Have we finally cracked the nuclear force problem?

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INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

N-N Interaction & the Nuclear Many-Body Problem

November 18-27, 2010

Tata Institute of Fundamental Research, Mumbai 400 005

Invited speakers
H. Arenhovel (Mainz)
D. Bandyopadhyay (SINP)
S. Beane (New Hampshire)
R.K. Bhaduri (McMaster)
V.S. Bhasin (Delhi)
S. Gupta (TIFR)
A.K. Jain (IIT-Roorkee)
R.C. Johnson (Surrey)
S. Kailas (BARC)
N. Kalantar (KVI)
St. Kistryn (Jagiellonian)
T. Kuo (Stony Brook)
R. Machleidt (Idaho)
A.N. Mitra (Delhi)
E. Piasetzky (Tel Aviv)
A.R.P. Rau (Louisiana)
S. Reddy (Los Alamos)
J.-M. Richard (Grenoble)
S.K. Saha (Bose Inst.)
R. Shyam (SINP)
H. Shimizu (Tohoku)
R. Schwengner (FZR-Dresden)
A.W. Thomas (Adelaide)
J. Vary (Iowa)
H. Weller (North Carolina)

Topics
- N-N interaction
- Few-body problems in nuclear physics
- Modern shell model Calculations
- Nuclear physics of neutron stars

Organizing Committee:
Rajeev S. Bhalerao, Nilmani Mathur, Indranil Mazumdar, Subrata Pal, Amit Roy, Anthony W. Thomas

Outline

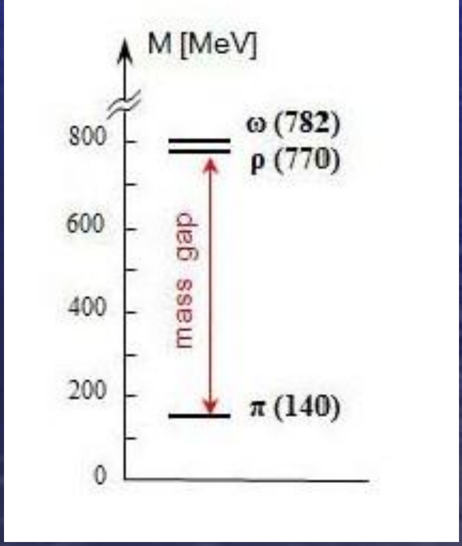
- Historical perspective
- Nuclear forces from chiral EFT: Overview & achievements
- Are we done? No!
- Sub-leading many-body forces
- Proper renormalization of chiral forces
- Have we cracked the problem?

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions $\equiv$ multi-pion resonances: $\sigma, \rho, \omega, \ldots$ The One-Boson-Exchange Model
1970's	Refine meson theory: More sophisticated meson-exchange models (Paris, Bonn, Williamsburg)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Meson Theory!** *But, with Chiral Symmetry*

From QCD to nuclear physics via chiral EFT (in a nutshell)

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales



- **Calls for an EFT**

  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$; pions and nucleon relevant d.o.f.
- **Low-energy expansion: $(Q/\Lambda_\chi)^\nu$**

  with ν bounded from below.
- **Most general Lagrangian consistent with all symmetries of low-energy QCD.**
- **π-π and π-N perturbatively**
- **NN has bound states:**

  (i) NN potential perturbatively

  (ii) apply nonpert. in LS equation.

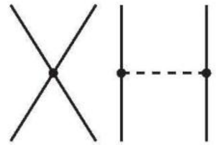
  (Weinberg)

The Hierarchy of Nuclear Forces

2N forces | **3N forces** | **4N forces**

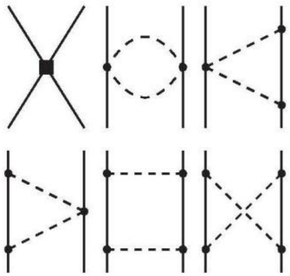
Leading Order

Q^0
LO



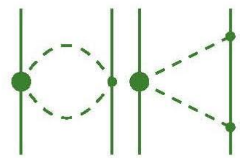
Next-to Leading Order

Q^2
NLO



Next-to-Next-to Leading Order

Q^3
N^2LO



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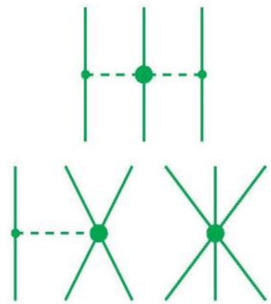
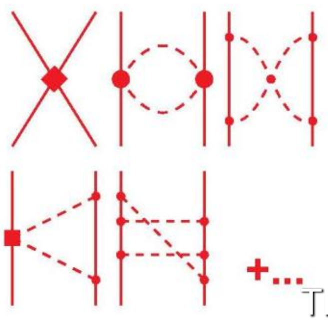
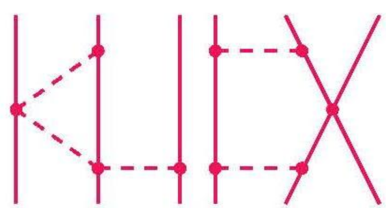


Next-to-Next-to-Next-to Leading Order

Q^4
N^3LO





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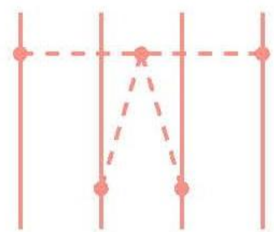
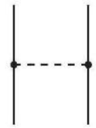
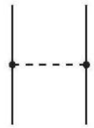
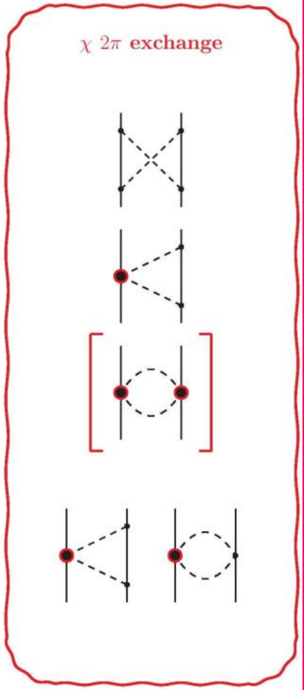
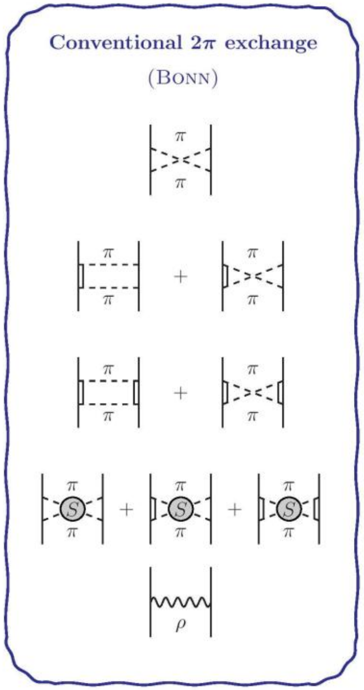
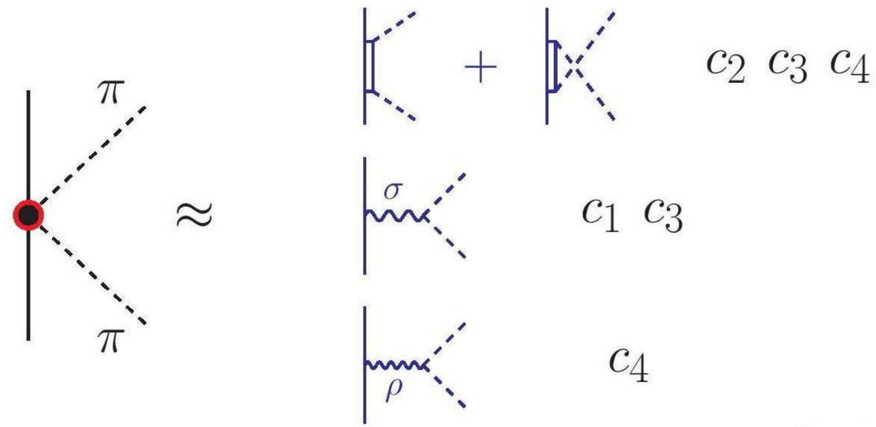


+...

