Exotic phases of hadronic matter

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VIRTUAL MEETING ON COMPACT STARS AND QCD
20th Aug 2020

Outline

- Matter in extreme conditions
- Color superconductors
- Meson condensation
- Conclusions

Some preliminary considerations

Why studying compact stars?

1. Because they are there.
2. They can help us to understand the properties of hadronic matter

The unavoidable problem

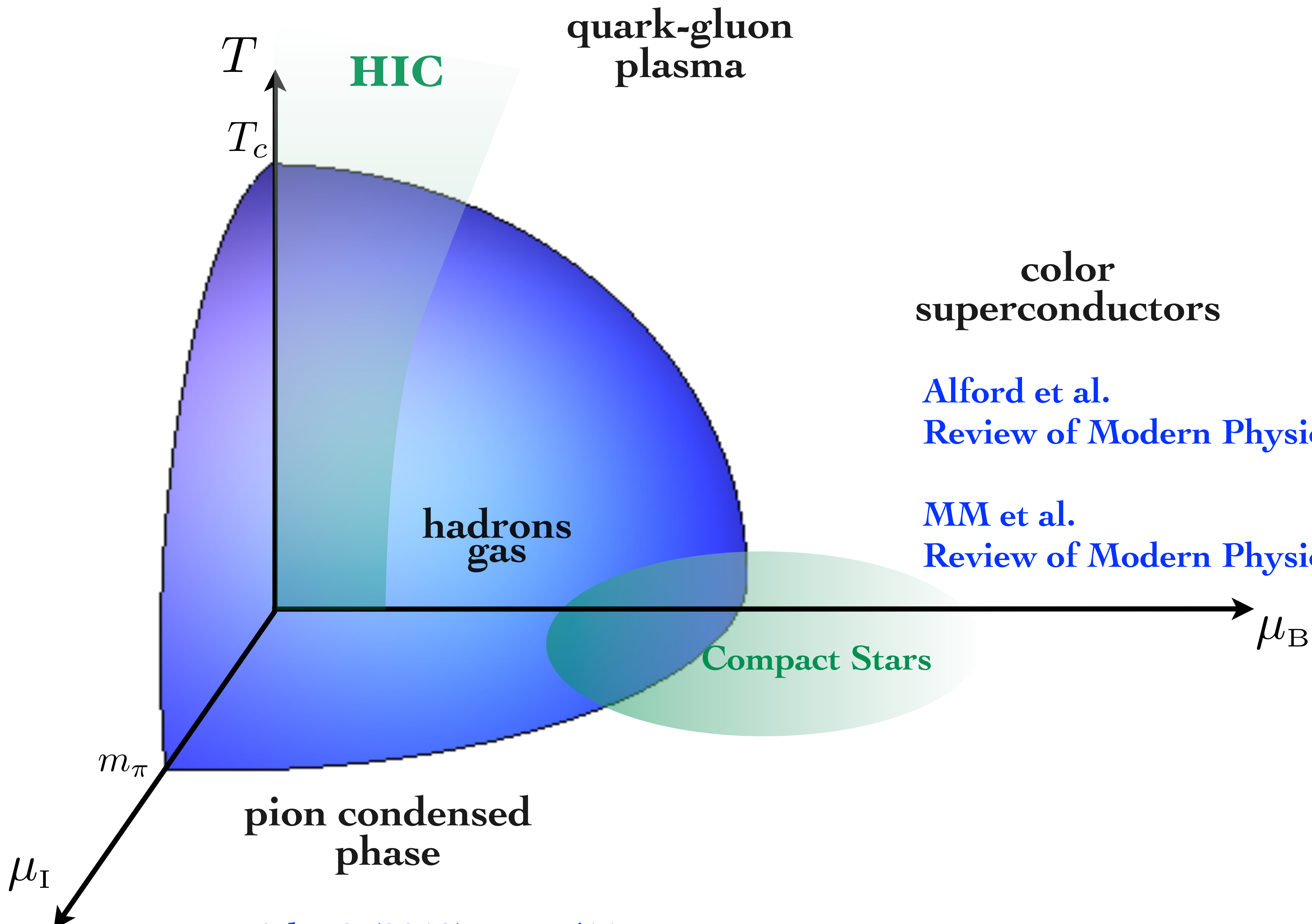
Compact stars are far away. We basically see their atmosphere and can measure few of their properties.

How to make progress

1) Careful and stoical observations
2) GW merging: since our childhood we know that smashing stuff and looking at the debris helps to understand how things work
3) Reasonable theoretical modeling

Matter in extreme conditions

The QCD phase diagram

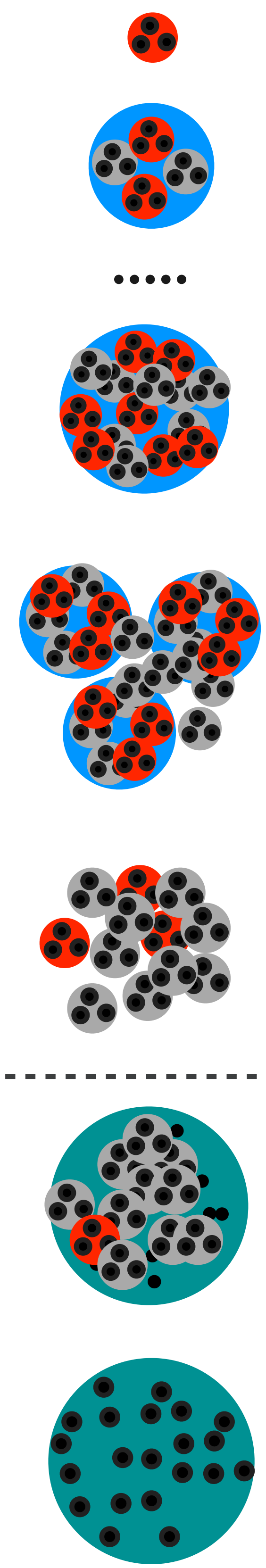


Alford et al.
Review of Modern Physics 80, 1455 (2008)

MM et al.
Review of Modern Physics 86, 509 (2014)

MM, Particles 2 (2019) no.3, 411

Increasing baryonic density



	Density	Compact Star	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H				
He	......	atmosphere		light nuclei
......				.....
Fe	10 g cm^{-3}	outer crust	confining	heavy nuclei
neutron drip	10^{11} g cm^{-3}	inner crust		
neutron proton soup	10^{14} g cm^{-3}	core		neutrons and protons
quark drip	very large		strong coupling	quarks and gluons Cooper pairs of quarks?
quark soup	extreme		weak coupling	Cooper pairs of quarks NGBs

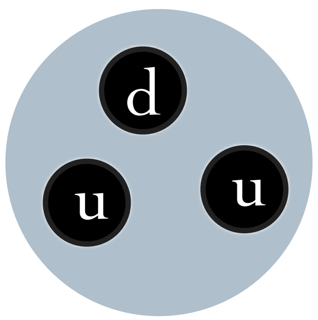
ρ

Quark modeling

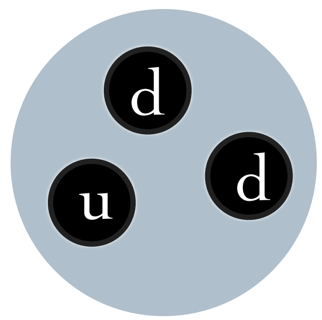
HADRONS

BARYONS

proton



neutron



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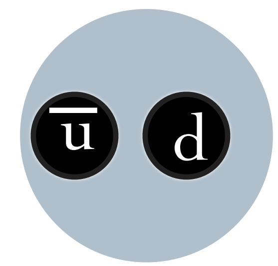
$$m_n \sim 1\text{GeV} \gg m_{u,d}$$

$$r_n \sim 1\text{fm} = 10^{-15}\text{m}$$

MESONS

pions



....

$$m_\pi \sim 135 \text{ MeV} \gg m_{u,d}$$

$$r_\pi \sim 0.7\text{fm}$$

Building blocks of hadrons are **quarks and gluons**

Q	Quarks (mass in MeV)		
$+2/3$	u (3)	c (1300)	t (170000)
$-1/3$	d (5)	s (130)	b (4000)

Internal degree of freedom called color. **Quarks can be: Red, Green or Blue.**
Gluons are the vector gauge bosons of the associated gauge group $SU(3)_c$

Quantum chromodynamics

$$\mathcal{L}_{\rm QCD} = \bar{\psi}(i\gamma^\mu D_\mu + \mu\gamma_0 - M)\psi - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$$

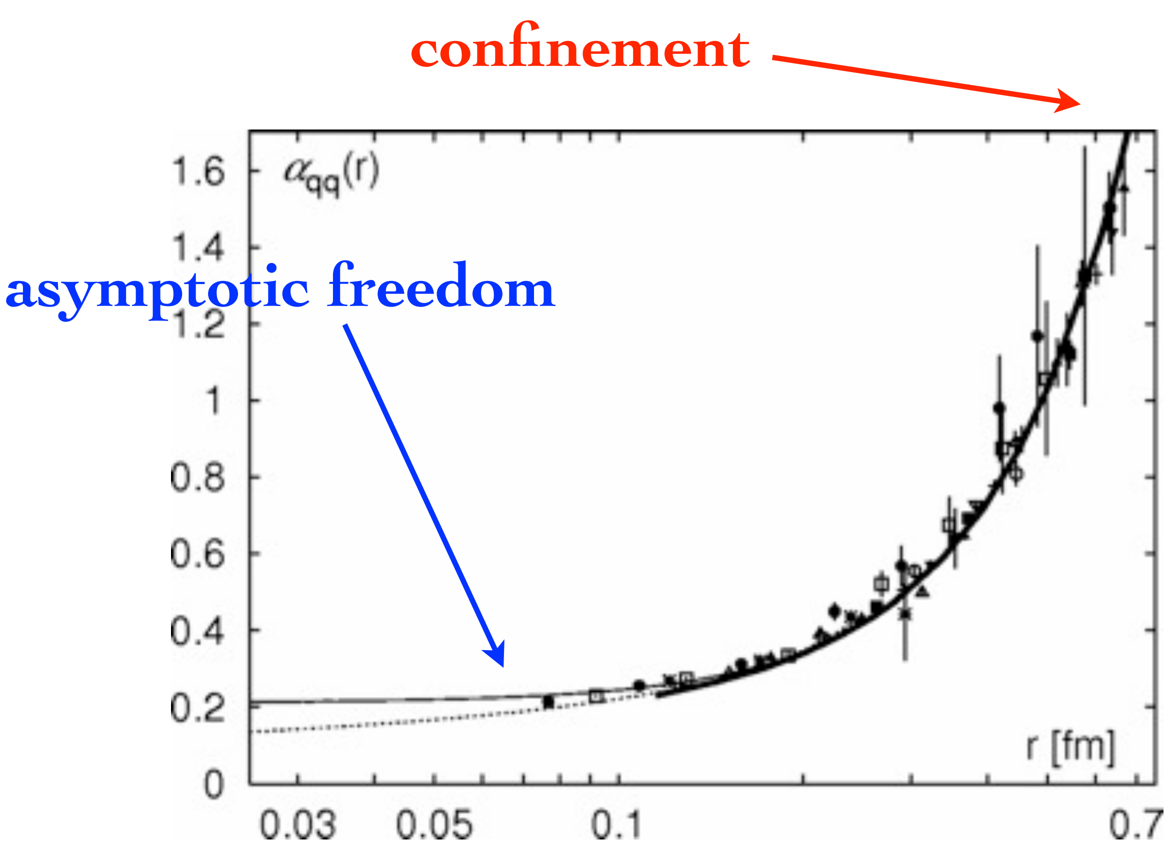
quark fields: $\psi_{\alpha,i}$

$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, 2, 3$ **flavor indices**

gluon gauge fields: $\mathbf{A}^a$

$a = 1, \ldots, 8$ **adjoint color index**



QCD non-Abelian gauge theory, non-perturbative at energy scales below the QCD energy scale $\Lambda_{\rm QCD} \sim 300\,{\rm MeV}$

Kaczmarek and Zantow
Physical Review D 71(11):114510

Symmetries of QCD

For three flavor massless quark matter

gauge group

global chiral symmetry

global baryonic number

$$SU(3)_{\rm c} \times \underbrace{SU(3)_{\rm L} \times SU(3)_{\rm R}}_{\supset\, [U(1)_{\rm e.m.}]} \times U(1)_{\rm B}$$

