Some Issues in Perturbative String Theory

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Plan

1. Mass renormalization problems in perturbative string theory

2. Solution — Roji Pius, Arnab Rudra, A.S.

3. Other possible applications

Perturbative string theory is based on the first quantized approach.

Instead of computing Feynman diagrams we compute correlation functions of the first quantized theory on Riemann surfaces.

A g-loop, n-point amplitude is given by integral of the moduli space of Riemann surfaces of genus g and with n punctures (marked points) where the vertex operators are inserted.

$$\int_{M_{g,n}} \left\langle \prod_{i=1}^{n} V_i \right\rangle$$

$M_{g,n}$: moduli space of Riemann surfaces with genus g and n punctures.

V_i: BRST invariant vertex operators

String amplitudes are supposed to compute on-shell S-matrix elements but this is not quite so.

The S-matrix elements in a QFT are given by the LSZ procedure

$$\lim_{k_i^2\to -m_{a_i,p}^2} G^{(n)}_{a_1\cdots a_n}(k_1,\cdots k_n)\prod_{i=1}^{n}\{Z^{-1/2}(k_i,a_i)(k_i^2+m_{a_i,p}^2)\}$$

$Z(k_i,a_i)$: wave-function renormalization factors

$m_{a_i,p}$: renormalized physical mass of the external state.

We define $Z(k_i,a_i)$ and $m_{a_i,p}$ by looking for poles in two point Green's function

$$G^{(2)}_{a_1,a_2}(k_1,k_2)=\delta_{a_1a_2}\,(2\pi)^D\delta(k_1+k_2)\frac{Z(k_1,a_1)}{k_1^2+m_{a_1,p}^2}$$

S-matrix elements of a QFT

$$\lim_{k_i^2 \to -m^2_{a_i,p}} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z^{-1/2}(k_i, a_i)(k_i^2 + m^2_{a_i,p})\}$$

In contrast, string amplitudes compute what in a QFT can be called ‘truncated Green’s function on classical mass shell’:

$$\lim_{k_i^2 \to -m^2_{a_i}} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m^2_{a_i})$$

m_{a_i}: <u>tree level</u> mass of the i-th external state carrying momentum k_i and other quantum numbers a_i.

The limit $k_i^2 \to -m^2_{a_i}$ is forced on us by world-sheet conformal invariance / BRST invariance.

(Need vertex operators of dimension (0,0)).

String amplitudes:

$$\lim_{k_i^2 \to -m_{a_i}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_{a_i}^2),$$

The S-matrix elements:

$$\lim_{k_i^2 \to -m_{a_i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z^{-1/2}(k_i, a_i)(k_i^2 + m_{a_i,p}^2)\}$$

The effect of $Z(k_i, a_i)$ can be easily taken care of, but the effect of mass renormalization is more subtle.

$\Rightarrow$ **String amplitudes compute S-matrix elements directly if $m_{a_i,p}^2 = m_{a_i}^2$ but not otherwise.**

This includes external massless gauge particles / BPS states.

A common excuse

"We can find the renormalized masses by examining the poles in the S-matrix of massless and/or BPS states which do not suffer mass renormalization."

Does not work when a conservation law prevents the appearance of the massive state under consideration as a single particle intermediate state in the scattering of massless states.

Example: In SO(32) heterotc string theory there is a massive state in the spinor representation of SO(32)

– cannot appear as a single particle intermediate state in the scattering of massless states which belong to adjoint or singlet representation of SO(32).

Even if we are not interested in massive states in string theory, this question is important for internal consistency of perturbative string theory.

A complete theory must be able to address all questions which can be asked within that theory.

How do we proceed?

1. Many indirect approaches to this problem have been discussed in the past. Weinberg; Seiberg; A.S.; Ooguri & Sakai; Das; Rey; $\cdots$

2. Direct approach is to define off-shell Green's function.

We can think of two routes:

a. String field theory

– many attempts but not much progress beyond tree level / bosonic string theory. Witten; Zwiebach; Berkovits; Berkovits, Okawa, Zwiebach

b. Pragmatic approach: Generalize Polyakov prescription without worrying about any string field theory origin.

Cohen, Moore, Nelson, Polchinski; Alvarez Gaumé, Gomez, Moore, Vafa; Polchinski; Nelson

String field theory may be needed to address big issues like finding non-perturbative vacuum.