	ChPT	Conventional meson theory
OPE		
TPE		

pi-N Lagrangian with two derivatives ("next-to-leading" order)

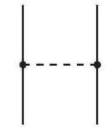
$$
\begin{aligned}
\mathcal{L}^{(2)}_{\pi N, c_i} = \; & \bar{N} \Big[2\, c_1\, m_\pi^2\, (U + U^\dagger) \\
& + \left(c_2 - \frac{g_A^2}{8 M_N} \right) u_0^2 \\
& + c_3\, u_\mu\, u^\mu \\
& + \frac{i}{2} \left(c_4 + \frac{1}{4 M_N} \right) \vec{\sigma} \cdot (\vec{u} \times \vec{u}) \Big] N
\end{aligned}
$$

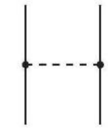


Bernard et al. '97

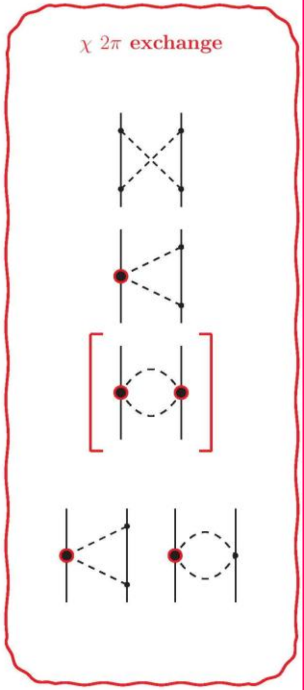
	ChPT	Conventional meson theory

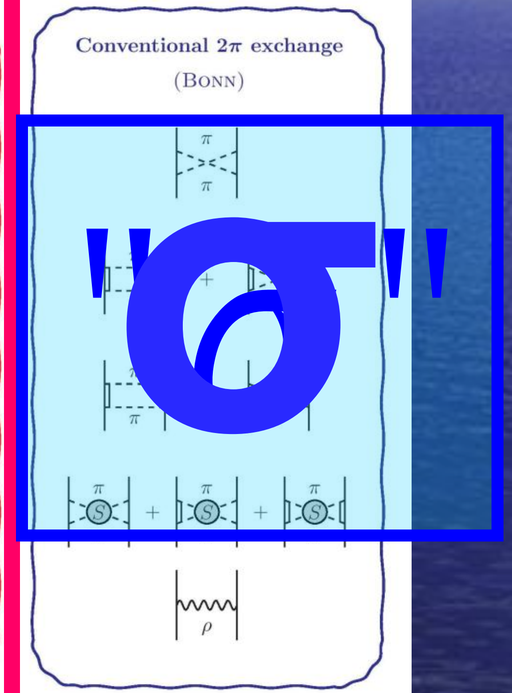
OPE



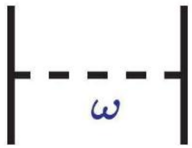


TPE



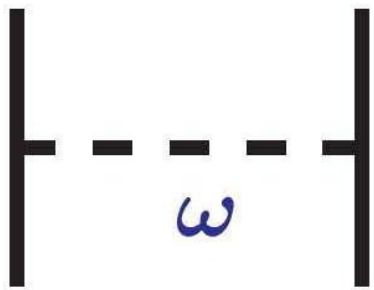


Short range

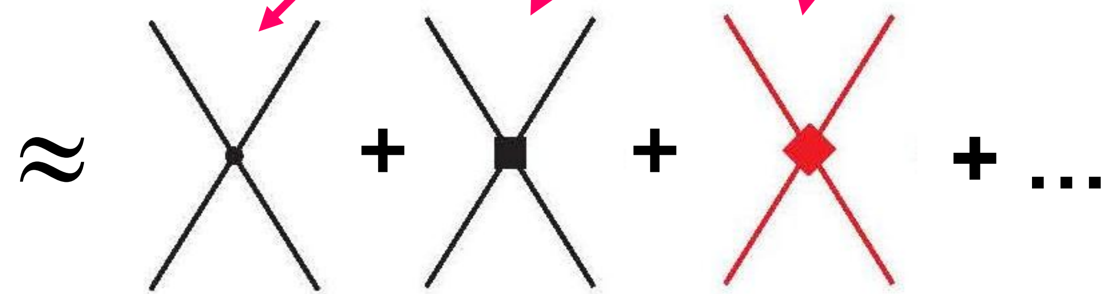


Resonance Saturation

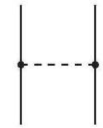
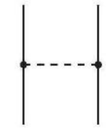
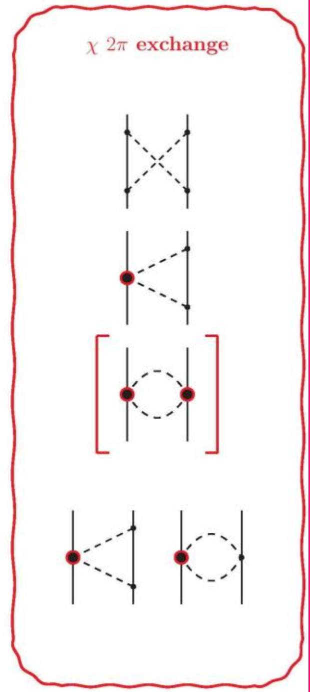
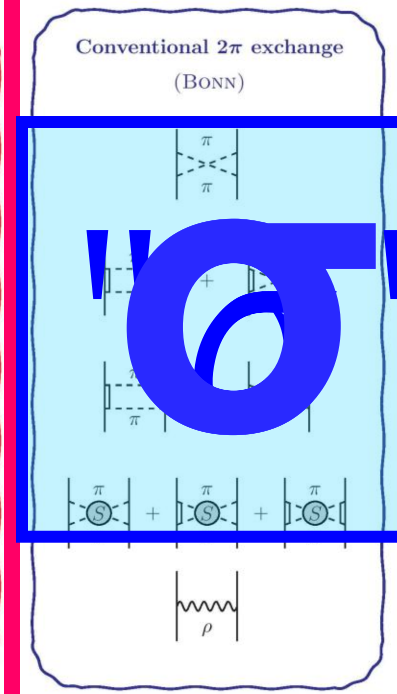
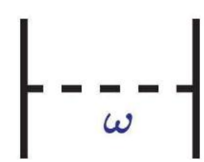
Consider the contribution from the exchange of a heavy meson



$$\frac{1}{m_\omega^2 + Q^2} \approx \frac{1}{m_\omega^2}\left[1 - \frac{Q^2}{m_\omega^2} + \frac{Q^4}{m_\omega^2} - \dots\right.$$



“Contact terms”

	ChPT	Conventional meson theory
OPE		
TPE		
Short range		

Resonance Saturation

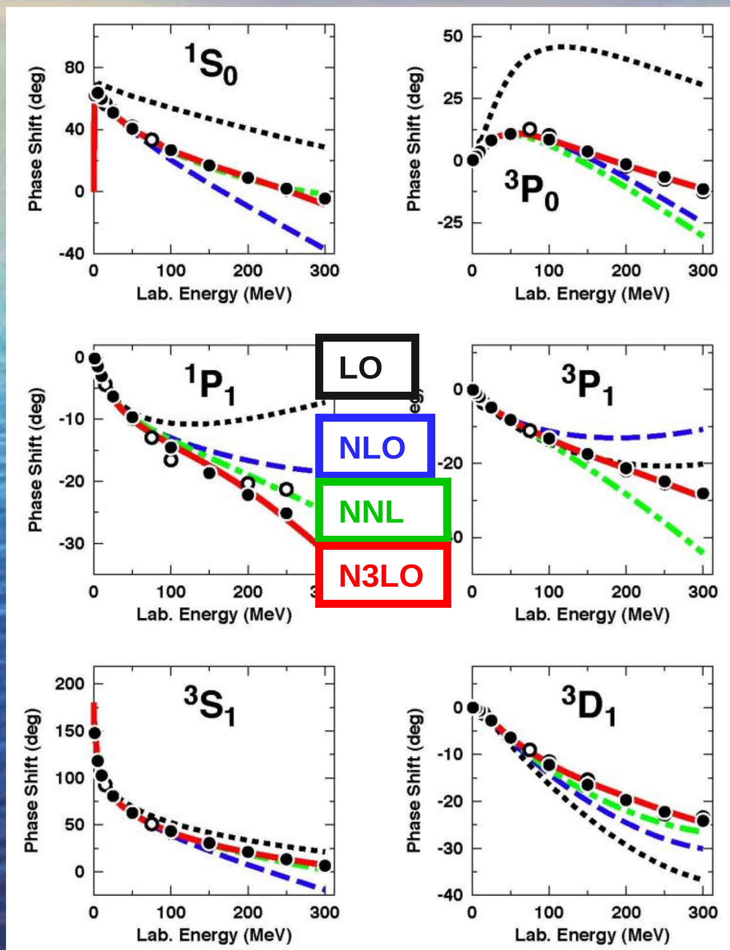
Question: When everything is so equivalent to conventional meson theory, why not just use meson theory?

