Renormalization group study of systems with quadratic band touching

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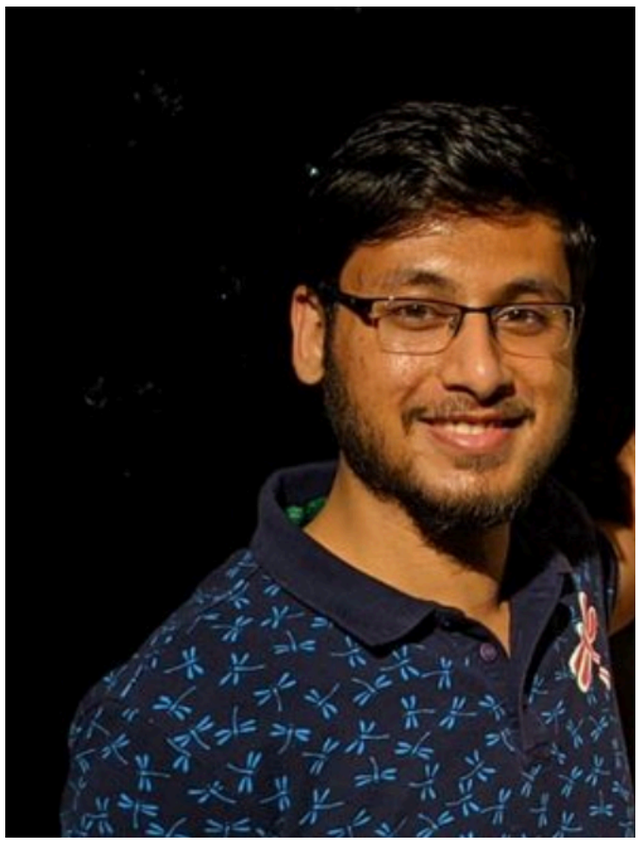


Jeet Shah

Currently a graduate student at the University of Maryland

Shah and Mukerjee, Phys. Rev. B, 103, 195118 (2021)

Thanks to Vijay Shenoy for discussions

Understanding the ground states of interacting Fermi systems

For weak interactions (coupling), a Wilsonian Renormalization Group (RG) can be adopted.

Successfully implemented to understand many ground states like Fermi liquids and certain types of non-Fermi liquids like Luttinger liquids.

The starting point is the non-interacting system which has a ground state with a well defined Fermi surface

The interactions scatter fermions from inside the Fermi surface to the outside.

For weak interactions only states close to the Fermi surface are scattered out of and into

Operationally

Define fields $\psi(\mathbf{k})$ corresponding to single particle momentum states

Fields are anticommuting Grassmann fields as opposed to real or complex fields for bosons and classical particles

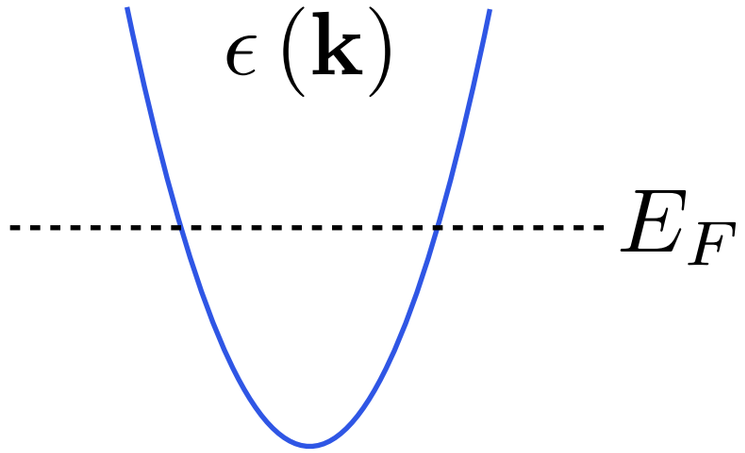
Write down Gaussian action for the non-interacting system in terms of fields with momenta $|\mathbf{k}| < \Lambda$ about the Fermi momentum.

Integrate out fields with momenta $\Lambda/s < |\mathbf{k}| < \Lambda$ and rescale fields to obtain invariant Gaussian action

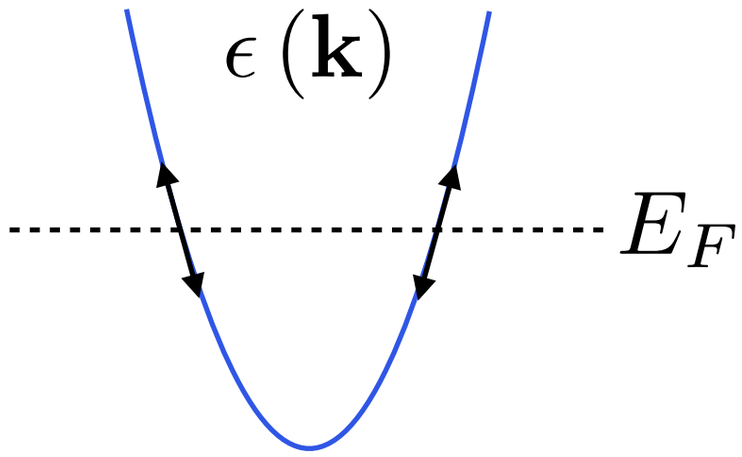
Obtain scaling dimension of the field and determine whether perturbations (interactions) are relevant, irrelevant or marginal at the Gaussian fixed point.

Perturbations assumed to be analytic (Taylor expandable) in momenta and frequencies

R. Shankar, Rev. Mod. Phys. 66, 129 (1994)



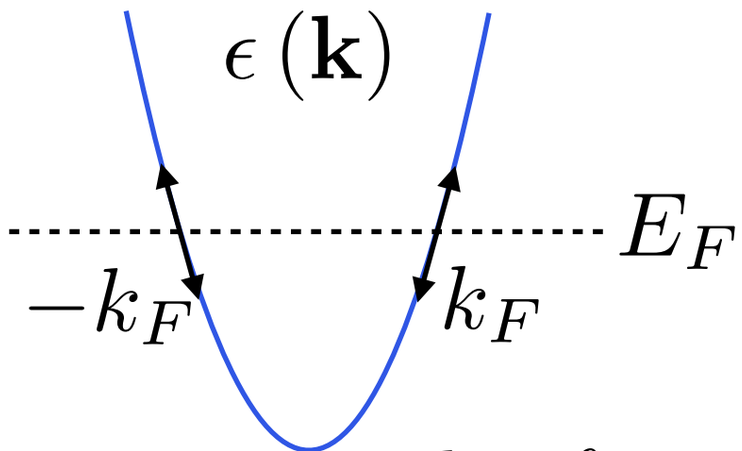
A low energy effective field theory specified by an action describes the system. The fields correspond to the low energy states of the system.



Linearize about Fermi surface

$$E(\mathbf{k}) \approx E_F + \nabla_{\mathbf{k}_F} \epsilon \cdot (\mathbf{k} - \mathbf{k}_F)$$

Example: Generic translationally invariant one dimensional system



Fermi surface is 2 Fermi points at $k_F \;\&\; -k_F$

Gaussian action **Linearization about Fermi points**

$$S_0 = \sum_{i=L,R} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{\psi}_i(\omega k)\,(i\omega - k)\,\psi_i(\omega k)$$

Scaling at the Gaussian fixed point

$$\begin{aligned} k &\to sk \\ \omega &\to s\omega \\ \psi &\to s^{-3/2}\psi \end{aligned}$$

Quadratic term

$$S_2 = \sum_{i=L,R} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mu(\omega k) \bar{\psi}_i(\omega k) \psi_i(\omega k)$$

Interaction

$$S_4 = \frac{1}{2!2!} \int_{K\omega} \bar{\psi}(4)\bar{\psi}(3)\psi(2)\psi(1) u(4,3,2,1)$$

$\mu(00) = \mu$ **relevant**

Single marginal quartic coupling

$$u = u_{LRLR} = u_{RLRL} = -u_{RLLR} = -u_{LRRL}$$

Crucially dependent on

- **Scaling dimension of the field**
- **Zero dimensionality of Fermi surface**

Flow equations to one loop

$$\frac{d\mu}{d\ln s} = \mu - \frac{u}{2\pi}$$
$$\frac{du}{d\ln s} = 0$$

μ at fixed point depends on u to keep filling fixed

u remains marginal

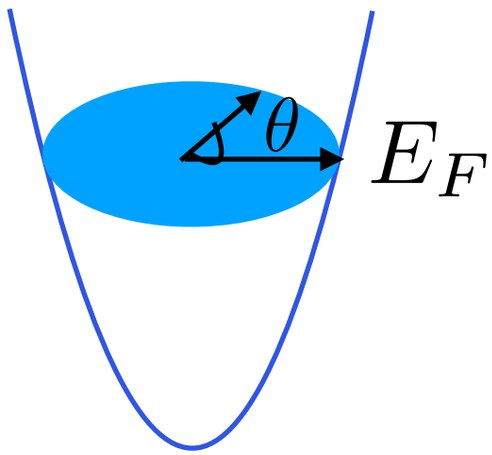
Luttinger liquid, which has no perturbative instabilities (different from a Fermi liquid)

Crucially important that there are two Fermi points

CDW nesting instability "cancels" BCS instability

R. Shankar, Rev. Mod. Phys. 66, 129 (1994)

2D system



Fermi surface is a closed curve (a circle)

Dispersion can still be linearized about E_F

Gaussian action

$$S_0 = K_F \int \frac{d\theta}{2\pi} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{\psi}(\omega k \theta) \left(i\omega - k\right) \psi(\omega k \theta)$$

$$\begin{aligned} k &\to sk \\ \omega &\to s\omega \\ \psi &\to s^{-3/2}\psi \end{aligned}$$

Quadratic term

$$S_2 = \int \frac{d\theta}{2\pi} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mu(\omega k) \bar{\psi}(\omega k) \psi(\omega k)$$

Interaction

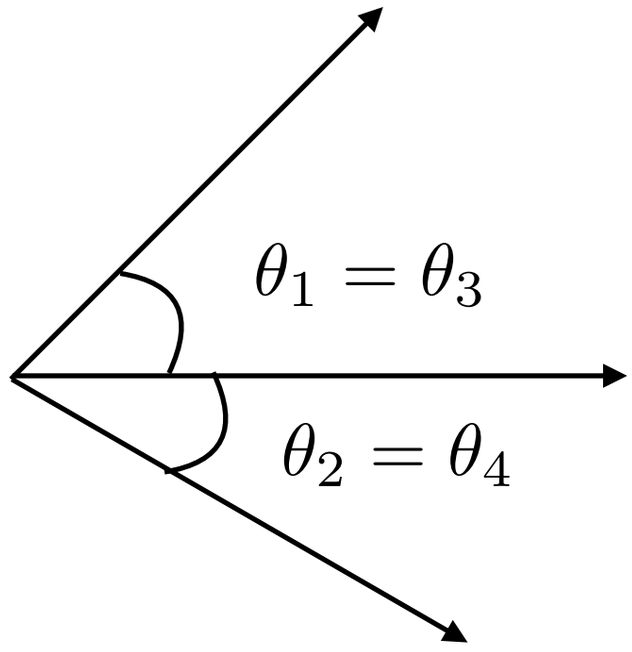
$$S_4 = \frac{1}{2!2!} \int_{K\omega} \bar{\psi}(4) \bar{\psi}(3) \psi(2) \psi(1) u(4,3,2,1)$$

