

Quantum simulations of parity-time symmetric Hamiltonians

Yogesh N. Joglekar

Indiana University - Purdue University Indianapolis (IUPUI)



Collaborators: Andrew Harter (IUPUI), Anthony Laing + group (Bristol)
Indianapolis area High-School Students

Outline

1. PT Hamiltonians: history.
2. PT = classical systems with “balanced” gain-loss.
3. Unitary dilation: simulate a quantum PT system.
4. Bosonic correlations.
5. Entropy evolution, quantum information evolution.
6. Conclusions.
7. Other examples: ultra cold gas, electrical circuits

Introduction: PT (quantum) theory



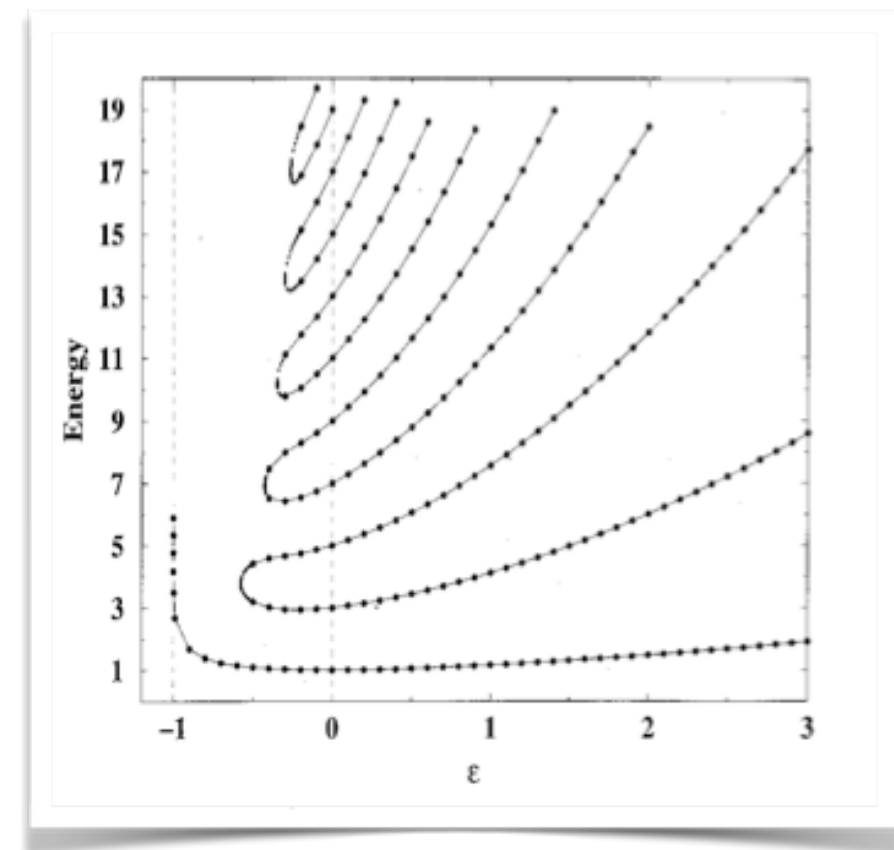
E Caliceti



Zinn Justin

Continuum models: $-\partial_x^2 + x^2(ix)^\epsilon$

- Hermitian, PT kinetic energy term
- Non-Hermitian, PT-symmetric potential
- Real spectra for “small” non-Hermiticity
- Complex eigenvalues for “large” non-H



PT symmetry breaking: emergence of complex conjugate eigenvalues.

$$H = -J\sigma_x + i\Gamma\sigma_z/2,$$
$$\lambda_{\pm} = \pm\sqrt{J^2 - \Gamma^2/4}$$

Real Eigenvalues: so what's next ?

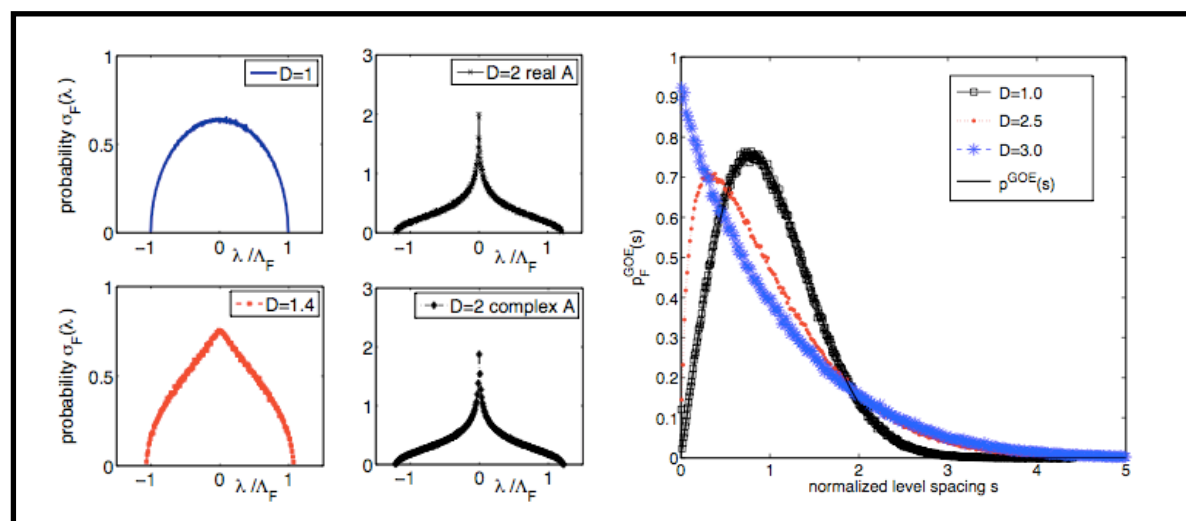


Carl Bender | 1998 PRL

Introduction: PT (quantum) theory

Option I: develop consistent quantum theory

- Similarity transform (“CPT”) to make H self-adjoint.
- Get unitary evolution: *claim fundamental theory*.
- Can define “PT” thermo, quantum brachistochrone etc.
- *Pseudo-Hermitian random matrices: interesting properties.*
- *Obtain “constants of motion” i.e. all intertwining operators*



M. Znojil Ali Mostafazadeh

Introduction: PT (quantum) theory

Consistent quantum theory: at what cost?

- Procedure valid only in PT-symmetric region.
- Intertwining operators depend on H .
- *Observables in the theory depend on H .*
- Continuum models: x, p no longer “observables”.
- Discrete models: S_k no longer “observables”.
- *Local version violates no-signaling principle.*

Y-C Lee et al, PRL 112, 1330404 (2014)

What is the second option?



