Instabilities in Viscosity-Stratified Flow

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Abstract

This review highlights the profound and unexpected ways in which viscosity varying in space and time can affect flow. The most striking manifestations are through alterations of flow stability, as established in model shear flows and industrial applications. Future studies are needed to address the important effect of viscosity stratification in such diverse environments as Earth's core, the Sun, blood vessels, and the re-entry of spacecraft.

1. INTRODUCTION

Viscosity is a function of space and time in a large variety of fluid flows, and its variation can have a dramatic effect on flow stability. Any flow in which the temperature or composition is not constant has a resulting variation in viscosity. In high-pressure flows, viscosity is a function of pressure. Viscosity can also depend on the shear and its history, as in non-Newtonian fluids. Blood and mucus are two such rheologically complex fluids. Most flows in the chemical or food industry and some in nature (e.g., glaciers, magma, and to a lesser extent in the ocean and atmosphere) involve viscosity stratification. Viscosity in Earth's outer core and within the Sun varies by orders of magnitude. In space shuttle re-entry, we are again confronted with immense viscosity variations, on short length scales. Our focus in this review is on the intimate interaction between viscosity stratification and large-scale shear in these flows, which acts to suppress instabilities or create new ones and thus influences the route to turbulence. **Figure 1** shows a small subset of present and possible future applications in which instabilities due to viscosity stratification are important. In some of these applications, viscosity stratification is inherent. In others, we may wish to impose stratification to promote or suppress turbulence to achieve better mixing or drag reduction, respectively.

The search for effective drag-reduction techniques led to several remarkable findings from the mid-twentieth century onward in disparate fields of engineering involving viscosity stratification. Significant among these are wall heating/cooling in boundary layers on solid bodies, aimed at drag reduction on aircraft wings; lubricated pipelining for oil extraction; and the famous polymer drag reduction. There is no evidence that the numerous researchers working in each of these directions during the early decades knew about each other's work, but to a modern worker in this area, the strong connection will be immediately obvious. Previous studies have often provided seemingly contradictory results: in some cases of huge destabilization and huge stabilization caused by a tiny stratification of viscosity and in other cases of large variations in viscosity that make no difference. A decrease in viscosity near the wall increases the critical Reynolds number enormously in some situations (Ranganathan & Govindarajan 2001), but such a decrease can also cause the opposite: instability at extremely low Reynolds numbers (Selvam et al. 2007). Surface tension sometimes stabilizes the flow, sometimes destabilizes it, and sometimes does both. Not all these contradictions are resolved, but below we explain those that are. We also discuss unexpected new modes of instability, for example, the short-wavelength instability on a strain rate interface (Hinch 1984, Hooper & Boyd 1983) or the double-diffusive (DD) mode in a stably stratified flow (Sahu & Govindarajan 2011).

2. THE PHYSICS OF VISCOSITY STRATIFICATION AND LINEAR INSTABILITY

How can the effect of viscosity stratification be big? One would at first sight not expect it to be, as these shear flows are typically at high Reynolds numbers and are therefore expected to be dominated by inertial effects.

To answer this question, let us consider a unidirectional shear flow whose velocity components may be written as $U = U(y)$, $V = W = 0$, respectively, along the streamwise coordinate x, wall-normal direction y, and spanwise coordinate z. These have been scaled by a reference velocity U_{ref}, usually either the maximum or the average value in the flow. The modified Orr-Sommerfeld and Squire's equations for viscosity-stratified flow, given by Equations 1 and 2 below (Sahu & Matar 2010b), describe the development of linear perturbations in parallel flows. They have been obtained by linearizing the Navier-Stokes equations upon splitting all flow quantities into a mean and a perturbation [e.g., $U_{\text{total}} = U(y) + \hat{u}(x, y, z, t)$] and expressing the perturbations in

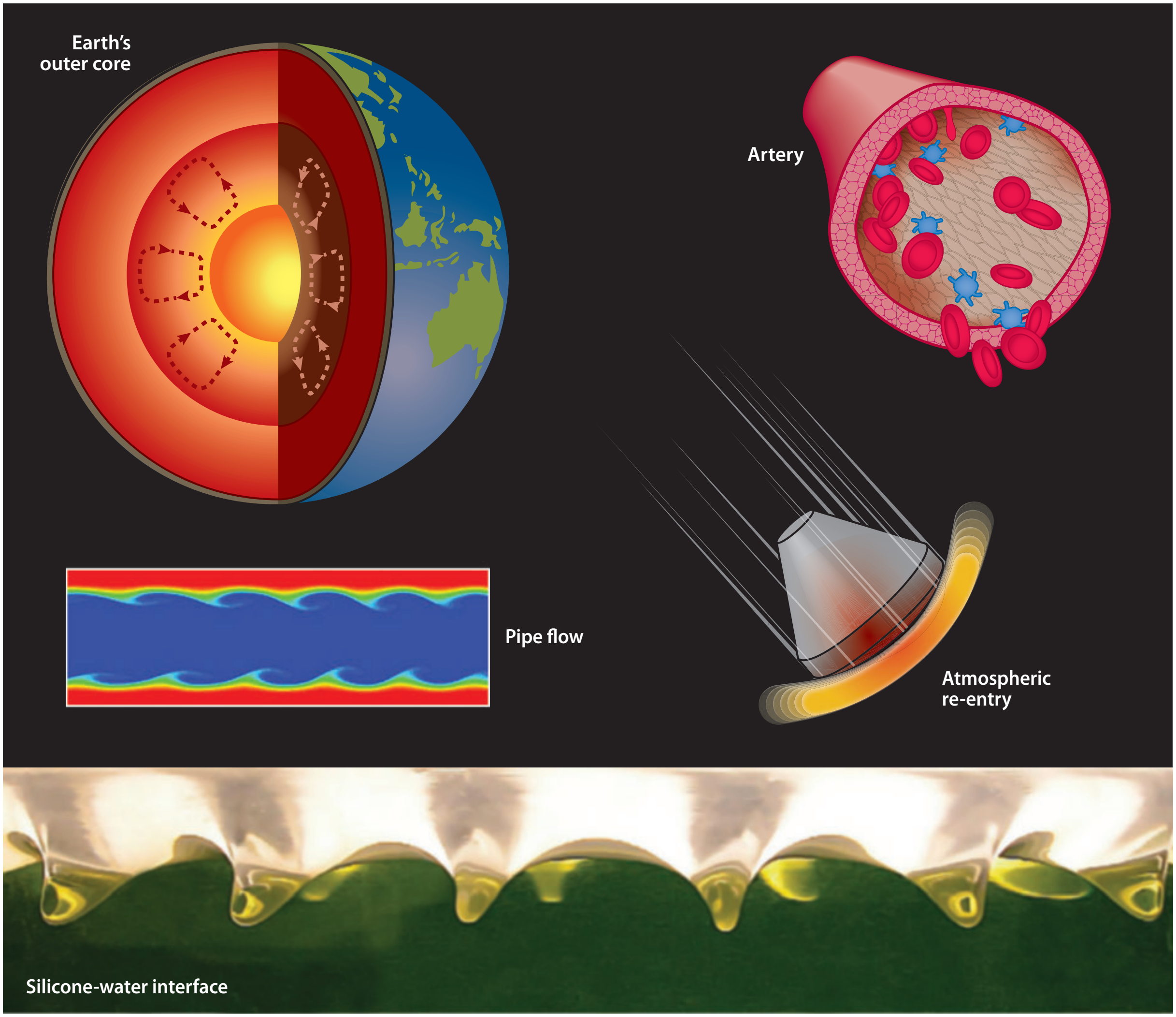

Figure 1

A small subset of situations in which viscosity stratification and instabilities driven by it are important: Earth's outer core, the inside of an artery with corpuscles and platelets, a re-entry vehicle (Reddy & Sinha 2009), pipe flow (Sahu et al. 2009a), and an oscillatory silicone-water interface (Yoshikawa & Wesfreid 2011). Figure after sketch by Manoj Tripathi.