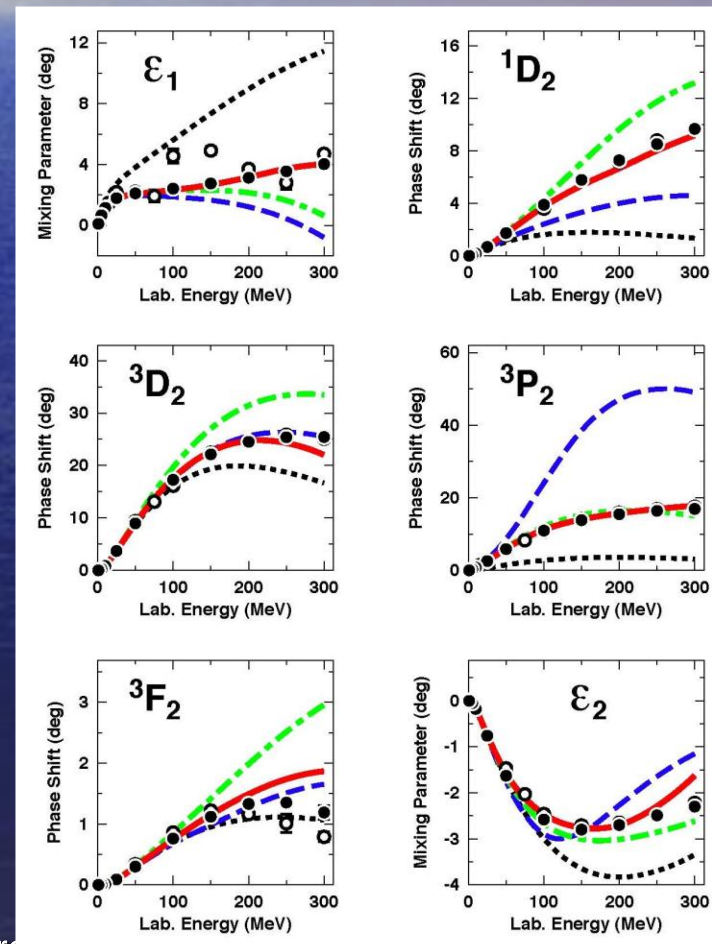
Answer: In ChPT, there is an organizational scheme ("power counting") that allows to estimate the size of the various contributions and the uncertainty at a given order (i.e., the size of the contributions we left out). Moreover, two- and many-body force contributions are generated on an equal footing.

In conventional meson theory, we go by range.

NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
Green dash-dotted line: NNLO Potential, and
blue dashed line: NLO Potential
by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).





χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Applications of the chiral NN potential at N3LO

PRL **101,** 092502 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions

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[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[3]*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway*
(Received 20 June 2008; published 29 August 2008)

We compute the binding energies, radii, and densities for selected medium-mass nuclei within coupled-cluster theory and employ a bare chiral nucleon-nucleon interaction at next-to-next-to-next-to-leading order. We find rather well-converged results in model spaces consisting of 15 oscillator shells, and the doubly magic nuclei ^{40}Ca, ^{48}Ca, and the exotic ^{48}Ni are underbound by about 1 MeV per nucleon within the coupled-cluster singles-doubles approximation. The binding-energy difference between the mirror nuclei ^{48}Ca and ^{48}Ni is close to theoretical mass table evaluations. Our computation of the one-body density matrices and the corresponding natural orbitals and occupation numbers provides a first step to a microscopic foundation of the nuclear shell model.

Chiral NN potential at N^3LO underbinds by ~1MeV/nucleon. (Size extensivity at its best.)

Nucleus	**ΔE / A [MeV]**
^{4}**He**	**1.08 (0.73^{FY})**
^{16}**O**	**1.25**
^{40}**Ca**	**0.84**
^{48}**Ca**	**1.27**
^{48}**Ni**	**1.21**

PHYSICAL REVIEW C **82**, 034330 (2010)

Ab initio coupled-cluster approach to nuclear structure with modern nucleon-nucleon interactions

G. Hagen,[1] T. Papenbrock,[1,2] D. J. Dean,[1] and M. Hjorth-Jensen[3]
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(Received 17 May 2010; revised manuscript received 20 August 2010; published 30 September 2010)

We perform coupled-cluster calculations for the doubly magic nuclei ^{4}He, ^{16}O, ^{40,48}Ca, for neutron-rich isotopes of oxygen and fluorine, and employ "bare" and secondary renormalized nucleon-nucleon interactions. For the nucleon-nucleon interaction from chiral effective field theory at order next-to-next-to-next-to leading order, we find that the coupled-cluster approximation including triples corrections binds nuclei within 0.4 MeV per nucleon compared to data. We employ interactions from a resolution-scale dependent similarity renormalization group transformations and assess the validity of power counting estimates in medium-mass nuclei. We find that the missing contributions from three-nucleon forces are consistent with these estimates. For the unitary correlator model potential, we find a slow convergence with respect to increasing the size of the model space. For the *G*-matrix approach, we find a weak dependence of ground-state energies on the starting energy combined with a rather slow convergence with respect to increasing model spaces. We also analyze the center-of-mass problem and present a practical and efficient solution.

… including the
chiral 3NF
at N2LO

PRL **105,** 032501 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Three-Body Forces and the Limit of Oxygen Isotopes

Takaharu Otsuka,[1,2,3] Toshio Suzuki,[4] Jason D. Holt,[5] Achim Schwenk,[5] and Yoshinori Akaishi[6]

