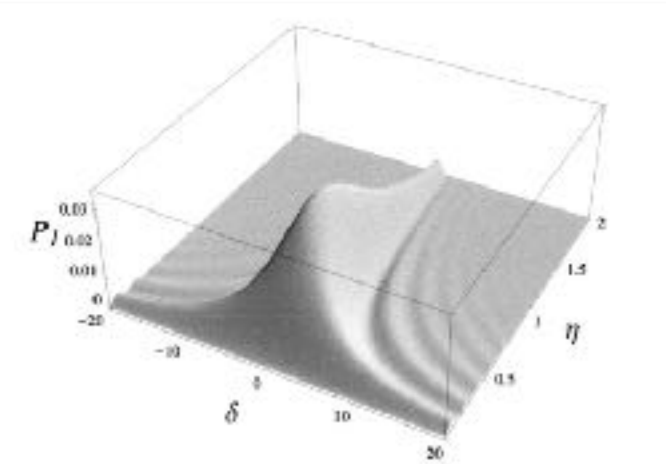
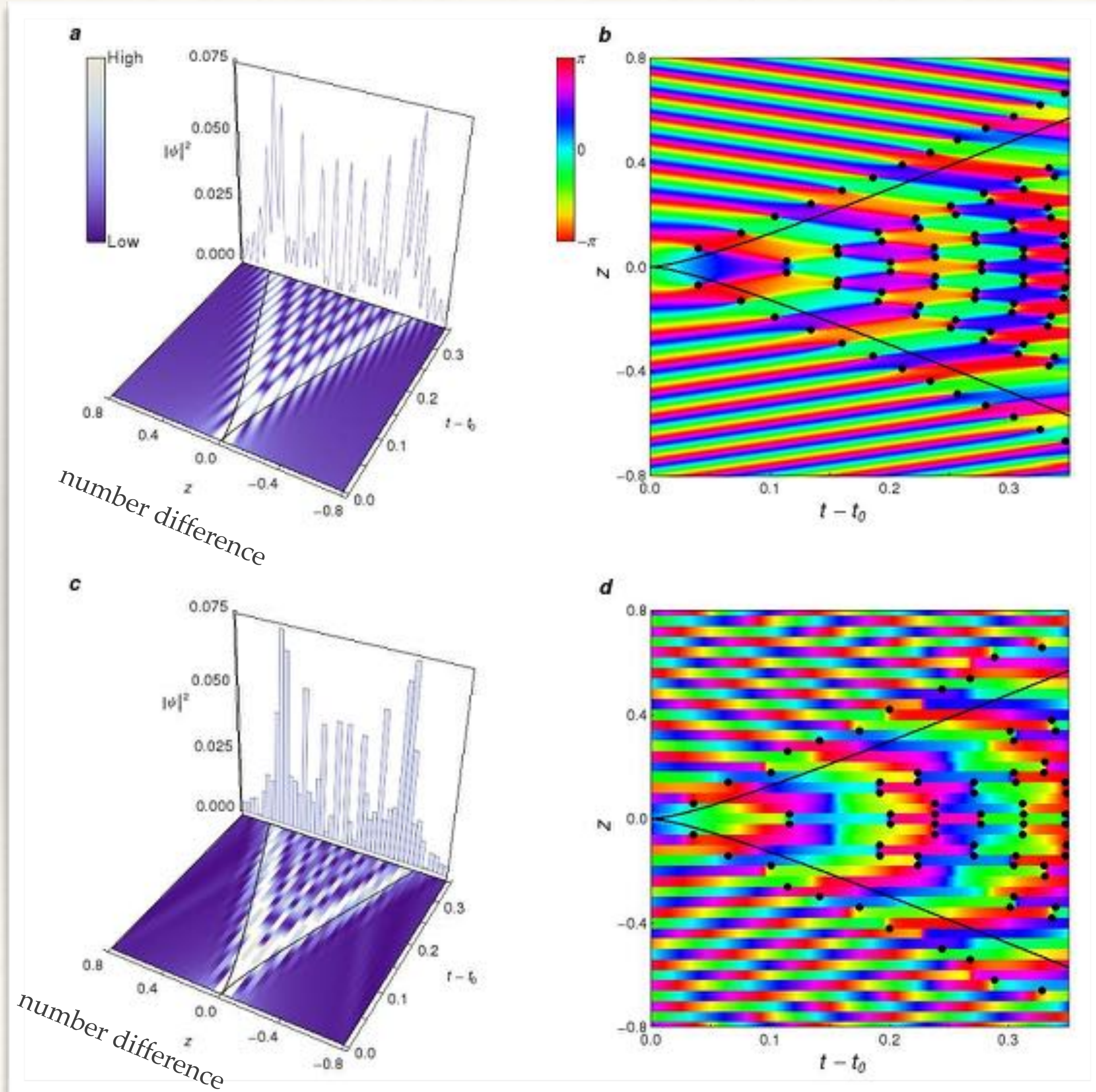


Figure 1. Intensity P_1 (equation (17)) of the first Bragg-reflected beam for an imaginary grating as a function of the thickness and angular deviation variables η and δ defined by (16).

Morphology of a quantum catastrophe

(Mumford, Turner, Sprung, D.O'D) arXiv:1610.08031v1



standard wave catastrophe

(continuum approximation)

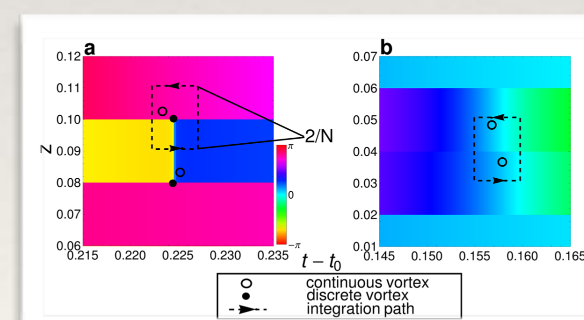
$$[\hat{\phi}, \hat{z}] = 2i/N$$

where: $$z \equiv \frac{n_l - n_r}{N}$$

$$\Psi\left\{Y, X; N_2\right\} = \left(\frac{N_2}{N_1}\right)^{\beta} \Psi\left\{\left(\frac{N_2}{N_1}\right)^{\sigma_1} Y, \left(\frac{N_2}{N_1}\right)^{\sigma_2} X; N_1\right\}$$

quantum catastrophe (discrete)

$$\oint \nabla\theta \cdot d\mathbf{l} = \pm 2\pi$$



Coreless vortices!

Inverse square potential

Inverse square potential

$$V(r) = \frac{g}{r^2}$$

$$H = \frac{1}{2}m\dot{x}^2 + \frac{g}{r^2}$$

Scale invariance: $x \to \lambda x \qquad t \to \lambda^2 t \qquad H \to H/\lambda^2$