PHY-207.5 Advanced Classical Electromagnetism
(ICTS Core Course for both Int.PhD.,PhD.)

Instructor : R. Loganayagam.

January 10, 2022

Basic Info :

- **Instructor name :** R.Loganayagam,
- **Tutors name :** Godwin Martin, Shivam Sharma .
- **Venue :** Online (Zoom) for now. Hopefully we can move to a hybrid mode in a few weeks.
- **Timings :** Tuesday/Thursday : 11:30 - 13:00 Hrs
  Tutorials on Saturday : 11:30 - 13:00 Hrs
- **First Class (Introduction) :** Monday (16:00 - 19:00 Hrs), 24th January, 2022
  (Note unusual timing for the first class.)
- **Drop test :** Jan 29th 2022 (tentative)
- **The grading policy** will be based on the following weightage :
  - Assignments : 35% for PhD students, 40% for I.PhD students
  - Mid term Exam : 30%
  - End term Exam : 30%
  - Term paper (a thorough review of a topic in electromagnetism not covered in the text-books mentioned below. I will send in a list of term paper suggestions later.) : 5% Extra credit (Compulsory for PhD Students)

Note that Assignments form a central part of this course, since one of the main aims of this course is to train students to solve problems.

Course Objectives:

The objective of this course is to train students to

1. Understand the variational principles, symmetries and their associated conservation laws in electromagnetism and apply it to solve problems.

2. Solve electrostatic/magnetostatic boundary value problems using the methods of Green functions, eigenfunction expansions, multipole expansion, etc.

3. Solve EM radiation problems using waveguide theory, far-field approximations, Larmor formula, Hertz-Debye potentials, vector multipole expansions etc.

4. Solve EM scattering/diffraction problems in long/short wavelength approximations as well as Mie scattering at intermediate wavelengths.

5. Construct and use simple statistical mechanical models of electric/magnetic susceptibility, ohmic conductivity, Debye screening and electrochemical activity.

6. Derive commonly used constitutive relations and make simple order of magnitude estimates of electric/magnetic properties of matter.

If done in full detail (say as done in any of the main textbooks below), this is a lot of material for a one (or even two) semester course ! We will try to cover at least as many of these topics we have tine for.

There will be a drop test testing on the above topics on Jan 29th 2022(tentative). I will require all of you to take it. If you do well, you do not have to credit the course.

Textbooks:

The main textbooks relevant to this course are

- Classical Electrodynamics by J.Schwinger, L. L. DeRaad Jr., K. A. Milton, and W.y. Tsai.
- Classical Electricity and Magnetism by Wolfgang K. H. Panofsky and Melba Phillips.
- Modern electrodynamics by A. Zangwill
- Classical Electrodynamics by J.D. Jackson

All of these are very well-written books (*including* the text by Jackson which is good if you treat it as a collection of encyclopaedia-type sections rather than a textbook).

I will not rigorously follow any of these textbooks in my lectures and I will supplement it with my own notes/other books. Nevertheless I will expect that you have read the relevant sections of these texts during my lectures, assignments and exams. Your aim should be to read and master $\geq 80\%$ of at least one of these textbooks by the end of this course.

I will recommend that the ICTS students who have to credit the course use their contingency grant try to buy a personal copy of either one or both of Schwinger et.al (Indian edition cost $\sim$ Rs.1000) and Jackson (Indian edition cost $\sim$ Rs.800). You will need them for the reading/problem sets, class/tutorials and the open book tests/exams. There is also a cheap edition of Panofsky and

Phillips (Indian edition cost $\sim$ Rs.700) available, in case you are interested. Zangwill, alas, has no Indian edition yet and is quite expensive ! In any case, you should be able to borrow copies of these texts from the ICTS Library.

Prerequisites:

I am assuming you are familiar with the following topics which are standard during a first course in electromagnetism:

- Vector analysis: gradient, divergence, curl, divergence theorem, Stokes' theorem.
- Electrostatics: Poisson equation, Boundary value problems, image problems, multipole expansion.
- Electric field and potential in matter, polarisation.
- Magnetostatics, magnetic field in matter, magnetisation.
- Electrodynamics: Faraday's law of induction
- Maxwell's equations/electromagnetic waves in free space and in matter.
- Geometric/ray Optics, wave optics.