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[4]*Department of Physics, College of Humanities and Sciences, Nihon University, Sakurajosui 3, Tokyo 156-8550, Japan*
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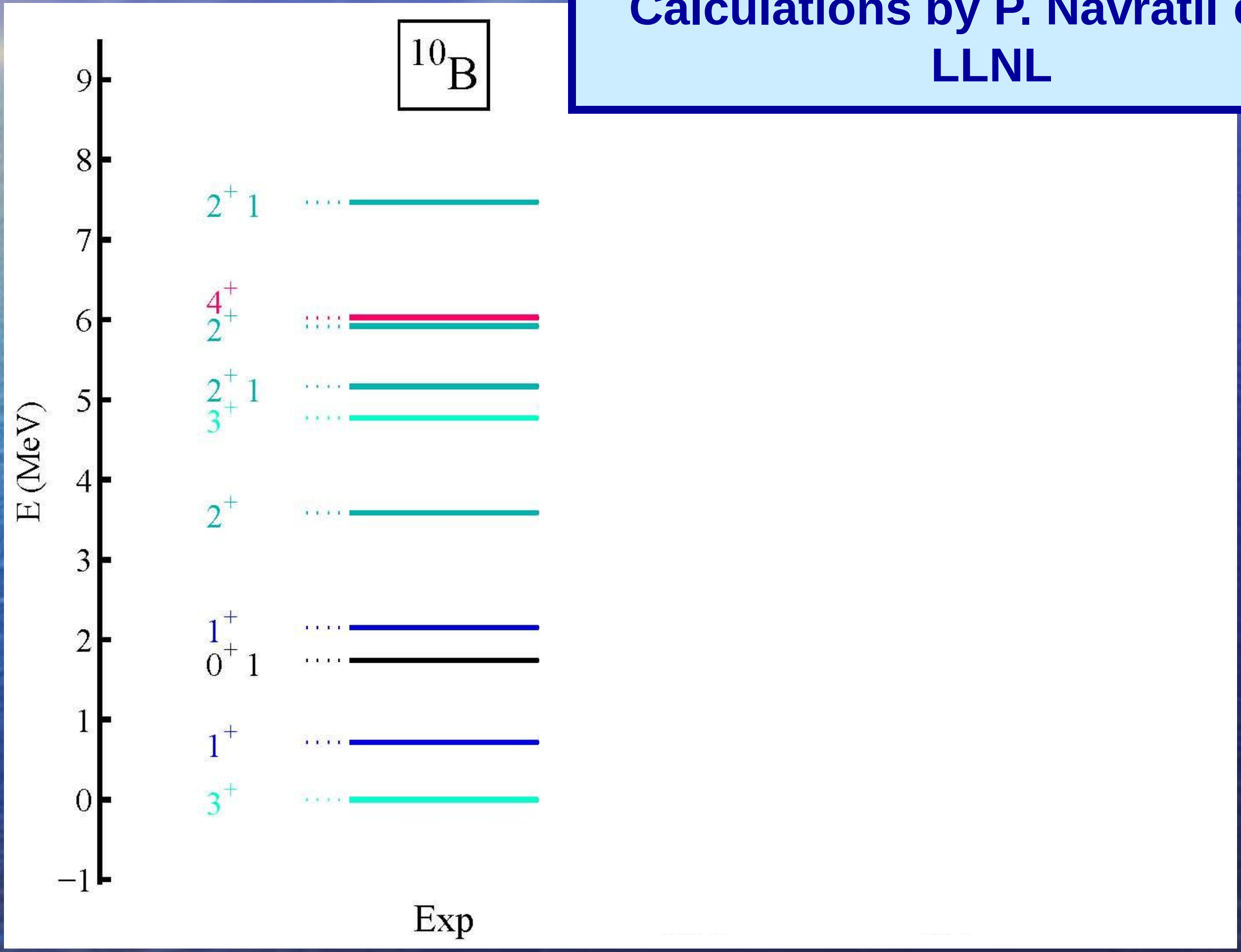
(Received 17 August 2009; published 13 July 2010)

The limit of neutron-rich nuclei, the neutron drip line, evolves regularly from light to medium-mass nuclei except for a striking anomaly in the oxygen isotopes. This anomaly is not reproduced in shell-model calculations derived from microscopic two-nucleon forces. Here, we present the first microscopic explanation of the oxygen anomaly based on three-nucleon forces that have been established in few-body systems. This leads to repulsive contributions to the interactions among excess neutrons that change the location of the neutron drip line from ^{28}O to the experimentally observed ^{24}O. Since the mechanism is robust and general, our findings impact the prediction of the most neutron-rich nuclei and the synthesis of heavy elements in neutron-rich environments.

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



Calculating the properties of light nuclei using chiral 2N and 3N forces

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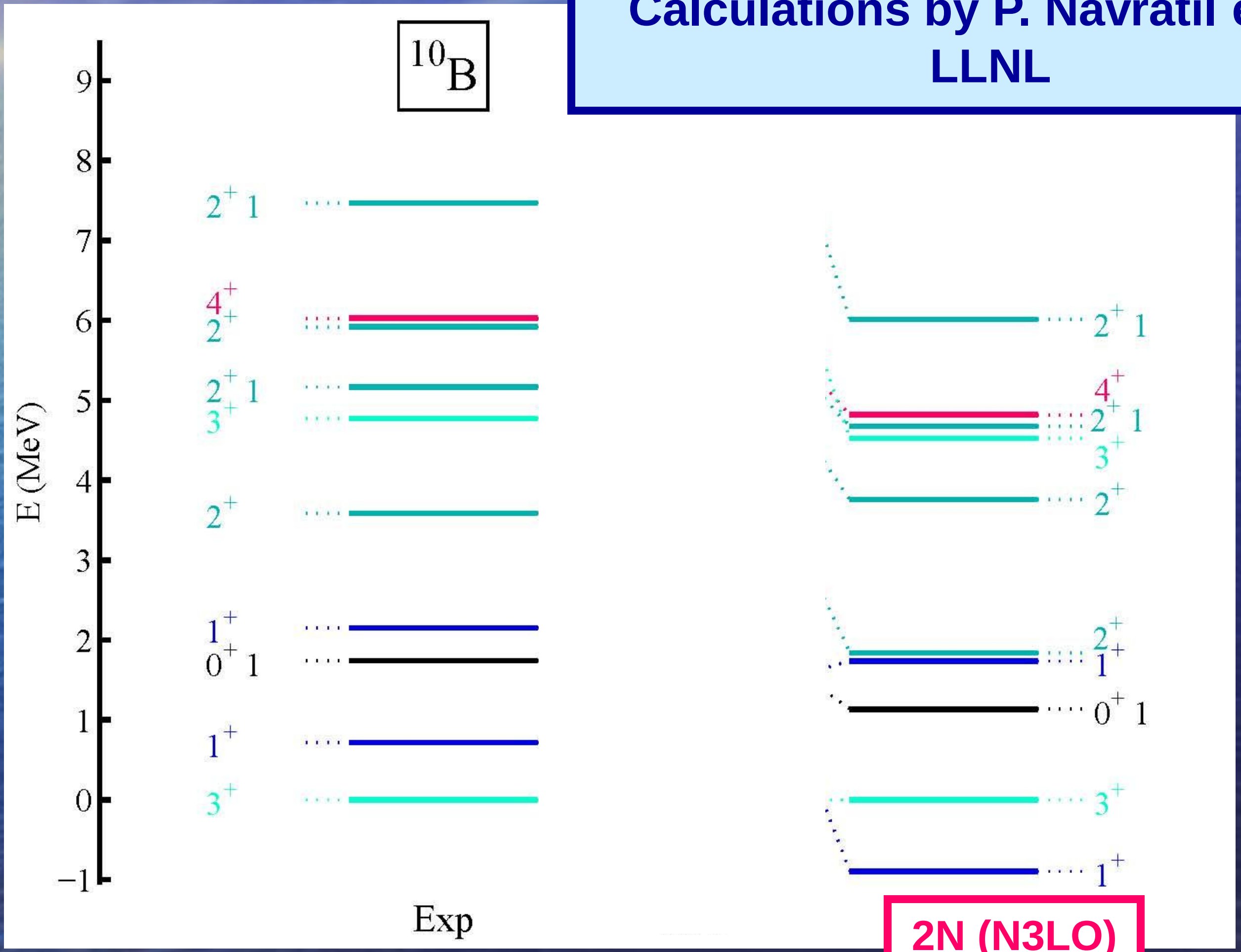
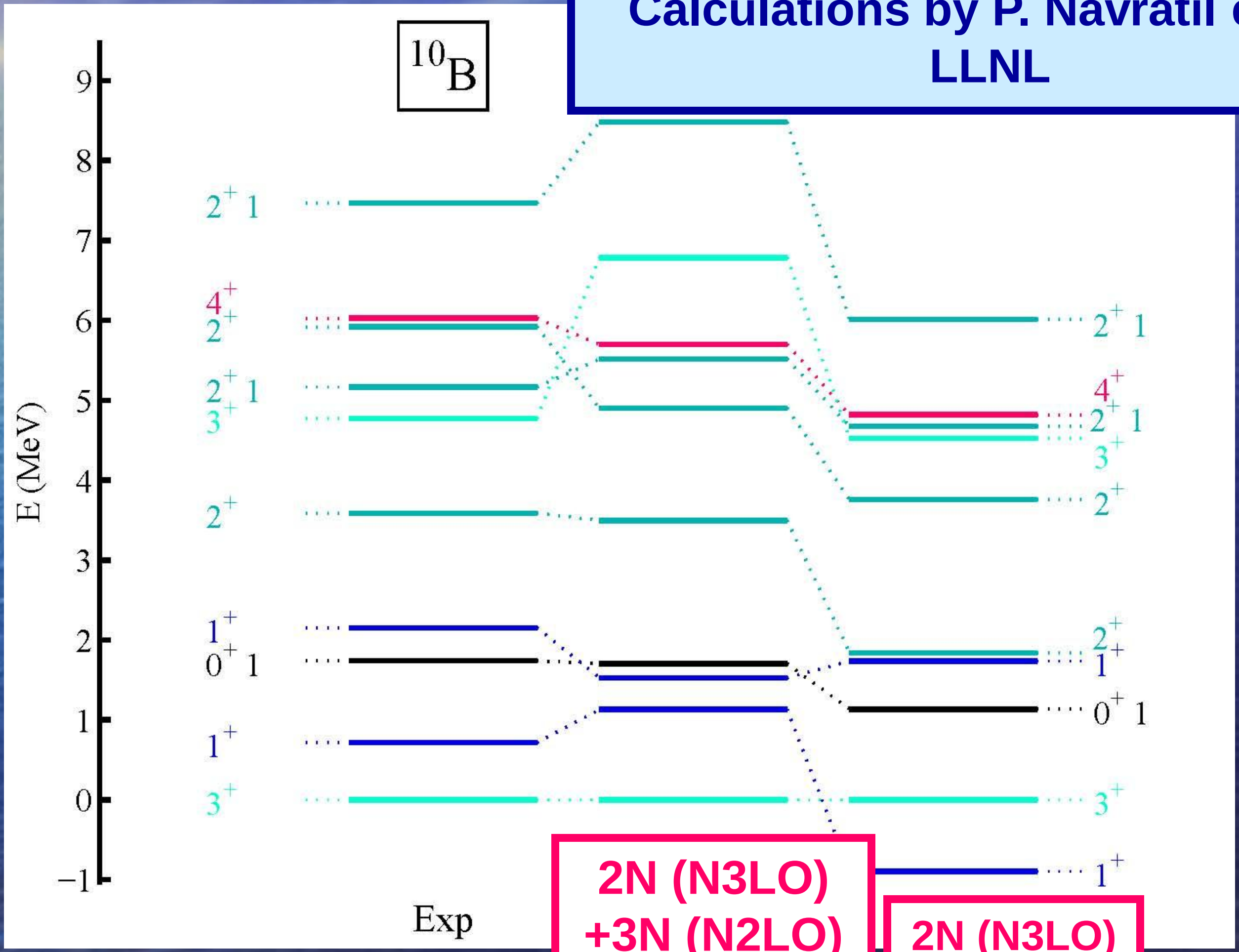