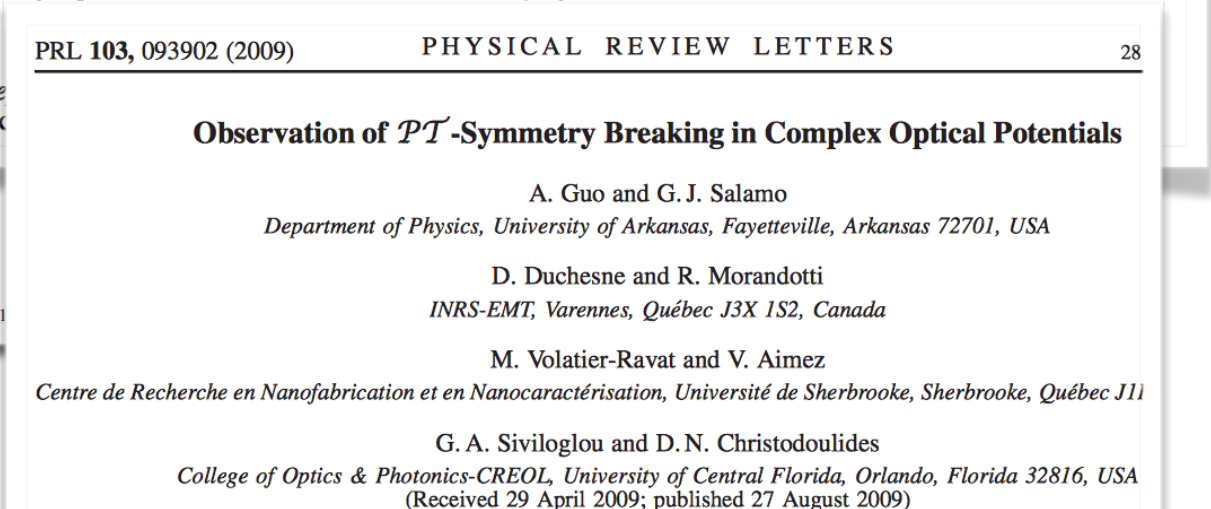
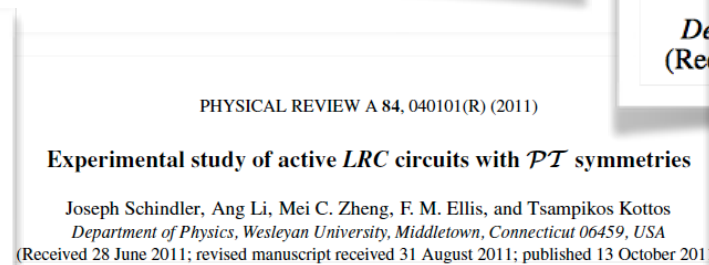
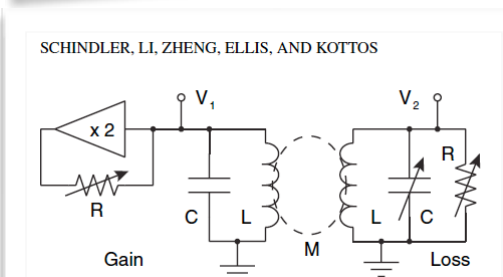
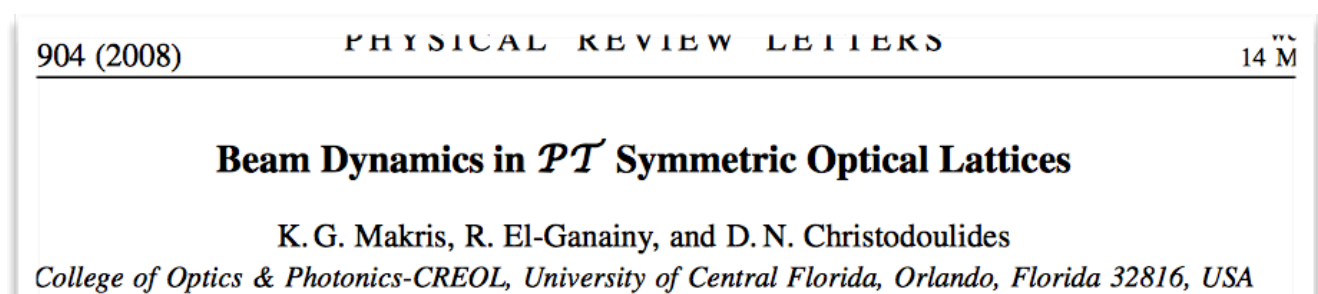
Demetrios Christodoulides

Option 2: effective model for open systems

Schrodinger equation : paraxial approximation to ME

real potential: index of refraction $n(x)$

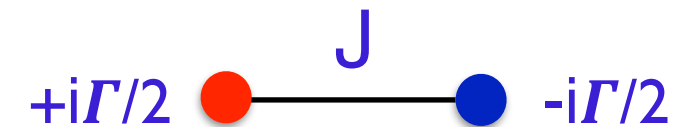
imaginary potential: gain or loss, complex $n^*(-x)=n(x)$



PT: (classical) balanced gain and loss

PT systems = balanced gain and loss

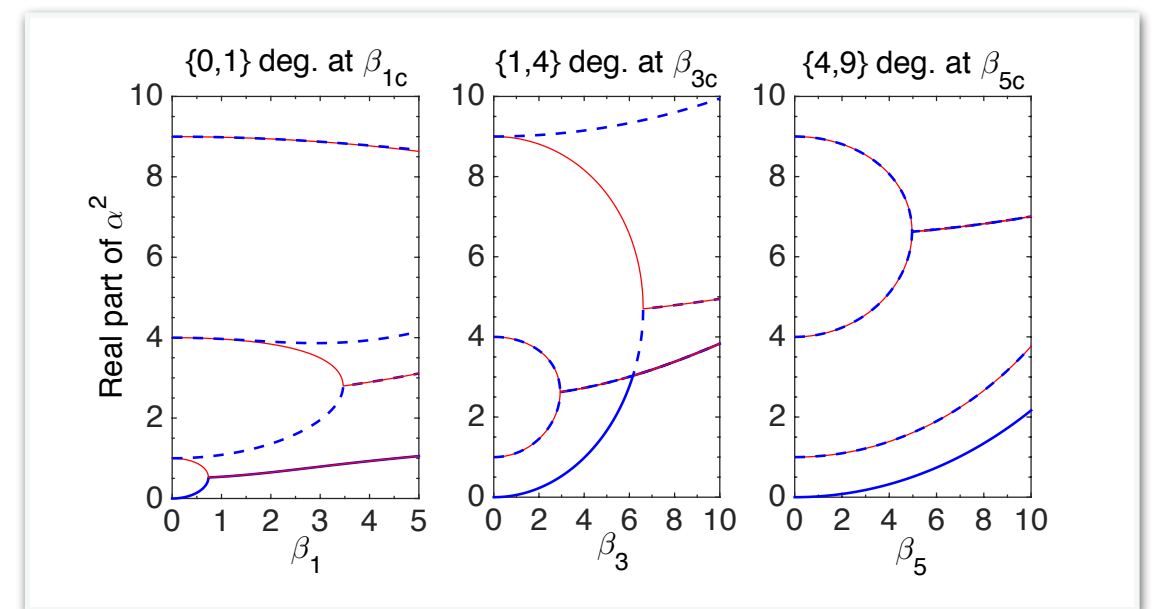
$$|\psi(t)\rangle = e^{-iH_{PT}t}|\psi(0)\rangle$$
$$\langle\psi(t)|\psi(t)\rangle \neq \langle\psi(0)|\psi(0)\rangle$$



$$H_{PT} = -J\sigma_x + i\Gamma\sigma_z/2$$

$$\lambda_{\pm} = \sqrt{J^2 - \Gamma^2/4}$$

PTS (real): intensity
oscillations, PT eigenfunctions
EP(degenerate):
power-law growth.
PTB (complex): exponential
growth, not eigenfunctions of
PT

