Mathematics of Planet Earth

BENGALURU 2013

AN EXHIBITION

MATHEMATICS FOR THE BILLION

ICTS

International Centre *for* Theoretical Sciences

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

INTERNATIONAL CENTRE for THEORETICAL SCIENCES

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In partnership with:

tifr CENTRE FOR APPLICABLE MATHEMATICS

VISVESVARAYA INDUSTRIAL AND TECHNOLOGICAL MUSEUM

National Council of Science Museums, Ministry of Culture, Government of India

Exhibition Design

SRISHTI School of Art, Design and Technology

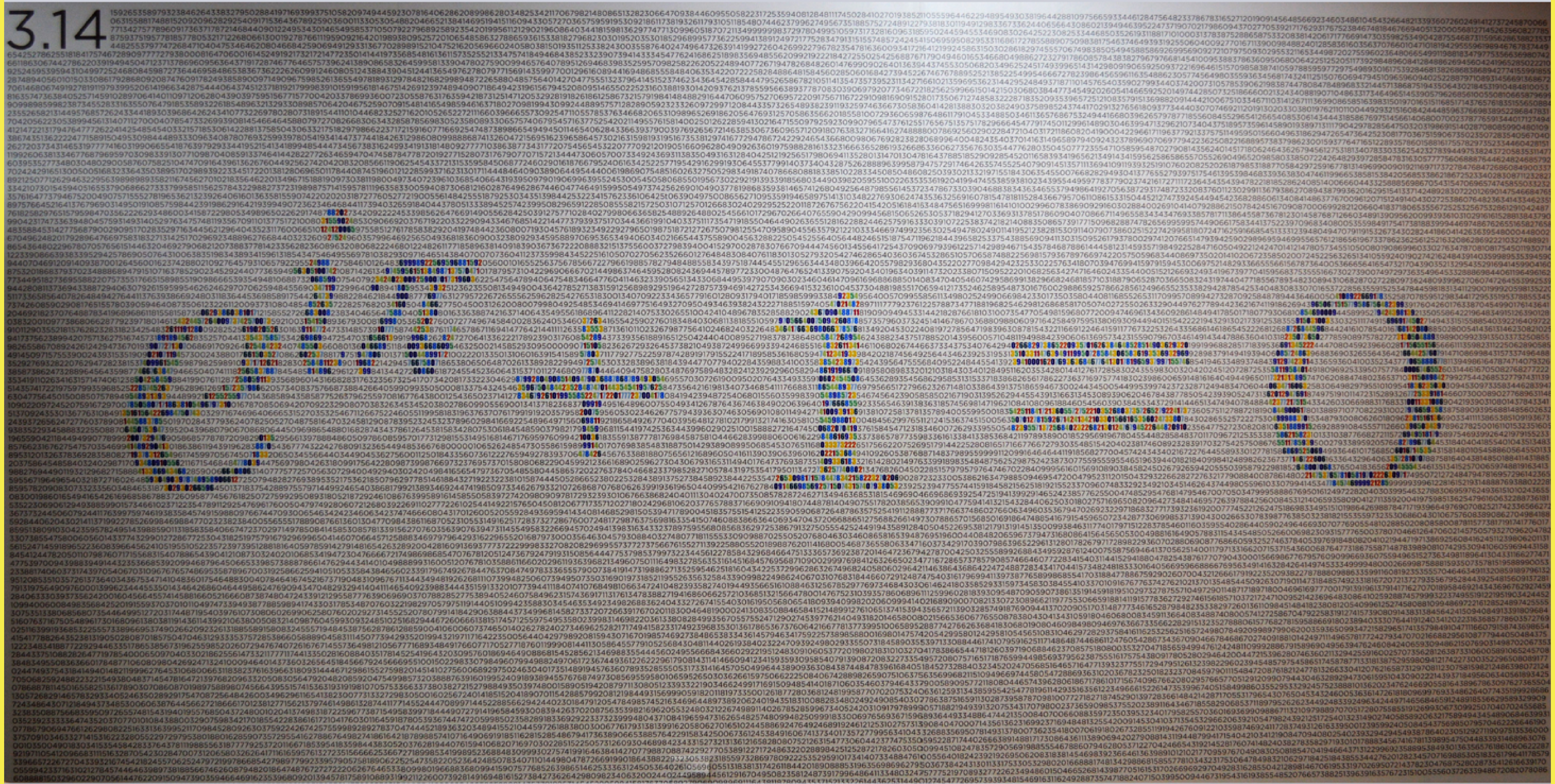
Book Design: *Kumari MC*

Preface

The Mathematics of Planet Earth 2013 (MPE-2013)

was an initiative of more than 100 scientific societies, universities, research institutes, and organizations all over the world, and the partnership now continues beyond the year 2013. MPE's mission is to increase engagement of mathematical scientists, researchers, teachers, and students in issues affecting the earth and its future. The International Centre for Theoretical Sciences of the Tata Institute of Fundamental Research (ICTS-TIFR), Bangalore, as the main Indian partner of MPE, organized an exhibition, during 22 November - 01 December, 2013, containing around 30 exhibits on mathematical themes such as structures, networks, oscillations, optimization, waves, etc. This booklet contains a description of the mathematical ideas as well as many of the design elements that went into these exhibits. The last chapter contains a more detailed description of MPE activities and the Indian partners that participated in the organization of this exhibition.

Note: The image on the following pages contains approximately 30000 digits of π.


3.14

contents

1 structures

The broad theme of structures is about setting up environments to serve a purpose, and one is naturally led to looking at natural and man-made ingenious designs to optimize various aspects like utility, economy, robustness, aesthetics and such.

Where does one begin? It is very illuminating to look at the myriad forms of nature which have evolved time-tested solutions that have effectively met many of these aspects. Nature continues to amaze humans, and it is no wonder that it has inspired them to mimic and model their structures: The Khalifa tower in Dubai would invariably remind one of an anthill and the Olympic stadium in Munich or the elaborate portals in the modern day airports would bring to focus the picture of spider webs. For anyone of any age group, it is hard to ignore the beauty of flowers, leaves, trees, the corals, seashells, butterflies, insects, their ornate dwellings - the list goes on and on. It has fascinated poets, musicians, artists, artisans, scientists and mathematicians alike.

Some of the exhibits under this theme are inspired by a couple of illuminating examples, like spider webs and honeycomb, and intend to showcase some of their structural wonders. They focus mainly on the following aspects of these structures: tensegrity, stress-distribution, as well as optimization.

1.1

Tensegrity Stool

Tensegrity is an amalgamation of two words 'Tension' and 'Integrity.' The tensegrity concept is mainly about the judicious use of available material to create a structure that is both stable and spans a large area; it is used to build structures which employ an economy of material and obtain optimal results.

Tensegrity structures contain two types of components to achieve this purpose: the struts which lend strong support, that is the integrity part, and the cables that maximize the span by stretching or tension. In other words, strong struts connected together by flexible cables hold the structure together in optimal tension.

Where do such structures occur? In nature, spider webs demonstrate this concept effectively: they contain strong structural supports, the radial threads and a few polygonal threads, and the rest of the web is woven around it.

Mathematically, tensegrity is a configuration of points, or vertices, that satisfy some simple distance constraints. Cables keep vertices close together and struts hold them apart.

Another main purpose in building structures employing this concept is also to achieve quick assembling and dissembling so as to enable ease of transportation.

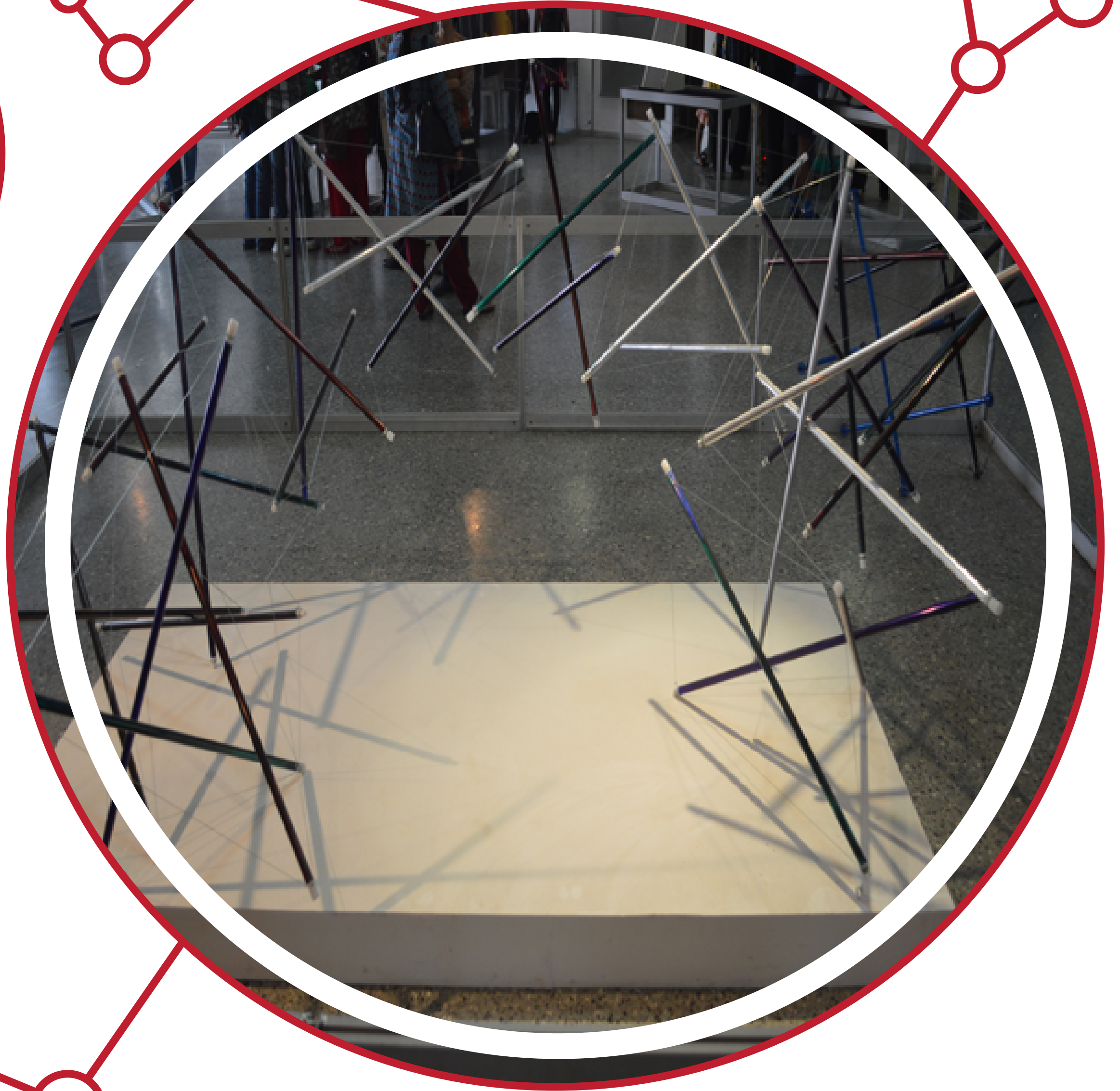
The tensegrity stool that was exhibited is one example of this structural concept. It basically consists of rods and strings connected together to support either a human or an object placed on it.

1.2

Tensegrity Tower

This exhibit is an extension of the tensegrity structure concept that brings out a further advantage or effectiveness, namely robustness and structural stability. For the structure to weather a storm or a tremor, it needs to absorb sudden shocks and regain the original form.

Where do these occur? Spider web example again relates this aspect to the fact that the web is not easily destroyed by strong winds or by violent disturbances caused at the anchored ends. Being one of the most commonly found designs in nature, the radial threads connected to fixed objects are criss-crossed by the spiral threads to form an optimal design. It is strong enough to hold a large prey and at the same time flexible enough to sustain strong wind blows. Recent studies show that if few local threads are broken, the overall loading capacity of the web is in fact increased, proving the robustness of the structure. When a prey is trapped, the spider gets the message through the vibrations of threads. The patterns of interwoven threads enables the spider to spot the prey in an efficient manner.



1.3

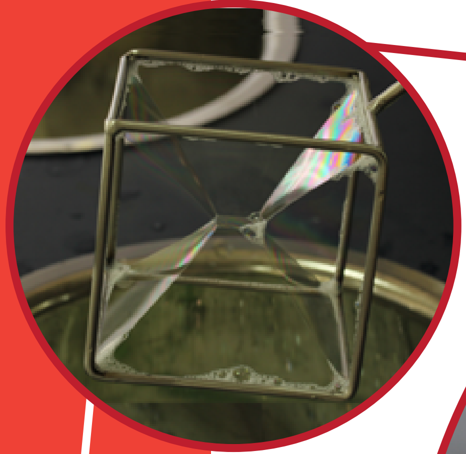
Soap Bubbles

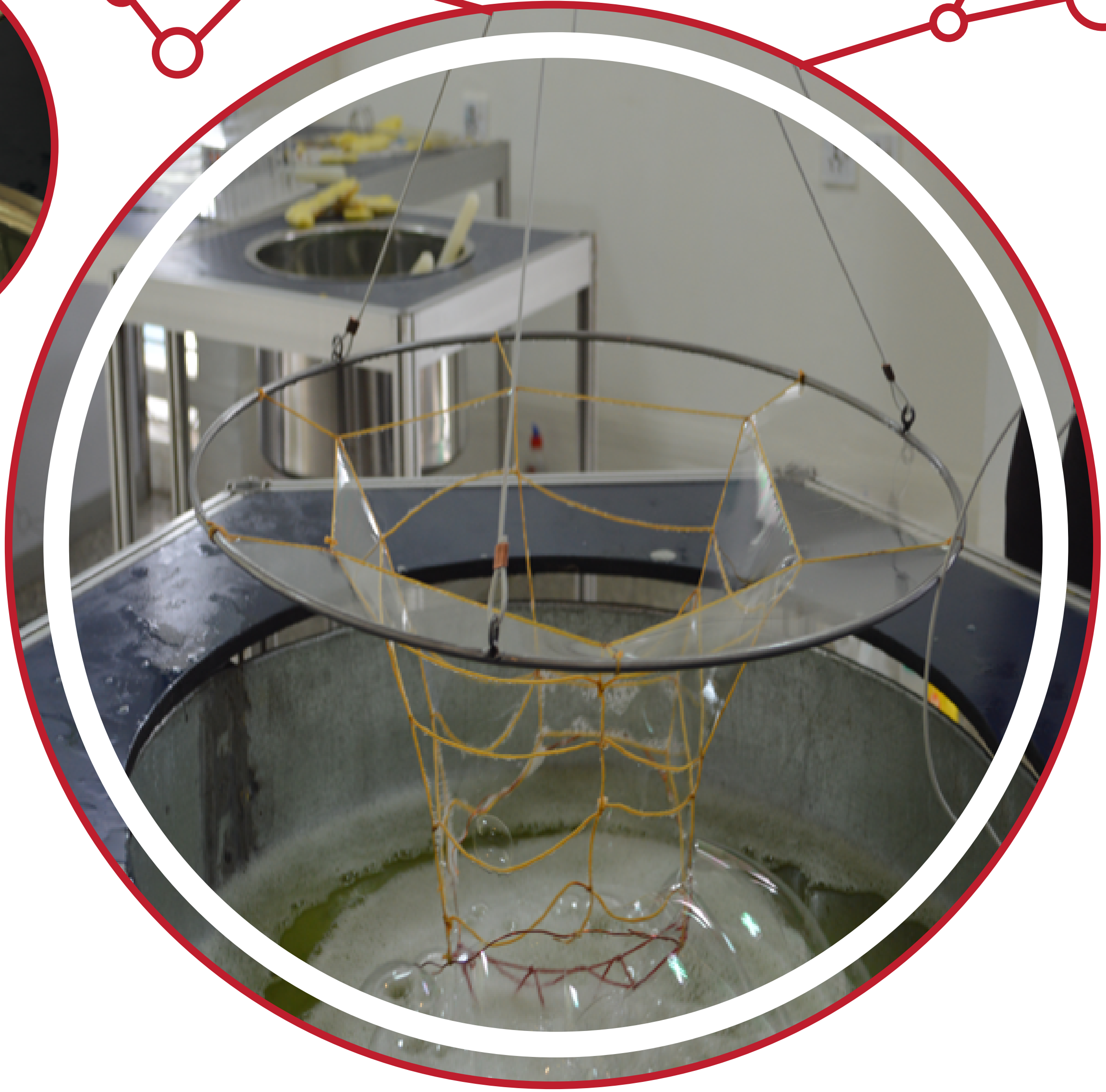
Soap bubbles are formed by the simple physical principle of surface tension, whose mathematical manifestation is the concept of "minimal surfaces" – the idea that soap films always minimize their surface area.

This seemingly simple principle leads to visually appealing and mathematically interesting shapes: individual bubbles are spherical in shape maximizing the volume enclosed, the bubble clusters settled on a plane surface display a hexagonal tessellation, the soap films formed by irregular shapes can show a dazzling variety.

Mathematically, some of the soap bubbles are classic examples of surfaces with negative curvature, like a saddle, while others are classic examples of surfaces with constant positive curvature, like the spherical bubbles. Soap bubbles have led to some intriguing mathematical discoveries as well, as we will see below.

In a state of equilibrium, the surface tension on a soap film is the same at all points, and its surface area will have a minimal value; this minimum area property of soap films can be used to solve some mathematical minimization problems.





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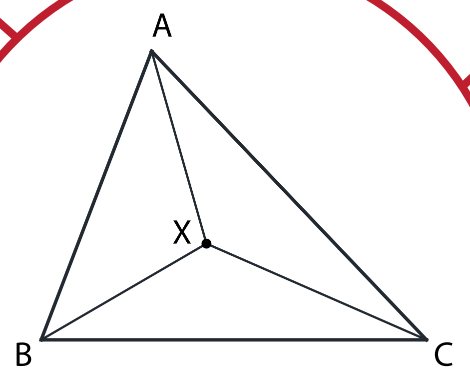


Figure 1. X is the centroid of the triangle ABC.

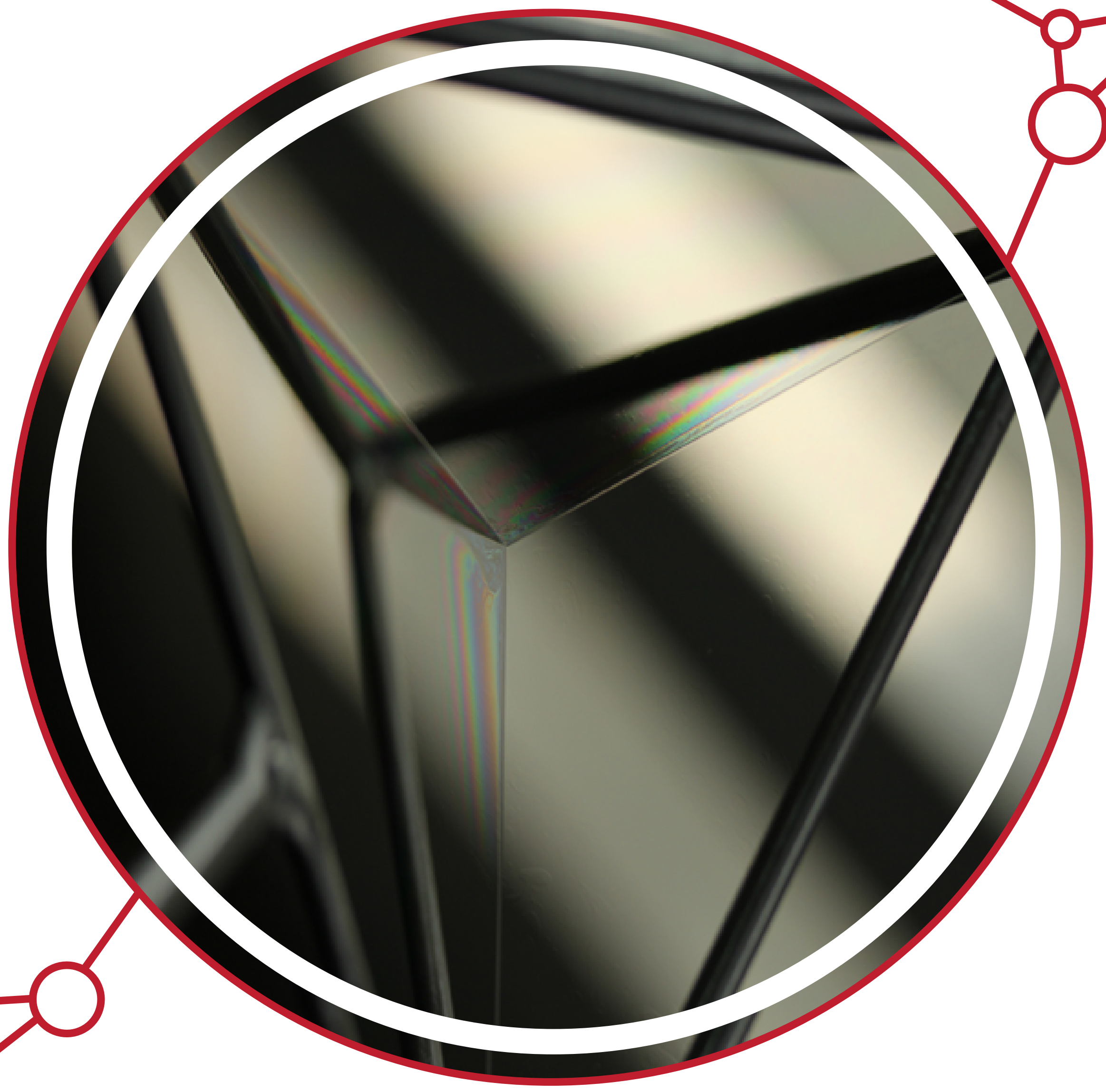
One such minimization problem is the problem of linking n points on a plane by the shortest possible path. For example, consider linking three towns A, B and C situated at the corners of a triangle by a road. What would you think is the shortest path linking the three towns? Or, what about four towns situated at the four corners of a square?

Such shortest paths are along straight lines forming a number of intersections: each intersection always contains three straight lines meeting at angles of 120 degrees. Figure 1 on the left shows a simple minded solution for linking three points at positions A, B and C that the intersecting soap films achieve. Suppose x is the location of the intersection. Then we basically want to minimize the sum of length of these three line segments, which is written as follows.

Find x that minimizes $f(x) = \|x - A\| + \|x - B\| + \|x - C\|$

The critical point (that is, the minimum) is given by $\nabla f(x) = 0$;

$$\frac{x-A}{\|x-A\|} + \frac{x-B}{\|x-B\|} + \frac{x-C}{\|x-C\|} = 0.$$



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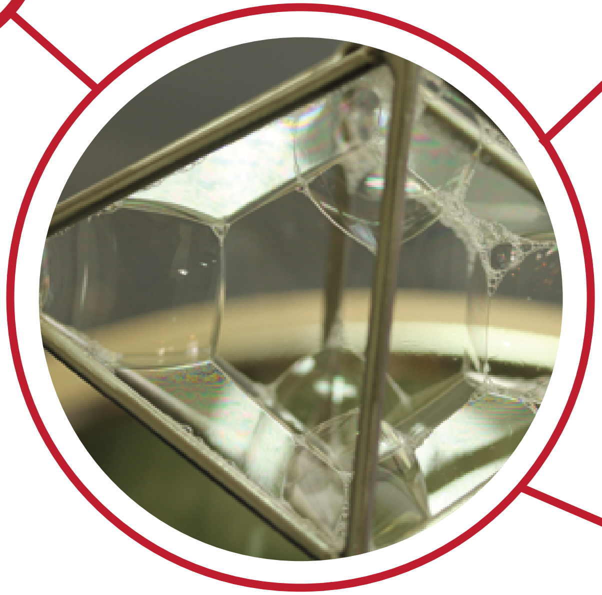
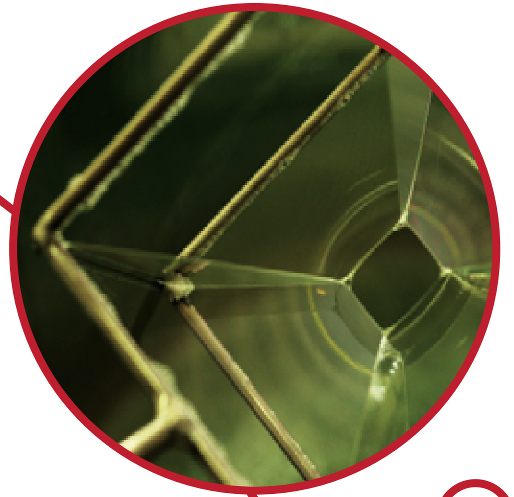
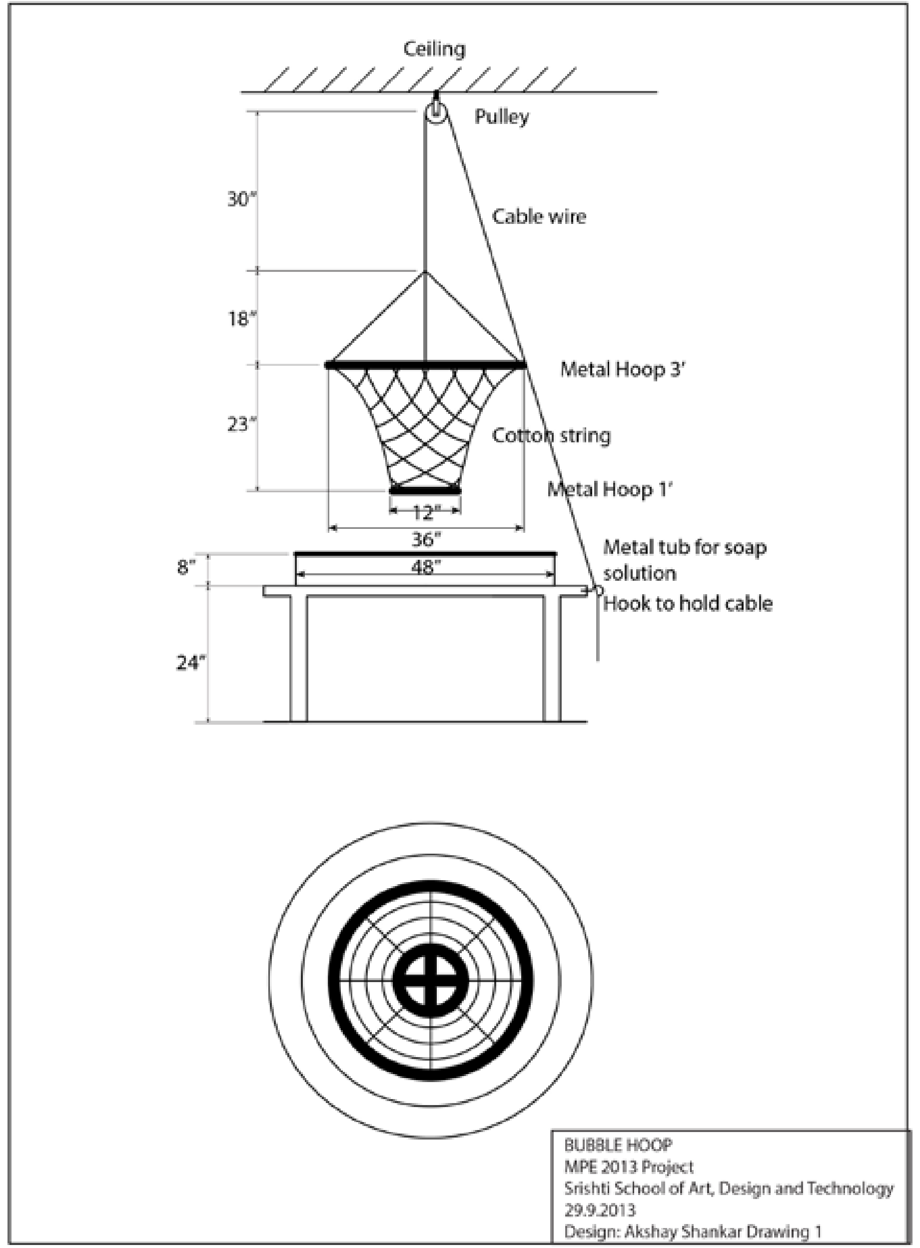


This means that we have three vectors of unit length adding up to zero. Connecting the vectors head to tail means that they form a cycle, which is a triangle. Since they all have the same length, the triangle is equilateral. Thus we see that the angle between the lines is 120 degrees.

Some of the soap bubble frames in this exhibit demonstrate this phenomenon. Even the surfaces of soap films always seem to meet at 120 degree angle, but this mathematical problem is still a challenge to researchers.