If we do not use QCD we want theories that preserve (part of) its symmetries

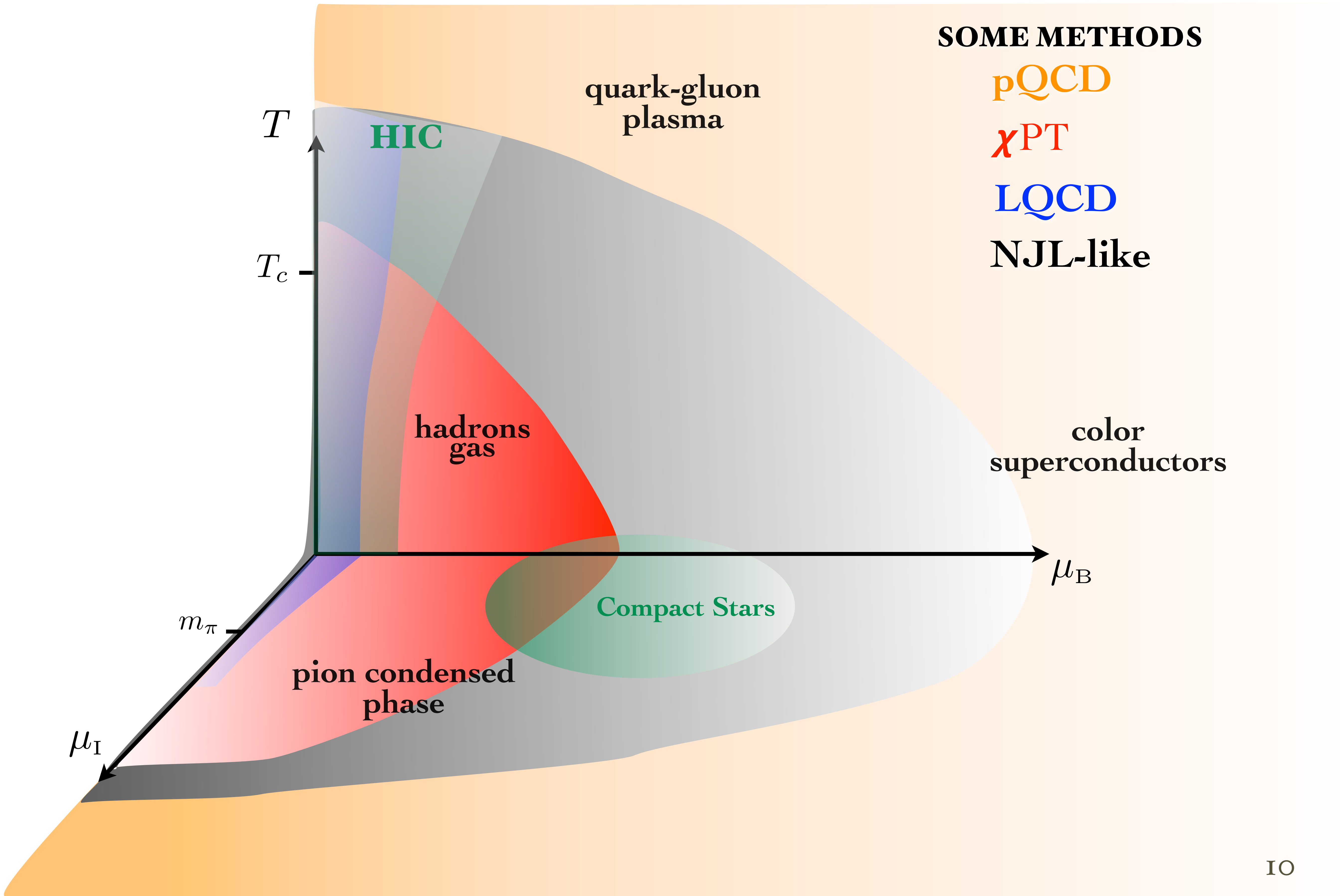
Lattice QCD

Discretization on a lattice.
Does not work at large densities

Effective field theories

Preserve global symmetries of QCD
Lack the gauge field dynamics

Phases of quark matter



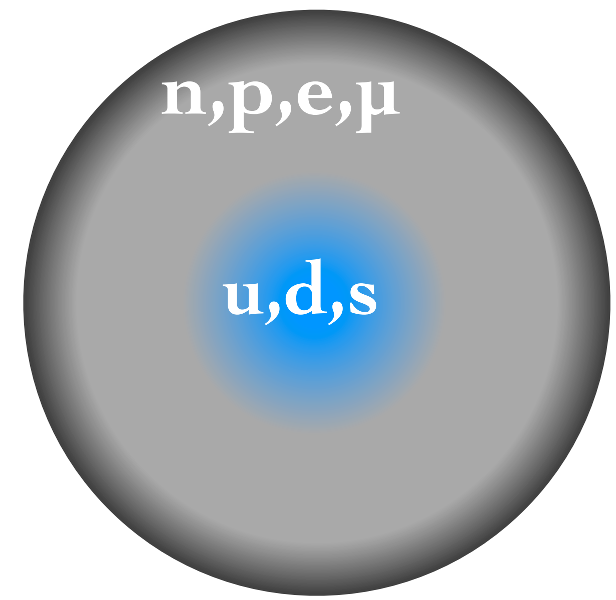
Color superconductors

Taxonomy of compact stars

Neutron star

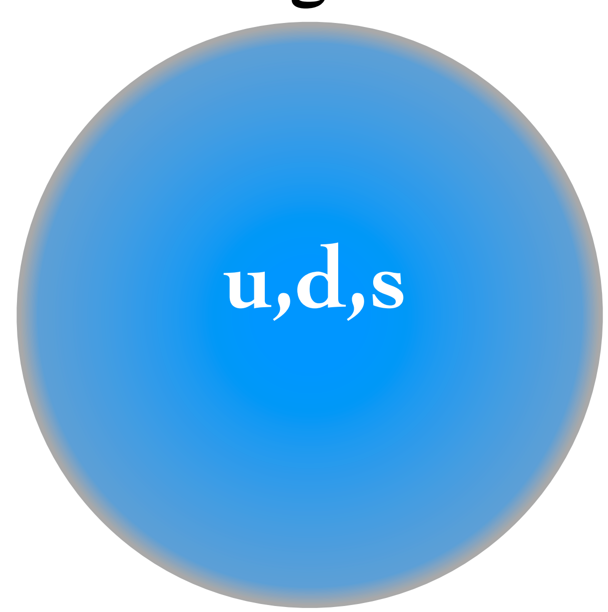
$R \sim 10$ km $\;M \simeq 1 - 2\,M_\odot$

Hybrid star



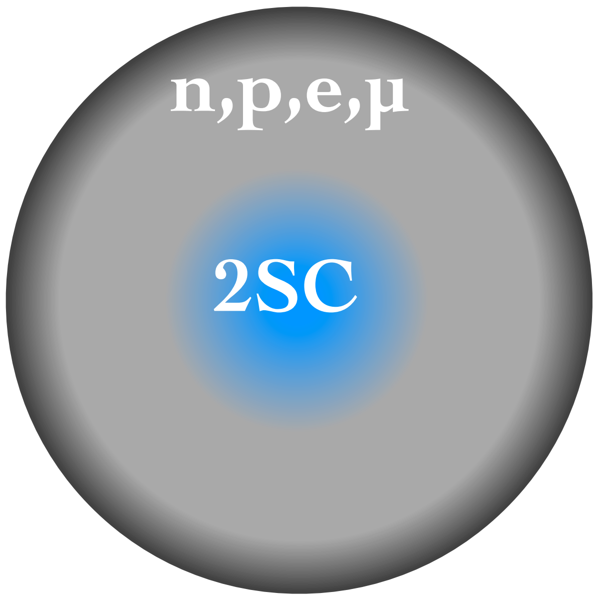
$R \sim 10$ km $\;M \simeq 1 - 2\,M_\odot$

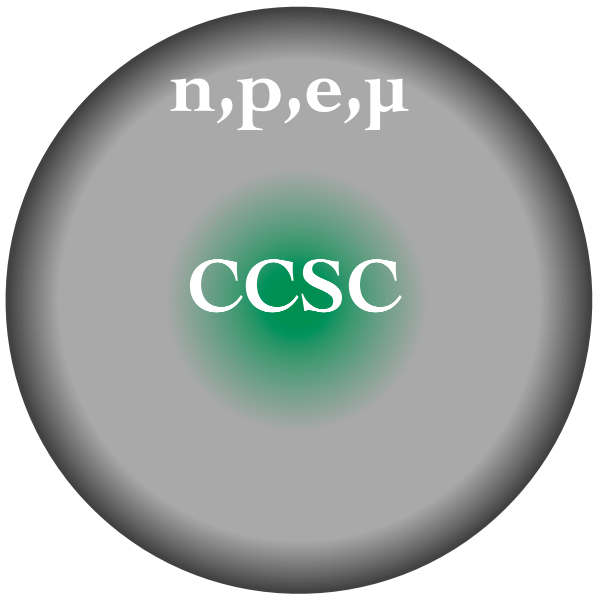
Strange star



$R \sim 0 - 10$ km $\;M \lesssim 2\,M_\odot$

And even more options...

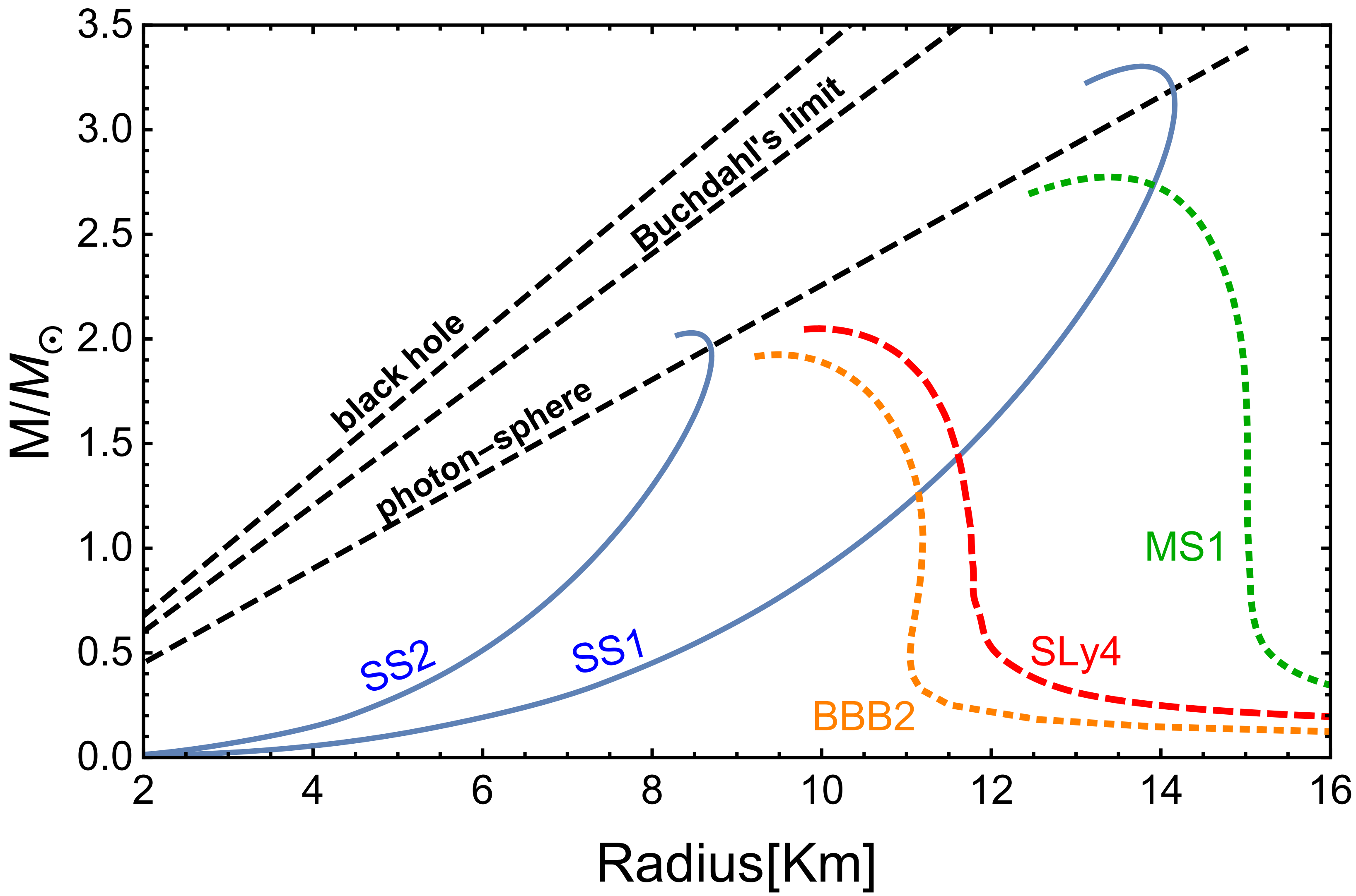




1. There are several possibilities

2. The transition densities
   are not strongly constrained

Some mass-radius comparisons



Hard to distinguish unless

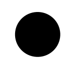
1. Precise radius measurement, see Jim's and Cole's talks

2. Identification of an object with small mass and radius

3. Identification of an object with large compactness, see MM and Tonelli, Phys.Rev.D 97 (2018)

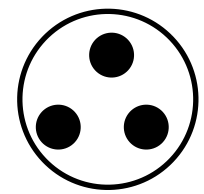
Basics of color superconductors

quark



point-like

baryon

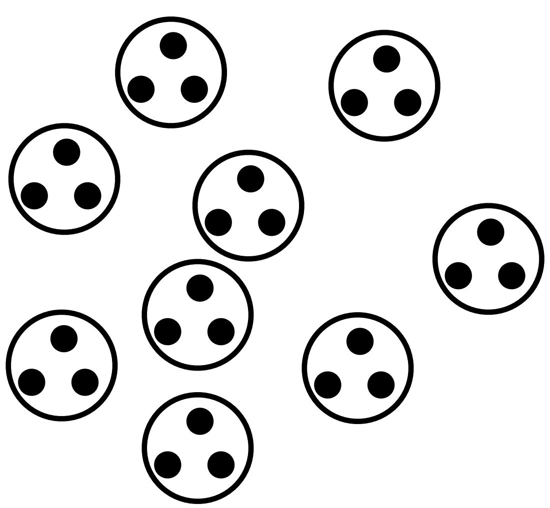


~1 fm

diquark

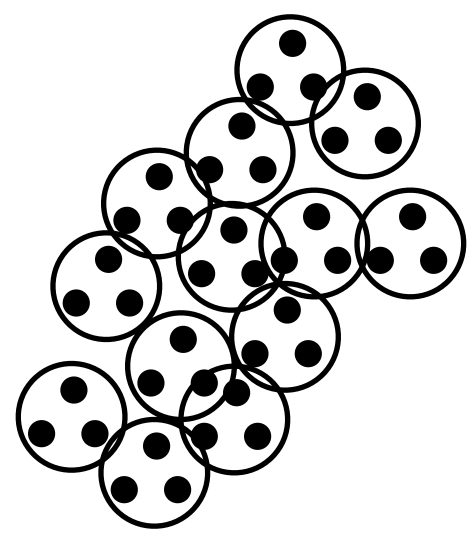
~few fm

"low density"



