Classical Electromagnetism (ICTS Core Course for both Int.PhD.,PhD.)

Instructor : R. Loganayagam.

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Basic Info :

- **Instructor name :** R.Loganayagam,
- **Tutors name :** Junaid Majeed, Basudeb Mondal .
- **Venue :** Feynman Lecture Hall, ICTS Campus, Bangalore
- **Timings :** Tuesday and Fridays, 11:00-12:30 AM(Tentative)
  Tutorials on Wednesday 3:00-4:00 PM(Tentative)
- **First Class (Introduction) :** Friday (02:00 pm - 05:00 pm), 3rd January, 2020
  Emmy Noether room (Note unusual venue/timing for the first class.)
- **Drop test :** Jan 25th 2020 (tentative)
- **The grading policy** will be based on the following weightage :
  - Quiz/Tests during Tutorials : 10% for Int.PhDs, 5% for PhD. students
  - Assignments : 30%
  - Mid term Exam : 30%
  - End term Exam : 30%
  - Term paper (a thorough review of a topic in electromagnetism not covered in textbooks below, see below for suggestions) : 5% Extra credit (Compulsory for PhD Students)

Note that Assignments form a central part of this course, since one of the main aims of this course is to train students to solve problems. .

Here are some of the candidate topics for the term paper :

1. Kinetic theory of Collisionless plasmas and Landau damping
2. Alfven's theory of charged particle orbits and application to charge motion in Earth's field
3. Electromagnetic theory of Rainbows/Glories
4. Physics of Lightning

5. Magneto-Hydro dynamics(MHD), anti-Dynamo theorems and dynamo models of Earth's magnetic field
6. Hodgkin-Huxley model, ion channels and electrical conduction in nerves
7. Electrophysiology of electric fishes
8. Berry phase and Modern theory of polarisation
9. Chern-Simons terms/Axionic electrodynamics, topological insulators and Hall effect
10. Magnetic monopoles, Witten effect and electric-magnetic duality
11. Magnetic vortex solutions in Ginzburg-Landau model of superconductivity (Abelian Higgs model)
12. Gaussian beams and Laser resonators
13. Born-Infeld electrodynamics and its solutions
14. Tensor multipole expansion for gravitational radiation
15. Electrodynamics as a conformal field theory, generating solutions using conformal transformations

If you want to write a term paper on a topic which does not appear in this list, meet me and we can discuss the possibility.

Course contents:

The traditional topics in a course of this type include

- Static boundary value problems : Green functions, multipole methods in electrostatics/magnetostatics
- Radiation theory, multipole expansion methods and basics of scattering/diffraction
- Introduction to Special Relativity, Covariant formulation of electrodynamics
- Symmetries, Conservation laws, Lagrangian and Hamiltonian formalism in electrodynamics
- Electromagnetic properties of matter

There will be a drop test testing on the above topics on Jan 25th 2020(tentative). I will require all of you to take it. If you do well, you do not have to credit the course.

If done in full detail (say as done in any of the main textbooks below), this is a lot of material for one (or even two) semester course ! We will try to cover at least four of these five topics in some detail. The main texts which cover this material are

- Classical Electrodynamics by J. Schwinger, L. L. DeRaad, Jr., K. A. Milton, and W-y. Tsai.
- Classical Electricity and Magnetism by Wolfgang K. H. Panofsky and Melba Phillips.
- Modern electrodynamics by A. Zangwill

- Classical Electrodynamics by J.D. Jackson

All of these are very well-written books (except perhaps Jackson which is good if you treat it as a collection of encyclopaedia-type sections rather than a textbook). But I will not rigorously follow any of these textbooks and I will supplement it with my own notes/other books.

I will recommend that the ICTS students who have to credit the course use their contingency grant try to buy a personal copy of either one or both of Schwinger et.al (Indian edition cost $\sim$ Rs.1000) and Jackson (Indian edition cost $\sim$ Rs.800). You will need them for the reading/problem sets, class/tutorials and the open book tests/exams. There is also a cheap edition of Panofsky and Phillips (Indian edition cost $\sim$ Rs.700) available in case you are interested, but Zangwill alas has no Indian edition yet and is quite expensive !