normal-mode form:

$$[v'' - (\alpha^2 + \beta^2)v](U - c) - U''v = \frac{1}{i\alpha Re}\{\bar{\mu}[v'''' - 2(\alpha^2 + \beta^2)v'' + (\alpha^2 + \beta^2)^2 v] + 2\bar{\mu}'[v''' - (\alpha^2 + \beta^2)v'] + \bar{\mu}''[v'' + (\alpha^2 + \beta^2)v] - i\alpha U'[\mu'' + \mu(\alpha^2 + \beta^2)] - 2i\alpha U''\mu' - i\alpha U'''\mu\}, \tag{1}$$

$$\eta(U - c) + \frac{\beta}{\alpha}U'v = \frac{1}{i\alpha Re}\left\{\bar{\mu}\left[\eta'' - (\alpha^2 + \beta^2)\eta\right] + \bar{\mu}'\eta' + i\beta U'\mu' + i\beta U''\mu\right\}, \tag{2}$$

where $Re (\equiv U_{\mathrm{ref}} H\rho/\mu_{\mathrm{ref}})$ is the Reynolds number, with ρ and H the density and a typical length scale in the y direction, respectively. $\bar{\mu}(y)$ is the base flow viscosity, rendered nondimensional by scaling with a reference viscosity μ_{ref}. It is a function of the basic variation $\bar{s}(y)$ of a scalar quantity (e.g., temperature or a solute concentration). The primes denote differentiation with respect to y. Perturbations in the wall-normal velocity, wall-normal vorticity, and viscosity are given by $(\hat{v}, \hat{\eta}, \hat{\mu}) = (v(y), \eta(y), \mu(y)) \exp[i(\alpha x + \beta z - \omega t)]$, respectively, wherein α, β, ω, and $c \equiv \omega/\alpha$ are the streamwise and spanwise wave numbers, the frequency, and the phase speed of the disturbance mode, respectively. The flow is temporally unstable when $\omega_i > 0$. Upon setting the viscosity to zero, the Orr-Sommerfeld equation reduces to the Rayleigh equation.

To maintain generality, we have written the equations in terms of viscosity perturbations, which may be written as $\hat{\mu} = (d\bar{\mu}/d\bar{s})\hat{s}$. The perturbation amplitude s of the scalar quantity will satisfy

$$\mathrm{i}\alpha(U - c)s - v\bar{s}' = \frac{1}{Pe}[s'' - (\alpha^2 + \beta^2)s], \tag{3}$$

where $Pe(\equiv U_{\text{ref}} H/\mathcal{D})$ is the Péclet number, with $\mathcal{D}$ the diffusivity of the species. The ratio of the species and momentum diffusivities is the Schmidt number, $Sc(\equiv \mathcal{D}\rho/\mu_{\text{ref}})$, an important player in the instability process. For temperature, the Prandtl number Pr, with thermal diffusivity in the place of $\mathcal{D}$, appears instead of Sc. When $\bar{s}$ stands for temperature, $\bar{\mu}(\bar{s})$ for liquids and gases are usually prescribed by the Arrhenius law (Nahme 1940) and the Sutherland law (White 1991), respectively. When viscosity variations are effected by the non-Newtonian nature of the fluids, pressure, or suspended particles, appropriate equations will replace Equation 3. The boundary conditions, including those that need to be introduced for a sharp interface within the domain, are flow specific and are not listed here. However, usually the eigenfunction of the disturbance is localized in y, and Dirichlet boundary conditions on the velocity and species amplitudes are prescribed.

The corresponding equation for the disturbance kinetic energy $\mathcal{E} \equiv (1/2)\langle \hat{u}^2 + \hat{v}^2 + \hat{w}^2 \rangle$, where angle brackets represent averaging in x and z, is given by

$$\frac{\partial \mathcal{E}}{\partial t} = \mathcal{P} - D + \mathcal{T} + \mathcal{S}, \tag{4}$$

where $\mathcal{P} = -\langle \hat{u}\hat{v} \rangle U'$ is the production of disturbance kinetic energy by Reynolds stresses, and

$$\mathcal{T} = \nabla \cdot \left[-\frac{\langle \tilde{u}\hat{p} \rangle}{\rho} + 2\frac{\mu}{Re}\nabla\mathcal{E} \right] \quad \text{and} \quad \mathcal{S} = \frac{1}{We}\Big\langle \Big[\hat{v}(\hat{h}_{xx} + \hat{h}_{zz}) \Big]_{y=\bar{h}} \Big\rangle \tag{5}$$

are the stress transport and surface tension effects, respectively. The Weber number is defined by $We \equiv \rho H U_{\text{ref}}^2/\sigma$, where σ is the surface tension, $\hat{h}$ represents the perturbation in the interface height, and tilde indicates a perturbation vector. $D = \frac{1}{Re}\langle |\nabla \tilde{u}|^2 \rangle$ is the viscous dissipation, which is evidently always positive, so viscosity introduces damping of the disturbance kinetic energy (see, e.g., Hu & Joseph 1989). For a miscible fluid, $\mathcal{S} = 0$, and whereas $\mathcal{T}$ is nonzero locally, its net contribution across y is zero for miscible flows and the boundary conditions considered here.

Regarding the effect of viscosity, one would intuitively expect two things, both of which are firmly belied: (*a*) that viscosity must always have a stabilizing effect and (*b*) that viscous effects, including those of stratification, must be small at high Reynolds number. Viscosity does indeed contribute dissipation, but it is capable of increasing production even more, resulting in net destabilization. We note that $\mathcal{P} = 0$ when $\hat{u}$ and $\hat{v}$ oscillate out of phase. That $\mathcal{T}$ is not zero locally means that viscosity diffuses momentum across the flow. Thus it changes the phase between $\hat{u}$ and $\hat{v}$. A stratification of viscosity will modify the diffusion, and thus the phase, production, and stability. Dissipation is usually concentrated near walls, whereas production is concentrated in the critical layer, in which the phase speed of the given perturbation is close to the base flow velocity (i.e., $U \sim c$).

Viscous effects can be large at high Reynolds numbers simply because they constitute a singular perturbation in the Orr-Sommerfeld equation (Drazin & Reid 1985, Lin 1946, Schmid

& Henningson 2001): Their introduction increases the order of the equation from two to four. This means that somewhere in the flow, viscous terms contribute an $O(1)$ effect, however high the Reynolds number, and in these portions of the flow, the Rayleigh equation cannot produce anything resembling the correct solution. It is easy to imagine that one such region must be the vicinity of a wall because viscous effects prevent slip near a wall. An examination of Equation 1 reveals that viscous effects are also important in the critical layer. Stratification can modify the singular terms significantly and can dramatically alter flow behavior. Studying a modified Yih flow in the small wavenumber limit, Craik (1969) was the first to realize that a continuous viscosity stratification at the critical layer can play a lead role in changing stability. Critical layer effects enter the balance at $O(Re^{-1/3})$ (Govindarajan 2004, Lin 1946), whereas wall effects enter at higher order, $O(Re^{-1/2})$. Thus it is the critical layer, rather than the wall layer, in which stratification has the biggest effect. Consequently, stratification hugely alters the disturbance production rather than dissipation.

Another role of viscosity stratification is in altering the base flow. By a fortunate circumstance, the Rayleigh-Fjørtoft criteria for inviscid stability act as qualitative indicators for viscous flows as well. Taken together (Fjørtoft 1950, Rayleigh 1880), the necessary condition for inviscid instability is that the velocity profile must contain a vorticity maximum. Tollmien argued that these criteria constitute sufficient conditions for the instability of symmetric channel profiles or monotonic boundary-layer profiles (Drazin & Reid 1985). In general, a given alteration will stabilize the flow if the velocity profile moves away from being inflectional (i.e., made fuller) and will destabilize it if U'' goes toward, or displays, a sign change. Viscosity variation can add an all-important inflection point to the velocity profile.

Small-amplitude instabilities produced by shear at high Re in boundary layers and channels are known as Tollmien-Schlichting (TS) modes. Under constant viscosity conditions, TS waves within a certain frequency range in plane Poiseuille flow display growth beyond $Re = 5{,}772.2$ (based on the channel half width), whereas in Couette or pipe flow, they decay at any Reynolds number. In reality, however, all these flows undergo transition to turbulence at $Re \sim O(10^3)$ (Schmid & Henningson 2001), even though they are linearly stable. That the stability operators are not self-adjoint implies that a superposition of stable eigenmodes can lead at early times to algebraic growth of the disturbance energy (see, e.g., Butler & Farrell 1992, Trefethen et al. 1993). This transient growth can be large enough for nonlinearities to set in, so the asymptotic (time $t \to \infty$) linear stable state is never reached, and the transition to turbulence occurs at a Reynolds number well below the critical value. The introduction of viscosity stratification can upset the balance and make exponential growth the dominant mechanism, at a far lower Reynolds number than in constant viscosity. Otherwise, it can alter transient growth in a significant way (Jerome et al. 2012).