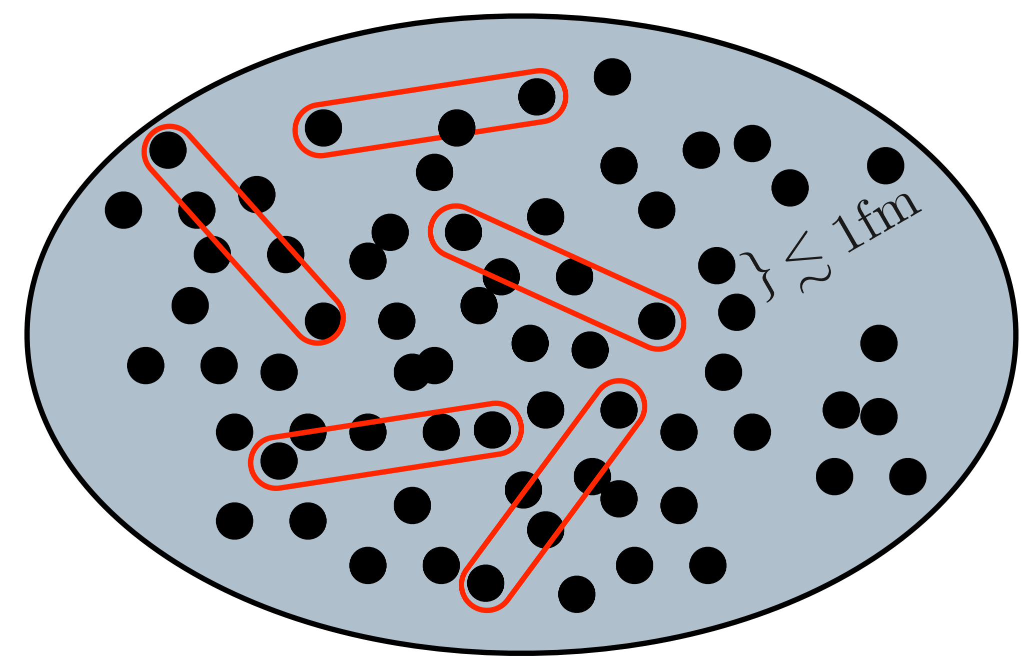
Gas of nucleons

High density



Liquid of overlapping nucleons

Very high density



Liquid of deconfined quarks with correlated diquarks

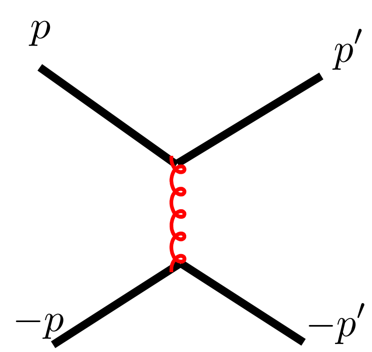
Asymptotic density

The two main aspects of the gluon interaction

Attractive perturbative interaction

$3 \times 3 = \bar{3}_A + 6_S$

attractive channel

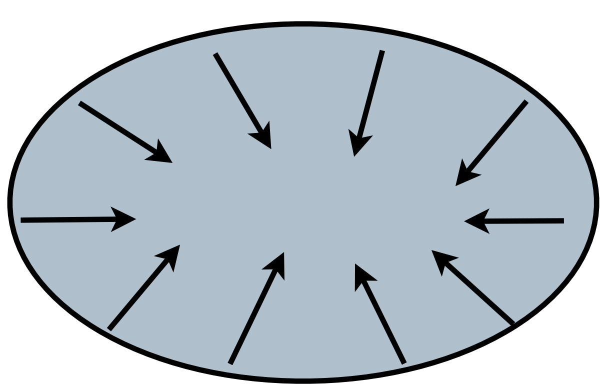


$p, p' \simeq p_{\rm F}$

Provides a channel to form correlated pairs.
We assume it works at any density

AND

Confining bag



Provides a mechanism to make quark matter self-bound.
No need of gravity to make quark matter stable.

General expression of the condensate

General color superconducting condensate:

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \sum_{I=1}^{3} \Delta_{\rm I} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

$\alpha, \beta = 1, 2, 3$ **color indices**

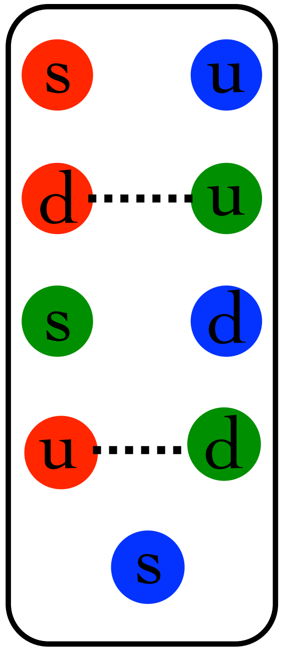
$i, j = 1, 2, 3$ **flavor indices**

$\Delta_{\rm I} \sim 10 - 100\,\mathrm{MeV}$

"gap parameters" for quarks whose flavor and color is not I

The condensate has a color charge, has a "flavor" charge and a baryonic charge
The corresponding symmetries can be broken or mixed.
A zoo of color superconducting phases.

2 Flavor color superconductor (2SC)



Suppose the strange quark mass is "large": $m_s > \mu_{u,d}$
Strange quarks decouple

$$\Delta_3 > 0\,,\ \Delta_2 = \Delta_1 = 0$$

Symmetry

$$SU(3)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_S}_{\supset U(1)_Q} \to SU(2)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}} \times U(1)_S}_{\supset U(1)_{\tilde{Q}}}$$

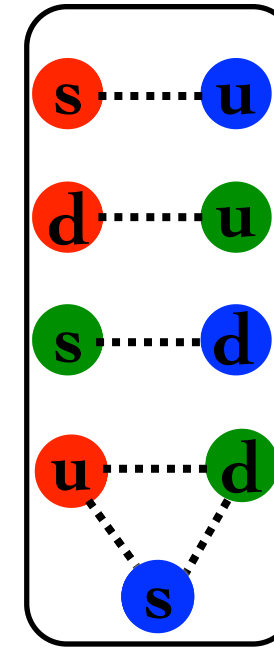
- **Higgs mechanism, 5 gauge bosons acquire a mass: COLOR SUPERCONDUCTOR**
- **No chiral symmetry breaking**
- **No global symmetry is broken: NOT A SUPERFLUID**
- **The photon is rotated (mixed with gauge and global symmetries)**

The system is an "electrical" conductor

Color Flavor Locked phase

Pairing of quarks of all flavors and colors

Alford, Rajagopal, Wilczek Nucl.Phys. B537 (1999) 443



Symmetry breaking

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

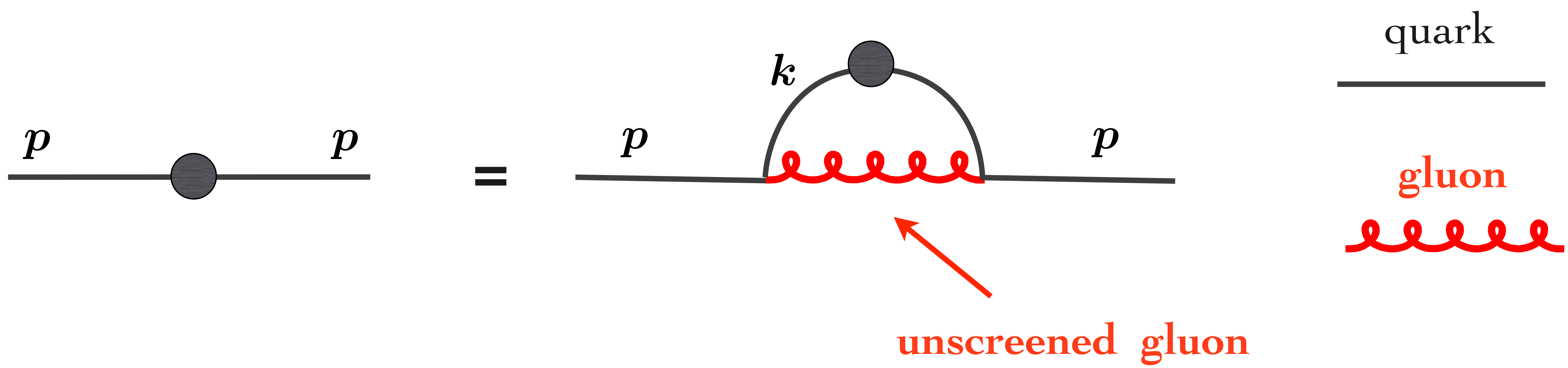
- **Higgs mechanism, 8 gauge bosons acquire a mass: COLOR SUPERCONDUCTOR**
- **χSB: 8 (pseudo) Nambu-Goldstone bosons (NGBs)**
- **$U(1)_B$ breaking, 1 NGB: SUPERFLUID**
- **"Rotated" electromagnetism, mixing angle** $\cos\theta = \frac{g}{\sqrt{g^2 + 4e^2/3}}$ **(analog of the Weinberg angle)**

Quantitative analysis

Suppose we use QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}(i\gamma^\mu D_\mu + \mu\gamma_0 - M)\psi - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$$

We want to measure the strength of the correlation using the Gap equation



$$\Delta \propto g^2 \int d\theta\, d\xi \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \frac{\mu^2}{\theta\mu^2+\delta^2} \quad \Rightarrow \quad \Delta_{\rm QCD} \propto \exp(-const/g)$$

collinear divergency

δ is the scale of Landau damping

Barrois, B. C., 1979, Ph.D. thesis,
Son, D. T., 1999, Phys. Rev. D59, 094019.

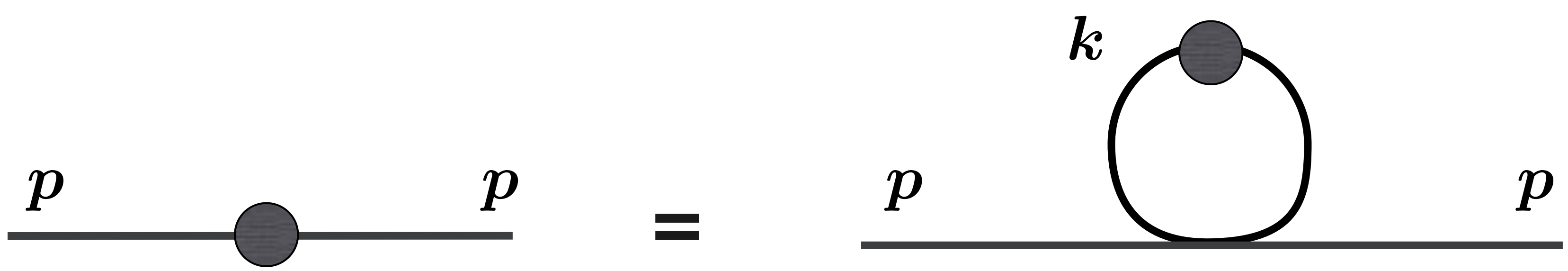
Gap equation in NJL-like models

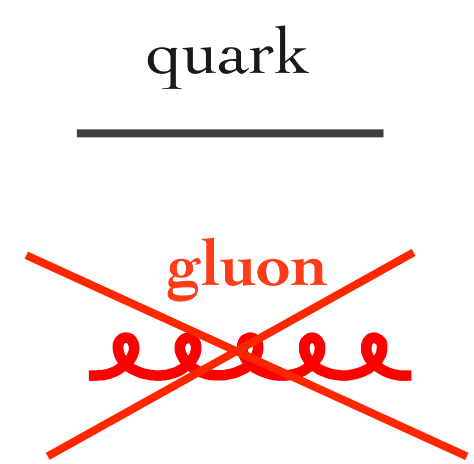
Contact interaction

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma^\mu \partial_\mu + \mu\gamma_0 - M)\psi - g(\bar{\psi}\lambda^\mu\psi)(\bar{\psi}\lambda_\mu\psi)$$

Free Lagrangian **Contact interaction**

Gap equation



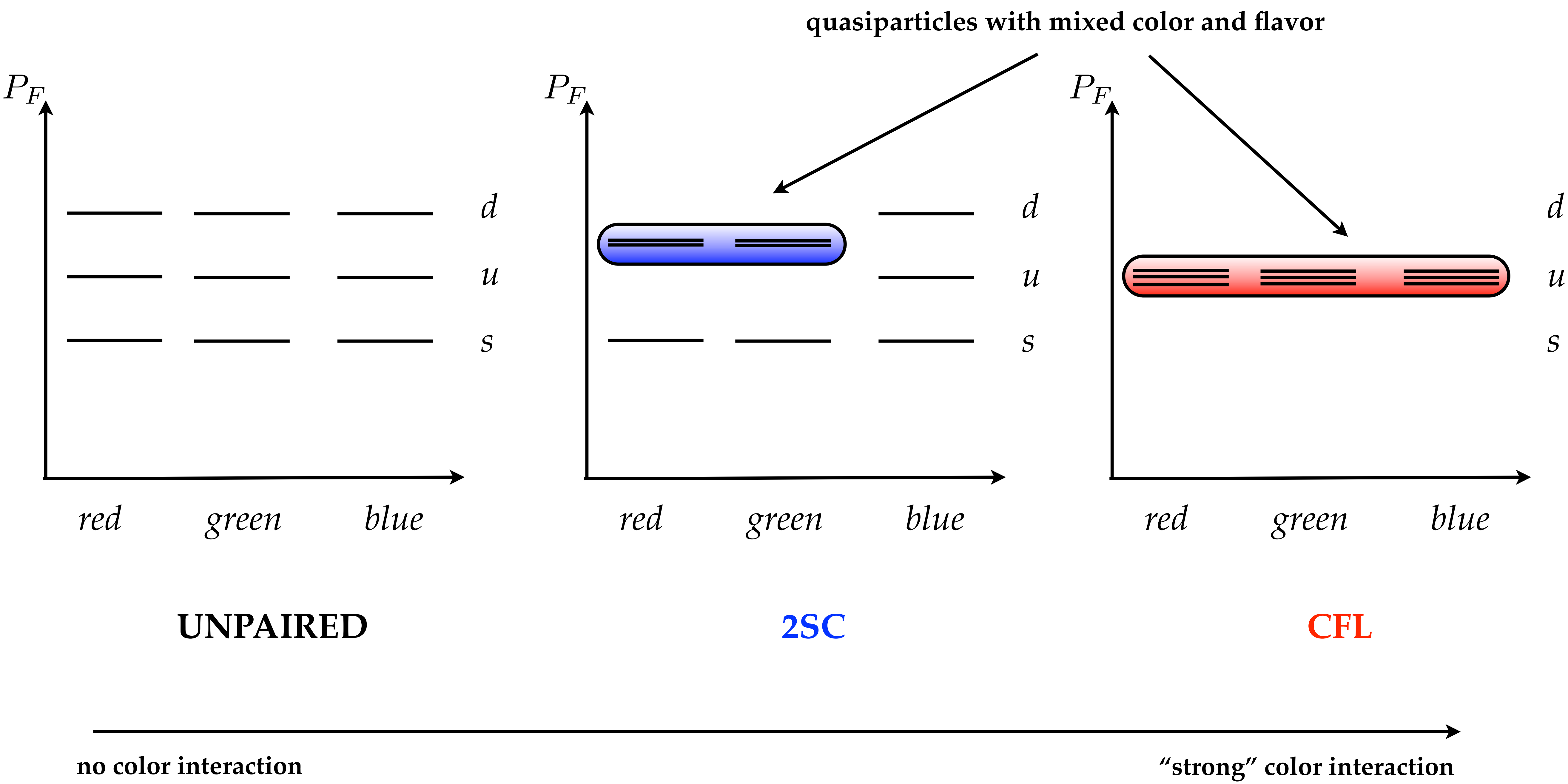


$$\Delta \propto g^2 \int d\theta \, d\xi \frac{\Delta}{\sqrt{\xi^2 + \Delta^2}} \quad \Longrightarrow \quad \Delta_{\rm NJL} \propto \exp(-const/g^2)$$

“Wrong” parametric dependence on the coupling

CRYSTALLINE COLOR SUPERCONDUCTORS

Fixed mismatch, increasing coupling



Whenever there is BCS pairing, the Fermi surfaces have to match.

Quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

→ **mismatch of Fermi momenta**

For simplicity, no strong interaction

weak interactions

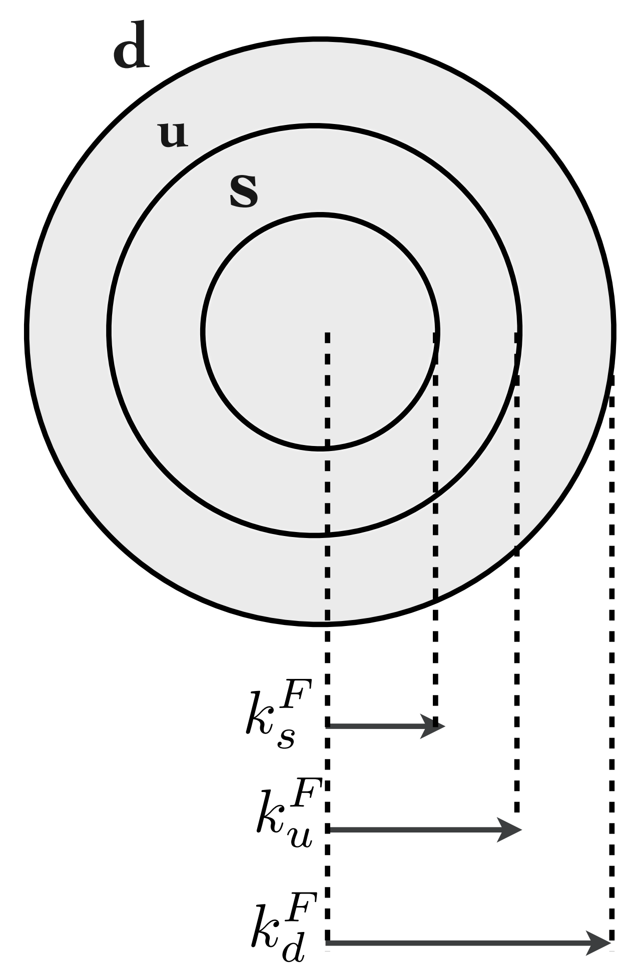
$u \to d + \bar{e} + \nu_e$
$u \to s + \bar{e} + \nu_e$
$u + d \leftrightarrow u + s$

→ $\mu_u = \mu_d - \mu_e$

$\mu_d = \mu_s$

electric neutrality → $$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$

Fermi spheres



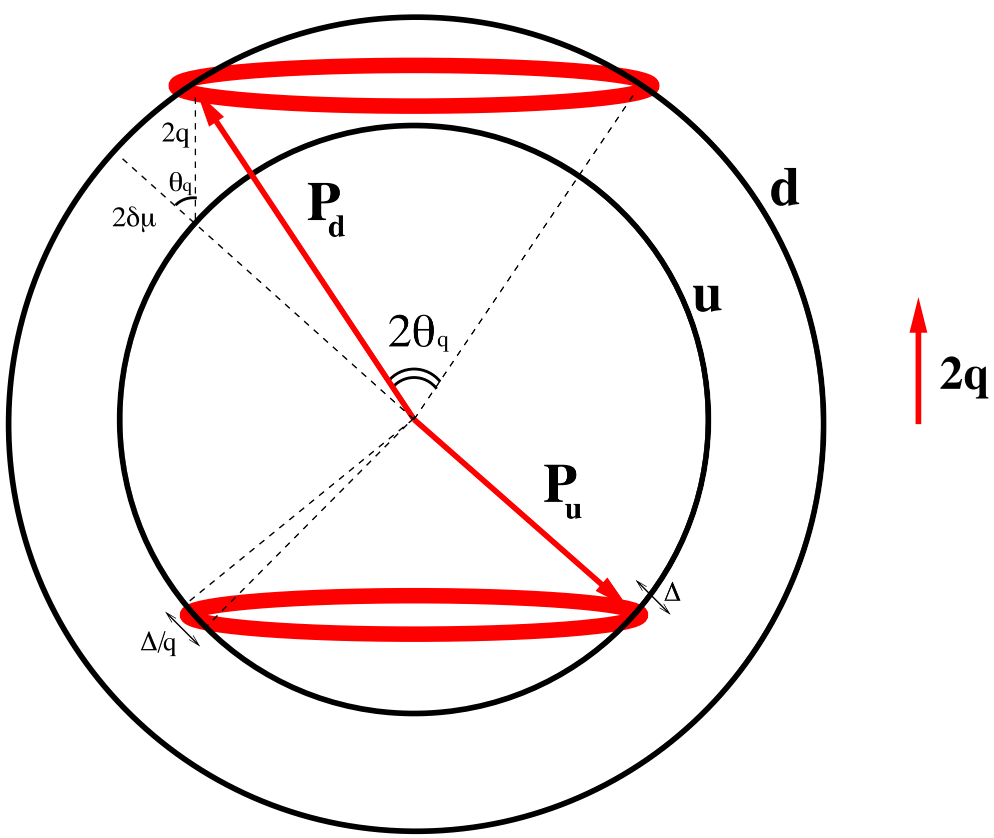
Fermi momenta $k_u^F = \mu_u \quad k_d^F = \mu_d \quad k_s^F = \sqrt{\mu_s^2 - m_s^2}$

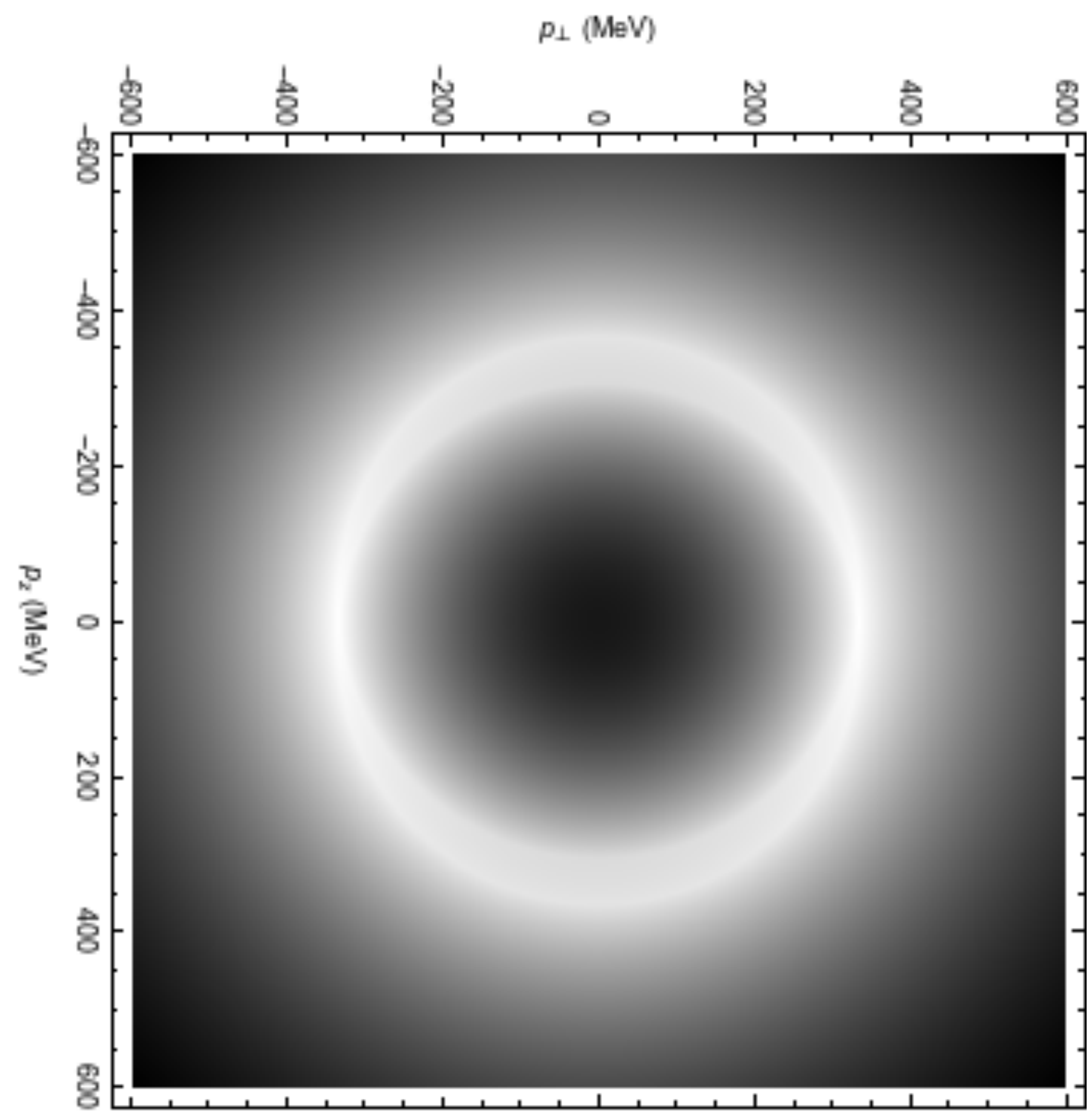
If the mismatch is too large, pairing cannot occur. The largest chemical potential mismatch which allows paring is named the **Chandrasekhar-Clogston limit**

Crystalline quark matter

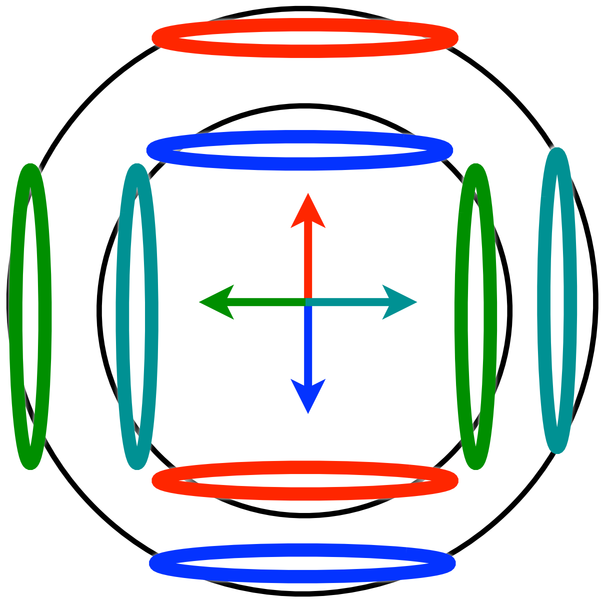
Beyond the Chandrasekhar-Clogston limit, the homogeneous BCS pairing is not possible

Non-isotropic LOFF phase: quark pairing only along certain directions

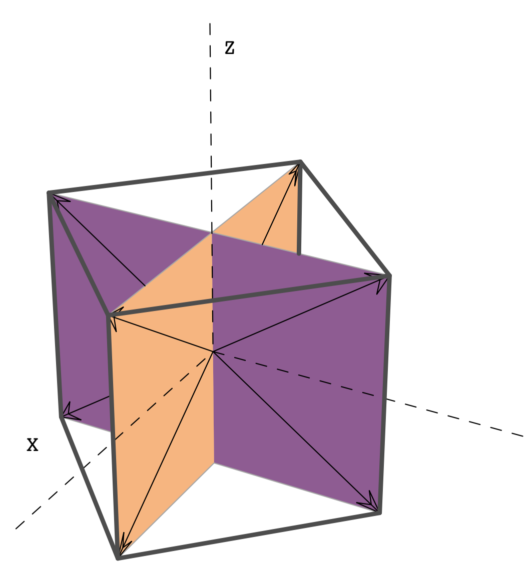


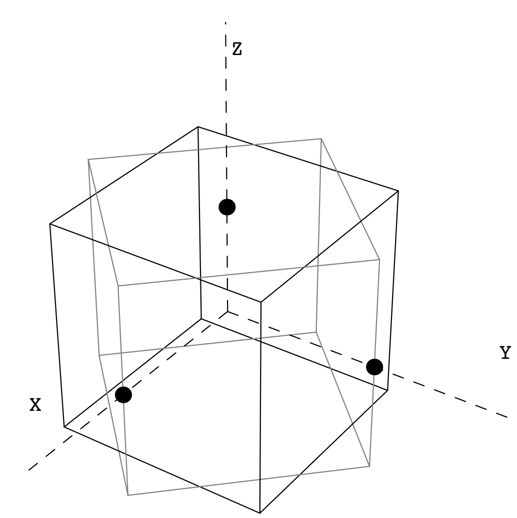


Crystalline structures: CCSC phase



- Complicated structures can be obtained combining more plane waves
- "no-overlap" condition between ribbons





- Three flavors

$$< \psi_{\alpha i} C\gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I^m\}} e^{2i\mathbf{q}_I^m \cdot \mathbf{r}} \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