$\mu(00) = \mu$ **relevant**

μ **at fixed point depends on** u **to keep filling fixed**

Marginal quartic coupling functions $u\left(\theta_1, \theta_2, \theta_3, \theta_4\right)$

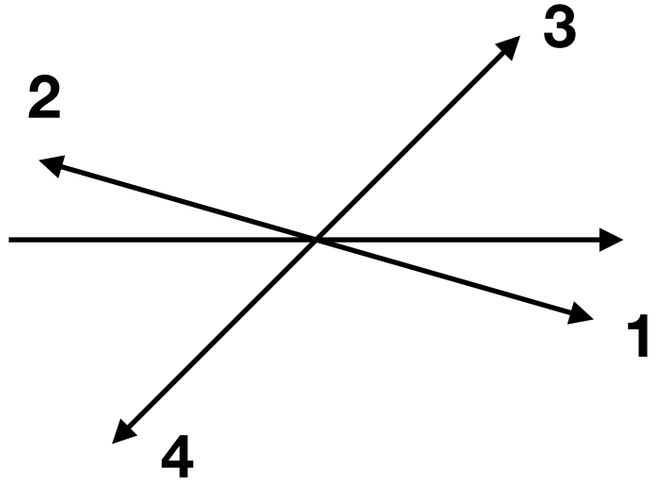
Fermi liquid



$$u\left(\theta_1, \theta_2, \theta_1, \theta_2\right) = F\left(\theta_1 - \theta_2\right)$$

Forward scattering amplitudes

Landau f functions



$$u\left(\theta_1, -\theta_1, \theta_3, -\theta_3\right) = V\left(\theta_1 - \theta_3\right)$$

Backward scattering amplitudes

At one loop

μ flows to keep filling constant

$$\frac{dF}{d\ln s} = 0$$

$$\frac{dV_m}{d\ln s} = -\frac{V_m^2}{4\pi} \qquad V(\theta) = \sum_m V_m e^{im\theta}$$

Landau f parameters remain marginal

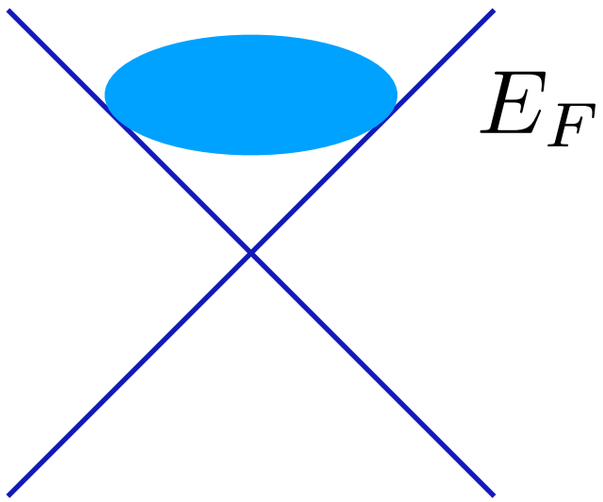
$V_m < 0$ **attractive interaction producing superconductivity**

BCS instability

Fermi liquid has perturbative instabilities unlike the Luttinger liquid

R. Shankar, Rev. Mod. Phys. 66, 129 (1994)

Linear dispersion

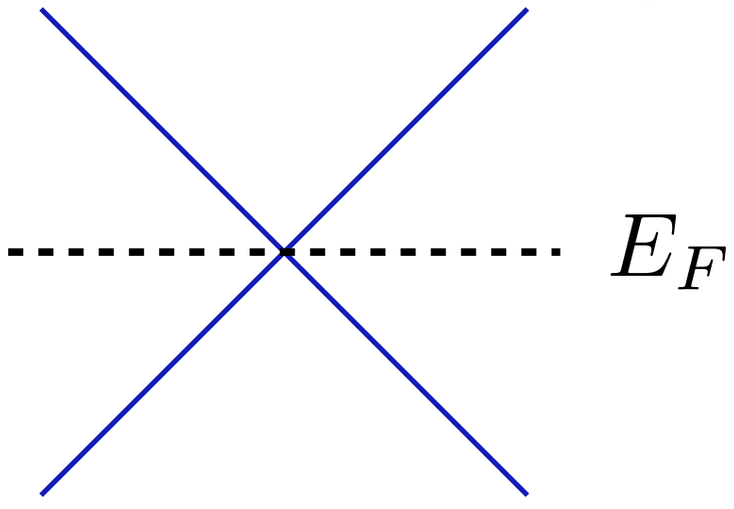


$$S_0 = K_F \int \frac{d\theta}{2\pi} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{\psi}(\omega k \theta) \left(i\omega - k\right) \psi(\omega k \theta)$$

Circular Fermi surface

2D Fermi liquid

$$\begin{aligned} k &\rightarrow sk \\ \omega &\rightarrow s\omega \\ \psi &\rightarrow s^{-3/2}\psi \end{aligned}$$



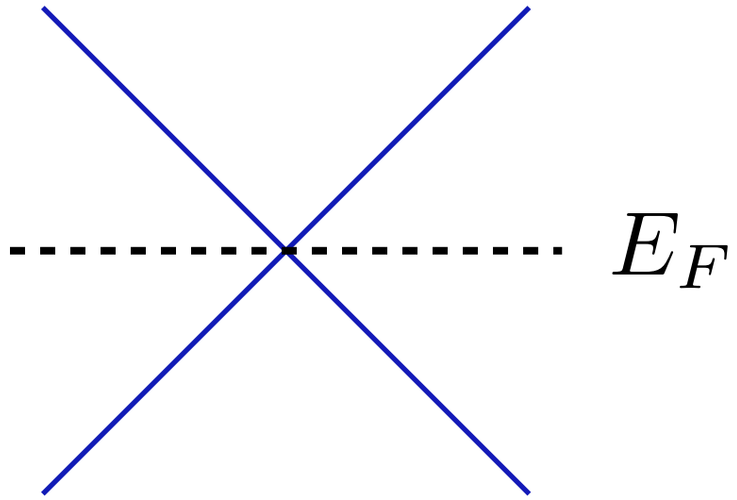
$$S_0 = \int \frac{d\theta}{2\pi} \int_{0}^{\Lambda} k \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{\psi}(\omega k \theta) \left(i\omega - \sigma.\mathbf{k}\right) \psi(\omega k \theta)$$

ψ **- two component Grassmann field**

$$\begin{aligned} k &\rightarrow sk \\ \omega &\rightarrow s\omega \\ \psi &\rightarrow s^{-2}\psi \end{aligned}$$

Point Fermi surface

Scaling dimension of field different from that of 2D Fermi liquid or 1D Luttinger liquid



$$S_4 = \frac{1}{2!2!}\int_{K\omega} \bar{\psi}(4)\bar{\psi}(3)\psi(2)\psi(1)u(4,3,2,1)$$

$u(4,3,2,1)$ **is irrelevant**

Density of states $g(E_F = 0) = 0 \Rightarrow$ **no screening of Coulomb interaction**

However, Coulomb interactions are marginal (when thought to be mediated by a gauge field)

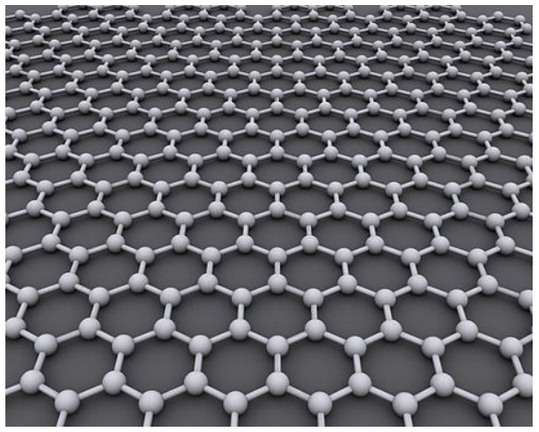
"Dirac liquid" distinct from a Luttinger liquid and Fermi liquid

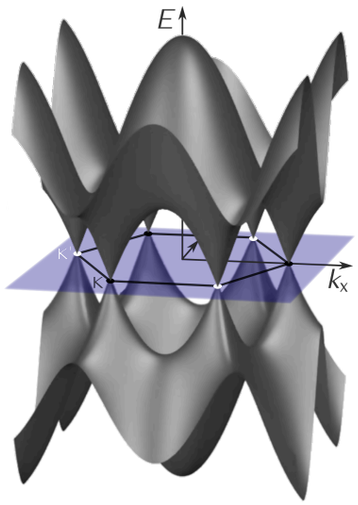
Fermi velocity renormalization

$$v(E) = v(E_0)\left[1 - g\ln\left(\frac{E}{E_0}\right)\right]$$

Gonzalez, Guinea and Vozmediano, Nucl. Phys. B 424, 595 (1994)

Barnes, Hwang, Throckmorton and Das Sarma, Phys. Rev. B 89, 235431 (2014)