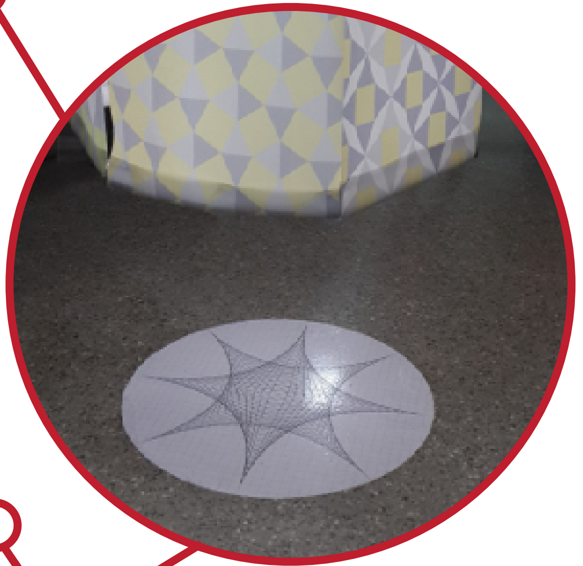


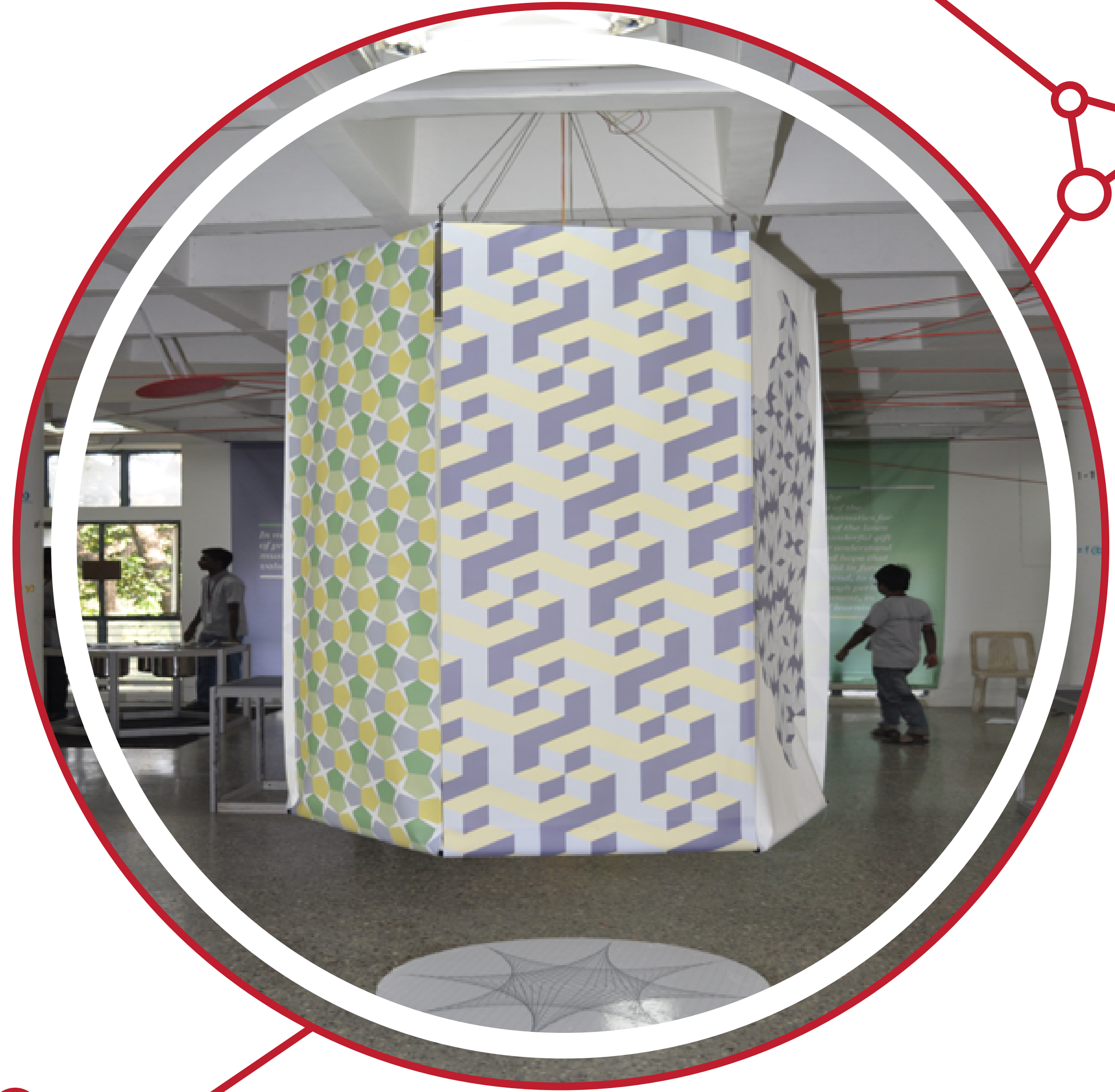


1.4

Escher Lamp

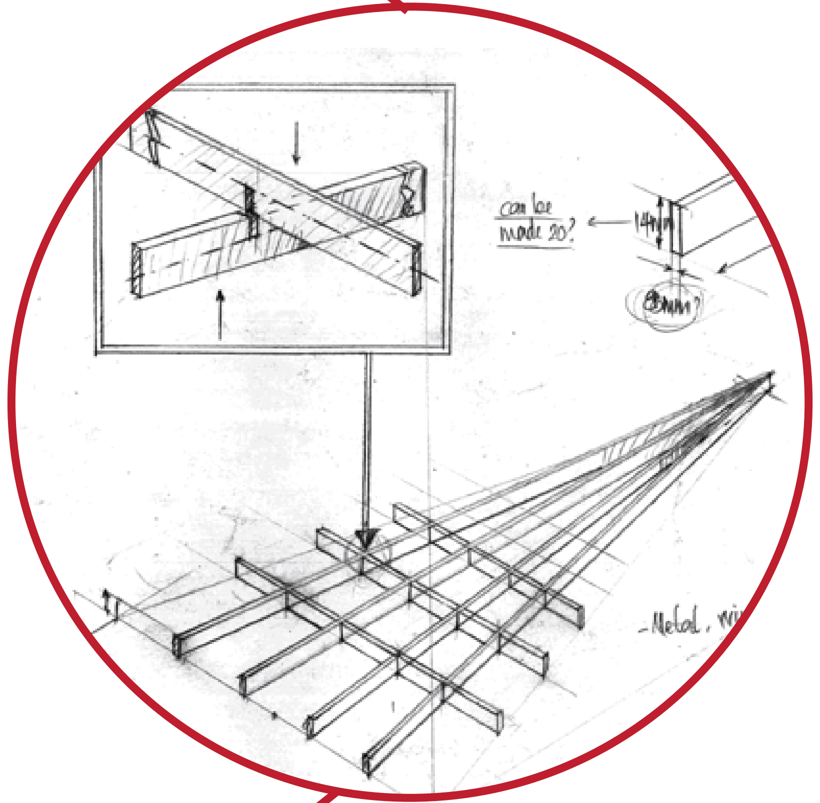
The great Dutch graphic artist M.C. Escher is known for his often mathematically inspired artworks. Much of his various works on tessellations employed the basic mathematical fact: among all the regular polygons, only the triangle, square, and hexagon can tessellate the plane (More on tessellations later in section 1.6.). Escher exploited these basic patterns in his tessellations, applying what geometers call reflections, glide reflections, translations, and rotations to obtain a great variety of patterns.





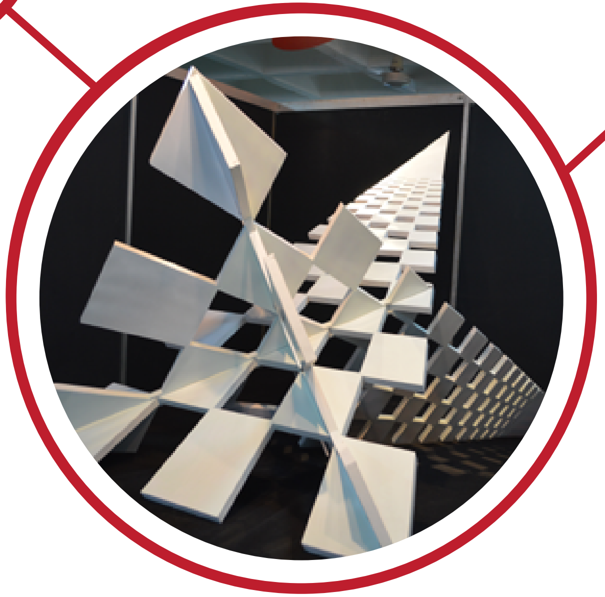
1.5

3D Escher

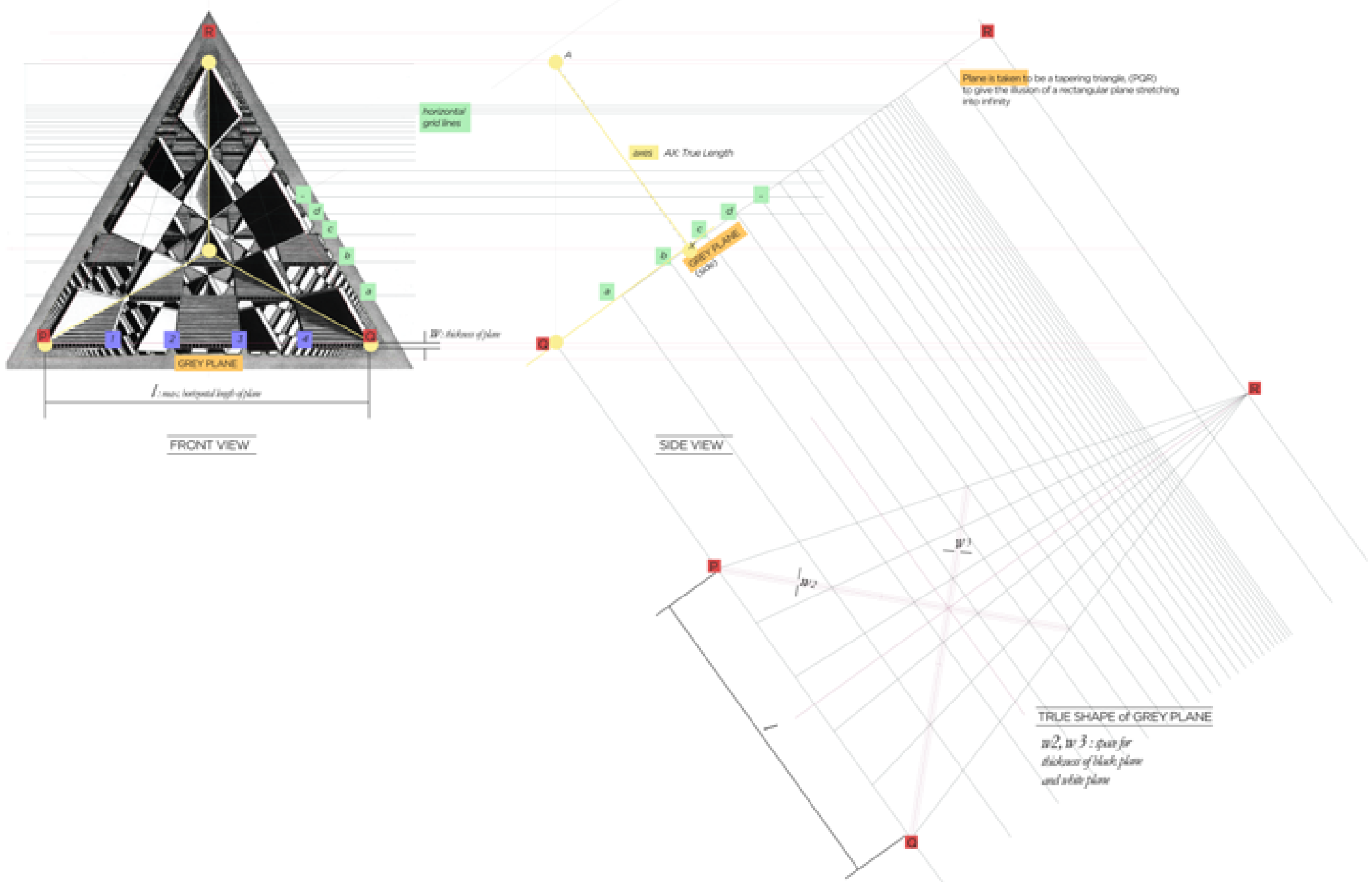


Escher also worked on regular solids, known as polyhedra, that he extensively explored in many of his works. His woodcut “Three Intersecting Planes” is an especially illuminating example of this.

1.5



In the present exhibit, the simple fact that two planes meet along an edge and that three planes meet at a vertex is demonstrated in an aesthetically appealing way.



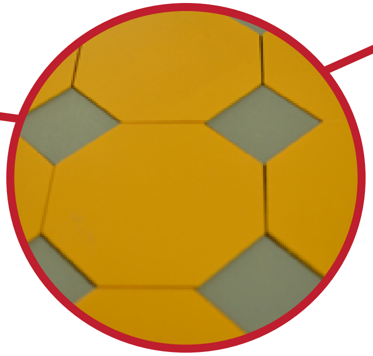
1.6

Tessellations

A *tessellation* is a way of covering a (flat) surface with tiles in such a manner that there are no gaps or overlaps. The idea behind building a tessellation is simple; take a pattern or tile and repeat it over the entire surface. Just like a mason tiles a floor! There are many different types of tessellations – different shapes can be used to tile a surface. The most commonly seen tiles are polygons.

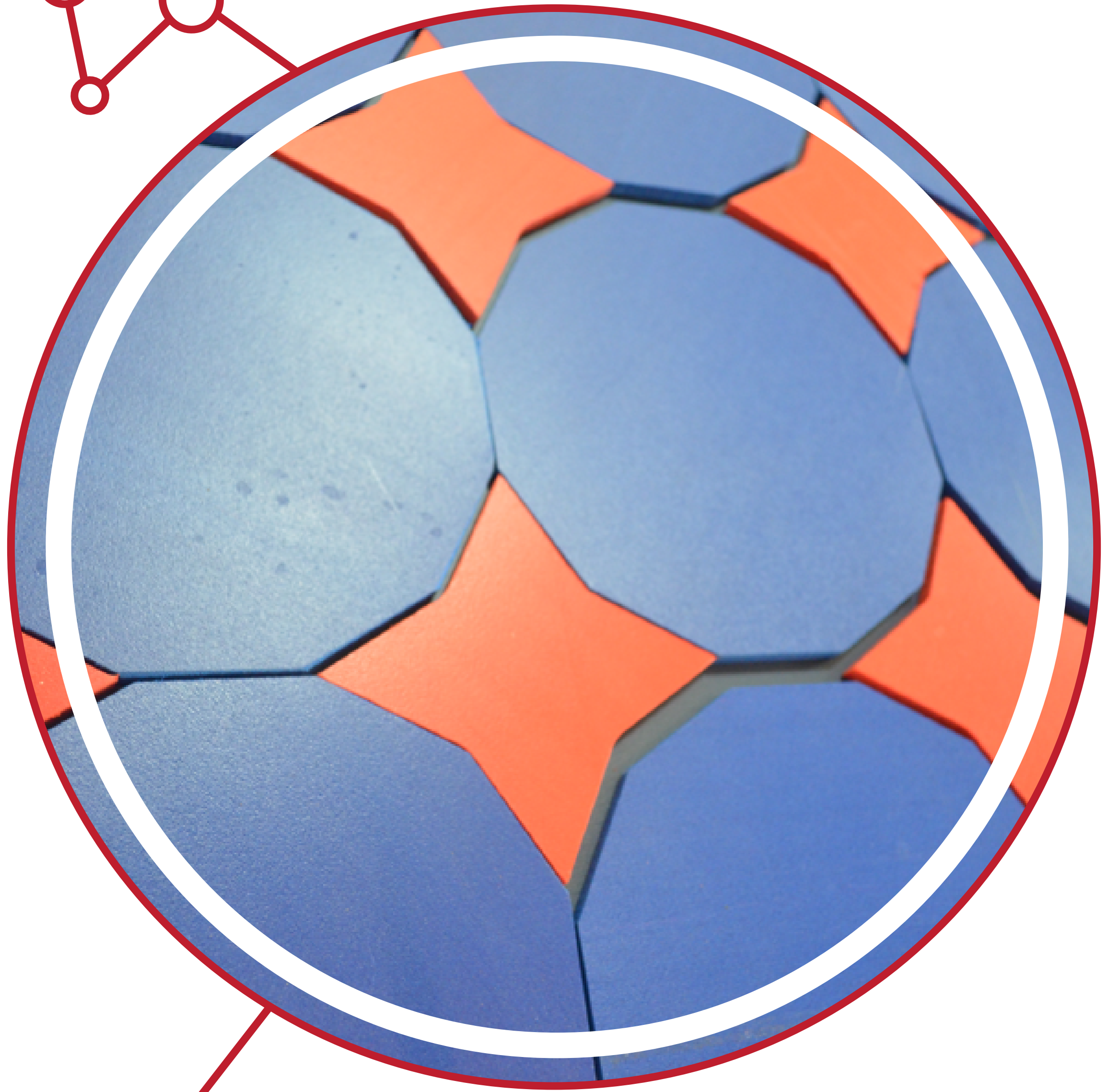
Symmetry abounds in nature. In every glance our eyes notice, almost seek out, symmetry. Nature shows us symmetry in abundance. Whether we look at the shell of a tortoise, a honeycomb (beehive) or even plant tissue through a microscope we find a discernible pattern. A pattern that is repeated over and over. It is this symmetry that is reflected in almost all designs made by humans as well.

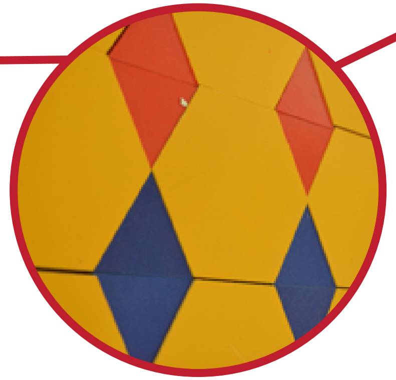
1.6.1 Regular Tessellations



A regular tessellation is a pattern which uses polygons to tile a planar surface. A polygon with n sides is called an $n-gon$. For instance, a triangle is a 3-gon; a square, rectangle or a quadrilateral is a 4-gon; a pentagon is a 5-gon; and so on. Strictly speaking a regular tessellation uses exactly one type of tile, like the patterns usually seen on the floor, or in a honeycomb. The simplest way to tile a surface is to use a regular $n-$gon as a tile. A regular $n-$gon is an $n-$gon where all sides have equal length, like an equilateral triangle or a square. Ever wonder why we don't see pentagons used to tile a surface? As illustrated by the exhibit, when one uses equilateral triangles, squares or regular hexagons then it is possible to construct a tessellation. But it doesn't work with regular pentagons. Here is why.

The interior angle of a regular $n-$gon is given by $\left(\frac{n-2}{n}\right)180^{\circ}$. So for an equilateral triangle the interior angle is 60°, for a square 90°, for a pentagon 108°, and 120° for a hexagon. Suppose we tile the plane with a regular $n-$gon. Look at the corner of one of the tiles. We see some other tiles meeting it at this corner. Imagine standing at such a corner and twirling around. One can complete a full turn. Mathematically speaking, one can sweep an angle 360° at this corner. So if tiles meet at this corner, the interior angle for each tile must add up to 360°. If we try with an equilateral triangle then we see that $6 \times 60 = 360$, thus we can construct a tessella-

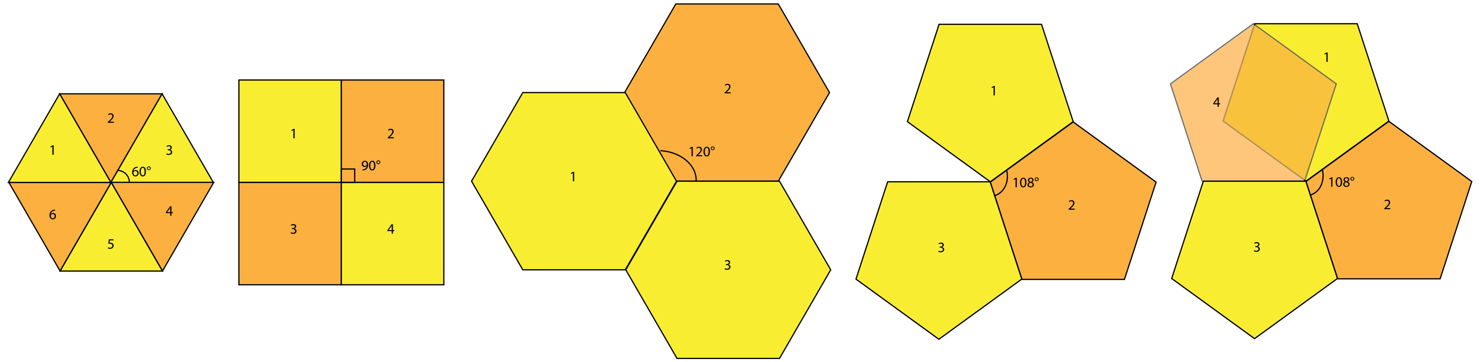




tion where 6 equilateral triangles meet at a corner. Similarly, since 4 × 90 = 360 and 3 × 120 = 360 we see that regular triangles, squares, or hexagons can be used to tile the plane.

Lets try the same with regular pentagons. If we put two pentagons together the angle at a corner is 108º+108º = 216º. If we place another pentagon, the angles add up to 324º. There isn't enough room to add another pentagon. So it is not possible to tile a planar surface using pentagons. Using the formula for the interior angle of a regular $n-$gon given above, one can see that the only regular $n-$gons that can tile the plane are equilateral triangles, squares and regular hexagons. This is illustrated in Figure 2.

We can also work with different tiles of different shapes to construct a tessellation. For example one can use equilateral triangles, squares and regular hexagons to construct a tessellation. Such a tessellation is called an Archimedean tessellation. Say we use r different polygons, $P_1,\ldots,P_r$ to construct a tessellation; where P_i is an n_i-gon. At a corner we see exactly one of P_1, one of P_2 and so on up to one of P_r then once again, the interior angles must all add up to 360º. So we see that;



Six triangles meeting at a corner in a regular tessellation

Four squares at a corner

Three hexagons at a corner

Three pentagons leave a gap, Four of them overlap

Figure 2. Three, four, and six sided polygons can tessellate a plane since their internal angles divide 360º but other polygons such as pentagon cannot.

1.6.1

$$\left(\frac{n_1-2}{n_1}+\frac{n_2-2}{n_2}+\dots+\frac{n_r-2}{n_r}\right)180^\circ = 2\times 180^\circ$$

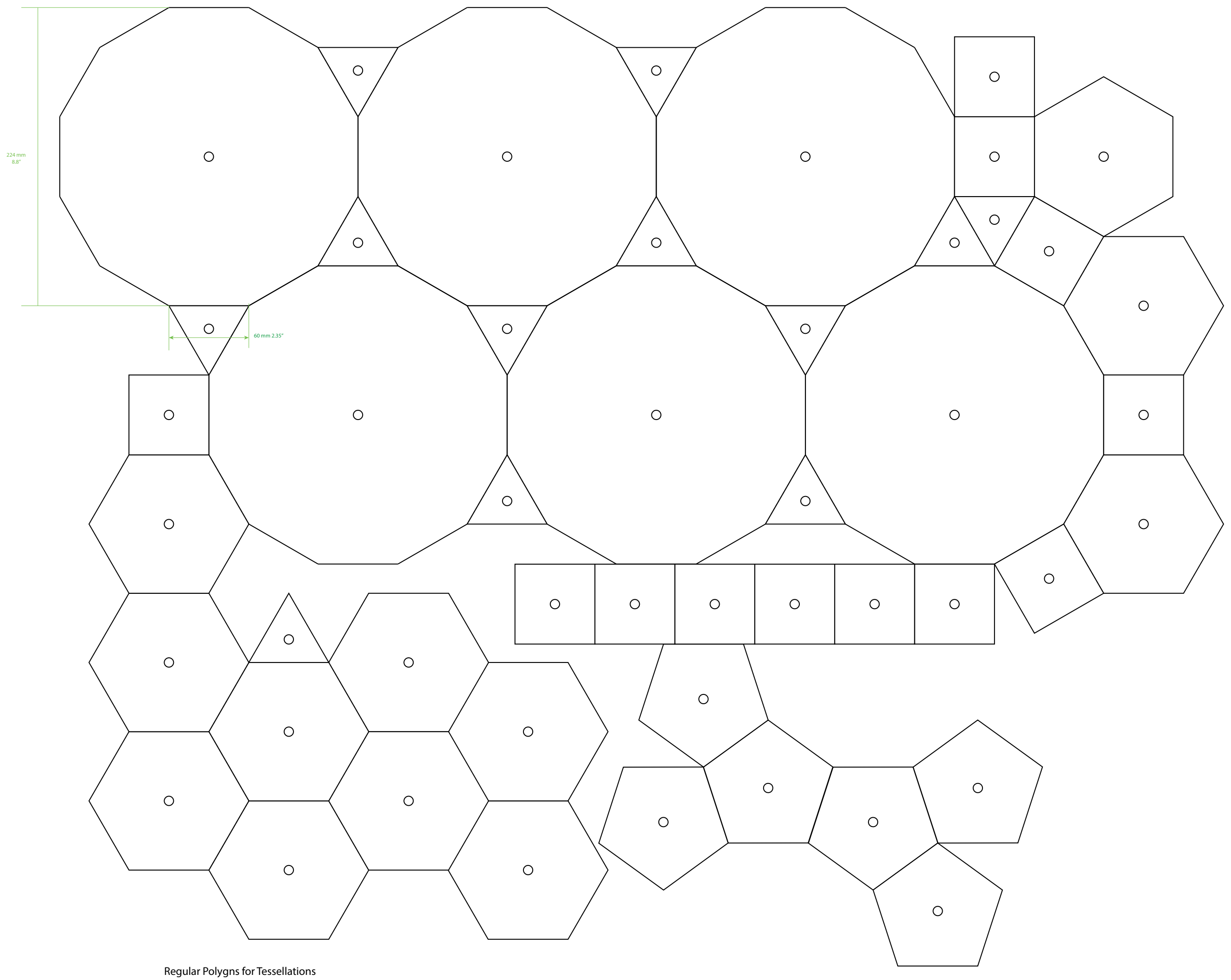
Simplifying the equation we see that $n_1, n_2, \dots, n_r$ must satisy the equation

$$\frac{1}{n_1}+\frac{1}{n_2}+\dots+\frac{1}{n_r}=\frac{r}{2}-1$$

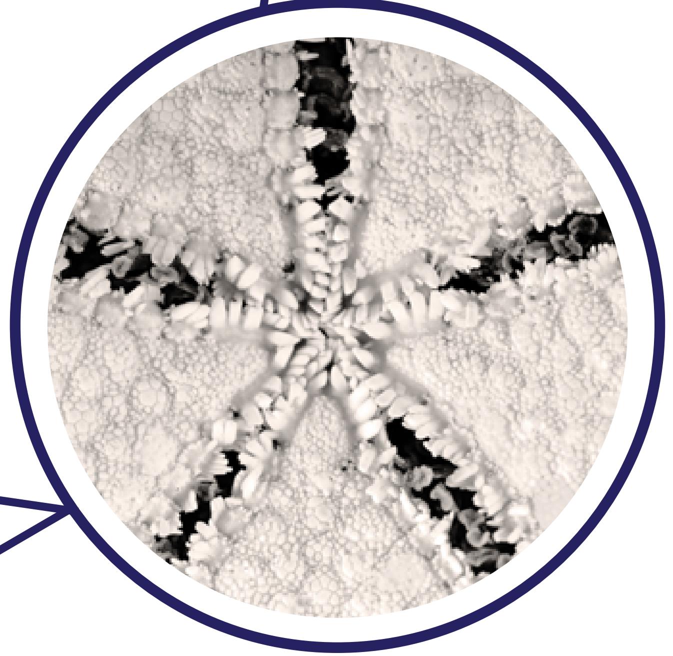
This gives us a nice way to know which patterns might be constructed. For more details we refer to the following blog maintained by John Carlos Baez : http://johncarlosbaez.wordpress.com/2012/02/05/archimedean-tilings-and-egyptian-fractions/

Tessellations come equipped with a natural symmetry. To construct a planar tessellation one starts with a single tile and translates it all over the planar surface. One can also construct different patterns by rotating a tile around a corner, or reflecting a tile about an edge. Regular tessellations are usually constructed in this manner. The Dutch artist M. C. Escher exploited this idea to construct beautiful patterns. Some of which were on display at the exhibition. Such patterns are called *periodic* tessellations. But these are not the only types of tessellations.

Aperiodic tessellations cannot be constructed the way periodic ones are constructed. In the sense that one does not start with a pattern and then translate it across the entire planar surface. Roger Penrose constructed an aperiodic tessellation, which was displayed near the exhibit.



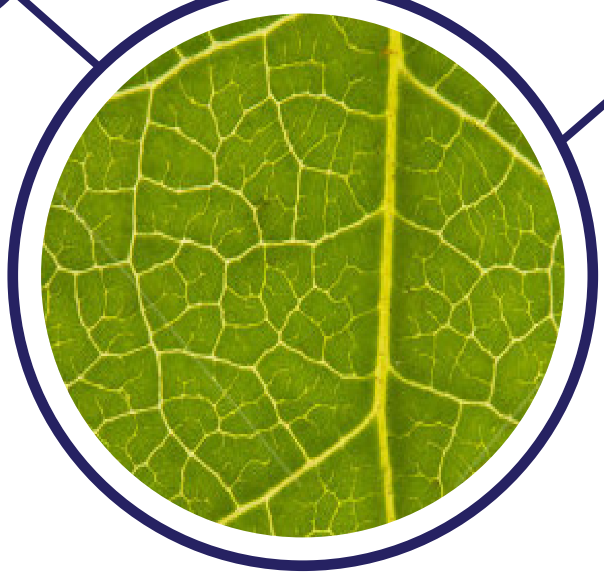
Regular Polygons for Tessellations



2 networks

How do you plan a journey from a small village in Kerala to one in Arunachal Pradesh? How does an email reach from the computer on which it is typed to the one on which it is read? How does your hand recoil from a very hot pot? How does overfishing in a river affect coral reefs in an ocean? All these questions can fruitfully be analysed by understanding the network underlying these systems. The connection, pun intended, between these varied questions and networks may not be obvious at first glance, so let us look at it in detail. The above questions are naturally related to, respectively, railway network that connects distant places together by railway lines and trains running on them; the internet network that connects computers together and moves data over optical fibers and copper wires to deliver emails and webpages; the neurons that are connected in a dazzling network which carries signals back and forth between our organs and the brain, which itself is one of the most complex networks yet known; the eco-systems of which food webs are a part, are also a complex network of interdependence between animals, plants, bacteria, and all living beings, and their interactions with the environment in which they live.