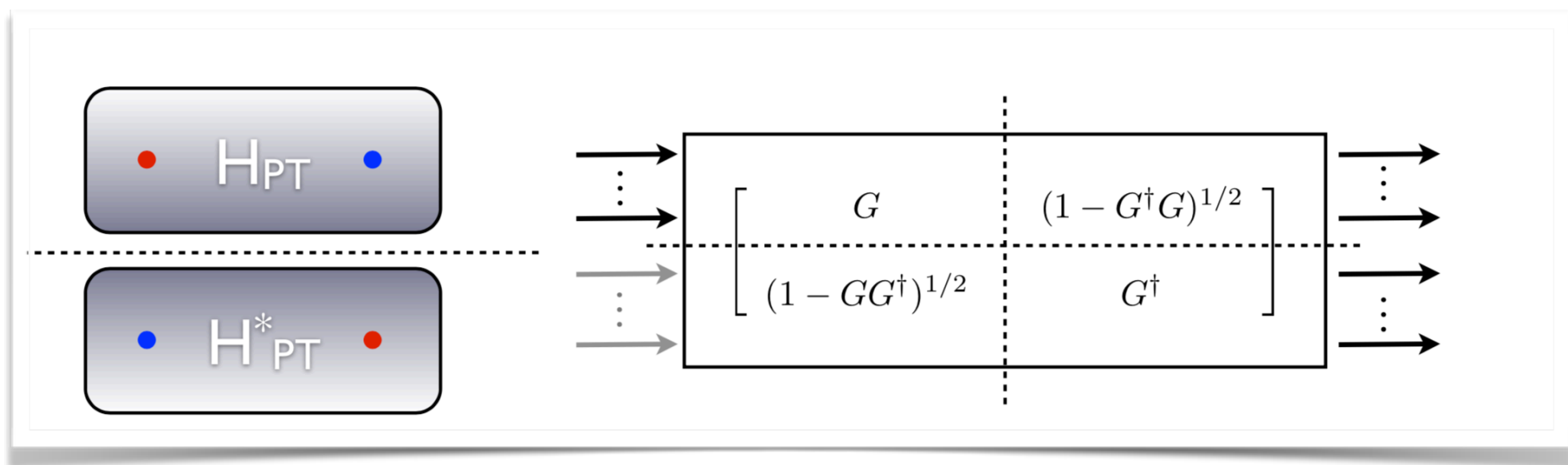


Typical flow of Re[Eigenvalues] vs. non-Hermiticity

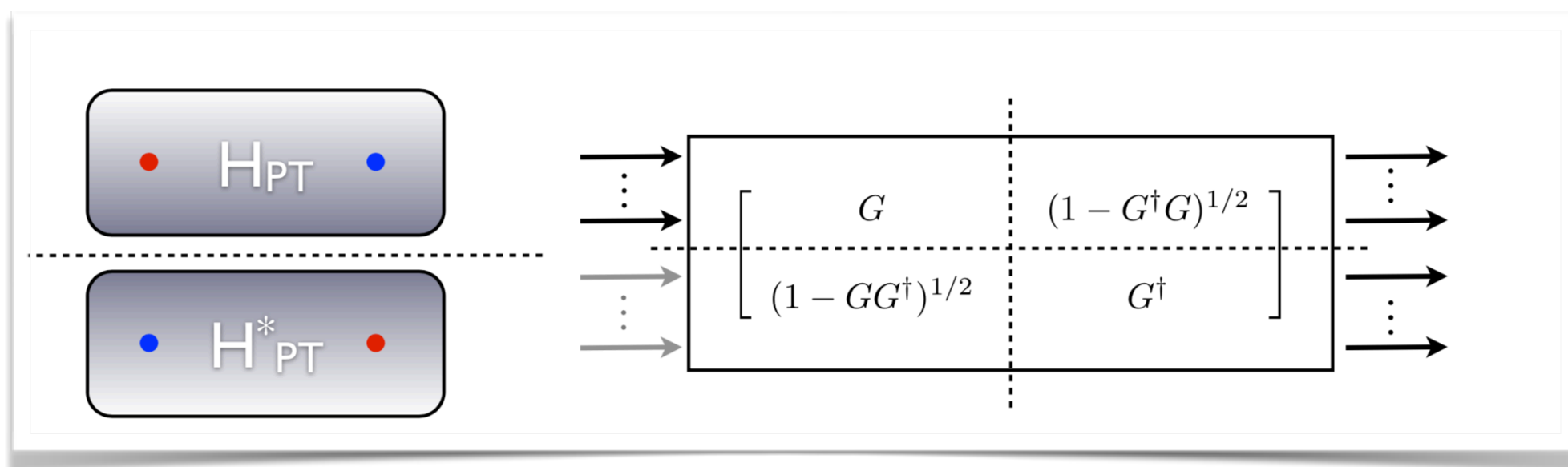
Quantum simulations of PT systems

- Optics: loss linear, but gain non-linear, random ✗
- Simulate gain-loss Hamiltonian (CTQW) ???
- Simulate loss-only Hamiltonian: quantum possible ✓
- Simulate time-evolution operator (DTQW) ✓