2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



2N (N3LO) +3N (N2LO) forces

2N (N3LO) force only

Analyzing Power Ay

Calculations by the Pisa Group

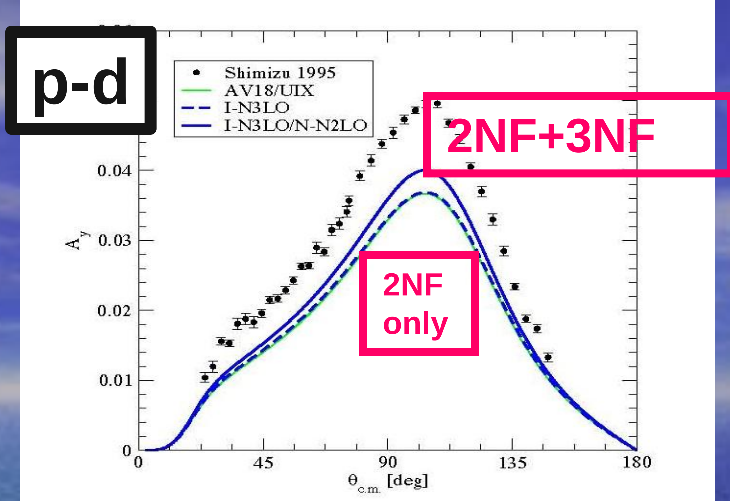


Fig. 9. $p-d$ observable A_y at $E_p = 3$ MeV calculated with the I-N3LO (blue dashed line), I-N3LO/N-N2LO (blue solid line) ... AV18/UIX (thin green solid line) interaction mod... MeV. The experimental data are from Ref. ...

The Ay puzzle is NOT solved by the 3NF at NNLO.

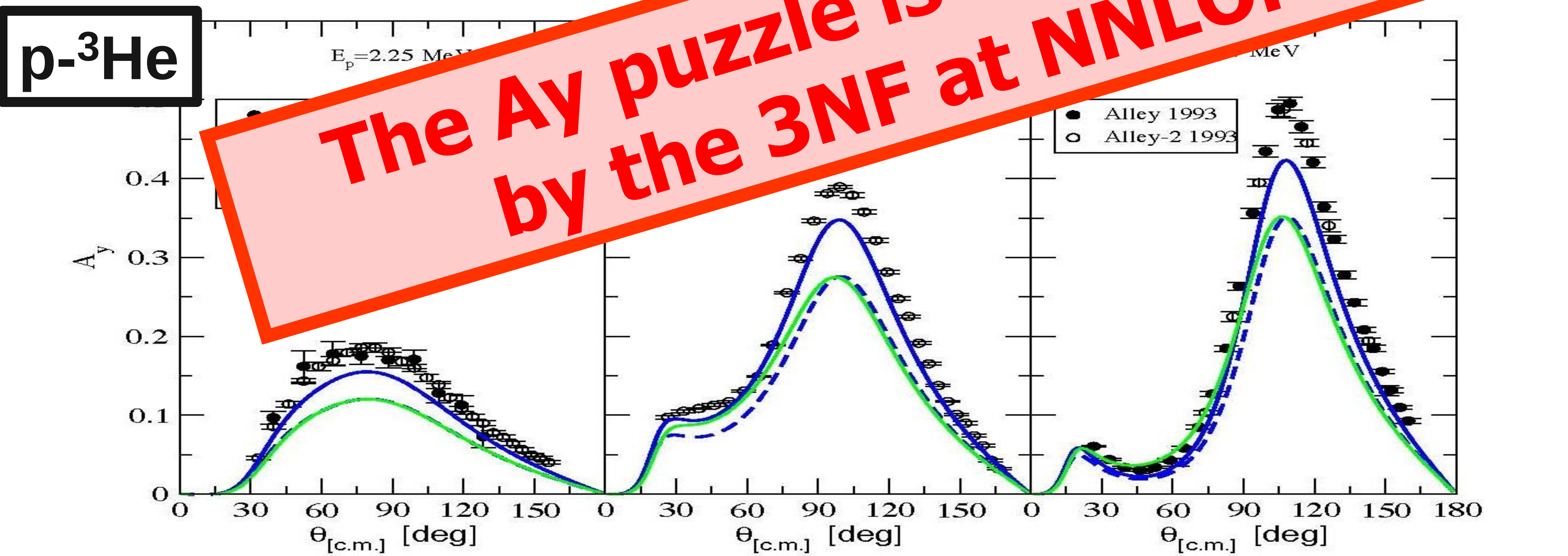
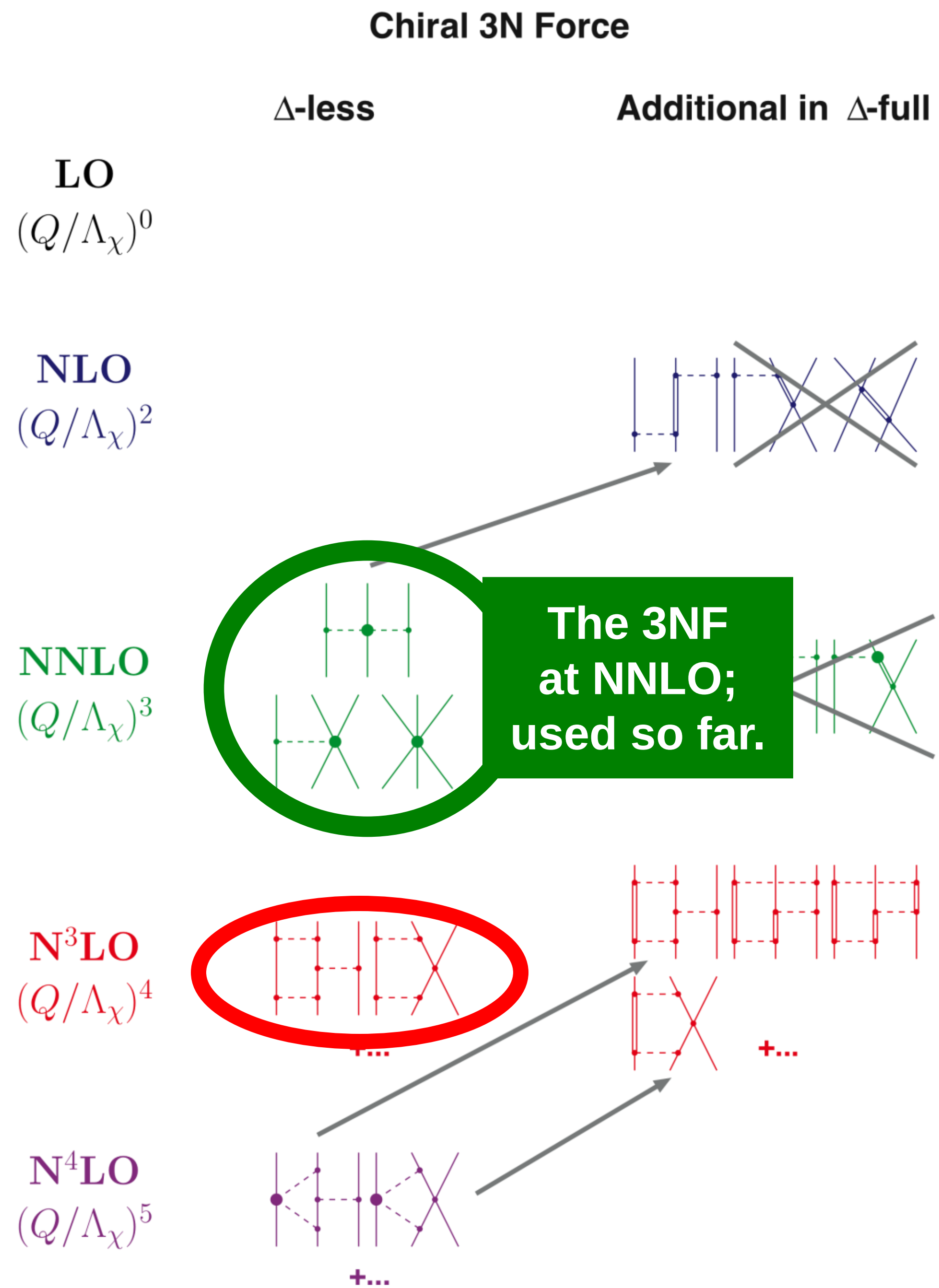


