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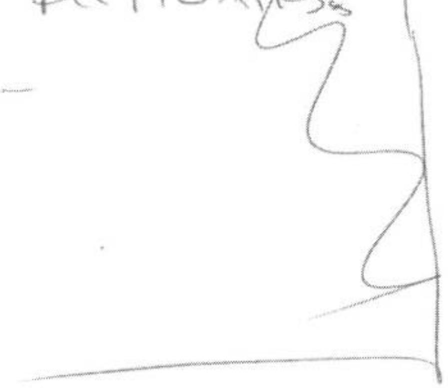
ICTS TALK:

D2-Branes from Periodic M2-branes

w/ I. Jeon & P. Richmond.

INTRO – Usual reduction of $M2 \to D2$ by "Novel Higgs Mechanism"

Motivations:



– But the most naive way would be to compactify the transverse space $\mathbb{R}^8 \to \mathbb{R}^7$.

Easy to arrange for a periodic array in ABJM

$$\langle Z^{A'} \rangle = 0 \qquad Z^4 = 2\pi i R \begin{pmatrix} \ddots & & & \\ & 2 & & \\ & & 1 & \\ & & & 0 \\ & & & & -1 \\ & & & & & -2 \end{pmatrix} \xrightarrow{} \text{Block matrix}$$

$A = 1,2,3,4 \quad , \; A' = 1,2,3$ $\qquad \Delta$

But the problem is to impose the discrete orbifold on fluctuations.

$$Z^A = 2\pi i \, \delta^A_4 R \Delta + \phi^A$$

want ~~ϕ~~ $\quad \phi_{mn} = \phi_{m+1\,n+1}$

Translation is not a symmetry of ABJM for

But on the other hand, at least for large k,

ABJM is weakly coupled.

- something should work

→ So here we proceed with "blind faith"

SHORT MOTIVATIONS: 1) Resolve a puzzle

- cf. W. Taylor + D-branes.

periodic array.

(T-duality D(p+1)-brane on S^1

- Infinite KK tower.

→ Don't expect such states here

2) Explore T-duality in M-theory

important M2 on T^3 array

= M5 on T^3.

Close to published but a few points

remain:

PLAN: 1) INTRO ✓.

2) SET-UP

3) Evaluate the Lagrangian

4) Compare to 5D MSYM

5) Further T^2 array to ~~5DSYM~~ D4-5?

6) Conclusions.

2) So we impose.

$$Z^A_{mn} = 2\pi i R \delta^A_4 + Z^A_{m-1\, n-1}$$

$$A^{L/R}_{\mu mn} = A^{L/R}_{\mu\, n-m} \quad \text{etc.}$$

$\Rightarrow$ leads to divergences.

e.g. $tr(D_\mu Z D^\mu \bar{Z})$

$$= \sum_{m,n} tr(\partial Z_{mn} \partial Z^+_{nm})$$

$$= \sum_{m,n} tr(\partial Z_{n-m\,0}\, \partial Z^+_{m-n\,0})$$

$$= \sum_p \sum_q tr(\partial Z_{p0}\, \partial Z_{-p0})$$

$$= \Big(\underbrace{\sum_q}_{|\Gamma| = \infty}\Big) \sum_p tr(\partial Z^A_{p0}\, \partial Z_{A p0})$$

This arises in the D-brane case but is harmless as $|\Gamma|$ just multiplies the whole Lagrangian.

- We will find other divergences e.g.

$$\sum_q q^2 = \cancel{\tfrac{2}{3}N^3 + \dots}$$

$$\sum_p p^2 = \cancel{2N + \dots}$$

~~+ also~~ $\cancel{\sum_q q = 2\sqrt{N} \quad \Omega = \tfrac{2}{\sqrt{3}}}$!

To handle these we consider a large but finite array $n = -N, \dots, 0, \dots +N$

- normalize fluctuations. e.g. $\sum_q q^2 = \frac{2}{3}N^3 \dots$

$$Z^A_{mn} = 2\pi i R\, \delta^A_4\, n\, \delta_{mn} + \frac{1}{\sqrt{2N}}\, \phi^A_{n-m}$$

$$A^{L/R}_{\mu mn} = a^{L/R}_{\mu\, n-m}$$

$$\Rightarrow \operatorname{tr}(\partial Z \partial Z) = \sum_p \operatorname{tr}\left(\partial \phi^A_p\, \partial \phi^A_{-p}\right)$$

But crucially also let k & R scale $\sim N$.

We still find. divergences

1st div. from $\mathrm{Tr}(DZ\,D\bar{Z})$

$$\rightsquigarrow \sum q^2\, a^{-}_{\mu p}\, a^{-\mu}_{-p}$$

$$a^{+}_{\mu} = \tfrac{1}{2}(a^{L}_{\mu} + a^{R}_{\mu})$$

$$\Rightarrow \quad A^{L}_{\mu} = A^{R}_{\mu}$$

Bifund.

$$U(N)\times U(N) \longrightarrow U(N) \text{ adjoint.}$$

2nd from $\mathrm{Tr}(DZ\,D\bar{Z})$

$$\sim \sum_{k\bar{N}} \frac{R^4}{k^2} q^2 \left(p^2 \phi^{A'}_{p}\, \phi_{A' p} \right) \sim \frac{R^4 N^2}{k^2}$$

$$\Rightarrow \quad \phi^{A'}_{p} = 0 \quad ?\ \forall c$$

– resolves the one puzzle

~~So the~~

So we set $\begin{cases} \phi^{A'}_{p} = 0 & p \neq c \\ a^{L}_{\mu p} - a^{R}_{p} = 0 \end{cases}$

$\Rightarrow$ Constraints.