$$G_{PT}(t) = \exp(-iH_{PT}t) \neq G_{PT}^{\dagger -1}(t)$$

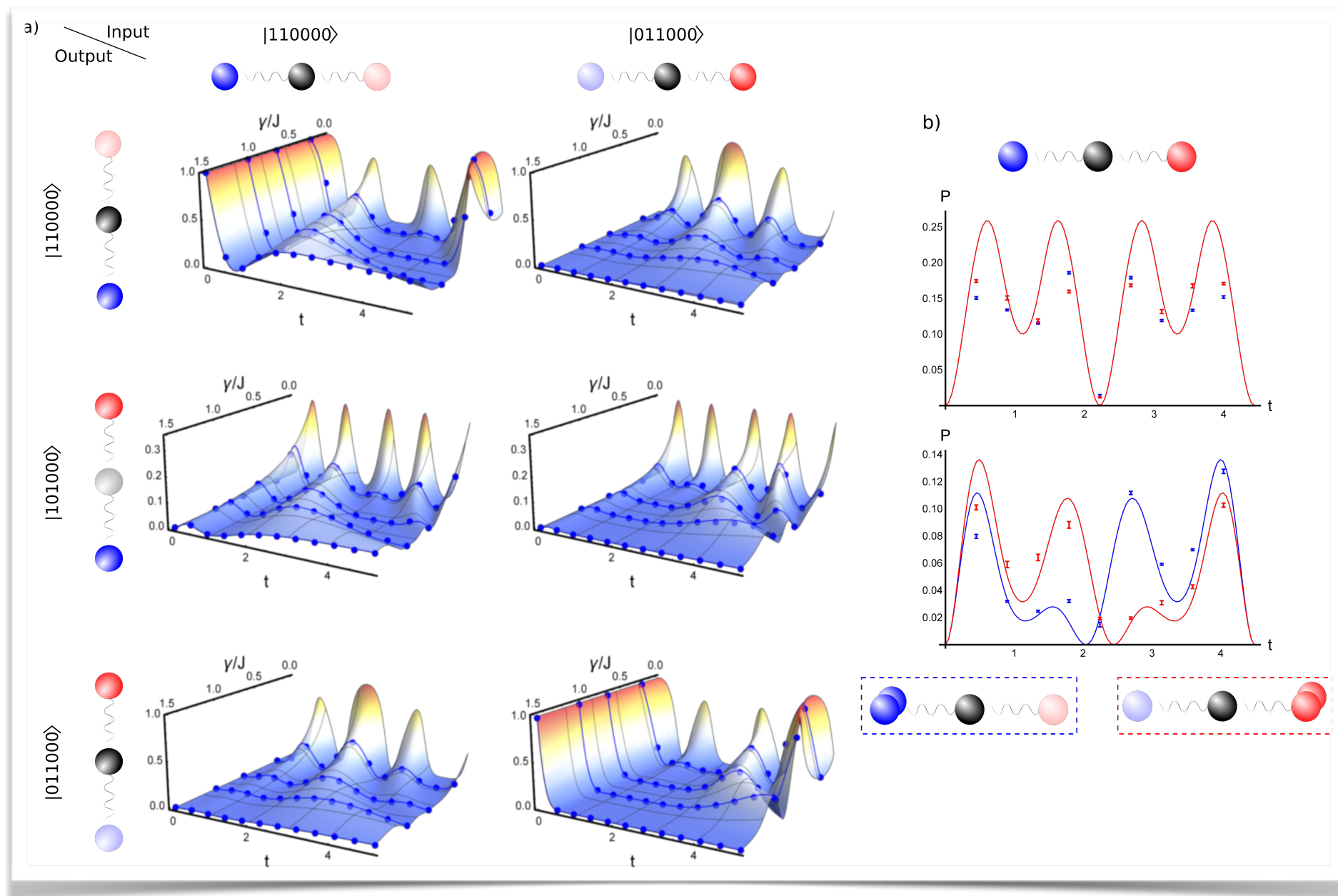


Unitary dilation approach to quantum simulation



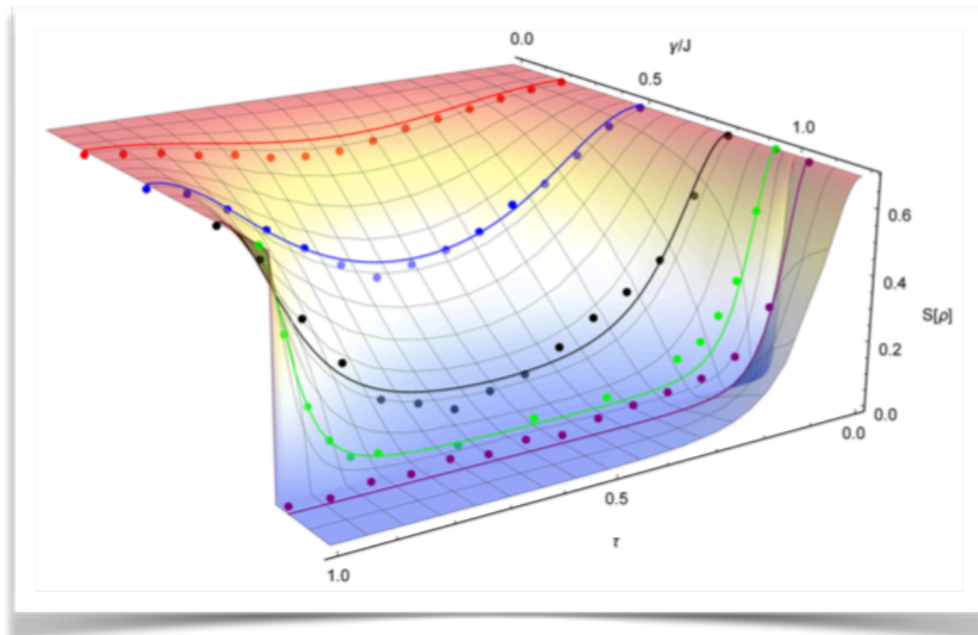
- Applicable in both PT symmetric + broken regimes.
- Use $2m \times 2m$ unitary for m -dimensional PT system.
- Not a Naimark or other Hamiltonian dilation.
- Measure coincidences, or full density matrix.
- Obtain quantum information properties.

Bosonic correlation results



Gain loss “interchangeable” in PT symmetric phase
Gain loss distinct in the PT broken phase

