

# **DFT+DMFT, oxide superlattices and the metal-insulator transition: the good, the bad and the ugly**

**A. J. Millis**

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Support:

US-DOE-ER-046169

US-NSF-DMR-1006282

**Hyowon Park, Andrew J. Millis, and Chris  
A. Marianetti, PRL 109, 156402 (2012)**

**E. Gull, O. Parcollet and A. J. Millis, PRL  
110 216406 (2013)**

**H. T. Dang, A. J. Millis and C. Marianetti,  
arXiv:1309.2995**

**Bangalore 2014**



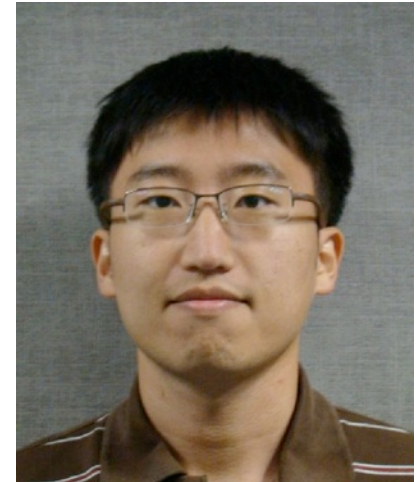
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# Collaborators

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**\*Hyowon Park**



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**M-J. Han (CU->ANL->KAIST)**

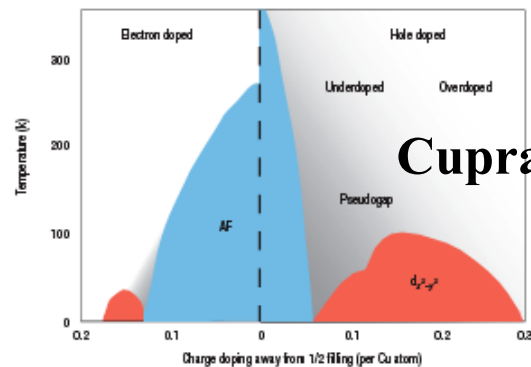
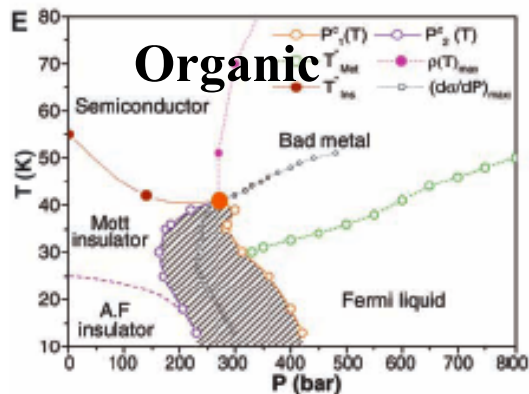
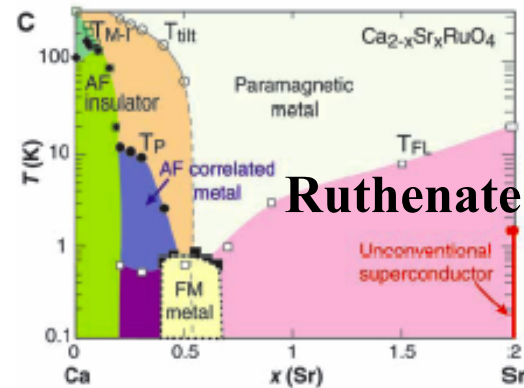
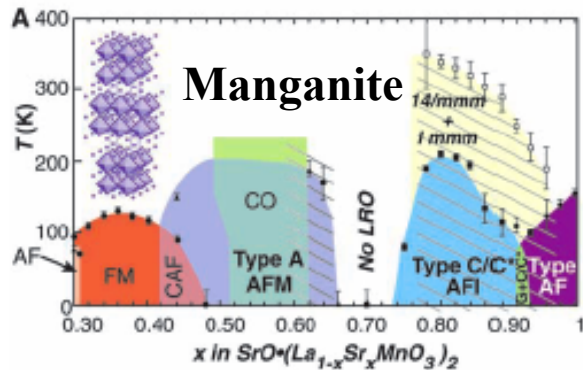
**X. Wang (CU->U-Md)**

**L. deMedici (ESPCI)**



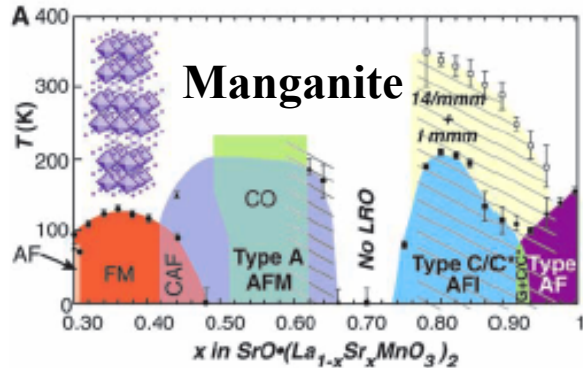
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# Materials with Strongly Correlated Electrons Do Interesting Things

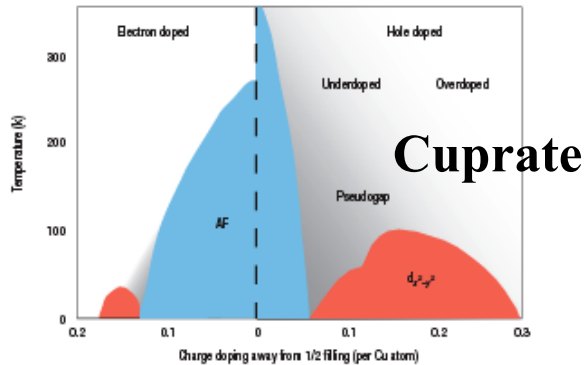


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# One would like to

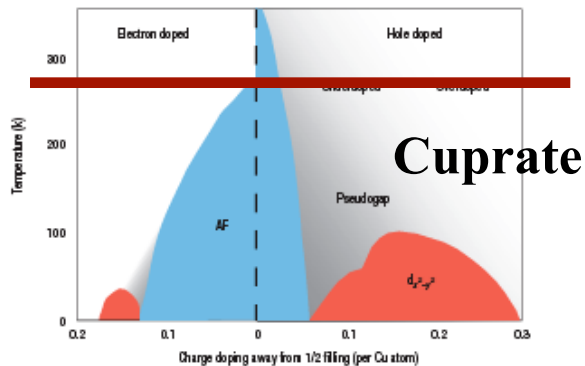
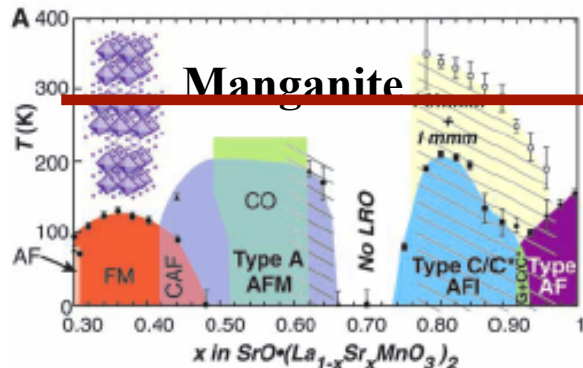


- **Design materials with desired behavior**
- **Control the phases with external fields**



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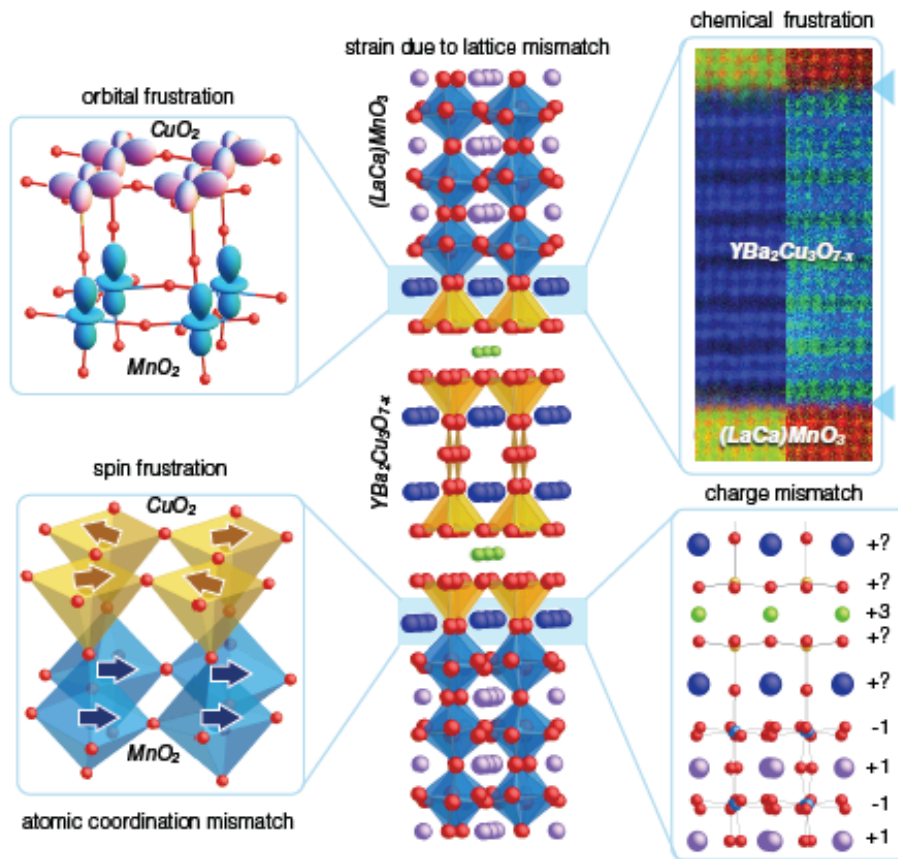
# One would like to



- Design materials with desired behavior
- Control the phases with external fields
- Move the interesting behavior closer to (or above) **room temperature**



# Oxide epitaxy: Can now combine ‘interesting’ materials on atomic scale



## Control (in principle)

- Charge density
- Strain
- Local geometry (coordination/bond angles)
- Proximity (of different kinds of orders)

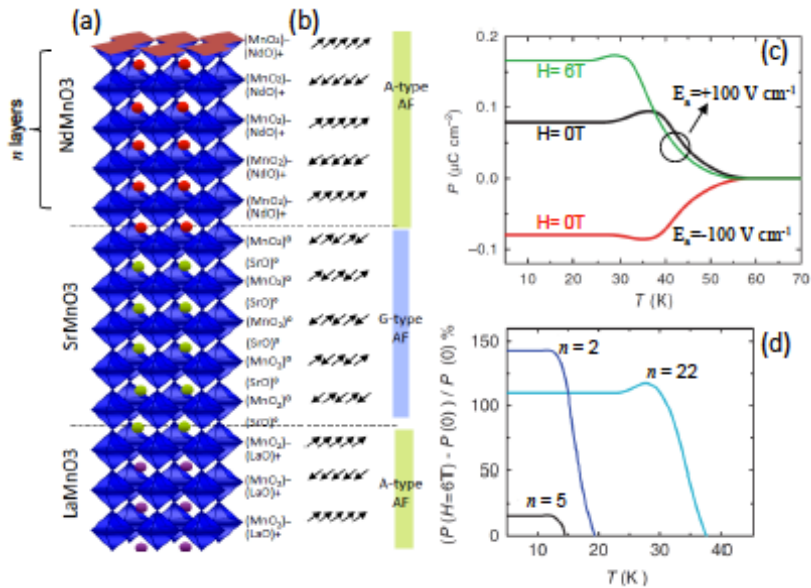
**=> Access (in principle) new physics**