The most unstable transient growth mode is usually three dimensional. The fastest growing linear mode at some Re could also be three dimensional (Sahu & Matar 2010b). However, when looking for the critical Reynolds number, per Squire's (1933) theorem, one will find it sufficient to study two-dimensional perturbations in planar single-fluid flows. Schaflinger (1994) and Yih (1955) showed that this is true for planar viscosity-stratified flows as well, whereas Azaiez & Homsy (1994) found an equivalent condition for viscoelastic fluids obeying the Oldroyd-B model: that at $Re \to \infty$ and high elasticity, with an $O(1)$ ratio of polymer relaxation and viscous timescales, the equations do not distinguish between two- and three-dimensional perturbations. In pipes, Squire's theorem does not apply.

In disparate applications, viscosity stratification is viewed as a means to stabilize the flow, but stability analyses most often yield the reverse answer. For a better understanding of the conflicting contributions of stratification, it is instructive to divide the discussion into three levels of viscosity

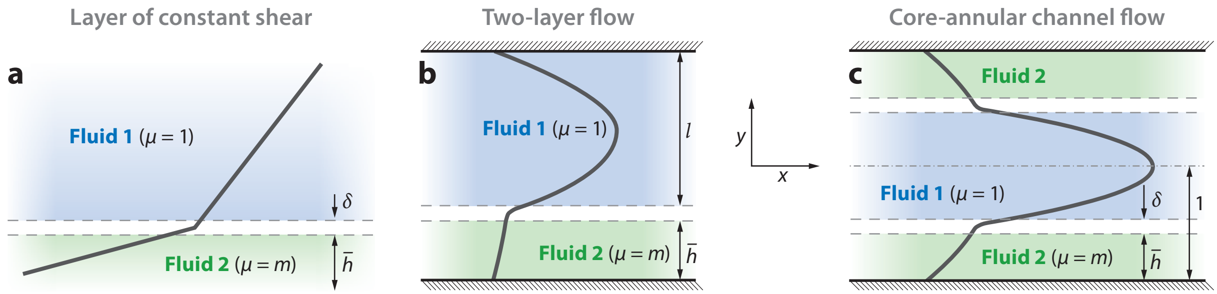

Figure 2

Velocity profiles in simple geometries, with viscosity variation confined to a layer of thickness δ. The viscosity of the layer closest to the wall, or of the lowest layer, as appropriate, is denoted as m. (*a*) An elbow profile created by a change in viscosity in constant shear-stress flow. It may be bounded, semibounded, or unbounded. (*b*) Two-layer flow. (*c*) Core-annular channel flow.

variation based on the interface thickness, δ: (*a*) sharp interfaces $\delta \to 0$, which occur in immiscible fluid flow; (*b*) thin miscible interfaces $0 < \delta \ll 1$, which occur, for example, at high Péclet numbers; and (*c*) continuous stratification of temperature or solute concentration (i.e., $\delta = 1$) (**Figure 2**). Viscosity varies within a layer of thickness δ and has different constant values on either side.

3. IMMISCIBLE FLUIDS SEPARATED BY SHARP INTERFACES

In this section, we present various planar flow geometries and then discuss pipe flows. We begin by considering the simplest viscosity-stratified flow: a unidirectional flow of constant shear stress, with a jump in viscosity at $y = 0$ (**Figure 2*a***). This flow, with or without walls, is neutrally stable by inviscid theory to perturbations of any wave number. In a remarkable demonstration of the power of viscosity stratification to destabilize flow, Yih (1967) showed, from the Orr-Sommerfeld equation, that long waves on a viscosity interface can be unstable, that too at any Reynolds number, for such a flow within walls. This finding (**Figure 3*a***) set in motion decades of research. Apart from the viscosity ratio m, the location $\bar{h}$ of the interface is an important parameter. When the narrower layer is the more viscous one, the flow is prone to long-wave instability. Why is that? Charru & Hinch (2000) offered a mechanism as a possible explanation: The disturbance vorticity is created on either side of the interface and is advected by the mean inertia to produce vortex pairs out of phase with the interface height. The pair in the narrow layer is too weak to have an effect. The pair in the wider layer, when this layer is less viscous, transports fluid from the elevated to the depressed regions of the interface (taking the wider layer to be lying above the narrow one) and is therefore destabilizing. It similarly acts to stabilize the flow if the wider layer is more viscous. The semibounded $\bar{h} \to 0$ limit is consistent with this picture, being unstable for $m > 0$ (Hooper 1985), whereas unbounded flow is stable to long waves (Hooper & Boyd 1983).

Apart from this long-wave mode, perturbations of wavelengths smaller than the viscous length scale—i.e., of large ξ [$\equiv \alpha^2\mu/(\rho\gamma)$, where γ is a reference strain rate]—are unstable at vanishingly small Reynolds numbers for any m and $\bar{h}$ (Hooper & Boyd 1983). The waves are so short that the geometry does not matter. In other words, a viscosity-jump interface always acts to destabilize itself at short wavelengths. This would seem incredible but for the argument of Hinch (1984), given broadly along the lines of that for long waves, except that now the less viscous layer, rather than the wider layer, is the driver.

In an intermediate–wave number range, Hooper & Boyd (1987) and Renardy (1985) found a distinct instability mode, when $m < 1$ and Re is sufficiently high, which vanishes at both $\alpha \to 0$

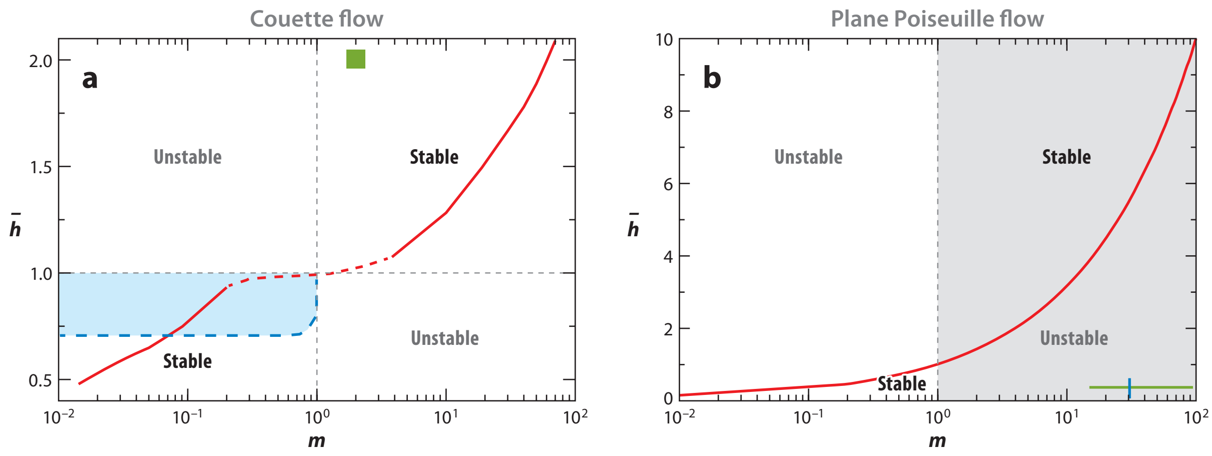

Figure 3

Stability boundaries in immiscible two-layer and core-annular flow. The planar flows shown here are unstable at $Re \to 0$ over the entire domain to waves shorter than the viscous length scale $\sqrt{\mu/(\rho\gamma)}$, where γ is the strain rate. (*a*) Couette flow. The red line represents Yih's (1967) results in the limit $\alpha Re \to 0$. The green box represents an example of the intermediate–wave number instability of Hooper & Boyd (1987). The light blue shaded region is stable in core-annular pipe flow (Joseph et al. 1984). (*b*) Plane Poiseuille flow. The red line is for two-layer flow (Yiantsios & Higgins 1989). The gray shaded region is unstable to in-plane core-annular (three-layer) flow (Than et al. 1987). The flow is absolutely unstable along the green horizontal (Sahu & Matar 2011) and blue vertical (Valluri et al. 2010) lines for a range of wave numbers and Reynolds numbers greater than $O(10)$.

and $\alpha \to \infty$. This instability is caused by the interaction of the viscosity interface with the wall layer. Classifications of these instabilities are found in Boomkamp & Miesen (1996) and Charru & Hinch (2000). The situation in Poiseuille flow (**Figure 2*b***) is quite similar to Couette flow, as seen, for example, in the long-wave results of Yiantsios & Higgins (1988, 1989) (**Figure 3*b***). Thus, two-layer flow, in the absence of surface tension, is unstable to one interfacial mode or another under any condition. The only stable flow at $Re \to 0$ is a lubricating one (i.e., when the narrower layer is less viscous) with a finite surface tension (Preziosi et al. 1989). All unstable modes at $Re \to 0$ must be interfacial modes. At higher *Re*, they may coexist with unstable TS modes. Such coexistence of interfacial and TS modes is evident at high Reynolds numbers in two-fluid boundary-layer flows as well (Ozgen 2008). Here, as $\bar{h}$ increases, the two modes show a tendency to coalesce; in other words, the neutral boundaries of the modes go from being disjointed to interlinked. We present examples of mode coalescence below.