Simply put, the most basic description of a network is in terms of a collection of nodes (places, computers, organs and brain, species, in the examples above) joined by links called edges (railway lines, optical and copper fibers, neurons, predator-prey links, respectively, in those



examples).

What is the math? One of the simplest ways to represent a network is by a matrix each row represents a node and so does each column. When one node (station) is connected to another node (station) by an edge (railway line), we put the number 1 in the corresponding row and column location. Such a matrix is called the “adjacency matrix”. Combining such a description in terms of a matrix with the powerful mathematical ideas from linear algebra and other fields helps us answer many questions about networks, connectivity. The whole field of graph and network theory is devoted to such study and is now an indispensable tool in many subjects such as biology, economics, computer science, and many others.

Can you try to write down the adjacency matrix of the different localities in Bangalore or your own city or nearby villages with edges indicating whether there is a direct bus service from one locality or village to another? Can you draw such a network on a map? What do you learn about the connectivity?

The above description has hopefully made it amply clear that networks occur everywhere literally! Just like waves , networks are ubiquitous: a family tree is a vast network of humans, especially if you include both paternal and maternal links; the chemical reactions linking our genes and proteins inside our cells forms a network with different chemicals as nodes and the chemical reactions forming the links; the mobile phones connecting so many of us together.

2.1

Adjacency Matrix

– the network of exhibits

We can even consider the various exhibits in this book as nodes of a network. One way to build a network from these nodes is the following: if the mathematical concepts behind two exhibits are closely related, then these two nodes can be connected by an edge. For example, the standing waves on the spring are mathematically very closely related to the oscillations of a pendulum, so these two nodes in the "network of exhibits" are connected by an edge, but there is no immediate mathematical connection between these standing waves and the anamorphic maps, so they are not connected by an edge. The matrix representation is obtained by putting the number 1 when an edge (or a connection) exists between the two concepts and the number 0 when it does not. Such a matrix is shown in the left side of the figure 3. Empty entries indicate a 0.

This can now be used in many ways. For example, by using the powers of this matrix (the multiplication of this matrix with itself), we can obtain the minimum number of edges that are required to go from one node (i.e. mathematical concept) to another. This is shown in the right side of the figure 3.

One thing that is clear from this network is that mathematics has a great unifying power, because in many cases, the mathematical description of seemingly unrelated topics brings out a natural and close relation between them. For example, the fractal coastlines of a country may seem far removed from the sine wave exhibit, but from the network of exhibits, we see that fractals are closely related to chaotic pendulum, which in turn is a form of oscillation, which in turn is related to the sine wave, so there is a "degree 3" connection between fractals and sine waves! This is shown by the number 3 in the 15^{th} row (fractal maps) of the 1^{st} column (sine waves).

This phenomenon is not entirely surprising. Take the example of finding a relation between you and a randomly chosen person in a village in Australia. If you write down a list of all your friends and relatives on one line, then the list of their relatives and friends on the second line and so on and so forth, and if the other person in Australia starts making such a list, the surprising aspects of such lists seems to be that there will be common names appearing in both the list, usually within the first 3-6 lines! So you are related to everyone else on the earth with only about 6-7 links!

20	1			1															1	
19						1											1			1
18	1				1															
17						1	1												1	
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3	1	1		1								1								
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1		1	1	1		1	1											1		1
Code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Exhibit Name	Sine Wave	Soliton Wave	Chladni + Wave	Oscillations	Chaotic Pendulum	SARS Networks	Adjacency Matrix	Knapsack	Minimal Surfaces	Tessellation tables	3D Escher	Tensegrity Sculpture	Collapsible stool & Tensegrity Sculpture	Anamorphic maps	Fractal maps	Kakeya needle	Permeability	Ecosystem	Weigh the Odds	Walk the function

20	1	2	2	1	2	2	2	5	5	5	6	3	4	4	3	5	2	2	1	0
19	2	3	3	2	2	1	2	6	5	5	6	4	5	4	3	5	1	3	0	1
18	1	2	2	2	1	2	2	5	4	4	5	3	4	3	2	4	3	0	3	2
17	2	3	3	2	2	1	1	6	5	5	6	4	5	4	3	5	0	3	1	2
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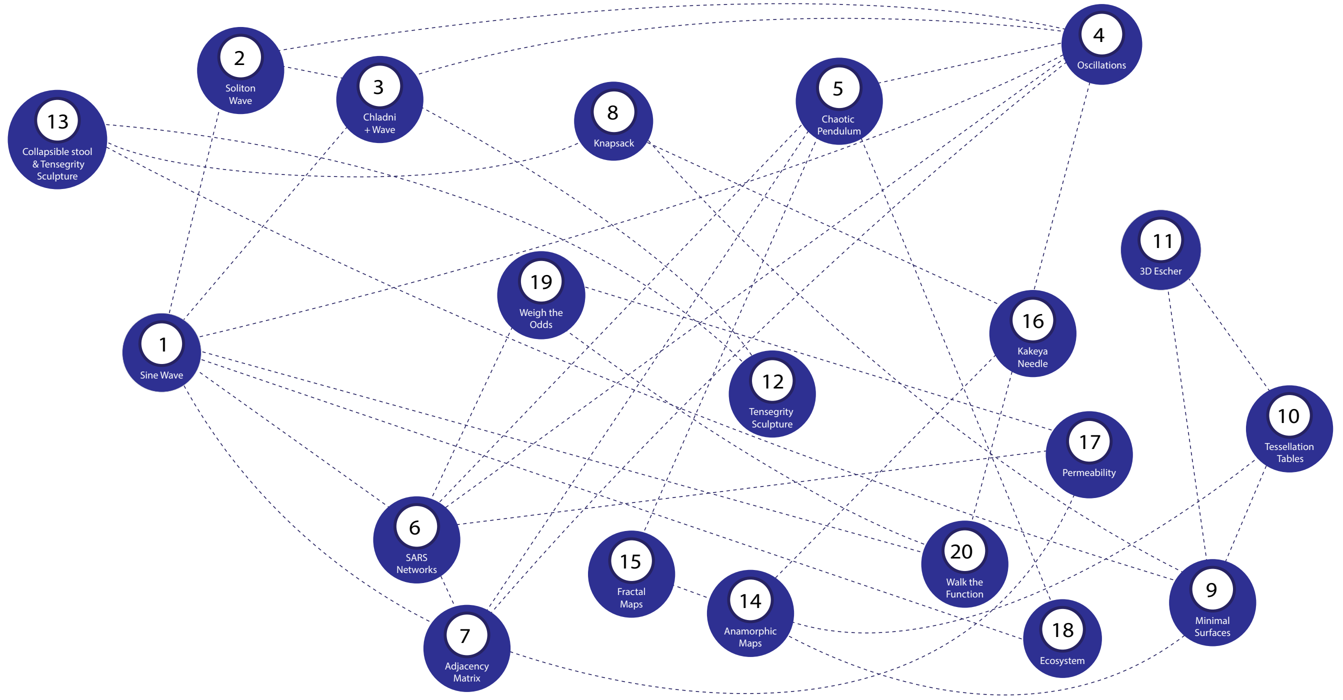
1	0	1	1	1	2	1	1	4	4	5	5	2	3	4	3	5	2	1	2	1
Code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Exhibit Name	Sine Wave	Soliton Wave	Chladni + Wave	Oscillations	Chaotic Pendulum	SARS Networks	Adjacency Matrix	Knapsack	Minimal Surfaces	Tessellation tables	3D Escher	Tensegrity Sculpture	Collapsible stool & Tensegrity Sculpture	Anamorphic maps	Fractal maps	Kakeya needle	Permeability	Ecosystem	Weigh the Odds	Walk the function

Figure 3. The adjacency matrix of the network of exhibits.



2.2

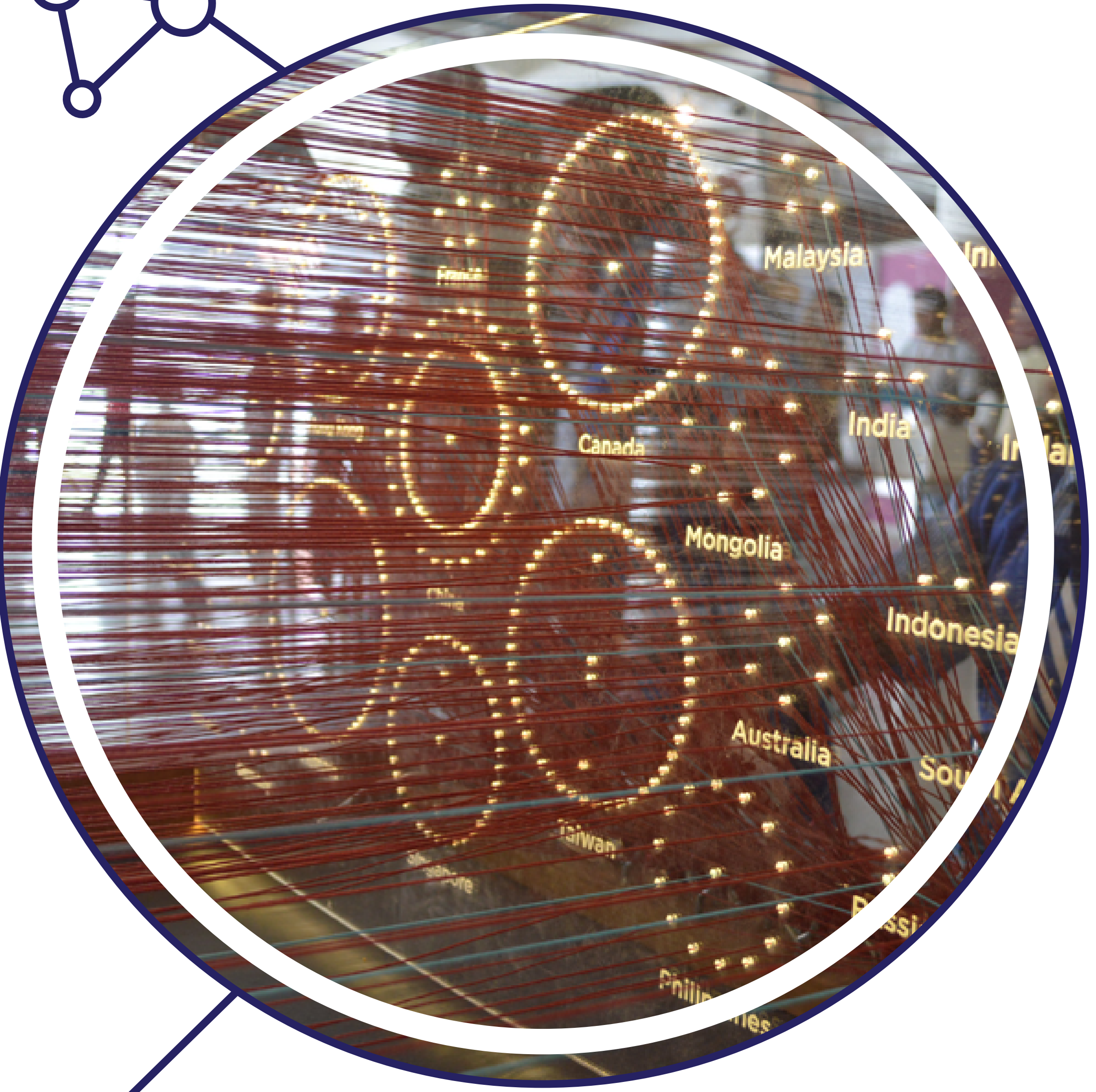
SARS Network

Have you ever wondered whether there is a mathematical description of the way diseases spread in a population, either locally such as in the case of a dengue epidemic in a city such as Singapore or Delhi, or globally such as in the case of the SARS epidemic around the world? In fact there is not just one, but many different such descriptions. One of them is in terms of networks. This could be in terms of places through which the disease spreads (as in the case of the actual exhibit described below); Or it could be in terms of the network of individuals who get infected or may infect others (some times through a "carrier" such as a mosquito in the case of dengue).

Severe acute respiratory syndrome (SARS) is a viral respiratory disease caused by the SARS coronavirus. An outbreak of SARS in Southern China caused an eventual 8,273 cases and 775 deaths in multiple countries between November 2002 and May 2003. Within weeks, SARS spread from Hong Kong to infect individuals in 28 countries in early 2003.

The installation contains a total of nine panels representing the duration across which the epidemic spread. Along with displaying the network of the spread of the epidemic, it also shows the exponential nature of its growth. What we see is the spatio-temporal spread (in space across the globe, and in time across just a few months) of the disease.

In the installation, there are 8 panels each showing the country-wise number of SARS cases during 8 months from November 2012 to June 2013. The strings cumulatively

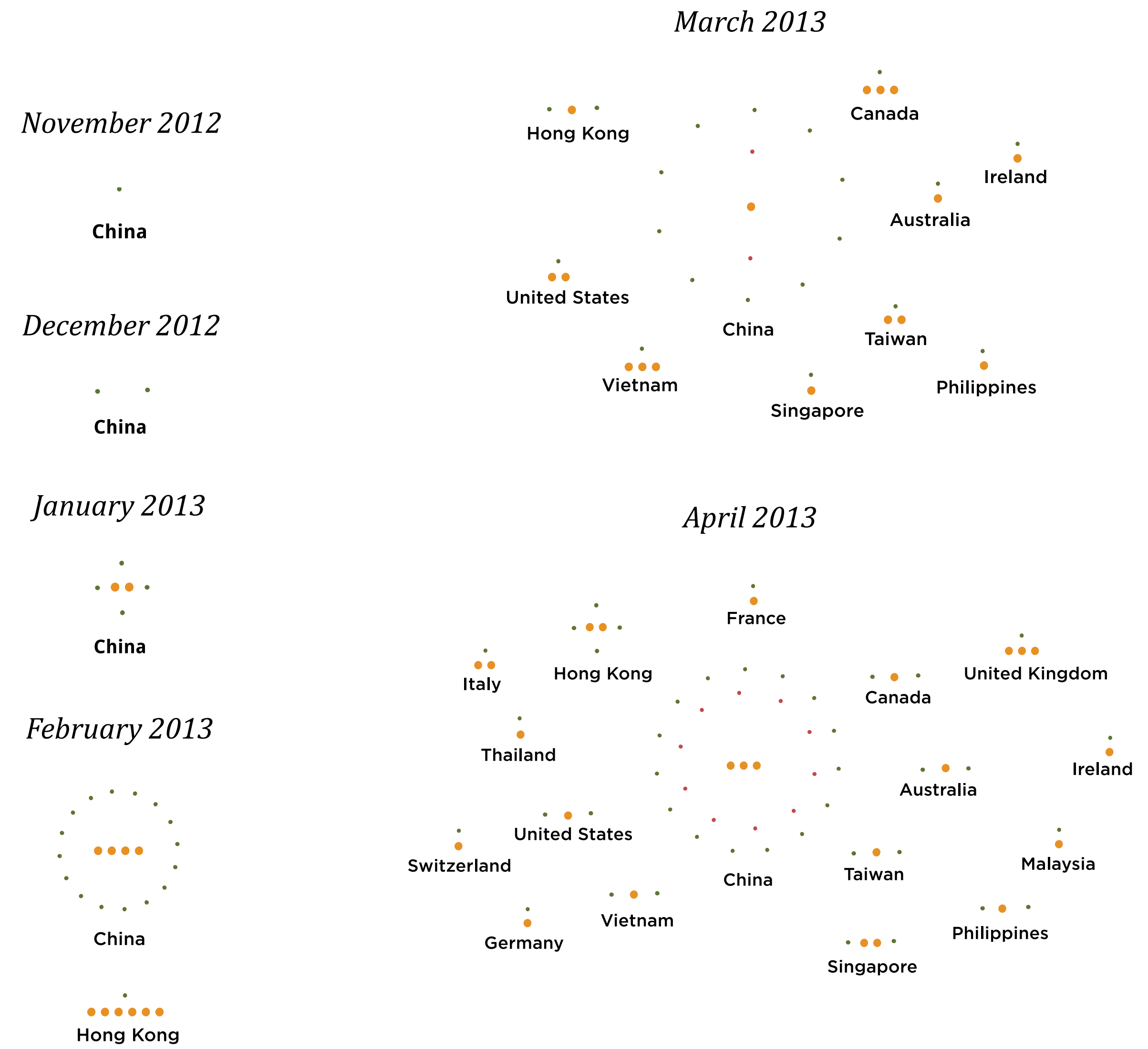


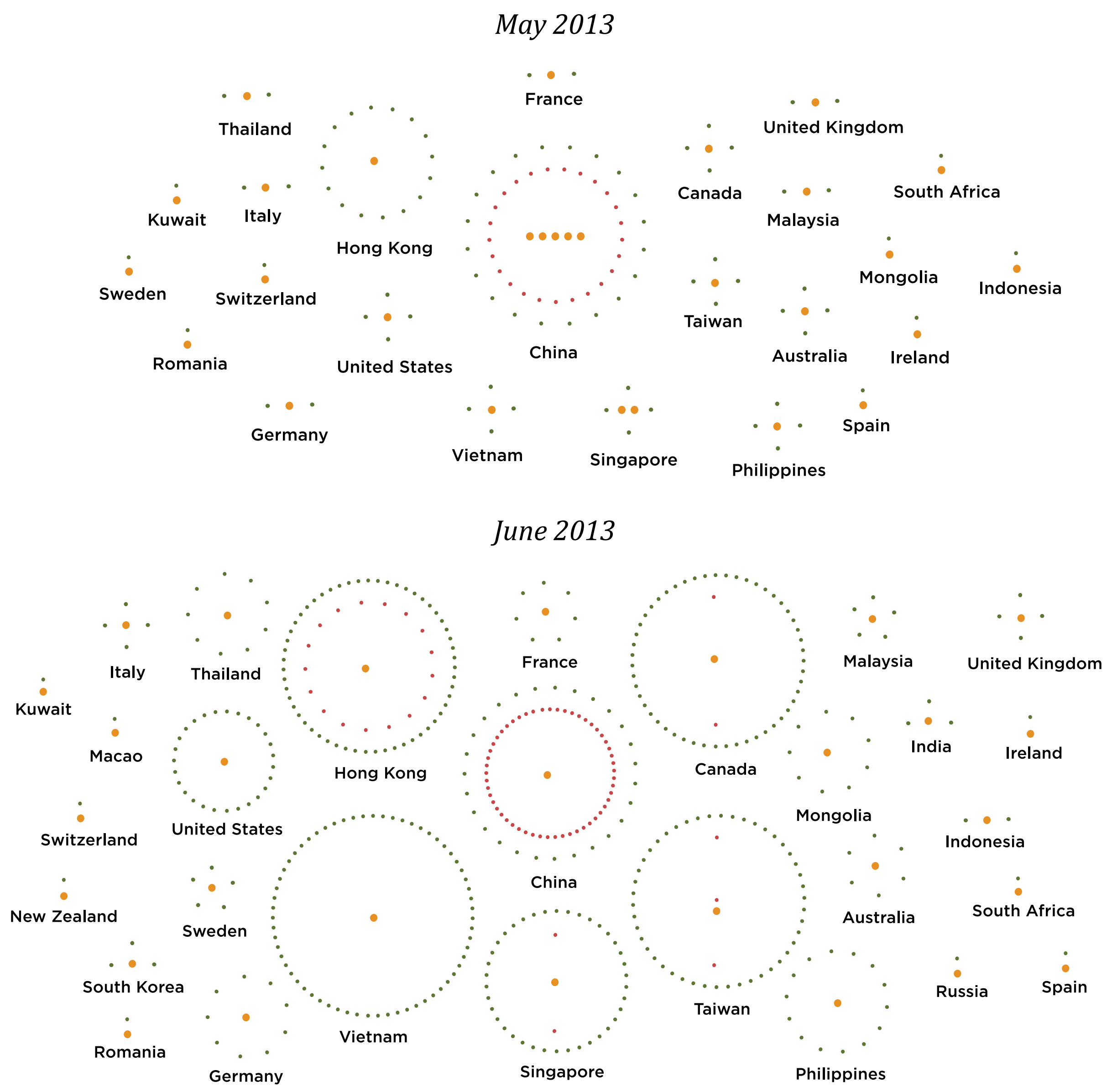
move towards the last panel in concentric circles. The central string represents the probable source from where the virus was imported into the country. The inner circle of strings signifies the number of cases in the country x 100 and the outer circle signifies the number of cases in the country x 1.

All the statistical data used to build this installation has been derived from the Global Alert and Response (GAR) post, dated 31st December 2003 on the official World Health Organization (WHO) website. See http://www.who.int/csr/sars/country/table2004_04_21/en/.

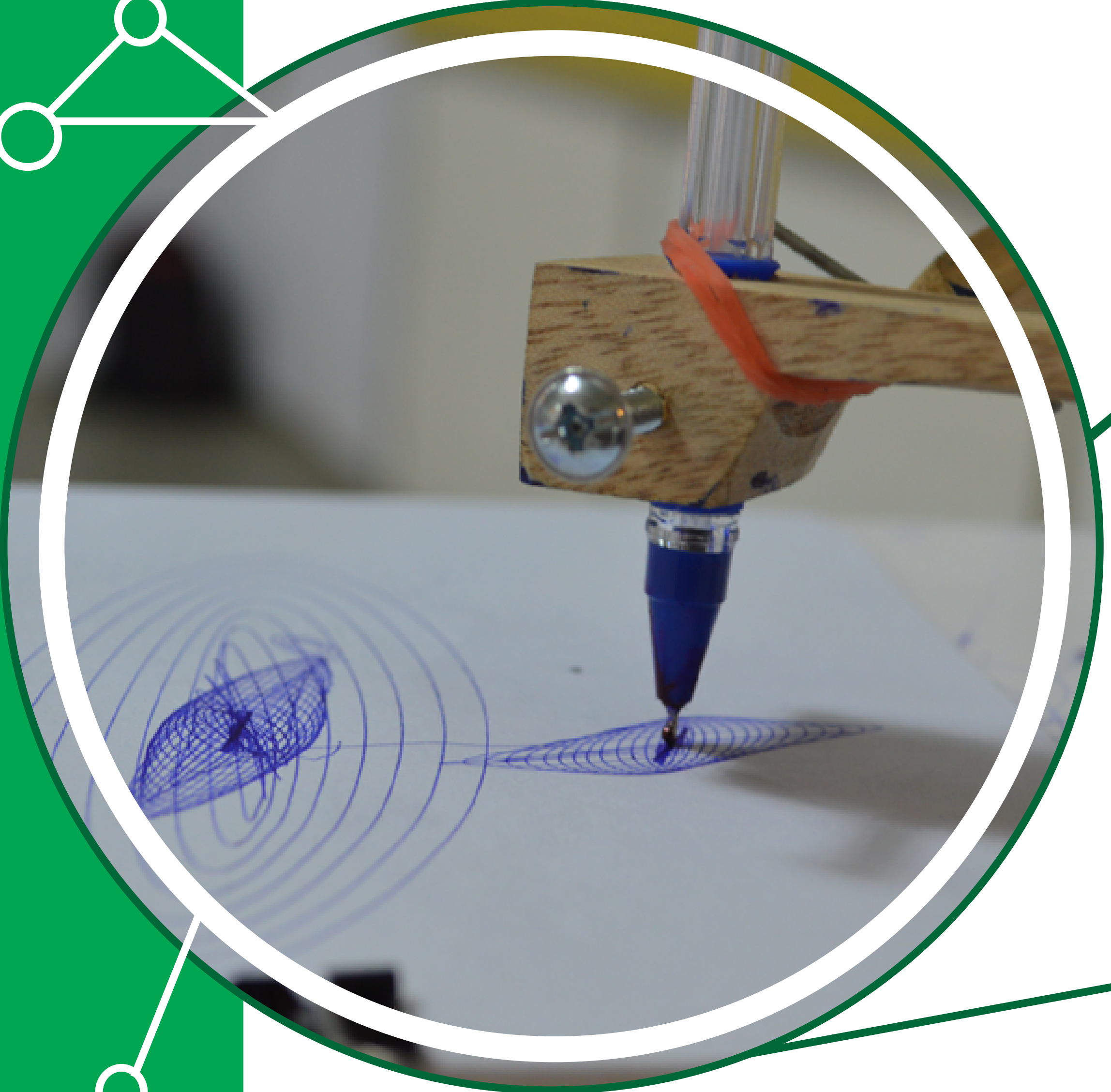
Country	Cases	Country	Cases
Australia	6	Philippines	14
Canada	251	Romania	1
China	5327	Russia	1
France	7	Singapore	238
Germany	9	South Africa	1
Hong Kong	1755	South Korea	3
India	3	Spain	1
Ireland	1	Sweden	5
Indonesia	2	Switzerland	1
Italy	4	Taiwan	346
Kuwait	1	Thailand	9
Macao	1	United Kingdom	4
Malaysia	5	United States	27
Mongolia	9	Vietnam	63
New Zealand	1		

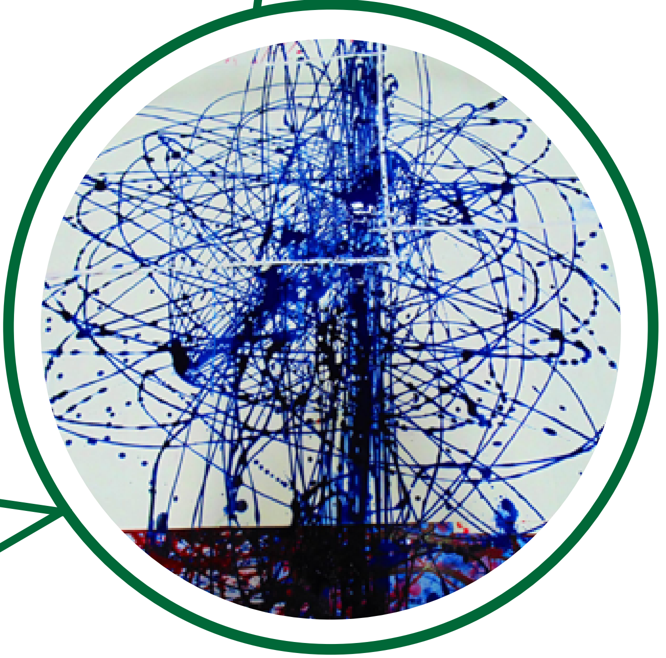
Table 1. Summary of probable SARS cases from 15th November 2002 to 15th May 2003





The drawing above shows the eight panels for eight months from November-2012 to June-2013, each one showing the country-wise number of SARS cases.





3 oscillations

Regular patterns like those formed when you swing in the park are good examples of oscillations. The main characteristic of oscillations is that one pattern repeats in time, with identical events separated by equally spaced time periods. A canonical example is of the oscillations of a pendulum in the good old wall clocks. These oscillations are exactly like those created by the harmonograph, one of the exhibits described below. But there are other, more complicated oscillations as well - for example, when two pendulums are attached to one another they may create patterns that are not repetitive in time. Such *chaotic* oscillations are illustrated in another exhibit described later.

Where do such system occur? The pendulum in a clock, or the motion of a cricket ball, or of a car engine, or even the earth going around the sun are examples of regular, oscillatory motion.

All these systems are "non-chaotic" in the sense that small changes in the input cause only small changes in the output; If two harmonographs are started with almost the same initial position, they will create almost identical patterns. Thus, even though the pattern may look complicated, it is relatively easy to create the same pattern again and again. We will later see examples of systems which do exactly the opposite: they are called chaotic.

Where is the mathematics? Oscillations are described by their amplitude – how far away from the "centre" or the "sta-

tionary" point does the oscillation go, by a frequency – how many times in a second does it go back and forth, and by a phase – where the oscillation begins. The simplest of oscillations is given by either of the following two equivalent equations:

(1) $\omega(t) = A\sin(2\pi ft + p)$ or $\frac{d^2\omega(t)}{dt^2} + (2\pi f)^2\omega(t) = 0.$

These equations are equivalent in the sense that the solution of the ordinary differential equation on the right is the equation on the left. Here A is the amplitude, f is the frequency, and p is the phase. We will later see a deceptively similar looking equation for a wave.

The power and beauty of mathematics can be illustrated by the following fact: any oscillation can be described by simply a sum of many different oscillations with different values of the amplitude A and the frequencies f. This is a very powerful and useful result that is at the basis of a lot of mathematical ideas as well as their applications, studied under the umbrella of Fourier analysis, named after the French mathematician-physicist Joseph Fourier, who initiated such investigations.