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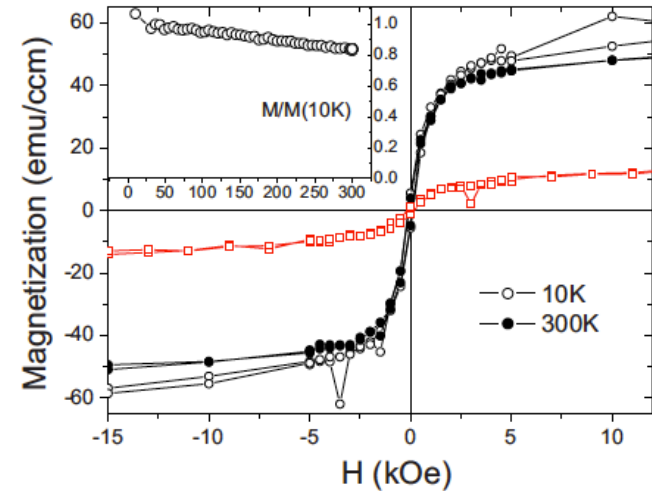
# Examples

## Multiferroicity



Rogdakis...Panagopolous, Nat. Comm 3 1064 (12)

## ?Room Temperature Ferromagnetism?



Luders et al Phys Rev B80 241102 (2009)



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# Challenge for Theory

**The Stuart Parkin question: What do I have to grow to get a room temperature superconductor??**

**More prosaically: need theory to**

- (a) determine electronic properties of model systems**
- (b) predict structures (esp oxygen positions)**
- (c) connect physical and chemical structure to properties**

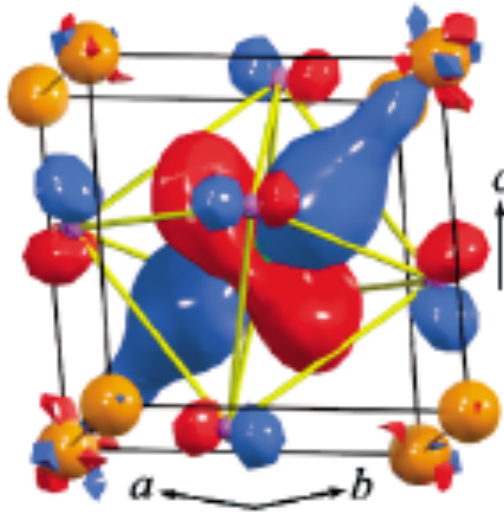
**=> DFT+DMFT**



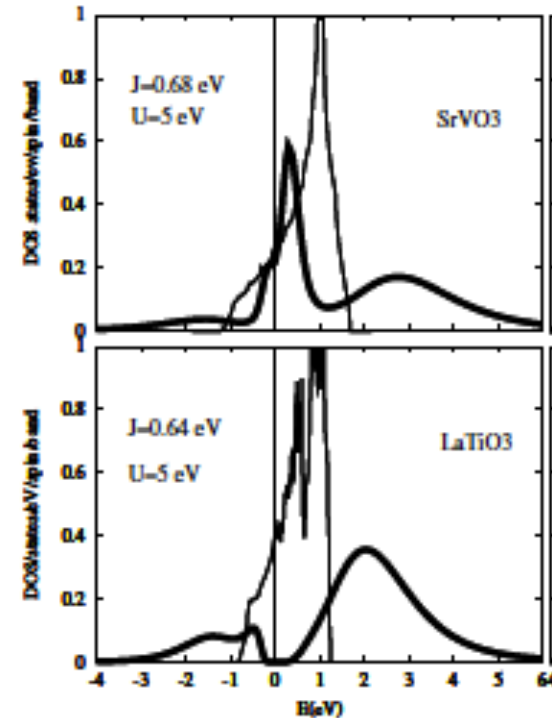
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# structure and properties (1a): local crystal structure

example:  $d^1$  perovskites



**Pavarini//Georges 2004:  $\text{GdFeO}_3$   
rotational distortion essential to  
insulating phase of  $\text{LaTiO}_3$**



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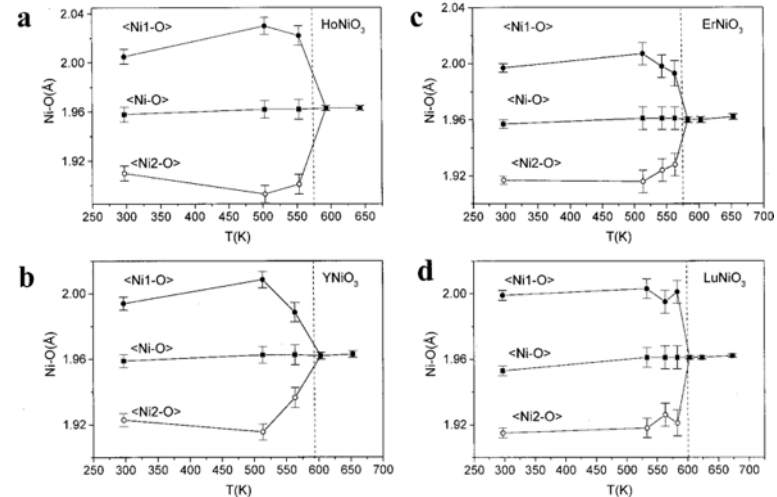
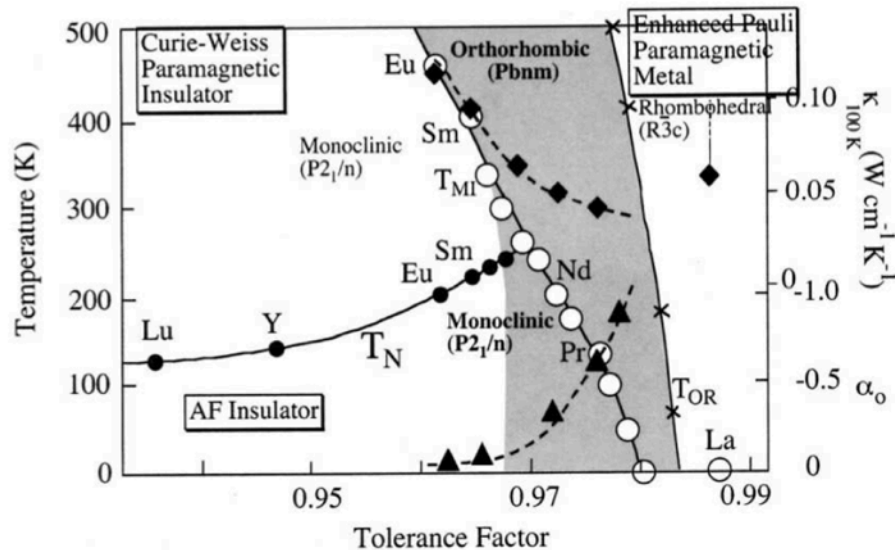
# To understand metal-insulator transition

**incorporate physics of crystal structure (esp. octahedral rotations) on equal footing with many-body effects**



# structure and properties (1b): oxygen displacements

example: rare earth nickelates



**Metal-insulator transition coincident  
with change in Ni-O bond-length.**



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# **To understand metal-insulator transition in nickelates**

**incorporate physics of changes in transition metal-oxygen bond lengths on equal footing with many-body effects**

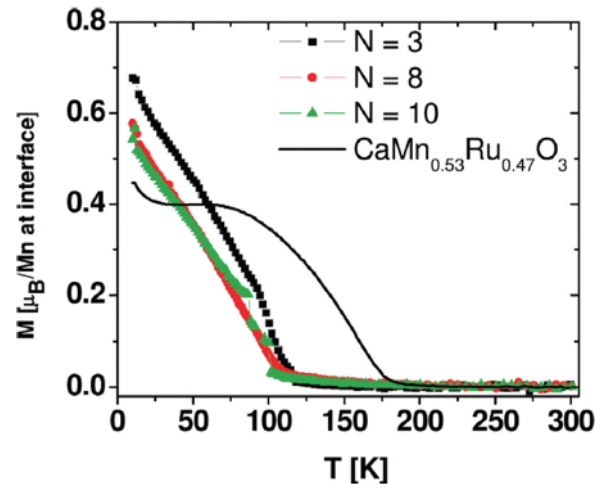
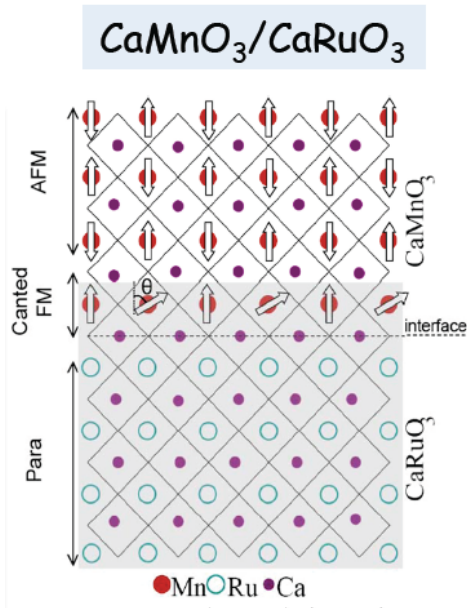


# Structure and properties (2): Charge transfer



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# Ferromagnetism at interface between paramagnet, antiferromagnet



He et al JAP 109 07D729 (2011)  
(see also Takahashi, Kawasaki and Tokura  
APL 79 1324 (2001))

**Idea: charge flow across interface dopes CaMnO<sub>3</sub>  
?how much charge moves? ?in which direction?**



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# To understand oxide superlattices

understand charge flow between different materials => work functions, atomic physics contribution to electronegativity and long-ranged Coulomb interaction (charge balance) on equal footing with many-body energetics

**=>DFT+DMFT with full charge self consistency**



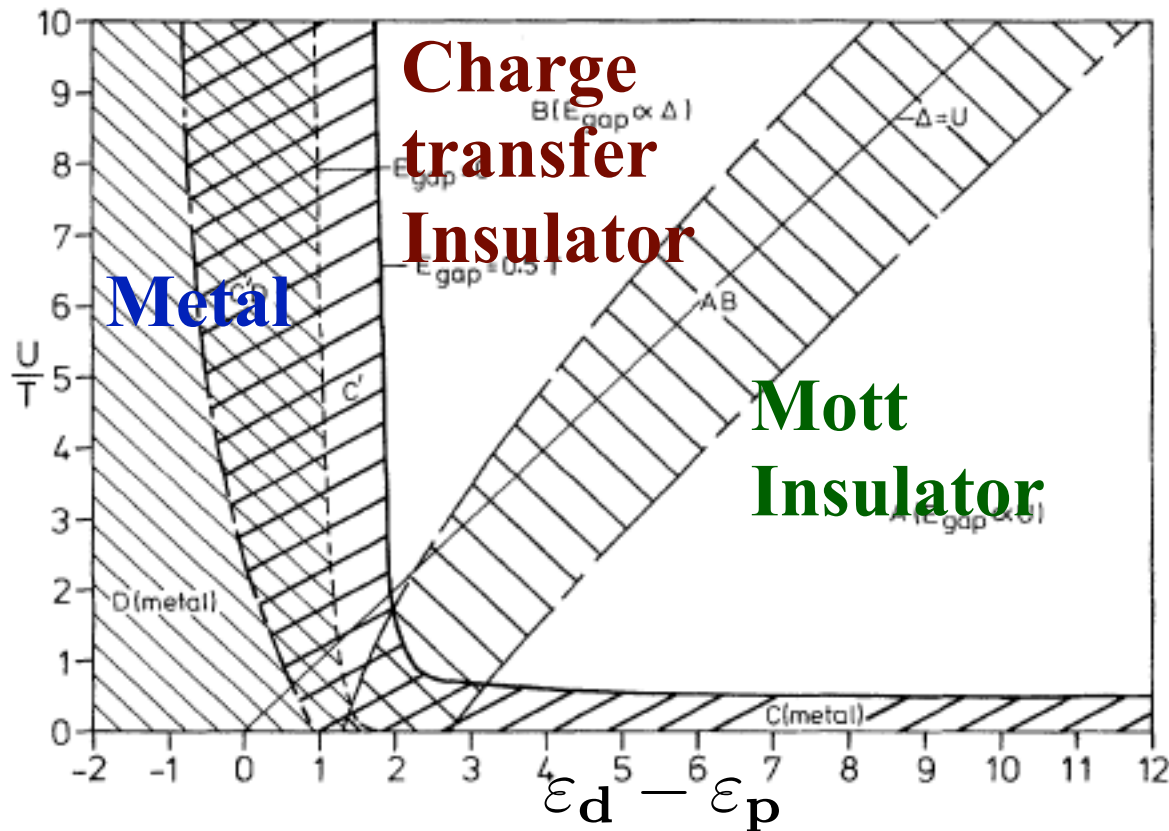
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# Important point