Fig. 6. $p-{}^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

Why do we need 3NFs beyond NNLO?

- The 2NF is N3LO;
  consistency requires that all contributions are at the same order.

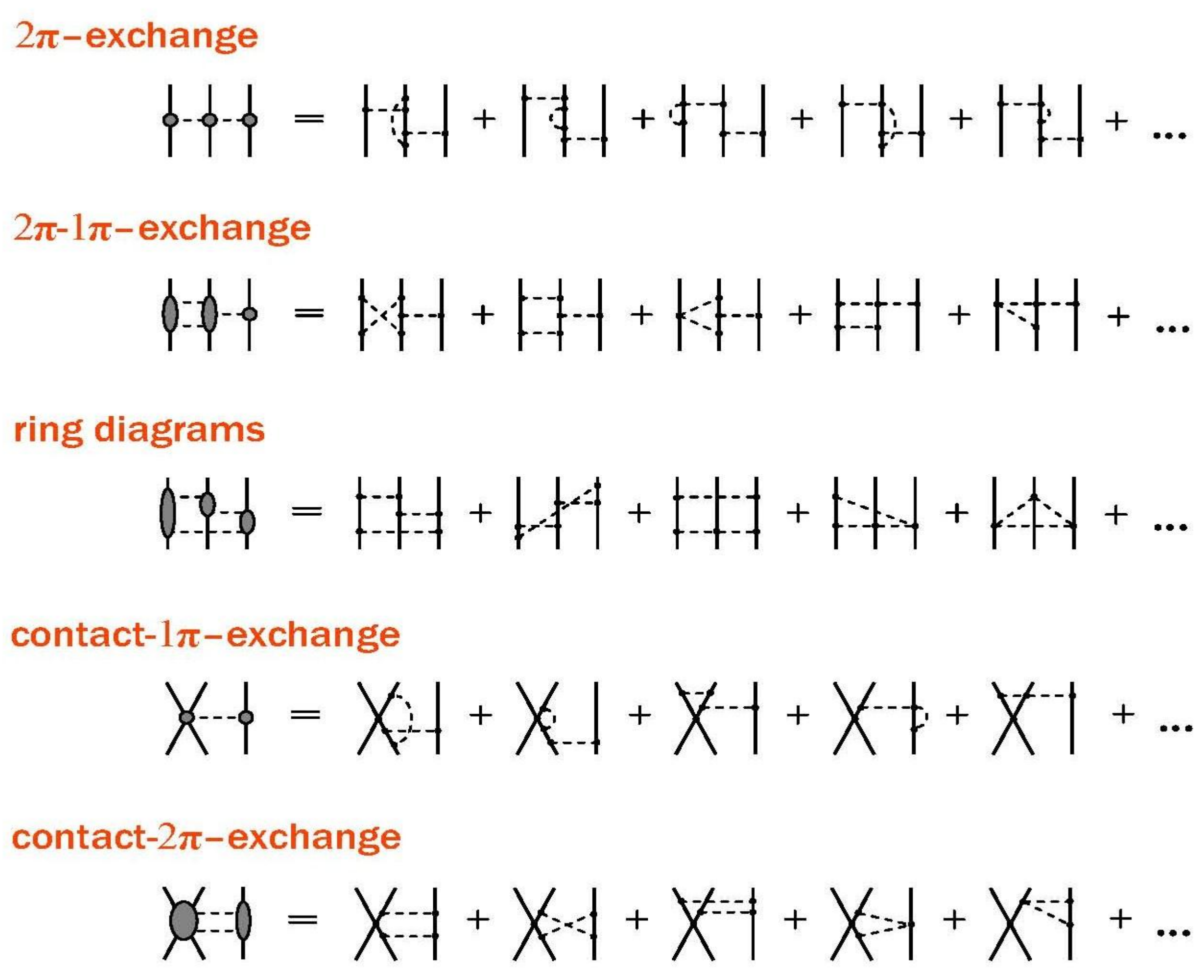
- There are unresolved problems in 3N, 4N scattering and nuclear structure.



See also contribution to this workshop by E. Epelbaum.