Some more modern aspects which I hope to at least touch upon briefly :

- Mathematical aspects of Gauge invariance, Charged fields, Superconductivity, Josephson junctions
- Materials with electromagnetic response beyond the dielectric-magnet-conductor triad (piezoelectrics, Hall insulators, topological insulators etc.), emergent abelian gauge fields
- Plasma physics, MHD, atmospheric/geophysical/astrophysical electric and magnetic fields
- Electrokinetics, Nernst-Planck equation, ion transport electric double layers and electric fields in Biology (in nerves, heart, brain etc.)

But, no guarantees !

Prerequisites:

I am assuming you are familiar with the following topics which are standard during a first course in electromagnetism :

- Vector analysis: gradient, divergence, curl, divergence theorem, Stokes' theorem.
- Electrostatics : Poisson equation, Boundary value problems, image problems, multipole expansion.
- Electric field and potential in matter, polarisation.
- Magnetostatics, magnetic field in matter, magnetisation.
- Electrodynamics: Faraday's law of induction
- Maxwell's equations/Electromagnetic waves in free space and in matter.
- Geometric/Ray Optics , wave optics.

This is a short summary of the detailed pre-requisites discussed during our meeting in November. If you want the detailed version, ask your tutors.

The basic texts that cover these pre-requisites are

- Feynman Lectures on Physics Vol 2 by Feynman, Leighton and Sands

- Electromagnetic Fields and Energy by Hermann A. Haus and James R Melcher
- Introduction to electrodynamics by David J.Griffiths
- Classical Electrodynamics by Greiner
- Electricity and magnetism by Edward M.Purcell and David M.Morin

If you feel that in a particular topic you are underprepared I will strongly recommend to look at the relevant chapters in these books. You should be roughly be able to read and comprehend first 75% of these books. Let me know if that is not the case.

There will be a set of preliminary tests (take home) (first one will be on 3rd January) so that I can have an idea of where the class stands regarding these topics.

Apart from these, it is good to revise various things from previous courses

- Classical mechanics : Lagrangian, Hamiltonian, Canonical transformation, Noether theorem
- Quantum mechanics : Rotations, Spherical harmonics/theory of angular momentum, Scattering theory of waves
- Statistical mechanics : Basic kinetic theory, Partition functions etc.
- Mathematical physics : Fourier/Laplace transforms, delta functions, complex analysis (especially residue theorem and contour integrals), PDEs (especially second order PDEs like wave equation, Poisson equation, Heat equation), Green functions, variable separation, Sturm-Liouville theory, Bessel functions, Legendre polynomials etc.

Course Philosophy :

Broad motivation :

I think of the theoretical physics graduate course curriculum as being divided into two halves : first, an 'un-compromisable' core minimum of five subjects in which a student should have a thorough critical and analytic skills. These are

1. Classical mechanics (at the level of the standard textbook by Goldstein or Landau-Lifshitz Vol. 1)
2. Quantum mechanics (at the level of the standard textbook "Modern Quantum Mechanics" by Sakurai)
3. Statistical mechanics (at the level of the standard textbook by M. Kardar or R. K. Pathria)
4. Mathematical Physics (at the level of the standard textbook by K. F. Riley, M. P. Hobson and S. J. Bence)
5. Classical electrodynamics (at the level of standard textbooks mentioned above)

I feel that it is very difficult to go very far in theoretical physics without a deep and thorough understanding of these subjects. To work on a research problem, this needs to be supplemented by 3 to 4 courses in the appropriate research area.

My reason for this opinion is as follows : each of these core subjects introduce a particular worldview which has been very successful in our understanding of physical reality. And these viewpoints recur again and again in theoretical physics (in fact, I would define a theoretical physics problem as one where this happens). So, let me begin by describing the worldview of classical electrodynamics.

Importance :

First some quick history. It is often said that electrodynamics was historically born out of unification of two forces - electric and magnetic. In fact, I think a more historically precise statement is that it combines *six* different phenomena known to our human ancestors :

1. **Magnets**, magnetic compass and magnetism of the earth. Magnets and the fact that they seem to act on each other at a distance mystified and fascinated the ancients.