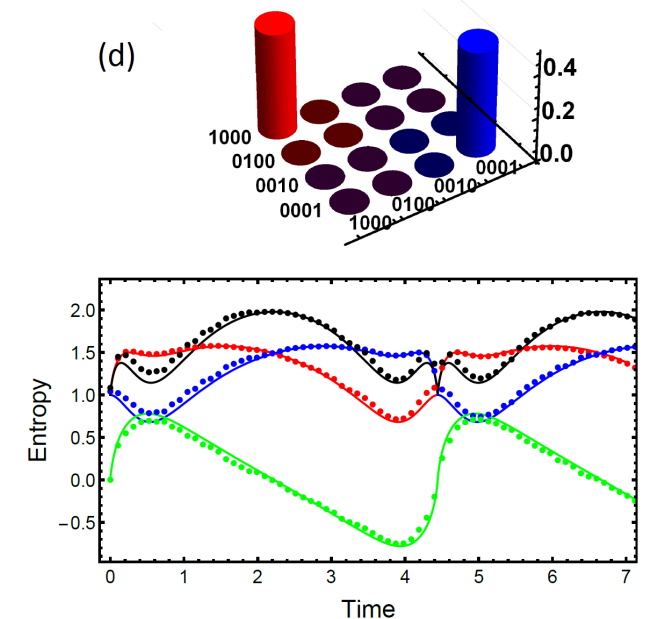
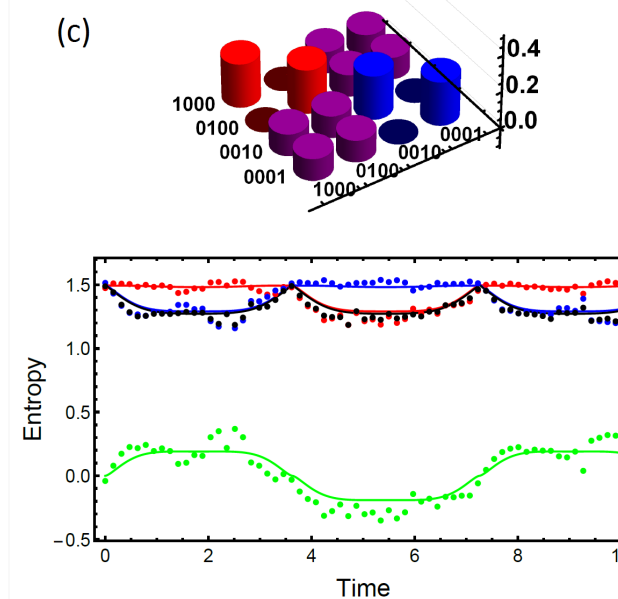
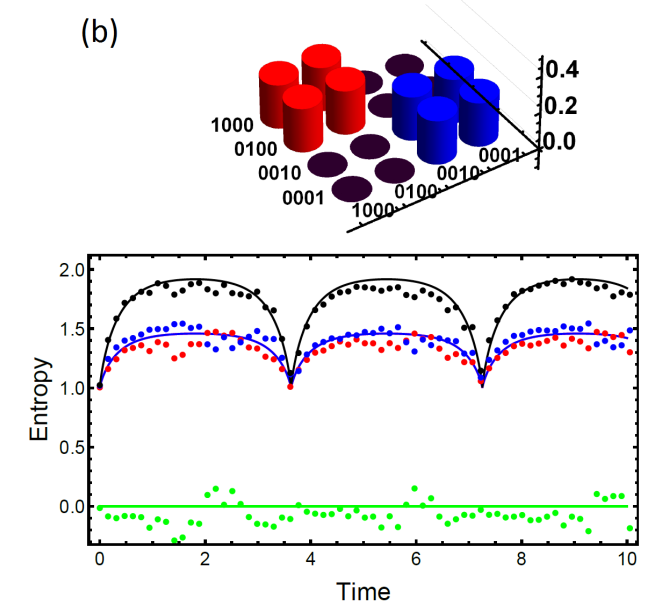
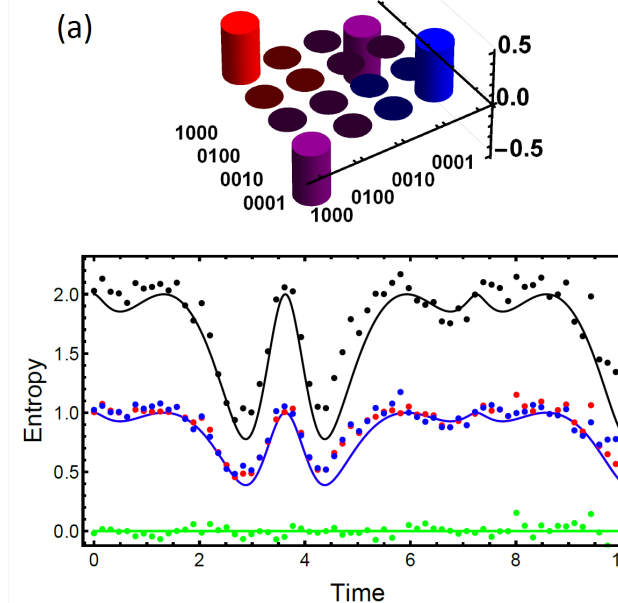
Quantum information across PT transition



Access to all phases.

Entropy is oscillatory or steady state.

Q Information metrics show similar behavior.

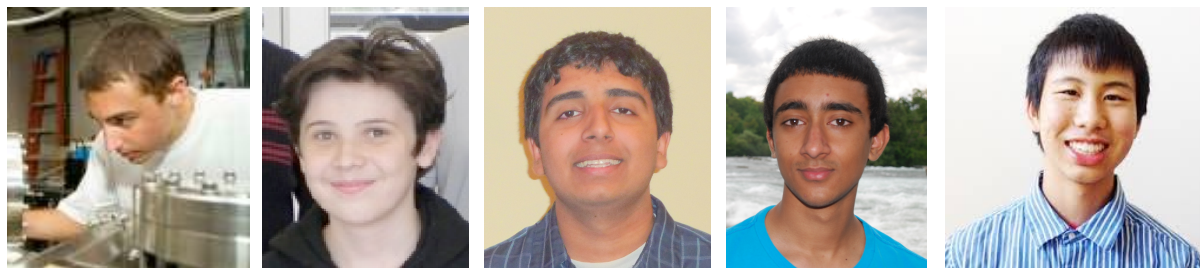


K. Kawabata et al. PRL 119, 190401 (2017)

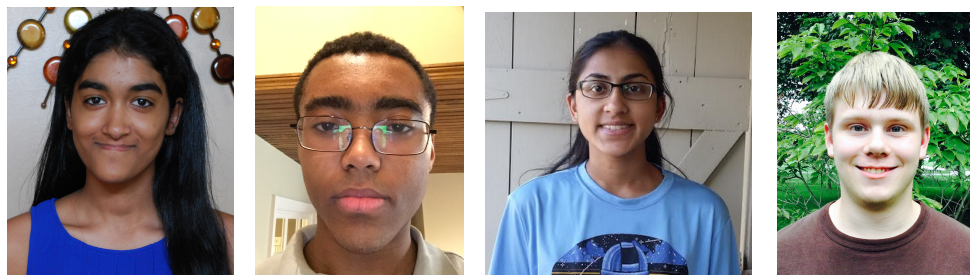
N. Maraviglia, A. Harter, ..., Y.N.J., Anthony Laing, in preparation

Conclusions and Outlook

- Unitary dilation allows quantum simulation.
- Q: Hamiltonian realizations? Lossy-system realizations?
- Highly accessible to young kids!



PRA 83, 030103(R) (2010).
PRA 84, 024103 (2011).
PRA 84, 043826 (2011).
PRA 87, 044101 (2013).
PRA 89, 030102(R) (2014).