However the pragmatic approach should be sufficient to address issues within the perturbative domain, like mass renormalization or small shifts in the vacuum.

We shall follow this pragmatic approach.

Main problem: The off-shell amplitudes are not invariant under a Weyl rescaling of the metric, or equivalently, under conformal transformations.

Example: Off-shell tree level 3 tachyon amplitude in closed bosonic string theory

$$\mathbf{A} = \langle \mathbf{V_1(z_1)V_2(z_2)V_3(z_3)} \rangle, \quad \mathbf{V_i} = \mathbf{c\,\bar{c}\,e^{ik_i\cdot X}}$$

$$\mathbf{A} = |\mathbf{z_1} - \mathbf{z_2}|^{\delta_3 - \delta_1 - \delta_2} |\mathbf{z_2} - \mathbf{z_3}|^{\delta_1 - \delta_2 - \delta_3} |\mathbf{z_1} - \mathbf{z_2}|^{\delta_2 - \delta_1 - \delta_3}$$

$$\delta_i = \frac{1}{2}k_i^2 - 2\,.$$

On-shell condition: $\delta_i = 0$

Unless the tachyon is on-shell the result depends on the choice of coordinate system.

– not invariant under

$$\mathbf{z} \to (\mathbf{az} + \mathbf{b})/(\mathbf{cz} + \mathbf{d}), \quad \mathbf{ad} - \mathbf{bc} = 1$$

Conclusion

Off-shell amplitude depends on spurious additional data like the world-sheet metric, or equivalently the choice of world-sheet coordinates in which the metric is flat.

– looks problematic at the first sight.

However this is not very different from the situation in a gauge theory where off-shell Green's functions of charged fields are gauge dependent.

Nevertheless the renormalized mass and S-matrix elements computed from these are gauge invariant.

Can the story be similar in string theory?

Strategy: Pius, Rudra, A.S.

1. Find a systematic way to characterize the additional data on which the amplitudes depend.

2. Show that the renormalized mass and S-matrix elements do no depend on this additional data.

Characterization of the additional data

Nelson

Given a Riemann surface we introduce two types of coordinates:

z: some reference coordinate system on the Riemann surface

w_i: local coordinate used to insert the i-th vertex operator V_i into the correlator.

$$z = f_i(w_i)$$

$z_i \equiv f_i(0)$: location of the i-th vertex operator in z-coordinates.

Genus g string amplitude:

$$\int_{M_{g,n}} \left\langle \prod_i f_i \circ V_i(0) \right\rangle$$

M_g: moduli space of genus g Riemann surface with n-punctures

$f_i \circ V_i(0)$: Conformal transform of V_i by f_i

e.g. if V_i is a primary of dimension (h,h) then

$$f_i \circ V_i(0) = |f_i'(0)|^{2h} V_i(f_i(0)) = |f_i'(0)|^{2h} V_i(z_i)$$

The correlation function $\langle \cdots \rangle$ is computed using z-coordinate system.

The result depends on the choice of local coordinates w_i but is independent of the choice of reference coordinate z.

Example: Tachyon 3-point function on the torus

$z = f_i(w_i)$

$$\begin{aligned}
A &= \langle f_1 \circ V_1(0) f_2 \circ V_2(0) f_3 \circ V_3(0)\rangle \\
&= |f_1'(0)|^{\delta_1} |f_2'(0)|^{\delta_2} |f_3'(0)|^{\delta_3} \langle V_1(z_1) V_2(z_2) V_3(z_3)\rangle \\
&= |f_1'(0)|^{\delta_1} |f_2'(0)|^{\delta_2} |f_3'(0)|^{\delta_3} \\
&\quad |z_1 - z_2|^{\delta_3-\delta_1-\delta_2} |z_2 - z_3|^{\delta_1-\delta_2-\delta_3} |z_1 - z_2|^{\delta_2-\delta_1-\delta_3}
\end{aligned}$$

Under $z \to z' = (az+b)/(cz+d) \equiv h(z)$, $f_i(z) \to h(f_i(z))$.