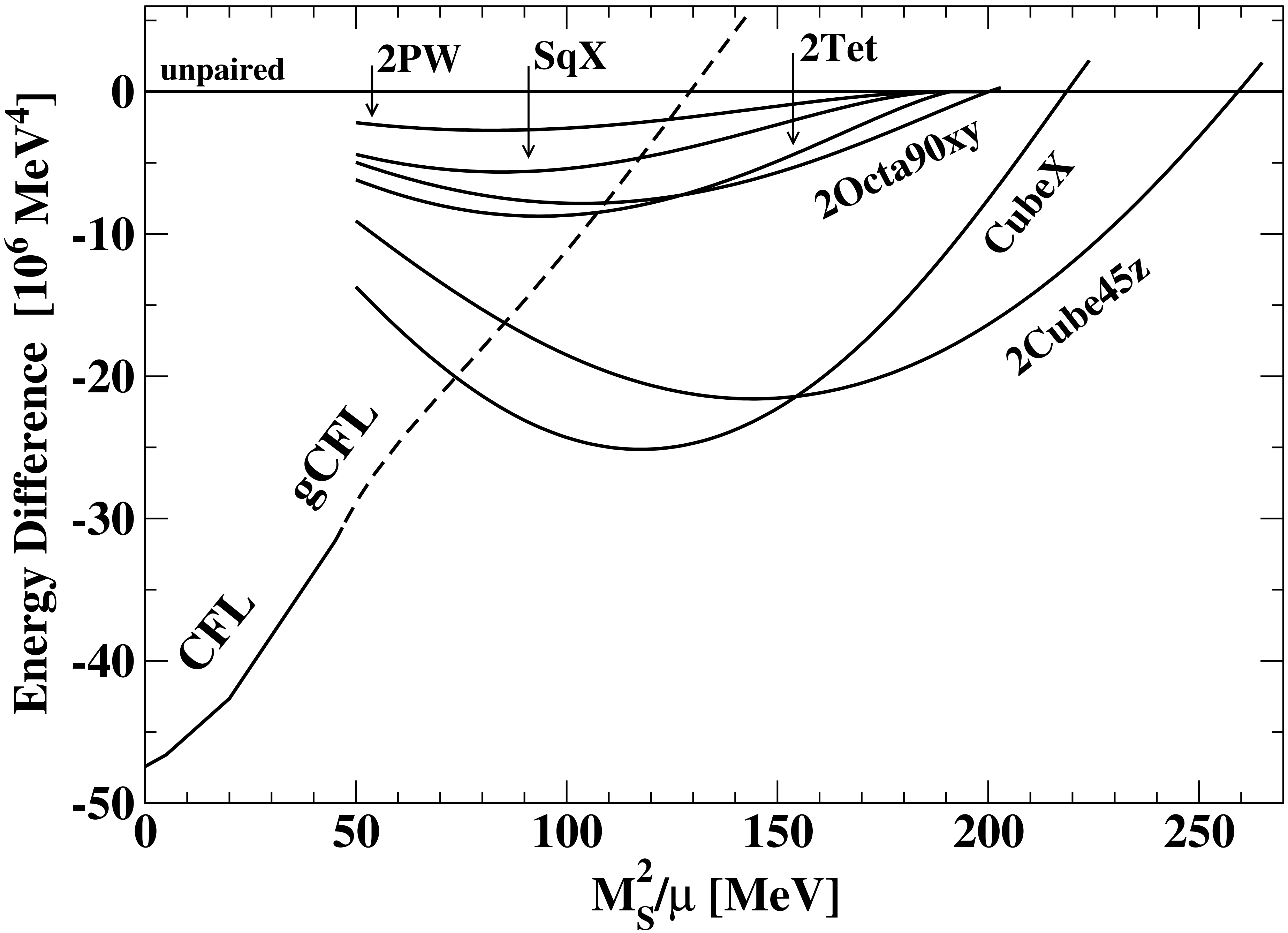
simplifications

$$\mathbf{q}_I^m = q\,\mathbf{n}_I^m$$

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Free energy estimate

NJL + GL expansion!!



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Fermionic dispersion laws

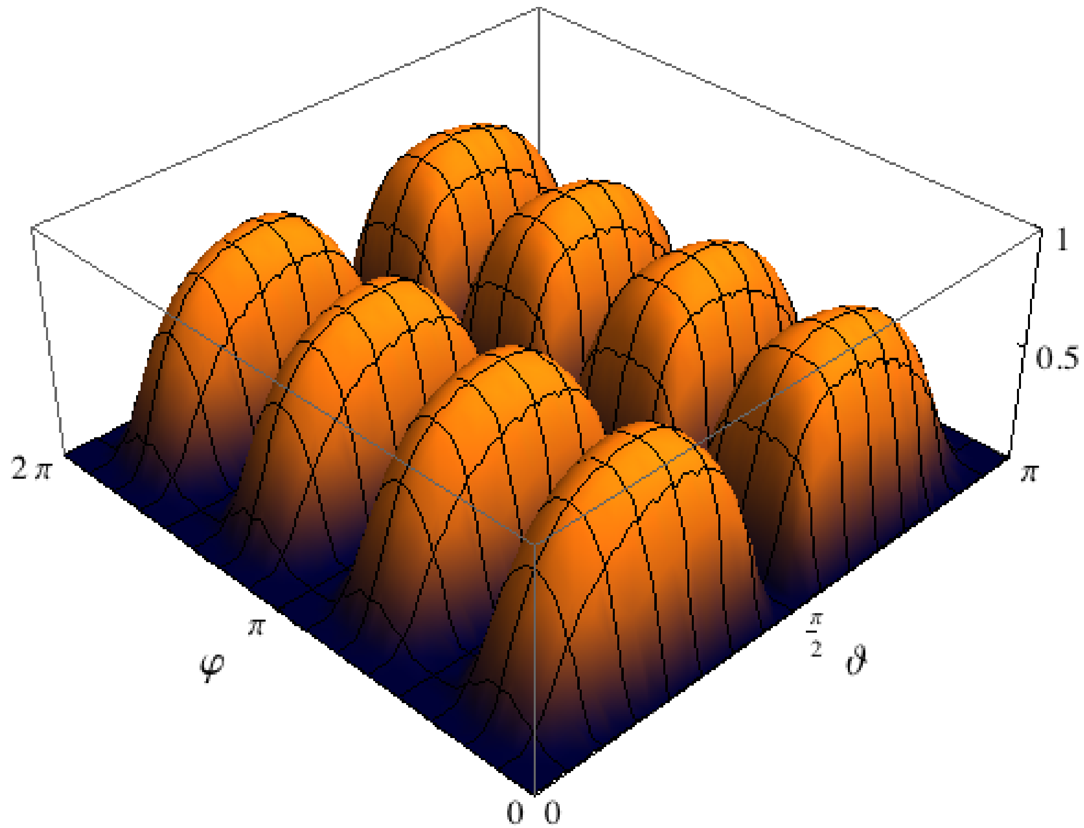
Quark quasiparticles have an anisotropic gapless dispersion law:

direction dependent velocity

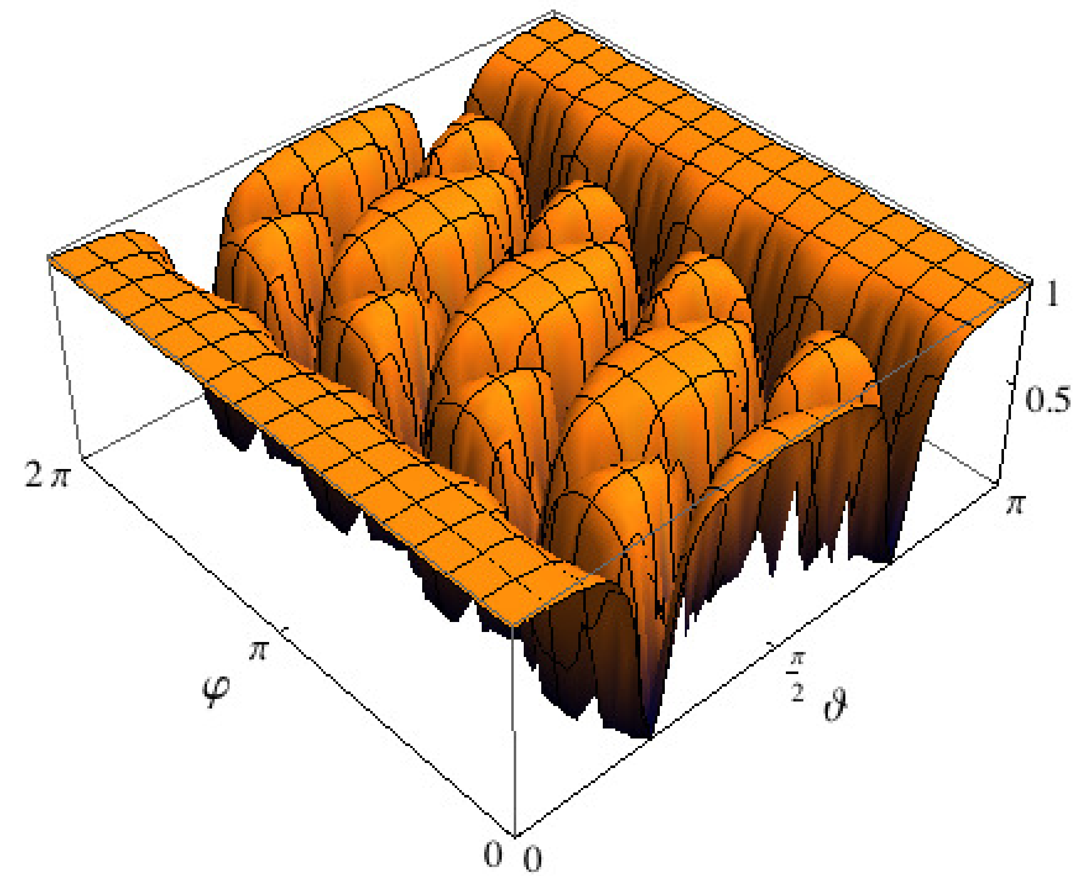
$$E = c(\theta, \phi)\, \xi$$

Velocity of fermions in two different structures

BCC



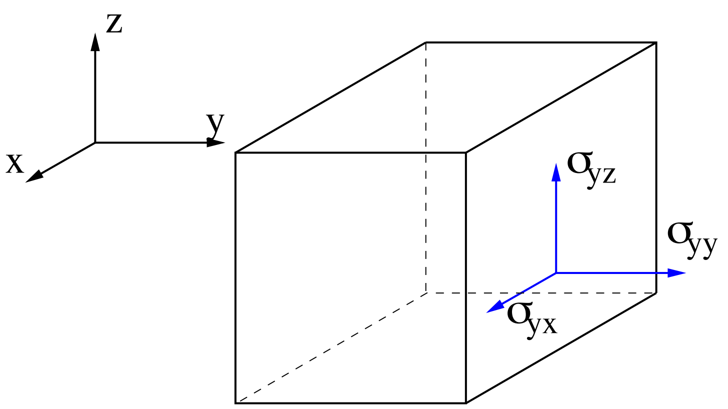
FCC



MM et al. Review of Modern Physics 86, 509 (2014)

Displacement of the crystal structure

Elastic deformation of a stressed crystal (Landau Lifsits, vol. 7)



displacement vector $u_i = x'_i - x_i$

deformation tensor $u_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$

stress tensor $\sigma_{ij} = K u_{kk}\delta_{ij} + 2\nu \left(u_{ij} - \frac{1}{3} u_{kk}\delta_{ij} \right)$

compressibility (K) — shear modulus (ν)

- **Crystalline structure given by the spatial modulation of the gap parameter**
- **It is this pattern of modulation that is rigid (and can oscillate)**

$\nu_{\rm CCSC} \sim 2.47\,{\rm MeV/fm}^3$ **20 to 1000 times more rigid than the crust of neutron stars**

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

ASTROPHYSICAL OBSERVABLES

Gravitational waves

Rotating and/or oscillating strange/hybrid stars with a CCSC crust/core, which are somehow deformed to nonaxisymmetric configurations, can efficiently emit gravitational waves

Lin, Phys.Rev. D76 (2007) 081502,
Phys.Rev. D88 (2013)

Haskell et al. Phys.Rev. Lett.99. 231101 (2007)

Knippel et al. Phys.Rev. D79 (2009) 083007

Rupak and Jaikumar Phys.Rev. C88 (2013) 065801

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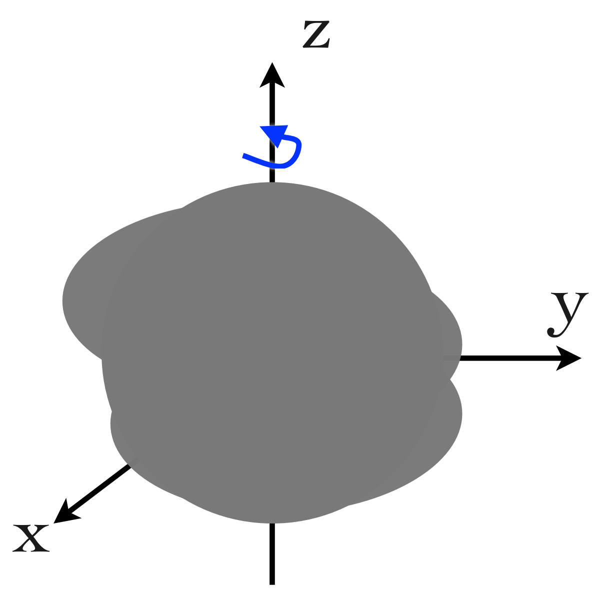
Gravitational waves from "mountains"



If the star has a non-axial symmetric deformation (mountain) it can emit gravitational waves

ellipticity

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

GW amplitude

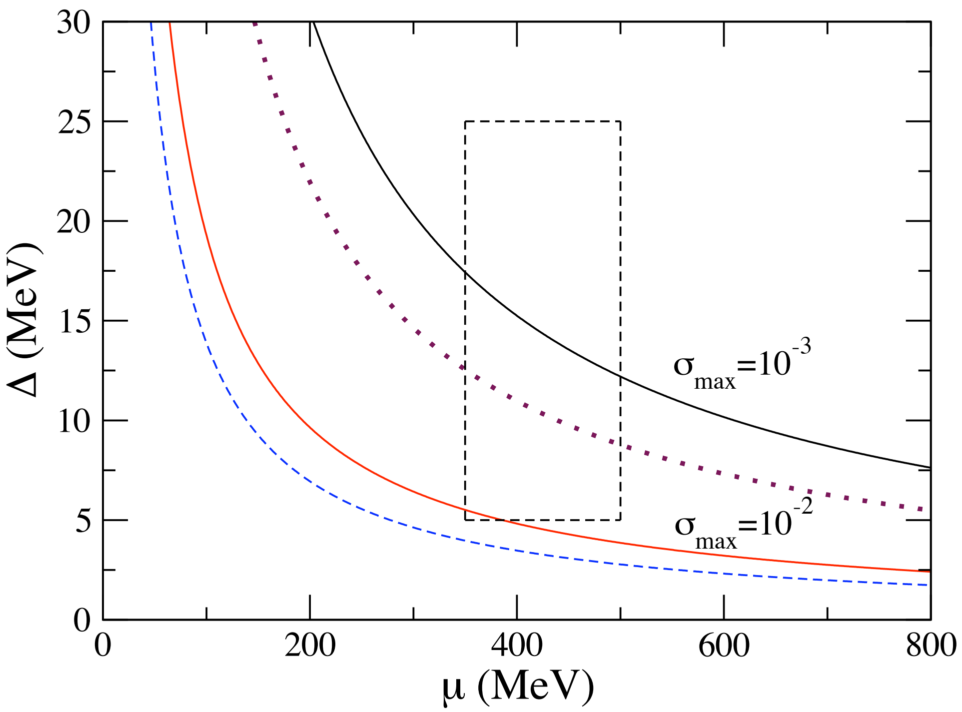
$$h = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{r}$$

- The deformation can arise in the crust or in the core
- Deformation depends on the breaking strain and the shear stress