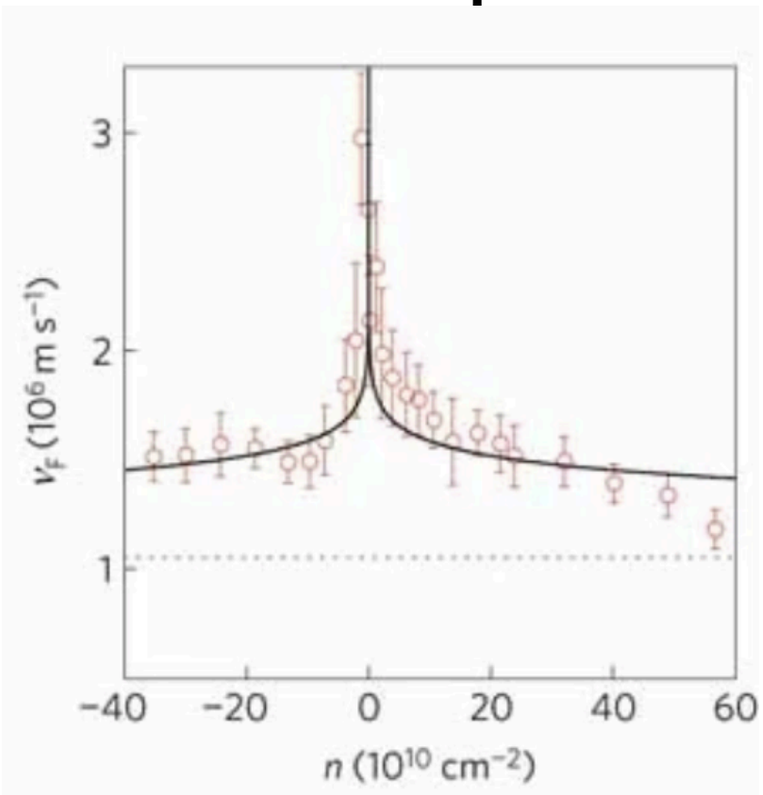


Naturally occurs in graphene

Image taken from Wikipedia

$$v(E) = v(E_0)\left[1 - g\ln\left(\frac{E}{E_0}\right)\right]$$

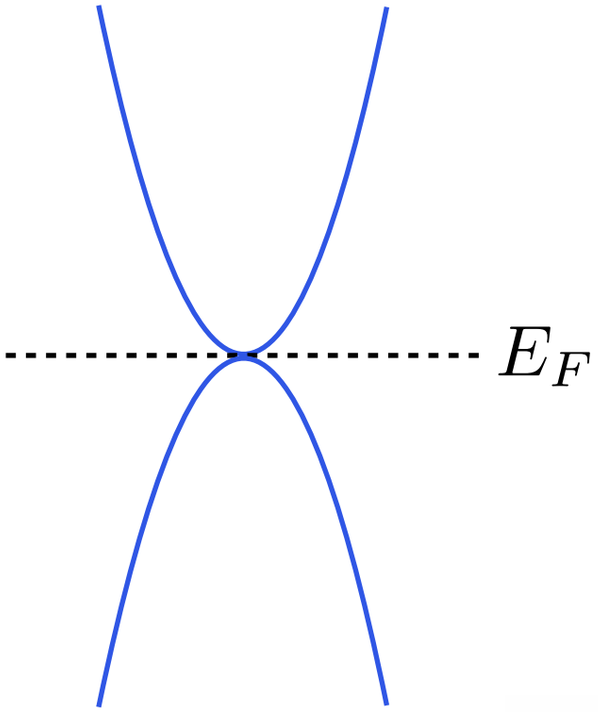
Observed experimentally



Fermi energy E depends on carrier density n

D. C. Elias et. al, Nat. Phys. 7, 701 (2011)

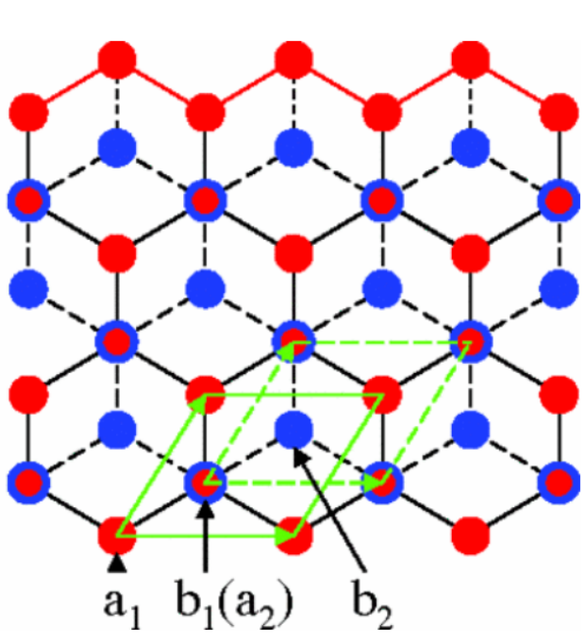
Quadratic band touching

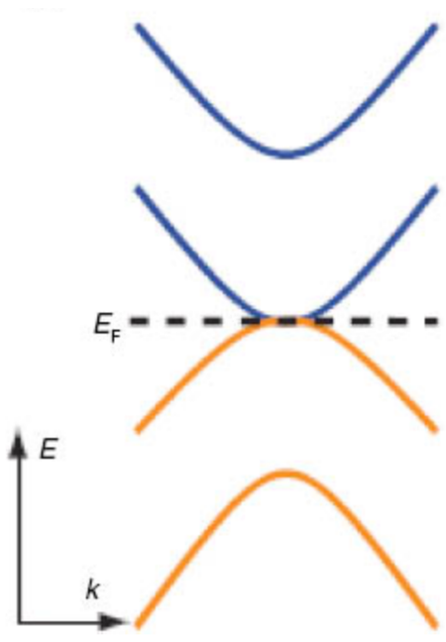


Density of states (DOS) at $E_F \neq 0$

Coulomb interactions are screened (short-ranged)

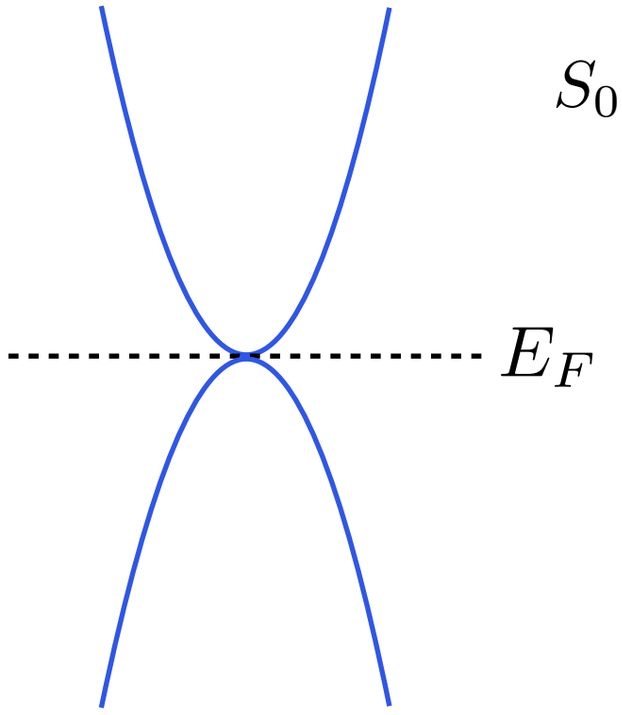
Occurs naturally in Bernal stacked bilayer graphene





From H. Yan, Nanophotonics 2015; 4:115–127

Quadratic band touching and two inequivalent points in the Brillouin zone. Valley quantum number.



$$S_0 = \int \frac{d\theta}{2\pi} \int_0^{\Lambda} k \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \bar{\psi}(\omega k \theta) \left(i\omega - H(\mathbf{k}) \right) \psi(\omega k \theta)$$

$H(\mathbf{k})$ **is quadratic in** $\mathbf{k}$

$$\begin{aligned} k &\to sk \\ \omega &\to s^2 \omega \\ \psi &\to s^{-3} \psi \end{aligned}$$

$$S_2 = \int \frac{d\theta}{2\pi} \int_0^{\Lambda} k \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mu(\omega k \theta) \bar{\psi}(\omega k \theta) \psi(\omega k \theta)$$

μ - chemical potential, relevant

$$S_4 = \frac{1}{2!2!} \int_{K\omega} \bar{\psi}(4) \bar{\psi}(3) \psi(2) \psi(1) u(4,3,2,1)$$

u marginal

Only coupling constants and not coupling functions

Not a Fermi liquid, Luttinger liquid or Dirac liquid

Systems with quadratic band touching

Topological stability of touching point and RG analysis of instabilities

Sun, Yao, Fradkin and Kivelson, Phys. Rev. Lett.
103, 046811 (2009)

Bilayer graphene

Two quadratic Fermi points unstable to splitting into four Dirac points due to interactions between valleys. Yields a nematic state.

Yang and Vafek, Phys. Rev. B 81, 041401 (R) (2010)

Marginal Fermi liquid

Barlas and Yang, Phys. Rev. B 80, 161408(R) (2009)

Identification of magnetic and pairing instabilities

Vafek, Phys. Rev. B 82, 205106 (2010)

Nandkishore and Levitov, Phys. Rev. B 82, 115431 (2010)

Fairly extensive theoretical studies performed over a decade ago

So, why revisit the problem of quadratic band touching?

Ans: To try to understand quadratic band touching in its simplest form without the accompanying “complications” arising from valley degeneracy and topological stability.

We will see that the basic physics can also be understood in terms of a simple microscopic model (although with the fine tuning)

The above microscopic model can be tuned to go from a gapless phase with an excitonic condensate instability to an insulator through the quadratic band touching point.

This is different from the physics of bilayer graphene and has not been investigated before (to the best of our knowledge)

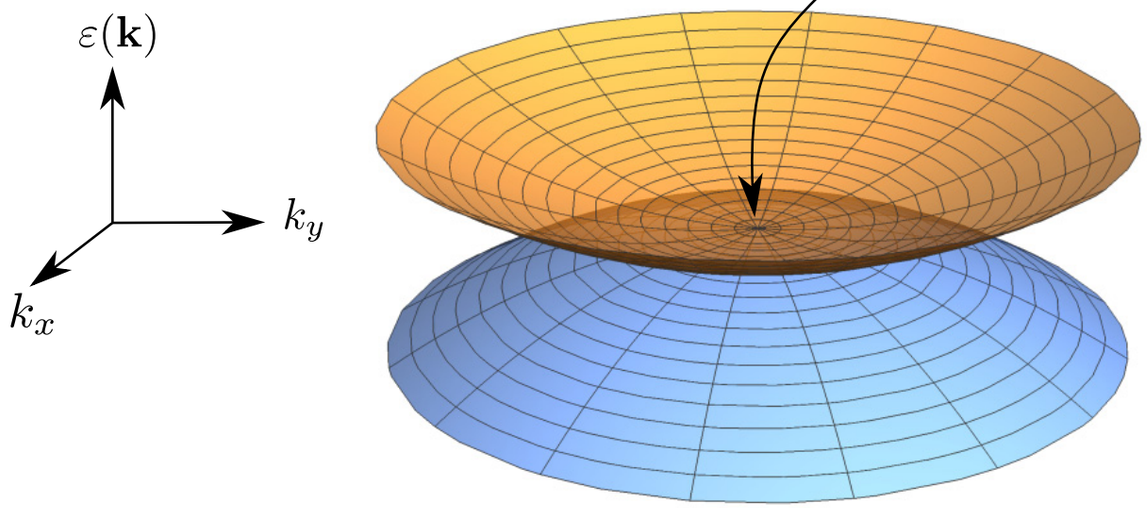
Two simple models

Model A

$$H_A(\mathbf{k}) = \begin{pmatrix} 0 & \frac{k^2}{2m} e^{-2i\theta} \\ \frac{k^2}{2m} e^{2i\theta} & 0 \end{pmatrix}$$

Model B

$$H_B(\mathbf{k}) = \begin{pmatrix} \frac{k^2}{2m} & 0 \\ 0 & -\frac{k^2}{2m} \end{pmatrix}$$

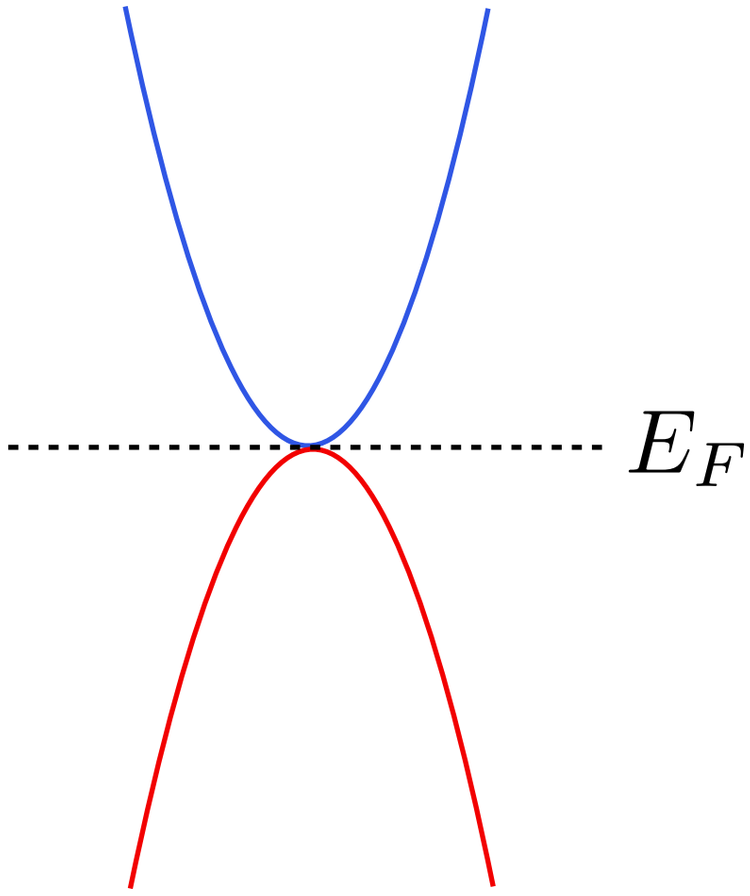


Two spatial dimensions

Gaussian action

$$S_0 = \int_{|\mathbf{k}|<\Lambda} \frac{d\mathbf{k}}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \psi^\dagger(\mathbf{k},\omega)[-i\omega + H(\mathbf{k})]\psi(\mathbf{k},\omega)$$