The 3NF at N3LO explicitly

One-loop, leading vertices

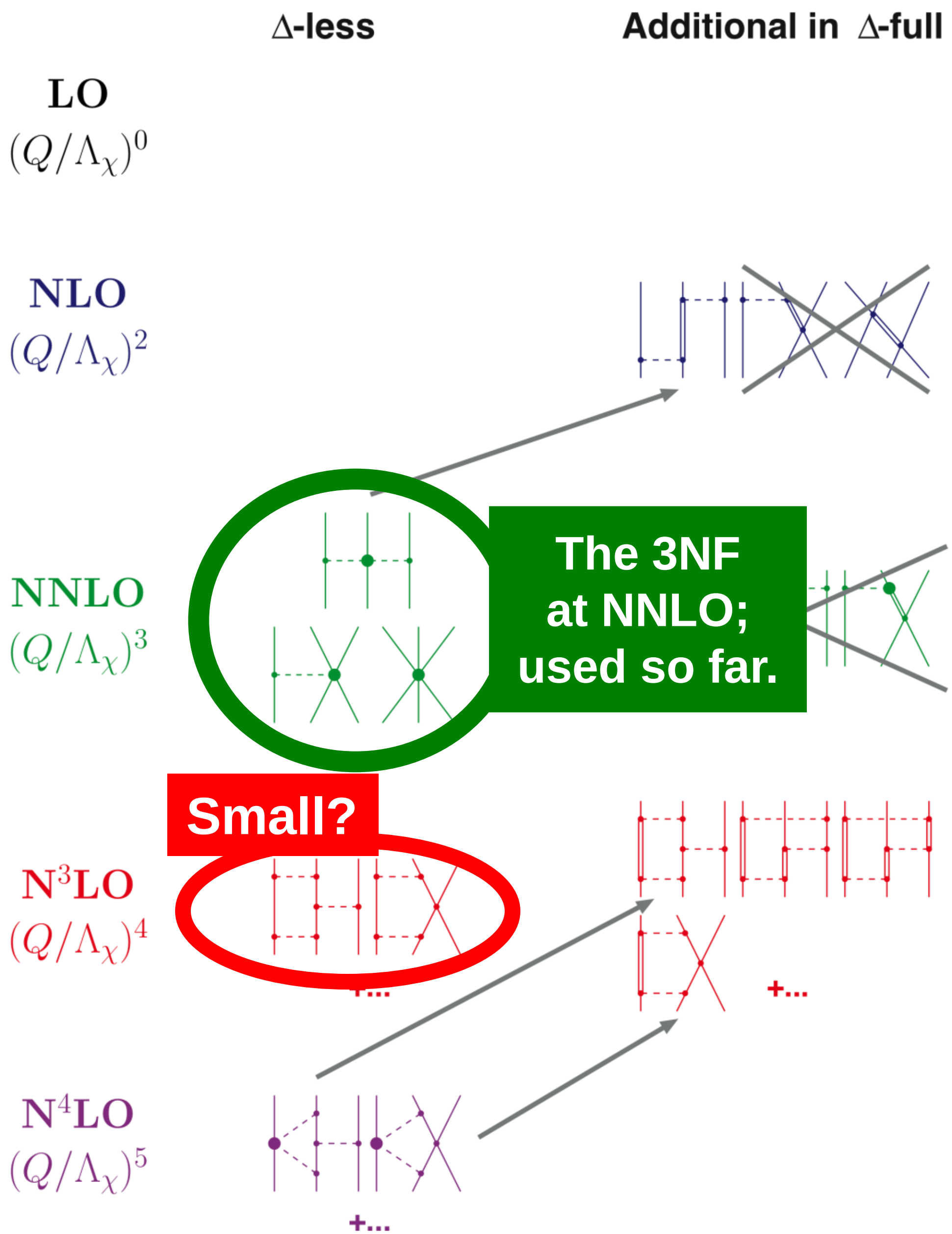


Ishikawa & Robilotta, PRC 76, 014006 (2007)

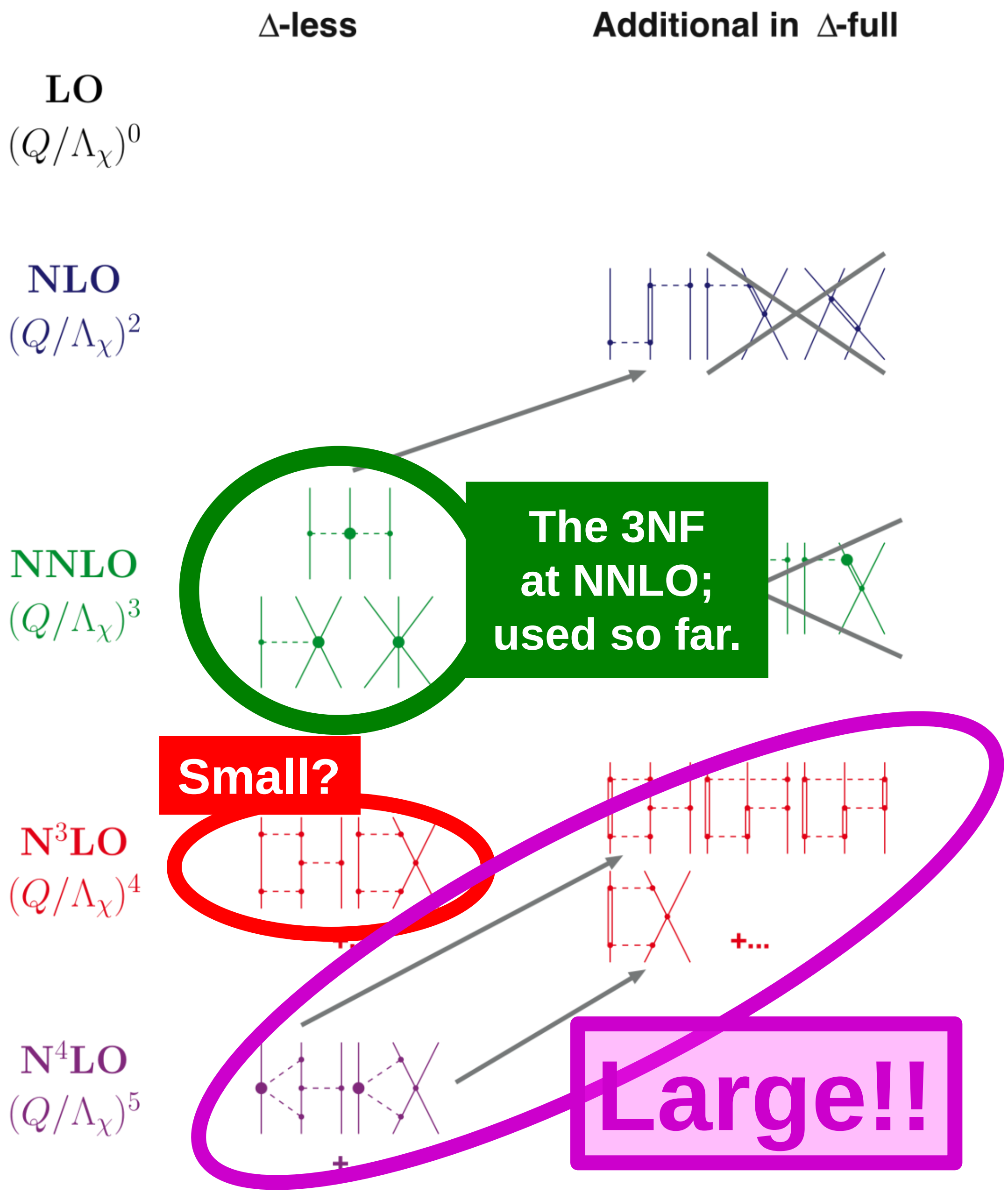
Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)

In progress

Chiral 3N Force



Chiral 3N Force



So, we are obviously not done!

Some of the more crucial open issues:

- **Subleading few-nucleon forces: N4LO in Δ-less or N3LO in Δ-full.**
- **Renormalization of chiral nuclear forces**

I will focus now on this one.

"I about got this one renormalized"

The issue has produced lots and lots of papers; this is just a small sub-selection.

D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl. Phys. **B478** (1996) 629; Phys. Lett. B **424** (1998) 390; Nucl. Phys. **B534** (1998) 329,

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M. C. Birse, Phys. Rev. C **74** (2006) 014003; Phys. Rev. C **76** (2007) 034002.

S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, Nucl. Phys. **A700** (2002) 377.

M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **72** (2005) 054002.

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R. Machleidt, P. Liu, D. R. Entem, and E. Ruiz Arriola, Phys. Rev. C **81** (2010) 024001.

E. Epelbaum and J. Gegelia, Eur. Phys. J. **A41** (2009) 341.

G. P. Lepage, *How to Renormalize the Schrödinger Equation*, nucl-th/9706029.

So, what’s the problem with this renormalization?

The EFT approach is not just another phenomenology. It's field theory.

The problem in all field theories are divergent loop integrals.