Some unstable flows can behave as self-sustained oscillators and display a catastrophic breakdown of steady flow, with the group velocity of the dominant perturbations going to zero. These are called absolutely unstable flows, as opposed to convectively unstable flows, which behave merely as disturbance amplifiers (Huerre & Monkewitz 1990), with the disturbance energy being convected away from the place it was created. What is the nature of instabilities created by a viscosity interface? **Figure 3** shows some regimes of absolute instability, obtained by Sahu & Matar (2011) and Valluri et al. (2010). The former also conducted a direct numerical simulation of the full Navier-Stokes equations, which is in good agreement with linear theory. Nonlinearities in viscosity-stratified flows have been studied in many numerical simulations, but we do not discuss them individually here because they usually lead to a final saturated state that carries the signature of its linear origins.

What do experiments say? Charles & Lilleleht (1965) were the first to observe waves on the interface in an oil-water channel flow. Some time later, in their oil-water flow in a rectangular channel, Kao & Park (1972) found instability, but only at relatively high Reynolds numbers. They therefore characterized this as a TS-type mode. Yiantsios & Higgins (1988), however, showed that Kao & Park's mode was consistent with an interfacial mode of the type discussed above, but stabilized up to $Re \sim 30$ by gravity and surface tension. Khomami & Su (2000) too attributed the problem with Kao & Park's experiment to the reduction of the growth rates by the surface tension and instead used small–surface tension fluids to demonstrate that interfacial instabilities indeed occur. They obtained good quantitative agreement with theoretical predictions. Two sets of experiments on two-layer rotating Couette flow (Sangalli et al. 1995, Barthelet et al. 1995) obtained conditions under which long and short waves are seen and gave further credence to theoretical predictions. In both experiments, the long waves are of the Yih type, whereas the shorter waves typically occur at higher *Re* and the wavelengths are $O(1)$, so they are likely to be of the intermediate type discussed above. Sangalli et al. (1995) demonstrated that as the more viscous fluid occupies less and less of the container, the dominant wavelength increases. The wavelengths are in agreement with linear theory, and the amplitudes are in agreement with weakly nonlinear theory. Barthelet et al. (1995) found that in regimes in which short and long waves coexist, the shorter waves subsume the long ones. They also found that dispersion effects are significant and that the long-wave instability is subcritical. Sangalli et al.'s experiment involved density-matched fluids, whereas Barthelet et al.'s did not. In summary, the experiments agree with theory, in that the predicted modes are seen, and viscosity stratification indeed destabilizes the flow in different ways.

We now turn our attention to pipe flows of immiscible fluids. Because they have been previously reviewed (Joseph & Renardy 1993, Joseph et al. 1997), the focus here is on contrasting them with planar flows and with miscible flows and on newer work. The indication thus far, that viscosity-stratified flows are almost always unstable, raises concerns about lubricated pipelining. However, various circumstances act in favor of stabilization in this case. First, very long (Chen & Joseph 1991) or short waves rarely exist in real flow because the region of parallel flow is finite, and either the surface tension or miscibility is nonzero. Then Hooper & Boyd's (1987) analysis demonstrating intermediate–wave number instability is not valid for $m \to 0$, which is the limit of interest in lubrication. Additionally, the curvature of the pipe could stabilize the flow. The result is a small bounded regime of stability in the m-Re plane, at intermediate *Re* and $m < 1$ (Preziosi et al. 1989), which fortunately is accessible to lubrication applications. In unstable pipe flows, the final flow pattern assumes a lot of variety. Experiments (Charles et al. 1961) and theory (Preziosi et al. 1989) find that below a critical speed, the annular configuration gives way to oil droplets in water, and above a second speed, water emulsifies in oil. Bai et al. (1992) observed bamboo waves in upflow and corkscrew waves in downflow. The former pattern was also obtained in simulations (Kouris & Tsamopoulos 2001). The flow is chaotic for $m \sim 1$, whereas for a large viscosity ratio, well-organized wave trains are seen in the simulations. The strongly nonlinear theory of Kerchman (1995), with a modified Kuramoto-Sivashinsky equation for the surface tension terms, also shows a variety of solutions, from chaos to quasi-steady waves.

When the annular fluid is more viscous, regardless of the size of $\bar{h}$, long waves in core-annular flow in pipes (Hickox 1971), and in channels (Than et al. 1987), grow at a rate $O(\alpha^2 Re)$. Hu & Joseph (1989) found no stable wave number or Reynolds number for this flow, in agreement with the experiments of Aul & Olbricht (1990). Kouris & Tsamopoulos (2002) found sawtooth waves in their simulations. Axisymmetric disturbances are always the most unstable kind, and surface tension aids in the destabilization. In a microfluidics experiment, Guillot et al. (2007) showed that this configuration of more viscous annular fluid can actually be stabilized if the core fluid is in the form of a fast jet. But decreasing the inner viscosity increases the droplet regime at the expense of the jet regime. Their droplet and jet regimes agree perfectly with their theoretical predictions of absolute and convective instability.

Surface tension and viscosity stratification sometimes work together and at other times at cross purposes. Intuition tells us that surface tension should erase high–wave number perturbations. Indeed, in planar flow, its contribution to the energy equation appears with a negative sign and scales with the cube of the wave number, so increasing the surface tension always stabilizes the flow, with a stronger effect at shorter wavelengths. In the weakly nonlinear regime as well, surface tension is seen to stabilize short waves (Hooper 1985), with the flow going to a stable, a nonlinearly saturated, or a quasi-periodic state. In pipe flow, surface tension has the contrasting effect of stabilizing short waves (Joseph et al. 1984) but destabilizing longer waves by the Rayleigh-Tomotika instability. At $Re \to 0$, the most destabilized wave number depends on the viscosity ratio (Tomotika 1935): $\alpha = 0$ grows fastest for $m \to 0$ or ∞, and maximum growth is attained for $m = 0.91$, $\alpha = 0.568$. An increase in the speed of the inner fluid suppresses the Rayleigh-Tomotika instability (Chen et al. 1990, Guillot et al. 2007), helping to attain the counterintuitive situation discussed above of a stable jet within a more viscous annular fluid.

In an appealing example in which the surface tension destabilizes and the jump in viscosity stabilizes the flow, Ooms et al.'s (1983) theory and Papageorgiou et al.'s (1990) experiments in pipes showed that chaotic flow is organized into waves upon the introduction of a viscosity difference between two fluids. The latter study models the nonlinear evolution of the interface by the Kuramoto-Sivashinksy equation.

Surface tension also stabilizes nonaxisymmetric waves in pipes (Preziosi et al. 1989). This is important because theoretical results that ignore surface tension often find the helical mode to be the least-stable mode, whereas experiments most often observe the axisymmetric mode. In the presence of surface tension, an interface must end on the wall upon completion of the nonlinear dynamic stage, so bamboo waves, rather than helical ones, are promoted (Joseph et al. 1997).

Frenkel & Halpem (2002) showed that Marangoni effects from a surfactant at the interface can trigger the growth of interfacial waves in two-layer channel flow. The experiments of Blyth et al. (2006) in a pipe, again using an insoluble surfactant, demonstrated that Marangoni effects destabilize the flow by increasing the growth rate and widening the unstable wave number range. This new instability acts counter to the Tomotika mode, in that without a mean flow, surfactants increase stability, but when there is a mean flow, they reduce stability. That thermocapillary effects can dominate over shear has also been shown theoretically (Wei 2006).

