Maths Circle Challenge

ICTS-RRI Maths Circle

July 8, 2025

Dividing a Field

Problem 1. A farmer's field is irregularly shaped, as shown in the figure. The terrain is flat and, the boundary lines are all in the North-South or East-West direction. The farmer has two sons and wants to divide the land into two pieces of land with equal area. Each son must be able to walk from any part of his land to any other part of his land without leaving his land. The farmer has before him a map of the land, a pencil and an old ruler, whose markings are faded and unreadable. Can you help the farmer divide the land with the instruments he has?

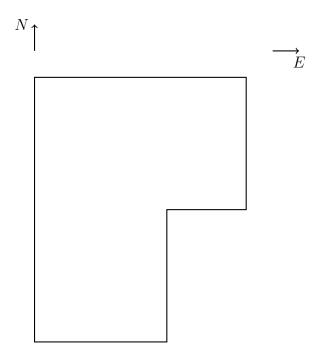


Figure 1: Figure shows an irregular piece of land owned by the Farmer. The problem is to divide the land into two connected pieces of equal area using only a straight edge.

Problem 2. Prove that for any integer n, $n^3 + 11n$ is a multiple of 6.

- **Problem 3**. Let us make the following definition. We call any finite sequence of English letters "a word" (whether or not it can be found in a dictionary). For example, we can form six words using the letters A, B, and C each exactly once: ABC, ACB, BAC, BCA, CAB, and CBA. In the following calculate the number of different words that can be obtained by rearranging the letters of the word.
 - (a) MESOPOTAMIA
 - (b) SCRAMBLE
 - (c) JUXTAPOSITION
 - (d) VIOLIN
 - (e) MISSISSIPPI
- Problem 4. Mr. and Mrs. Sharma have four children three boys and a girl who each likes one of the colours blue, green, red, yellow and one of the letters P, Q, R, S. The oldest child likes the letter Q. The youngest child likes green. Aditya likes the letter S. Bhanumati, the girl, has an older brother who likes R. The one who likes blue isn't the oldest. The one who likes red likes the letter P. Chetan likes yellow. Based on the above facts, Deepak is the
 - A. youngest child
 - B. third child
 - C. second child
 - D. oldest child
- **Problem 5**. When we throw a die, the numbers one to six are all equally likely. When we throw a pair of dice, the "outcome" of the throw is usually defined as the sum of the two numbers appearing on top of the dice. Let us change the rules and define the "outcome" as the product of the two numbers.

How many distinct outcomes are there? What is the chance (probability) that the outcome is

- (a) a prime number
- (b) a perfect square
- (c) a triangular number
- (d) an even number
- (e) an odd number
- (f) both triangular and square