ψ **- two component Grassmann field**



Density of states (DOS) at $E_F \neq 0$

Coulomb interactions are screened

Interaction part of the action

$$S_I = \frac{g}{2}\int_{\mathbf{k},\omega} \bar{\psi}_1(4)\bar{\psi}_2(3)\psi_1(2)\psi_2(1)$$

$$\int_{\mathbf{k},\omega} = \int_{|\mathbf{k}_i|<\Lambda} \prod_{i=1}^{4}\left[\frac{d\mathbf{k}_i d\omega_i}{(2\pi)^3}\right](2\pi)^3\delta(\mathbf{k}_4+\mathbf{k}_3-\mathbf{k}_2-\mathbf{k}_1)$$
$$\delta(\omega_4+\omega_3-\omega_2-\omega_1).$$

Interaction marginal and thus single coupling constant (only interband) due to fermionic statistics

$g>0$ **attractive** $g<0$ **repulsive**

$$S = \int_{\mathbf{k},\omega} \bar{\psi}\left[-i\omega + H(\mathbf{k})\right]\psi + \frac{g}{2}\int_{\mathbf{k},\omega} \bar{\psi}_1(4)\bar{\psi}_2(3)\psi(2)\psi(1)$$

Non-interacting Green's functions

$$G(\mathbf{k}\omega) = [-i\omega + H(\mathbf{k})]^{-1}$$

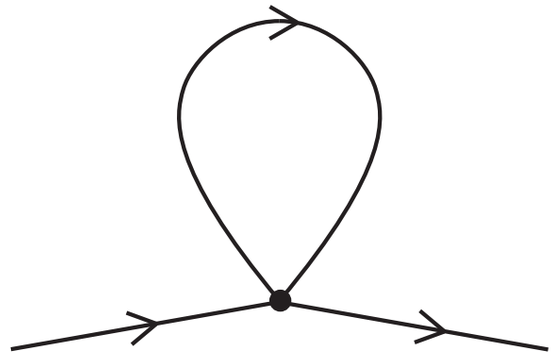
For model A

$$G(\mathbf{k}\omega) = \frac{1}{\omega^2 + \left(\frac{k^2}{2m}\right)^2} \begin{pmatrix} i\omega & k^2 e^{-2i\theta}/2m \\ k^2 e^{2i\theta}/2m & i\omega \end{pmatrix}$$

For model B

$$G(\mathbf{k}\omega) = \begin{pmatrix} \frac{1}{-i\omega + k^2/2m} & 0 \\ 0 & \frac{1}{-i\omega - k^2/2m}. \end{pmatrix}$$

Green's functions are used to calculate the flows of the different couplings



Tadpole diagram - renormalizes the chemical potential

$$\delta S_T = \frac{-g\Lambda^2(s^2-1)}{16\pi}\int_{|\mathbf{k}|<\Lambda}\frac{d\mathbf{k}d\omega}{(2\pi)^3}\psi^\dagger(\mathbf{k}\omega)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\psi(\mathbf{k}\omega).$$

for model A

$$\delta S_T = \frac{-g\Lambda^2(s^2-1)}{8\pi}\int_{|\mathbf{k}|<\Lambda}\frac{d\mathbf{k}d\omega}{(2\pi)^3}\psi^\dagger(\mathbf{k}\omega)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\psi(\mathbf{k}\omega).$$

for model B

Different renormalizations at one loop of the quadratic term for models A and B

$$S = \int_{\mathbf{k},\omega} \bar{\psi} \left[-i\omega + H(\mathbf{k}) \right] \psi + \frac{g}{2} \int_{\mathbf{k},\omega} \bar{\psi}_1(4) \bar{\psi}_2(3) \psi(2) \psi(1)$$

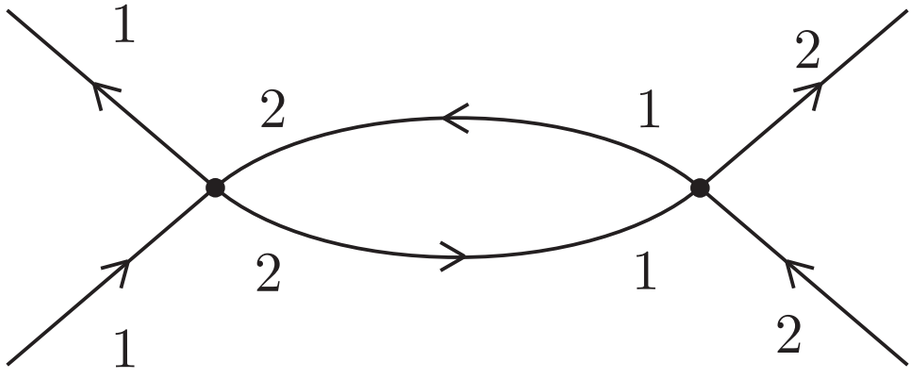
Interactions between electrons in bands 1 and 2

Model B, single particle eigenstates are diagonal in the 1-2 basis

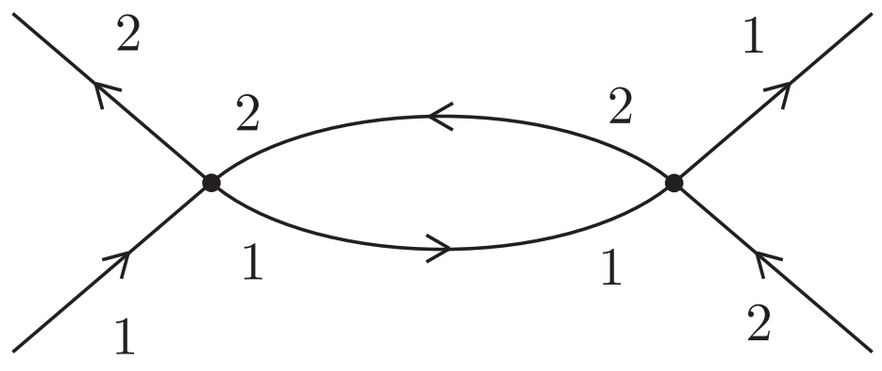
Model A, single particle eigenstates are linear combinations of the 1-2 basis

Renormalization of the coupling at one loop

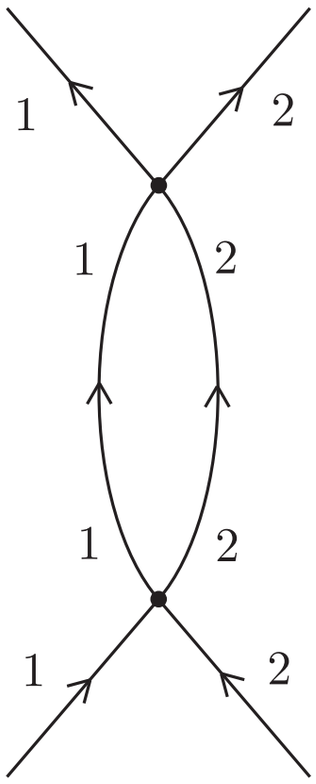
Diagrams



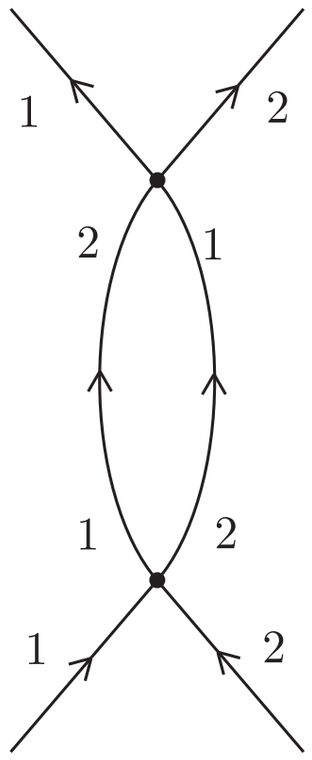
ZS diagram



ZS' diagram



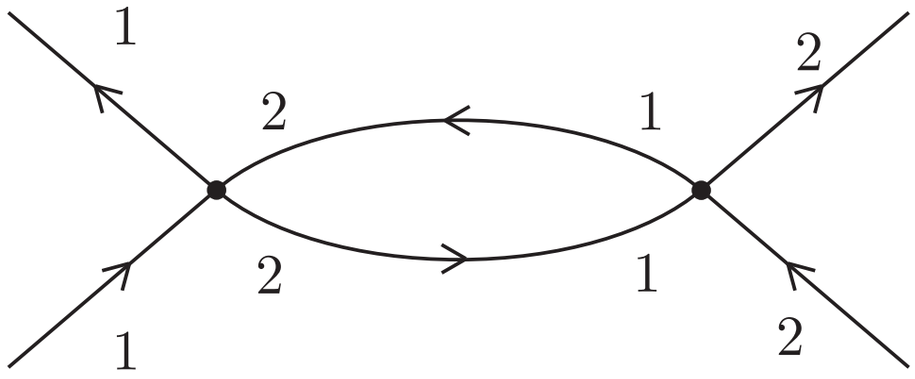
BCS diagrams



Renormalization of the coupling at one loop

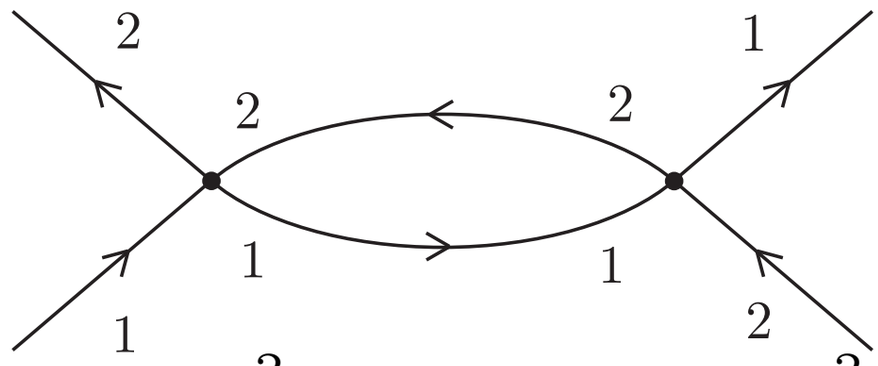
Diagrams

ZS diagram



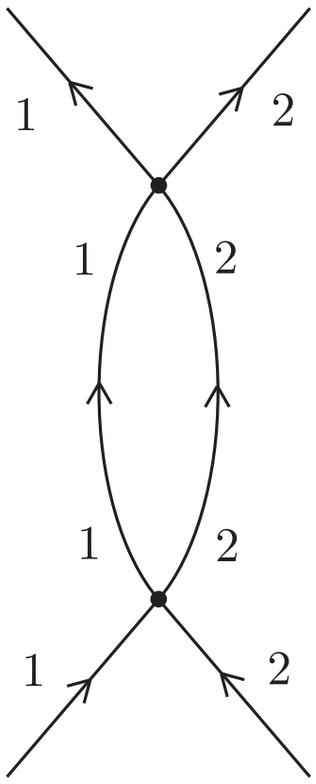
Model A $-\frac{g^2 m}{8\pi}$ **Model B** 0