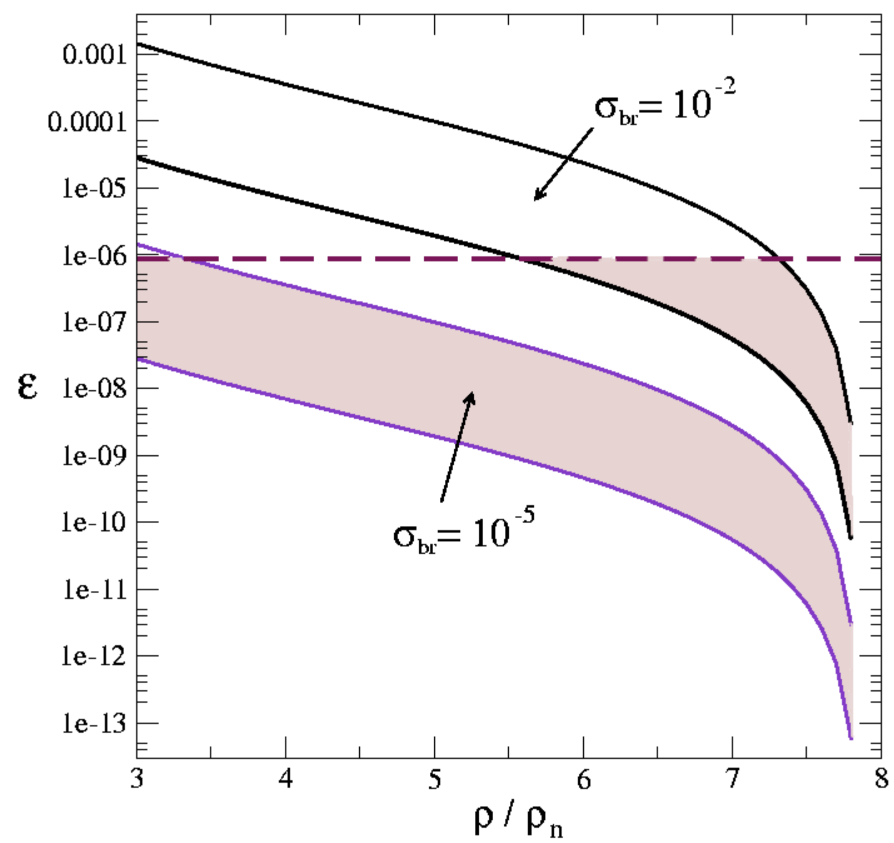
To have a "large" GW amplitude	• Large shear modulus • Large breaking strain

Gravitational waves from mountains

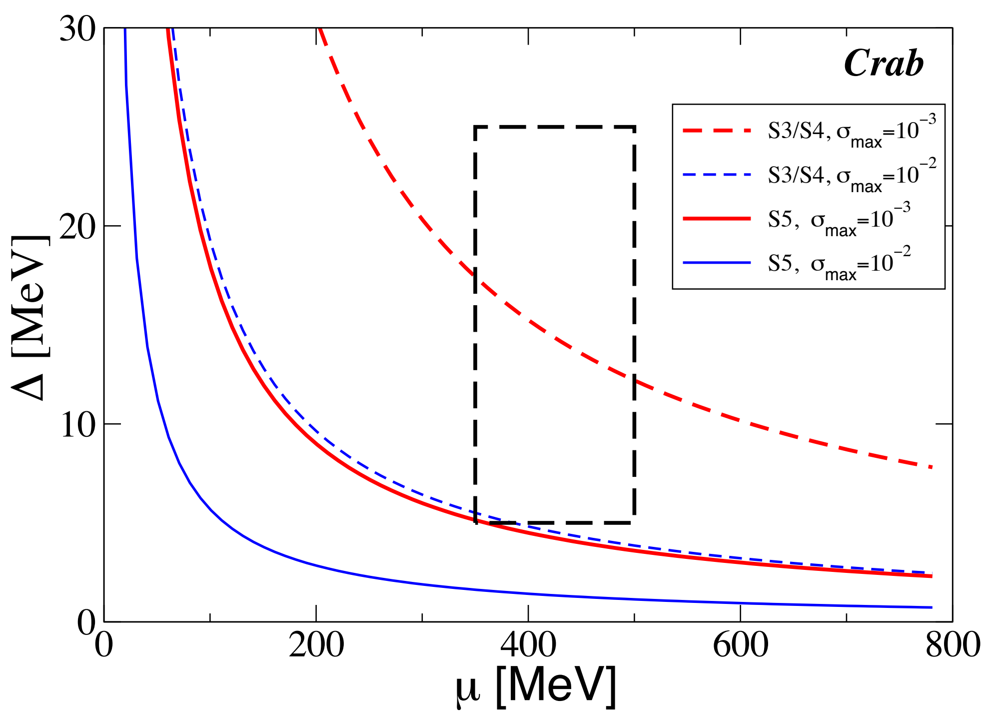
Exclusion plots from no GW signal from the Crab



Lap-Ming Lin, Phys.Rev. D76 (2007) 081502



Andersson et al. Phys.Rev. Lett.99. 231101 (2007)



MM et al.
Review of Modern Physics 86, 509 (2014)

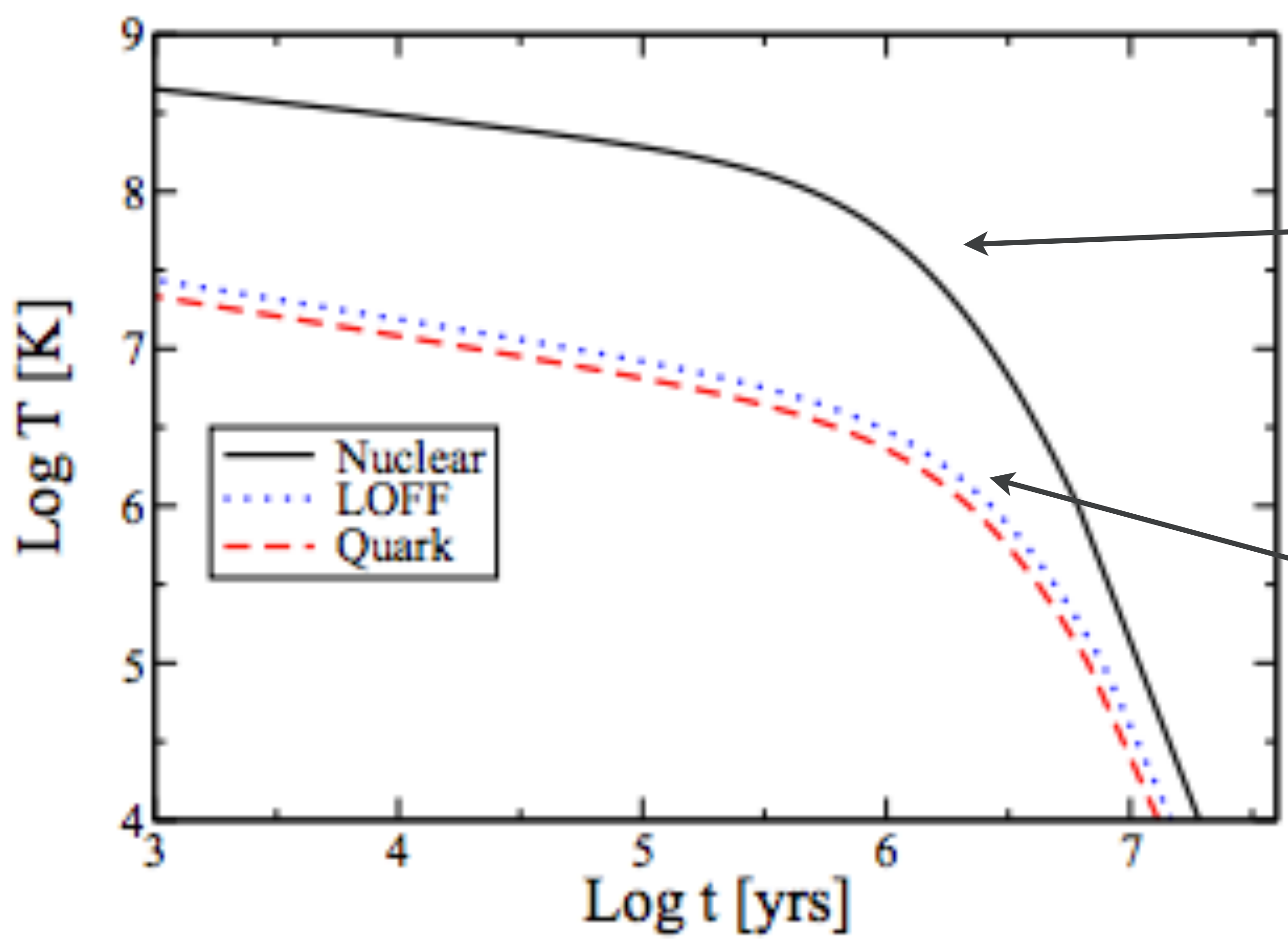
Unfortunately the results depend on

1. The breaking strain, which is not known
2. Whether the magnetic field (or any other agent) can maximally deform the star

Cooling

Since the crystalline phase has gapless modes it can quickly cool by direct Urca processes

Toy model calculations using uncompressible star models



$M = 1.4 M_{\odot}$

$R = 12\text{km}$

Hybrid stars

Core $R_1 = 5\,\text{km}$

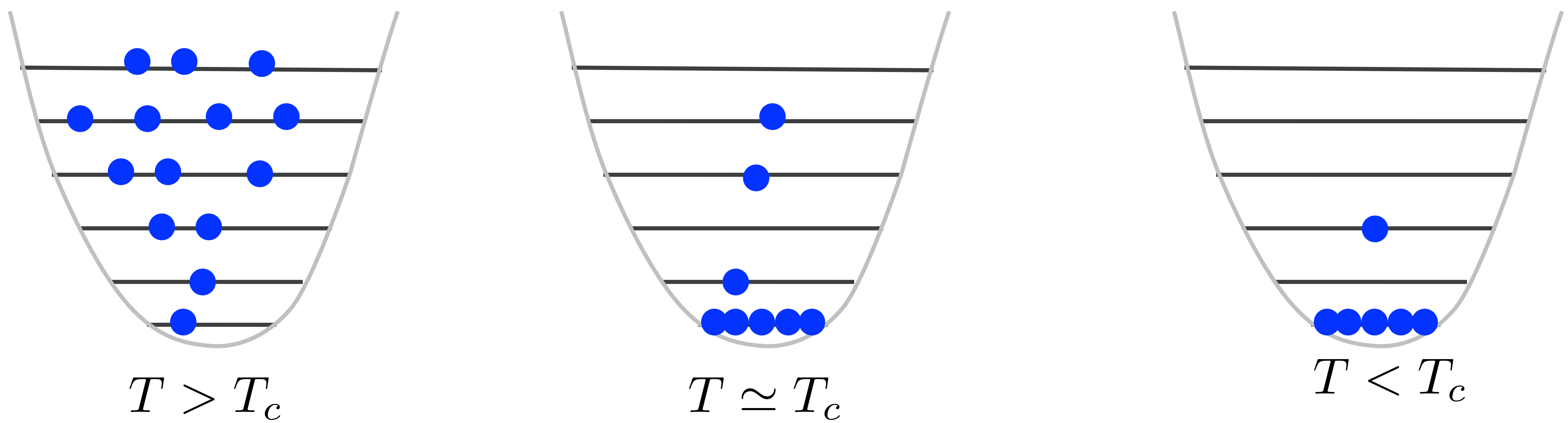
Anglani, MM, Nardulli, Ruggieri Phys.Rev.D 74 (2006) 074005

Meson condensation and compact stars

Bose-Einstein condensation

The Bose-Einstein condensate (BEC) is a **coherent state of matter.**
A "thermodynamically" large number of particles occupy the same quantum state

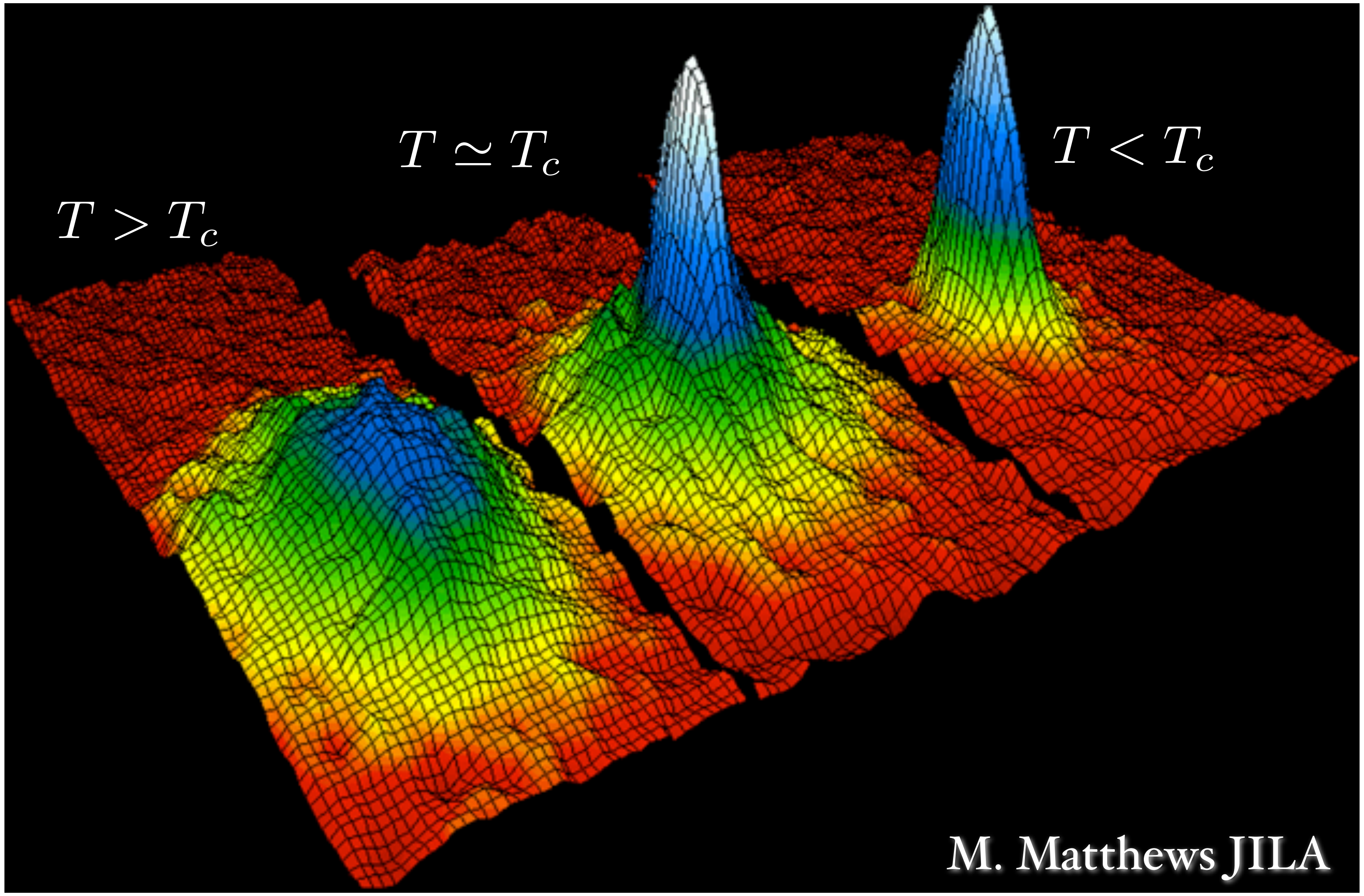
BOSONS@ low temperature in an harmonic potential