**the many body physics of the metal-insulator transition and the charge flow in superlattices are intimately related**



**p-d splitting governs both metal-insulator transition and superlattice charge flow.**



**Conventional Wisdom**

**Oxygen energy: DFT  
band theory**

**d-level energy:  
correlation physics**

VOLUME 55, NUMBER 4

PHYSICAL REVIEW LETTERS

22 JULY 1985

**Band Gaps and Electronic Structure of Transition-Metal Compounds**

**Zaanen Sawatzky Allen**



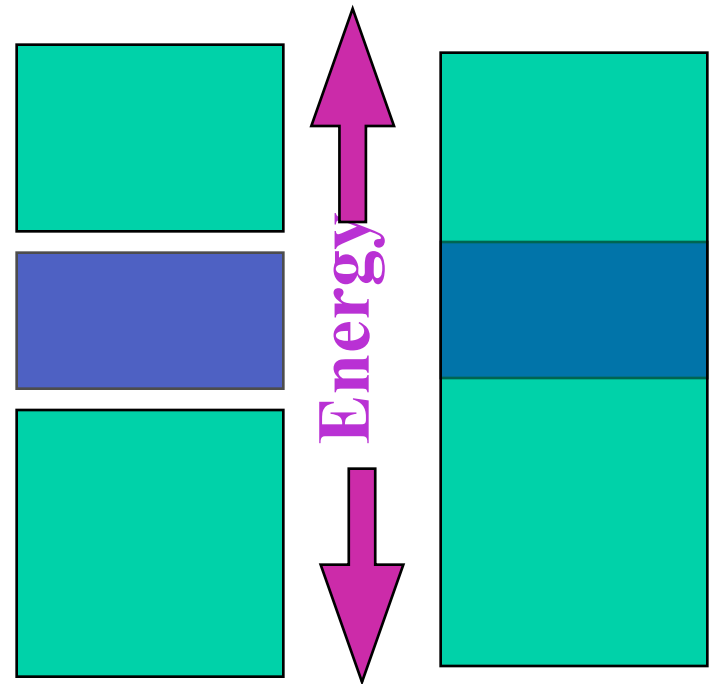
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# General Approach to Many-Body Electronic Structure

- =>partition d.o.f. into  
``**correlated subspace**'' (active space) (atomic-like d orbitals) and ``**background**''

- treat **correlated subspace** by many-body method (**DMFT**); treat **background** by mean field method (**DFT**)

- embed** active space into background (**DMFT self-consistency+double counting +charge self consistency**)



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# Correlated subspace: transition metal d orbitals

**Interaction: Full multiplet structure of d-shell  
(here written for  $t_{2g}$  or  $e_g$ )**

$$\begin{aligned} H = & U \sum_a n_{a\uparrow} n_{a\downarrow} + (U - 2J) \sum_{a>b, \sigma=\uparrow, \downarrow} n_{a\sigma} n_{b\sigma} \\ & + (U - 3J) \sum_{a \neq b\sigma} n_{a\sigma} n_{b\bar{\sigma}} - J \sum_{a \neq b} c_{a\uparrow}^\dagger c_{a\downarrow}^\dagger c_{b\uparrow} c_{b\downarrow} + c_{a\uparrow}^\dagger c_{b\downarrow}^\dagger c_{b\uparrow} c_{a\downarrow} \end{aligned}$$

**U: screened in solid--phenomenological or ‘c-RPA’ calculations.**

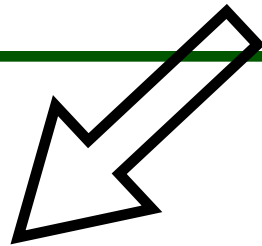
**J: close to atomic limit value**



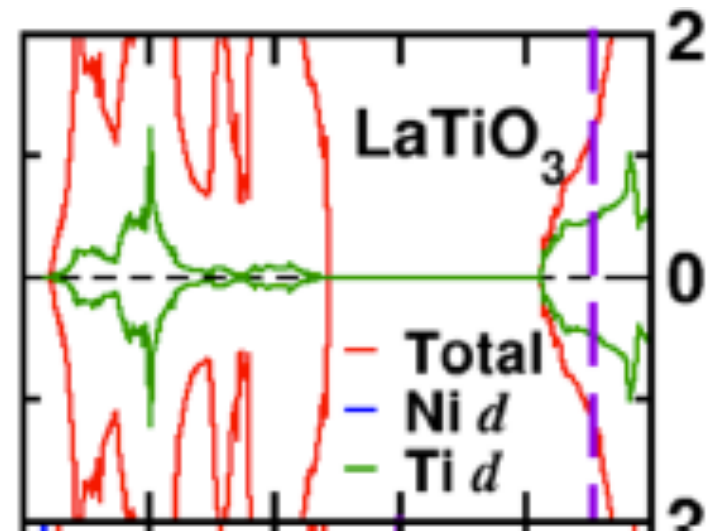
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# DFT+DMFT

$$\mathbf{G}^{-1} = \left[ \omega \hat{1} - \hat{\mathbf{H}}_{\text{DFT}} - \hat{\mathbf{P}}_d \left( \hat{\Sigma}_d(\omega) - \hat{\Sigma}_{dc} \right) \hat{\mathbf{P}}_d \right]$$



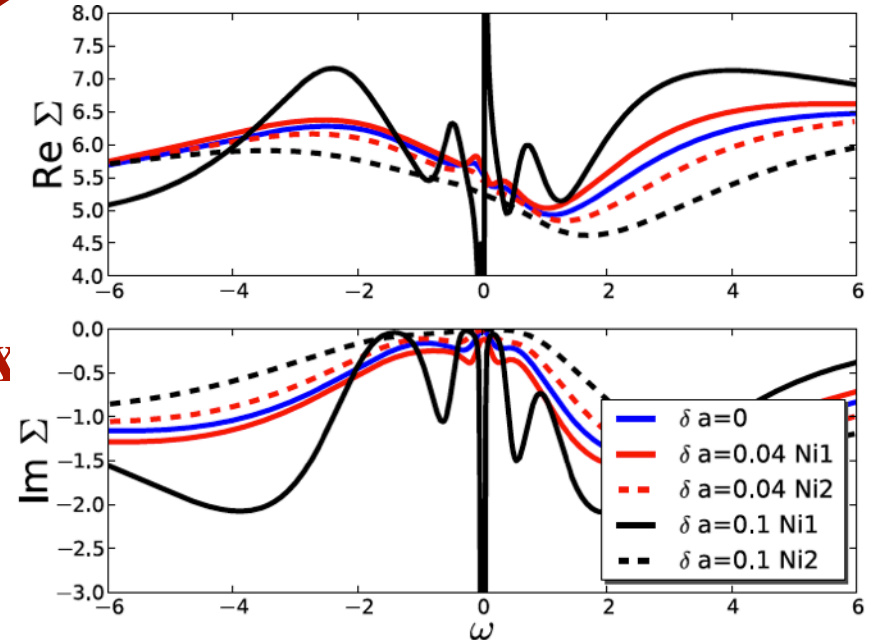
**Defines p and (bare) d level energies, and (via Wannier or projector method) d-orbital wave function**



# DFT+DMFT

$$\mathbf{G}^{-1} = \left[ \omega \hat{\mathbf{1}} - \hat{\mathbf{H}}_{\text{DFT}} - \hat{\mathbf{P}}_{\text{d}} \left( \hat{\boldsymbol{\Sigma}}_{\text{d}}(\omega) - \hat{\boldsymbol{\Sigma}}_{\text{dc}} \right) \hat{\mathbf{P}}_{\text{d}} \right]$$

**Many-body physics of d-level:**  
real part implies Hartree shift  
of d-states relative to DFT  
**DFT+U: static Hartree approx**



(H. Park: LaNiO3 in ideal and disproportionated state)

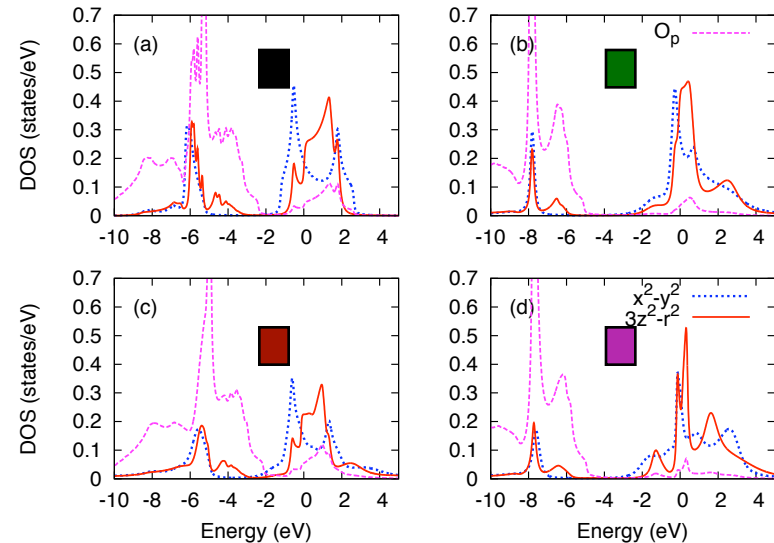
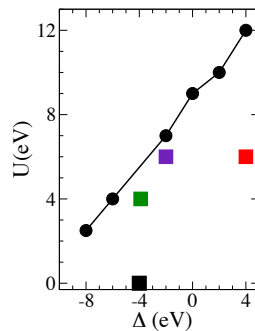


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# DFT+DMFT

$$\mathbf{G}^{-1} = \left[ \omega \hat{\mathbf{1}} - \hat{\mathbf{H}}_{\text{DFT}} - \hat{\mathbf{P}}_{\text{d}} \left( \hat{\boldsymbol{\Sigma}}_{\text{d}}(\omega) - \hat{\boldsymbol{\Sigma}}_{\text{dc}} \right) \hat{\mathbf{P}}_{\text{d}} \right]$$

**Double counting correction:  
cancels portion  
of Hartree shift:  
controls physical  
p-d splitting.**



(X Wang PRL 2011 LaNiO3 with different U and double counting)



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# Crucial question for superlattice and metal-insulator transition in bulk materials

**??What is the double counting??**



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# Accepted choice: 'FLL'

Added  
interaction:

$$H_{\text{int}} = \frac{1}{2} \sum_{m,m',\sigma} U_{mm'} n_{m\sigma} n_{m'-\sigma} + \frac{1}{2} \sum_{m,m',m \neq m',\sigma} (U_{mm'} - J_{mm'}) n_{m\sigma} n_{m'\sigma}$$

Atomic limit  
exp. val.