$f_i'(0) \Rightarrow h'(z_i)\, f_i'(0) = f_i'(0)/(cz_i + d)^2$

$(z_i - z_j) \Rightarrow (z_i - z_j)/\{(cz_i + d)(cz_j + d)\}$

The amplitude A remains invariant.

$$\begin{aligned} A \;=\;& |f_1'(0)|^{\delta_1} |f_2'(0)|^{\delta_2} |f_3'(0)|^{\delta_3} \\ & |z_1 - z_2|^{\delta_3-\delta_1-\delta_2} |z_2 - z_3|^{\delta_1-\delta_2-\delta_3} |z_1 - z_2|^{\delta_2-\delta_1-\delta_3} \end{aligned}$$

Consider change in local coordinates $w_i \to \widetilde{w}_i$ with

$$w_i = h_i(\widetilde{w}_i), \quad h_i(0) = 0$$

$$z = f_i(w_i) = f_i(h_i(\widetilde{w}_i)) \equiv \widetilde{f}_i(\widetilde{w}_i)$$

$$\widetilde{f}_i'(0) \to f_i'(0) h_i'(0), \quad z_i \to z_i$$

$$A \to A\, |h_1'(0)|^{\delta_1} |h_2'(0)|^{\delta_2} |h_3'(0)|^{\delta_3}$$

Thus A depends on the choice of local coordinates.

Local coordinate system near the punctures is the spurious data on which the off-shell amplitude depends.

Goal: Prove that renormalized mass and S-matrix elements are independent of the choice of local coordinates.

However instead of working with most general choice of local coordinates we work within a restricted class.

We add an extra condition – gluing compatibility – on the choice of local coordinates.

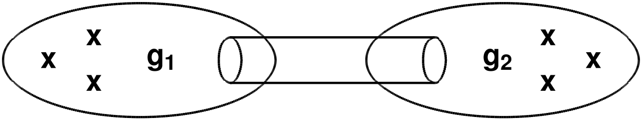
(Inspired by bosonic string field theory) Zwiebach

Consider a genus g_1, m-punctured Riemann surface glued to a genus g_2, n-punctured Riemann surface by plumbing fixture at one each of their punctures:

$$w_1 w_2 = e^{-s+i\theta}, \quad 0 \le s < \infty, \quad 0 \le \theta < 2\pi$$

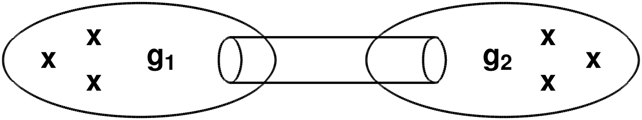
w_1, w_2: choice of local coordinates at the punctures which are glued.

Corresponds to removing a disk around $w_1 = 0$ on the first Riemann surface and a disk around $w_2 = 0$ on the second Riemann surface and gluing them at the boundaries to get a new Riemann surface.



This gives a family of genus $g_1 + g_2$ Riemann surface with (m+n-2) punctures.

Since the original Riemann surfaces were equipped with choices of local coordinate system around each puncture, gluing induces a choice of local coordinate system around each of the (m+n-2) punctures of the new Riemann surface.



Our demand: Choice of local coordinates at the punctures of the genus $g_1 + g_2$ Riemann surface must agree with the one induced from the local coordinates at the punctures on the original Riemann surfaces.

Goal: Prove that renormalized mass and S-matrix elements are independent of the choice of local coordinates within this class.

Gluing compatibility allows us to divide the contributions to off-shell Green's functions into 1-particle reducible (1PR) and 1-particle irreducible (1PI) contributions.

Two Riemann surfaces joined by plumbing fixture

$\Updownarrow$

Two amplitudes joined by a propagator

Riemann surfaces which cannot be obtained by plumbing fixture of other Riemann surfaces contribute to 1PI amplitudes.

1PI amplitudes do not include degenerate Riemann surfaces and hence are free from poles in the external momenta.

We can now carry out the usual field theory manipulations with this.

Example: Two point function

At genus 1, all amplitudes are 1PI (ignoring tadpoles).

(A 2-punctured torus cannot be obtained by gluing two lower genus surfaces).

At genus 2, we can get a subset of the Riemann surfaces by gluing two 2-punctured tori using plumbing fixture

– declared to be 1-particle reducible.

Identify the contribution from the rest of the Riemann surfaces as 1PI.