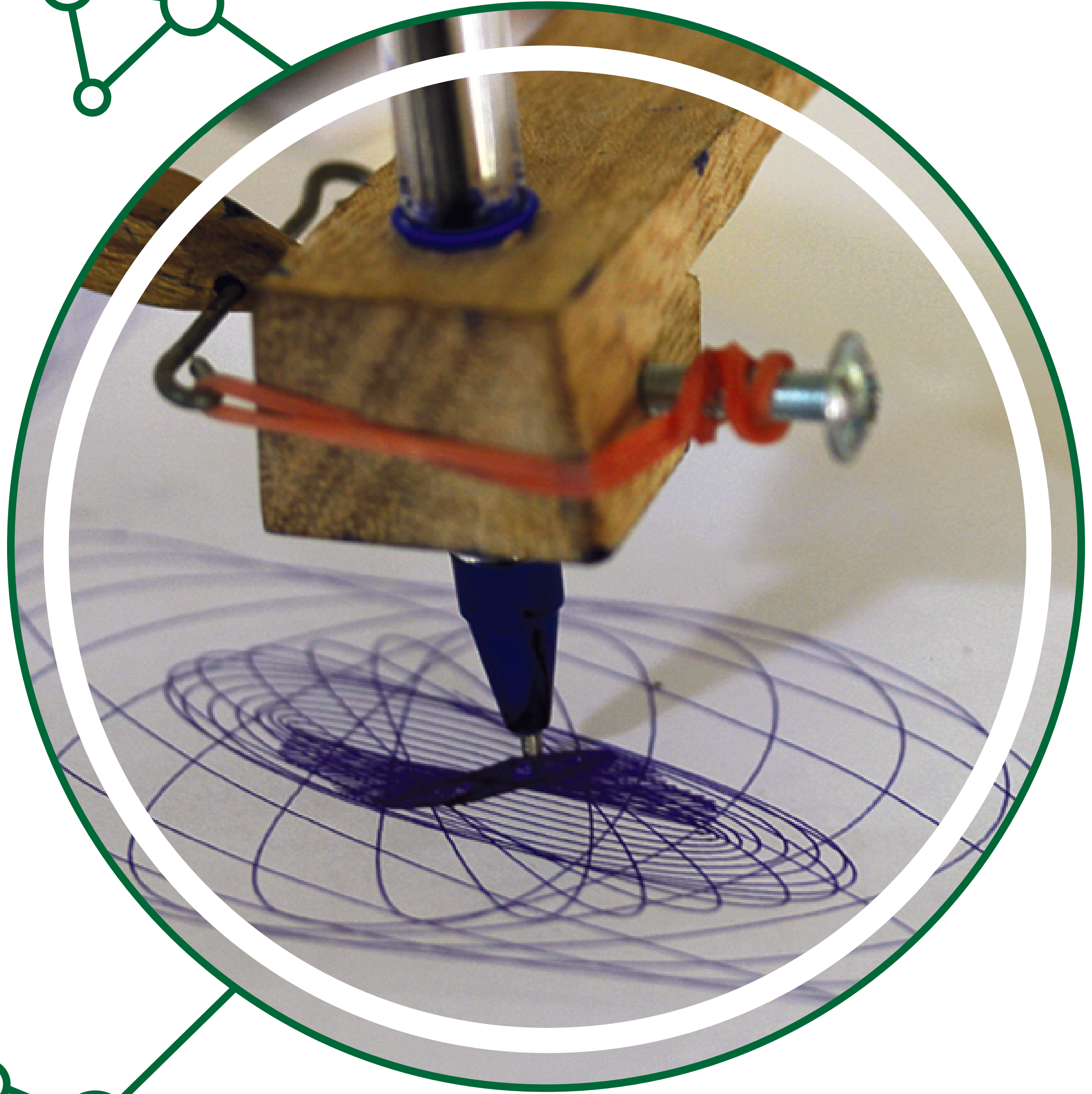
Let us now look at some of the exhibits related to oscillations.

3.1

Harmonograph and Lissajous' figures

Harmonograph is essentially a compound pendulum with multiple separate pendulums operating at right angles to each other. The combined oscillations of these pendulums give rise to a pattern called Lissajous' figures. When the different weights hanging from the harmonograph are set swinging at the same time, the observer sees superposition (that is, the sum) of the oscillations of these pendulums. This superposition can give rise to the complex patterns that are drawn on the paper. Different phases and amplitudes of the pendulums give rise to an endless variety of patterns. The visitor starts the pendulums swinging and controls their relative phase.

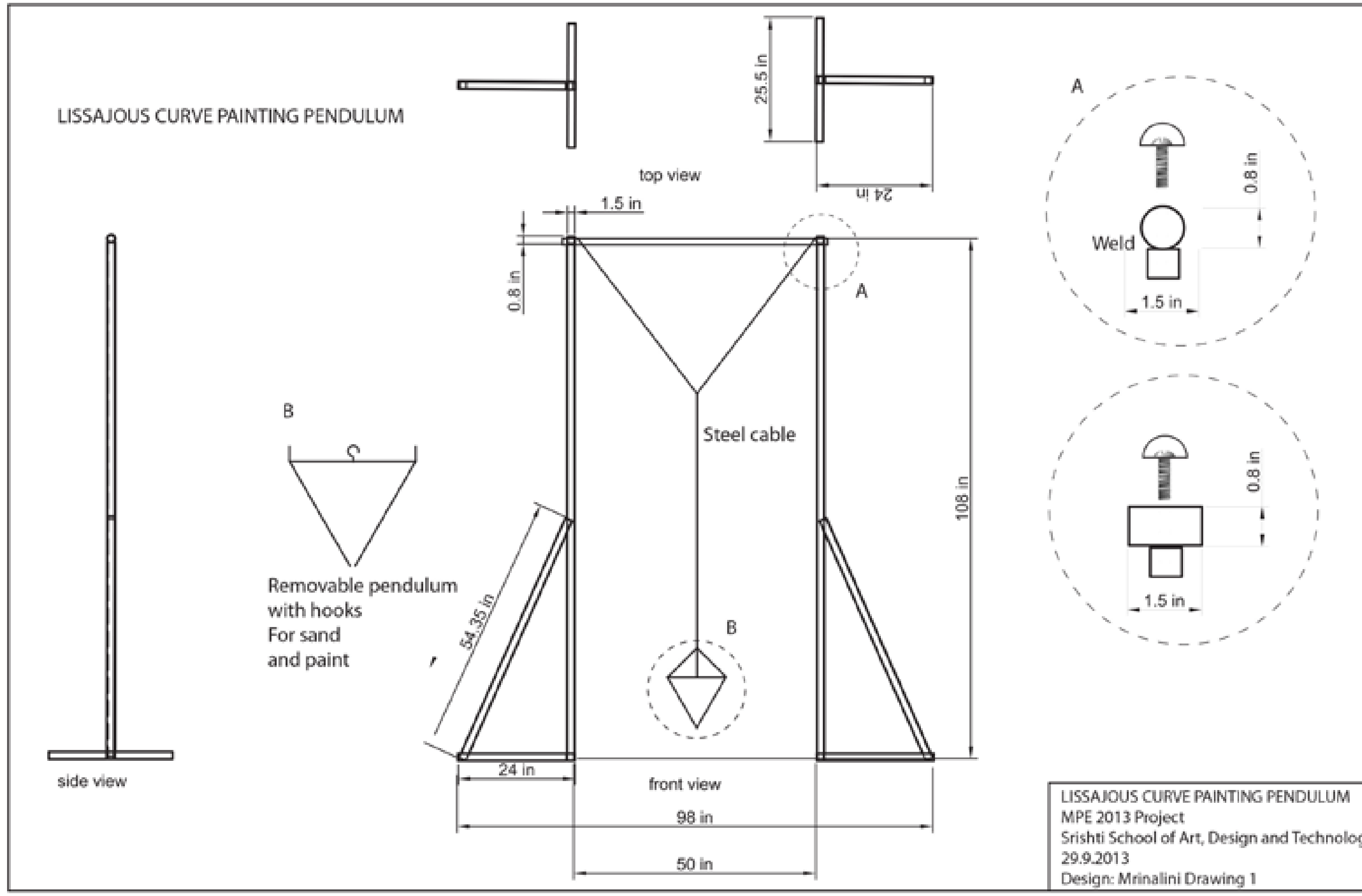
The exhibit on Lissajous' figures is similar to the harmonograph. The two different oscillations at right angles to each other are achieved by a special arrangement of the "Y" shaped yoke from which the pendulum hangs. When the pendulum is swung, the special pivot enables the same pendulum to operate as two pendulums with different lengths at right angles. As a result, the painting pendulum traces patterns known as Lissajous' figures.



(2) $x(t) = A\sin(2\pi f_x t + p_x)$ and $y(t) = A_y \sin(2\pi f_y t + p_y)$

Just out of these two simple equations, we can produce all the patterns that you see on the paper below the oscillating pendulum. The equations for the patterns on the harmonograph are similar, but consist of more than one sine term in each of the above equations.[1]

To understand the mathematics behind this, let us call the positions of the pendulum in two perpendicular directions to be $x(t)$ and $y(t)$ - e.g. it could be the distance from the two adjacent sides of the room. Each oscillation has the same form as above:



[1]Some references that discuss this in detail are:
i) http://www.karlsims.com/harmonograph/ ii) http://www.wikihow.com/Make-a-Three-Pendulum-Rotary-Harmonograph

3.2

Chaotic Pendulum

A pendulum is just a swinging object, for example, a swing. In this exhibit, the swinging object itself has swinging parts - called a double or triple pendulum. Such pendulums can have “chaotic” - not random - motions. You will notice that small changes in the initial position of the two identical pendulums causes a very large variation in their path.

You would think that chaos is a phenomenon observed at any busy intersection in any city in India! No – not quite – that is just disorder and confusion. Chaotic systems have the essential characteristic that very small changes at one time lead to very large changes at later times. That’s why they are hard to predict – we need very accurate knowledge of the system to predict it without a large uncertainty. A prime example is the weather: we cannot really predict the weather even a few days in advance.

Where do such chaotic systems occur? There are many examples: changes in weather (temperature, rainfall), motions of some of the comets, the pattern of heartbeats, the economic activity, the movement of a stream flowing down a mountain, and many others. Contrast this with the regular motion of a fan, or of the harmonograph or the Lissajous pendulum (which are all non-chaotic), or changes in the stock market (which are essentially random).

What is the mathematics? Nonlinearity is an absolutely essential feature of all chaotic systems – doubling the input does not double the output, tripling the input does not triple the output. Of course not all nonlinear systems are chaotic, e.g. motion of a single pendulum, or of the parts of a car is also described by nonlinear equations, but they are regular, not chaotic.

4 optimization

Optimization simply refers to the process of finding a maximum or minimum of certain quantity. It is a process that goes on in many (arguably, all) natural and man made systems - all fundamental laws and theories of physics are based on optimization. It is also the driving force behind evolution of biological systems. Most designs attempt optimization of energy or time or material used or some other appropriate quantity or combination of these. Mathematically, optimization could be carried out by simply enumerating all possibilities and finding one that is “best” (in some appropriate definition of what is “better”) or by using calculus as some of you may know or will learn in your classes. The exhibits below describe two specific classes of optimization problems that are quite ubiquitous.

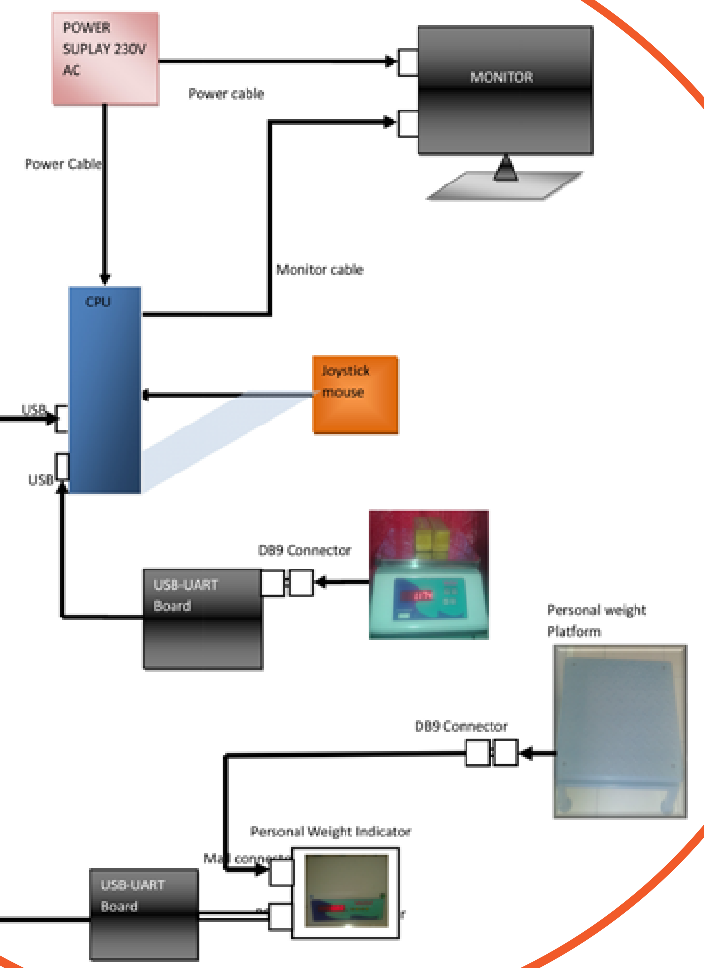
4.1 Knapsack Problem

The original inspiration for the knapsack problem is a puzzle that we are all familiar with. It is one that a vegetable vendor has to solve every day: the vendor goes to the wholesale market to pick up vegetables to sell on a particular day. Of course she cannot carry more than a certain weight, and the total volume cannot exceed the volume of the basket. There are several, at least 10-20, different vegetables she may choose. She knows the profit she can make per kilo of each of the vegetables, and the volume per kilo for each of these as well. Given all this data, she has to choose vegetables that will fit in her basket and are within the weight she can carry, but at the same time will give her maximum profit. This is indeed a complex mathematical problem, but most vendors find a solution for such a problem every day!

More generally, we are all familiar with situations where people have to manage with limited resources. So these problems are of great importance in all walks of life: a homemaker deciding to buy food and groceries for her family for the entire month; the amount of time available to a student to answer a set of questions; scheduling the available manpower to get a job done. In each of these situations, the decision about the best possible manner in which to achieve a stated objective has to be arrived at, while carefully considering the constraints and conditions to be satisfied.

This kind of problem is known as "constrained optimization": the limits on weight and volume are the "constraints" that cannot be exceeded, whereas "optimization" refers to the aim of trying to maximize the profit. "Knapsack problem" is a colloquial

Connection diagram of a knapsack system



name for a problem that a thief may face, very similar to that of the vegetable vendor.

A slightly simpler version of this problem is illustrated in this exhibition.

In the exhibit on display, there are several bars each of which has a specific weight and is assigned a value. The visitor stands on the scale and the “constraint” is that he or she cannot choose more than a fifth of his or her body weight. She gets a specified amount of time, say 30 seconds, to choose the weights so as to remain within this weight limit but at the same time maximizing the total value.

Where is the mathematics? Actually there is a whole mathematical field devoted to study these types of problems. The simplest form is as follows. Suppose there are N objects which we will simply number $k = 1, 2, 3, \ldots, N$. Let the k-th object be of weight ω_k with value v_k. Suppose the limit on the maximum weight is W. So we have to choose some of these objects, say p of them $k_1, k_2, \ldots, k_p$, such that

$\omega_{k_1} + \omega_{k_2} + \ldots \omega_{k_p} \leq W$ such that $v_{k_1} + v_{k_2} + \ldots v_{k_p}$ is maximum.

What this really means is that if we had chosen any other set of $\overline{p}$ objects $\overline{k_1}, \overline{k_2}, \ldots \overline{k_{\overline{p}}}$ with a total weight less than W, then their total value would be less than the set of objects $k_1, k_2, \ldots, k_p$:

$v_{\overline{k_1}} + v_{\overline{k_2}} + \ldots v_{\overline{k_p}} < v_{k_1} + v_{k_2} + \ldots v_{k_p}$.

The set of weights and values that were actually used in this game are recorded in Fig. 4.

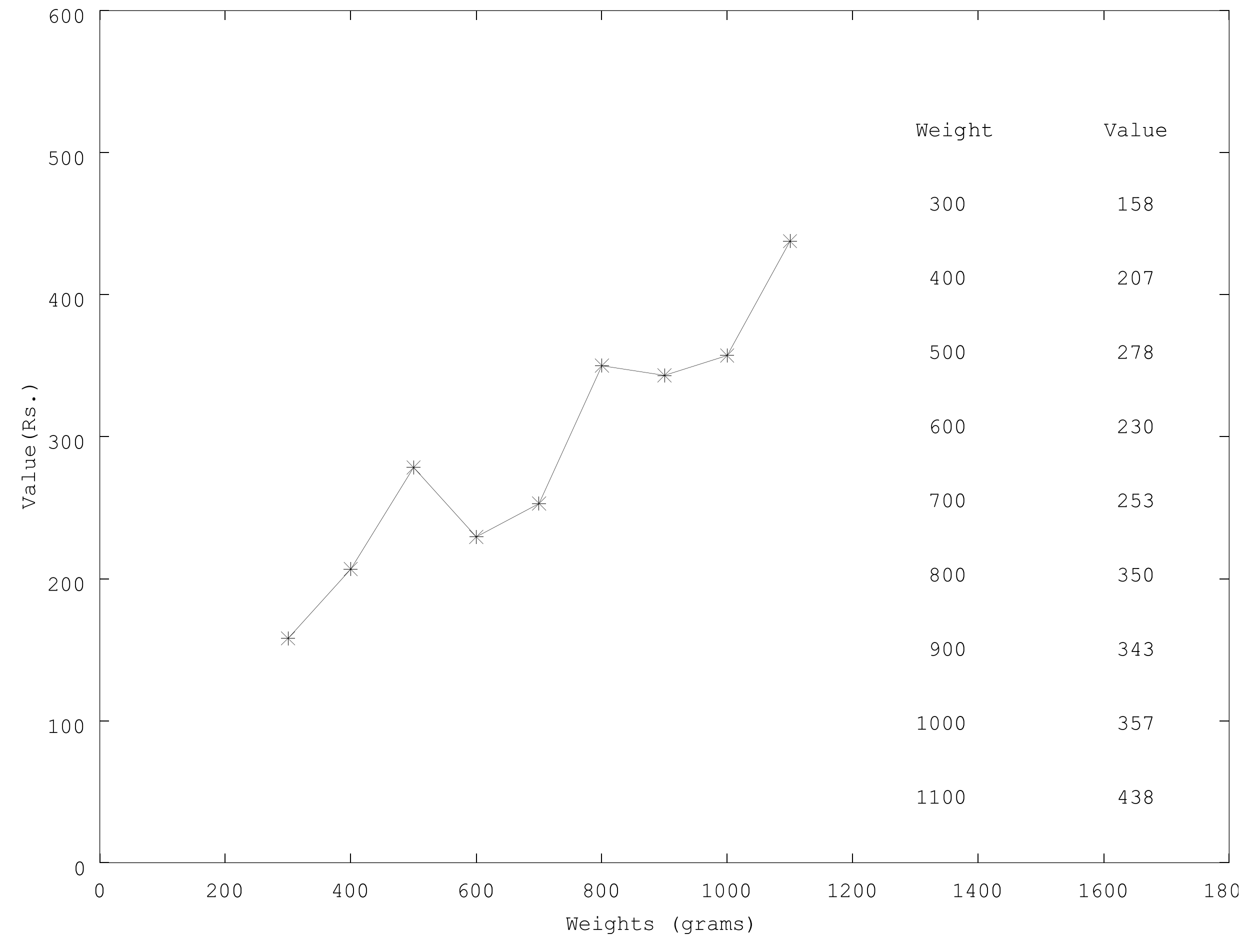


Figure 4. The weights and values assigned to the blocks in the knapsack exhibit

4.2

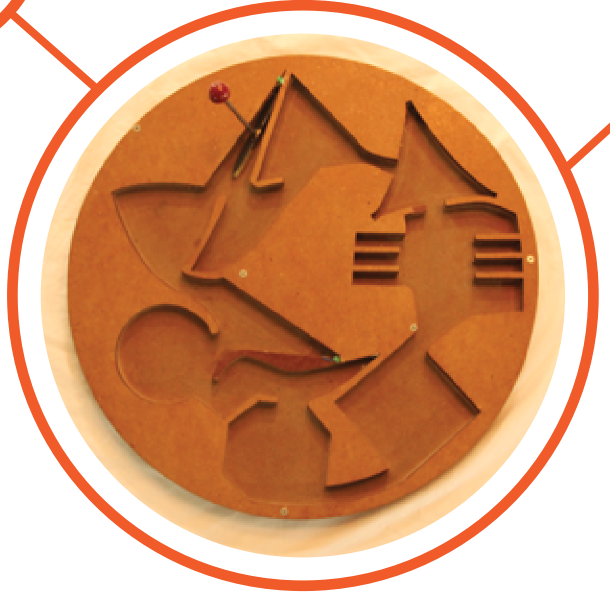
Kakeya Needle Problem

Imagine having to move a ladder, or a large stick, from one room to another or suppose one is trying to park a car in a tight spot. Most often in such a situation one has to navigate through a rather cramped space. Both tasks require a great deal of ingenuity and dexterity.

In the year 1917, a Japanese mathematician S. Kakeya considered the problem of finding the area of the smallest convex set in the plane inside which a needle of unit length can be reversed, i.e., turned around through 180º. (A "convex set" is nothing but an area which has no "spikes" e.g. a circle or a triangle is convex whereas a star in *not* convex.)

For example, one easy possibility for a convex set inside which a needle of unit length can be rotated through 180º is a semi-circle of radius 1 whose area is $\pi/2$. To do better than this, consider a circle of diameter 1; placing the needle along a diameter and rotating it through 360º around the centre gives us a set whose area is $\pi/4 = 0.78$. To do even better than this, consider an equilateral triangle ABC of height 1 and suppose that the needle lies on the side AB with its tip at vertex B; then, rotating the needle around the tip by 60º, sliding it backwards on the side BC, rotating around C by 60º, sliding it forward along side AC and finally rotating it through 60º around the vertex A and sliding it backwards along AB one can see that the needle has been reversed inside the triangle ABC. We have done better because the area of this triangle is $1/\sqrt{3} = 0.58$, which is smaller than the area of the circle of diameter equal to 1.

Indeed, Kakeya conjectured that an equilateral triangle of unit height is the smallest such set; he also observed that if the con-



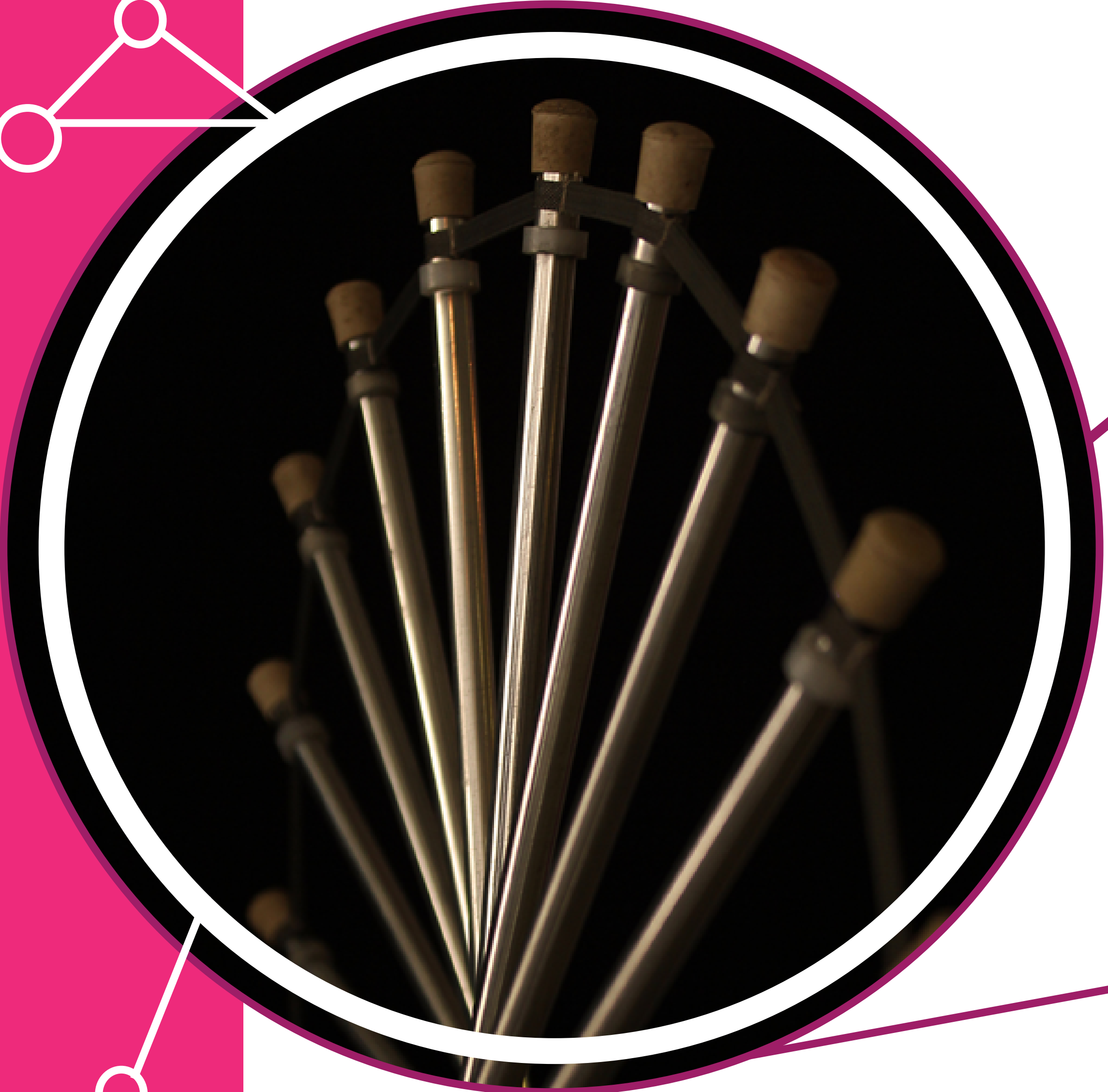
vexity condition is dropped, that is, if we allow for shapes such as stars or octopus, with "spikes," then a smaller set is possible. The more interesting question of finding the smallest set (without the convexity condition) in which to turn the needle came to be known as the *Kakeya needle problem*.

Let us first exhibit a non-convex set, of area smaller than that of an equilateral triangle of unit height, inside which one can reverse a unit needle. This can be achieved inside a three-cornered hypocycloid ABC, one of the shapes in the exhibit tray. It is possible to reverse the needle inside this region in a manner similar to the procedure followed in the case of the equilateral triangle of unit height. Here, start by placing a needle inside the hypocycloid with its tip at B. Then slide it tangentially along the curve BC until the other end of the needle is now tangential at C. Next, follow this procedure along the curve AC until the needle sits with its tip tangential at A. Finally, repeat the same procedure once again so that the other end of the needle sits tangentially at B thereby moving back to its original position with its ends switching positions as desired. The area of this hypocycloid ABC swept by the needle turns out to be equal to $\pi/8 = 0.39$, which is smaller than the area of the triangle ABC discussed above.

The conjecture with the convexity assumption on the set was proved in 1921 by J. Pál. The resolution of the Kakeya needle problem in 1928, without the convexity assumption, by Besicovitch has a curious history. Working on an entirely different problem, Besicovitch had constructed, in 1919, a certain set that

contains a unit segment in every direction. It was realized in 1928 that a simple modification to the Besicovitch set yielded a solution to the Kakeya problem that there exists sets of arbitrarily small area inside which it is possible to reverse a unit segment. How this solution was arrived at was indicated in the associated short computer animation. (This is available on the website.)

Where? Some of the techniques used in Besicovitch's solution are exactly similar to the methods used in reversing of a vehicle in a cramped space. But this exhibit also illustrates that some very simple looking questions may lead to very rich mathematics that eventually is then used in many other areas of mathematics itself and in applications.



5 waves

Waves allow us to receive television and radio signals and watch live telecasts of our favourite events half way across the globe, see the light from distant stars, see the minute structures in a cell in our body, send text messages to our friends, detect oil and gas hidden far below the surface of the earth, and communicate all this to each other!

Simply put, a wave is a rhythmic undulation that moves in space with a finite speed. A periodic movement of one part causes nearby parts to start oscillating which in turn incite its neighbors, and so it continues, thereby creating a wave.

Contrast this with pure oscillation, which is just periodic motion of any object or any quantity in time, without the propagation in space at finite speed which are the main defining characteristics of a wave. (Think of the wave of motion of cars as the light turns green at a road signal – the first set of cars start moving, then after a delay the next set behind them moves, and so on. If only most drivers realize that the speed of such a wave is much less than the speed of the green light waves which instantly reach the drivers' eyes, much of the honking at the signals will reduce drastically – a real example of how understanding of mathematics may change our everyday lives!)

Where do waves occur? Everywhere – literally! Wave phenomena is ubiquitous: waves on a still pond, in the deep ocean producing tsunami, in the atmosphere, in the dense plasma in our Sun, the light that

lets us see, the sound that we hear, the patterns on the tabla. The list is only limited by our imagination – literally!

What's the mathematical description of these phenomena? All waves have a wavelength – the distance after which it repeats itself; A frequency – the number of times it repeats itself in a second; An amplitude – how loud it is; And a phase – where does the wave start from? Can you notice the similarities and the differences with the characteristics of an oscillation? The equation for the undulating pattern of the wave is written in terms of some quantity W that is varying periodically as a function of position x and time t. For example, W could be electric field (e.g. for light), variation of pressure (e.g. for sound or plasma waves), or distance (e.g. waves on a spring). The simplest of such oscillations is given by

(3) $$W(x,t) = A\sin\left[2\pi\left(\frac{x}{L} - ft + p\right)\right].$$

Here, L is the *wavelength*, f the *frequency*, p is the *phase*, while A is the *amplitude* of the wave. Another important quantity is the *wave speed* $c = Lf$. Exactly as in the case of oscillations, any wave, however complicated, is simply a sum of waves of various frequencies, amplitudes, and phases.

Notice this looks deceptively like the equation for an oscillation which depends only on time t, but whereas the waves depend on the position x and time t.

Just like in the case of the oscillation, the above equation for waves is itself a solution of many different types of equations, all of which are partial differential equations. The most basic of these equations describing the waves is, not surprisingly, the wave equation!