$$E^{\text{at lim}} = \frac{1}{2} U N(N-1) - \frac{1}{2} J N_{\uparrow}(N_{\uparrow}-1) - \frac{1}{2} J N_{\downarrow}(N_{\downarrow}-1)$$

$$E = E_{\text{DFT}} + \langle H_{\text{int}} \rangle - E_{\text{at}}$$

=> hartree contribution  
to d-level self energy

$$\hat{\Sigma}_{\sigma}^{\text{dc}} = U \langle N \rangle - J \langle N_{\sigma} \rangle$$

Czyzyk and Sawatzky, PRB49 14211 (1994)

M. Karolak et al, J Elect Spectr.181 p. 11 (2010)



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# Note 1/2 in energy

**Added  
interaction:**

$$H_{\text{int}} = \frac{1}{2} \sum_{m,m',\sigma} U_{mm'} n_{m\sigma} n_{m'-\sigma} + \frac{1}{2} \sum_{m,m',m \neq m',\sigma} (U_{mm'} - J_{mm'}) n_{m\sigma} n_{m'\sigma}$$

**Atomic limit  
expectation  
val.**

$$E^{\text{at lim}} = \left( \frac{1}{2} U \right) N(N-1) - \frac{1}{2} J N_{\uparrow}(N_{\uparrow}-1) - \frac{1}{2} J N_{\downarrow}(N_{\downarrow}-1)$$

$$E = E_{\text{DFT}} + \langle H_{\text{int}} \rangle - E_{\text{at}}$$

**=> hartree contribution  
to d-level self energy**

$$\hat{\Sigma}_{\sigma}^{\text{dc}} = U \langle N \rangle - J \langle N_{\sigma} \rangle$$

Czyzyk and Sawatzky, PRB49 14211 (1994)

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# Charge flow



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# To understand charge transfer across oxide interface

**1. correctly place  
oxygen states on each  
side of interface**



# To understand charge transfer across oxide interface

1. correctly place  
oxygen states on each  
side of interface
2. determine p-d gap



# To understand charge transfer across oxide interface

**1. energy of oxygen  
states on each side of  
interface**

**2. determine p-d gap**

**3. account for effects  
of charge flow**

## Conventional wisdom

**Basic (uncorrelated)  
electronic structure**

**Many body physics  
of transition metal  
d-orbital**

**Full charge self-  
consistency**

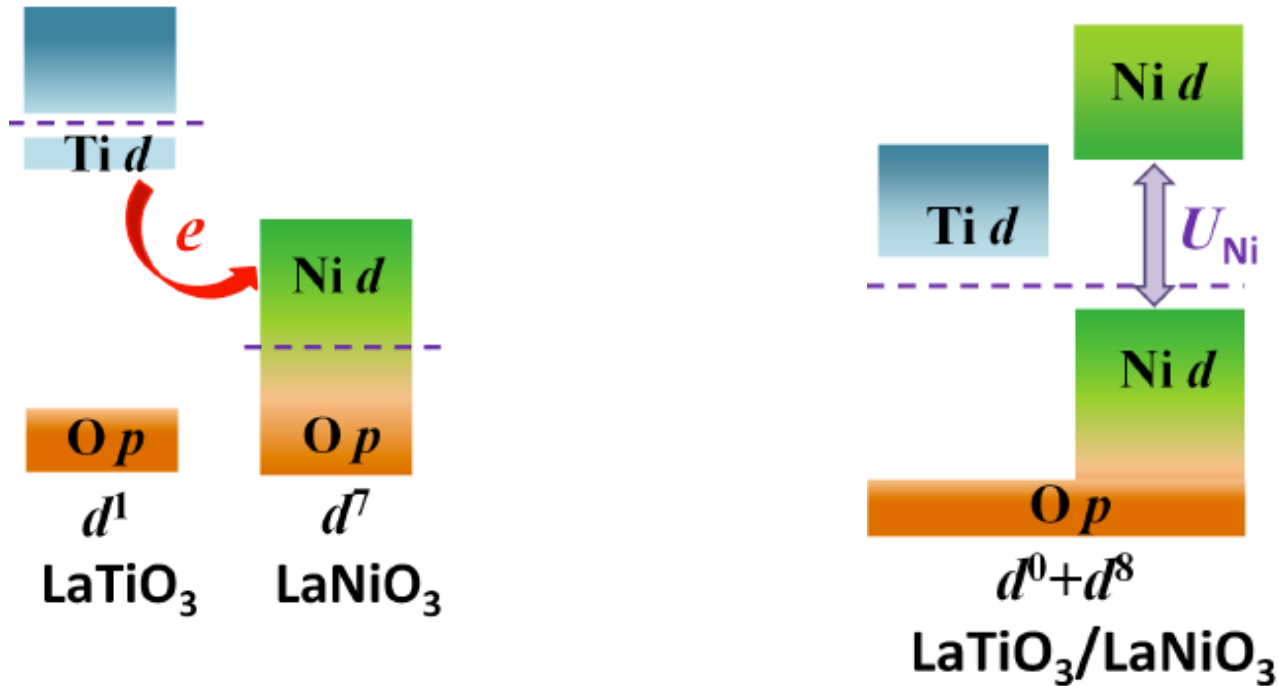


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# LaTiO<sub>3</sub>/LaNiO<sub>3</sub> (001)

LaTiO<sub>3</sub>:  $d^1$  'Mott' insulator    LaNiO<sub>3</sub>:  $d^7$  correlated metal

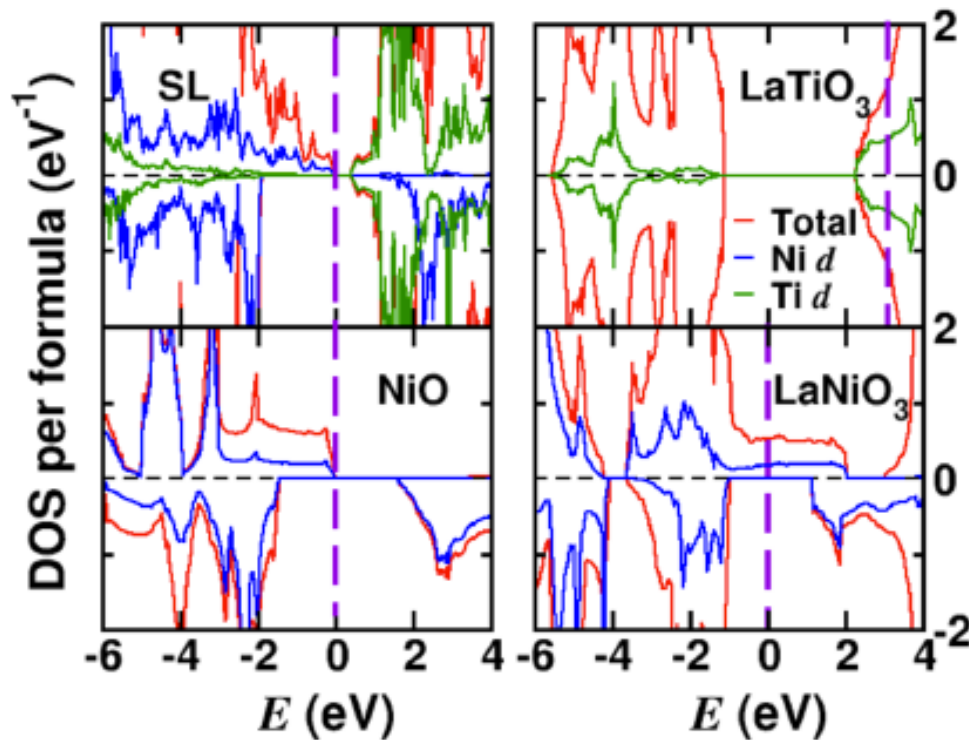
Idea: if put them together, complete charge transfer Ti→Ni  $\Rightarrow S=1$  (Ti  $d^0$  Ni  $d^8$ ) Insulator



H. Chen, AJM and C. Marianetti  
Phys. Rev. Lett. 111, 116403 (2013)



# Calculation: DFT+U (FLL double counting)



(FLL double counting)

**\*oxygen bands line up  
(approximately)**

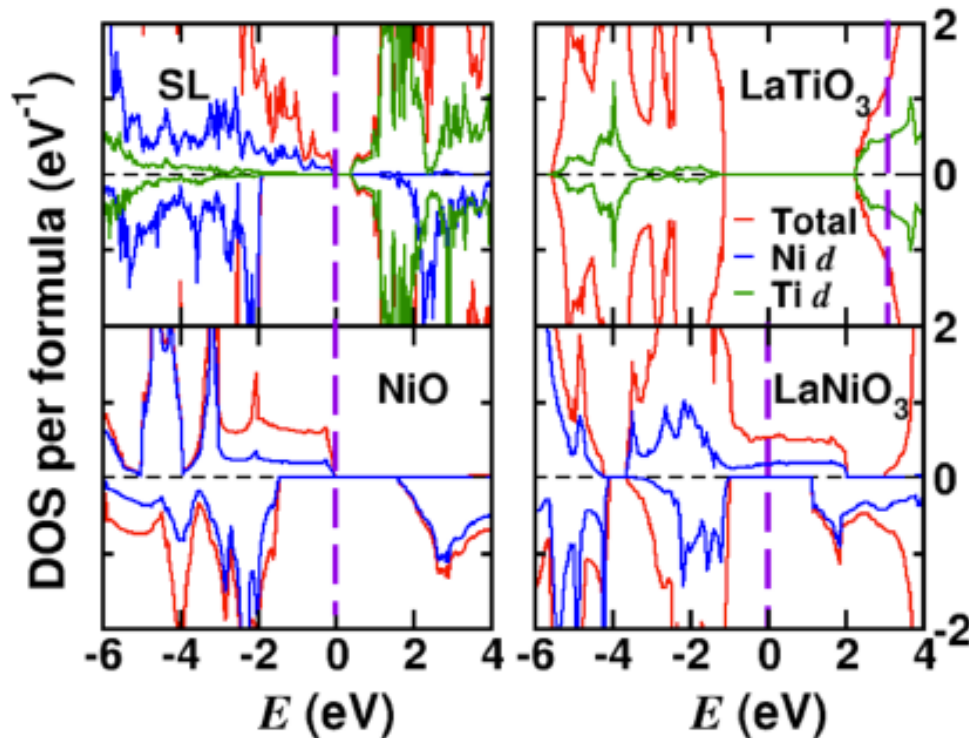
**\*local physics fixes d  
level energy relative to O**

**\*d-level energy  
difference drives  
complete charge transfer**

H. Chen, AJM and C. Marianetti,  
Phys. Rev. Lett. 111, 116403 (2013)



# Side remark: actual charge density almost unchanged

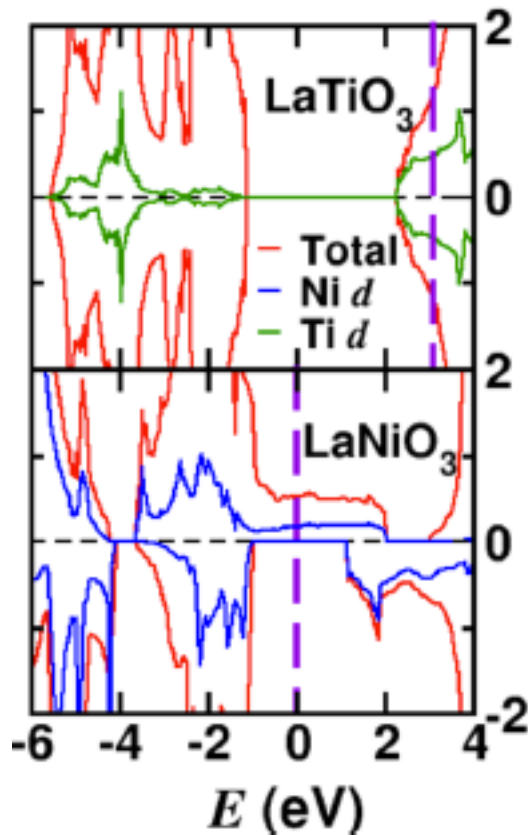


**`rehybridization':  
degree of mixing of  
other (uninteresting)  
states changes to  
compensate transfer of  
near fermi surface  
charge**