The net contribution to two point amplitude

In bosonic string theory and Neveu-Schwarz (NS) sector of superstring and heterotic string theories we can convert this into an algebraic expression for the 2-point amplitude:

$$\mathcal{F} = \widehat{\mathcal{F}} + \widehat{\mathcal{F}}\Delta\widehat{\mathcal{F}} + \cdots = \widehat{\mathcal{F}}(1 - \Delta\widehat{\mathcal{F}})^{-1} = (1 - \widehat{\mathcal{F}}\Delta)^{-1}\widehat{\mathcal{F}}$$

$\mathcal{F}$**: Full 2-point amplitude**

$\widehat{\mathcal{F}}$**: 1PI contribution to two point amplitude**

Δ**: tree level propagator**

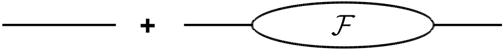
$$\Delta \propto \int \mathbf{dsd}\theta\mathbf{exp}\left[-\mathbf{s}(\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0}) + \mathbf{i}\theta(\mathbf{L_0} - \bar{\mathbf{L}}_\mathbf{0})\right] \propto (\mathbf{L_0} + \bar{\mathbf{L}}_\mathbf{0})^{-1}\delta_{\mathbf{L_0},\bar{\mathbf{L}}_\mathbf{0}}$$

(represents the effect of gluing two Riemann surfaces using plumbing fixture)

Two point amplitude

$$\mathcal{F} = \widehat{\mathcal{F}}(1 - \Delta\widehat{\mathcal{F}})^{-1} = (1 - \widehat{\mathcal{F}}\Delta)^{-1}\widehat{\mathcal{F}}$$

Full propagator



$$\Pi = \Delta + \Delta\mathcal{F}\Delta = (\Delta^{-1} - \widehat{\mathcal{F}})^{-1}$$

If k is the momenta carried by external states then poles of Π in the $-\mathbf{k}^2$ plane give the renormalized mass^2.

Are these independent of the choice of the local coordinate system?

$$\Pi = \Delta + \Delta \mathcal{F} \Delta = (\Delta^{-1} - \widehat{\mathcal{F}})^{-1}$$

Since string theory has infinite number of states, the matrices Π, Δ, $\mathcal{F}$, $\widehat{\mathcal{F}}$ etc. are all infinite dimensional.

To study the propagator of states with tree level mass m we can ‘integrate out’ all states at other mass levels and dump their contribution with the 1PI amplitude.

Δ has block diagonal form:

$$\Delta = \begin{pmatrix} * & & & & & & & & \\ & \cdot & & & & & & & \\ & & \cdot & & & & & & \\ & & & * & & & & & \\ & & & & (\mathbf{k^2} + \mathbf{m^2})^{-1}\,\mathbf{I} & & & & \\ & & & & & * & & & \\ & & & & & & \cdot & & \\ & & & & & & & \cdot & \\ & & & & & & & & * \end{pmatrix}$$

Define P_T: Projection operator into states of tree level mass m.

$$\bar{\Delta} \equiv \Delta - (\mathbf{k^2} + \mathbf{m^2})^{-1}\mathbf{P_T}$$

$$\Rightarrow \bar{\Delta} = \begin{pmatrix} * & & & & & & & & \\ & \cdot & & & & & & & \\ & & \cdot & & & & & & \\ & & & * & & & & & \\ & & & & \mathbf{0} & & & & \\ & & & & & * & & & \\ & & & & & & \cdot & & \\ & & & & & & & \cdot & \\ & & & & & & & & * \end{pmatrix}$$

Important point: $\bar{\Delta}$ has no poles at $k^2 + m^2 = 0$, even though it has poles at other mass levels.

Define

$$\bar{\mathcal{F}} \equiv \widehat{\mathcal{F}} + \widehat{\mathcal{F}}\bar{\Delta}\widehat{\mathcal{F}} + \cdots = \widehat{\mathcal{F}}(1 - \bar{\Delta}\widehat{\mathcal{F}})^{-1} = (1 - \widehat{\mathcal{F}}\bar{\Delta})^{-1}\widehat{\mathcal{F}}$$

$$\widetilde{\mathbf{F}} \equiv \mathbf{P_T}\bar{\mathcal{F}}\mathbf{P_T}, \quad \mathcal{V}_\mathbf{T} = \mathbf{P_T}\,\Pi\,\mathbf{P_T}$$

$\bar{\mathcal{F}}$ and $\widetilde{\mathbf{F}}$ also have no poles at $k^2 + m^2 = 0$.

