6. Nonequilibrium steady state

bulk detailed balance H, $c_{x x\pm 1}$ $e^{-\beta H + \mu N}$

boundary chemical potential μ_+, μ_- flip rate $0 \leftrightarrow 1$ $c_-(\eta), c_+(\eta)$

detailed balance $e^{-\beta H + \mu_\pm N}$

(no boundary jumps)

unique NESS

$\mu = \mu_+ = \mu_-$ then $e^{-\beta H + \mu N}$ stationary

$\mu_+ \neq \mu_-$ NESS

problem $\langle \eta_x \eta_y \rangle_{ss} - \langle \eta_x \rangle \langle \eta_y \rangle$ N large

hydrodynamics: $\partial_q D(\rho) \partial_q \rho = 0$ $\rho(0) = \rho_-$, $\rho(1) = \rho_+$

method fluctuation theory (Landau-Lifschitz) (poor man effects)

solution $\rho_s(q)$

INSERT Langevin equation, Gaussian fluctuation theory

$\phi(x,t)$ A, B, C linear operators in $x \in \mathbb{R}$

$$\partial_t \phi(x,t) = A\phi(x,t) + B\xi(x,t)$$ modifies noise

$\xi(x,t)$ Gaussian white noise

$\langle \xi(x,t)\rangle = 0$

$\langle \xi(x,t)\xi(x',t')\rangle = \delta(x-x')\delta(t-t')$

$A^T A \leq 0$ decay

$\rightsquigarrow$ stationary measure $\langle\ \rangle_{ss}$ $\langle\phi\rangle = 0$

$$C(x,x') = \langle \phi(x,0)\phi(x',0)\rangle_s$$ static covariance

general fact

$$-(AC + CA^T) = BB^T \qquad C = \int_0^\infty dt\, e^{At} BB^T e^{A^T t}$$

---

our application with $\rho = $ const (equilibrium)

$A = D\partial_x^2$, C is continuum limit of $\langle \eta_i \eta_j\rangle_\rho - \rho^2 = g(i-j)$ rapid decay

$C(x,x') = \chi\, \delta(x-x')$ large scale $\quad \chi = \sum_j (\langle \eta_0 \eta_j\rangle_\rho - \rho^2)$ $\quad 0 < \chi < \infty$

$\rightsquigarrow$

$-2\chi D\partial_x^2 = -2BB^* \partial_x^2$ ok

$$B = -\sqrt{2\chi D}\,\partial_x = -\sqrt{2\sigma}\,\partial_x, \quad B^T = \sqrt{2\sigma}\,\partial_x \quad BB^T = -2\sigma\partial_x^2$$

$\rightsquigarrow$ steady state, relation holds locally, $\rho \to \rho_s(x)$

$$A = D(\rho_s(x)) \qquad B = -\partial_x\sqrt{2\sigma(x)}$$

A linearization

$B = -\partial_x\sqrt{2\sigma}$ conductivity

$\rightsquigarrow$ conservation law, fluctuating current

$$\partial_t \phi + \partial_x\left(-\partial_x D(\rho_{ss})\,\phi + \sqrt{2\sigma}\,\xi\right) = 0 \quad !!$$

$\sigma(\rho_{ss}(x))$ changes spatially

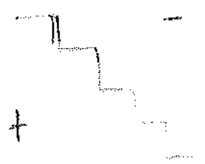
$$\lim_{t\to\infty} \langle \phi(x,t)\phi(y,t)\rangle$$