Requirements:

1. Particles must be **bosons** or boson-like, e.g. Cooper pairs in BCS
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

Ultracold atoms in an optical trap

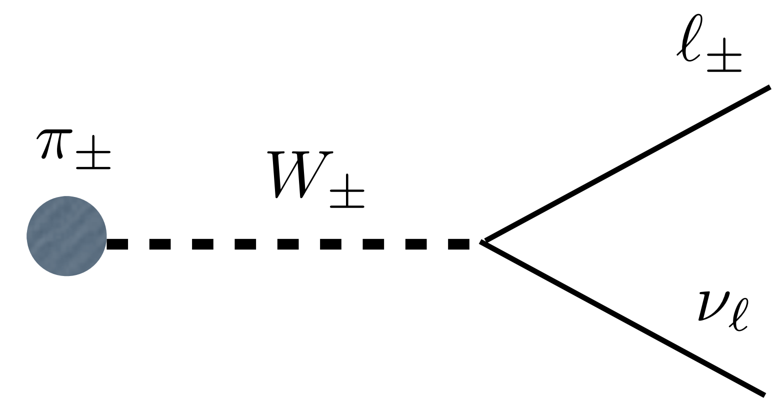


Velocity distribution of ^{87}Rb atoms $\qquad$ $T_c \simeq 200$ nK

1. ^{87}Rb is **bosonic**
2. can be **cooled**
3. has a lifetime of about **10^{10} years** (the experiment lasts $\sim 10^3$s)

Pion decay

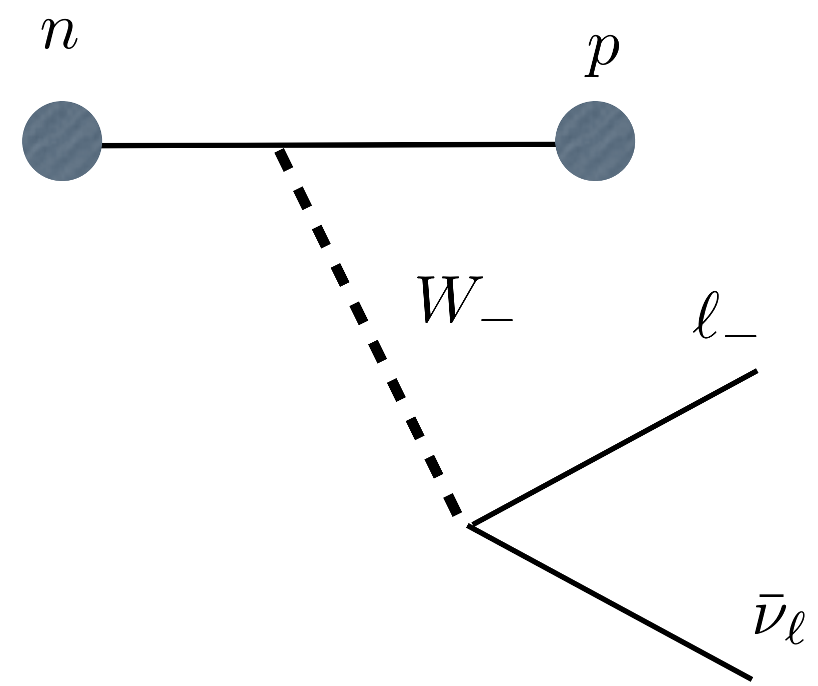
1. Pions are **bosons**
2. Can be produced at **low temperature** in compact stars
3. $\pi^{\pm}$ has a lifetime of about 10^{-8} s



It seems hard to have degenerate pions...

Similar problem arises with neutrons in vacuum lifetime of about 800 s

In medium neutrons can be "statistically" stabilized by Pauli blocking + charge neutrality



Increasing asymmetry in cold matter

Favored isotopes in the NS crust

Isotope	Z/A	$\rho_t (\mathrm{g/cm^3})$	μ_e (MeV)
^{56}Fe	0.464	7.96×10^6	0.95
^{62}Ni	0.452	2.71×10^8	2.61
^{64}Ni	0.437	1.3×10^9	4.31
^{66}Ni	0.424	1.48×10^9	4.45
^{86}Kr	0.419	3.12×10^9	5.66
^{84}Se	0.405	1.10×10^{10}	8.49
^{82}Ge	0.390	2.80×10^{10}	11.4
^{80}Zn	0.375	5.44×10^{10}	14.1
^{78}Ni	0.359	9.64×10^{10}	16.8
^{126}Ru	0.350	1.29×10^{11}	18.3
^{124}Mo	0.339	1.88×10^{11}	20.6
^{122}Zr	0.328	2.67×10^{11}	22.9
^{120}Sr	0.317	3.79×10^{11}	25.4
^{118}Kr	0.305	4.31×10^{11}	26.2

Neutron rich matterinside a NS

many electrons

neutron drip

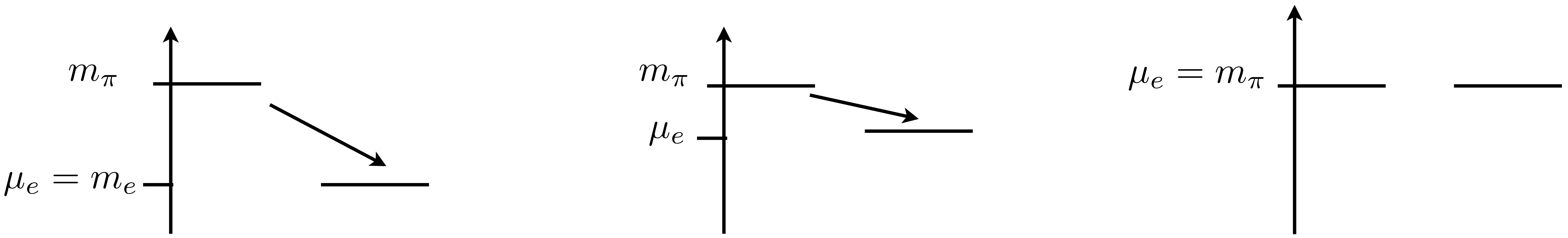
Haensel and Pichon
Astron.Astrophys. 283 (1994) 313

When there are many unbound neutrons $Z/A \sim 0.1$

In medium pions

The decay is Pauli blocked if $\mu_e > m_\pi$ and the pion becomes stable

Increasing electron density



A similar effect can stabilize kaons if the kaon effective mass is below the kinematic threshold for decay.

Caveats

- Need to take into account electromagnetic interactions
- Larger effective pion mass from strong interaction with baryons
- Fight between hyperons and pions

Some general considerations on meson condensation

Effect of the isospin

Mesons

Energy spectrum splitting
Stark-like effect

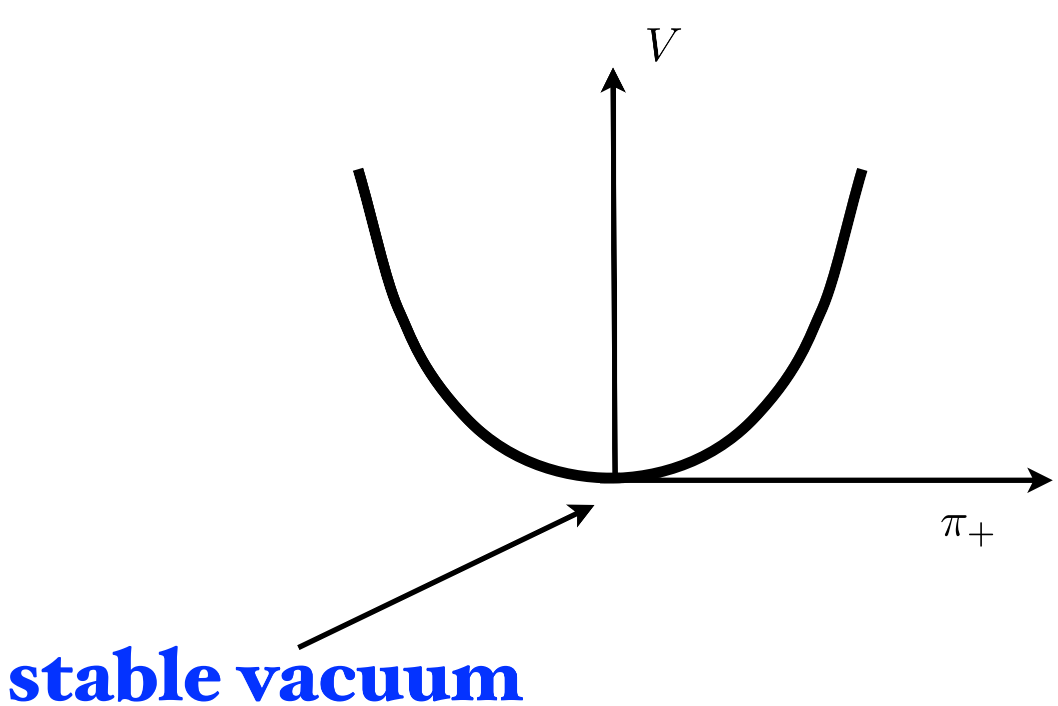
$$E_{\pi^0} = \sqrt{m_\pi^2 + p^2}$$
$$E_{\pi^-} = +\mu_I + \sqrt{m_\pi^2 + p^2}$$
$$E_{\pi^+} = -\mu_I + \sqrt{m_\pi^2 + p^2}$$

$$m_{\pi^+}^{\text{eff}} = m_\pi - \mu_I$$

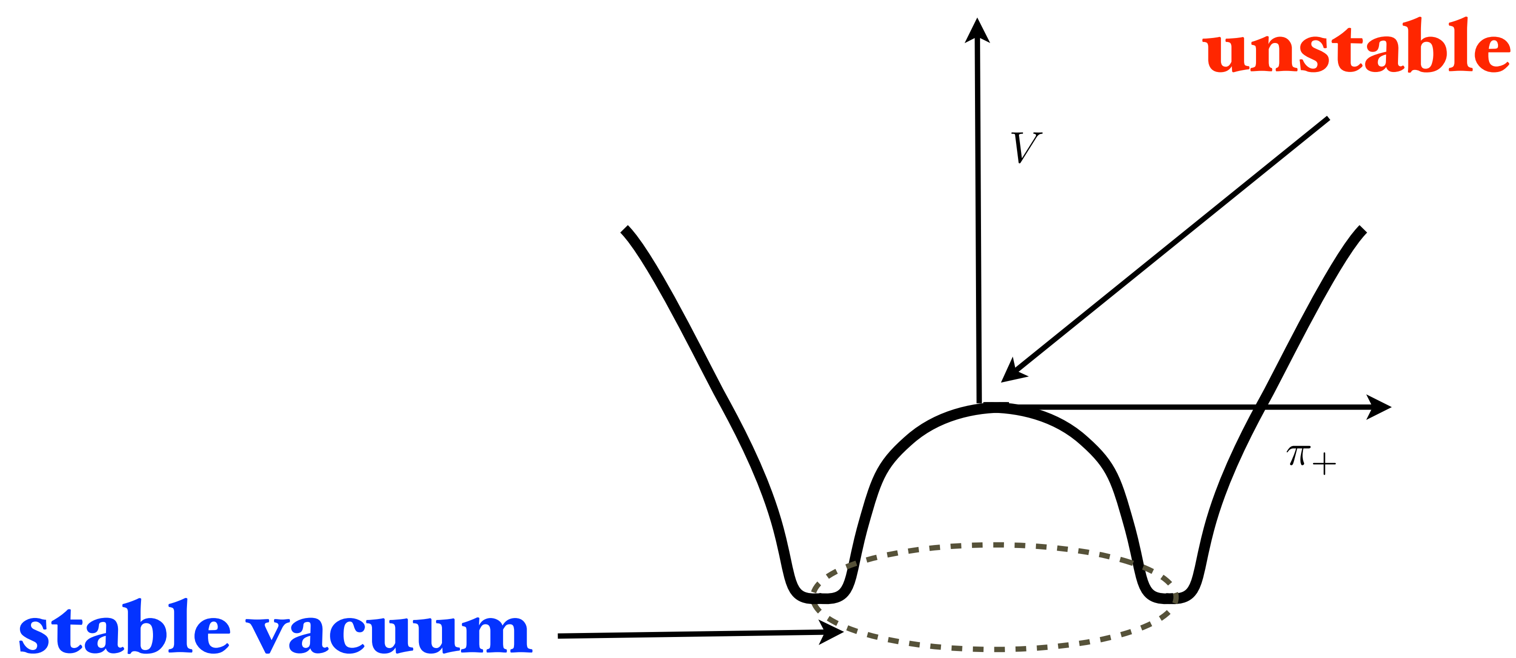
What happens for $\mu_I > m_\pi$?