4. MULTILAYER FLOWS OF MISCIBLE FLUIDS

Most combinations of fluids are at least slightly miscible. Some of the earliest stability studies of miscible interfaces are on the fingering instability, which ensues when a less viscous fluid, such as water, displaces a more viscous one, such as sugar syrup. Studies of fingering in miscible situations date back to Hill (1952) and are still investigated today (John et al. 2013, Oliveira & Meiburg 2011). Fingering instabilities in flow through porous media, and in Hele-Shaw cells, are discussed in an appealing review by Homsy (1987), so we restrict our discussion here to shear flows, of the type discussed in the previous section, in which the viscosity variation is predominantly in the wall-normal direction. This section is devoted to thin mixed layers, where $0 < \delta \ll 1$.

In Section 3, we see that immiscible interfaces become unstable at any Reynolds number, and whereas the broad features of the geometry are important, the details of the shear flow are not. The behavior of miscible interfaces is vastly different. The resemblance to single-fluid flow is now more apparent. Instabilities in these flows have been researched only over the past decade or so, beginning with the core-annular channel flow shown in **Figure 2*c*** (Ranganathan & Govindarajan 2001). In immiscible fluids, the interface determines the location of the critical layer, and the two are usually well separated. However, in miscible flows, the two are usually independent, as the critical layer is driven by the mean shear, and this independence leads to a rich range of possibilities. When the stratified layer is distinct from the critical layer of the dominant disturbance, the balance at the lowest orders in the critical layer is unaffected by viscosity stratification, and a modified TS mode governs stability. However, when the two layers overlap, the response is very large (Govindarajan 2004, Ranganathan & Govindarajan 2001). The relative importance of the different terms may be obtained by an extension of the singular perturbation approach of Lin (1946). The dominant balance within this layer now depends on the relative thicknesses of the viscosity-stratified layer (δ), the momentum critical layer $(\alpha Re)^{-1/3}$, and the concentration critical layer $(\alpha Pe)^{-1/3}$. The diffusivity of the solute species is now an important parameter. On either side of a cross-over Schmidt number ($Sc_p \sim Re^{2/3}$), different effects are important. At moderate to high diffusivity ($Sc \leq Sc_p$), the balance is between the viscosity stratification and advection of momentum. Here a 10% decrease in the wall viscosity ($m = 0.9$) can increase the critical Reynolds number for linear instability by an order of magnitude (from 5,772.2 to approximately 88,000), whereas an increase by a similar amount in the wall viscosity causes a correspondingly large destabilization. It can be explicitly demonstrated (Govindarajan et al. 2001) that the energy production $\mathcal{P}$, which is localized in the critical layer, is dramatically modified by viscosity stratification, whereas the dissipation, being concentrated close to the wall and not in the region of viscosity stratification, is practically unaffected. When the outer fluid is more viscous, a new mode of instability appears (**Figure 4*a***). This is termed the overlap mode, as it occurs when the critical layer and the viscosity-stratified layer overlap over a significant extent in y. In fluids in which $Sc \gg Sc_p$, the poor diffusivity of species is the controlling factor, and the overlap mode loses its identity to a broadband instability that sets in at low Reynolds numbers. This forms the miscible analog of the Yih/Hooper/Joseph modes. In fact, in the limit of infinite Schmidt number, this flow is shown to be unstable in the Stokes flow regime (Talon & Meiburg 2011).

Ern et al. (2003) were the first to study the finite diffusivity and interface thickness effects of this flow. Increasing the interface thickness has a uniformly stabilizing effect, whereas increasing the Péclet number at a constant Reynolds number has a large destabilizing effect. As $\delta \to 0$ and $Pe \to \infty$, one would expect to approach the most unstable solution, but remarkably, finite δ and Pe can be more unstable than a sharp interface (**Figure 4*b***). By varying Sc over a vast range in a Hele-Shaw cell, Goyal & Meiburg (2006) and Rakotomalala et al. (1997) also showed theoretically and experimentally that weak diffusion destabilizes the flow. They confirmed that

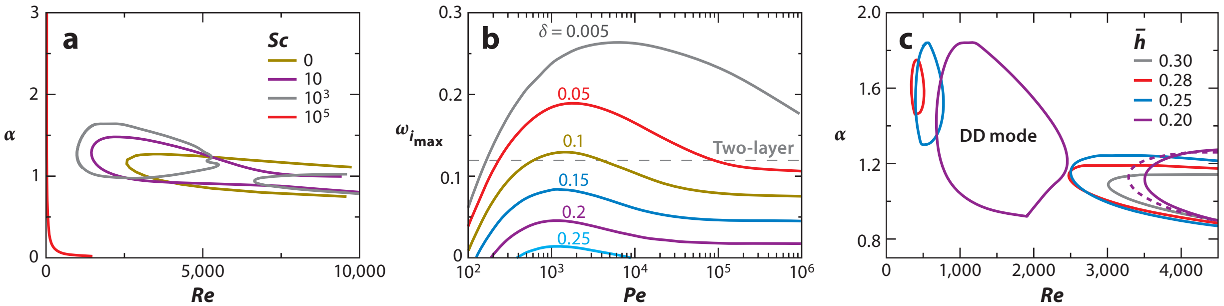

Figure 4

The effect of diffusivity on stability. (*a*) At a Schmidt number of 10^3, the overlap mode of instability (closed region) is distinct from the high–Reynolds number Tollmien-Schlichting mode, which appears in unstratified channel flow (Govindarajan 2004). (*b*) Result from Ern et al. (2003) showing that a stratified layer of finite thickness and diffusivity can be more unstable than an interface. $\omega_{i,\max}$ represents the growth rate of the least-stable eigenmode. (*c*) Double-diffusive (DD) instability in a stably stratified system (Sahu & Govindarajan 2011).

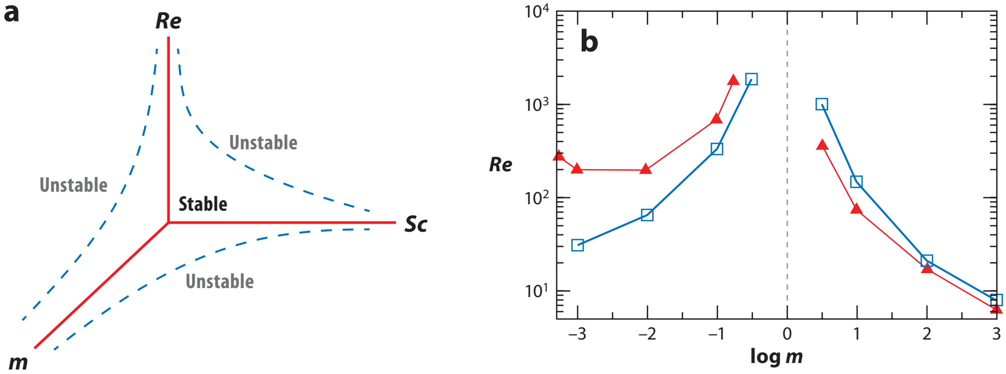

Figure 5

(*a*) Miscible planar flows are stable in the region close to the origin and are unstable at high values of any of the three parameters shown. (*b*) Pipe flows are unstable to axisymmetric (*blue squares*) and helical (*red triangles*) modes above the lines shown (Selvam et al. 2007). Here, $Sc = 1$, and the critical Reynolds number decreases monotonically with an increase in Sc.

viscosity stratification can destabilize the flow even when the Reynolds number is as small as $O(1)$ and that increasing the viscosity contrast increases the disturbance growth rate. Increasing the viscosity ratio or the Péclet number shortens the wavelength of the dominant instability.

Miscible planar core-annular flows may thus be summarized qualitatively as shown in **Figure 5**. Increasing m, Re, or Sc makes the flow progressively more unstable. The last is true in a range of geometries.