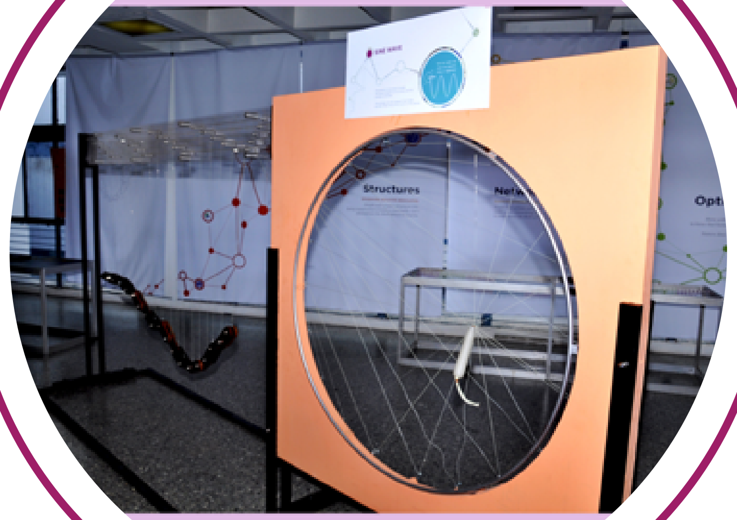
(4) $$\frac{\partial^2 W}{\partial t^2} = (Lf)^2 \frac{\partial^2 W}{\partial x^2}.$$

A large number of other equations – those describing light, or electrons, or flow of air and water, or traffic, and many others – can be transformed to describe waves. This is the power of mathematical modeling of the world around us: it affords a unified description of a diverse set of observations!

There are several exhibits which show different manifestations of these phenomena of waves. Let us discuss them one by one.

5.1

Sine Wave



Turn the handle and watch the patterns. What is happening? Movement of one piece causes the piece hinged to it to move, and the process continues. The strings not only support the hinges, but also help control the motion of the pieces. In a way, this pattern is not exactly a sine wave as written in the equation above, but is only an approximation that visually resembles a sine wave.

5.2

Soliton

Have you heard of this exotic wave: a solitary wave or soliton? This exhibit gives a visual representation of what a soliton wave may look like (even though it is not exactly a soliton wave itself).

These soliton waves (i) maintain their shape as they move, and (ii) can emerge unscathed after a collision with another soliton. They are also special, because the "louder" they are, the faster they move.

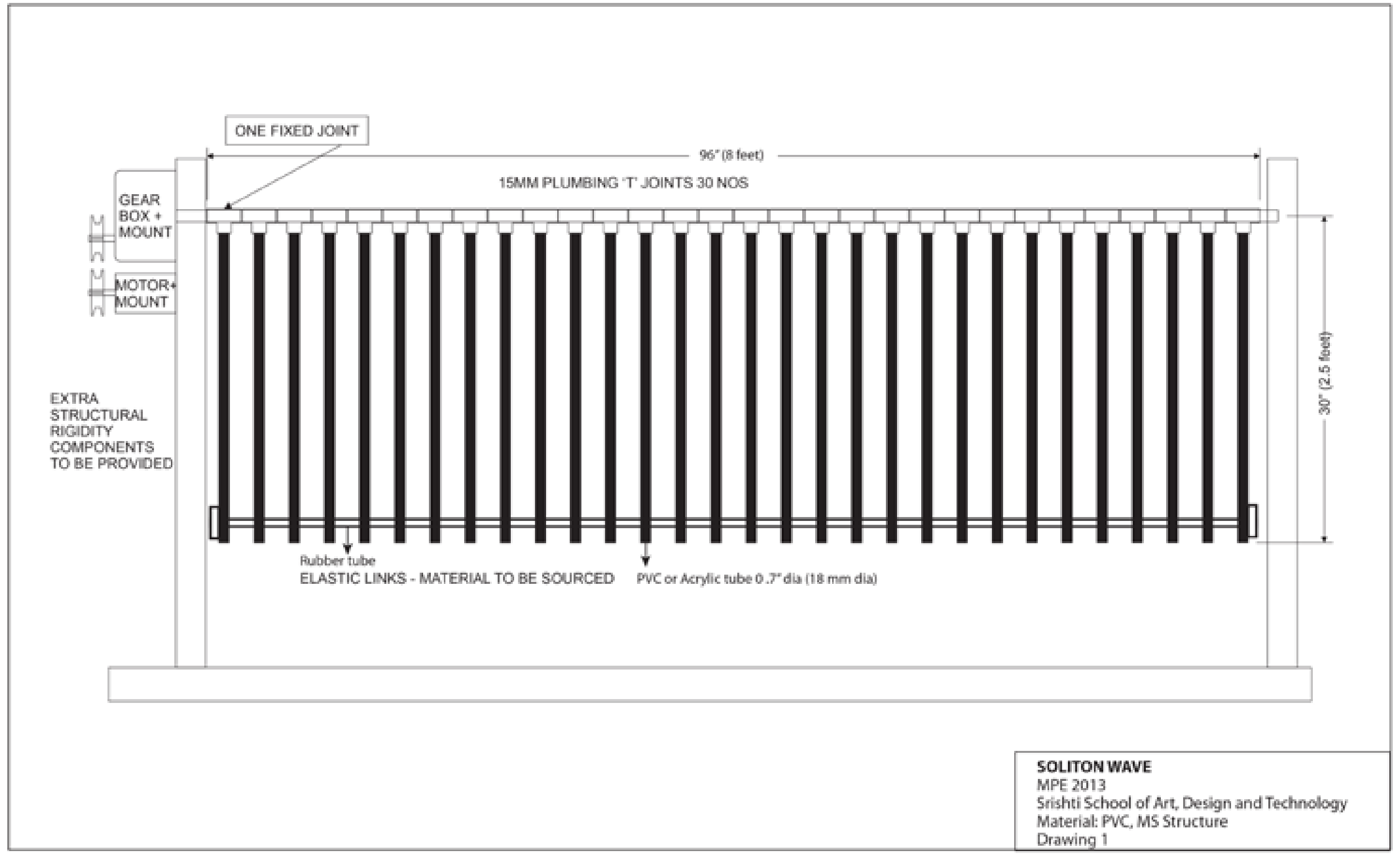
Contrast this with the "usual" waves such as the sound or the light – their speed does not depend on their loudness.

Where do solitons occur? Solitons were first observed on surface of water, but they also occur in optical fibres used to form the network that carries internet data, in some DNA and other biopolymers.

What is the mathematics? Soliton solution were discovered in very special types of equations which have a property that, even though they are nonlinear equations, they can be solved exactly, in contrast to the chaotic pendulum or the long oscillating spring. One of the examples of equations which has solutions that are solitons is known after two mathematicians, the Korteweg-de Vries equation (KdV equation for short):

$$\frac{\partial W}{\partial t} + \frac{\partial^3 W}{\partial x^3} + 6W\frac{\partial W}{\partial x} = 0. \tag{5}$$

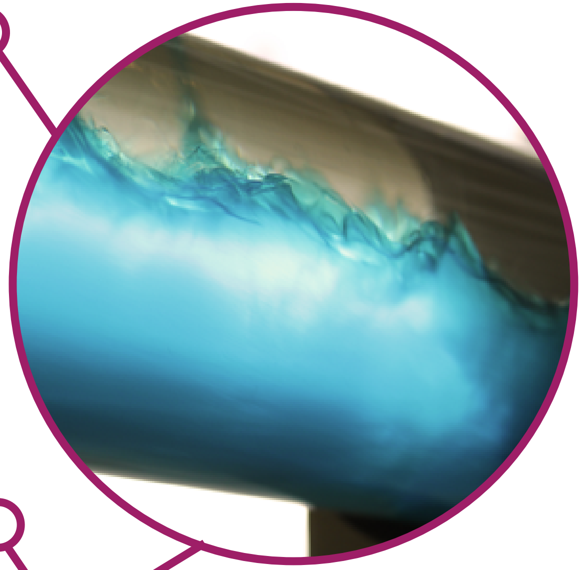
There is a lot of beautiful mathematical ideas related to such waves and many of them are still being explored by mathematicians and physicists.

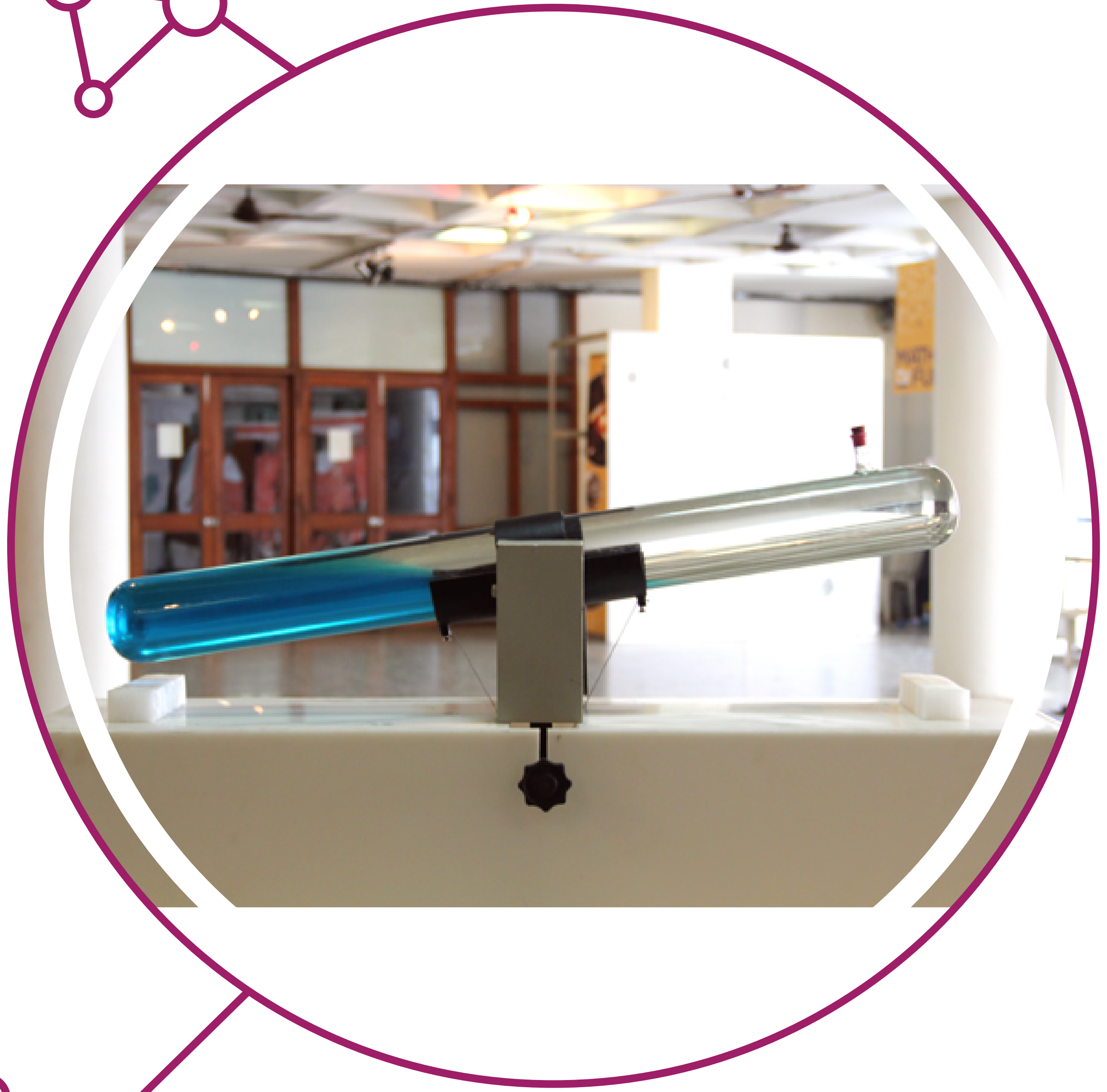


5.3

Wave Tube

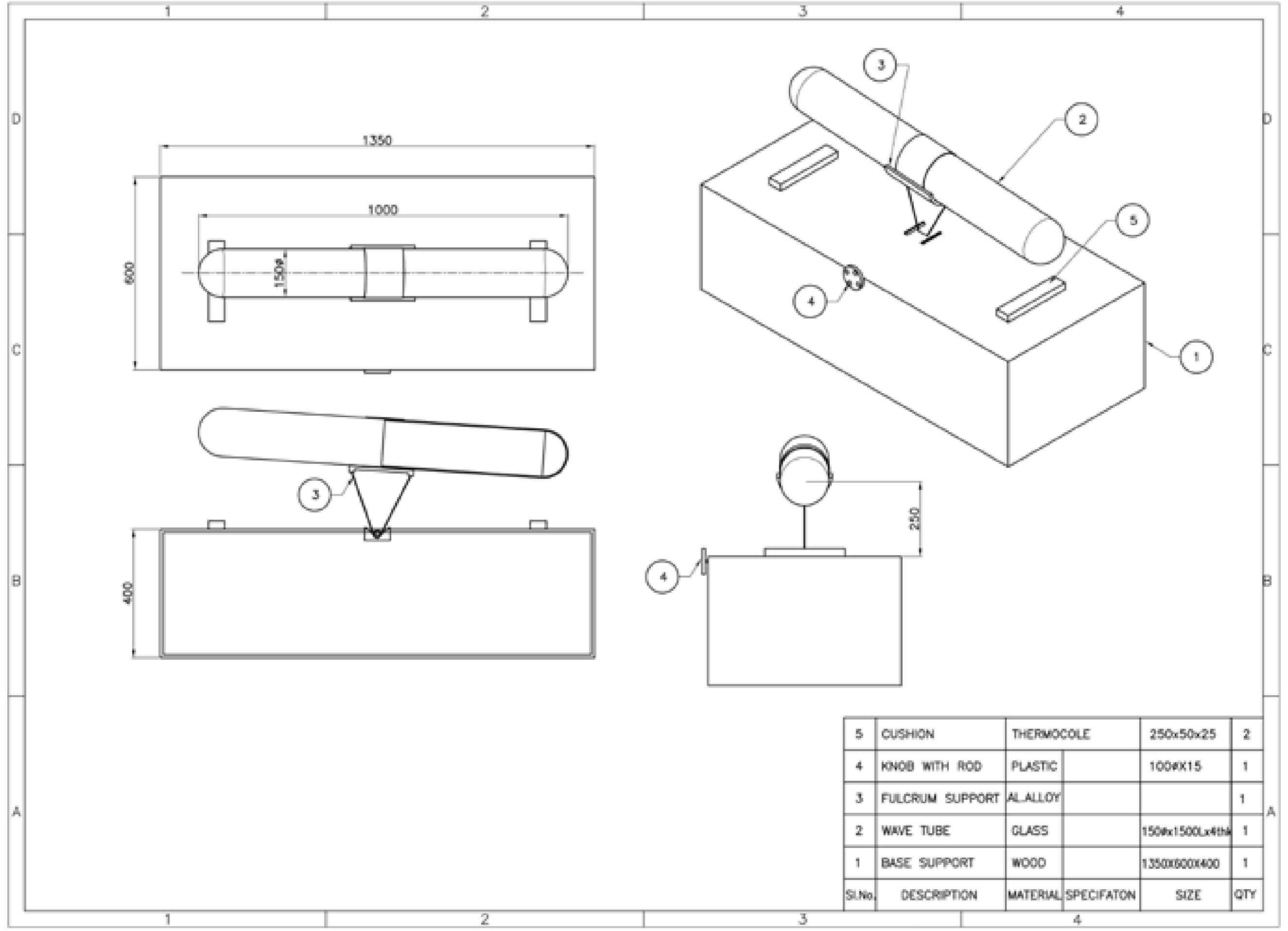
The waves that we see on the interface of the two fluids in this exhibit clearly remind us of waves that you can see on a river, on the surface of a lake or the sea, even in the clouds in the sky – or if you stretch your imagination a little, a tsunami (though tsunami is quite a different type of wave than those seen here). There are many different descriptions of these various types of waves, but all of them can be mathematically described by a combination of the basic type of wave described in the wave equation above.





you see any similarity between these waves and the waves on the surface of the ocean?

Turn the knob to tilt the glass tube to one side and observe the formation of waves in the blue liquid. The upper half of the tube is filled with a lighter, colourless liquid - kerosene, while the lower half contains a viscous liquid - tinted glycerol. The waves formed in the interface of the two immiscible liquids demonstrate the characteristics of both the transverse and the longitudinal waves at the same time. Do



5	CUSHION	THERMOCOLE		250x50x25	2
4	KNOB WITH ROD	PLASTIC		100ØX15	1
3	FULCRUM SUPPORT	AL.ALLOY			1
2	WAVE TUBE	GLASS		150Øx1500Lx4thk	1
1	BASE SUPPORT	WOOD		1350X600X400	1
Sl.No.	DESCRIPTION	MATERIAL	SPECIFATON	SIZE	QTY

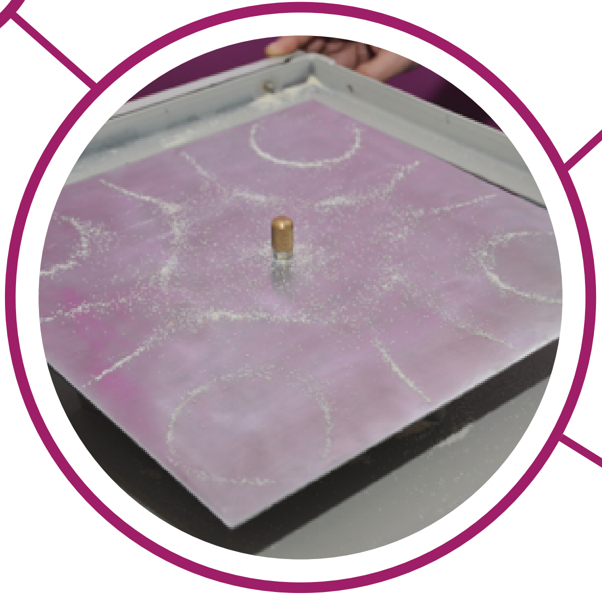
5.4

Chladni's Plate and the Standing Waves on the Spring

Vibrations of any structure - a building, a bridge, a violin string - show characteristic patterns and characteristic frequencies. These patterns depend on the shape and size of the structure, on the material used, on its mass. When winds or earthquakes shake these structures at these characteristic frequencies, they "resonate" and this resonance can lead to large oscillations.

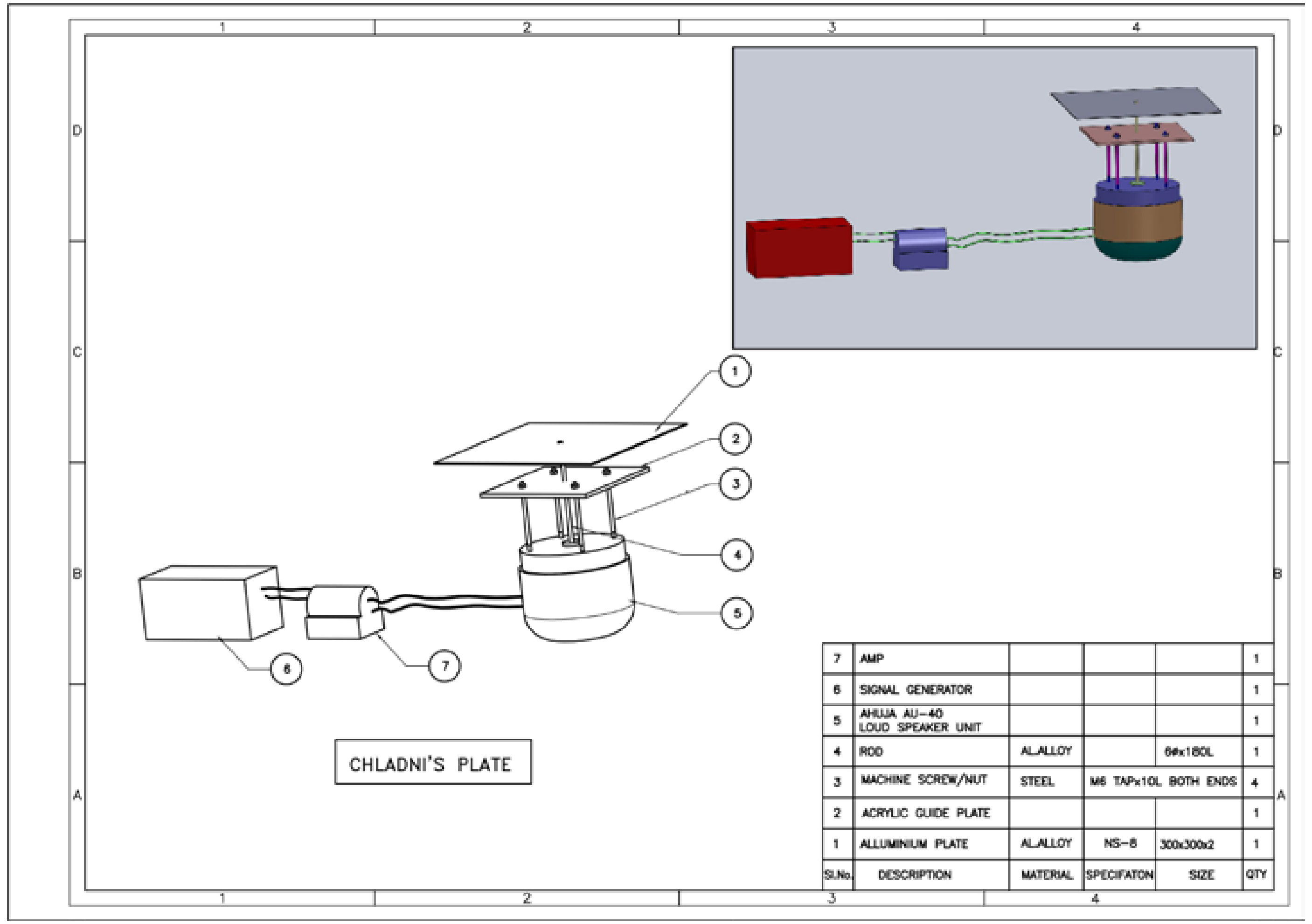
Knowledge of these vibrations, which can be calculated using mathematical equations describing them, helps engineers build structures which can withstand earthquakes, or strong winds and waves.

5.4



This exhibit shows the vibrating patterns of a square aluminum plate. Add a little sand (sooji!) and adjust the frequency and see how the patterns are formed.

There is a natural relationship between these vibrations and waves. As we saw earlier, vibrations of one part of the plate cause adjacent parts to start vibrating, and this leads to waves moving on the surface of the plate. What we are seeing here is the result of many different waves coming together to cause what are known as standing waves.



SI.No.	DESCRIPTION	MATERIAL	SPECIFATON	SIZE	QTY
7	AMP				1
6	SIGNAL GENERATOR				1
5	AHUJA AU-40 LOUD SPEAKER UNIT				1
4	ROD	AL.ALLOY		6#x180L	1
3	MACHINE SCREW/NUT	STEEL	M6 TAPx10L BOTH ENDS		4
2	ACRYLIC GUIDE PLATE				1
1	ALLUMINIUM PLATE	AL.ALLOY	NS-8	300x300x2	1

5.4

The standing waves on the plate have interesting, complicated shapes. The simplest form of standing waves is seen on the spring. The equation for them is very similar to the basic equation for the wave we saw earlier. In particular, for the spring, if x is the distance of a point along the length of the spring, and y is the distance by which that point is displaced from its original position of being in a straight line, the equation for y as a function of x and time t is given by

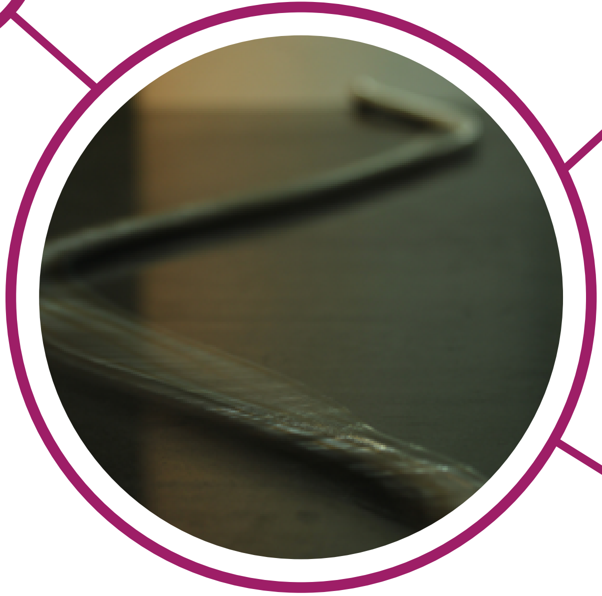
(6) $$y(x,t) = A\sin(2\pi ft)\sin\left(\frac{2\pi nx}{L}\right),$$

where n is the number of points on the spring that are not moving at all (called the *nodes*), and the frequency is related to n by the following relation: $f = nc/L$, with c being the speed of the wave on the spring.

The strings on a sitar, a guitar, or any string instrument, or the air pressure in a flute, a shehnai, or most wind instruments vibrate according to exactly this same equation above! As you may imagine, the equation for the Chladni's plate or for the membrane of a tabla or mridangam is a bit more complicated, but the basic idea remains the same.

One way to look at the standing waves is as follows: the far end of the spring is fixed while the front end is sliding free. Waves generated from the free end reflect from the fixed end. Hold the handle and create a transverse wave pulse with one quick snap of the wrist. The pulse will travel the other end of the spring and reflect back.

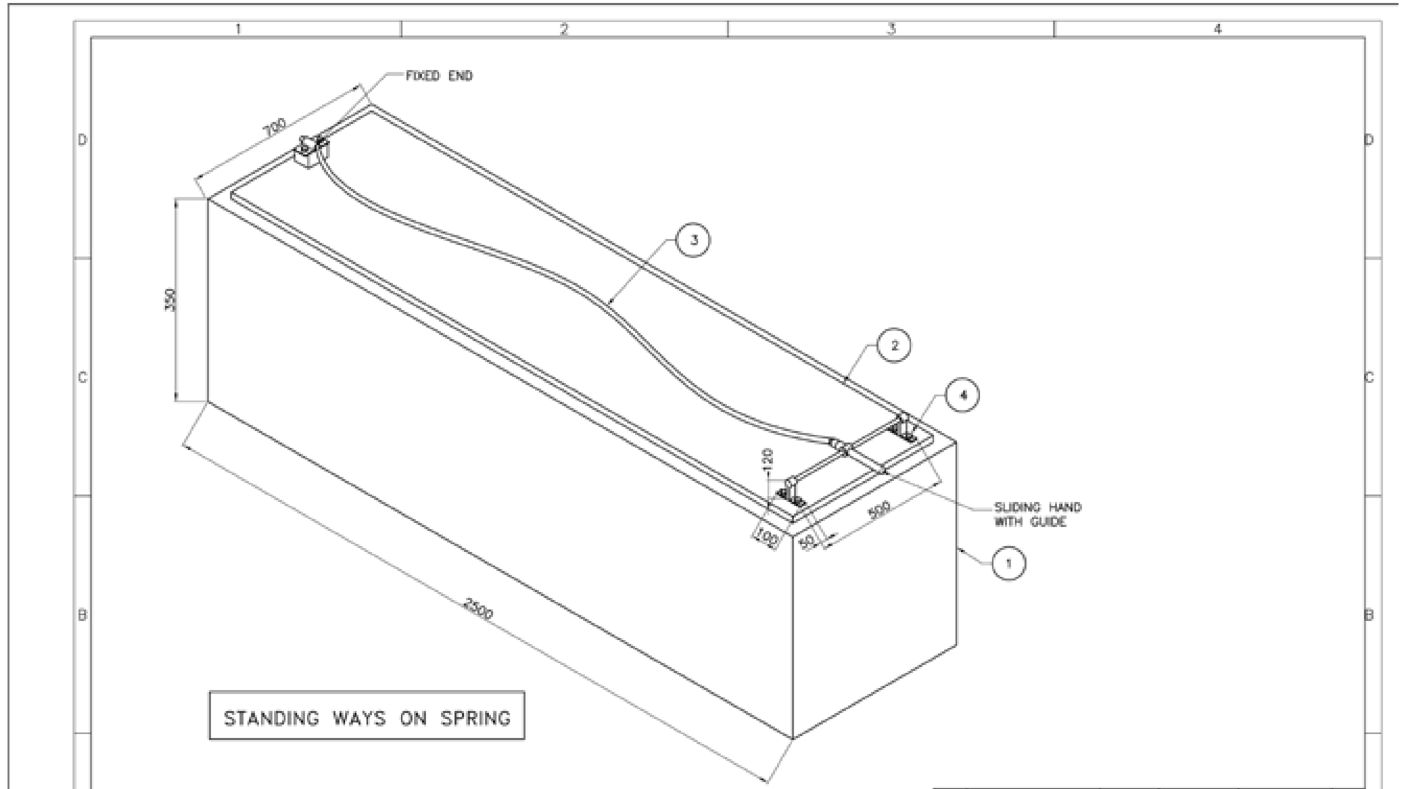
5.4



Does the reflection pulse return on the same side as the original pulse or on the opposite side?

Now slide the handle in quick succession to create waves that reflect from the other end. Standing waves can be created when the reflected waves interfere with the incident waves at specific frequencies.

Find the right rhythm such that the incident waves and the reflected waves interfere to produce standing waves. How many standing waves can you create? By measuring the length of the spring, the frequency with which you oscillate it, and counting the number of nodes, can you find the speed of this wave?