This is a short summary of the detailed pre-requisites discussed during our meeting in December. If you want the detailed version, ask your tutors.

The basic texts that cover these pre-requisites are

- Feynman Lectures on Physics Vol 2 by Feynman, Leighton and Sands.
- Electromagnetic Fields and Energy by Hermann A. Haus and James R Melcher
- Introduction to electrodynamics by David J.Griffiths
- Classical Electrodynamics by Greiner.
- Electricity and magnetism by Edward M.Purcell and David M.Morin.

If you feel that you are under-prepared in a particular topic, I will strongly recommend to look at the relevant chapters in these books. You should be roughly able to read and comprehend the first 75% of these books. Let me know if that is not the case.

There will be a set of preliminary tests (take home) so that I can have an idea of where the class stands regarding these topics.

Apart from these, it is good to revise various things from previous courses such as

- Classical mechanics: Lagrangian, Hamiltonian, canonical transformation, Noether theorem.
- Quantum mechanicsbe: Rotations, Spherical harmonics/theory of angular momentum, Scattering theory of waves,
- Statistical mechanics: Basic kinetic theory, partition functions, etc.

- Mathematical physics : Fourier/Laplace transforms, delta functions, complex analysis (especially residue theorem and contour integrals), PDEs (especially second order PDEs like wave equation, Poisson equation, Heat equation), Green functions, variable separation, Sturm-Liouville theory, Bessel functions, Legendre polynomials, etc.

Policy on online classes/privacy :

The online nature of the course that has been forced on us by these pandemic times. While it has opened up new opportunities, its predominant effect has been negative: it has has quite an adverse effect on the student interactions/feedback and on the assessment of student progress. A recognition of these limitations and a commitment to overcome them (both on the side of students as well as the instructor/tutors) is necessary for a course like this to succeed. I hope the students take a pro-active role.

In order to encourage frank assessments and discussions, this course will be restricted to present ICTS graduate students. Especially, it is expected that the Zoom links and video recordings of classes would be access-restricted and I expect the students to respect the online privacy of their peers in keeping it so.

Course Philosophy :

Broad motivation :

I think of the theoretical physics graduate course curriculum as being divided into two halves : first, an 'un-compromisable' core minimum of five subjects in which a student should have a thorough critical and analytic skills. These are

1. Classical mechanics (at the level of the standard textbook by Goldstein or Landau-Lifshitz Vol. 1).
2. Quantum mechanics (at the level of the standard textbook "Modern Quantum Mechanics" by Sakurai).
3. Statistical mechanics (at the level of the standard textbook by M. Kardar or R. K. Pathria)
4. Mathematical Physics (at the level of the standard textbook by Arfken or K. F. Riley-M. P. Hobson-S. J. Bence).
5. Classical electrodynamics (at the level of the standard textbooks mentioned above).

I feel that it is very difficult to go very far in theoretical physics without a deep and thorough understanding of these subjects. To work on a research problem, this needs to be supplemented by three to four courses in the appropriate research area.

My reason for this opinion is as follows: each of these core subjects introduce a particular worldview which has been very successful in our understanding of physical reality. These viewpoints recur again and again in theoretical physics (in fact, I would define a theoretical physics problem as one where this happens). So, let me begin by describing the worldview of classical electrodynamics.

History :

First some quick history. It is often said that electrodynamics was historically born out of the unification of two forces - electric and magnetic. In fact, I think a more historically precise statement is that it combines *six* different phenomena known to our human ancestors :

1. **Magnets**, magnetic compass and magnetism of the earth. Magnets, and the fact that they seem to act on each other at a distance, mystified and fascinated the ancients. As a first force which was recognised to act at a distance, magnetism became the template on which the Newtonian theory of universal gravitation was constructed.

