String Theory I (ICTS Reading Course)

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Basic Info :

- **Instructor name :** R.Loganayagam,
- **Venue :** Feynman Lecture Hall, ICTS Campus, Bangalore
- **Timings :** Wednesday and Friday, 11:00-12:30 AM(Tentative)
- **First Class :** Wednesday (10:00 am), 7th August, 2019
- **Structure of the course :** The reading course has three components : Presentation/Class participation, assignments and exams.

  Presentations will be twice a week (1.5-2hrs each) where all students take turns in reading the assigned text and presenting them. I will start off the course with a set of 6 to 8 lectures (i.e, $3 - 4$ weeks) giving a brief survey at the level of basic textbooks mentioned below.

  Assignments will be a set of problems on various modules which need to be handed over by those who are crediting the course. Since I do not really have a TA for this course, I want the students who credit this course to grade each others' assignments.

  There will be a mid-semester and an end-semester exam (the latter can be replaced by a term-paper, see below for details).
- **The grading policy** will be based on the following weightage :
  - Class presentation/participation : 20%
  - Assignments : 40%
  - Mid term Exam : 20%
  - End term Exam (or) Term paper : 20%

Term paper :

Term paper should be a detailed review of a topic in string theory (see below for suggestions). A first draft of the term paper should be submitted by the middle of the semester to qualify for an exemption from the end term exam.

Here are some of the candidate topics for the term paper :

1. Topological string theory and Chern-Simons theory
2. Large N 2d QCD as String theory
3. String theory and non-commutative geometry
4. Heterotic string theory and its spectrum and dualities
5. Boundary state for D Branes and Orientifolds
6. Integrability of Classical string theory in $\mathrm{AdS}_5\times \mathrm{S}^5$
7. $\mathcal{N} = 2_{2d}$ minimal models and Gepner construction
8. Calabi-Yau compactifications and their low-energy physics
9. Seiberg-Witten theory and its stringy origins
10. D-Brane couplings to RR fields : anomaly cancellation and T duality
11. Pure spinor formulations of superstring
12. BMN limit of $\mathcal{N} = 4_{4d}$ SYM and strings in plane wave background
13. stable non-BPS states and Tachyon condensation in string theory
14. Proof of Unitarity within string field theory
15. Super-Riemann surfaces and their moduli space
16. Mirror symmetry and enumerative geometry
17. Classification of supersymmetric and superconformal algebras
18. AdS_3, symmetric orbifold and its higher spin symmettries

If you want to write a term paper on a topic which does not appear in this list, meet me and we can discuss the possibility.

As fun, here are a set of open problems in free string theory. If you solve any one of them, you can write a paper on it and submit it to arXiv. I will be happy to consider that as a term paper. But, do remember that most of these are difficult problems which have eluded the best string theorists for many decades...

1. Free String theory dual of large N QCD and its spectrum
2. Quantum Superstring spectrum in a background with finite Ramond-Ramond flux

3. Finding the stable vacuum of closed bosonic string theory
4. Quantisation of membranes
5. Derivation of Vassiliev higher spin gauge theories from String field theory
6. Schwinger Keldysh path integrals/out-of time-ordered correlators in free string theories

Pre-requisites:

I will assume a thorough familiarity with the five foundational subjects of theoretical physics: Classical mechanics, Quantum mechanics, Statistical mechanics, Electrodynamics and Mathematical physics. This means you should be able to solve any problem that is taken from standard books on the subject.

Apart from that, the main pre-requisites are Quantum field theory and General Relativity, at the level of following textbooks.

- QFT textbooks : Mark Srednicki, Matthew D. Schwartz, Michael E.Peskin-Daniel V. Schroeder, Steven Weinberg
- GR textbooks : Sean Carroll, Robert M. Wald, Charles W. Misner-Kip S. Thorne-John Archibald Wheeler .