PHHQP 2020

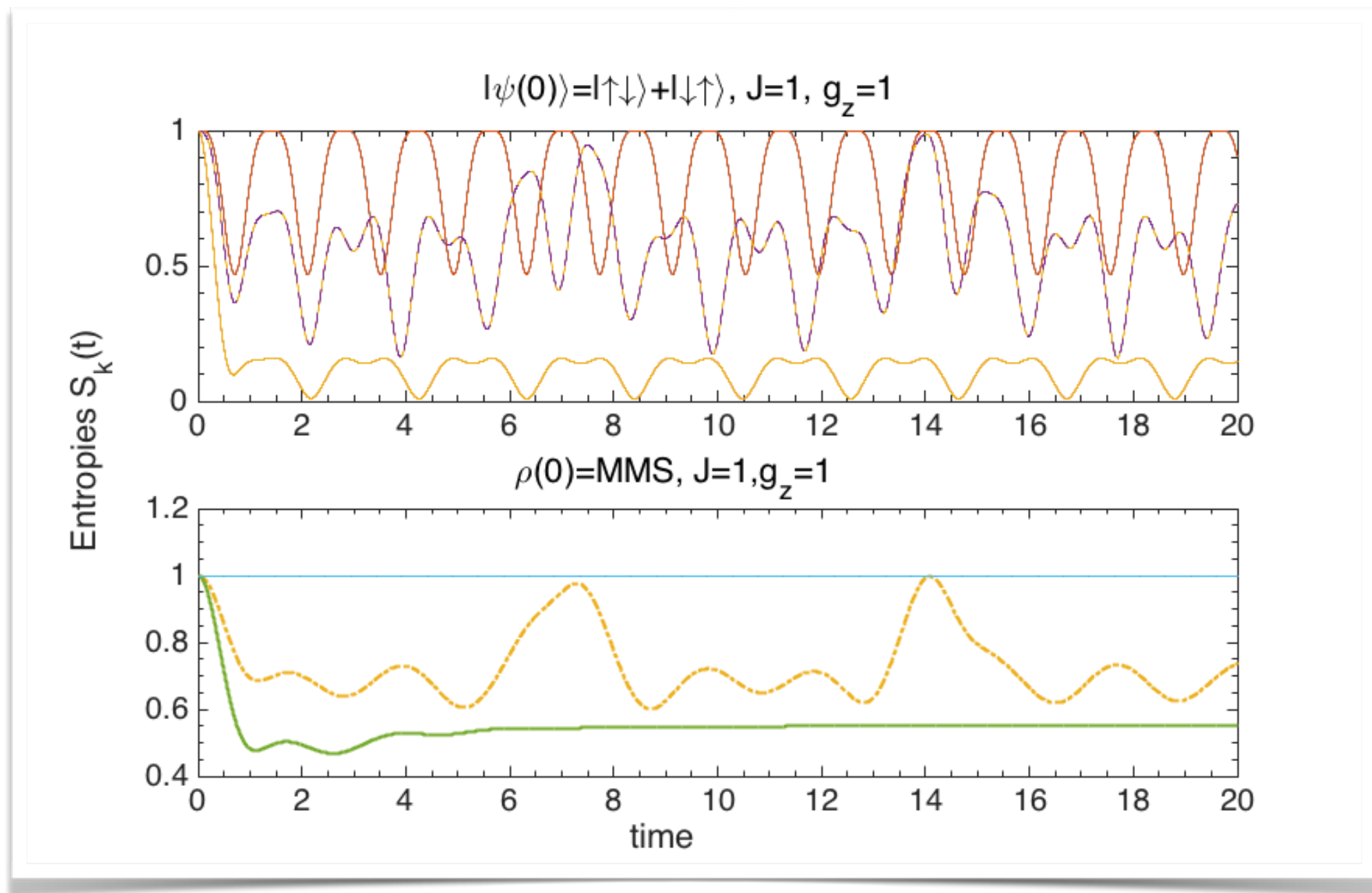
Yogesh Joglekar and Avadh Saxena

Santa Fe, New Mexico
June 28 - July 3, 2020



Quantum entanglement in coupled qubits: proposal

$$H = -J(X_1 + X_2) + g_z Z_1 Z_2 - i\Gamma_1(1 - Z_1)/2$$



“Passive” PT systems = neutral and localized loss

PTS: decay-rate increases with Γ .

PT transition: fastest decay.

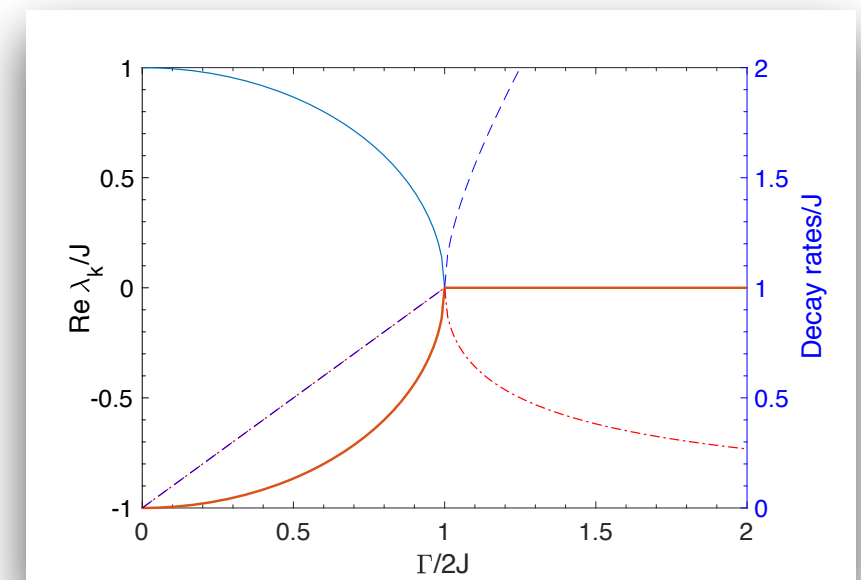
PTB: decay-rate *decreases* with Γ .

Scaled norm shows oscillatory to exponential time dependence.