In a similar study of core-annular flow through a pipe, Selvam et al. (2007) showed that in the range $0.9 < m < 1.1$, flow with a thin mixed layer is stable at any Reynolds number, in sharp contrast to the immiscible case. In this range, the behavior is consistent with the pipe flow of a single fluid. Above this value, the flow can be unstable, unlike the situation in a plane channel, even when the core fluid is more viscous. Similar to the planar case, miscible pipe flow can be more unstable than the immiscible case in some parameter ranges. Unlike the case for channel flow, there is no sudden change in the stability with the radial location of stratification (i.e., critical layer effects are not sharply distinguished). Remarkably, when the less viscous fluid is in the core, the flow is unstable only at higher Re, in direct contrast to immiscible flows. This fact is reassuring because in a miscible flow at $Re \rightarrow 0$, the dissipation of perturbation energy must diverge while production must remain finite. In this limit, if Sc is finite, the parallel flow approximation breaks down, as diffusion is rapid compared to downstream advection, so the flow consists of two mixing layers, for species and for momentum, growing spatially at different rates. At very high Sc, however, parallel flow results present a contradiction to physics, predicting instability at $Re \rightarrow 0$. Global stability analyses are needed to resolve this paradox.

The axisymmetric and corkscrew modes are dangerous when the more viscous fluid is in the annulus and core, respectively, but both coexist over a range. D'Olce et al. (2008) observed pearl and mushroom patterns (**Figure 6**) in a neutrally buoyant core-annular horizontal pipe flow at high Schmidt numbers and Reynolds numbers in the range $2 < Re < 60$. The more viscous fluid is in the annulus, so the axisymmetric nature of the instability is consistent with theory. The theoretical study went up to $m \sim 20$, and just beyond this ratio, at $m = 24$, Cao et al. (2003) presented experimental images of instability at comparable Re, which look like the corkscrew mode. They too found that $Re \rightarrow 0$ is stabilizing.

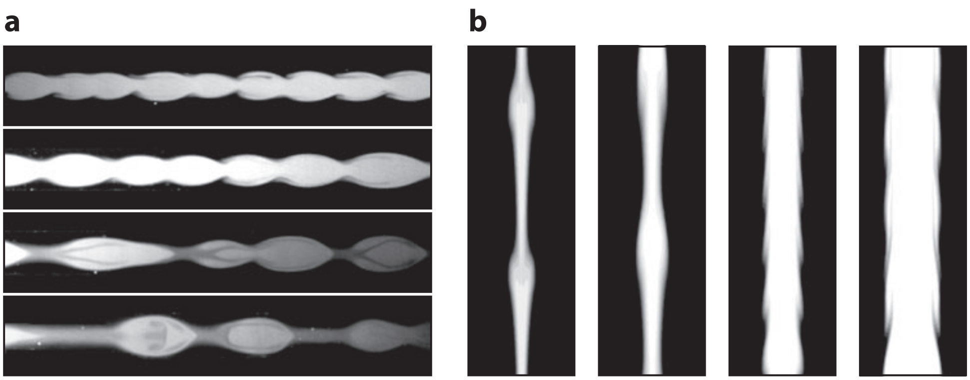

Figure 6

Pearl and mushroom patterns (*a*) in the experiments of d'Olce et al. (2008), with *Re* increasing from top to bottom from 5 to 18, and (*b*) in the simulations of Hormozi et al. (2011), with the coflow of a Newtonian and a non-Newtonian fluid.

To discuss absolute instability in miscible flows, we briefly return to the planar geometry to mention that for $m > 25$, planar flow becomes absolutely unstable in a range of $\bar{h}$ (Naraigh & Spelt 2010, Sahu et al. 2009a, Valluri et al. 2010). In pipes, d'Olce et al. (2009) experimentally and Selvam et al. (2009) theoretically showed that high viscosity contrasts with weak diffusion can make the flow become absolutely unstable, as evidenced by self-sustained synchronized oscillations. Viscosity perturbations were found to be essential for this. The requirement of a high viscosity ratio for absolute instability is the same as in immiscible flows. We remark that a flow taken from a completely stable to absolutely unstable mode, by viscosity stratification alone, is a testament to the influence of this property on the flow. Sahu et al. (2009a) showed the convective and absolute instabilities arising in a three-layer configuration when the highly viscous fluid occupies the near-wall region for $m = 2$ and $m = 30$, respectively (see **Figure 7**). The transient growth of perturbations in a linearly stable situation also depends strongly on the viscosity ratio of the fluids (Yecko et al. 2002). This effect is significant at moderate values of mixed layer thickness δ. As δ decreases, a mode analogous to the interfacial mode destabilizes the flow linearly, and disturbance growth is no longer transient (Malik & Hooper 2005).

Displacement flows deserve another mention, mainly because of the contrast they present to steady flow: Displacement flow through a pipe is always stable when the invading fluid is more viscous than the resident fluid (Joseph et al. 1997). This paper again provides evidence, and this time a whole body of it, that it is stabilizing to put the core into fast jet-like flow. The mechanism needs to be understood and rests perhaps in the spatial (and temporal) development of displacement

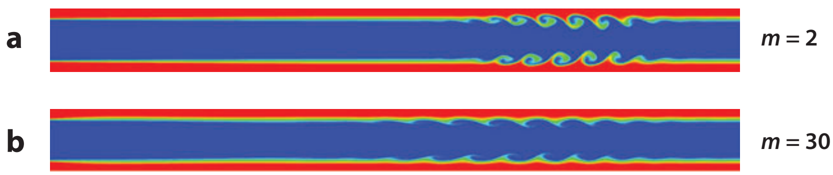

Figure 7

The concentration contours for viscosity ratio (*a*) $m = 2$ and (*b*) $m = 30$ for $Re = 500$, $Sc = 100$, and $h = 0.3$ in a three-layer configuration. Figure taken from Sahu et al. (2009a).

flows, whereas our intuition is based on a parallel flow assumption. When the displacing fluid is less viscous, in the miscible case roll-up structures appear at the interface separating the fluids, as seen experimentally (Taghavi et al. 2012) and theoretically (Sahu et al. 2009a,b), whereas in the immiscible case, sawtooth structures appear, as in the numerical simulations of Redapangu et al. (2012).

Viscosity stratification could be achieved by two solutes (or a single solute and temperature), with vastly different diffusivities and stratified in opposite senses, namely in a DD system. The difference in diffusivities triggers a new mode of instability (Sahu & Govindarajan 2011, 2012) in a classically stable system, with the less viscous fluid in the near-wall region (**Figure 4*c***). This is analogous to the well-known DD instabilities in gravity-driven flow. A DD instability was also obtained in the direct numerical simulations of a pressure-driven displacement flow of a less viscous fluid by a high-viscosity one containing two solutes of different diffusivities (Mishra et al. 2012). A new cap-type instability was obtained at the leading edge, whereas such a flow would be completely stable in the case of only one scalar.

We summarize that miscible core-annular flows of finite diffusivities, unlike their immiscible counterparts, are stable at $Re \to 0$. They display high–Reynolds number instability in the form of modified TS waves. Under overlap conditions, they display a new instability at low Reynolds numbers, but this is not the same as the interfacial instability because it depends on local shear, and besides, the latter does not support overlap conditions. At infinite *Sc*, one may have instability in the Stokes flow regime (Talon & Meiburg 2011), but for any nonzero diffusivity, one must remember that the parallel flow approximation fails in this limit.

Density stratification has been extremely well studied in terms of stability characteristics, but viscosity stratification has traditionally received less attention. The role of density is usually simpler to predict than that of viscosity. From the early part of the twentieth century, with the discovery of Rayleigh-Bénard convection (see Chandrasekhar 1981), it has been known that a density increase in the direction of gravity is stabilizing, whereas one in the reverse direction is destabilizing. Talon et al. (2013) demonstrated this in miscible three-layer flow, in which the top-heavy interface is unstable in the Stokes flow limit, and the bottom-heavy one is stable. The two stratifications of density and viscosity can produce an interesting range of instabilities and can work cooperatively or adversarially. For example, Renardy & Joseph (1985) showed that a top-heavy flow can be stabilized by appropriate viscosity stratification, whereas Hooper & Grimshaw (1985) showed the reverse (i.e., a bottom-heavy flow destabilized). The experiment of Scoffoni et al. (2001), on a vertical miscible displacement flow through a pipe with a lighter and less viscous displacing fluid in the core, demonstrated corkscrew modes at smaller density and viscosity contrasts, and axisymmetric modes at high contrasts, with stable flow at low speeds. A very large body of work on gas-liquid flows exists (see, e.g., Miles 1957, Tammisola et al. 2011), and these are beyond the scope of this review.