It would be good to brush up especially on following topics :

- Poincare Lie algebra and its massless and massive particle/field representations
- Relativistic field theories of
  - real/complex scalar (Klein-Gordon) field,
  - Dirac/Weyl/Majorana spinor fields,
  - Maxwell/Yang-Mills vector (gauge) fields

  their action, equations of motion, symmetries and Noether currents, classical solutions and quantisation
- Notion of propagator, Wick's theorem, perturbation theory via Feynman rules/diagrams to compute correlators, computation of S-Matrix
- Loops, counter-terms, renormalisation, beta functions, anomalous dimensions etc.
- Path integral formalism for quantum mechanics (especially harmonic oscillators), coherent path integrals
- Grassman number calculus, path integrals over Grassman odd fields
- Gauge fixing, Faddeev-Popov ghosts, BRST invariance in gauge theories
- Goldstone theorem, Anderson-Higgs mechanism

- Basic differential geometry of spacetime (Covariant/contravariant tensors, geodesics in curved spacetime, Christoffel symbols, Riemann tensor, Bianchi identity, Killing vectors, Lie derivatives, maximally symmetric spacetimes etc.)
- Einstein equations, Linearised Einstein equations and their solutions

There will be a initial take-home test on these topics around 14th Aug, 2018. Let me know if you are not comfortable with any of the topics above. If a substantial majority of class feels that way, within reason, we can have some extra lectures to be given by students and I can design some practice problem sets.

Perspectives from particle physics, condensed matter theory, cosmology etc., while much useful to a practicing string theorist, show up only somewhat implicitly at a course at this level. However, you may appreciate some of the things better with that kind of background.

On the other hand, perspectives from advanced Mathematical physics (by which I mean topics like Grassmann calculus and supersymmetry, gauge theories, BRST-BV formalism and related cohomological techniques via ghosts, moduli space of Riemann surfaces, representation theory of finite/infinite Lie algebras which are not covered in the basic mathematical physics books) often turn up and you should be willing to pick them up as we go along.

Course contents:

The primary aim of the course would be to familiarise the student with relativistic free (i.e., non-interacting) strings, their quantisation and spectrum. We will also try to set the stage for learning about interacting strings, but this is difficult in a single semester.

To situate the context, let me add that it is useful to think of learning string theory as going through various stages :

- Free string theory using world-sheet methods
- Weakly interacting string theory (including string perturbation theory, supergravity)
- Strongly interacting string theory and the network of string dualities
- Field theory - String theory dualities (especially AdS/CFT correspondence and the methods of conformal field theory)
- Topological string theory, Mirror symmetry etc.

Each of these is a semester worth of course by itself. However, no textbook takes on this entire breadth of topics yet. Contemporary string theory courses often cover a subset of these topics laying the foundations and leave the rest for the student to learn on their own/at schools .

Our goal here this semester is to focus on the first topic in this list with the aim of establishing a foundation on which the rest of the topics could be learnt. Next semester, I plan to have string theory II covering the second topic.

Why does studying a free theory (which is essentially a study of harmonic oscillators) occupy most of a semester ? One reason is that students are usually somewhat unfamiliar with the main technical tools necessary for free string theory : QFT in first quantisation, BRST formalism, 2d CFT methods and supersymmetry (including Fermions in higher dimensions). Most of the time in a first course is soaked up in learning this stuff. At a more conceptual level, even when there are no interactions, relativistic string is a notoriously fragile system and difficult to quantise without quantum fluctuations destroying it.

One can successfully quantise a string only when special conditions are met. For example, the spacetime that it lives in should not have too many dimensions : as we increase the dimensions of space, the string has more directions to oscillate in and the quantum fluctuations become wilder. Above a high enough dimension called the critical dimension, quantum corrections destroy the string. Mathematically, this shows up as a statement that a quantum Hilbert space of states can no more be constructed, if the dimensions are too high. This anti-higher-dimensional nature of string theory should be contrasted with particle field theories which do not have any such restriction.

There are many other constraints that need to be met to make sense out of a free string theory. Another important constraint is that the fields/objects that string couples to should also be quantised : whether it be a gauge field or the metric in the spacetime or it be a 'Dirichlet Brane' (D-Brane), i.e., a membrane surface on which the open strings end (one can often think of these as membrane shaped black-holes). This leads naturally to quantum gauge theories, quantum gravity or quantum theories of relativistic membranes. While quantum gauge theories are understood to some extent, we still struggle a lot with the latter two : so, this is indeed fortunate. Again, all this should be contrasted with particle theories, where it is not impossible to think of a particle moving in curved spacetime without gravity being dynamical or a test charge moving in a non-dynamical electric field. Our focus would be on a detailed study of conditions that a string requires before it is quantised and how trying to satisfy them lead to the emergence of dynamical membranes/gravity/gauge theories.