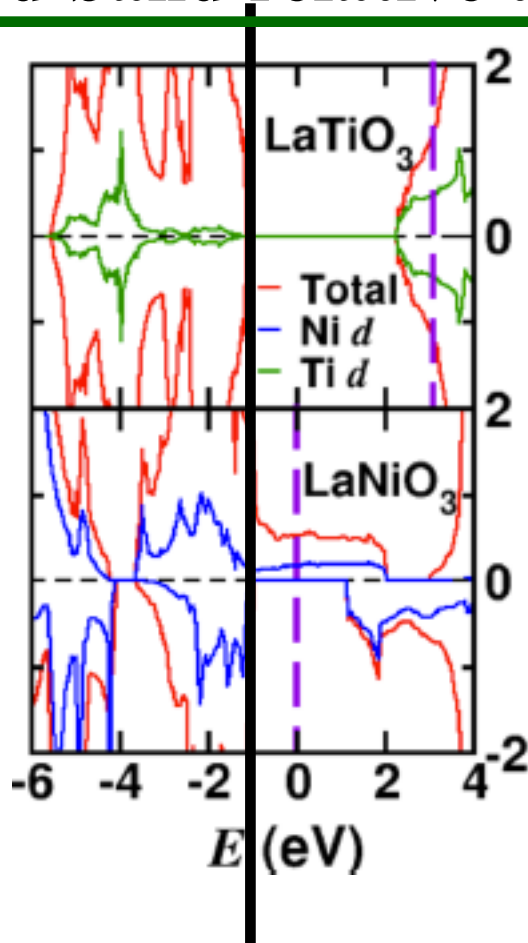


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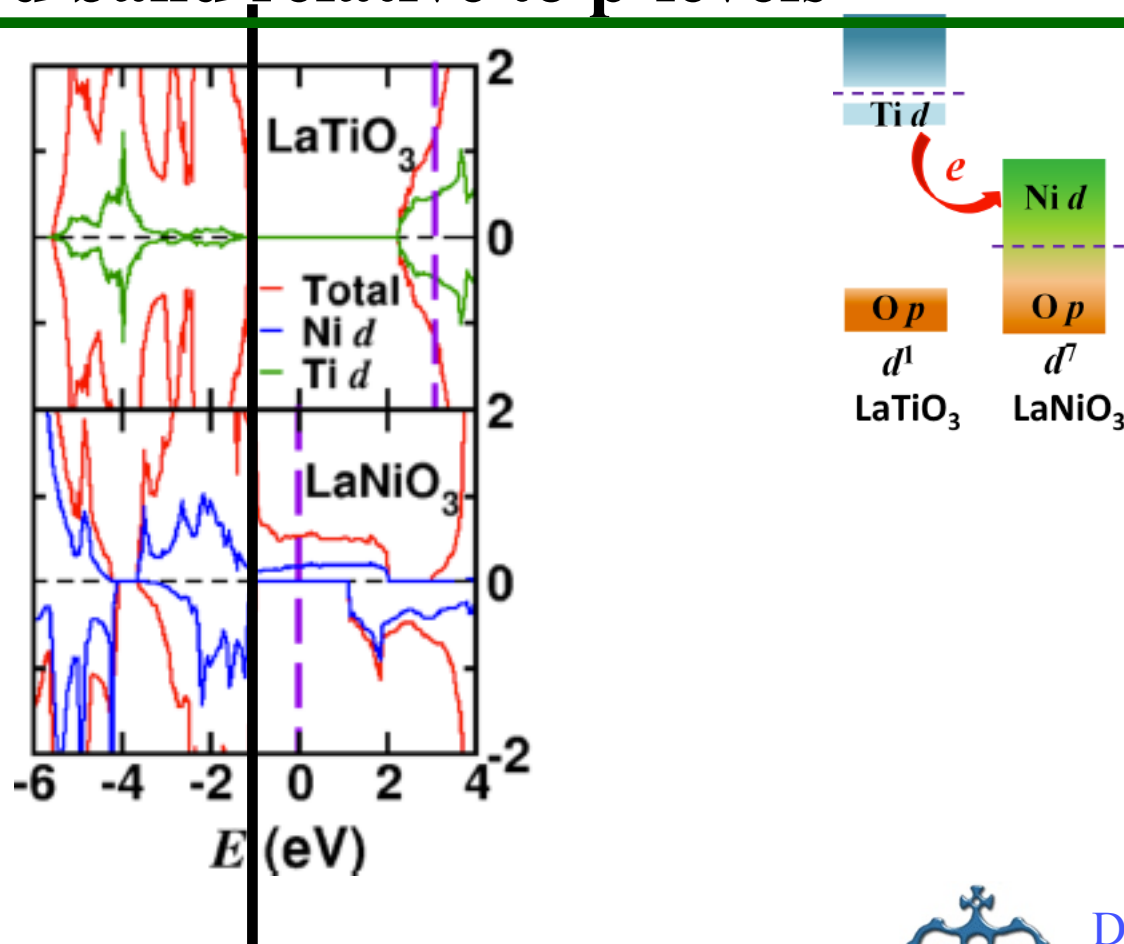
**Rough physical picture:  
oxygen states line up.  
Charge transfer controlled by energy of  
d band relative to p-levels**



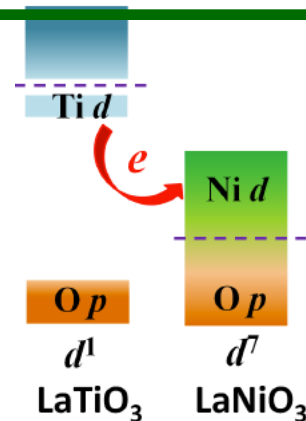
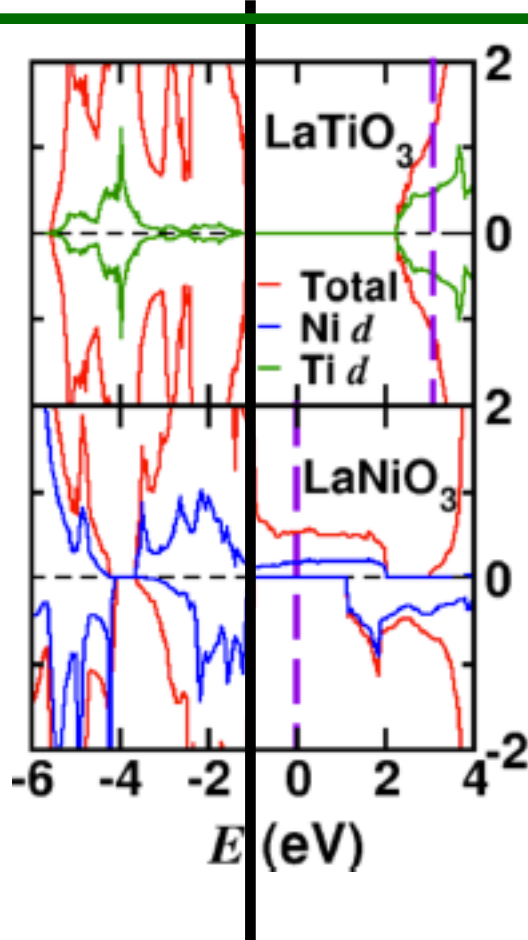
**Rough physical picture:  
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**Rough physical picture:  
oxygen states line up.  
Charge transfer controlled by energy of  
d band relative to p-levels**



**Rough physical picture:  
oxygen states line up.  
Charge transfer controlled by energy of  
d band relative to p-levels**



**(Note: thicker interface  
=>Schottky barrier effects.)**



**?Are the estimates right?**



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# Two examples

**NiO**

**LaTiO<sub>3</sub>**

**Experimentally:  
both charge transfer insulators**

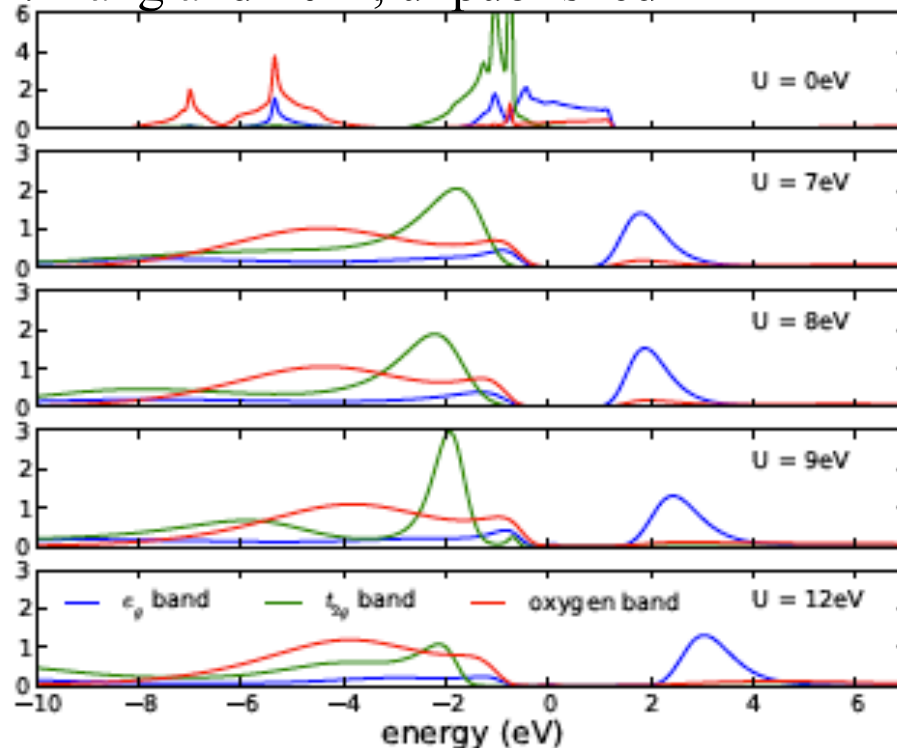
**Theory: DFT+DMFT, defining d orbitals from  
MLWF defined over energy range of p-d manifold**



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# NiO: Full charge sc +FLL double counting produces insulator