$$\bar{\Delta} \equiv \Delta - (\mathbf{k}^2 + \mathbf{m}^2)^{-1} \mathbf{P_T}$$

$$\bar{\mathcal{F}} = \widehat{\mathcal{F}}(1 - \bar{\Delta}\widehat{\mathcal{F}})^{-1} = (1 - \widehat{\mathcal{F}}\bar{\Delta})^{-1}\widehat{\mathcal{F}}$$

$$\mathcal{F} = \widehat{\mathcal{F}}(1 - \Delta\widehat{\mathcal{F}})^{-1} = (1 - \widehat{\mathcal{F}}\Delta)^{-1}\widehat{\mathcal{F}}$$

$$\Pi = \Delta + \Delta\mathcal{F}\Delta = (\Delta^{-1} - \widehat{\mathcal{F}})^{-1}$$

$$\mathcal{V}_\mathbf{T} = \mathbf{P_T}\,\Pi\,\mathbf{P_T}, \quad \widetilde{\mathbf{F}}_\mathbf{T} = \mathbf{P_T}\,\widehat{\mathcal{F}}\,\mathbf{P_T}$$

From these is easy to derive the following relations

$$\begin{aligned}
\mathcal{V}_\mathbf{T} &= \mathbf{P_T}(\mathbf{k}^2 + \mathbf{m}^2 - \widetilde{\mathbf{F}}_\mathbf{T})^{-1}\mathbf{P_T} \\
&= \mathbf{P_T}\Big[(\mathbf{k}^2 + \mathbf{m}^2)^{-1} + (\mathbf{k}^2 + \mathbf{m}^2)^{-1}\widetilde{\mathbf{F}}_\mathbf{T}(\mathbf{k}^2 + \mathbf{m}^2)^{-1} \\
&\quad + (\mathbf{k}^2 + \mathbf{m}^2)^{-1}\widetilde{\mathbf{F}}_\mathbf{T}(\mathbf{k}^2 + \mathbf{m}^2)^{-1}\widetilde{\mathbf{F}}_\mathbf{T}(\mathbf{k}^2 + \mathbf{m}^2)^{-1} + \cdots\Big]\mathbf{P_T}
\end{aligned}$$

$$\mathcal{V}_T = P_T(k^2 + m^2 - \widetilde{F}_T)^{-1} P_T$$

$\mathcal{V}_T$ is a finite dimensional matrix labelled by the states which have tree level mass m.

Renormalized mass^2: Values of $-k^2$ where $\mathcal{V}_T$ has poles

– i.e. $k^2 + m^2 - \widetilde{F}_T$ has zero eigenvalues.

Are the locations of these poles independent of the choice of local coordinates?

Some added complications

At a given tree level mass, string theory contains physical as well as unphysical states.

e.g. a vertex operator of the form $c\bar{c}V$ with V dimension (1,1) matter sector primary operator represents physical state.

Other vertex operators with same L_0, $\bar{L}_0$ eigenvalues represent unphysical states (secondaries in matter sector and/or ghost excitations).

Off-shell, quantum corrections cause mixing between physical and unphysical states.

$\Rightarrow$ Only renormalized physical masses can be expected to be independent of the choice of local coordinates.

How do we sort out the renormalized physical masses from the renormalized unphysical masses?

– done in several steps.

1. Identify a set of special states which do not mix with unphysical states at the same mass level due to symmetries.

Example: States on the leading Regge trajectory

For these states the mixing problem is absent and we can prove that the renormalized masses and S-matrix elements are independent of the choice of local coordinate system.

2. For general states quantum corrections mix physical, unphysical and pure gauge states.

Physical $\Leftrightarrow$ BRST invariant

Pure gauge $\Leftrightarrow$ BRST trivial

Unphysical $\Leftrightarrow$ BRST non-invariant

We develop an algorithm to block diagonalize the quantum corrected propagator and identify the quantum corrected physical block.

– satisfies the requirement that in the zero coupling limit the quantum corrected physical states approach a linear combination of tree level physical states and pure gauge states.

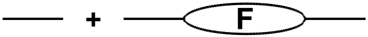
3. We show that in the scattering amplitudes of massless, BPS and special states only physical poles appear as intermediate states.

Since the former are independent of the choice of local coordinates, this shows that the physical masses are independent of the choice of local coordinates.