5. CONTINUOUS STRATIFICATION OF VISCOSITY

5.1. Stratification Due to Temperature Variation

In many flows in the chemical industry, lubrication, tribology, food processing, polymer processing (Pearson 1985), and supersonic flight, a continuous temperature, and hence viscosity, gradient is inherent, and we have $\delta = 1$. From a parallel base flow given by

$$\frac{1}{\rho}\frac{dP}{dx} = \mu(T)\frac{d^2U}{dy^2} + \frac{d\mu(T)}{dT}\frac{dT}{dy}\frac{dU}{dy}, \tag{6}$$

we may predict the effect that wall heating or cooling will have on the stability. By Rayleigh's criterion, a velocity profile will become fuller and therefore more stable as the second term on the right-hand side in Equation 6 becomes more positive. Because the viscosity of gases increases with temperature, cooling of the wall will produce a stabilizing effect. However, in the case of liquids, the temperature coefficient of viscosity is typically negative, so wall heating will produce a less viscous flow near the wall and will therefore stabilize the flow.

Apart from inherent viscosity variations, the above discussion suggests that heating or cooling may be imposed at the wall to achieve flow control, and this idea has been explored for decades in the aerospace industry and appears in many reviews (see, e.g., Saric et al. 2011). In incompressible flow, the effectiveness of modifying the boundary-layer profile using temperature is well established. Lees & Lin (1946) demonstrated this long ago using inviscid stability theory for subsonic air boundary layers, and Mack (1984) included viscous effects to show a complete stabilization by wall cooling. Experimentally too, especially in water tunnels, many authors (e.g., Barker & Gile 1981) have shown that wall heating is effective in flow stabilization, leading to drag reduction. However, given the low thermal conductivity of air, this strategy has been more difficult to use in real flight (W.S. Saric, private communication), prompting Saric et al. (2011) to comment that this method of flow control "remains in the lab." In supersonic flow, we have another problem. The first mode, which is the TS mode, is indeed strongly suppressed, but wall cooling in air has a destabilizing effect on the higher modes (Mack 1984), especially the sedan or acoustic mode (Reed & Balakumar 1990). The experiments of Iysenko & Maslov (1984) confirmed this. It is not clear, however, if it is time to give up on this control strategy. Because the instabilities are significantly modified, we may dare hope for an innovative approach to deal with the modified spectrum. On a cone at a Mach number of 6, for example, Kara et al. (2008) demonstrated that wall cooling takes the dominant modes to much higher wave numbers and frequencies. Also, the effects are very different on other geometries. For example, Couette flow, which is linearly stable when flow is incompressible, is destabilized when deliberately maintained at a constant temperature, for a range of Mach numbers (Malik et al. 2008). However, compressibility brings with it an inherent tendency to stratify temperature across any shear layer, giving rise to a hot wall. This distorts the Couette flow profile away from linearity owing to viscosity stratification, and amazingly, the resulting profile is far more stable than the undistorted Couette flow. In this geometry in compressible flow, it is transient nonmodal growth that triggers turbulence. Malik et al. (2008) showed, interestingly, that this too is suppressed significantly by viscosity variations arising from the inherent temperature variation.

Thus, whether flow control in flight is achieved by wall cooling remains to be seen, but meanwhile, the effect of inherent wall heating must be understood so that we are able to make good predictions of instability and transition to turbulence, which are crucial to those applications. In hypersonic re-entry vehicles, the temperature can vary by thousands of degrees in the vicinity of the body and also changes extremely rapidly with time, with corresponding variations of over two orders of magnitude in viscosity and of diffusivity (K. Sinha, private communication), over a few centimeters. Stability studies (e.g., Friederich & Kloker 2012) and direct numerical simulations (e.g., Muppidi & Mahesh 2012) almost always use Sutherland's law to model viscosity variations. However, the effect of this viscosity variation has not been understood in isolation. Moreover, the validity of Sutherland's law in hypersonic flow needs to be called into question. That viscosity variation effects are not discussed in a recent review paper (Zhong & Wang 2012), whose main purpose is to highlight the importance of instability and transition predictions in hypersonic flows, is a sign that much of this work lies in the future. With the likely advent of aerospace planes, such studies could take center stage fairly soon.

We now turn our attention back to channel flows. When the temperature profile is symmetric, it is easy to predict that a less viscous wall fluid will become stable, and this is indeed what happens

(Sameen & Govindarajan 2007). The stabilization due to decreasing near-wall viscosity is very large, whereas the destabilization due to increasing near-wall viscosity is small. If the temperature gradient across the channel is linear, one wall must be destabilized and the other stabilized. Which wins? Potter & Graber (1972) claimed that destabilization does, but Pinarbasi & Liakopoulos (1995), Schäfer & Herwig (1993), and Wall & Wilson (1996) obtained the contradictory result. Sameen & Govindarajan (2007) pointed out that this was because the first study was based on constant power input, whereas the others maintained constant Reynolds number. Weak diffusion, which was so powerful in destabilizing thin stratified layers because of the overlap with the critical layer, seems at first to not be a player in continuous stratification. Varying *Pr* over many decades does not change the answer. But this is only with regard to the least-stable eigenmode. Decreasing the diffusivity increases the transient nonmodal growth by over an order of magnitude (Sameen et al. 2011). Buoyancy, as before, has a significant destabilizing or stabilizing effect depending on whether the fluid is top heavy or bottom heavy. In the first case, we have Rayleigh-Bénard-Poiseuille convection. Jerome et al. (2012) conducted an interesting study showing that streamwise independent structures, as in unstratified flow, emerge in heated flow as well, but their study is for constant-viscosity flows. Zonta et al. (2012) were the first to conduct direct numerical simulations to obtain the effect of viscosity variations, isolated from gravity effects, of incompressible flow in a heated channel. They confirmed the predictions of Sameen & Govindarajan (2007), showing that turbulence is promoted in the cold side of the channel and suppressed on the hotter side. They also confirmed that it is the modification of disturbance production, rather than dissipation, that achieves this effect. On the hot wall, it is confirmed that production decreases and dissipation increases, whereas the reverse happens at the cold wall. A new optimal initial structure appears in heated channel flow (**Figure 8**). A nonparallel version of this flow was studied by Helfrich (1995), whose experiments were a simplified model of the flow of hot magma through a gap with cold walls. A fingering instability occurs here, and hot low-viscosity fingers speed up and spout out at the exit.

When the fluids are very viscous, heating due to viscous dissipation, which has been neglected in our discussion so far, plays an important role. This is easy to imagine if one has experienced a blender collapse while grinding batter for *idli* (a breakfast food). We now have an additional strong coupling between the energy and the momentum equations, with the momentum dissipation appearing as a source term in the heat equation (Equation 3). The boundary-layer profile including this effect was first obtained by Busemann (1931) and Crocco (1932) for $Pr = 1$ and is known by their names. A range of studies ensued (the most recent ones include Costa & Macedonio 2005, Pinarbasi & Imal 2005, Sahu & Matar 2010a, Sahu et al. 2010, and Wylie & Huang 2007). The main conclusion is that viscous heating has a destabilizing influence. Based on everything we have seen so far, this is counterintuitive because dissipation is highest near the wall, and in a liquid this should result in reduced viscosity there and should be stabilizing. This calls for further investigation.

5.2. Stratification Driven by Variations in Pressure and Shear

From the informative paper of Malek & Rajagopal (2001), we learn that Stokes (1845), in his famous but hard-to-find paper, had already recognized that viscosity is in general a function of pressure. However, by experiments under normal pressure conditions, he neglected such variation, and it is now accepted that viscosity is a sensitive function of pressure at high pressure. Fukui et al. (2010) found that the viscosity of ethanol-water mixtures at room temperature is a monotonically increasing function of pressure. This is consistent with Bridgman (1926), but at higher temperatures, of approximately 340 K, at tens of atmospheres, Bridgman showed that the viscosity