H. T. Dang and AJM, unpublished

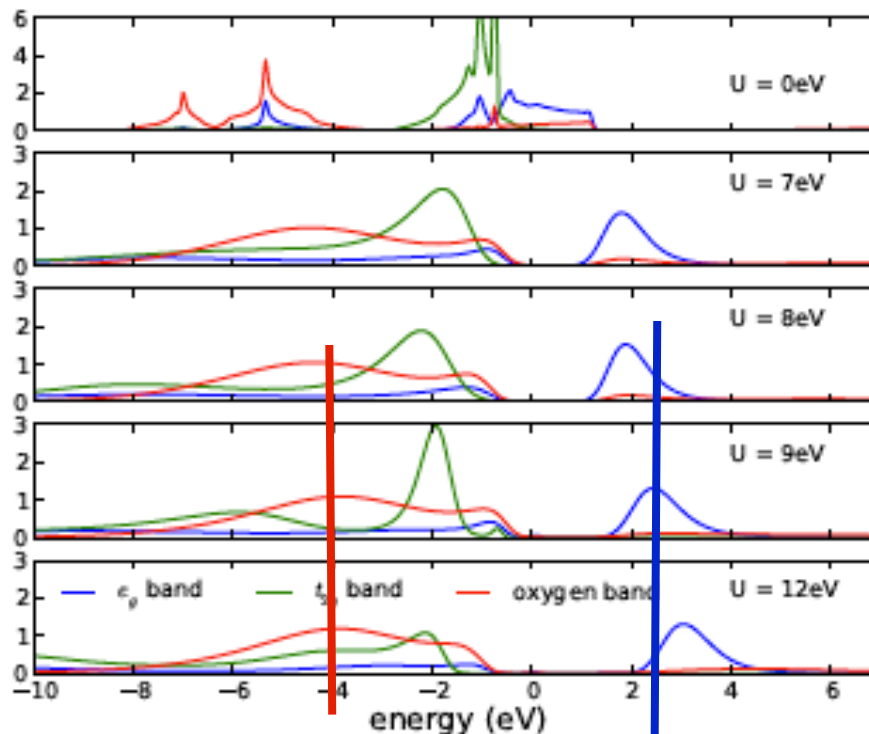
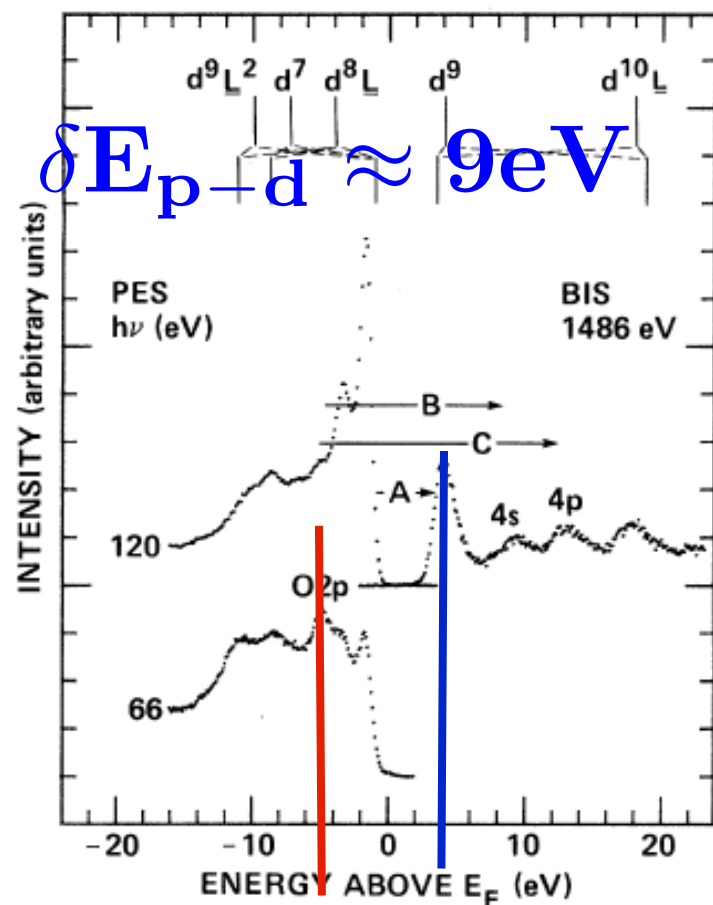


- insulator
- gap (3eV)<expt (4eV)
- p-d splitting only
- weakly U-dependent
- p bands too close to d-bands

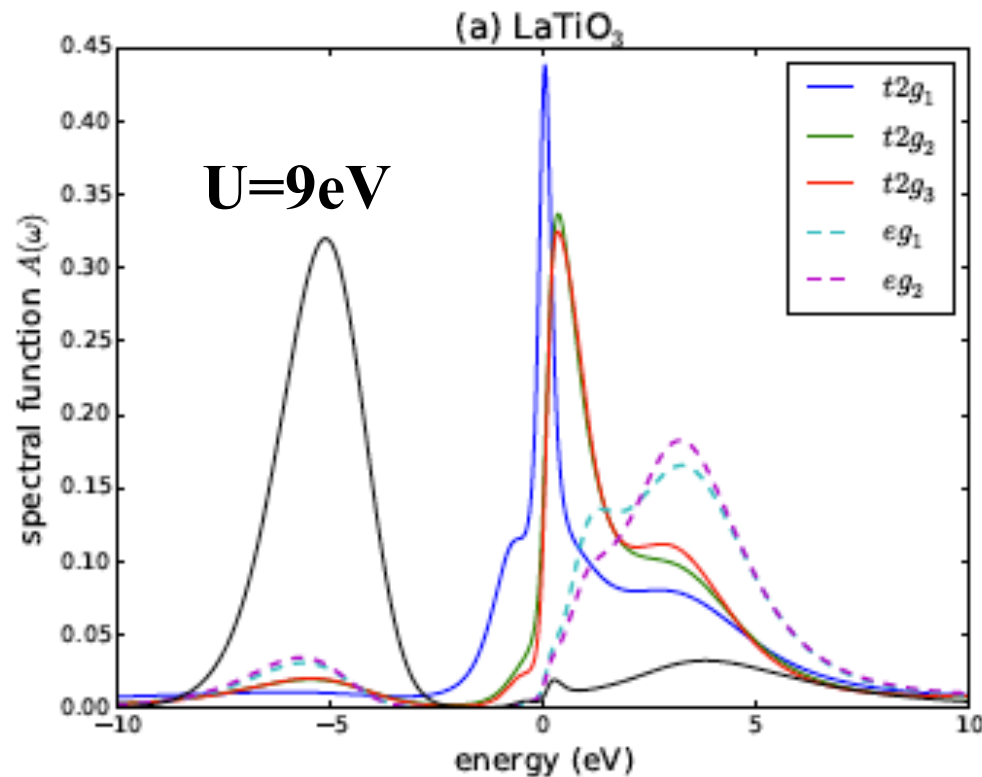
FLL double counting for NiO also discussed in  
M. Karolak et al, J Elect Spectr.181 p. 11 (2010)



# p-d splitting: DFT/DMFT vs Expt



# LaTiO<sub>3</sub>: Full charge sc +FLL double counting produces metal



## Experimental structure

H. T. Dang, C. Marianetti and AJM, unpublished



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**?What is going on?**

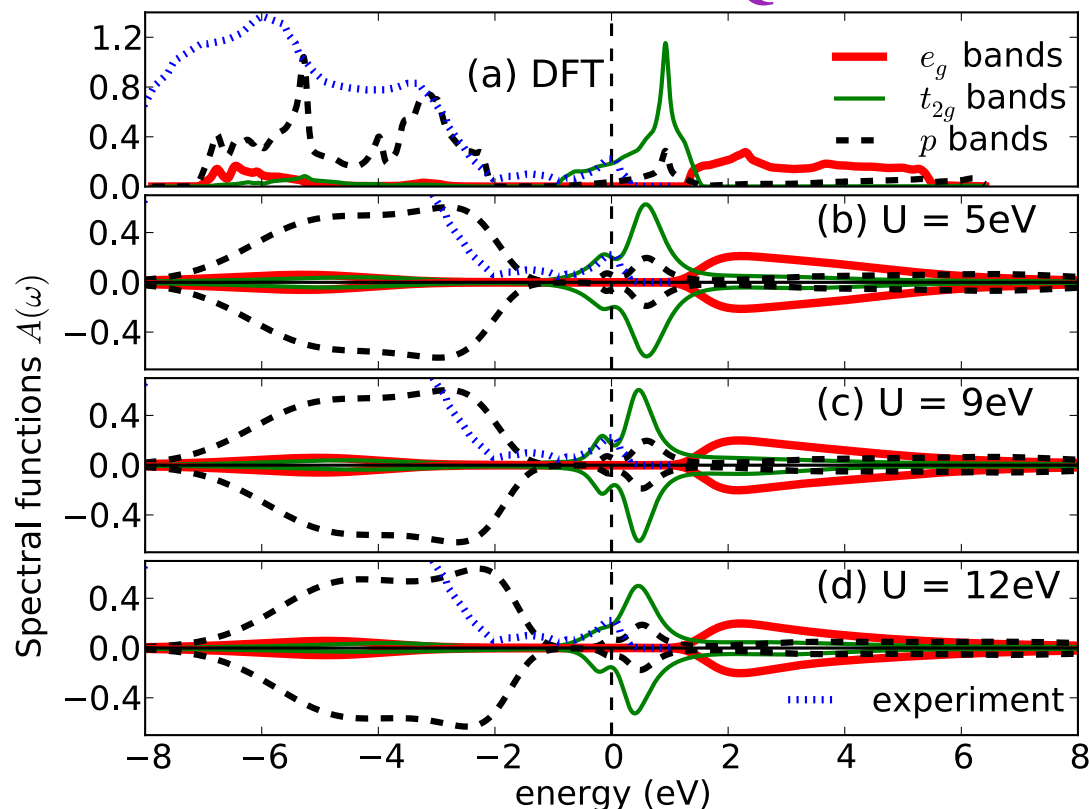


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# SrVO<sub>3</sub>: an instructive example

Cubic, moderately correlated metal

DFT+DMFT TRIQS Code



**FLL+ full charge self consistency keeps relative separation of p and d bands  $\sim$  constant and  $\sim 1$  eV smaller than expt value for all  $U$**

**Without charge self consistency, p-bands stay at DFT position**

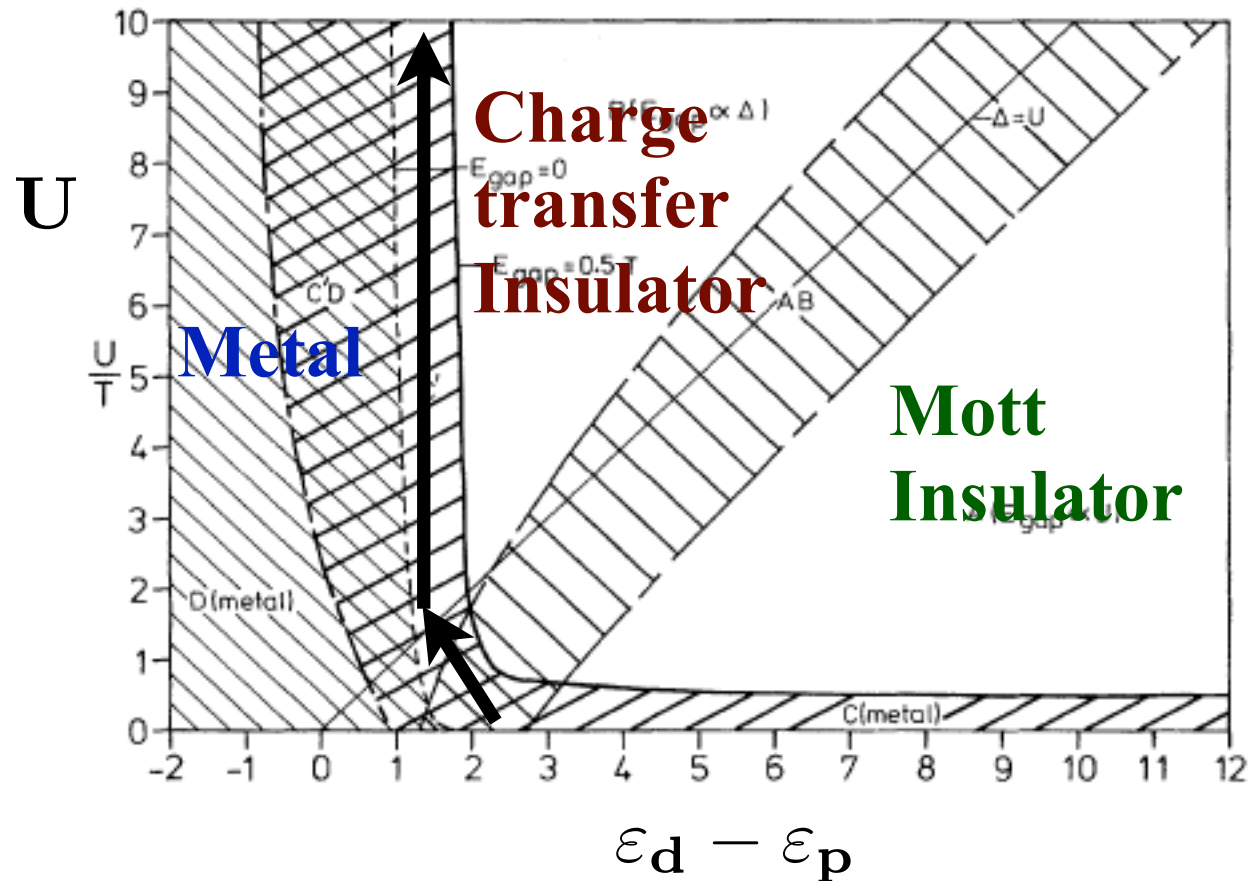
H. T. Dang, AJM and C. Marianetti  
arXiv:1309.2995