ZS' diagram



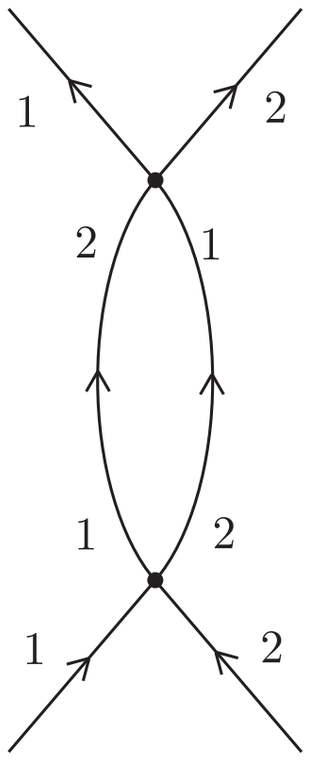
Model A $-\frac{g^2 m}{8\pi}$ **Model B** $-\frac{g^2 m}{4\pi}$

BCS diagrams



Model A 0

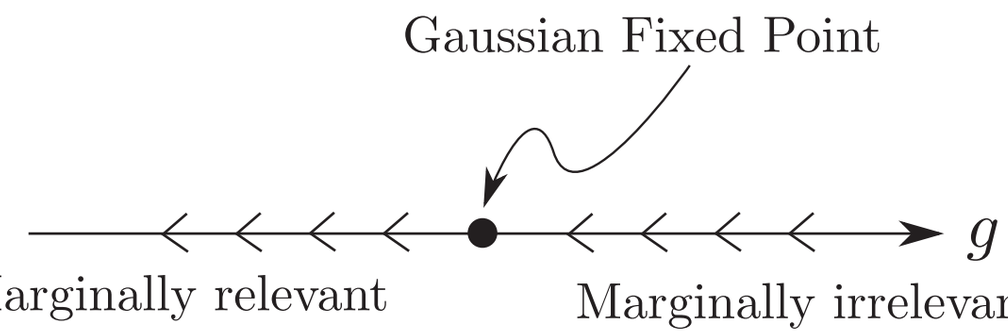
Model B 0



Renormalization of the coupling at one loop

$$\frac{dg}{d\ln s} = -\frac{g^2 m}{4\pi}$$

$g > 0$ **attractive (marginally irrelevant)** $g < 0$ **repulsive (marginally relevant)**



Agrees with

Sun, Yao, Fradkin and Kivelson, Phys. Rev. Lett. 103, 046811 (2009)

Reminder: 2D Fermi liquid

Attractive interactions: marginally relevant (BCS instability)

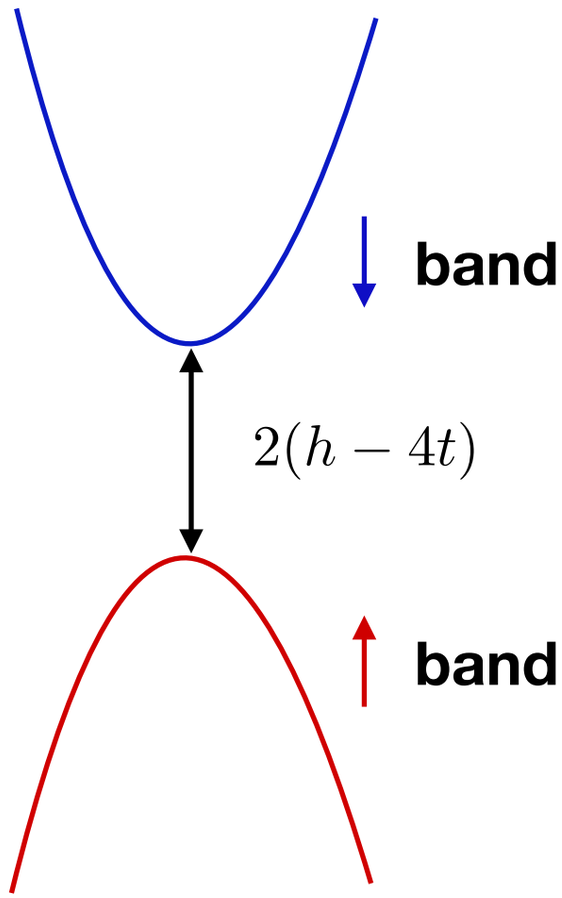
Repulsive interactions: marginal or marginally irrelevant

What state does this system flow to with repulsive interactions?

Microscopic model

Hubbard model on a square lattice with a Zeeman field

$$H = -t \sum_{\langle ij \rangle} \left[\left(c^\dagger_{i\downarrow} c_{j\downarrow} - c^\dagger_{i\uparrow} c_{j\uparrow} \right) + \text{h.c.} \right] - h \sum_i \left(n_{i\uparrow} - n_{i\downarrow} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Gapless for $h = 4t$

Quadratically touching bands

$$\epsilon_{\mathbf{k}\uparrow} \approx -tk^2$$

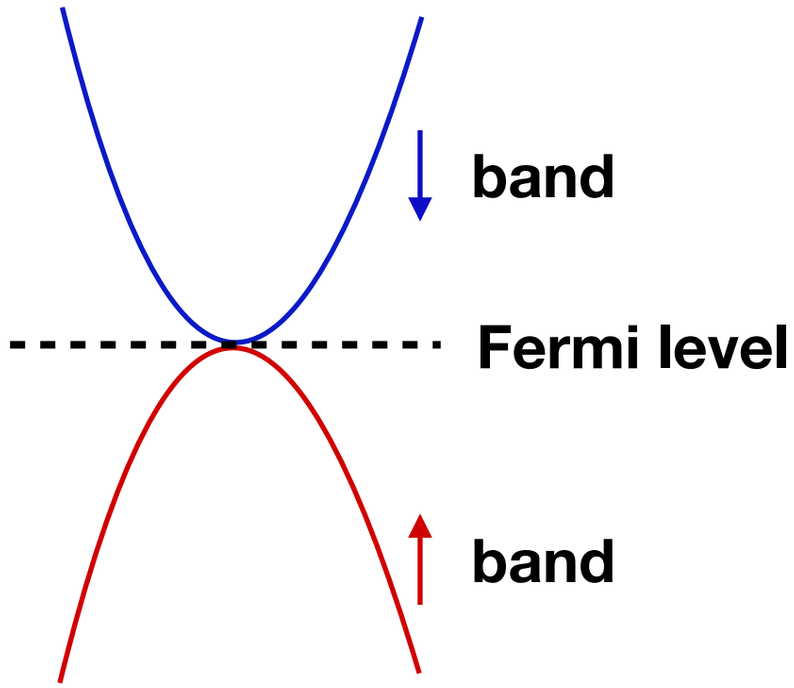
$$\epsilon_{\mathbf{k}\downarrow} \approx tk^2$$

for small k

Yields model B in the continuum limit

Microscopic model

Gapless system. Fermi level at band touching point



Ground state for $U > 0$ is the same as for $U = 0$ ↑ band filled

A repulsive interaction is marginally relevant and causes a flow to a state with the same ground state but gapped excitations

Green's function

Can be obtained exactly for the microscopic model since the exact interacting ground state is known

$$\langle \mathcal{T} c^*_{\mathbf{k}\sigma}(\tau) c_{\mathbf{k}'\sigma'}(\tau') \rangle = \begin{cases} \langle \mathrm{GS} | \left[e^{\tau H} c^\dagger_{\mathbf{k}\sigma} e^{-\tau H} \right] \left[e^{\tau' H} c_{\mathbf{k}'\sigma'} e^{-\tau' H} \right] | \mathrm{GS} \rangle & \text{for } \tau > \tau' \\ -\langle \mathrm{GS} | \left[e^{\tau' H} c_{\mathbf{k}'\sigma'} e^{-\tau' H} \right] \left[e^{\tau H} c^\dagger_{\mathbf{k}\sigma} e^{-\tau H} \right] | \mathrm{GS} \rangle & \text{for } \tau < \tau' \end{cases}$$

$$G(\mathbf{k}, \omega; \mathbf{k}', \omega') = 2\pi \delta(\omega - \omega') \delta_{\mathbf{k},\mathbf{k}'} \begin{pmatrix} \frac{1}{-i\omega + \varepsilon_{\mathbf{k}\downarrow} + U} & 0 \\ 0 & \frac{1}{-i\omega + \varepsilon_{\mathbf{k}\uparrow}} \end{pmatrix}$$

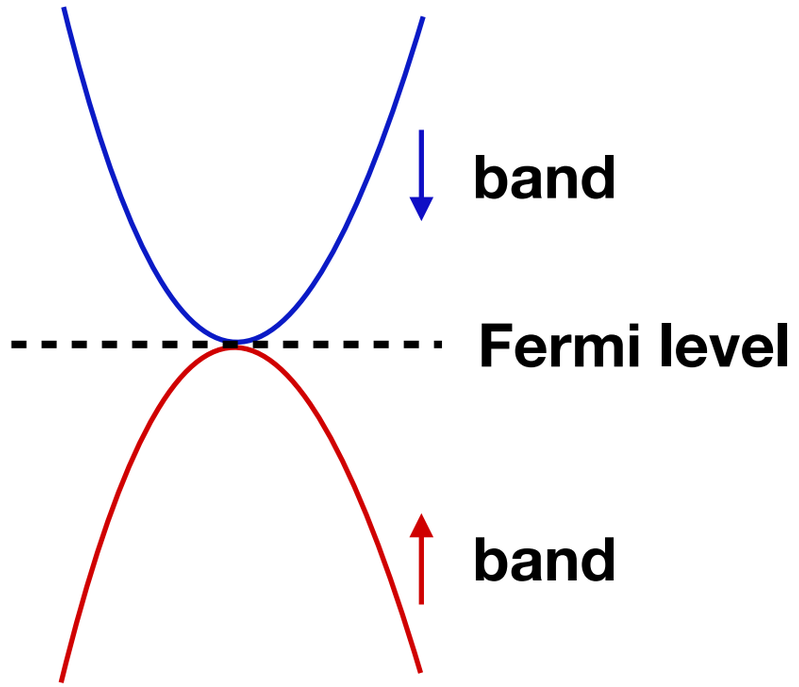
matches with the perturbative Green's function up to one loop from the field theory

$$G_I(2;1) = \delta(2-1) \begin{pmatrix} \frac{1}{-i\omega + \frac{k^2}{2m} - \frac{g\Lambda^2}{8\pi}} & 0 \\ 0 & \frac{1}{-i\omega - \frac{k^2}{2m}} \end{pmatrix}$$