A vanishing "effective mass" may imply the onset of an instability because $(m_\pi^{\text{eff}})^2 \sim \frac{\partial^2 V}{\partial \pi^2}$

$\mu_I < m_\pi$



$\mu_I > m_\pi$



Effect of the isospin and strangeness

Both strangeness and isospin effectively split the energy spectrum

$$m_{\pi^0} = m_\pi \,,$$

$$m_{\pi^\pm} = m_\pi \mp \mu_I \,,$$ ← **Charged pions splitting**

$$m_\eta = \sqrt{\frac{4m_K^2 - m_\pi^2}{3}} \,,$$

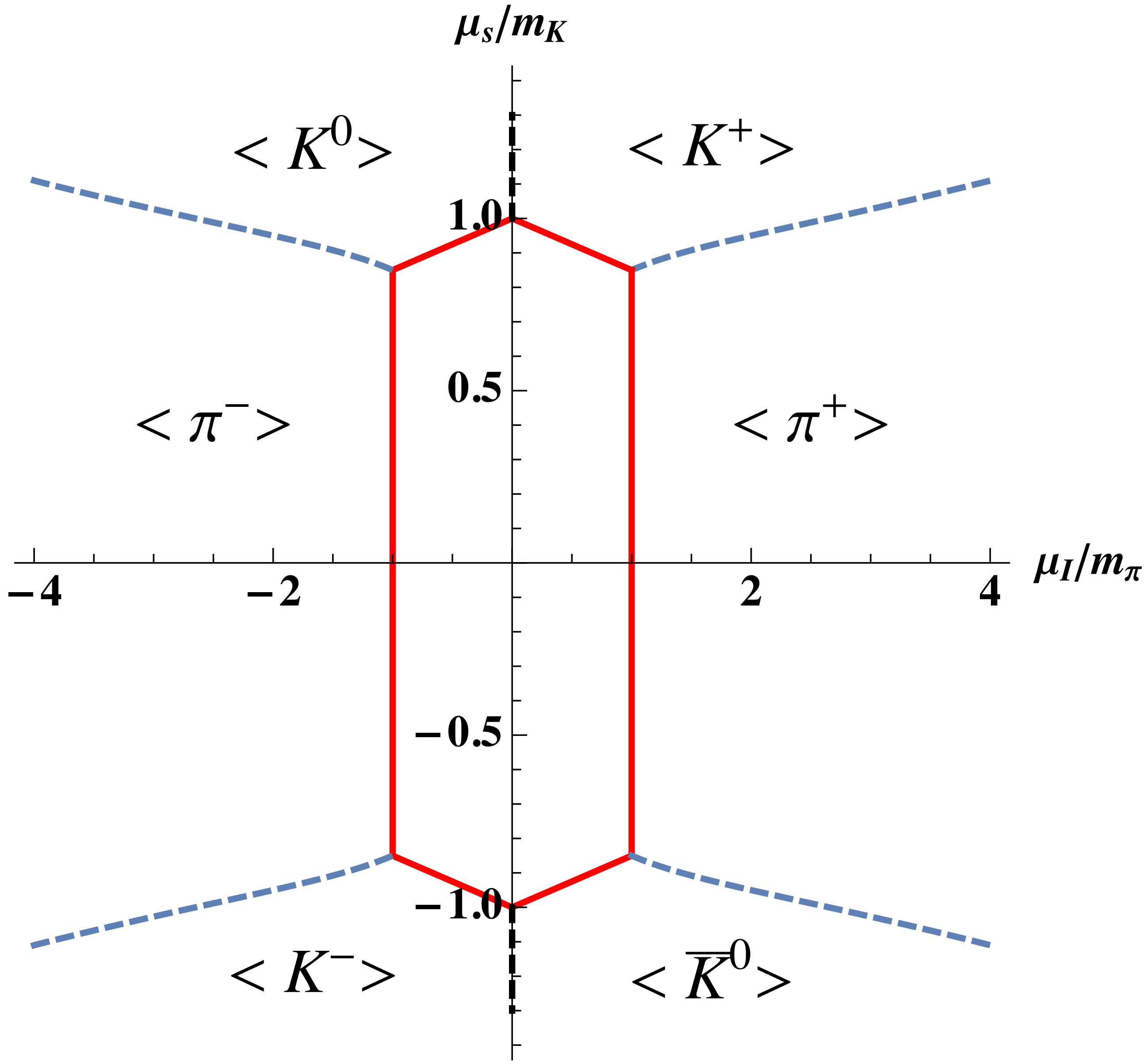
$$m_{K^\pm} = m_K \mp \frac{1}{2}\mu_I \mp \mu_S \,,$$ ← **Charged kaons splitting**

$$m_{K^0/\bar{K}^0} = m_K \pm \frac{1}{2}\mu_I \mp \mu_S \,,$$ ← **Neutral kaons splitting**

A vanishing "effective mass" implies the onset of an instability

Phase diagram

Plotting the lines of vanishing mass and considering the mode that becomes massless, we find...



solid lines: second order

dotted lines: first order (competition between different condensates)

Kogut and Toublan PhysRevD.64.034007

Chiral perturbation theory
(χPT)

“Low momentum” EFT

QCD is a confining and strongly interacting theory.
χPT is a realisation of hadronic matter at soft energy scales

$$p \ll \Lambda_\chi \sim 1\,\mathrm{GeV}$$

Qualitative recipe

Variationally derive the **nonperturbative vacuum** and **expand** around that vacuum for small momenta.

Since you are expanding, you have **control parameters**

We do not include baryons and vector mesons

$$|\mu_B| \lesssim 940 \text{ MeV} \qquad |\mu_I| \lesssim 770 \text{ MeV}$$

Leading order pion Lagrangian

The $\mathcal{O}(p^2)$ Lorentz invariant Lagrangian density for pseudoscalar mesons

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger) + \mathrm{Tr}(M\Sigma^\dagger + M^\dagger \Sigma)$$

low energy constants (LECs)

Meson field

Radial field

For SU(2)

$$\Sigma = \cos\rho + i\hat{\varphi}\cdot\boldsymbol{\sigma}\sin\rho$$

"Angular" field

Pauli matrices

A general $\mathrm{SU}(2)$ static and homogeneous vev

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

variational parameters

Static Lagrangian

$$\mathcal{L}_0(\alpha, \mu_I, n_3) = F_0^2 m_\pi^2 \cos\alpha + \frac{F_0^2}{2}\mu_I^2 \sin^2\alpha(1 - n_3^2)$$

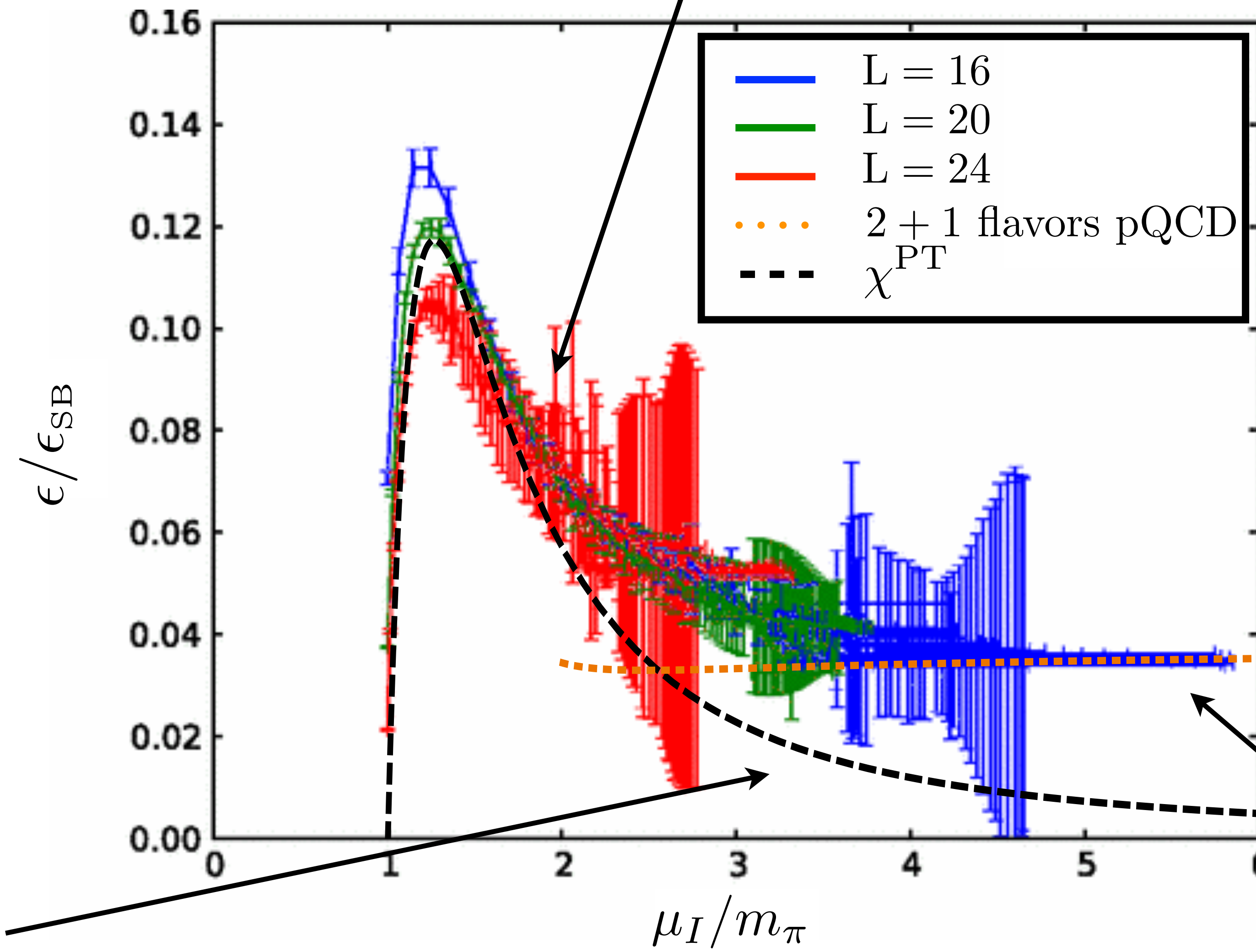
Maximising the Lagrangian

for $\mu_I < m_\pi$	$\cos\alpha = 1$	$\mathcal{L}_0$ independent of $\boldsymbol{n}$
for $\mu_I > m_\pi$	$\cos\alpha_\pi = m_\pi^2/\mu_I^2$	$n_3 = 0$ residual $O(2)$ symmetry

The vacuum has been tilt in some direction in isospin space

Results for the energy density

Lattice QCD simulations
W. Detmold, K. Orginos, and Z. Shi, Phys. Rev. D86, 054507 (2012)



$$\epsilon_{SB} = \frac{N_c N_f}{4\pi^2}\mu_I^4$$

factor $\sim \frac{1}{16}$ missing

χPT
S. Carignano, A. Mammarella, MM Phys.Rev. D93 (2016) no.5, 051503

pQCD
T. Graf, et al. Phys. Rev. D 93, 085030 (2016)

χPT gives an ANALYTIC expression for the peak

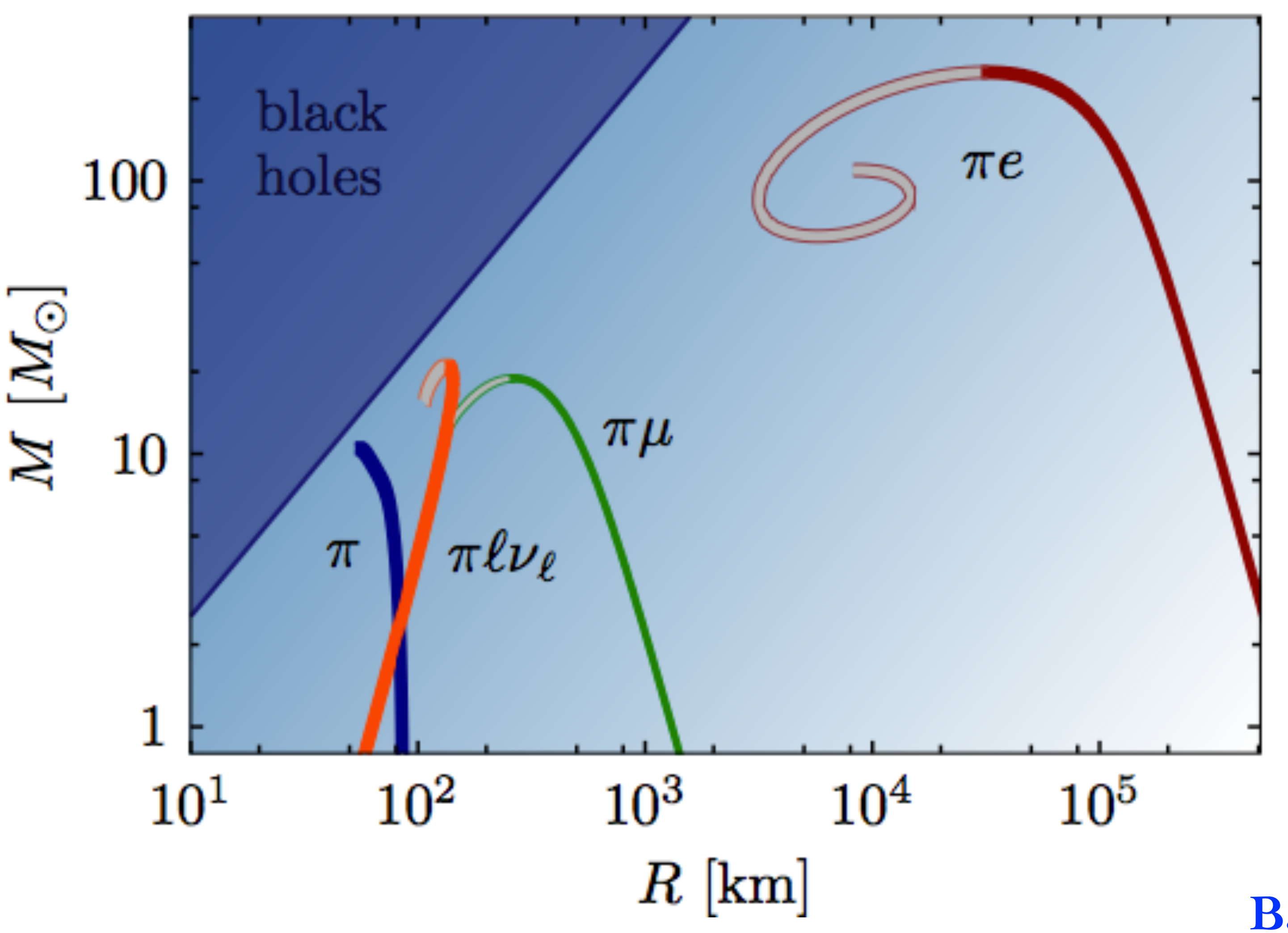
$$\mu_{I,\mathrm{LQCD}}^{\mathrm{peak}} = \{1.20, 1.25, 1.275\} m_\pi \qquad \mu_{I,\chi\mathrm{PT}}^{\mathrm{peak}} = (\sqrt{13}-2)^{1/2} m_\pi \simeq 1.276\, m_\pi$$

Pion stars

Pion stars

Since the pion condensed phase is stable, it is possible to form a star entirely made of pions

MM et al. Eur.Phys.J.A 53 (2017) 2, 35



B. Brandt et al.
Phys.Rev.D 98 (2018) 9, 094510

Summary and outlook

- Compact stars are excellent matter squeezers

- Exotic phases are intriguing, if quark matter is deconfined some nontrivial phase could appear

- We have to be patient, not easy to rule in/out models

- Try to make predictions and compare with observations