Our aim is to cover both bosonic and various types of fermionic/supersymmetric string theories, both open and closed strings, both oriented and un-oriented strings. We will go over the three traditional methods of quantisation which proceed via the world-sheet of the string : Light-cone quantisation (LCQ), old covariant quantisation (OCQ) (also termed Gupta-Bleuler quantisation) and BRST formalism. The fermionic/supersymmetric string theories have two main versions : the Ramond-Neveu-Schwarz (RNS) formalism and Green-Schwarz (GS) formalism. A new method for supersymmetric string theories called 'pure spinor formalism' inspired by GS formalism also has gathered a lot of attention in recent years and we will try to look at it briefly.

An active area of current research is string field theory, which is a second quantised way of looking at string theory and is necessary, for example, for understanding things like renormalisation in interacting string theory. Many unsolved problems in free string theory (some of which we will go over) seem to require

Each of these methods have their merits and demerits with the first few giving a quick intuition for the physics. The latter formalisms (e.g., BRST formalism) are more elaborate in the mathematical machinery they employ, but still the most useful when dealing with interactions. The latter formalisms also lay a greater emphasis on world-sheet conformal/super-conformal field theory, a subject which will occupy most of the course. Apart from the world-sheet methods, we will try to cover as much as possible about D-Branes on which open strings end.

A slight variant of free string is a string in a box (in complete analogy with free particle in a box considered in Quantum mechanics). If the box has periodic boundary conditions, one encounters a surprising duality called T duality. This is a good example to learn how 'stringy geometry' differs from classical geometry and is the physical setup behind the mathematical idea of mirror symmetry. They also are the simplest examples of the rich web of dualities that connect various string theories. They can be studied within free string theory (though their importance really stems from how they hold even in the presence of interactions). Another set of topics beyond world-sheet methods is supergravity (i.e., supersymmetric theories of gravity). This requires us to learn some amount of supersymmetry, and is somewhat time-consuming. So, I am not sure how much of it we would be able to do, but we can try.

Textbooks :

We list a set of useful references for this course below. We will mainly use the textbooks by Zwiebach, Green-Schwarz-Witten part I and Peter West for this course. But, many other references will be used as we go along.

An introductory survey of some of the topics we intend to cover can be found in

- A First Course in String Theory by Barton Zwiebach
  Covers in detail the free Bosonic string theory in LCQ
- Introduction to Strings and Branes by Peter West
  Focus is on free string theory.
- An Introduction to String Theory and D-Brane Dynamics by Richard J. Szabo
  An overall survey ending in derivation of D-Brane world-volume theory
- String Theory Demystified by David Mcmahon
- A Primer on String Theory by Volker Schomerus
- Lectures on String Theory by David Tong
  `http://www.damtp.cam.ac.uk/user/tong/string.html`
  These two are good overall surveys with more details than Szabo's book.
- Lectures on String Theory by Dieter Lüst, Stefan Theisen
  `https://link.springer.com/book/10.1007%2FBFb0113507`

Advanced Textbooks which go into more depth are

- Superstring Theory (2 vols) by Michael B. Green, John H. Schwarz, Edward Witten
- String theory (2 vols) by Polchinski
- Basic Concepts of String Theory by Ralph Blumenhagen, Dieter Lüst, Stefan Theisen
  `https://link.springer.com/book/10.1007%2F978-3-642-29497-6`
- String Theory in a Nutshell by Elias Kiritsis
- String Theory and M-Theory: A Modern Introduction
  by Katrin Becker, Melanie Becker, John H. Schwarz
- D-Branes by Clifford V. Johnson

Some very good reviews which are as good as textbooks :

- Sigma Models and String Theory by Curt Callan and Larus Thorlacius
  from David Tong's site
  `http://www.damtp.cam.ac.uk/user/tong/string/sigma.pdf`

- Introduction to String and Superstring Theory II by Michael E.Peskin
  `http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-4251.pdf`
- Notes On String Theory And Two-dimensional Conformal Field Theory by Daniel Friedan
  `http://inspirehep.net/record/227089`
- Conformal invariance, supersymmetry and string theory
  by Daniel Friedan, Emil Martinec, Stephen Shenker
  `http://www.physics.rutgers.edu/pages/friedan/papers/Nucl_Phys_B271_93_1986.pdf`
- The geometry of string perturbation theory by Eric D'Hoker and D. H. Phong
  `https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.60.917`

There are also various texts/notes on 2d conformal field theory which you might want to refer :