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**DFT+DMFT: increasing  $U$  moves you more or less vertically in the ZSA phase diagram.**



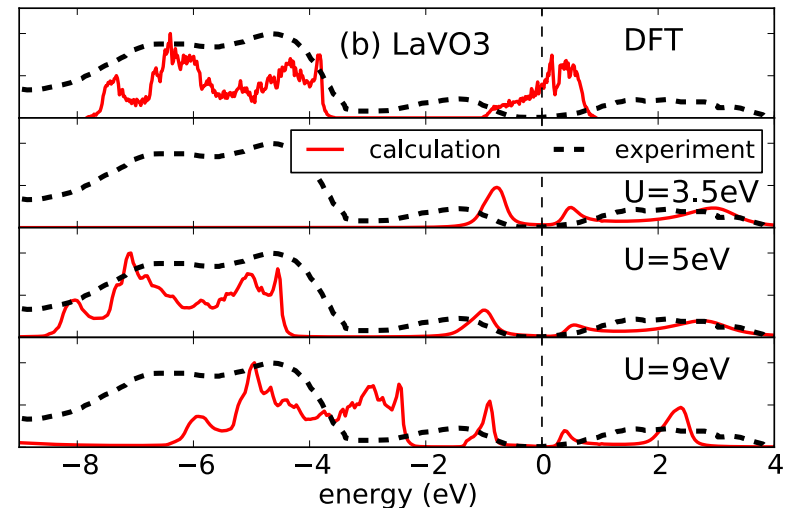
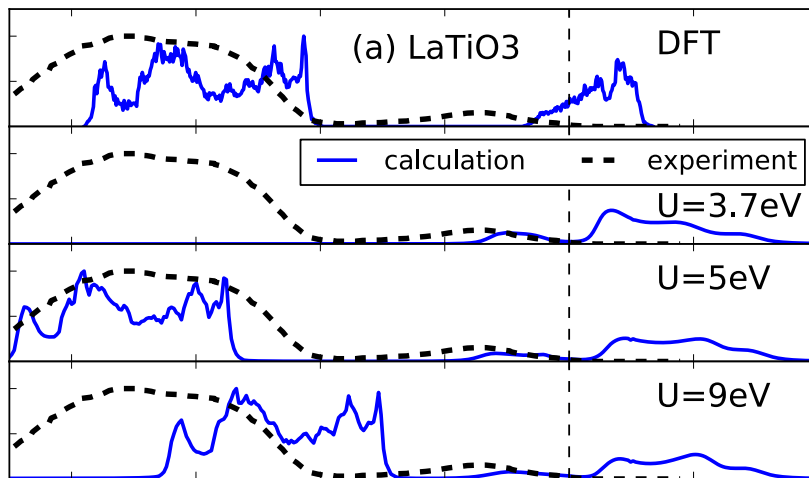
**If the calculation starts with too small a p-d energy splitting, it remains in the metallic regime as  $U$  increases**



# Titanates/Vanadates

For each  $U$ , adjust  $E_{dc}$  by hand so calc gives insulating gap of size found in experiment. Compute oxygen bands. **Relatively narrow  $U$ -range consistent with expt**

**$U \sim 5\text{-}6\text{eV}$  is found in c-RPA calculations**

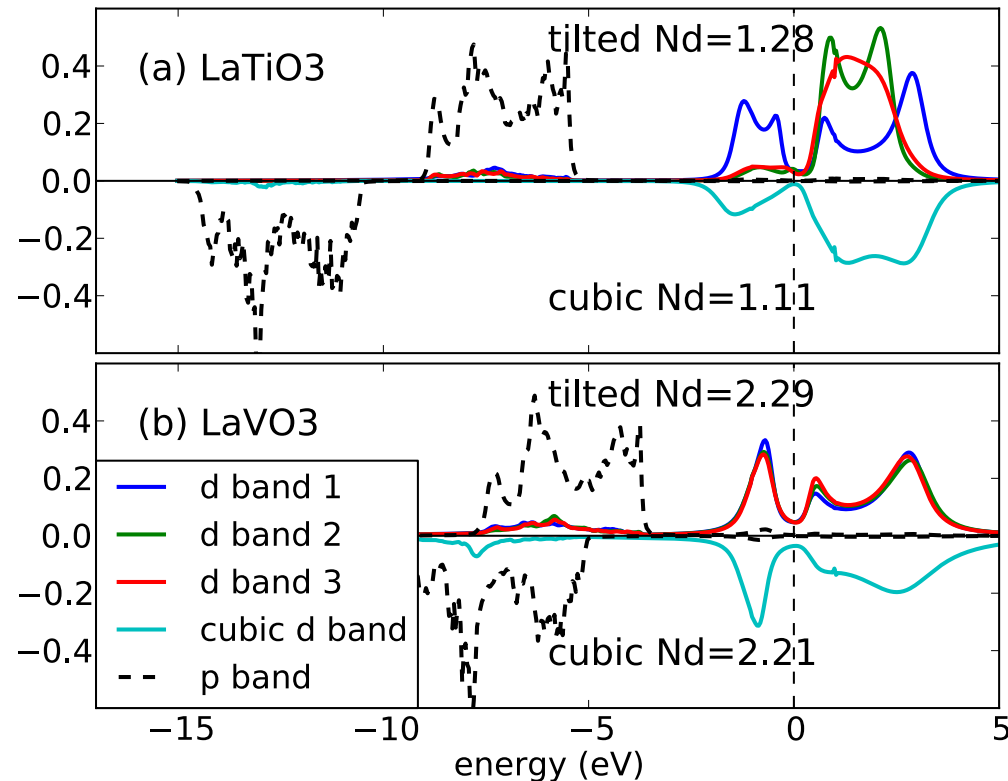


**Single-site DMFT +DFT “works” \*provided\* oxygen levels are correctly positioned and reasonable  $U$  chosen**



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# Aside: $\text{GdFeO}_3$ -distorted (tilted) structure essential for insulator



**$\text{GdFeO}_3$  structural distortion (and associated level splitting) essential for insulating behavior of  $\text{LaTiO}_3$  (as previously found by Pavarini, Georges et al); much less so for  $\text{LaVO}_3$**

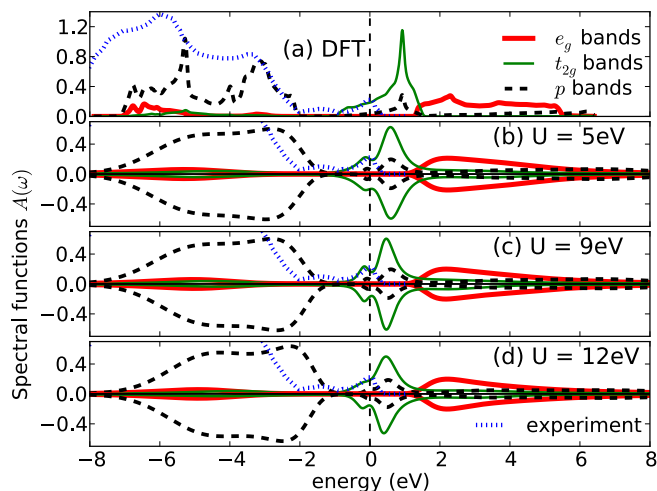
H. T. Dang, C. Marianetti and AJM, unpublished



# ?How to position oxygen levels?

Fully charge self consistent DFT +DMFT pins p-d splitting to value (slightly) smaller than DFT value.

=>2 options



**Modify band theory**

**\*Better approx than DFT (?GW?)**

**Modify  $E_{DC}$**

**\*Haule/Kotliar**

**\*Park/Marianetti**

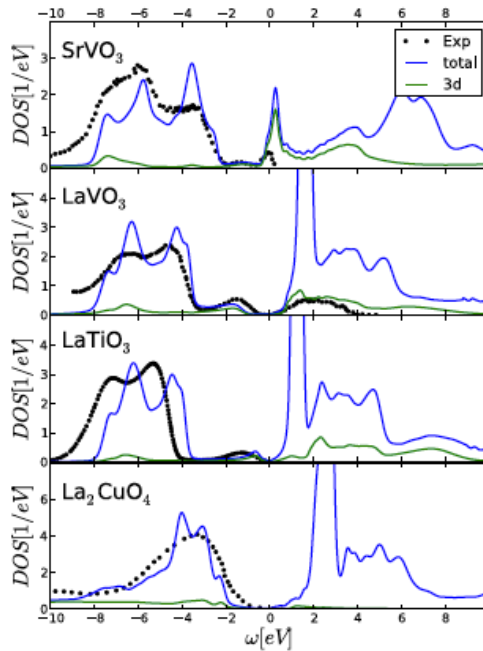


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# Modify $E_{DC}$ (1)

Haule et al arXiv:1310.1158

$$E_{DC} = N_d \left( U - \frac{J}{2} \right) \left( N_d^0 - \frac{1}{2} \right) \quad N_d^0: \text{formal valence}$$



$U=10\text{eV}$

**Good agreement with data**



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# But: big problem with energetics

**FLL double counting**

$$E^{\text{at lim}} = \frac{1}{2}U N(N-1) - \frac{1}{2}JN_{\uparrow}(N_{\uparrow}-1) - \frac{1}{2}JN_{\downarrow}(N_{\downarrow}-1)$$

**Haule double counting**

$$E_{\text{DC}} = N_d \left( U - \frac{J}{2} \right) \left( N_d^0 - \frac{1}{2} \right)$$

**Haule double counting is level shift, not interaction energy=>no factor of 1/2 => problem for energetics**



# Alternative (ad hoc)

**Park, AJM, Marianetti arXiv:1310.5772**

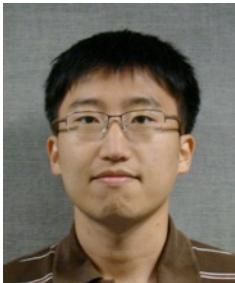
$$E_{\text{DC}} = \frac{1}{2} \mathbf{U}' N_{\text{d}} (N_{\text{d}} - 1) - \frac{1}{2} \mathbf{J} \sum_{\sigma} N_{\sigma} (N_{\sigma} - 1)$$