with $U = -\frac{g\Lambda^2}{8\pi}$

Microscopic model

$$U < 0$$

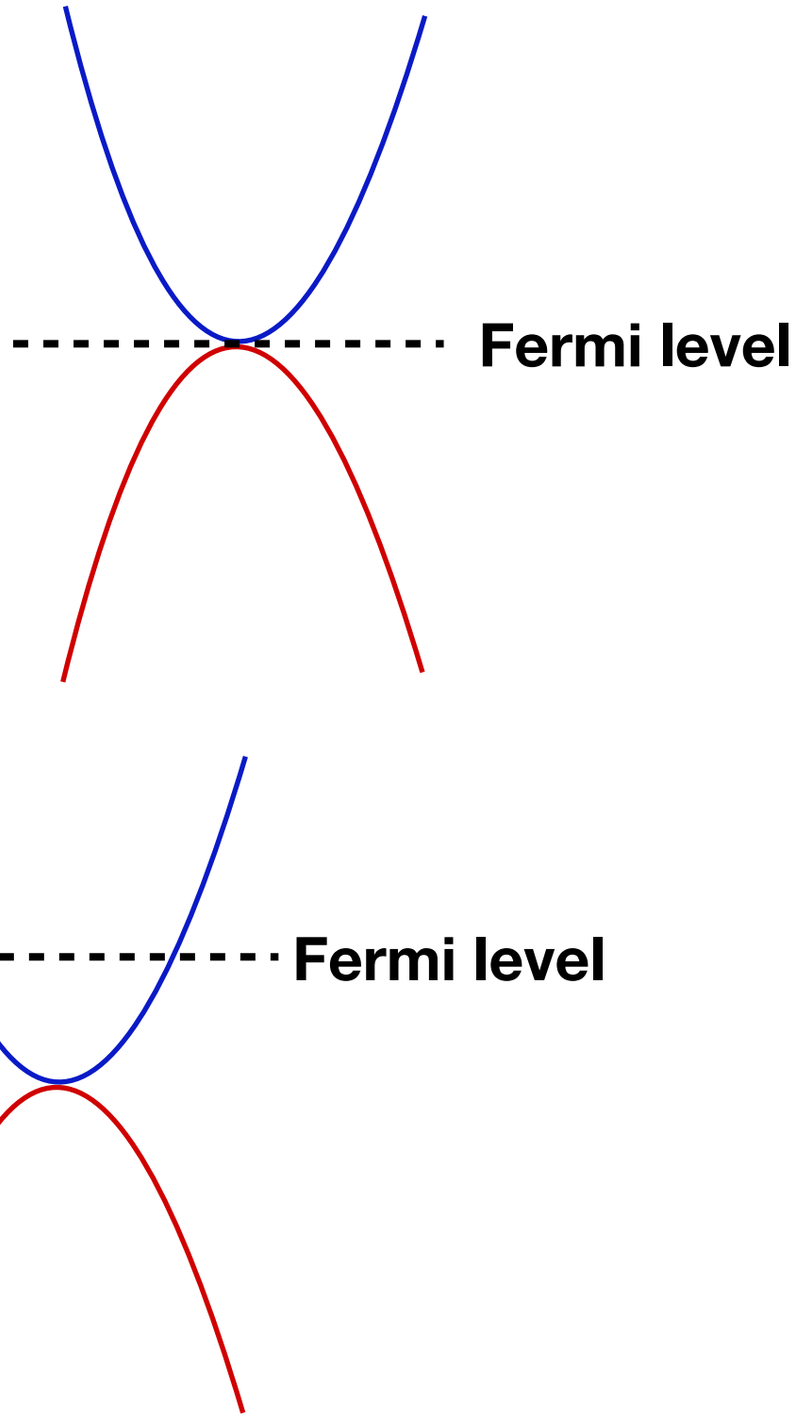


Interaction energy can be lowered by exciting from the ↑ band to the ↓ band

But this increases the kinetic (band) energy

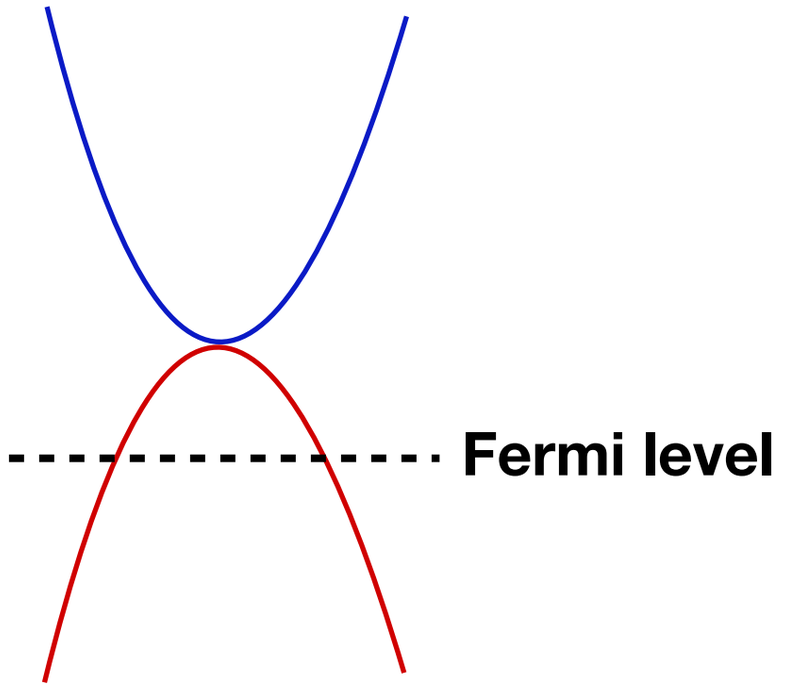
Energetically favourable to excite a fraction $f < \frac{|U|/t}{1+|U|/t}$

In the perturbative limit, this fraction is negligible consistent with the marginal irrelevance of $U < 0$



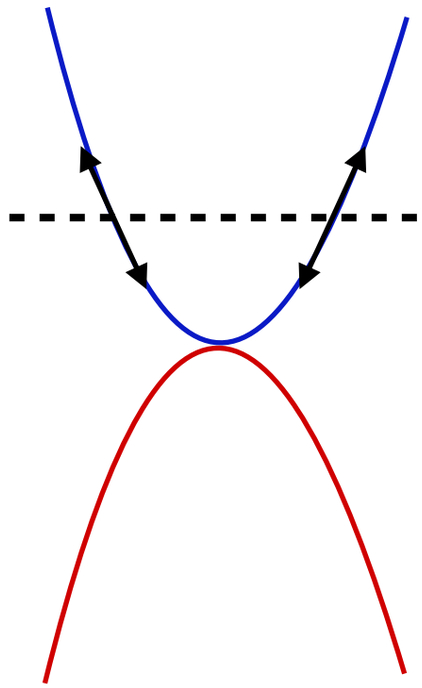
Fermi surface is a point

or



Fermi surface is a circle.

Back to the field theory



Fermi level

Fermi surface is a circle.

Dispersion can be linearized about Fermi momentum

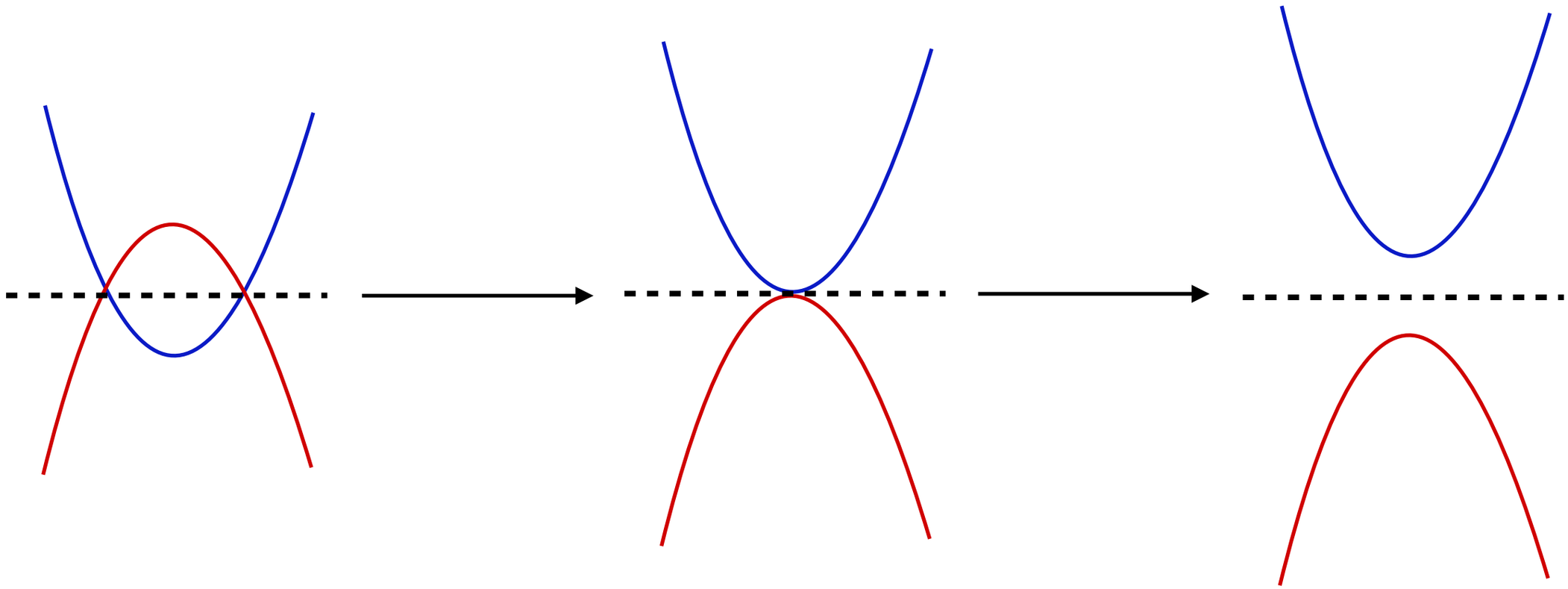
Scaling dimension of the fields changes

Couplings not single parameters but forward scattering amplitudes (Fermi f parameters) and backward scattering amplitudes

A regular 2D Fermi liquid is obtained.

Lifshitz like transition out of regular 2D Fermi liquid when Fermi energy is at the band touching point

Another possibility for a Lifshitz-like transition



Fermi surface is circular

Metallic

Fermi surface is a point

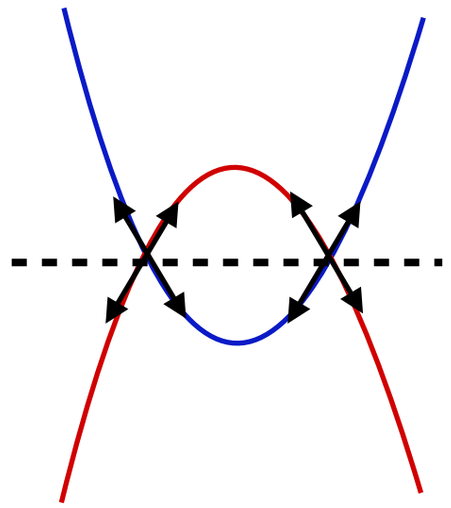
Non Fermi-liquid

Fermi surface not properly defined

Insulator

$$H = -t \sum_{\langle ij \rangle} \left[\left(c^\dagger_{i\downarrow} c_{j\downarrow} - c^\dagger_{i\uparrow} c_{j\uparrow} \right) + \text{h.c.} \right] - h \sum_i \left(n_{i\uparrow} - n_{i\downarrow} \right)$$

Tune $h - 2t$



Circular Fermi surfaces for both bands

Linearize the dispersion with speed c

$$G(\mathbf{K},\omega) = \begin{pmatrix} \frac{1}{-i\omega+ck} & 0 \\ 0 & \frac{1}{-i\omega-ck} \end{pmatrix}$$

Linearized dispersion changes scaling dimension of the fields

Allows intraband interactions in addition to interband interactions

$$S_1 = \int_{\mathbf{k},\omega} g_1(4321)\bar{\psi}_\downarrow(4)\bar{\psi}_\downarrow(3)\psi_\downarrow(2)\psi_\downarrow(1),$$
$$S_2 = \int_{\mathbf{k},\omega} g_2(4321)\bar{\psi}_\uparrow(4)\bar{\psi}_\uparrow(3)\psi_\uparrow(2)\psi_\uparrow(1),$$
$$S_{12} = \int_{\mathbf{k},\omega} g_{12}(4321)\bar{\psi}_\downarrow(4)\bar{\psi}_\uparrow(3)\psi_\downarrow(2)\psi_\uparrow(1).$$