- Introduction to Conformal Field Theory with Applications to String Theory
  by Ralph Blumenhagen, Erik Plauschinn
  `https://link.springer.com/book/10.1007%2F978-3-642-00450-6`
- Conformal Field Theory : Lecture notes by A.N. Schellekens
  `https://www.nikhef.nl/~t58/CFT.pdf`
- Conformal Field Theory by Sergei V Ketov
- Conformal Field Theory by Phillippe Francesco, Pierre Mathieu, David Senechal
- Quantum Non-linear Sigma-Models by Sergei V Ketov
  `https://link.springer.com/book/10.1007/978-3-662-04192-5`
- A Mathematical Introduction to Conformal Field Theory by Martin Schottenloher
  `https://link.springer.com/book/10.1007%2F978-3-540-68628-6`
- Non-Perturbative Field Theory: From Two Dimensional Conformal Field Theory by Yitzhak Frishman, Jacob Sonnenschein
- An Introduction to Two-dimensional Quantum Field Theory with (0,2) Supersymmetry by Ilarion V. Melnikov
  `https://link.springer.com/book/10.1007%2F978-3-030-05085-6`
- Boundary Conformal Field Theory and the Worldsheet Approach to D-Branes by Andreas Recknagel, Volker Schomerus

Here are some texts/notes on supersymmetry and supergravity which you might want to refer :

- Introduction to Supergravity by Yoshiaki Tanii
- Introduction to Supersymmetry and Supergravity by Peter C. West
- Ideas and Methods of Supersymmetry and Supergravity, Or a Walk Through Superspace
  by I. L. Buchbinder, Sergei M. Kuzenko
- Superspace Or One Thousand and One Lessons in Supersymmetry by S.J.Gates Jr., M.T.Grisaru, M. Rocek, W.Siegel,

- Modern Supersymmetry: Dynamics and Duality by John Terning
- N=2 Supersymmetric Dynamics for Pedestrians by Yuji Tachikawa

There are a variety of lecture notes of varying depth and detail available online on string theory. One can find many online resources at String Wiki (https://www.stringwiki.org/wiki/String_Theory_Wiki). Another source of references is the resource letter by Donald Marolf https://arxiv.org/abs/hep-th/0311044.

Some of the online lecture notes that I liked include

- Graduate Course in String Theory by Angel M. Uranga
  https://members.ift.uam-csic.es/auranga/firstpage.html
- Introduction to String Theory by Thomas Mohaupt
  http://cds.cern.ch/record/573740/files/0207249.pdf
- Introduction to String Theory by Gerard 't Hooft
  http://www.staff.science.uu.nl/~hooft101/lectures/stringnotes.pdf
- Introduction to String Theory by Timo Weigand
  https://www.thphys.uni-heidelberg.de/~weigand/Skript-strings11-12/Strings.pdf
  https://www.thphys.uni-heidelberg.de/~weigand/Strings-2015.html
  https://www.thphys.uni-heidelberg.de/~weigand/StringsII-2016.html
- Introduction to String Theory by A.N. Schellekens
  https://www.nikhef.nl/~t58/StringLectures2016.pdf

Also useful are the following lecture notes

- Kevin Wray's notes on the course by Kostas Skenderis
  https://math.berkeley.edu/~kwray/papers/string_theory.pdf
- Notes by Bartholomew Andrews on the course by Paul Townsend
  http://www.bartholomewandrews.com/teaching/notes/year5/string_theory/Lectures.pdf

Most of these cover the material relevant to this course.

A set of video lectures on string theory are available online at

- Video lectures by Shiraz Minwalla roughly following Polchinski's book
  (2007-2009) : http://theory.tifr.res.in/~minwalla/
  (2018-2019) : https://www.youtube.com/playlist?list=PL3PVFGnaPl_sCp2A87NVD8GT5Z8Oqw3Yr
- Introduction to the Bosonic String by Alex Buchel (2009)
  http://perimeterinstitute.ca/video-library/collection/introduction-bosonic-string
- String theory lectures by Davide Gaiotto (2017-2018)
  https://perimeterinstitute.ca/video-library/collection/psi-2017/2018-string-theory-gaiotto
- PSI Reviews of String theory by
  Freddy Cachazo, Katrin Becker,Melanie Becker (2009) : http://pirsa.org/C09047/
  Freddy Cachazo (2012) : http://pirsa.org/C12007
  Barton Zweibach (2013) : http://pirsa.org/C13003

Broad Course Philosophy

String theory is what string theorists do. But, who are string theorists ? In my opinion, string theorists are a set of people who are broadly interested in the set of questions appearing under the topics listed below. Further, they often take as a working hypothesis that the tools and techniques towards answering these questions are so inter-related that it is useful to have a common community addressing them.