gain-loss = localized loss



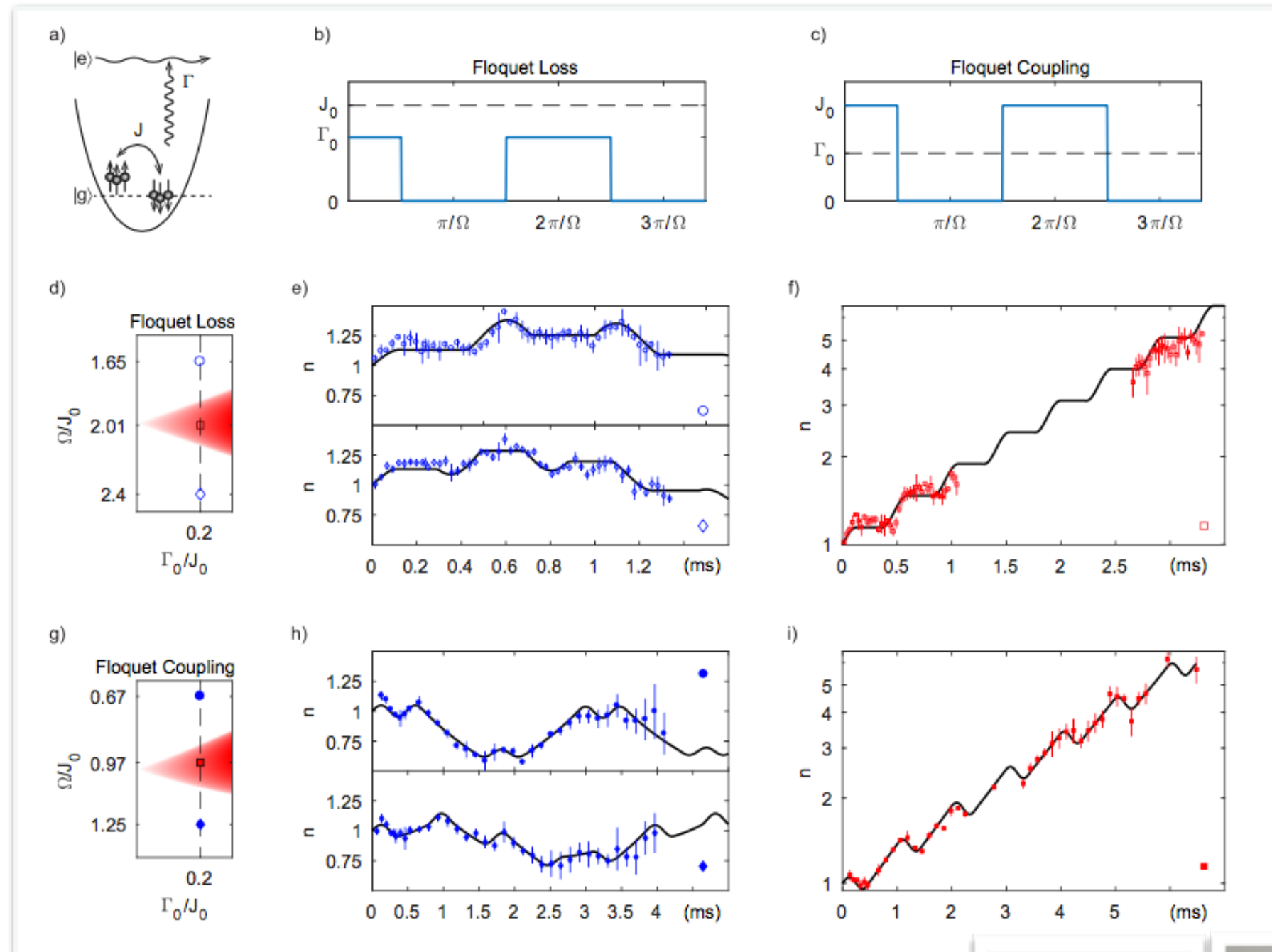
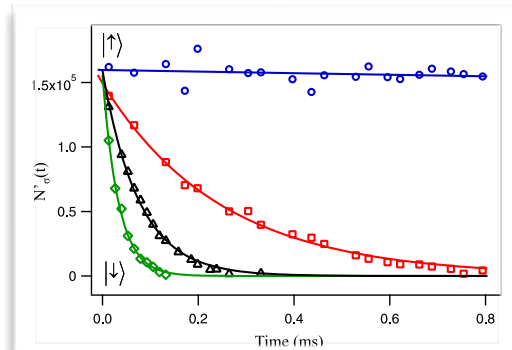
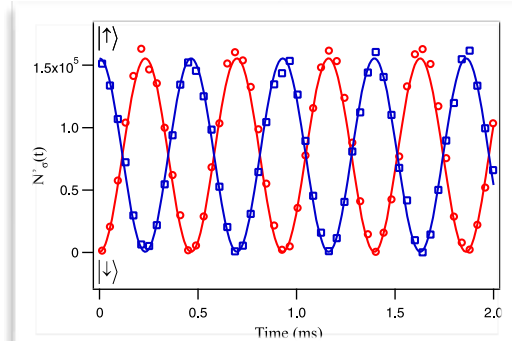
$$H = -J\sigma_x - i\Gamma|\uparrow\rangle\langle\uparrow|$$
$$\lambda_{\pm} = -i\Gamma/2 \pm \sqrt{J^2 - \Gamma^2/4}$$



Passive PT: quantum realizations possible

Floquet PT-breaking in a ultra cold gas (^6Li)