Sl.No.	DESCRIPTION	MATERIAL	SPECIFATON	SIZE	QTY
4	M6 BOLT	STEEL	COMMERCIAL	M8x20L	6
3	WAVE SPRING	SPRING STEEL	25#x1mm WIRE DIA ID=16, OD=18.5, 4'LONG		1
2	GRANITE SLAB			2400x600x25	1
1	BASE SUPPORT	WOOD		2500x700x350	1

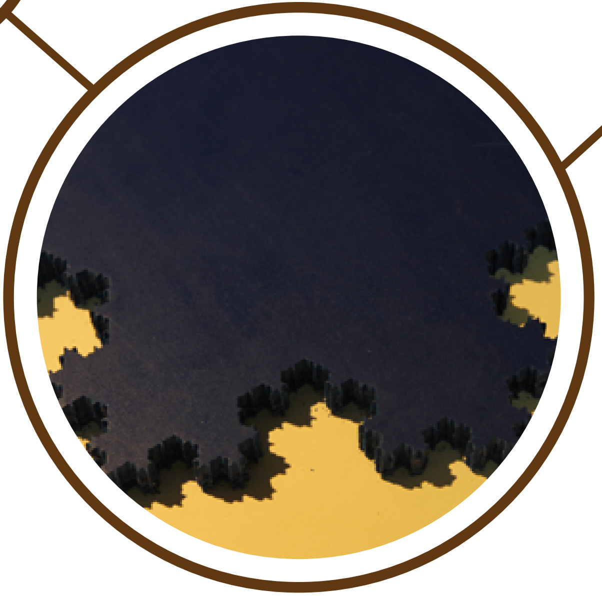
6 other exhibits

Probability, geometry, numbers ...

We are all familiar with the concept of chance, the idea that something may or may not happen, as opposed to the concept of certainty - that something will certainly happen or will not happen. At first glance, it seems that mathematical ideas are all certain - one plus one is always two, the area of a circle is always πr^2 , etc. and these facts (or theorems) are always true and never false! So how does one describe the outcome of the throw of a dice or of a coin toss at the beginning of a cricket match? That is the subject of probability theory, and the basis for understanding the two exhibits involving the dice, and the rolling balls in a maze.

The most basic aspect of probability theory is an *experiment*, and the collection of all possible outcomes of the experiment is called the *sample space* corresponding to the experiment. Two of the simplest experiments, which form the basis of much of probability theory are: tossing a coin and throwing a die. The corresponding samples spaces are {Head, Tail} and {1,2,3,4,5,6}, respectively. One can mix and match these experiments and construct loads of probabilistic games. In the exhibition, two of the exhibits were based precisely on the above basic experiments of tossing of coin and throwing of dice.

Where does such a theory get used? All of us use *probability* in daily life without ever noticing it - most colloquial statements containing words such as “chance,” “luck,” “fortune,” “random” are invariably statements about probabilities! In par-



ticular, we often use *laws of large numbers* to put forth our observations regarding a *typical* behaviour. Have you noticed how you use probability? Think about deciding whether to take a rickshaw or just wait for a bus to arrive in time. The decision can be based on calculations that use probability theory. These days, probability theory gets used everywhere from mobile phone networks, to IPL cricket matches, to climate studies.

The notions of uncertainty (even when presented in terms of numbers such as 20% chance of rain) can be easily contrasted with the sense of certainty provided by geometrical objects such as squares, circles, tetrahedra, etc. and by numbers such as even or odd, rational or irrational, real or complex numbers. Some of the exhibits described below, in particular those on fractals, maps, and Fibonacci numbers describe some possibly unusual but interesting aspects of these familiar concepts of geometry and numbers.

The fractals are the kind of geometrical objects that defy the usual notions of length or area and are closely related to some deep properties of numbers and to probabilistic concepts. The distinction between fractal and non-fractals is one of the main highlights of the exhibit on fractals using shapes such as a circle, a snow flake, or the map of Karnataka.

The next exhibit on maps shows the various geometrical concepts that underlie the transformation of the spherical surface of the earth onto a flat piece of paper in order to make maps that we can fold and put in our pockets, or see on a flat computer screen!

The Fibonacci exhibit shows two of the myriad patterns that occur in the realm of numbers, namely, two sequences consisting of the "usual" Fibonacci numbers and of "generalized" Fibonacci numbers. In general, the notion of sequences is so ubiquitous in mathematics, that it is impossible to over-emphasize its importance. Keeping this in mind, the exhibits on Fibonacci numbers also show the concept of convergence of a sequence.

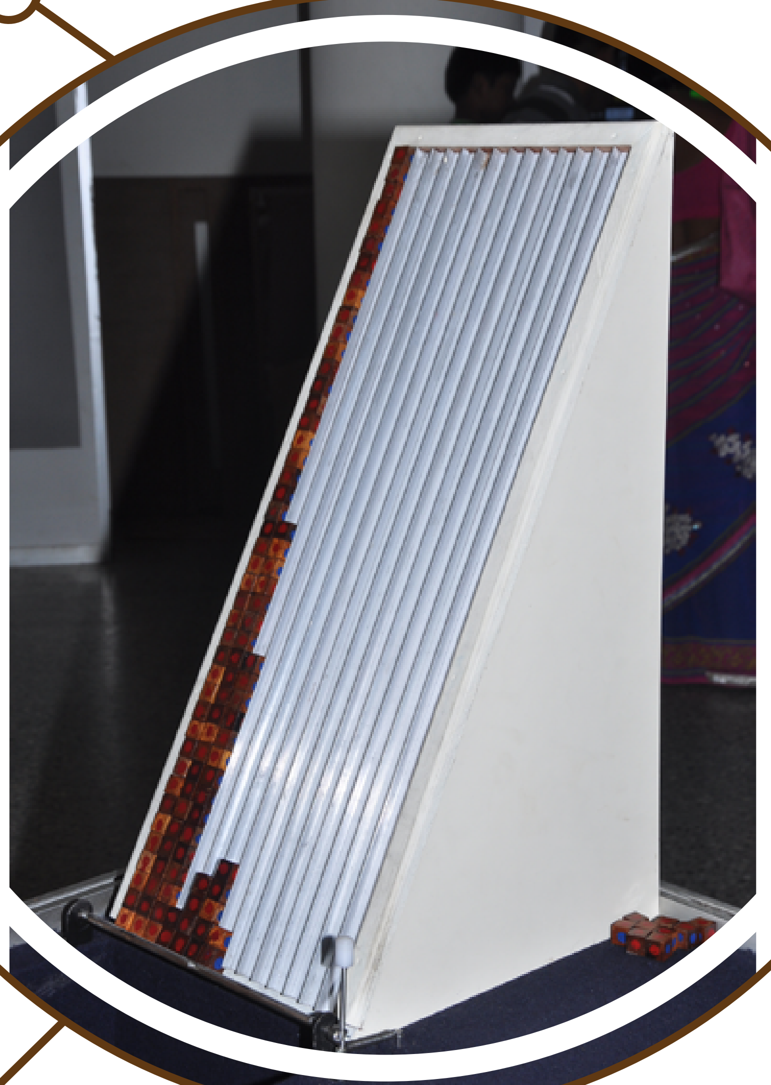
The last exhibit (borrowed from the museum Mathematikum) asks the visitor to experience the mathematical ideas of a function and its graph by "walking the function" and tracing a curve on the computer screen in the process.

6.1

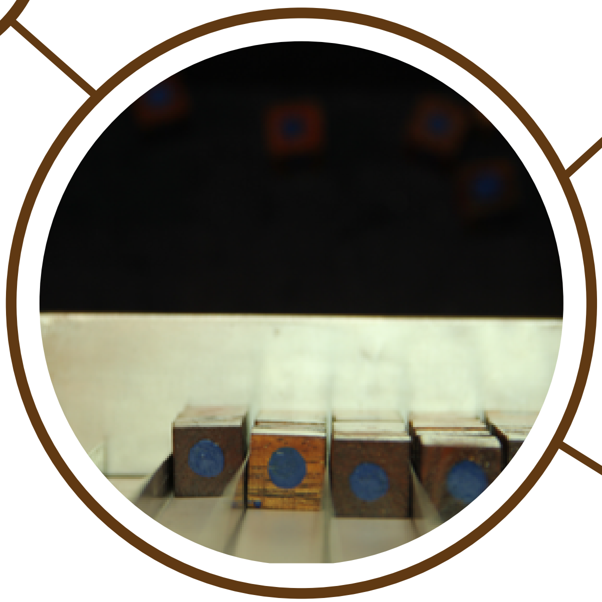
Dice

The dice used in this experiment are a bit special: some of their sides have a red dot while all others have a blue dot, for example, 2 sides with red dots and 4 sides with blue dots. The game goes as follows: A large number of dice are thrown, and ones showing the red color are collected, and stacked in a column. The remaining dice are thrown again, and the ones showing red are again collected, and stacked in a column next to the earlier column. This process is repeated until one exhausts all the dice. Watch how the height of the columns vary!

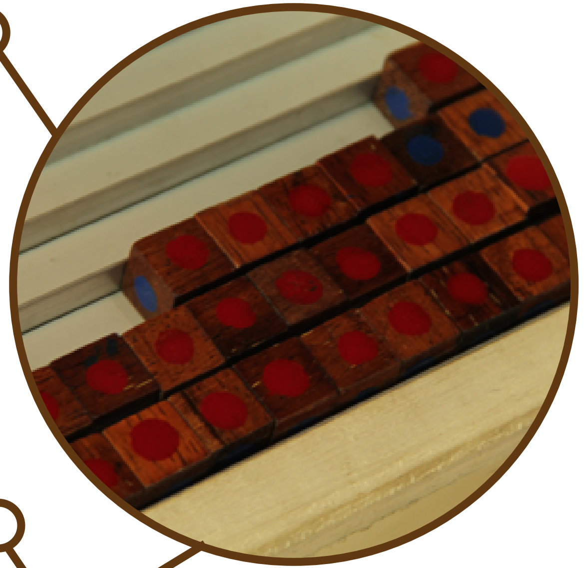
What is the math? This game highlights the definition of *probability of an event* and also illustrates what are called *laws of large numbers* in the probability theory. We all know that a fair die shows up the number 1 with probability 1/6. Does it mean that if we throw a die six times, we must observe the number 1 exactly once out of the six throws? No. What it really means is that if we throw a die a large number of times, then approximately one sixth of the times we shall observe 1, which is a consequence of what are known as *laws of large numbers*.

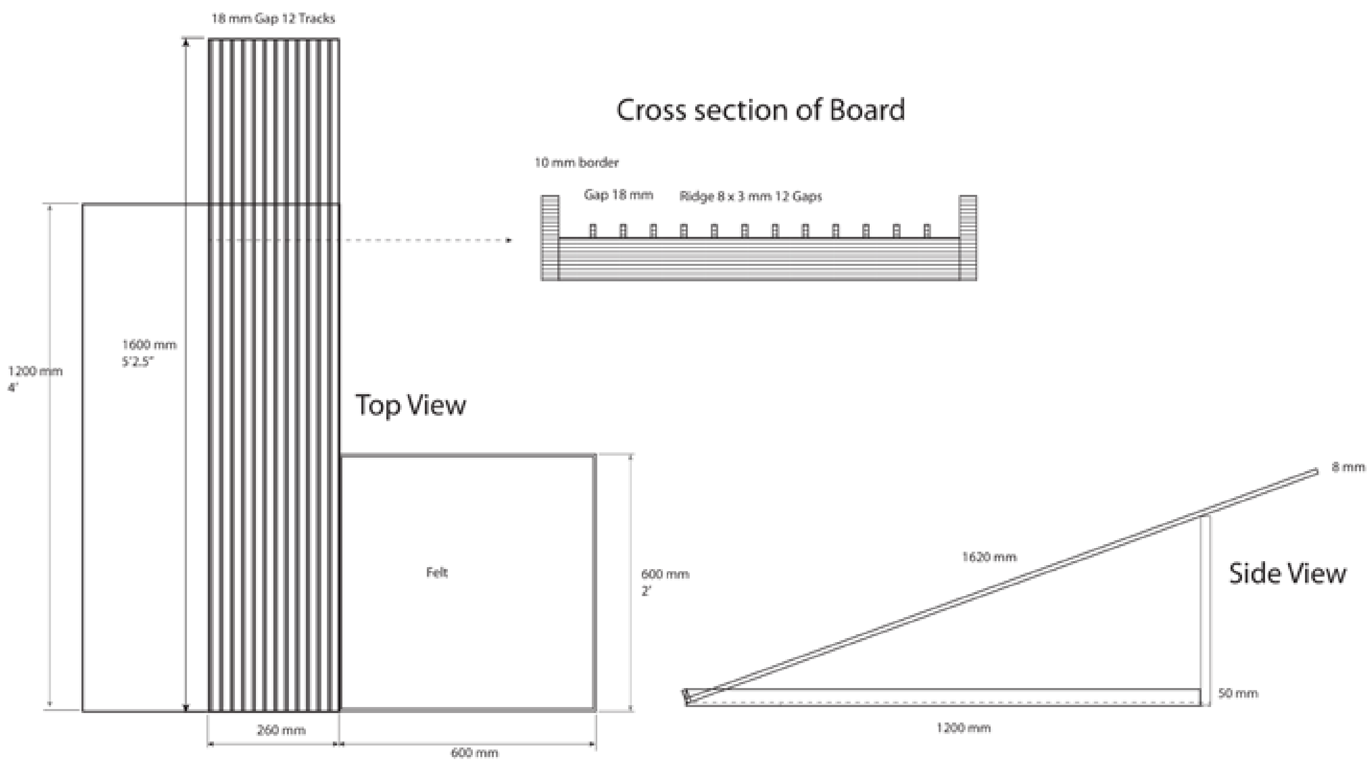


6.1



Thus, if the number of dice (N) is large, then approximately $N\frac{1}{6}$ of the dice are going to show up 1 in the first throw of dice. Then, at the second stage, of the remaining $N\frac{5}{6}$ dice, approximately $N\frac{1}{6}\frac{5}{6}$ are going to show 1 in the second throw of dice. Thus, at the k-th stage, approximately $N\frac{1}{6}\left(\frac{5}{6}\right)^{k-1}$, which represents an exponential function in the variable k. Thus this game naturally introduces the exponential function as well.





Dice Game showing Exponential Decay
3 Tables with 1/6, 3/6 and 5/6 Dice

6.2

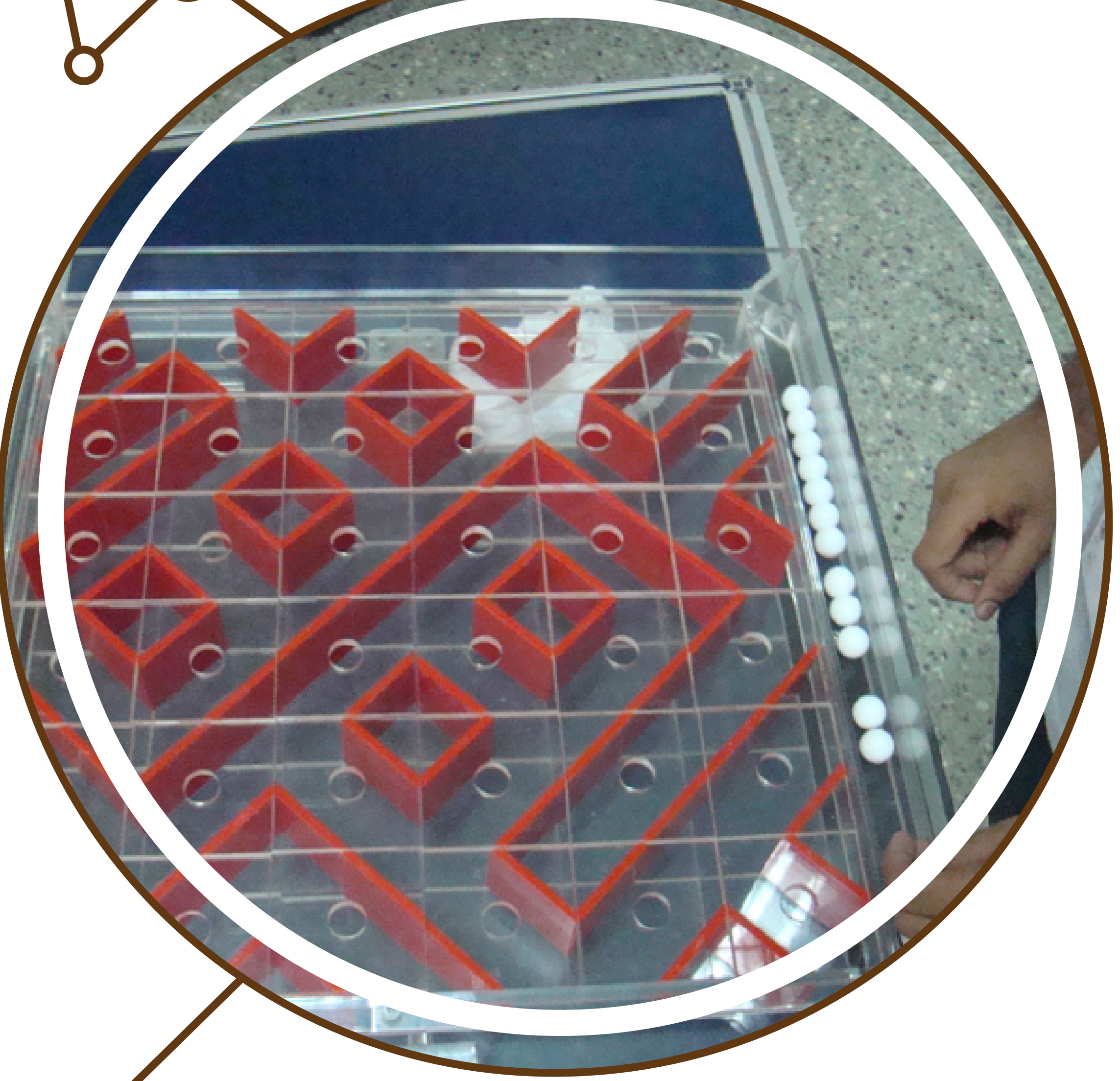
Permeability/ Percolation

As noted above, the sample space (which is the collection of all possible outcomes of an experiment) corresponding to the experiment of tossing a single coin is simply $\{H, T\}$, where H = Head and T = Tail. However, if we toss two coins then the sample space becomes $\{HH, HT, TH, TT\}$, whereas the sample space for 3 coin tosses is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. What if we toss 36 coins?

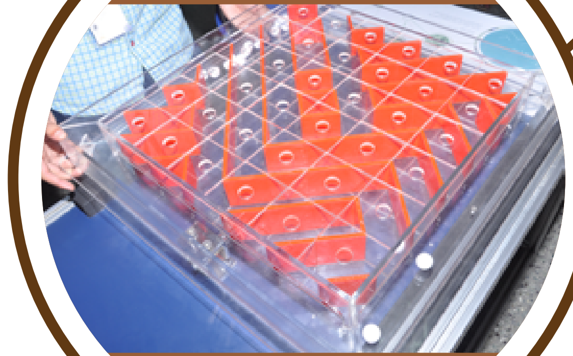
This was precisely at the heart of an exhibit which has 36 small squares arranged in a 6 × 6 pattern inside a large square. Each smaller square had a path going through it along its diagonal (any one diagonal). Each of these smaller squares could be turned by 90°. So effectively, each square has two distinct orientations. Now what are the total number of such configurations for all the 36 squares together?

The relation to coin toss is as follows: suppose we say that a head of a coin toss will correspond to the path going towards top left while a tail of the coin toss to the path going towards top right. So the configuration of all the 36 squares can be determined by throwing 36 coins and arranging the squares according to the outcome of these tosses.

What is the math? Try answering these questions for a smaller experiment, say with 3 × 3 square, or 2×2 square for that matter. Notice that the answers are hard to come by as the number of squares grow from 2 to 3, or from 3 to 4. Such combinatorial questions are at the center of contemporary research in *probability theory*.

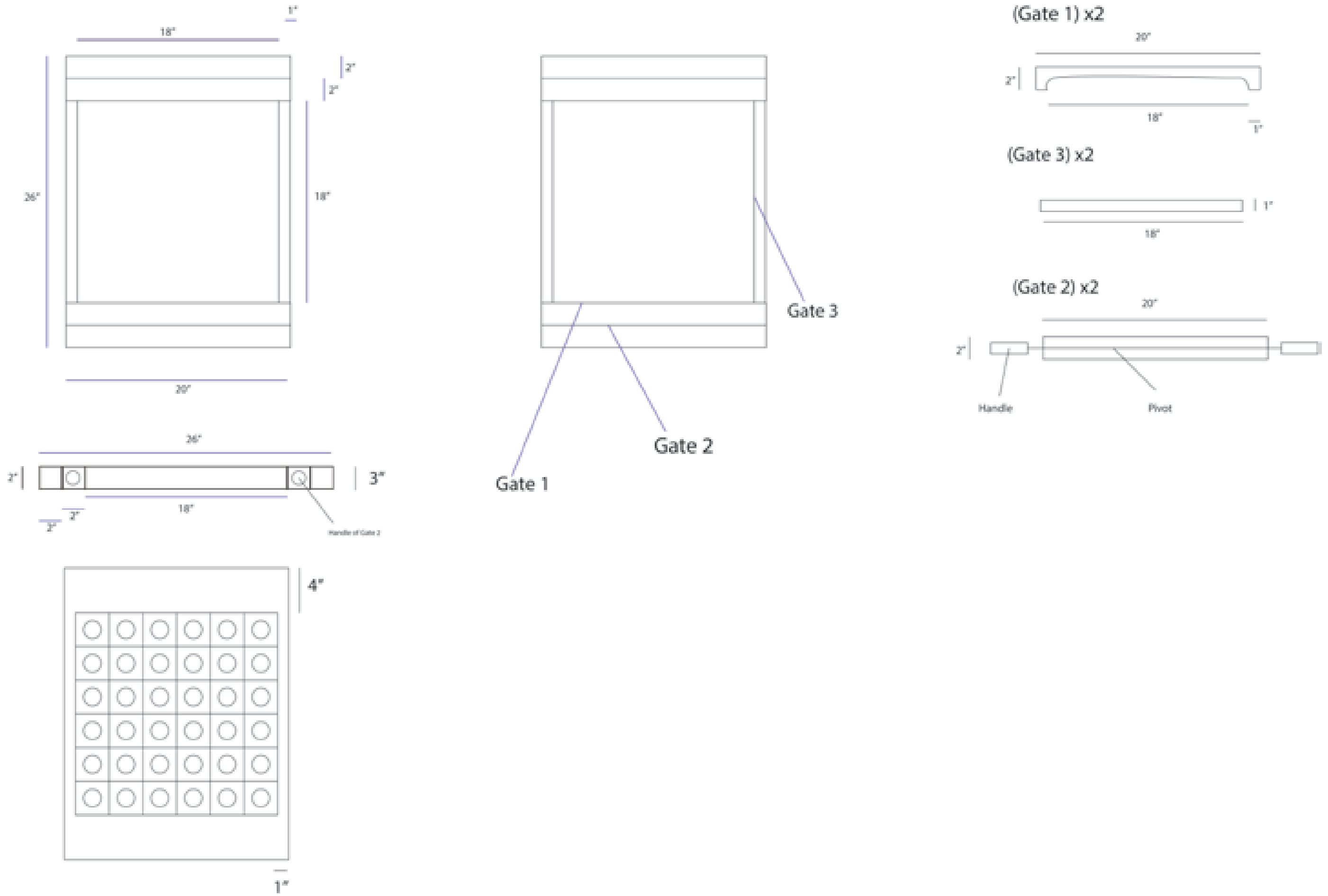


6.2



Where do such problems occur? Have you thought about how the water seeps through loose soil or a sponge but not through a hard stone? Think of the balls rolling down through the squares in the exhibit as water going through a rock. Whether the rock is permeable or not, that is whether the water can seep through the rock or not, is determined by whether it can find a path going from one end to the other. This is the phenomenon of percolation leading to the property of permeability. The simple probabilistic experiment in this exhibit illustrates this idea. Can you arrange the squares in a way so as to make this "rock" permeable or impermeable.

Finally, going back to the idea of the coin toss, how many of the combinations will lead to a permeable exhibit while how many will lead to an impermeable one? If you were using coins to decide the arrangements of these squares, will you be more likely to come up with a permeable arrangement?



6.3

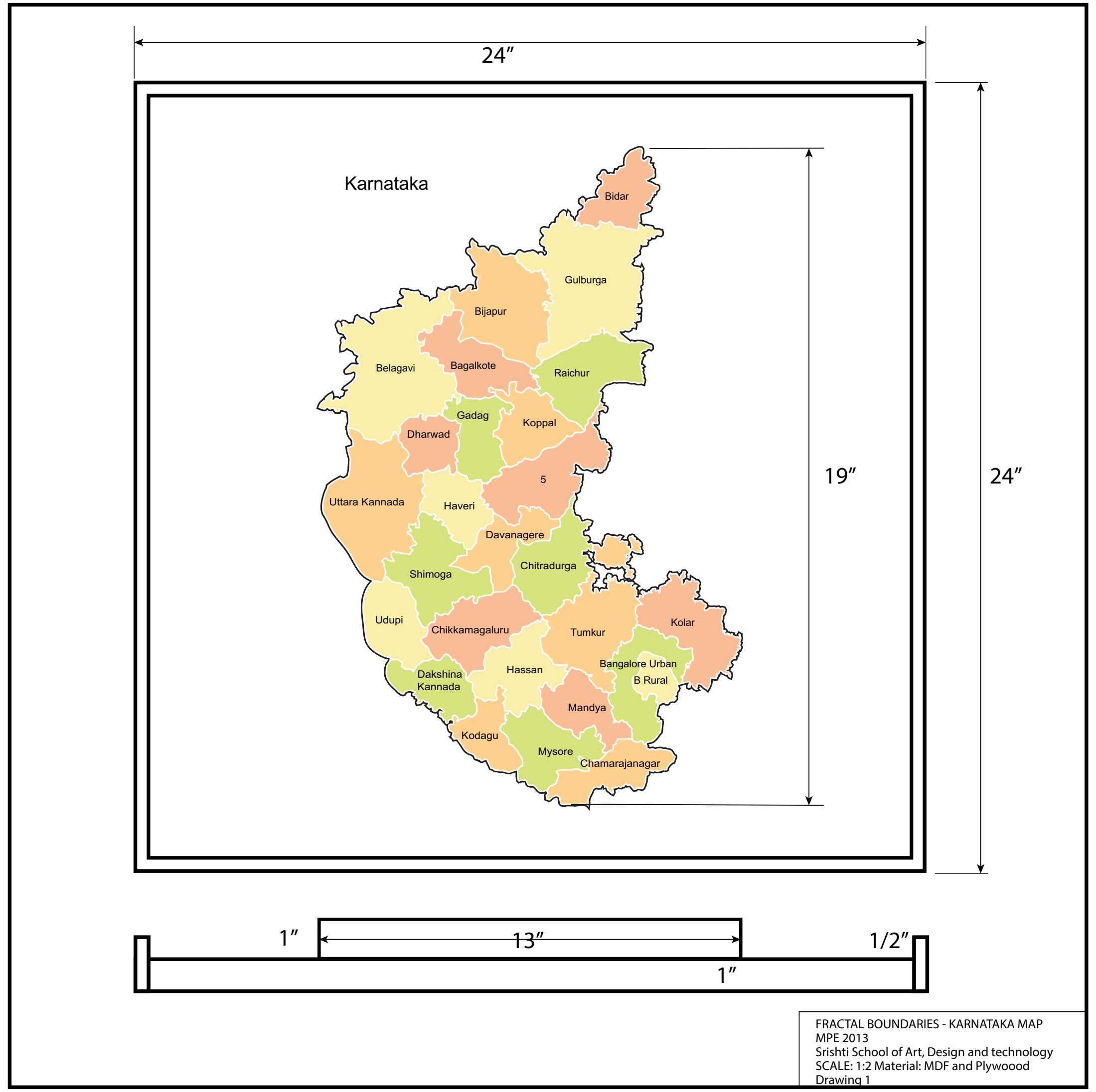
Fractals

This exhibit is designed to illustrate the difference between objects (in this case, curves) which are *fractals* from those which are not. Let us begin with commonly encountered objects which are *not* fractals: a football, the smooth surface of a computer screen or a book, the boundary of a cricket field, train tracks. Indeed, most man made objects are non-fractal objects. On the other hand, approximate fractal properties are observed in many natural objects, such as coastlines, mountain ranges, river networks, lungs, path of lightning, clouds, etc. A quick glance at this list may suggest that fractal properties are related to "non-smoothness" of an object and that is precisely the concept that is captured by the mathematical definition of a fractal, which we will look at after we describe the exhibits.

In the interactive exhibit, we will use rulers with different units. An easy way to make such rulers will be to use beads/thin glass tubes of different diameter/length. Collect few beads/thin glass tubes of diameter/length 1cm, 2cm, 4cm, and 8cm. Use a thread to connect all the beads/thin glass tubes of length 1cm to get a ruler of unit 1cm. Similarly other rulers of unit 2cm, 4cm and 8cm can be made.

There are two experiments to perform.

