Information Theory and its applications in Combinatorics and Computation (Outline)

Day 1: Shannon entropy

We will review the problem of information compression and show the role that entropy of a probability distribution plays in it. We will analyse compression in the one-shot and the iid settings, leading up to Shannon's source-coding theorem.

Key concepts: Prefix-free encoding, typical sequences, rate of a code.

Day 2: Conditional entropy and mutual information

We will consider the problem of transmitting information over a noisy channel, and arrive at Shannon's channel-coding theorem, in which the notion of mutual information of random variables plays a central role.

Key concepts: Conditional entropy, chain rule for mutual information, capacity of a channel.

Day 3: Entropy and its applications in combinatorics

We will see how standard inequalities about information lead to intuitive proofs of several combinatorial inequalities.

Key concepts: Loomis-Whitney inequality, Bregman's theorem, Moore bound

Day 4: Relative entropy in statistics

We will more carefully examine relative entropy and its properties, and understand its connections to other distance measures between distributions and its role in statistics.

Key concepts: Stein's lemma, Sanov's theorem, Pinsker's inequality, Fidelity.

Day 5: Communication complexity of functions

We will introduce the area of communication complexity and present some applications of information theory to this area.

Key concepts: Transcript, the communication matrix.

Resources

I will consult these sources, but we will not go through any of them cover to cover.

Days 1 & 2

A. I. Khinchin, *Mathematical Foundations of Information Theory*, New York: Dover, 1957. ISBN 0-486-60434-9.

Cover, Thomas; Thomas, Joy A. (2006). *Elements of information theory* (2nd ed.). New York: Wiley-Interscience. ISBN 0-471-24195-4.

Csiszar, I, Korner, J. *Information Theory: Coding Theorems for Discrete Memoryless Systems* Akademiai Kiado: 2nd edition, 1997. ISBN 963-05-7440-3.

Yeung, RW. *A First Course in Information Theory* Kluwer Academic/Plenum Publishers, 2002. ISBN 0-306-46791-7.

Himanshu Tyagi. Information Theory. NPTEL Course. https://onlinecourses.nptel.ac.in/noc20_ee96/preview

Day 3

David Galvin. *Three tutorial lectures on entropy and counting,* https://arxiv.org/abs/1406.7872.
Friedgut, Ehud. *Hypergraphs, Entropy, and Inequalities. The American Mathematical Monthly* 111, no. 9 (2004): 749–60. https://doi.org/10.2307/4145187.

Day 4

Sergio Verdu. *Relative Entropy.* NIPS Conference 2009 - Vancouver. https://videolectures.net/videos/nips09_verdu_re
Edward Witten (2020). *A mini-introduction to information theory.* La Rivista del Nuovo Cimento, 43(4), 187-227. https://arxiv.org/abs/1805.11965
Vlatko Vedral (2002). *The role of relative entropy in quantum information theory. Reviews of Modern Physics*, *74*(1), 197. https://arxiv.org/pdf/quant-ph/0102094
Clément L Cannone. *A short note on an inequality between KL and TV.* arXiv:2202.07198 (2022).

Day 5

Anup Rao, Amir Yehudayoff. *Communication complexity and applications.* Cambridge : Cambridge University Press, 2020.

The physics of information

Zack Savitsky. *What Is Entropy? A Measure of Just How Little We Really Know.*

https://www.quantamagazine.org/what-is-entropy-a-measure-of-just-how-little-we-really-know-20241213/#comments

Gregory Falkovich. *The physical nature of information: a short course*.
https://www.icts.res.in/sites/default/files/seminar%20doc%20files/PNIreduced.pdf
https://www.icts.res.in/program/bssp2024/talks