## Assignment

(a) Consider a network comprising 8 nodes (labelled $\mathrm{A}, \ldots, \mathrm{H}$ ) whose adjacency matrix is shown at right. Partition the network into two modules - i.e., show that the nodes can be divided into two groups such that the number of links between the nodes belong to the two groups is minimized.

Mention which nodes will form one group and which nodes will form the other.

What is least number of links that occur between the two modules ? Identify these links, i.e., mention the pair of nodes (belonging to the different groups) that each link connects.

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\mathbf{B}$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\mathbf{C}$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| D | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| $\mathbf{E}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| F | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| $\mathbf{G}$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\mathbf{H}$ | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |


(b) Perform the recursive core decomposition technique to identify the $k$-order cores of the network where $k=1,2,3, \ldots$ (recall that the $k$-core of a network is the subnetwork containing all nodes that have degree at least equal to $k$ ).

How many nodes are in (i) 1-core, (ii) 2-core, and (iii) 3-core?
F
(c) What is the order of the innermost core, i.e., the highest value of $k$ for which the corresponding core is non-empty? Identify the nodes in the innermost core.
(d) What is the size of the largest connected component of the 3-core?

## Answers

(a) Nodes A,B,F,G will form one group and nodes C,D,E,H will form the other group.

The least number of links occurring between two groups is 1 .
The link will connect the nodes C and F which belong to different groups

|  | A | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\mathbf{B}$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $\mathbf{C}$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| $\mathbf{D}$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| $\mathbf{E}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| $\mathbf{F}$ | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| $\mathbf{G}$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\mathbf{H}$ | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |


(a) 1-core: all 25 nodes

2-core: 13 nodes (B,C,E,G,K,L,M,N,U,V,W,X,Y)
3-core: 9 nodes (the two isolated clusters B,C,E,G,K and $\mathrm{V}, \mathrm{W}, \mathrm{X}, \mathrm{Y}$ )
(c) The order of innermost core is 4 , whose members are B,C,E,G,K
(d) The size of the largest connected component of the 3 -core is 5 . The nodes that belong to the largest connected component are B,C,E,G,K