(1) Experiment 1: This experiment contains a disc and few polygonal shapes with straight edges. Measure the perimeter of each object by wrapping the boundary by different rulers made as above and counting the number of beads/glass tubes. One notices that the perimeter of objects which is obtained by multiplying the number of units by the



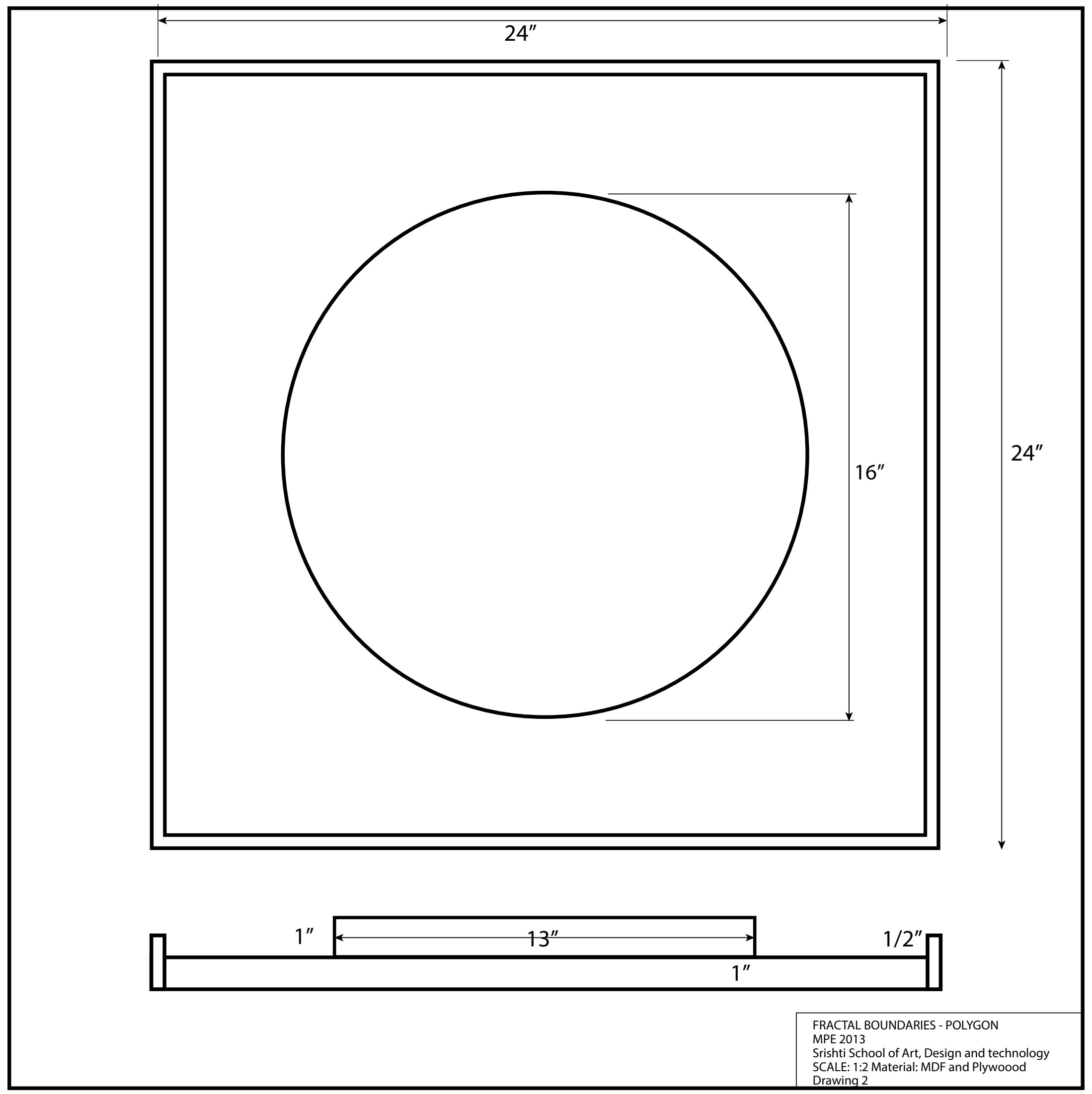
6.3

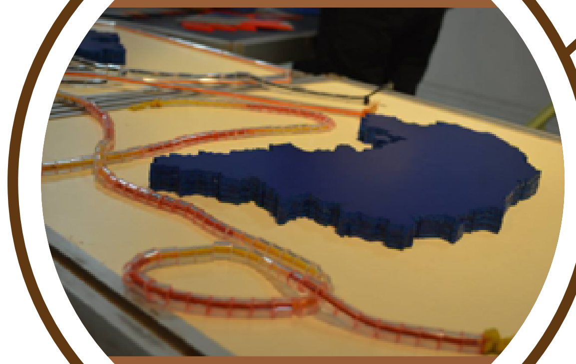
length of one unit remains approximately the same when measured using different rulers.

(2) Experiment 2: This experiment contains objects with highly irregular boundaries. A model of Karnataka (or Norway!) map and snowflake would be ideal choices for this purpose. Repeat the activity of measuring the perimeter with these objects and notice that with different rulers, one gets a different perimeter!

Let's look at the mathematics behind this exhibit to understand the concept of fractals. Let us denote the number of 8cm glass tubes needed to cover the full length of the perimeter of an object by n_8, the number of 4cm tubes needed by n_4, the number of 2cm tubes by n_2 and that of 1cm tubes will be written as n_1. For example, if n_8 is 10, then the perimeter of the object as measured with the 8cm tubes, or mathematically stated as *measured at length scale of 8cm*, would be $8n_8 = 8\times10 = 80$cm. If $n_2 = 42$, then the perimeter at length scale 2cm would be $42\times2 = 84$cm.

For non-fractal objects like a square or a triangle or a circle in the experiment 1, you will notice that the number doubles each time we halve the length scale. This relationship is written as: $n_4 = 2n_8$, and $n_2 = 4n_8$ and $n_1 = 8n_8$. So the number of rulers is inversely proportional to the length of the rulers. This is what we will expect intuitively anyway. This can be stated mathematically as follows: the number n_L of tubes with length L needed to cover the full length of the perimeter P would be $n_L = P/L$.



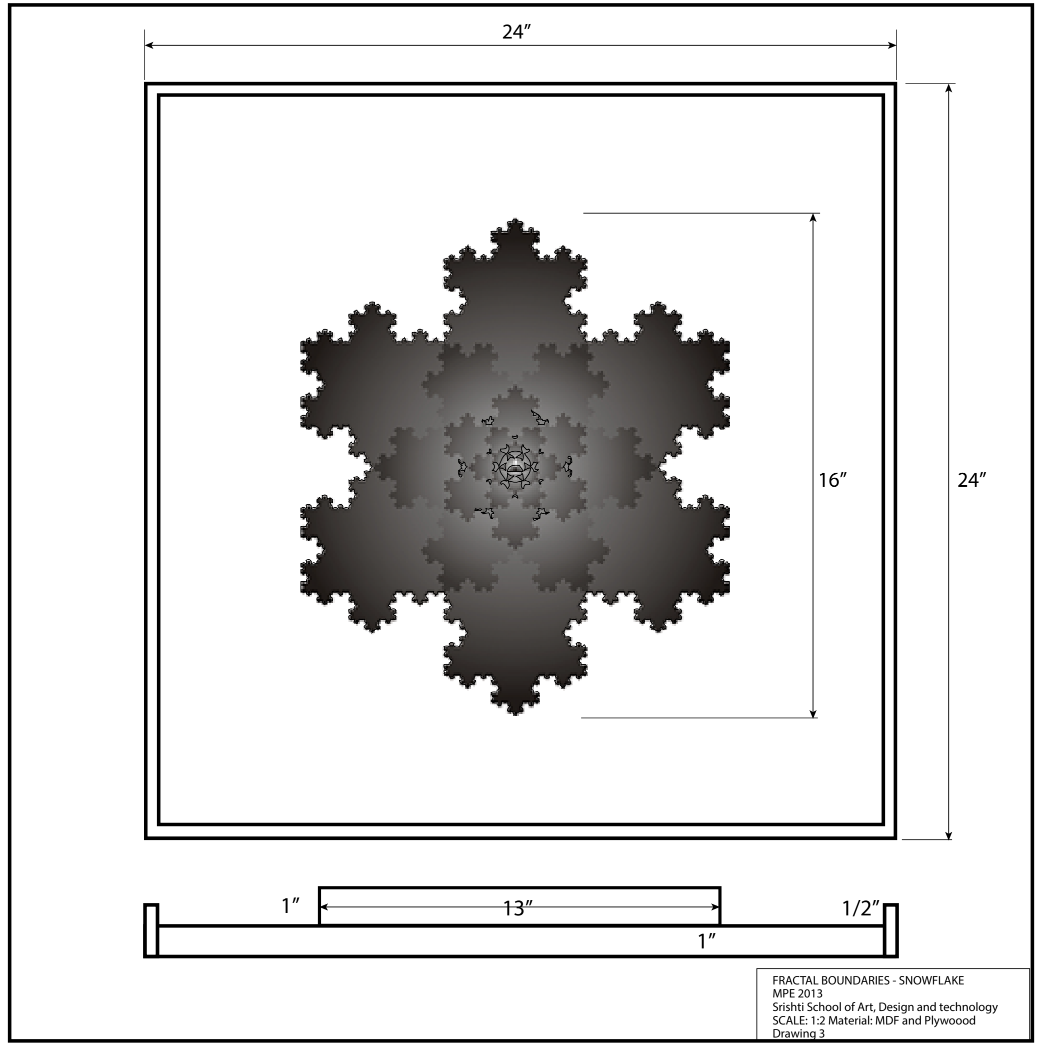


But for a fractal object like a snowflake or Karnataka boundary in experiment 2, the number of rulers needed to measure the perimeter and the length of rulers used do not follow such an intuitive "linear" relationship. In that case, n_4 will be greater than $2n_8$, and similarly, $n_2 > 4n_8$, and $n_1 > 8n_8$, and so on. Indeed, it will be seen that the number of rulers of length L will be given by $n_L = c / L^d$ with d some number between one and two: $1 < d < 2$. Note, that in the case of a non-fractal object, $d = 1$.

In the experiment 1, if the number of rulers n_L is plotted against the scale of the ruler L in a log-log graph, the slope of the line would be -1, but for experiment 2 it will be $-d$. The negative of this slope is called the "dimension" of the perimeter. The perimeter of a snowflake has a dimension which is more than 1 but less than 2, whereas the edge of a circle or a square is of dimension 1.

A similar phenomenon like in experiment 2, which is generally referred as coastline paradox, can be observed when one tries to measure the length of coastline of some countries. These extremely counter-intuitive phenomena of dependence of measurement on the scale is the defining characteristic of the fractal nature of the boundaries of these objects.

Many fractals have another important property called *self similarity* on all scales. If we zoom in, we see the same type of object again: a snowflake seen through different microscopes of different magnifications looks the same! Can you notice whether the snowflake is "self-similar" or not?



6.4

Anamorphic Maps and Foldabale Maps

We can begin with two experiments.

(1) Draw a triangle on a globe. Of course there are readymade triangles on the globe, described by any two longitudes and the equator. Now measure the three angles of this triangle and sum them up. Notice that their sum is never 180 degrees. Why is that so?

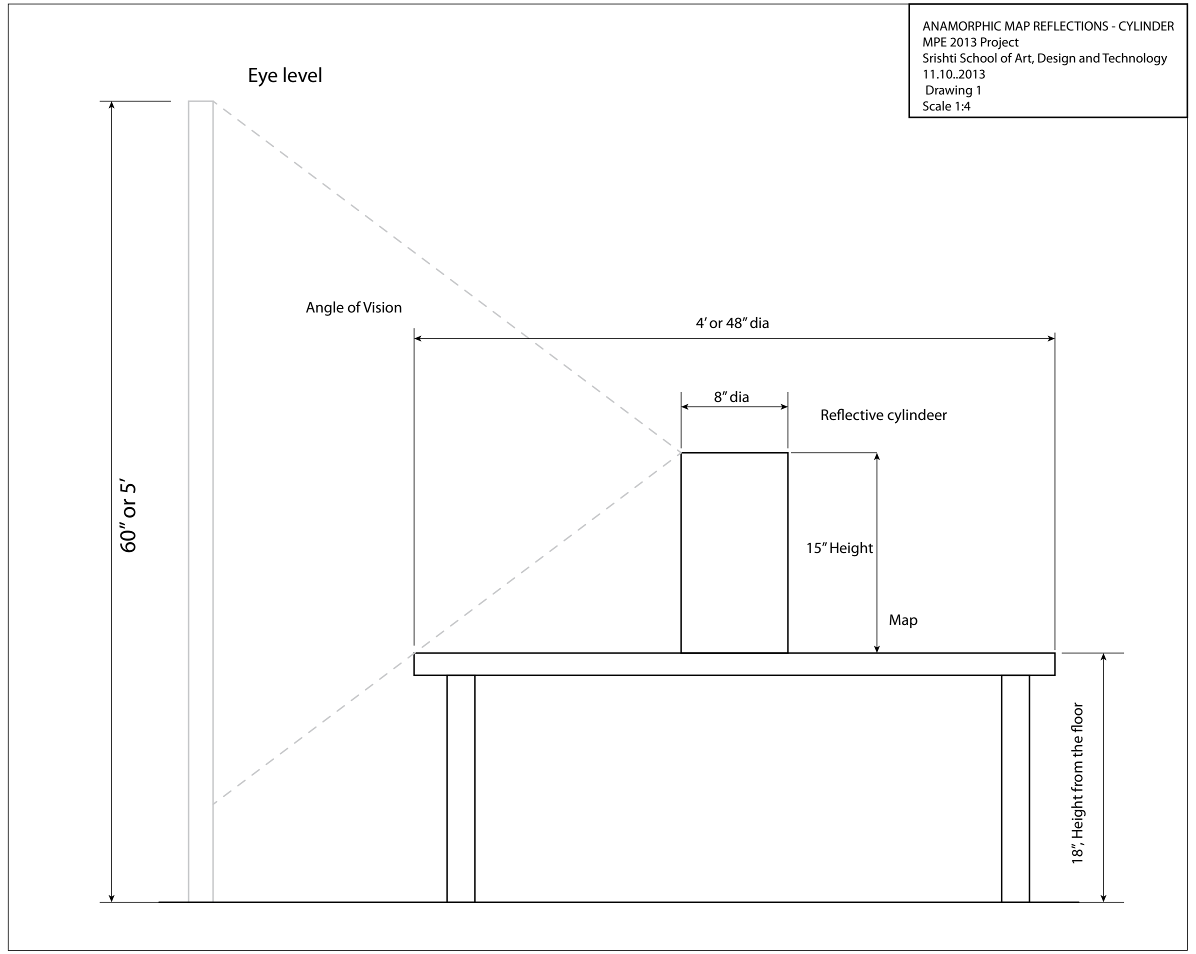
(2) Take a large sheet of paper, and try wrapping it around a football or a globe, such that the paper never gets wrinkled. Impossible, right? Next try cutting this sheet of paper into 5-8 pieces (say polygons such as hexagons), and try pasting these polygons on the globe, again without overlapping and no wrinkles. Not possible again! Now cut a large number (100s) of tiny polygons, and then you may notice that it appears possible to achieve this goal. More so because the wrinkles are not going to be visible on such small polygons. How does one understand this?

All the above are manifestations of the same phenomenon: that the earth or the football is not flat, but is spherical and curved! When we want to project such a curved surface onto a flat sheet, it cannot be done without some distortion. E.g. a triangle on the sphere whose angles do not add up to 180° cannot be projected onto a flat sheet, because if this could be done, its angle would sum up to 180°, which is clearly a contradiction! The anamorphic map exhibit shows precisely this phenomenon. Notice that the map as seen on the flat surface is highly distorted - the areas, the angles, and the distances are all wrong! But when you see its reflection on the curved surface of a cylinder, it looks just fine. Map-

6.4

The foldable maps exhibit illustrates the second phenomenon noted above. If we take large number of small flat polygons, we can approximate the earth surface with much more accuracy than with a small number of large flat polygons. Indeed this property defines the general class of objects called "manifolds" of which the surface of the earth is a familiar example. In general, you can think of "manifolds" as "curved surfaces." But the power of mathematics is that the same mathematical concepts that describe surfaces (which are two dimensional manifolds) also describe "higher dimensional manifolds" such as the universe in which we live!

makers use the mathematics behind such "reflections" or "projections" in order to make various kinds of maps which reduce one type of distortion or the other – either the angles or the distances or the areas. But no map on a flat piece of paper can reproduce all the three aspects exactly for the whole earth, and that's a mathematical theorem, so however hard you try, you will never be able to do it!



6.5

Fibonacci and other numbers and sequences

We are all familiar with numbers - even or odd, rational or irrational, real or complex. We also come across various patterns when dealing with the numbers and these patterns often manifest themselves in real life phenomena. For example, every other number starting from 2 is divisible by 2 (the even numbers). Another example is the sequence of prime numbers, which are not divisible by any other number except 1.

A sequence that has been studied by mathematicians for centuries and that has intrigued many people is the Fibonacci sequence: the sum of two consecutive elements of the sequence gives the next element. Written mathematically, if the n-th element of the sequence of is represented by F_n, then $F_n + F_{n+1} = F_{n+2}$, with $F_1 = 1$ and $F_2 = 2$.. The first few elements F_n of this sequence are listed below.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,
233, 377, 610, 987, 1597, 2584, 4181, 6765,
10946, 17711, 28657, 46368, 75025, 121393, ...

The ratio of the consecutive elements approaches a fixed number called the golden ratio. Thus the ratio $r_n = F_{n+1} / F_n$ for larger and larger values of n come closer and closer to $\gamma = (\sqrt{5}+1)/2 \approx 1.6180339887498949$. Here are the first few such ratios r_n.

1.000000000, 0.500000000, 0.666666667, 0.600000000,
0.625000000, 0.615384615, 0.619047619, 0.617647059,
0.618181818, 0.617977528, 0.618055556, 0.618025751,
0.618037135, 0.618032787, 0.618034448, 0.618033813,
0.618034056, 0.618033963, 0.618033999, 0.618033985,
0.618033990, 0.618033988, 0.618033989, 0.618033989,
0.618033989, 0.618033989,...

Do you see any pattern in these numbers? If you have taken a calculus course, does this bring to your mind words such as sequence, convergence, etc.?

The Fibonacci sequence and the golden ratio have been argued to occur in spiral shells, pine cones, and the arrangement of seeds in sunflowers. Furthermore, they both do have some very special mathematical properties.

But there are myriad such sequences which are equally important in mathematics. One simple modification of the above process of obtaining the Fibonacci sequence is the following: simply change the first two elements! Thus, we could have started with $F_1 = 2$ and $F_2 = 7$ and then applying the same operation of "sum of consecutive numbers is the next number", we would obtain the following sequence.

2, 7, 9, 16, 25, 41, 66, 107, 173, 280,
453, 733, 1186, 1919, 3105, 5024, 8129,
13153, 21282, 34435, 55717, 90152,...

The amazing fact is that the ratio F_{n+1} / F_n of consecutive elements of this new sequence also come closer to γ for larger values of n, as you may verify yourself!

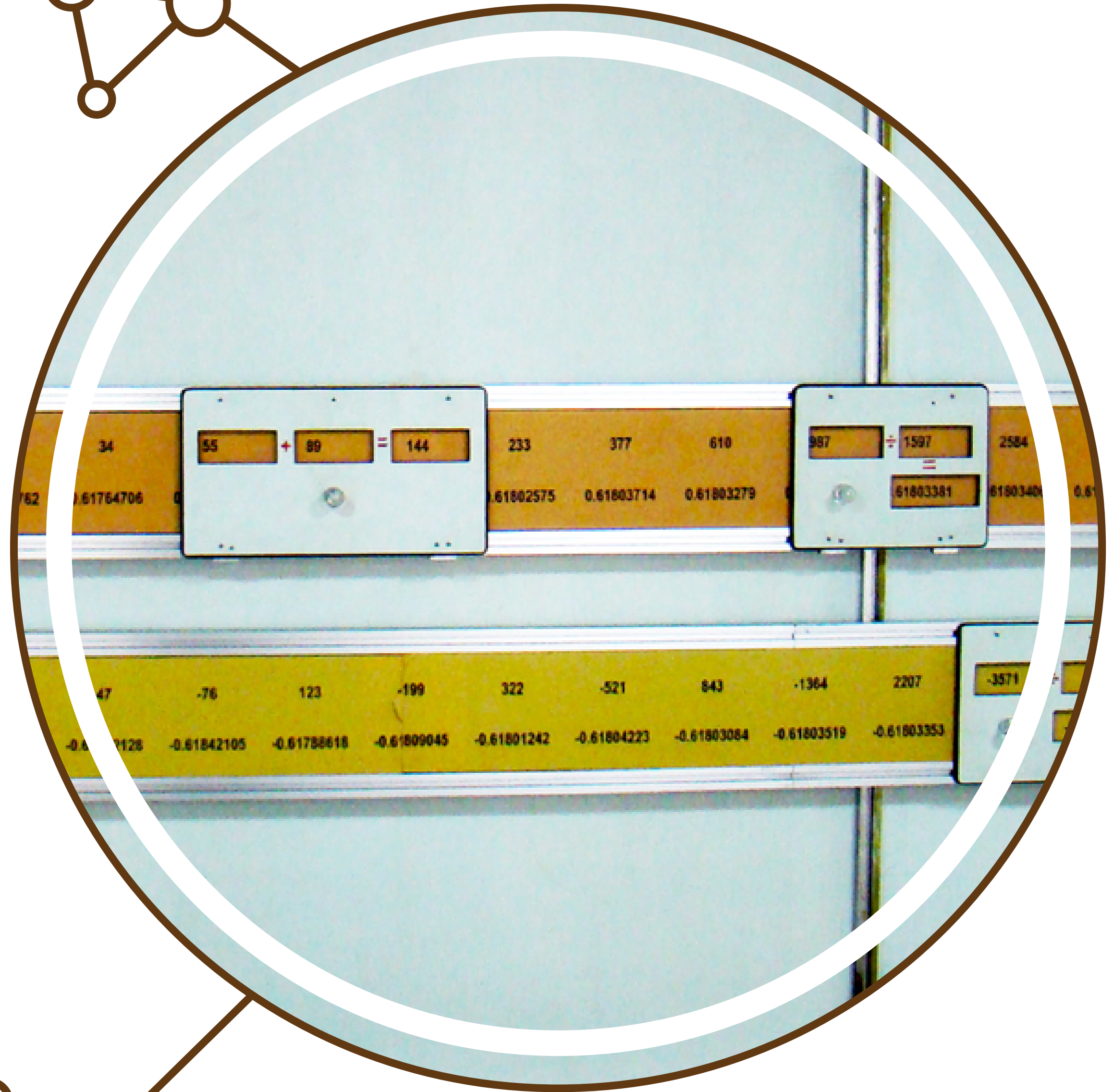
6.5

In fact, we can go further and device similar rules to generate other sequences. One such example is as follows. Start with $G_1 = 1$ and $G_2 = 2$ and define the rule that the difference of the consecutive elements gives the next element: $G_{n+2} = G_n - G_{n+1}$. This will generate the following sequence.

1, 2, -1, 3, -4, 7, -11, 18, -29, 47, -76,
123, -199, 322, -521, 843, -1364, 2207, -3571,
5778, -9349, 15127, -24476, 39603,....

Again, notice that starting with a different value for the first two numbers (G_1, G_2) will generate a different sequence. Another amazing fact is that the ratio G_{n+1}/G_n also approaches a fixed number and indeed this number is $-\gamma$, the negative of the golden ratio! We will leave it as a challenge to the reader to find out for herself or himself the reason for this "amazing" facts - or stated in a mathematician's language, the reader can *prove* these facts!

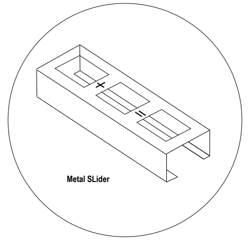
The Fibonacci exhibit illustrated the above mathematical facts. It was motivated by a similar exhibit in the museum *Mathematikum* in the German town of Giessen. There is a division slider and a addition or subtraction slider. The sliders can be placed over any consecutive numbers to see the above facts in action.

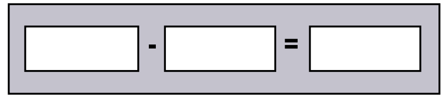


3" x 152"

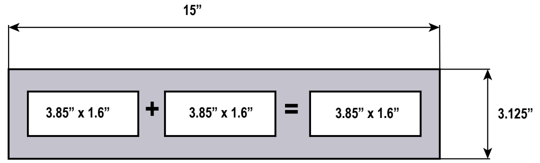
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	+	2584	=	4181	6765	10946	÷	17711	28657	...

0.00000000	1.00000000	0.50000000	0.66666667	0.60000000	0.62500000	0.61538462	0.61904762	0.61764706	0.61818182	0.61797753	0.61805556	0.61802575	0.61803714	0.61803279	0.61803445	0.61803381	0.61803406	0.61803396	0.61803400	0.61803399	= 0.61803399	0.61803399	...

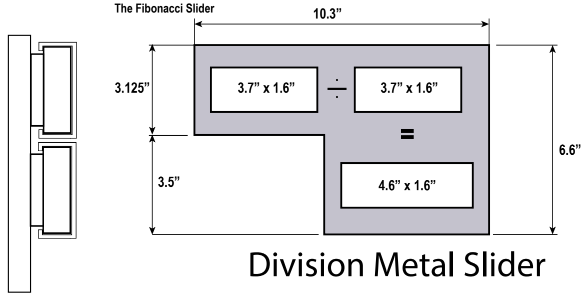




Minus Metal Slider



Plus Metal Slider



Division Metal Slider

1	2	-1	3	-4	7	-11	18	-29	47	-76	123	-199	322	-521	843	-1364	-	2207	=	-3571	5778	-9349	15127	÷	-24476	39603	...

0.50000000	-2.00000000	-0.33333333	-0.75000000	-0.57142857	-0.63636364	-0.61111111	-0.62068966	-0.61702128	-0.61842105	-0.61788618	-0.61809045	-0.61801242	-0.61804223	-0.61803084	-0.61803519	-0.61803353	-0.61803416	-0.61803392	-0.61803401	-0.61803398	= -0.61803399	-0.61803399	...

The drawing above shows the design of the sliders which illustrate how the sequences are generated and how the ratios approach a fixed value.

6.6

Early Warning Signals

Natural systems are dynamic. Trees bloom in spring and shed leaves in autumn. A forest composed of such trees changes character over seasons, years, decades. A region containing such forests, rivers, deserts, and other ecosystems reflects these changes. Even the whole earth is showing possibly irreversible signatures of climate change.

Some of these changes are simply due to the natural variability of dynamical systems - a pendulum swings due to gravity, the changing position of the sun causes seasons, the monsoon rains come and go every year. But external factors such as the human interactions with these systems, for example, construction of a dam or a factory, or reforestation of an area, also affect the progression of these changes. Some of these changes due to external factors can occur quite suddenly instead of gradually - something like the difference between going over a cliff instead of rolling down a slope! These sudden changes may be irreversible and are often called "tipping points." They may prove to be catastrophic, for example, in the context of ecosystems or other complex systems.

The physical exhibit on ecosystems and tipping points provided a illuminating example of such a phenomenon. Here the position of a light ball (a table tennis ball) indicated the state of a system, for example, say the amount of vegetation. We could think of valley on the right to correspond to a forest landscape (henceforth called forest valley) while the valley on the left to correspond to a desert landscape (to be called desert valley).

The blowers played the role of external factors such as cutting down trees or re-forestation. Two different types of systems were illustrated by two different "tracks" on which the ball moved.

• In one of them with a tall peak, the ball stayed in the valley which corresponded to large amount of vegetation, even in the presence of external factors. This showed the types of systems that do not show the "tipping point" behaviour.

• In another type of system with a short peak, the ball could "tip over" to the desert valley even with a small external push (from the blowers). This type of system is thus capable of the tipping point behaviour. Note that since the depth of the desert valley was deep, the ball would never revert back to the forest valley, showing irreversibility of this transition. (Of course we can pick up the ball and move it to the forest valley - that may correspond to building a canal from Ganga to Rajasthan!)

This exhibit was created by a team from Indian Institute of Science, Bangalore and it won the first prize in the intercollegiate competition for MPE exhibits, organized by ICTS-TIFR. The detailed report which describes the mathematical aspects of the problems is available on the ICTS webpage for this exhibition, to be soon linked from http://www.icts.res.in/program/MPE2013

6.7

Walk the Function

The concept of a function is one of the most fundamental ideas in mathematics, and in life in general. Like any functional unit we encounter, a mathematical function takes an input and gives an output. The inputs can be from a pre-defined set, for example, all integers, or all positive real numbers (or in case of a living being, food and air) and the outputs are also usually limited to a specific set, for example the set of even numbers, or the set of complex numbers (or in the case of a living being, air, water, solid waste).

We use mathematical functions all the time. When we look at a time-table for a train, it indicates the time as a function of location - you tell me the city (input) and the time table will tell you the time (output). Of course, if the input city is not in the set of cities where the train stops, there is no output!

Functions which depend only on one factor are most easily represented by a graph - the input is shown on the horizontal axes and the output is shown on the vertical axis. The “walk the function” exhibit, whose idea was borrowed from the Mathematikum in Giessen, Germany, is a way to make this idea of a “graph” perceptible - the visitor has to walk in such a way that the line indicating her/his position on the screen follows a predefined graph, or in mathematical terms, the graph of the position versus time would trace a predefined curve. More information about this exhibition is available on the ICTS webpage for this exhibit, to be soon linked from: http://www.icts.res.in/program/MPE2013

7 epilogue

: an impression about the exhibition

Written by Professor K.R. Yogendra Simha,
Department of Mechanical Engineering, Indian Institute of Science, Bangalore

As usual it was a busy day at the Visvesvaraya Industrial and Technological Museum (VITM). But, that day turned to be special for me and many other thousands of visitors including students from schools, colleges and research institutes. Under the theme of Mathematics of Planet Earth (MPE), the International Center for Theoretical Sciences (ICTS) and the Center for Applicable Mathematics (CAM), both centres of the Tata Institute of Fundamental Research (TIFR) had organized a unique festival projecting the hidden beauty of mathematics through colourful posters, artifacts and engaging exhibits. Thousands of students, teachers and the general public who filed through the exhibition enjoyed the experience but also expressed the need for a little booklet explaining the basic mechanics and mathematics underlying each exhibit. This proved to be a challenging and rewarding exercise for all the contributors, and this epilogue is a quick birds eye view of the gallery of exhibits item by item to help recapture the action and reaction of excited students, teachers and parents who participated in thousands each day from Friday 22 November to Sunday 1 December, 2013.