7. Moving interfaces, KPZ universality   Kardar, Parisi, Zhang 1987

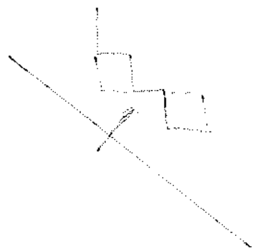
7.1 Interface motion

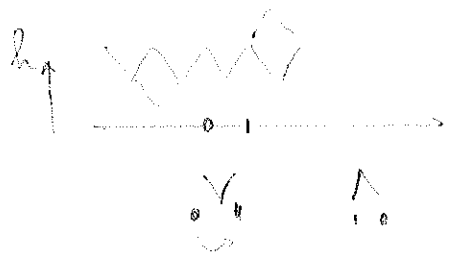
Glauber spin flip, low temperatures

initial condition



stable − stable





height $\wedge \leftrightarrow \vee$

= SSEP rates 1

Roughening

short cut   LL

$$\partial_t \rho + \partial_x \left(- \partial_x \rho - \xi \right) \qquad D, \sigma = 1$$

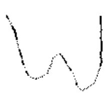
$$h' = \rho \quad \rightsquigarrow \quad \partial_t h(x,t) = \partial_x^2 h + \xi \qquad h(x,0) = 0$$

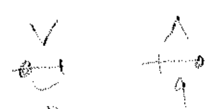
solution: $h(0,t) = \int_0^t ds \left(e^{\partial_x^2 (t-s)} \xi\right)(0,s)$ $\langle h(x,t) \rangle = 0$

$$\langle (h(0,t))^2 \rangle = \int_0^t ds \left(e^{2\partial_x^2 s}\right)(0,0) = \int_0^t ds \frac{1}{\sqrt{s}} = \sqrt{t}$$

$$h(0,t) \simeq t^{1/4}$$

stable − meta-stable





$p > q$ or $p = 1,\ q = 0$

TASEP

stable phase invades metastable one?

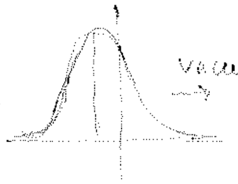
definition

$$\mathbb{P}(\xi_{TW} \le s) = \det\left(1 - P_{[s,\infty)}\, \mathbb{P}(H_{Ai} \le 0)\, P_{[s,\infty)}\right)$$

$$H_{Ai} = -\frac{d^2}{dx^2} + x$$

density

bulk ←



vacuum →

mean

$e^{-|x|^{3/2}}$

skewed

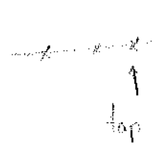
$e^{-|x|^3}$

Where does this come from?

GUE random matrix

X_{ij} iid complex Gaussians

semi circle Wigner



$A_{ij} = \frac{1}{\sqrt{2}}(X + \bar{X})_{ij}\sqrt{N}$

eigenvalue

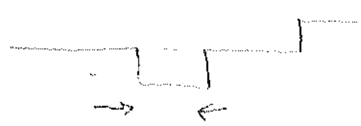
largest eigenvalue $2N + N^{1/3}\xi_{TW}$

7.4 The PNG model

again Glauber

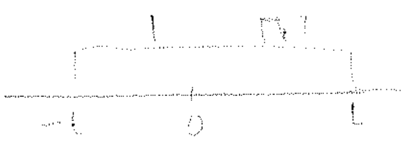
etc growth process

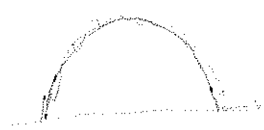
(i)



(ii) random nucleation

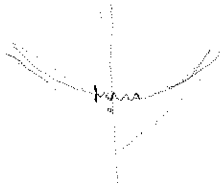
rounded



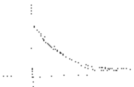


one knows more

$h(\xi, t)$

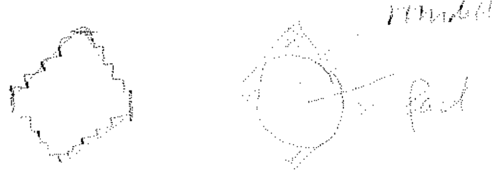


$h(\xi, t) - vt - \frac{\xi^2}{t}$ random in ξ spatial statistics



. Aztec diamond

tiling of square  random uniformly



equilibrium

facet edge KPZ universality