Marginal coupling functions instead of a single marginal coupling constant

Coupling constants correspond to forward scattering F

and backward scattering V parameters

Coupling constants correspond to forward scattering F

and backward scattering V parameters

To one loop

Intraband F and V identical to those of a 2D Fermi liquid in each band

$$\frac{dF_j}{d\ln s} = 0,$$

$$\frac{dV_{j,l}}{d\ln s} = \frac{V_{j,l}^2}{\pi c}$$

$$V_{j,l} = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{il\theta} V_j(\theta)$$

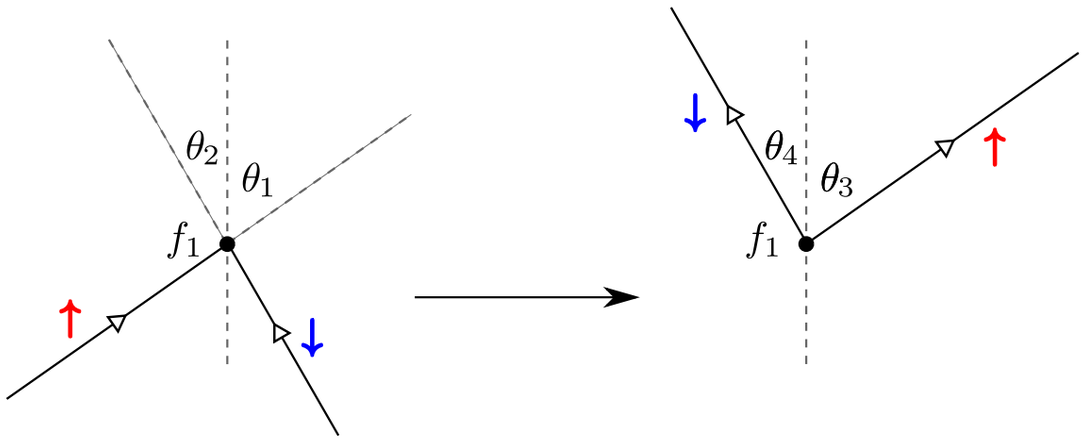
Positive V_j corresponds to intraband attractive interactions

Marginally relevant and gives rise to intraband superconductivity

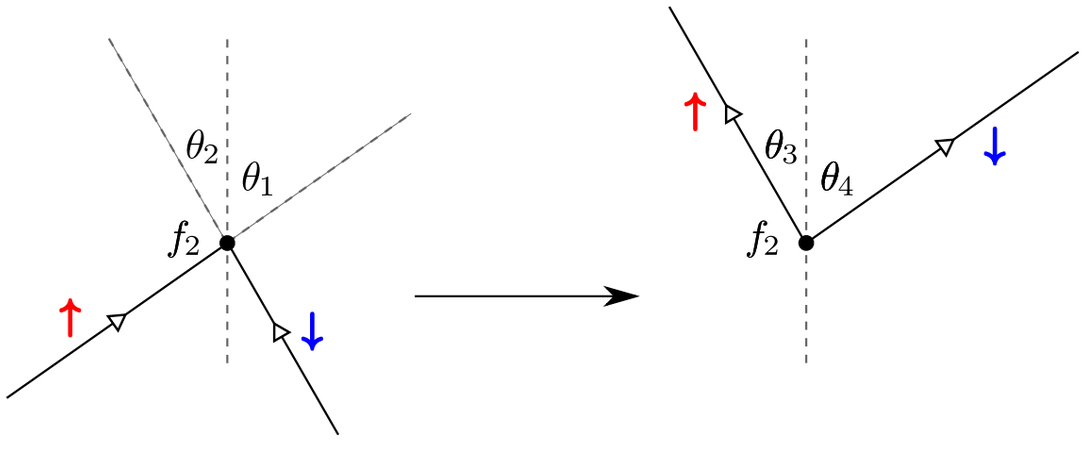
Interband coupling

$$S_{12} = \int_{\mathbf{k},\omega} g_{12}(4321)\bar{\psi}_{\downarrow}(4)\bar{\psi}_{\uparrow}(3)\psi_{\downarrow}(2)\psi_{\uparrow}(1).$$

Two types of forward scattering amplitudes



(a)



(b)

Initial and final momentum are the same for the two spins

Spins exchange initial and final momenta

$$\frac{df_1}{d\ln s} = 0,$$

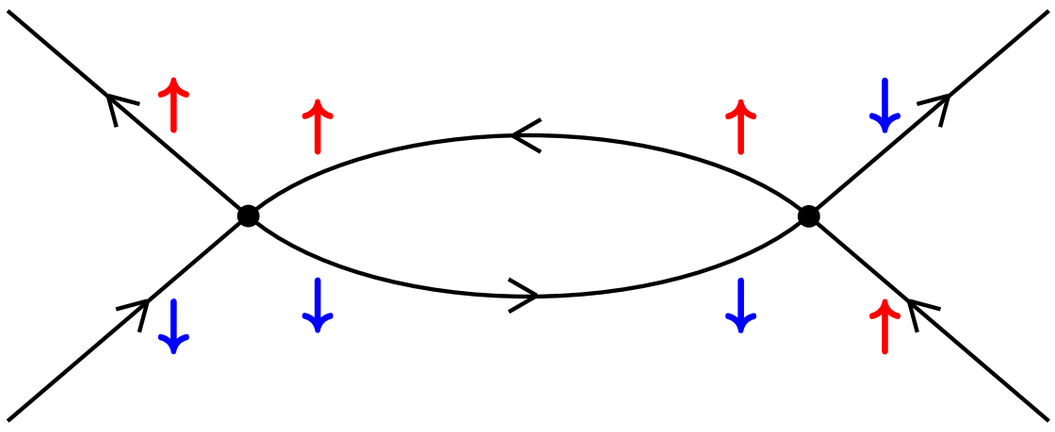
$$\frac{df_{2,l}}{d\ln s} = -\frac{f_{2,l}^2}{2\pi c},$$

$$\frac{dv}{d\ln s} = 0$$

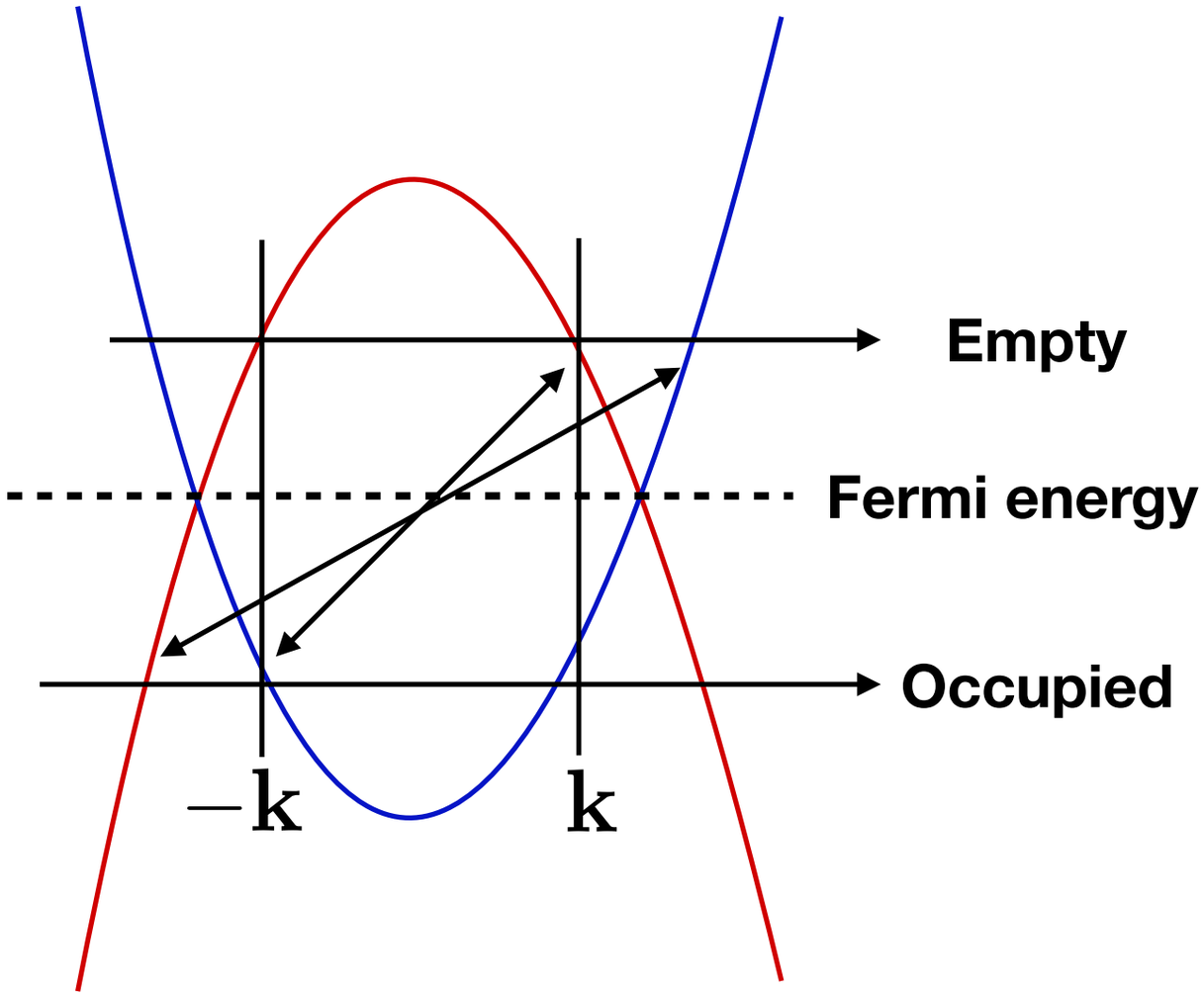
$$f_{2,l} = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{il\theta} f_2(\theta)$$

Interband forward scattering f_1 and backward scattering v do not flow

Interband forward scattering f_2 flows due to the ZS' diagram



The backward scattering amplitude v does not flow and hence does not produce BCS pairing



Both bands do not have occupied states at $\mathbf{k}$ & $-\mathbf{k}$

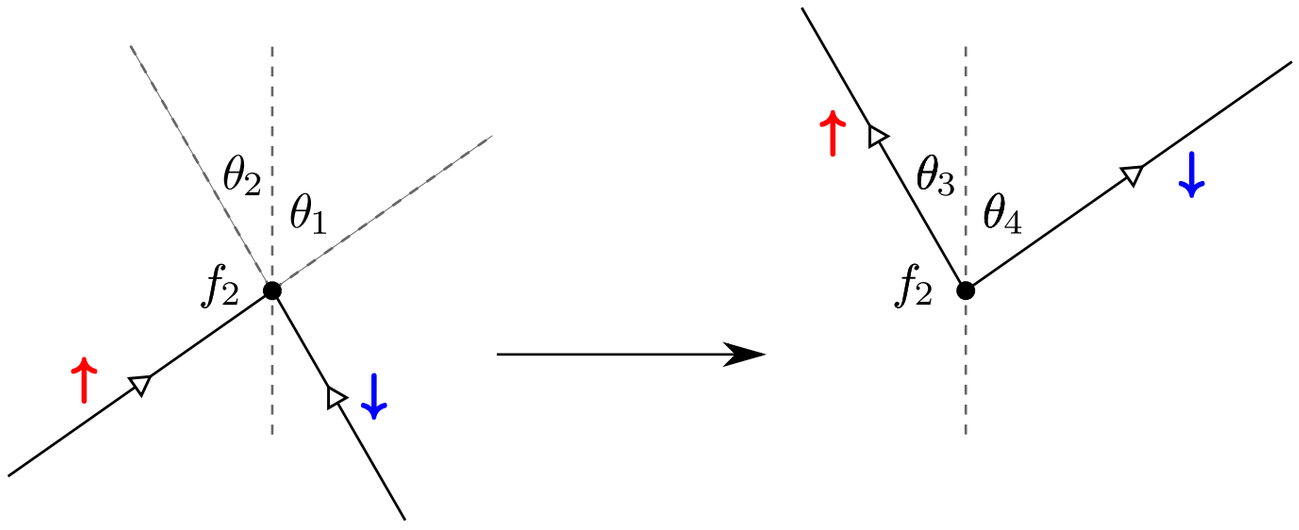
Hence no conventional BCS pairing induced by backward scattering