Some technical details

Let F(k) be the matrix describing the off-shell two point amplitude of special states at mass level m.

Then the special state propagator at mass level m is given by



$$(k^2 + m^2)^{-1} + (k^2 + m^2)^{-2} F(k)$$

This is expected to be of the form

$$Z(k)^{1/2} (k^2 + M_p^2)^{-1} Z^{1/2}(k)^{\dagger}$$

M_p^2: Diagonal physical mass^2 matrix.

$Z(k)$: Wave-function renormalization matrix with no pole near $k^2 = -m^2$.

$$(k^2+m^2)^{-1}+(k^2+m^2)^{-2}F(k)=Z(k)^{1/2}(k^2+M_p^2)^{-1}Z^{1/2}(k)^\dagger$$

Now suppose we change the local coordinate system.

We would want to test if it leaves M_p unchanged and only changes $Z(k)$.

Define $\delta Y(k)=\delta Z(k)^{1/2}Z(k)^{-1/2}$

Then we want

$$\begin{aligned} & (k^2+m^2)^{-1}+(k^2+m^2)^{-2}(F(k)+\delta F(k)) \\ = \; & (1+\delta Y(k))\{(k^2+m^2)^{-1}+(k^2+m^2)^{-2}F(k)\}(1+\delta Y(k)^\dagger) \end{aligned}$$

$$\Rightarrow \delta F(k)=\delta Y(k)(k^2+m^2+F(k))+(k^2+m^2+F(k))\delta Y(k)^\dagger$$

Note: Since $Z(k)$ is analytic near $k^2+m^2=0$, the same must hold for $\delta Y(k)$.

Computation of δF

– arises from variation of local coordinates at one of the two puctures where the vertex operator is introduced.

The variation of the vertex operators are $\propto (k^2 + m^2)$ since for $(k^2 + m^2) = 0$ they are dimension zero primaries.

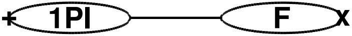
– call them $(k^2 + m^2)\delta H(k)$ and $(k^2 + m^2)\delta H(-k)$

$\delta F = (k^2 + m^2)$  $+$  $(k^2 + m^2)$

+: δH vertex,   x: ordinary vertex

$\delta \mathbf{F} = (\mathbf{k}^2 + \mathbf{m}^2)$  $+$  $(\mathbf{k}^2 + \mathbf{m}^2)$

 $=$ 

$+$ 

$$\equiv \delta \mathbf{Y} + \delta \mathbf{Y}\, (\mathbf{k}^2 + \mathbf{m}^2)^{-1}\, \mathbf{F}$$

$$\Rightarrow \delta \mathbf{F} = (\mathbf{k}^2 + \mathbf{m}^2)\, \delta \mathbf{Y} + \delta \mathbf{Y}\, \mathbf{F} + (\mathbf{k}^2 + \mathbf{m}^2)\, \delta \mathbf{Y}^\dagger + \mathbf{F}\, \delta \mathbf{Y}^\dagger$$

– the desired relation.

δY, being 1PI, has no pole near $\mathbf{k}^2 + \mathbf{m}^2 = 0$.

This proves that the renormalized masses of special states are independent of the choice of local coordinates.

Similar analysis can be used to prove the other results.

For the future

1. Extend the analysis to Ramond sector.

2. Use this algorithm to compute two loop renormalized mass of SO(32) spinors in heterotic string theory.

3. Many other problems in string theory require intermediate off-shell formalism even though eventually we want to compute on-shell quantities.

Apply the general off-shell formalism to those cases.

Example: In many compactifications of SO(32) heterotic string theory on Calabi-Yau 3-folds, one loop correction generates a Fayet-Iliopoulos term.

Net effect: Generate a potential of a charged scalar ϕ of the form

$$c(\phi^*\phi - Kg^2)^2$$

c, K: positive constants, g: string coupling

Dine, Seiberg, Witten; Dine, Ichinose, Seiberg; Atick, Dixon, A.S.

It is clear that there is a supersymmetric vacuum at $|\phi| = g\sqrt{K}$, but on-shell techniques do not tell us how to carry out systematic perturbation expansion around the new vacuum.

The general off-shell formalism we have discussed may be useful for giving a systematic algorithm for computing S-matrix around the shifted vacuum.

4. Other possible applications are likely to crop up as we understand this off-shell formalism better.