For conceptual convenience, I have arranged the ideas behind string theory under four major themes each with three topics under them :

- **Gravity and its quantisation :**
  - Perturbative gravity : mathematical structures and effective computational techniques (including supergravity)
  - Black holes : its entropy, formation, evaporation and their non-equilibrium dynamics. Questions about the dynamics of its interior. Effective descriptions in the presence of horizons. Their role in holography.
  - Cosmology of our universe : its initial conditions (may be with inflation ?), stability and its eventual fate. Origin of vacuum energy and its quantum dynamics.
- **Gauge theories : its emergence and dynamics**
  - Perturbative gauge theory : mathematical structures and effective theories, Long distance behaviour and computational techniques. This includes questions about the origin and properties of Standard model (which is a gauge theory with the gauge Lie-algebra

    $$\mathfrak{g}_{SM} \equiv su(3)_C \oplus su(2)_W \oplus u(1)_Y$$

    which controls the structure and interactions of vector fields in particle physics).
  - Fermions (especially chiral fermions) , their gauge interactions. This includes the flavor structure in Standard model which has 90 known fermionic particles which transform in three copies (called three 'generations') of a 30 dimensional representation

    $$R_{SM} \equiv (2,1)_{\text{Spin}(3,1)} \otimes [(3,2)_{+1} \oplus (\bar{3},1)_{+2} \oplus (1,2)_{-3} \oplus (\bar{3},1)_{-4} \oplus (1,1)_{+6}] \ ,$$

    of the Poincare algebra + the gauge Lie algebra $\mathfrak{g}_{SM}$. Understanding the origins of this chiral fermion representations has been an driving motivation for a variety of techniques with string theory, especially in the sub-field of heterotic string theory. This is also one of the main reasons for the physicist's interest in a class of geometrical objects named Calabi-Yau manifolds.

    More broadly, one is interested in the broader question of fermions subject to confinement and strong interactions.
  - Higgs, its stabilisation (via supersymmetry?). Questions about grand unification of observed gauge theories.
- **Matter/Gravity under strong interactions**

- Dualities, i.e., multiple equivalent classical descriptions of the same quantum system (think of wave-particle duality) : both perturbative and non-perturbative, supersymmetric dualities, strong weak dualities.
- Gauge-gravity dualities whereby the duality involves relating gravitational theory to a non-gravitational theory (usually a gauge theory), prominent non-perturbative example is AdS/CFT. More generally, dualities between field theory and string theory.
- Non-equilibrium dualities whereby systems away from equilibrium can be usefully recast into another simpler system. More generally, non-equilibrium phenomena in quantum field theory especially at strong coupling. More recently, relations to quantum information and the geometric structure of entanglement have been added to this list.

- **Mathematics-Physics interface**
  - Mathematical structure of Quantum Field Theory (QFT)
  - Integrable structures and exactly solvable systems (including systems with supersymmetry and/or conformal symmetry)
  - Dualities in algebra and geometry

The order/division here is somewhat arbitrary and this is by no means an exhaustive list. But, most string theorists I know, when asked to justify their work, start from some item in here and work their way towards others. Most talks in string theory I have been to, start from some motivation stated in here. Thus, I think, having this list in mind, helps one understand who string theorists are, what are they interested in and why they are interested in it.

Our main aim in this course would be to begin understanding these interconnections and their relation to the quantum mechanical study of one dimensional relativistic objects. The above list will hopefully help in understanding the broad motivations for various topics we will pursue in this course as we quantise strings. In particular, when we find a result pertinent to any of the topics above we will pause and ask what can that result tell us about these questions ?

Given this diverse list, the reader may quite justifiably wonder what is the relation between these ideas and string theory ? I do not know of a quick answer to that question, except that, as a fact, these ideas do get connected in surprising ways within string theory.

Perhaps, it is because quantising a string is a difficult business requiring powerful mathematical tools and those tools naturally lead us to the hidden overarching structure behind the ideas sketched above. Perhaps there is a better framework beyond current string theory in which the connections between these ideas would be obvious.