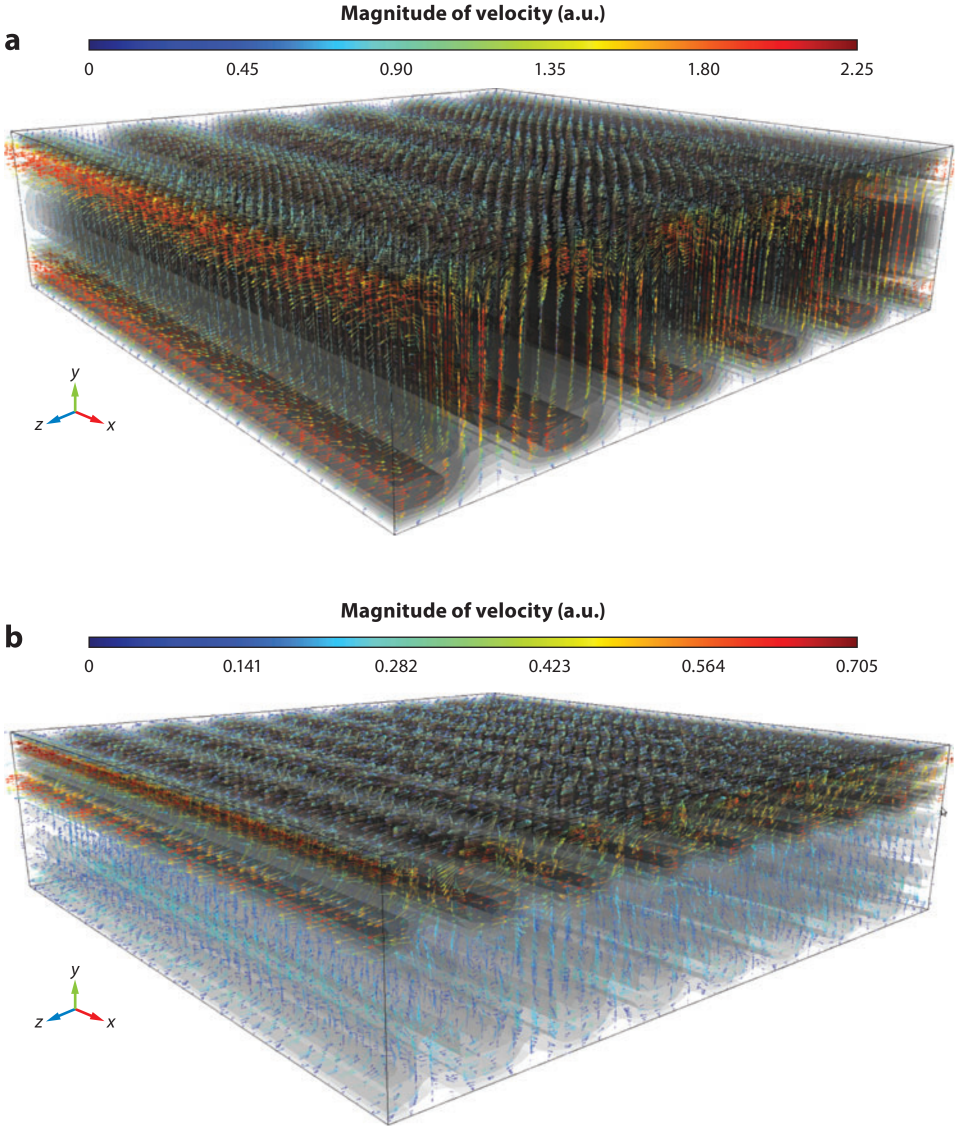

Figure 8

The rolls in panel *a* are the well-known structures yielding maximum transient growth in unstratified flow ($m = 1$), whereas the optimal initial conditions for a heated channel in panel *b* ($m = 1.1$) include a second set of smaller rolls in the region of low viscosity. Here $Re = 1{,}000$ and $Pr = 1$. A small buoyancy is included. Result taken from S. Jose, L. Brandt & R. Govindarajan (manuscript to be submitted).

of a range of liquids drops suddenly over two orders of magnitude, followed by a recovery, with an increase in pressure. What is the effect on flow stability near the ocean floor, in Earth's core, and in the mysterious interior of the Sun? Future work should tell us. A preview of a nonlinear instability in glacier flow with pressure-dependent viscosity is presented by Schoof (2007).

A more accessible situation than high pressures is that of fluids with shear-dependent viscosity (e.g., polymeric solutions and cornstarch), which show significantly modified stability behavior. The stability of inelastic non-Newtonian fluids has been studied extensively, and there is consensus that shear thinning is stabilizing and shear thickening is destabilizing. The high-shear regions are typically close to the wall, and shear thinning and thickening would make the velocity profiles fuller and closer to being inflectional, respectively. What happens to transient growth in the linearly stable range of parameters? As discussed above, the answer to this question can be important in flows that transition to turbulence by the algebraic-growth route. In plane Poiseuille flows of shear-thinning fluids, with viscosity perturbations ignored, transient growth is slightly decreased (Chikkadi et al. 2005). With viscosity perturbations obtained approximately by a simple shear-thinning model, transient growth is slightly increased (Nouar et al. 2007), whereas in Couette flow, transient growth is increased substantially in a shear-thinning fluid flow (Liu & Liu 2011). Thus shear thinning, which damps the leading eigenmode, can promote turbulence by the linear superposition of eigenmodes.

A range of non-Newtonian fluids that are more complex than the power-law fluids discussed above has been examined from this point of view, revealing a range of results, including those in which the sign of the effect can go in either direction. Interesting among these is the case of a free–shear layer flow of a viscoelastic fluid, studied by Azaiez & Homsy (1994). Here an Oldroyd-B fluid is seen to be far more stable than a Newtonian one, whereas the stability of a corotational Jeffrey fluid flow is unaffected by elasticity. The mechanism for elastic stabilization the authors provide is consistent with the normal stress differences in these models and relies again on a phase shift, created this time by elasticity. Viscoelasticity provides a slew of instabilities, such as the buckling of a thin sheet in shear flow described by Slim et al. (2012). In another example, in two-layer Poiseuille flow of two Bingham fluids, which have constant viscosities but finite yield stresses, Frigaard (2001) showed that interfacial instabilities are suppressed. The opposite result, namely that increasing the yield stress has a destabilizing influence, was obtained by Frigaard & Nouar (2003) and Sahu et al. (2007), with the latter showing that the direction of the effect depends on the amount the flow has yielded. To identify a possible pathway toward transition, Nouar & Bottaro (2010) revisited this problem and demonstrated that transient amplification is the main driver in this fluid as well.

There is another fundamental difference in the mechanism of stabilization in non-Newtonian fluids. In Newtonian fluids, we have seen how viscosity stratification primarily modifies the production of the disturbance kinetic energy. The nonlinear viscous terms in non-Newtonian flows also affect dissipation in a major way (Chekila et al. 2011, Lashgari et al. 2012). Combinations of Newtonian and non-Newtonian fluids (Hormozi et al. 2011) can yield new instabilities, sometimes akin to Newtonian flows (e.g., **Figure 6**).

Finally, we discuss the case of polymer drag reduction. In turbulent shear flows in which a long chain polymer, in minuscule concentrations, is added, dramatic drag reduction can be obtained. Much has been written about this intriguing phenomenon insofar as turbulent flow is concerned, including several review articles (Lumley 1973, Procaccia & L'vov 2008, White & Mungal 2008). Polymers produce a shear thinning combined with elastic effects, and in many of the mechanisms proposed, it appears that the role of elasticity in achieving this reduction is crucial, for example, in the attainment of the maximum drag-reduction asymptote (Ptasinski et al. 2003, Sreenivasan & White 2000, Xi & Graham 2010). If we are in the pretransition regime, however, there is little doubt that the shear-thinning aspects of polymers, via favorable viscosity stratification, are very effective in delaying transition (Rudman et al. 2004).

6. FUTURE DIRECTIONS

Most work in viscosity-stratified flow lies in the future. Many specific open questions, including large unexplored areas, would have revealed themselves to the reader in the discussion above, and we mention some others below. Even in the linear regime, the effect of viscosity stratification is not completely understood, and far less clear are the nonlinear stages of instability growth and the transition to turbulence.

Most work to date is for the parallel flow limit, which is not a valid approximation of most flows. Variations in geometry are as sensitive a lever for stability as those in viscosity. These variations occur together most of the time, and their combined effects need attention. That stratification survives despite turbulence brings in the necessity of studying stratification in combination with rotation and, more interestingly, the simple question of what stability means for a flow that is already turbulent. Chemical reactions result in viscosity stratification that can affect the progress of the reaction. A better understanding of laval and glacial flows is called for. Flows in which diffusivity and viscosity vary by many orders of magnitude, as in Earth's core, merit attention from this point of view. As research on the Sun matures further, it will become feasible to include the effects of viscosity stratification.

DISCLOSURE STATEMENT

The authors are not aware of any biases that might be perceived as affecting the objectivity of this review.

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