**$\mathbf{U}' < \mathbf{U}$  increases p-d splitting**



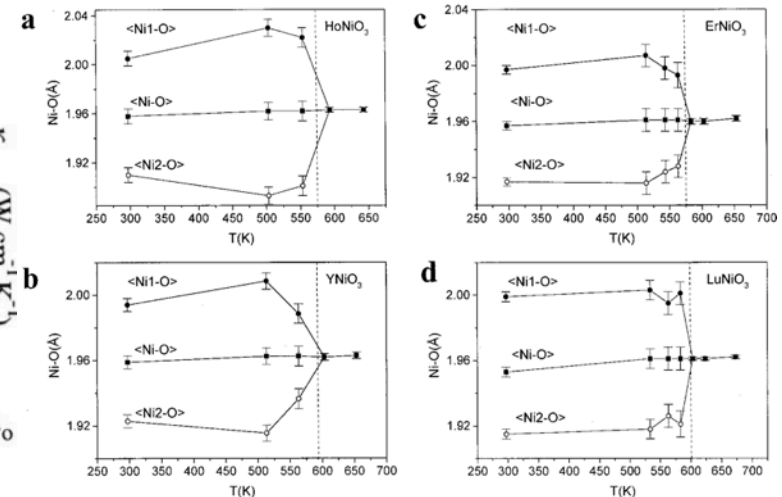
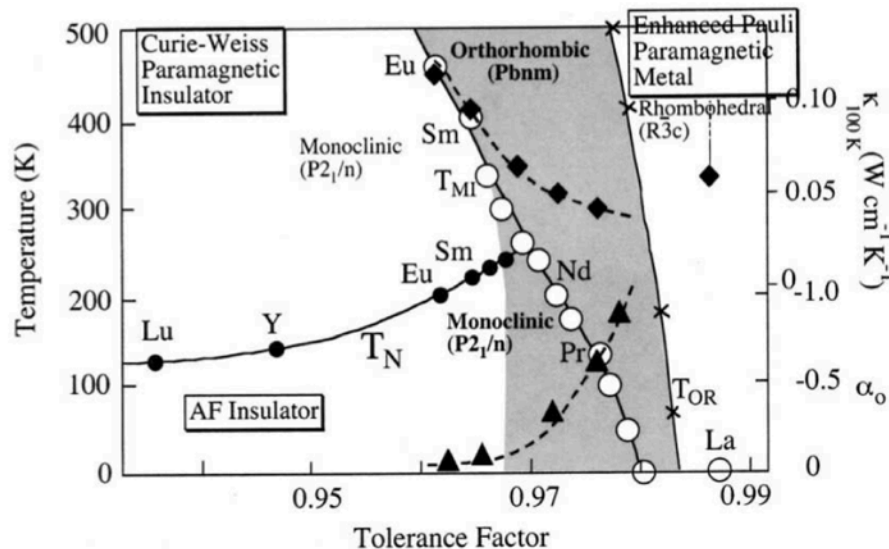
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# Total energy within DFT+DMFT: Application to rare earth nickelates



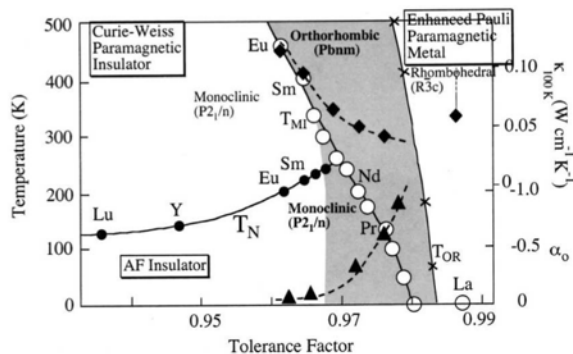
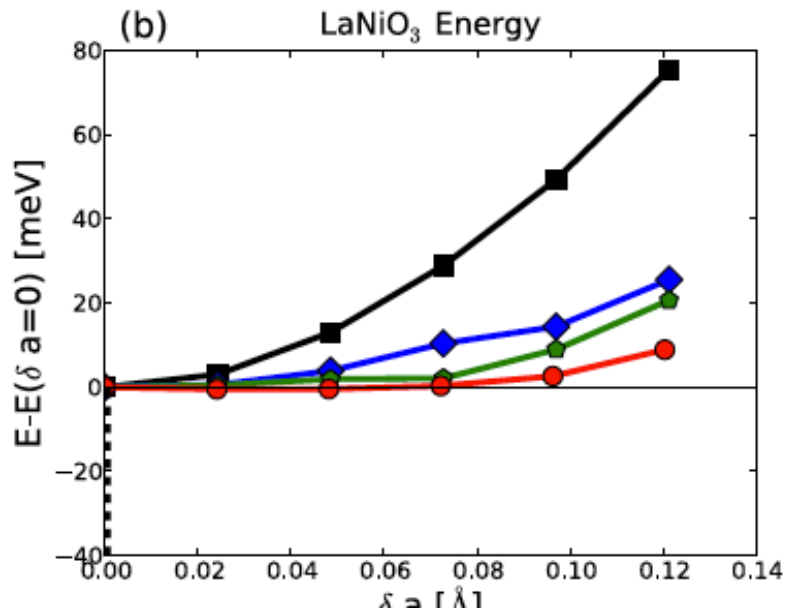
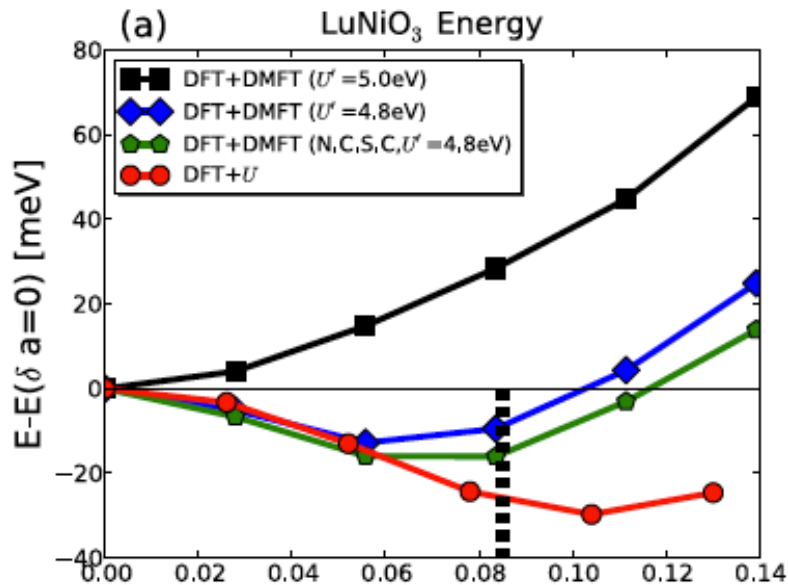
**\*Hyowon Park**

**PRL 109, 156402 (2012)**  
**arXiv:1310.5772**



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# Energy as function of distortion



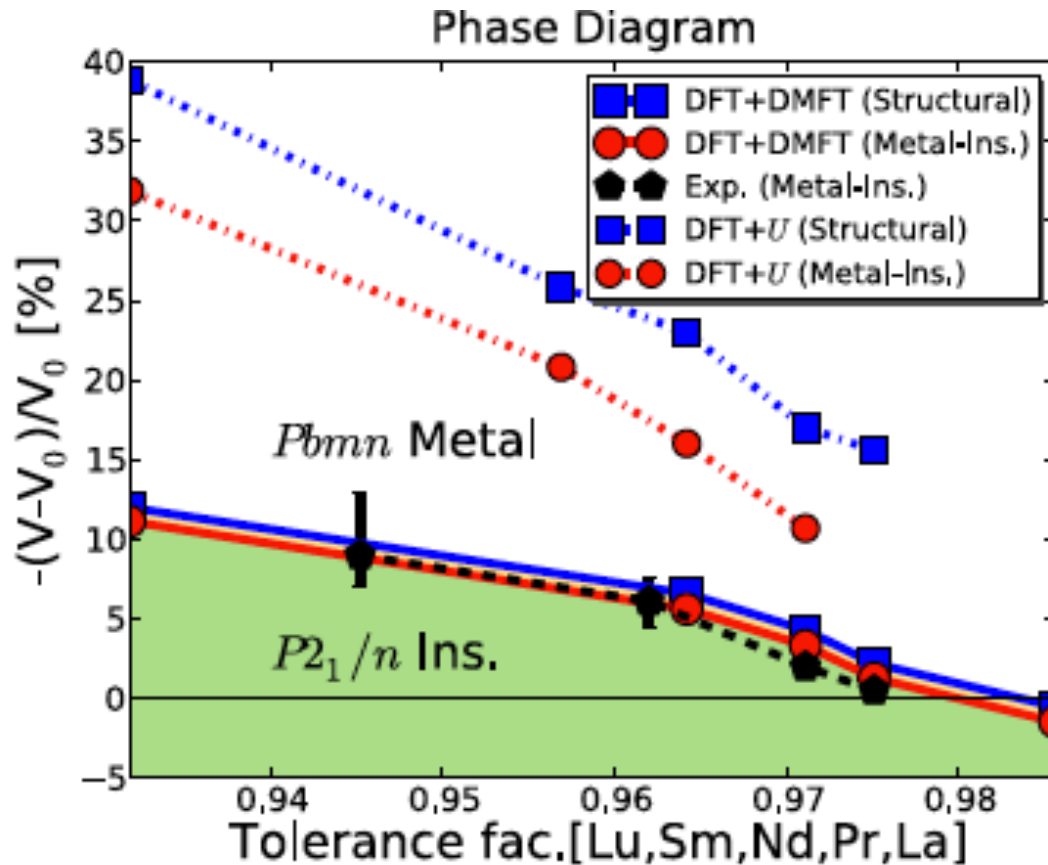
$U' = U$  No distortion

$U' = 0.96U$  correct distortion



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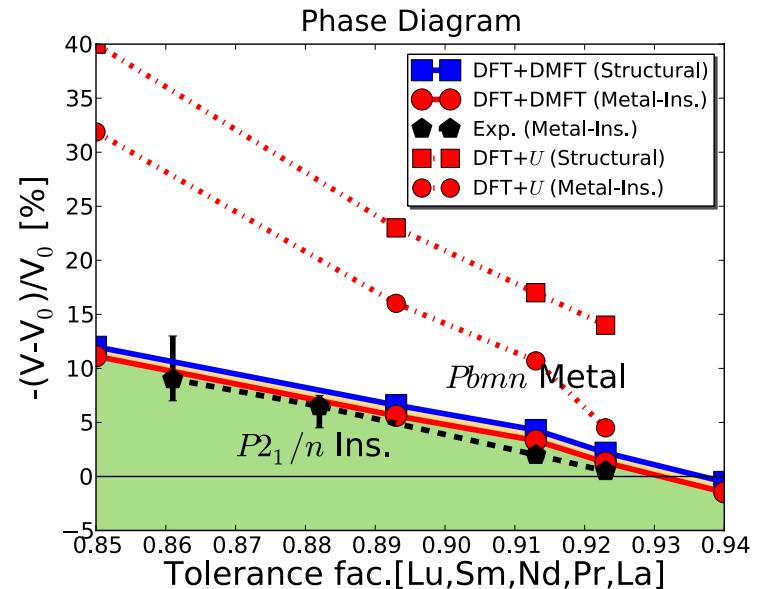
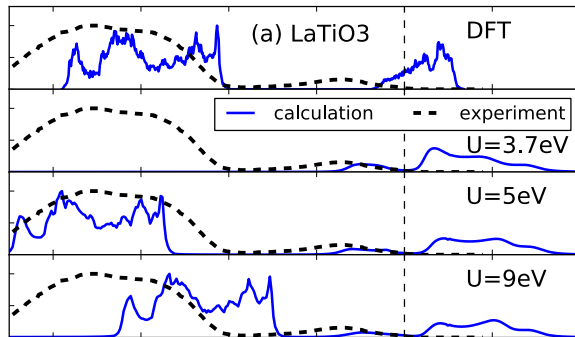
# Entire pressure-rare earth phase diagram given correctly



$$U' = 0.96U = 4.8\text{eV}$$



# Conclusion



**Key issue for DFT+DMFT: proper p-d energy splitting.**

**Is this a problem of double counting or band theory**



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