$\{H, C\} = C' \quad \Rightarrow$ more constraints

To make a long story short we find only zero-modes:

$$\phi^A_0 \,, \quad a^+_{\mu 0}$$

$$a^+_{\mu p} \;\&\; \mathrm{Im}\,\phi^4_p \;\propto\; \mathbb{1}$$

& can be integrated out

$$a^+_{\mu p} = \frac{1}{2\pi i}\frac{1}{R}\frac{1}{\sqrt{N}}\,\partial_\mu \mathrm{Im}\,\phi^4_p$$

So in the end we find:

$$\boxed{\begin{aligned} \mathcal{L} &= -\left(\mathrm{tr}\, D_\mu \phi^A_0 D^\mu \phi_{A0}\right) - V \\ V &= \tfrac{1}{2} g^2\, \mathrm{tr}\Big(\left|[\phi^A_0, \phi^B_0]\right|^2 + \left|[\phi^{A'}, \phi^{B'}]\right|^2 \\ &\qquad + 2\left|[\phi_0^{A'}, \mathrm{Im}\,\phi^4_0]\right|^2 \Big) \\ &\boxed{\begin{aligned} g^2 &\propto \frac{R^2 N}{k^2} \\ M_5^2 &= \frac{R^4 N^2}{k^2} \end{aligned}} \end{aligned}}$$

Consider the Fermions

here we see sums like

$$\sum_i q = ?$$

let us assume $\sum_i q = \Omega N^2$

might say $\Omega = a$

on the other hand tower of Fermions

$$\frac{\Omega^2}{4}\Omega \frac{N^2}{N} \sum_p p\, \psi^A_p \psi_{Ap}$$

$$\Rightarrow M_s \propto \Omega N \frac{\Omega^2}{4^2} = M_s \text{ if } \underline{\Omega = \tfrac{2}{?}}$$

then

$$\mathcal{L} = \mathcal{L}_b + -i \mathrm{tr}\left(\bar{\psi}_0 \gamma^\mu \cancel{D}_\mu \psi_0\right)$$

$$- \mathcal{L}_{\text{Yukawa}}$$

Furthermore it turns out that $\mathcal{L}_{\text{Yukawa}}$ is are exactly the same as 3D MSYM.

$$\Rightarrow \boxed{\mathcal{L}_{\text{cnny}} = -\tfrac{1}{2}\nabla_\mu X^I \nabla^\mu X^I - \tfrac{1}{2}\nabla_\mu Y \nabla^\mu Y + \tfrac{1}{4} g^2([X^I, X^J])^2 + \text{Fermions.}}$$

$Y = \text{Re}\, q^4{}_0. \quad I = 3, \dots, 9$

$\Rightarrow SO(7)$ invariant.

but no kinetic term for $A_\mu = \tilde{c}^{+}_{\mu\,0}$

But all is not lost: A_μ eq. of M.

$$i[X^I, \nabla_\mu X^I] + i[Y, \nabla_\mu Y] = 0.$$

$$\nabla^2 Y = 0.$$

with $\quad \nabla_\mu Y = -\frac{1}{2g}\epsilon_{\mu\nu\lambda} F^{\nu\lambda}$

$$\Rightarrow [\cancel{\nabla}_\mu, \nabla_\mu Y] = \frac{1}{g^2}\nabla^\mu F_{\mu\nu}.$$

$\Rightarrow$ recover 3D SYM.

ie all solutions to 3D SYM solve our system.

but in general there are extra "modes":

e.g. $A_\mu = i g^{-1} \partial_\mu g$

$Y = g Y_0 g^{-1}$ Y_0 = "moduli

⇒ position of M2's in 11th Dim.

$Y \sim$ Dual gluon.

Consider a further T-duality in $\mathbb{T}^2$

— follow usual procedure of W.Tay

$\mathcal{L}$

$$\boxed{\begin{aligned} S = \frac{-1}{g^2}\int d^5x \quad & \tfrac{1}{2}(\nabla^{\mu'} X^{I'})(\nabla_{\mu'} X^{I'}) - \tfrac{1}{4}\sum([X^{I}, \\ & + \tfrac{1}{2}(\nabla_\mu Y)(\nabla^\mu Y) + \tfrac{1}{2} F_{\mu\alpha} F^{\mu\alpha} + \tfrac{1}{4} F_{\mu\nu} \end{aligned}}$$

⇒ not Lorentz inv.

but again. $\nabla_\mu Y = -\frac{1}{2}\epsilon_{\mu\nu\lambda} F^{\nu\lambda}$

gives equation of motion of 5D MSYM.

This then might say

$\nabla_\mu Y = \frac{1}{2}\epsilon^{\alpha\beta} H_{\mu\alpha\beta}$

$Y_0 = \int B$?