Blending mathematics with arts and crafts has been an enduring passion for the human intellect. Archimedes, Euclid, Kalidasa, Shakespeare, da Vinci, Galileo,

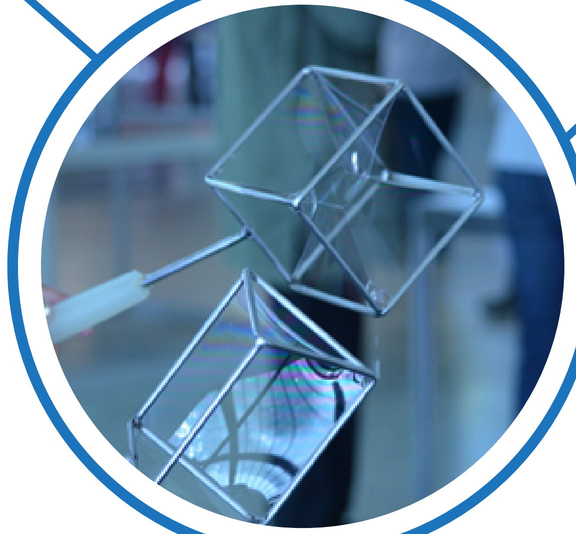
Newton, Gauss, Euler, Tyagaraja, Bach, Beethoven, Escher and Ramanujan created enduring science, prose, poetry, melody and paintings using numbers, colours and sounds. Perhaps mathematics provides the only means for understanding creativity in all its diversity. MPE-13 exhibition provided abundant opportunities for students to appreciate a wide range of concepts in arithmetic, plane and projective geometry, algebra, calculus, linear algebra, networks, differential equations, superposition principle and others. Over 25 exhibits classified under themes like Structures, Networks, Optimization, Oscillations, Waves and miscellany were displayed during the exhibition. Escher art exhibits injected a different feeling and colour to mathematics. The details of all these exhibits are compiled in this booklet with pertinent text, figures, drawings and photographs. Let us now briefly recapitulate each one of them adding some explanatory and appreciative notes, remarks and comments from a visitor's point of view.

Tensegrity structures at first appeared rather weak and fragile. Assembled from slender struts and strings, the tripod stool barely weighing a kilo easily bore the weight of a 100 Kg adult! Here you may also recall that the spokes of the common bicycle are always tense !! Several thousands of tension members like beams, bars and cables are deployed for building bridges, transmission towers and skyscrapers to make them safe, strong, stable and smart (meaning they are instrumented with sensors and alarms for detecting imminent danger!).

Escher art exhibits provide wonderful expressions of some deeply abstract mathematical notions of symmetry, inversion and perspective map. While science, law and sociology are subject to limits and constraints, mathematics like art, music and literature provide unlimited opportunities for creative thought. Thus, there is neither any need for limiting speed or direction for moving objects as long as they obey mathematical laws!

The next set of exhibits delineated mathematics underpinning the art and craft of tessellations or tiling. Tiling like tapestry has always been an enduring passion for all societies over thousands of years. That there can only be a finite number of ways of filling a given space with regualar polygons is the mathematical crux. Obviously, regular polygons are also preferable from an engineering production viewpoint as well as for estimating the cost of tiling a given area.

The exhibit on permeability is a simple teaching aid for understanding percolation theory dealing with flow of fluids such as oil and gas or electric current in porous media. Porous media in nature are not assembled from identical spherical grains. As a result, fluid flow paths can become uneven and tortuous like blood flow inside our brain. Only a few paths remain open in complicated networks. Predicting the scaling laws of percolation as a function of porosity forms a vital mathematical input for succesful oil and gas extraction from deep underground. Like phase transition behind a cloudburst there is a specific porosity threshhold for initiating the flow. Upon exceeding this threshold, the flow rate follows a power law. This exhibit, fashioned after a chessboard throws open a pandoras box of mathematical puzzles, permutations and combinations for opening and shutting the diagonal gates! Thus, for the 6x6 board with as many gates there exist 2^{36} permutations (approx. quantillion, i.e., 10^{18}). By the way, this figure roughly represents the number of bytes



of information generated everyday in the world today.

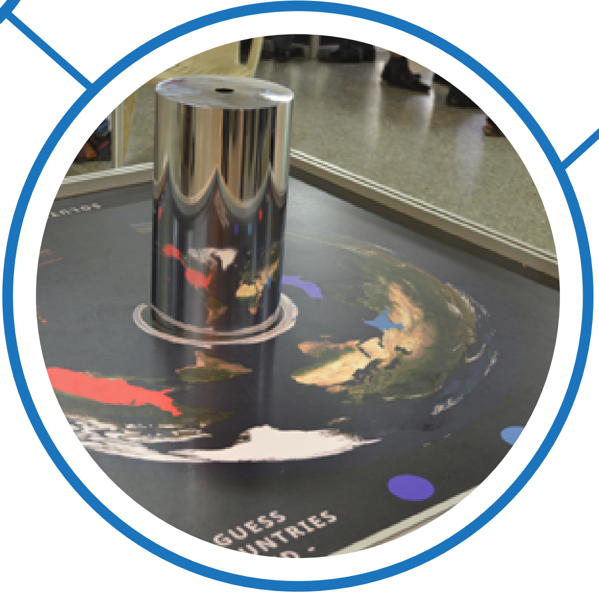
Networks exhibits highlight the complexity of dynamics over a set of interconnected nodes, be it a viral infection like SARS or information leaking on the internet. Infection and information spread out from a source rapidly but slow down eventually as the infected / informed population rises. Time scales of variation can range from a few weeks for endemics to few minutes for internet diffusions. This kind of logistic modeling is widely used in diverse fields such as economics, ecology and public policy matters pertaining to literacy, health and hygeine. Global models for epidemics are getting as complex as their counterparts for predicting climate change. The main challenge in global modeling stems from unpredictably rapid local fluctuations on the one hand with uncontrollable large-scale global events on the other. For example, high frequency megahertz transmissions are now commonplace for international trading of stocks and bonds.

It is difficult to assess the impact of invasive internet technologies for promoting social, educational and business networks. Instantaneous diffusion of information on a global scale offers tremendous advantages for ranking and rating products and services. Ranking and rating education and research have greatly altered the mindset of university students and professors. Fortunately, however, elementary schools and community colleges catering to local needs and custom often turn out to be a best bet for creative mathematics!

Facebook, YouTube and Twitter constituting the FUT triad has radically altered the means and modes of social, economic and cultural interaction. There is little doubt that technology and social media will dominate global business, politics, health care,tourism, trade and entertainment in the foreseeable future. And, hopefully, mathematicians will aid us rank all new information to assess their authenticity and educational merit.

The exhibit on dice furnishes a graphic illustration of exponential decay resulting from multilayered branching of beads wading through a calculated layout of pins. Eventaully the beads pile up following an exponential law of decay of radiactive materials; or, the loss of intensity of a light beam on a foggy day or inside a dusty room. Exponentials along with polynomials and trigonometric function account for a large bulk of available solutions for a large class of differential equations for modelling a wide variety of phenomena ranging from shapes assumed by soap bubbles and hanging cables to oscillating beams, plates and shells.

The exhibits under the theme of optimization are entertaining as well as illuminating at the same time. The first one concerns soap film shapes when stretched inside a wire frame. That all liquid films carry a small amount of tension is easy to prove. Just prick a soap bubble and see it explode like an inflated balloon. This explosion is a quiet affair owing to the extremely small pressure needed to inflate soap bubbles. Bubbles are spherical to minimize the surface area. Soap films drawn across wire frames with myriad shapes also accomplish the task of minimizing the surface area, but there is a key difference here. The wire frame acts like a constraining fence. Such problems classified as constrained optimization problems (COP) demanding enormous computing resources are the vanguard of finance, economics, insurance and investment banking.



The next exhibit further extends optimization fun. Referred to as the knapsack problem, the fun starts when you plan a picnic or a hiking expedition and you start packing for the trip. If you are allowed to carry only 20 kilos of food, cloth and water, how do you decide the amount of each item to carry, given a set of constaints? Though one can solve simple cases with a small list of items, the situation can rapidly turn into a numerical monster. There is a close link between this problem and the profound genius of Ramanujan who proposed a formula for partitioning a large number using integers.

The Kakeya needle exhibit seemed deceptively simple by challenging students to reverse a needle of a given length inside variously shaped spaces. A friend of mine who had accompanied me to the exhibition vividly recalled this exhibit and theorized how the Kakeya needle serves as an inspiring model for drivers while pulling their cars out of congested parking lots without rubbing or scratching other cars or cattle!

Maps and mapping have been a major part of mathematical history as vikings and pirates alike sailed the oceans, climbed mighty mountains and fought for land and fortune. Maps for keeping track of their exploits and treasures was a treacherous matter for the wise ones in those days. They devised ingeniously devious schemes and codes for tracking down town and country, islands and oceans not to mention other bygone relics like towers and pyramids. Some of their legendary secrets and codes have been immortalized through books, novels and movies. The exhibit on anamorphic maps offers a glimpse into the minds of those past masters who saw the world around them in different shapes and shadows creating spectacular charts and maps. Strategically exploiting the laws of projection and reflection anamorphic map was an instant favourite for all.

The exhibit on fractals introduced fractal geometry for mapping a rough, zigzag state boundary (Karnataka in this case). Fractal art and geometry shot to fame riding on the power of computer graphics. Fractal measures capture the intricacies of stochastic processes such as the brownian path followed by particles in turbulent flow; or, the volatile fluctuation of stock prices during wars and floods. With specific reference to this exhibit, the jagged state boundary can be measured to varying degrees of accuracy depending on the length scale being used in a particular measurement. The extreme case of a boundary which is everywhere continuous but nowhere differentiable is of infinite length enclosing a finite area! Similarly in three dimensions we can imagine a jagged solid of infinite surface area enclosing a finite volume. Some spiky seeds and fruits as also some leafy vegetables require fractal concepts to model their structure. These strange elusive concepts for describing objects only conceivable by mathematicians enormously help us in characterizing the structure of irregular shapes and contours.

The next exhibit continues with mapping the earth onto a foldable plane sheet of paper. This task requires carefully developing a curved surface into plane triangular segments of equal area. In this context, it is interesting to observe how 20 hexagons and 12 pentagons were sewn up to make soccer balls of yore (soccer balls today look different). Basically, a spherical surface can be tesselated with equilateral triangles. Here, the middle strip containing Asia, Russia and America also shows

the polar great circle passing through India. The earth is cut into 18 lunes just like cutting up a round fruit like melon or a cantaloupe into thin slices. This imaginative cut and fold view is called as Gauss-Kruger or transverse Mercator projection. For navigating seas or for surveying land, each great circle around the earth divided by 360 gives an angular length measure in degrees. When each degree is subdivided further into 60 minutes, each minute called a nautical mile measures 1.852 km. For example the distance along the great circle joining Bangalore and Denver can be estimated by supplying their latitudes and longitudes. Mapping the earth onto an octagon is another concept projected in this exhibit.

Modeling ecosystems is a prize-winning entry with seemingly smooth ecological variations, as one might find in a botanical garden. Reality, however, can be harshly different. Much of forest land of yesteryears has vanished or turned into desert just as rivers and oceans vanished under growing land mass. There is a serious threat of ocean waters inundating islands and shoreland in the event of global warming. Ecological shocks also imply changes in flora and fauna. The interactions between a large number of ecological variables are incredibly complex and codes which offer good short term predictions become unreliable for long term predictions.

Walk the function exhibit turned out to be good fun for putting some maths into human locomotion during walking, games or dance. It required us to walk on a straight line obeying a given sequence of velocity cues. These cues are commands for us to move along the command line both forwards and backwards like a drill. This will teach students how to program robots, toys, rockets and spacecraft to follow a given trajectory.

The remaining exhibits dealing with waves and oscillations celebrate one of the most passionate subjects in the long history of intellectual thought: mechanics or the study of motion. It is indeed amazing that powerful ancient observations and notions pertaining to the motions of planets and other heavenly bodies are remarkably true to this day. Great strides in celestial mechanics led to the even more deeper notions of waves and wave functions of quantum theory. Basically, however, the sensory world of physics is all about the flow of energy in space and time. Conversion and flow of energy manifest as waves and oscillations.

Studying the cyclic motion of a simple pendulum reveals a wealth of ideas to model the motion of astronomically massive objects ranging from satellites to neutron stars on the one hand to vanishingly small particles like electrons and photons. A general mix of linear and angular motion of objects persisting all the way down to the quantum scale causes strange patterns in space. The lissajous painting exhibit is an example of how two orthogonal wave motions can be combined creatively and artistically on paper.

The exhibit dealing with the oscillations of a chaotic pendulum demands much deeper mathematical understanding dealing with the initial state of motion or initial conditions. Understanding and modeling chaos is relatively recent when Lorentz wrote down a set of coupled differential equations for the velocity components in 1963. A massive amount of work has been done using super-

computers to model climate change.

The exhibit on harmonograph is another version of lissajous art that we saw in the pendulum painitng. Here, the two orthogonal oscillations damp out in time. The diminishing artistic loops appear like the vanishing shadow cast by a receding object.

The wave tube gave a graphic display of waves generated along the interface of two immiscible dissimilar liquids. Interfacial waves are genearted not only in liquids but also along solid interfaces. Invariably present in deep ocean as well as along tectonic plates, interface waves provide vital clues for monitoring sesmic or oceanic activity. Interfacial waves are almost always dispersive, but non-dispersive waves are possible in solids and liquids. Earthquakes and tsunamis propagate as surface waves wreak great havoc because they propagate at speeds too great to design early warning systems.

The exhibit on soliton was designed to look like a stable and sturdy chandelier. But when it was cranked up and released, it came alive with a spiralling soliton racing away from us like a dragon breathing fire. This eight feet long exhibit diplayed the terrific combination of size, shape and speed of a soliton to accomplish winning a marathon. Russel must have been indeed mesemerized by this unique power of a soliton as he chased one down astride his horse!

The Chladni plate is a vivid exhibition of modal shapes of thin plates and membranes used abundantly for designing musical instruments, chimes, toys and artifacts. We should also not forget that we create and sense sound using wafer thin skin, bone and tissue. The subect of studying sound or acoustics has been a major inspiration for physicists, philosophers and mathematicians. In this vein, well over two hundred years ago, Chladni inspired the mathematician and philosopher Sophie Germain. She formulated the laws of sound emanating from thin plates such as the one used in this exhibit. An interesting aspect of this exhibit concerned using bran for visualizing the modal patterns. These mode shapes evolve with the driving frequency in accordance with theory as long as the vibration amplitude is small. This is also true for mode shapes of the spring. Theoretical models for large amplitude waves and vibration are extremely difficult stemming from nonlinear effects. But there do exist some remarkably elegant solutions for nonlinear waves as in the magnificent case of the soliton.

The exhibit on sine wave is flexible sine wave wave model using tubes and strings hanging from a common support reminiscent of a suspension bridge in all its metallic glory of cables and columns. A common query by students is what makes a wave. This indeed is a profound question if one ponders on the first law of motion proclaimed by Newton to introduce the concept of inertia. Waves add another challenging dimension to reveal the physical nature of wave bearing solids, liquids and gases.

Thus, the exhibition visitors were truly swaying to the sights and strokes of a dizzy mix of exhibits on waves and oscillations. The interminable role of waves in everyday life extends all the way back to the explosive birth of our universe. Cosmologists are trying to record and reconstruct the first faint and distant signals of the Big Bang ripples propagating as gravitational waves 14 billion light years away. While the speed of light limits all physically sensible signals, mathematicians speculate

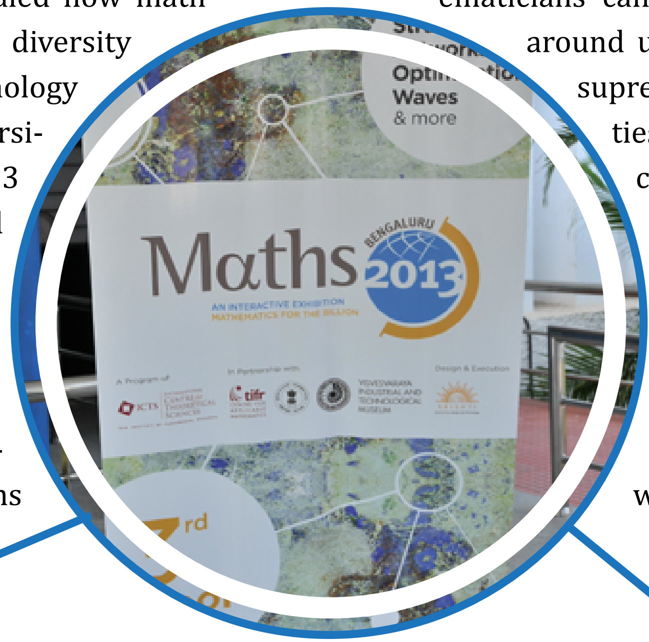
that the fragments hurled by the big bang might have initially moved much faster! There is no such uncertainty pertaining to waves and oscillations that we experience and witness in normal life. Our five senses for discriminating sight, sound, smell, taste and touch are primarily trained and tuned to classify and identify a specific wave type propagating at a speed (c), frequency (f) and amplitude (A). Large frequencies and amplitudes pose health hazards with the safety limits dictated by the product fA. The ratio of fA with respect to the wave speed fA/c is vital for designing safety goggles, gloves, guards, boots, ear muffs or helmets. While a gently swaying park swing can promote health and happiness, an impact generated by a high frequency jerk can easily break the spinal column.

Waves and oscillations enjoy a huge patronage from mathematicians for appreciating music, melody and art created by the fusion of light and sound waves in nature and instruments and engineering artifacts like musical fountains. Here again it is important to emphasize that music and melody are the result of artistically blended notes of different frequencies and amplitudes in the right sequence. Such sublime musical aspects have played a major role in the lives of many great scientists and mathematicians like Gauss, Euler, Raman and Feynman.

Depending on the media of propagation, waves of different frequencies can disperse away from the source; or, can steepen together into a shock. Dispersion implies a frequency dependent wave speed whereas the latter is a nonlinear process in which the wave speed increases with the wave amplitude. Under extremely special circumstances, the above two effects combine to generate a solitary wave or soliton.

Thus, in closing this epilogue, MPE-13 turned out to be surprise package for students, teachers and the general public alike. This exhibition dispelled some erroneous notions that mathematicians shy away from designing physical models for showcasing their ideas; Or another misconception that mathematics demands advanced computational tools, techniques and software; Or the misinterpretation that mathematics deals only with numbers, equations and theorems.

This exhibition revealed how mathematicians can creatively model all kinds of complexity and diversity around us. While global competition for science and technology supremacy has created many opportunities for universities and research institutes, exhibitions like MPE-13 can provide major inspiration for preparing and motivating creative young minds to pursue research in mathematics. In this regard, MPE-13 is a welcome whiff of fresh air for promoting mathematics. By hosting MPE-13, VITM also added another feather in its cap as it marches majestically through its Golden Jubilee celebrations which began in July 2014.



8 MPE 2013 - In India

Mathematics : a subject that is dreaded by an overwhelming majority of school children and a topic that is feared by an even larger majority of adults for the rest of their life. Of course, most people would also revere and admire mathematics as a beautiful and majestic subject, in much the same distant manner in which one would admire the cold and austere beauty of Everest, i.e., without having seen the object itself or without having been involved in anything close to the real experience.

But a similarly overwhelming majority of people do not realize that the mathematical sciences are indispensably woven into the modern social fabric. From economy to ecology, genetics to glacier dynamics, oceanography to optical fibre communication, physics to paleontology, telephone networks to traffic management, weather predictions to warehouse management - mathematics provides efficient and fundamental techniques to quantify the varied phenomena in the world around us, and also to understand and address challenges facing humanity.

Mathematics of Planet Earth 2013[2] **-**

MPE-2013 – was born out of the will of the world mathematical community to formulate the most urgent planetary problems that mathematics can address, to bring together world-class researchers to find solutions to these problems, and to engage the public in a dialogue about the significance of these problems. Over

[2]See the webpage http://www.mpe2013.org for further details.

100 organisations in more than 30 countries, including mathematical institutes, professional societies, research centres and teachers' associations, are partners in this initiative. In India, the International Centre for Theoretical Sciences of the Tata Institute of Fundamental Research (ICTS-TIFR), Bangalore, was a partner institution in MPE-2013.

As the MPE events around the world started materializing, the enormous response to these efforts and the scope of this initiative necessitated the continuation of MPE beyond the year 2013. Thus MPE will now be seen as a continuous effort towards advancing research on applications of mathematics to planetary issues, and towards articulating these applications in communications with the public through various media and with students through curricula in schools, colleges, and universities.

There are four broad themes of MPE:

(1) *A planet to discover* : Planet Earth and the solar system; Geodesy, geoscience, geography; Ocean, atmosphere, cryosphere; Weather, climate, natural resources

(2) *A planet supporting life* : Biodiversity; Ecology, evolution; Sustainability; Water, agriculture, aquaculture

(3) *A planet organized by humans* : Social-economic systems; Ecosystem management; Renewable energy; Transport and communications networks

(4) *A planet at risk* : Climate change; Natural disasters; Ecosystem regime shifts; Infectious diseases; Sustainable development; Invasive species,

The list above is only indicative and not exhaustive or exclusive, and will certainly evolve over time.

Public outreach activities formed a major portion of MPE-2013. One of the main MPE outreach events in India was an exhibition about mathematics and its applications. This interactive exhibition aimed at displaying applications of mathematical concepts through visual and physical models that most effectively express the vision of the MPE initiative. It was designed to make visitors with little mathematics background feel at ease with mathematical ideas, through participatory activities, and through the explanations given by enthusiastic facilitators.

The planning for this exhibition began almost three years before the actual exhibition, which was held from 22 November until 20 December 2013. It was an ICTS-TIFR program organized in collaboration with the TIFR Centre for Applicable Mathematics, Bangalore, the Visvesvaraya Industrial and Technological Museum, Bangalore, and the National Council for Science Museums, and was designed and executed by a joint team from these organizations along with the Srishti School of Art, Design and Technology, Bangalore.

This booklet is an attempt to capture the excitement and enthusiasm generated during the process of creating this exhibition and to present ideas that were displayed as interactive exhibits. We will introduce the main mathematical concepts as well as their relevance to many real world situations where they are applied. We will also present the designs and photographs of most of these exhibits, with the aim that they may be reproduced by any individual or group that may be interested.

The process of creation of these ideas and the physical exhibits themselves was an extremely enriching and stimulating experience for all of us involved in this process, and we would highly recommend the readers to make an attempt to create these exhibits for themselves. We would be even more delighted if you can improve upon or modify these designs to emphasize certain additional conceptual aspects, or just to clarify the concept more crisply, or to make the exhibit more appealing and interesting to the viewer. Please do share these ideas with us and we can include them on the website created specifically for this exhibition.

There were five main themes of the MPE exhibition.

(1) Structures
(2) Networks
(3) Oscillations
(4) Optimization
(5) Waves

Most of the exhibits were categorized under one of these themes and are presented in respective sections of this booklet. There were a few additional exhibits that are presented in the final section. Some reflections about the experience of going through this exhibition are contained in the epilogue, written by one of the visitors who is also an enthusiastic supporter of the creation of this booklet.

The team behind MPE exhibition.

The MPE planning meetings that took place at the American Institute of Mathematics, Palo Alto, CA in March-2011 and March-2012, and a festival of astronomy called "Kalpaneya Yatre" at the Jawaharlal Nehru Planetarium, Bangalore from 25 November to 5 December, 2010, were the two main catalysts in formulating the idea of an exhibition related to the theme of Mathematics of Planet Earth.

The exhibition was a close collaboration between various institutions.

- This event was a part of the MPE-2013 program (http://www.icts.res.in/program/MPE2013) of the International Centre for Theoretical Sciences of the Tata Institute of Fundamental Research (ICTS-TIFR), which is the main Indian partner of the MPE initiative and which provided the necessary resources to carry out all the activities related to the exhibition.

- TIFR Centre for Applicable Mathematics (CAM) was the main academic partner of this exhibition.

- The exhibition would not have taken place without the partnership with the National Council of Science Museums (NCSM), and the Visvesvaraya Industrial and Technological Museum (VITM), Bengaluru, which made available the whole of the third floor exhibition hall and the various venues for conducting the outreach workshops, and whose enthusiastic staff gave indispensable support during all stages and literally put and held the exhibition together in their workshops and the exhibition halls!

- The design and execution of the exhibition was done mainly by the Srishti School of Arts, Design and Technology, Bangalore, jointly with VITM.

- Jain University, St. Josephs College, Christ University, provided wonderful support for conducting the workshops related to the MPE exhibition, in particular for training the volunteers.

- Navnirmiti (Mumbai and Pune), the Vikram A. Sarabhai Community Science Centre (VASCSC - Ahmedabad), and the Bharat Gyan Vigyan Samiti were the partners in organization of the many outreach workshops conducted during the exhibition.

Here we would also like to make note of

the people behind this exhibition and the various roles they played. Of course, we understand the risk of inadvertently forgetting names while making such a list more than 15 months after the event, and with apologies to those who have been left out below, we venture ahead.

- Constant support and encouragement: Spenta Wadia and Avinash Dhar

- A series of discussions with a cross-section of people including scientists, designers, artists, etc., for developing the idea of the exhibition: B.S. Shylaja, Geetha Narayanan, H.R. Madhusudan, R. Niruj Mohan, R. Ramanujam, Ramana Raju, Rustam Vania, Sabina von Kessel, Seema Nanda, Sujatha Ramdorai.

- The core team, consisting of those who have contributed to writing signicant parts of this booklet and were the main source of most of the mathematical ideas, stolen or otherwise: Amit Apte, Binoy, C.S. Aravinda, Mythily Ramaswamy, Sreekar Vadlamani, Sudhir Rao, Sunil Kumar, Vikram Aithal.

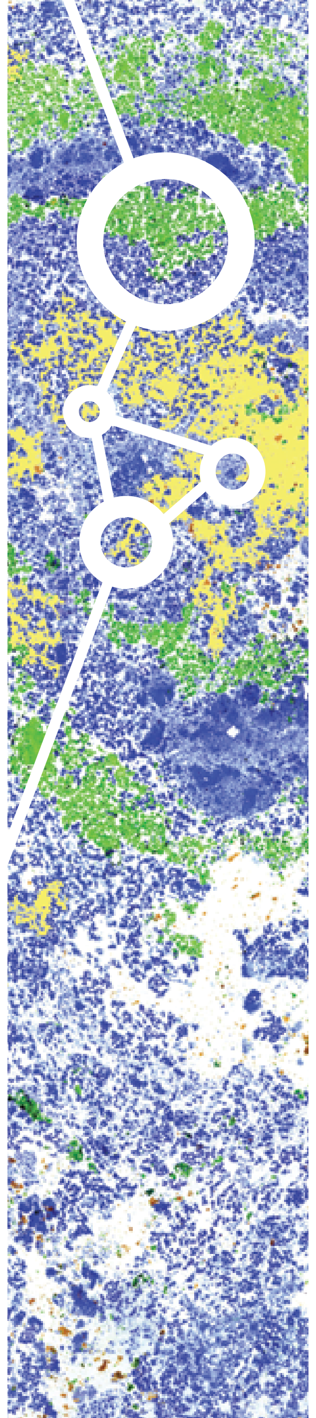
- The design team, mainly from the Srishti School of Arts, Design and Technology, that converted the mathematical ideas into insightful, stunning exhibits, created the informative and attractive posters, panels, and other visual material, and planned and executed the layout of the exhibition: Agnishikha Choudhuri, Ajai Narendran, Debjani Banerjee, Gautam Dayal, Lavanya, Mary Jacob, Naga Nandini, Sharath, Sonalee Mandke, Sudipto Dasgupta, Sunil Kumar.

- The Srishti team also included the students of Srishti who chose to work on the MPE exhibition as part of their courses, or voluntarily. They really made this process of converting “boring” mathematical ideas into exciting exhibits a lively and cheerful semester-long activity.

- A large set of the mathematical ideas were formulated during the 2013 summer visiting students research program of TIFR-CAM through the initiative of the students who took part in this program: Aishwarya Ramasethu, Akshay S.H., Ankit Jain, Arun Reddy, Debashis Chatterjee, Kavan Ganapathi, Naveen Kumar, Polala Arun Kumar, Priyankur Chaudhuri, Ravi Teja, Riya Thomas, Rohan Hemanth, Saniya Arora; And TIFR-CAM students and postdocs who participated in these discussions: Deep Ray, Sajini Anand, Surabhi Pandey.

- The contagious enthusiasm of everyone at VITM and NCSM throughout this whole partnership was invaluable: G.S. Rautela, K.G. Kumar, Madan Gopal, Muthu Kumar, Sajoo Bhaskaran.

- Special mention must be made of the enthusiastic team from WorldServe, for not only creating the “walk the function” exhibit and the workstations showing the



videos, but also staying at the exhibition almost the whole time in order to fix the many computer and other issues and volunteer to explain the exhibits to visitors: Sudhir Rao Rupanagudi, Ranjani B.S., Varsha G. Bhat, Vikas S., and Vivek C. Bharadwaj.

- The outreach activities including the workshops for the school children and teachers and others were received with enthusiasm and were a result of the amazing work of dedicated teams: Dilip Surkar, Hemaben Vasavada, Lata Torvi, Neelam Mishra from VASCSC, Ahmedabad; Vivek Monteiro, Vipula Abhyankar, Varsha Khanwelkar from Navnirmiti, Mumbai and Pune; Prashanth, Jaykumar from BGVS, Bangalore.

- The most important part of the exhibition was really the presence of volunteers from many different institutions from Bangalore: St. Joseph’s, Mount Carmel, MS Ramaiah Institute, Christ University, Jain University, IISc (UG students).

- The staff of the ICTS-TIFR and TIFR-CAM provided excellent support for all aspects of the program including logistics, accounting, public relations, and many others.

- The last person to be mentioned here played the most important role during this entire enterprise: the MPE scientific outreach coordinator, Bhakti Dhamdhere.

Volunteers for the MPE exhibit :

Christ University:

Anvitha K J
Bijal Modi
Divya Bharathi MS
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