2. **Lightning** and atmospheric phenomena like Aurora Borealis. (Eventually, the ionosphere)

3. Energy behind **biological dynamics.** e.g., forces applied by humans via their **nerves and tissues**. The ancient notions of 'life force', 'vital energy' etc., including talks about Ase (Nigeria), Qi (China), Manitou (North American tribes), elan vital (Europe), some notions of Prana (India) are all pre-Maxwellian theories of phenomena which are now known to be electric in their origin.

4. The forces born out of **materials and chemicals** including the forces behind **friction** and **chemical reactions**.It is human fascination with friction that led to the discovery of fire. The fascination with fire and transformations induced by it is in turn at the heart of alchemy.

5. **Electrostatic** forces obtained by rubbing various materials against each other.

6. Phenomena associated with **light** and optics in general.

In a remarkable turn of history filled with a long array of accidental experimental discoveries, these forces and energies were understood to be one and the same. This is perhaps the first lesson of electrodynamics : *much of the physical reality we see around us have common origins.* Textbook history of physics tends to somewhat overemphasise the history of gravity. But I believe there is a way of looking at the history of physics and even the entire history of humanity as mirrored in the history of electromagnetism.

What are the other lessons of electrodynamics ? They include

- Convenient description of reality is in terms of **fields**. These fields have a physical reality of their own, e.g., they carry energy and momentum. The fields evolve according to certain **partial differential equations** with appropriate **initial conditions/boundary conditions**.

- The microscopic forces except gravity are all described in fact by a pair of **electric** and **magnetic** fields acting between **charges** and **currents**. The charges produce fields and the fields in turn affect charges.

- Often, fields leave the charges to travel far in terms of **waves**. These waves can undergo **reflection, refraction, interference and diffraction** in various media.
- In vacuum, these waves can travel very fast (in fact, with the maximum possible speed) and are thus **relativistic**.
- In media, the **collective behaviour** of charges leads to an **effective/averaged description** whereby the forces are screened/enhanced/modified into something entirely new. The waves **disperse** in a frequency dependent way and their **effective speed of propagation** also becomes frequency dependent.
- At temperatures much below their frequencies, these waves start behaving like particles thus leading to **quantum behaviour**.
- When the reverse effect of particles behaving like waves is taken into account, one then forms **stable bound states/novel phases** (like atoms, molecules, solids, liquids etc.).

The reader will notice that these highlighted ideas are absolutely fundamental to how a physicist (especially theoretical physicist) views the world. It is a way of thinking, analysing and arguing about physical phenomena and one of the primary aims of an electrodynamics course is to train a student to do that.

Many textbooks give the false idea that classical electromagnetism is a finished subject. This is NOT true. The four oldest problems of electrodynamics : that of magnetic phases of matter, that of dynamics of geo-magnetic field, that of lightning, that of electric/magnetic field generation and detection by animals and plants are all active areas of research. Historically, these problems motivated the pioneers of electromagnetism from Gilbert to Galvani to Gauss, from Franklin to Faraday. Despite centuries of work, they still throw up puzzles which baffle us. For some unknown reason, many books pretend such questions do not exist.

This is apart from the new challenges which were unknown few decades ago. The electric/magnetic response of Hall systems from 1980s has brought into physics surprisingly sophisticated mathematical ideas. The theory of ferro-electrics (called Modern Theory of Polarization) was really understood only in early 1990s. The theory of insulators (usually thought of as boringly simple) threw up surprises over the last decade whose resolution has led to the theory of topological insulators. Ferromagnetic and anti-ferromagnetic order, of course, dominate so much of modern condensed matter. Emergent 'new' kind of electromagnetic fields and the physics of low dimensional electromagnetism continue to puzzle and fascinate us. So much of the modern world is built on our understanding of electromagnetism that I feel it ridiculous to even attempt at describing how much.

This course also serves as a stepping stone towards more advanced courses of the graduate curriculum. To name a few : quantum and statistical field theory, astrophysics (via plasma physics), condensed matter physics, General relativity, particle physics (more generally high energy theory including string theory). In reality, it is almost impossible to find something in physics where the tools and viewpoint of electromagnetism are not employed at least indirectly. To summarise, this is a **very important** course.