$\sim 0.16\text{M } ^6\text{Li}$
@ $T_f = 10\text{ }\mu\text{K}$

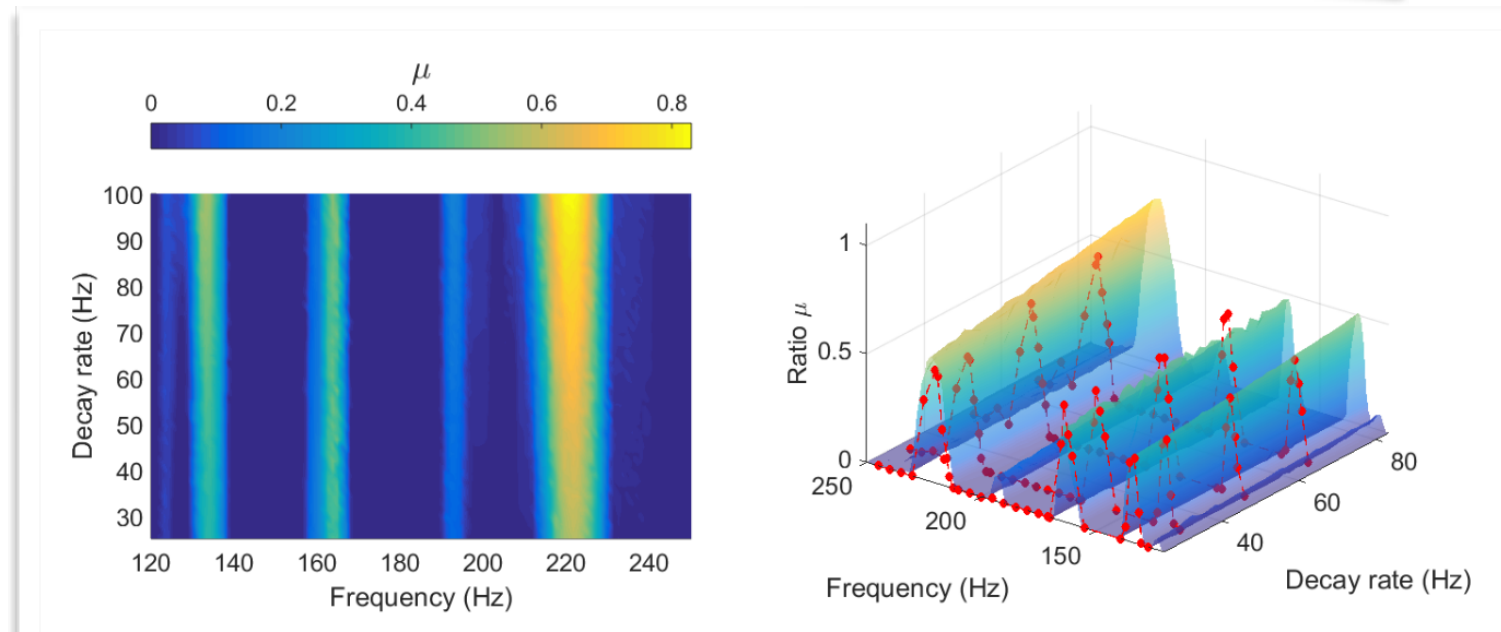
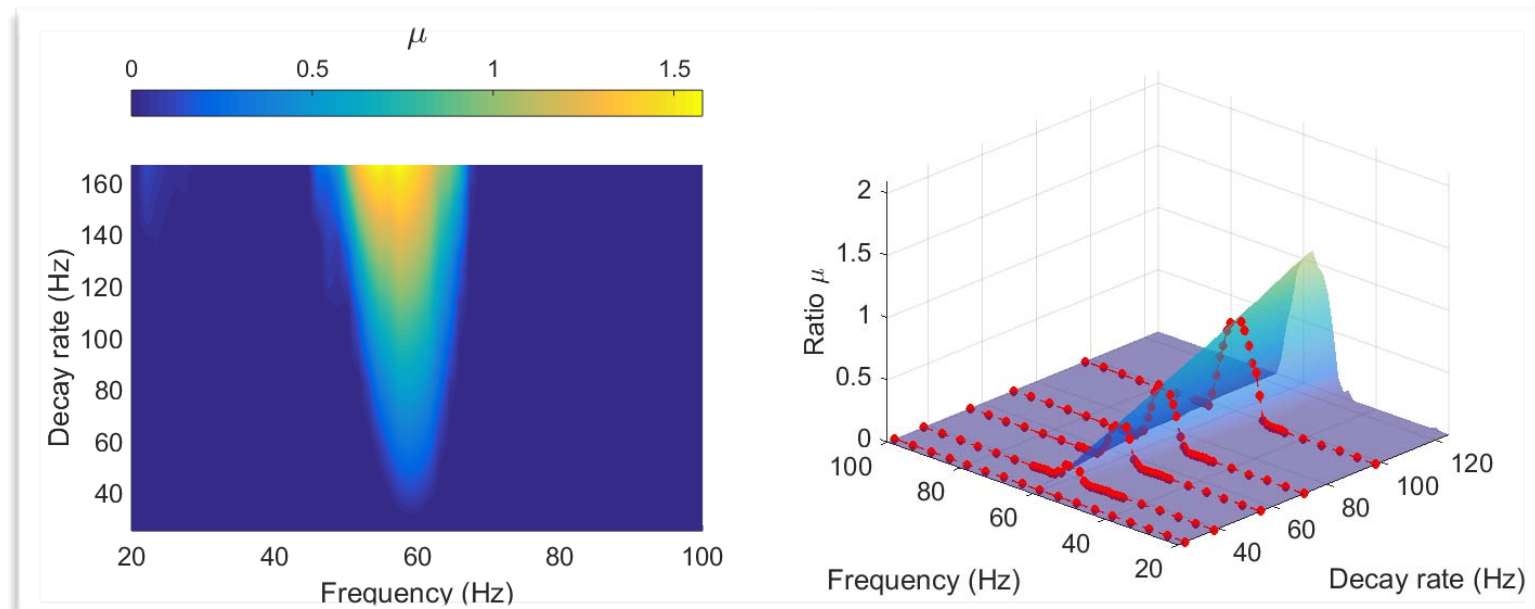
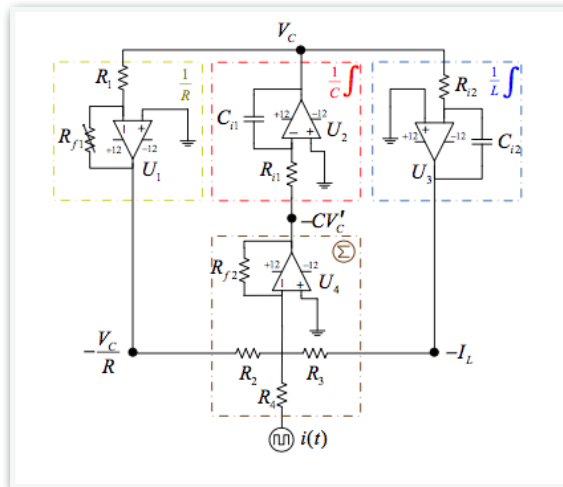
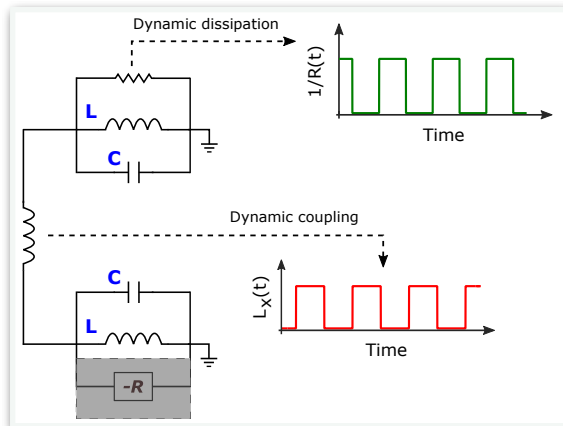


Access PT transitions with minimal loss!





PT transitions in synthetic electrical circuits



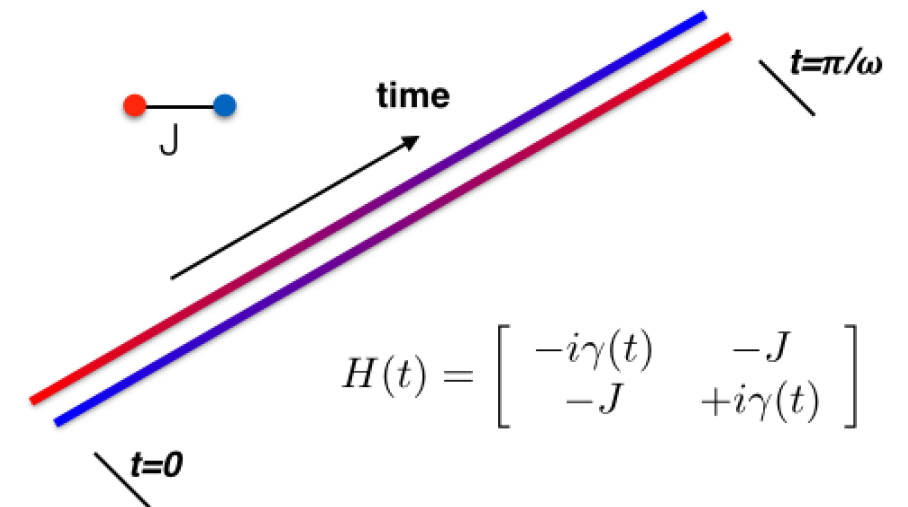
Nonlinearities in dissipation or time-delays possible

(Passive) PT Floquet model

Static case: single PT transition.

High frequencies ω : no PT breaking.

Resonance: PTB for small gain-loss.



$$H = -J\sigma_x + i\Gamma(t)\sigma_z/2$$

$$i\Gamma(t) = i\Gamma_0 \text{Square}(\Omega t)$$

