CAM-ICTS Joint PhD Program

Semester I	Semester II
Algebra & Linear Algebra (4 credits)	Probability Theory (4 credits)
Topology & Geometry (4 credits)	Partial Differential Equations (4 credits)
Measure Theory & Functional Analysis (4 credits)	Complex Analysis (4 credits)

Syllabus

Algebra & Linear Algebra

Group Theory: The Jordan-Holder Theorem, solvable groups, symmetric and alternating groups, nilpotent groups, groups acting on sets, Sylow Theorems, free groups.

Rings and Modules: Noetherian and Artinian conditions, the Hilbert Basis Theorem, principal ideal domains, unique factorization domains, inductive and projective limits of rings and modules, bilinear maps and forms, the tensor product.

Field Theory: Steinitz Theorem on algebraic closures, algebraic extensions, finite fields, Galois theory and applications.

Linear algebra: Modules over principal ideal domains, the minimal polynomial of an endomorphism, Jordan canonical form, the characteristic polynomial of an endomorphism, Cayley-Hamilton Theorem.

Topology & Geometry

Topology: Homotopy, retraction and deformation, fundamental group, Van Kampen theorem, covering spaces and their relations with fundamental group, universal coverings, automorphisms of a covering, regular covering.

Geometry: Differential geometry of curves and surfaces, mean curvature, Gaussian curvature, differentiable manifolds, tangent and cotangent spaces, vector fields and their flows, Frobenius theorem, differential forms, de Rham cohomology.

Measure Theory & Functional Analysis

Measure Theory: Abstract integration (Concept of Measurability, Elementary properties of Measures, Integration of positive functions), Positive Borel Measures (Riesz Representation Theorem, Properties of Borel Measures, Lebesgue Measure, Continuity properties of measurable functions), L^p spaces (basic properties, approximation by continuous functions), Complex Measures (Total variation, absolute continuity, Radon Nikodym Theorem, Riesz representation theorem), Differentiation (Derivatives of measures, fundamental theorem of calculus), ntegration on product spaces (Product measures, Fubini theorem, convolution, distribution functions)

Functional Analysis: Banach spaces and some properties, Hahn Banach Theorem and its consequences, Uniform Bounded Principle, Closed Graph Theorem, Open Mapping Theorem, Weak topologies, reflexive spaces, Hilbert spaces and some properties (dual of a Hilbert space, Theorems of Stampacchia and Lax-Milgram), Compact Operators and spectral decomposition of compact self-adjoint operators, Hille-Yosida Theorem

Probability Theory

Review of discrete Probability, Review of measure theoretic facts (distribution of a random variable, expectation, product measures, Fubini), Laws of large numbers (independence, sums of independent random variable, weak law of large numbers, Borel-Cantelli theorems, strong law of large numbers, random series) Central limit theorems (weak convergence of probability measures, characteristic functions, central limit theorem, infinitely divisible distributions), Conditional expectation, Martingales (uniform integrability, Doob's upcrossing lemma, martingale convergence etc), Introduction to Brownian motion

Additional topics (depending on time and taste of instructor): Random walks, Markov chains, ergodic theory.

Partial Differential Equations

Introductory PDE: First order PDE (solutions by method of characteristics), Analysis of Laplace, heat and wave equations, Cauchy Kowalewsky theorem, Holmgren's uniqueness theorem

Distribution Theory: Concept of distributions, differentiation of distributions, multiplication of a distribution by smooth functions, compactly supported distributions, tensor product of distribution, convolution, Schwartz kernel theorem, Fourier transforms and tempered distributions.

Sobolev spaces: Introduction to Sobolev spaces and elementary properties, approximation by smooth functions, extensions, traces, Sobolev and Poincare inequalities, compactness.

Study of second order elliptic equations: Weak formulation, Lax - Milgram Lemma, existence and regularity of solutions up to the boundary, Maximum principle, elementary variational inequality.

Additional topic (depending on time and taste of instructor): Study of linear evolution equations, existence of weak solutions, energy methods.

Complex Analysis

Cauchy's theorems (homology version), normal families, Riemann mapping theorem, conformal mappings, Schwarz-Christoffel theorem, Runge's theorem, entire functions (Jensen's formula, order of an entire function, Weierstrass factors, Hadamard Factorization theorem), hyperbolic metric on the unit disc and the upper half plane (definition, invariant form of Schwarz lemma, curvature of a metric, Alhlfors-Schwarz lemma, Picard theorems), Paley-Wiener theorem.

Additional topics (depending on time and taste of the instructor): Proof of the prime number theorem and connections to number theory, Riemann Surfaces, Asymptotic methods, topics from elliptic functions.