Pairing induced by forward scattering f_2

Model interaction

$$H_I = \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{k}'\downarrow} c_{\mathbf{k}\downarrow} c_{\mathbf{k}'\uparrow}$$

corresponds to f_2



The two species exchange momenta upon scattering

$$H_I = \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{k}'\downarrow} c_{\mathbf{k}\downarrow} c_{\mathbf{k}'\uparrow}$$

Define hole operators $a_{\mathbf{k}\downarrow} = c^\dagger_{-\mathbf{k}\downarrow}$

$$H_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} - \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow}$$

$$= \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} + \varepsilon_{\mathbf{k}} a^\dagger_{\mathbf{k}\downarrow} a_{\mathbf{k}\downarrow} - \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}}$$

$$H_I = -\frac{U}{V} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} = -\frac{U}{V} A^\dagger A$$

$A^\dagger = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow}$ **excitonic operator**

BCS gap equation

Excitonic condensate gap $\Delta = -U\langle A\rangle/V$

$$\Delta = \frac{U}{2V}\sum_{\mathbf{k}}\frac{\Delta}{\sqrt{\varepsilon_{\mathbf{k}}^2 + |\Delta|^2}}.$$

$$\Delta = 2c\Lambda e^{-\frac{1}{UN(0)}}$$

BCS gap equation

Competes with regular intraband BCS superconducting state

Occupied by particles

Fermi energy

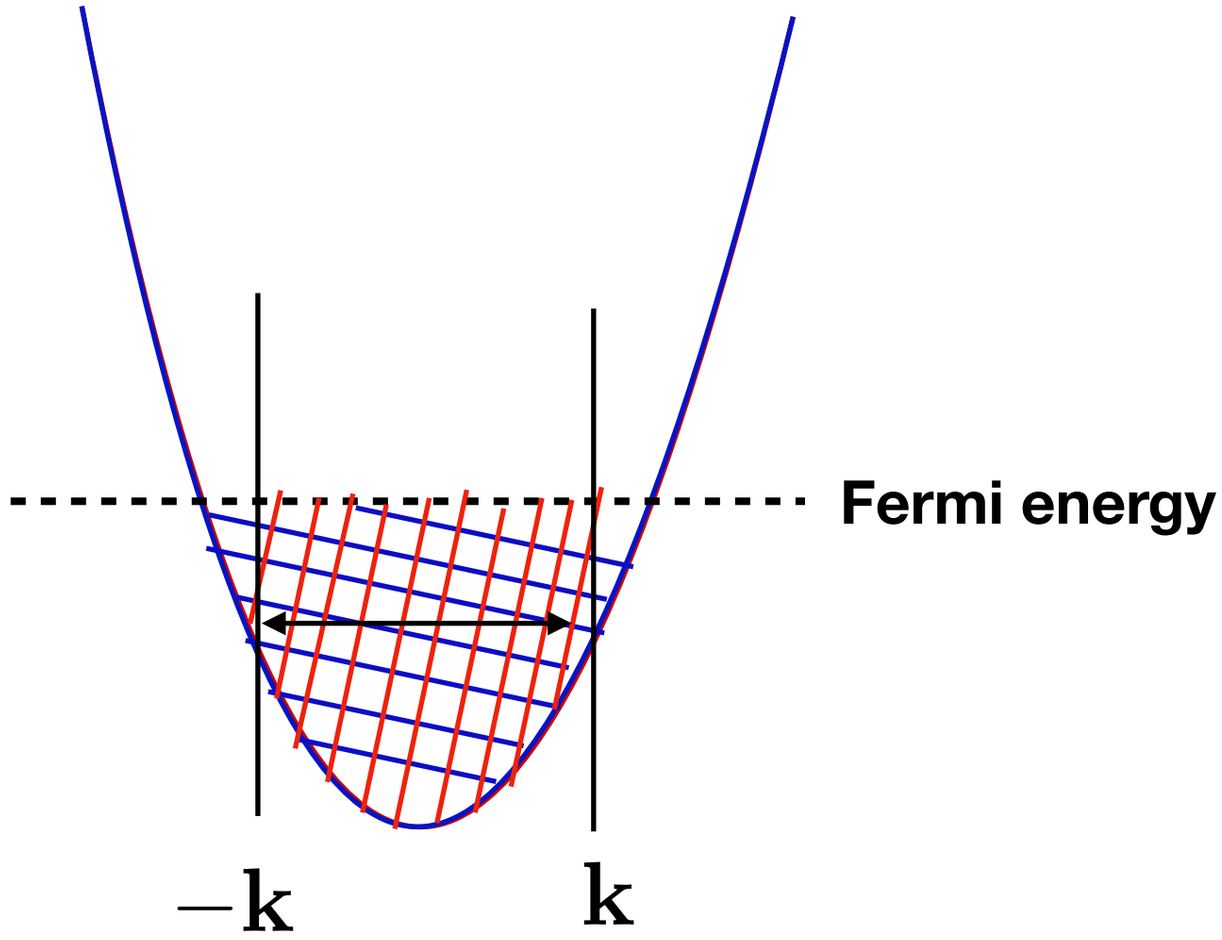
Particle hole transformation

$$a_{\mathbf{k}\downarrow} = c^{\dagger}_{-\mathbf{k}\downarrow}$$

Fermi energy

Occupied by particles

Occupied by holes



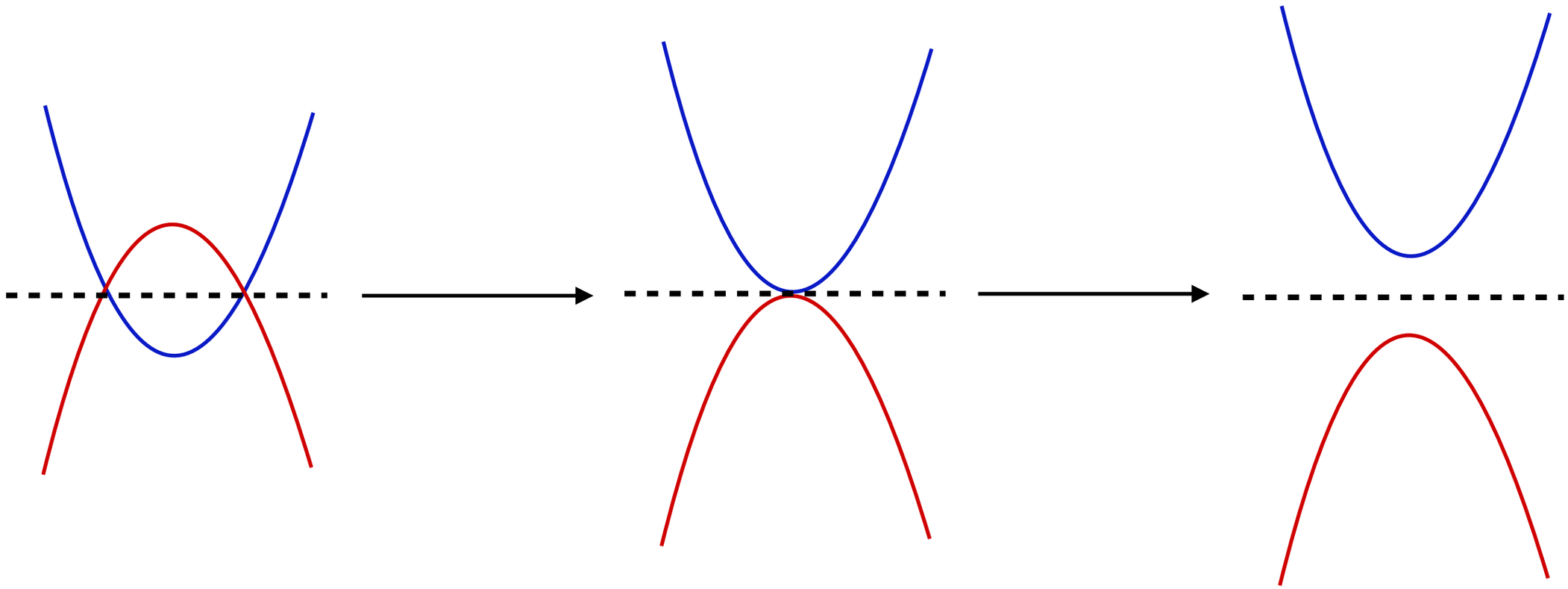
Cooper pairing between particle of momentum $\mathbf{k}$ and hole of momentum $-\mathbf{k}$

Excitonic condensate

$$A^\dagger = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow}$$

$$H_I = -\frac{U}{V} \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} = -\frac{U}{V} A^\dagger A$$

Conclusions



Fermi surface is circular

2D Fermi liquid with intraband BCS and interband excitonic instabilities

Fermi surface is a point

Non Fermi-liquid

But not a Luttinger liquid or a Dirac liquid

Fermi surface not properly defined

Insulator

Green's function of the interacting system

One loop

$$G_{I,\alpha\beta}(2;1) \approx -\langle \bar{\psi}_\beta(2)\psi_\alpha(1)\rangle_0 + \langle S_I \bar{\psi}_\beta(2)\psi_\alpha(1)\rangle_0 - \langle S_I\rangle_0 \langle \bar{\psi}_\beta(2)\psi_\alpha(1)\rangle_0$$

Model A

$$G_I(2;1) = \frac{\delta(2-1)}{\omega^2 + (k^2/2m)^2} \times \begin{pmatrix} i\omega - \tilde{g}\frac{\omega^2-(k^2/2m)^2}{\omega^2+(k^2/2m)^2} & \frac{k^2 e^{-2i\theta}}{2m}\left(1+\frac{2i\omega\tilde{g}}{\omega^2+(k^2/2m)^2}\right) \\ \frac{k^2 e^{2i\theta}}{2m}\left(1+\frac{2i\omega\tilde{g}}{\omega^2+(k^2/2m)^2}\right) & i\omega - \tilde{g}\frac{\omega^2-(k^2/2m)^2}{\omega^2+(k^2/2m)^2} \end{pmatrix} \qquad \tilde{g} = g\Lambda^2/16\pi$$

Model B

$$G_I(2;1) = \delta(2-1)\begin{pmatrix} \frac{1}{-i\omega+\frac{k^2}{2m}-\frac{g\Lambda^2}{8\pi}} & 0 \\ 0 & \frac{1}{-i\omega-\frac{k^2}{2m}} \end{pmatrix}$$