2. **Lightning** and atmospheric phenomena like Aurora Borealis. (Over the last century, the study of ionosphere)

3. Energy behind **biological dynamics.** e.g., forces applied by humans using their **tissues and nerves**. The brain and heart are now recognised to be quintessentially electric organs. The ancient notions of 'life force', 'vital energy' etc., including talks about Ase (Nigeria), Qi (China), Manitou (North American tribes), elan vital (Europe), some notions of Prana(India) are all pre-Maxwellian theories of phenomena which are now known to be electric in their origin. Many such ancient theories and arguments about soul have eventually been subsumed under modern electro-physiology.

4. The forces born out of **materials and chemicals** including those behind **elasticity, friction** and **chemical reactions**. The human fascination with friction led to the discovery of fire. The fire and the transformations induced by it further gave rise to alchemy and then modern chemistry.

5. **Electrostatic** forces obtained by rubbing various materials against each other.

6. Phenomena associated with **light** and optics in general.

In a remarkable turn of history filled with a long array of accidental experimental discoveries and brilliant insights, all these forces and energies have come to be understood as one and the same. This is perhaps the first lesson of electrodynamics : *much of the physical reality we see around us have common origins.*

Textbook history of physics tends to somewhat overemphasise the history of gravity and mechanics. But I believe there is a way of looking at the history of physics and even the entire history of humanity as mirrored in the history of electromagnetism.

Importance :

What are the other lessons of electrodynamics?

- Convenient description of reality is in terms of **fields**. These fields have a physical reality of their own, e.g., they carry energy and momentum. The fields evolve according to certain **partial differential equations** with appropriate **initial conditions/boundary conditions**.

- The macroscopic forces except gravity are all described in fact by a pair of **electric** and **magnetic** fields acting between **charges** and **currents**. The charges produce fields and the fields in turn affect charges.

- Often, fields leave the charges to travel far in terms of **waves**. These waves can undergo **reflection, refraction, interference and diffraction** in various media.
- In vacuum, these waves can travel very fast (in fact, with the maximum possible speed) and are thus **relativistic**.
- In media, the **collective behaviour** of charges leads to an **effective/averaged description** whereby the forces are screened/enhanced/modified into something entirely new. The waves **disperse** in a frequency dependent way and their **effective speed of propagation** also becomes frequency dependent.
- At temperatures much below their frequencies, these waves start behaving like particles thus leading to **quantum behaviour**.
- When the reverse effect of particles behaving like waves is taken into account, one then forms **stable bound states/novel phases** (like atoms, molecules, solids, liquids etc.).

The reader will notice that these highlighted ideas are absolutely fundamental to how a physicist (especially a theoretical physicist) views the world. In reality, it is almost impossible to find something in physics where the tools and viewpoint of electromagnetism are not employed at least indirectly. It is a way of thinking, analysing and arguing about physical phenomena. One of the primary aims of an electrodynamics course is to train a student in doing that.

Many textbooks give the false idea that classical electromagnetism is a finished subject. This is NOT true. The four oldest problems of electrodynamics: that of magnetic phases of matter, of dynamics of the geo-magnetic field (and other astrophysical magnetic fields), of lightning, that of electric/magnetic field generation and detection by animals and plants are all active areas of research. Historically, these problems motivated the pioneers of electromagnetism from Gilbert to Galvani to Gauss, from Franklin to Faraday. Despite centuries of work, these questions still throw up puzzles which baffle us. For some unknown reason, many books pretend that such questions do not exist any more.

This is apart from the new challenges which were unknown few decades ago. The electric/magnetic response of Hall systems from the 1980s has brought into physics surprisingly sophisticated mathematical ideas. The theory of ferro-electrics (called the Modern Theory of Polarization) was really understood only in the early 1990s. The theory of insulators (usually thought of as boringly simple) threw up surprises over the last decade whose resolution has led to the theory of topological insulators. Ferromagnetic and anti-ferromagnetic materials, of course, dominate so much of modern condensed matter physics. To conclude, so much of modern life revolves around electromagnetism: it is ridiculous to even attempt at describing how much.