The method to deal with them in field theories:

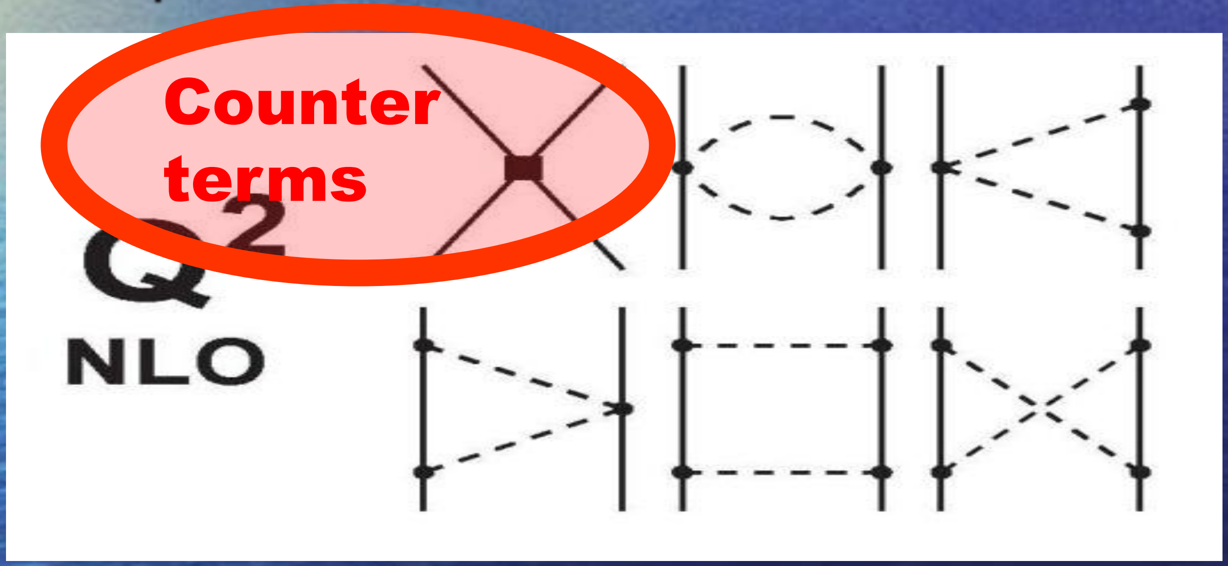
1. **Regularize the integral (e.g. apply a "cutoff") to make it finite.**
2. **Remove the cutoff dependence by Renormalization ("counter terms").**

For calculating pi-pi and pi-N reactions no problem.

However, the NN case is tougher, because it involves two kinds of (divergent) loop integrals.

The first kind:

- "NN Potential":
  irreducible diagrams calculated perturbatively.
  Example:



➢ **perturbative renormalization**
(order by order)

The first kind:

- "NN Potential":
  irreducible diagrams calculated perturbatively.
  Example:

➢ **perturbative renormalization**
(order by order)

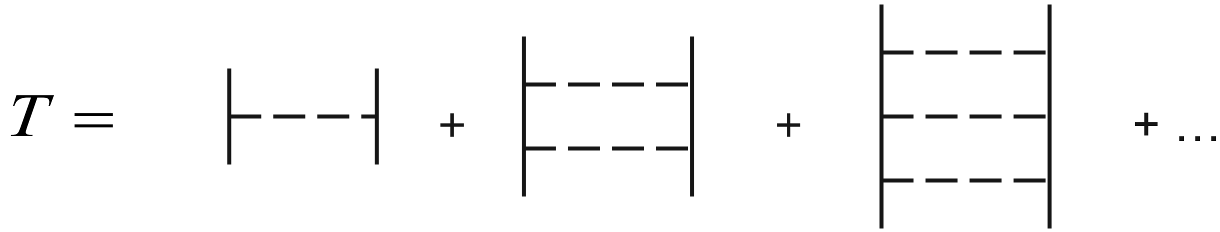
This is fine. No problems.

The second kind:

- Application of the NN Pot. in the Schrodinger or Lippmann-Schwinger (LS) equation: non-perturbative summation of ladder diagrams (infinite sum):

$$T(\vec{p}\,',\vec{p}) = V(\vec{p}\,',\vec{p}) + \int d^3p''\, V(\vec{p}\,',\vec{p}\,'')\, \frac{M_N}{p^2 - p''^2 + i\epsilon}\, T(\vec{p}\,'',\vec{p}) \,,$$

In diagrams:



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- Divergent integral.
- Regularize it:

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p})\; e^{-(p'/\Lambda)^{2n}}\; e^{-(p/\Lambda)^{2n}} \,.$$

- Cutoff dependent results.
- Renormalize to get rid of the cutoff dependence:

➢ **Non-perturbative renormalization**

The second kind:

- Application of the NN Pot. in the Schrodinger or Lippmann-Schwinger (LS) equation: non-perturbative summation of ladder diagrams (infinite sum):

$$T(\vec{p}\,', \vec{p}) = V(\vec{p}\,', \vec{p}) + \int d^3p''\, V(\vec{p}\,', \vec{p}\,'')\, \frac{M_N}{p^2 - p''^2 + i\epsilon}\, T(\vec{p}\,'', \vec{p})$$

- Divergent integral.
- Regularize it:

$$V(\vec{p}\,', \vec{p}) \longmapsto V(\vec{p}\,', \vec{p})\, e^{-(p'/\Lambda)^{2n}}\, e^{-(p/\Lambda)^{2n}}.$$

- Cutoff dependent results.
- Renormalize to get rid of the cutoff dependence:

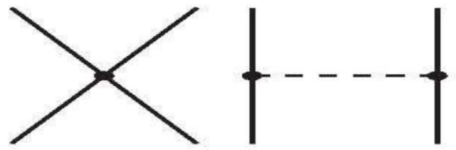
➢**Non-perturbative renormalization**

With what to renormalize this time?

Weinberg's silent assumption: The same counter terms as before.
("Weinberg counting")

Weinberg counting fails already in Leading Order (for Λ → ∞ renormalization)

- **Q^0**
  LO



$$V^{(0)}(\vec{p}', \vec{p}) = C_S + C_T \,\vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4f_\pi^2}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \,\frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2},$$

- 3S1 and 1S0 (with a caveat) renormalizable with LO counter terms.
- However, where OPE tensor force attractive:

  3P0, 3P2, 3D2, …

  a counter term

  must be added.

Nogga, Timmermans, v. Kolck PRC72, 054006 (2005):

"Modified Weinberg counting" for LO

Quantitative chiral NN potentials are at N3LO.
So, we need to go substantially beyond LO.

Renormalization beyond leading order –

Aspects

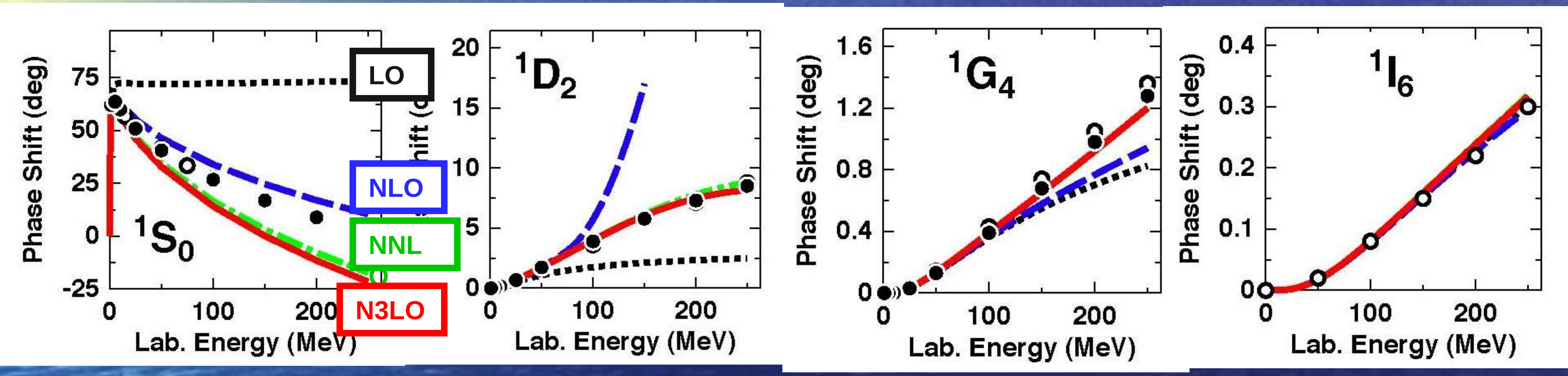
- Nonperturbative or perturbative?
- Infinite cutoff or finite cutoff?

Renormalization beyond leading order –

Options

1. Continue with the nonperturbative infinite-cutoff renormalization.
2. Perturbative using DWBA.
3. Nonperturbative using finite cutoffs $\leq \Lambda_\chi \approx 1$ GeV.

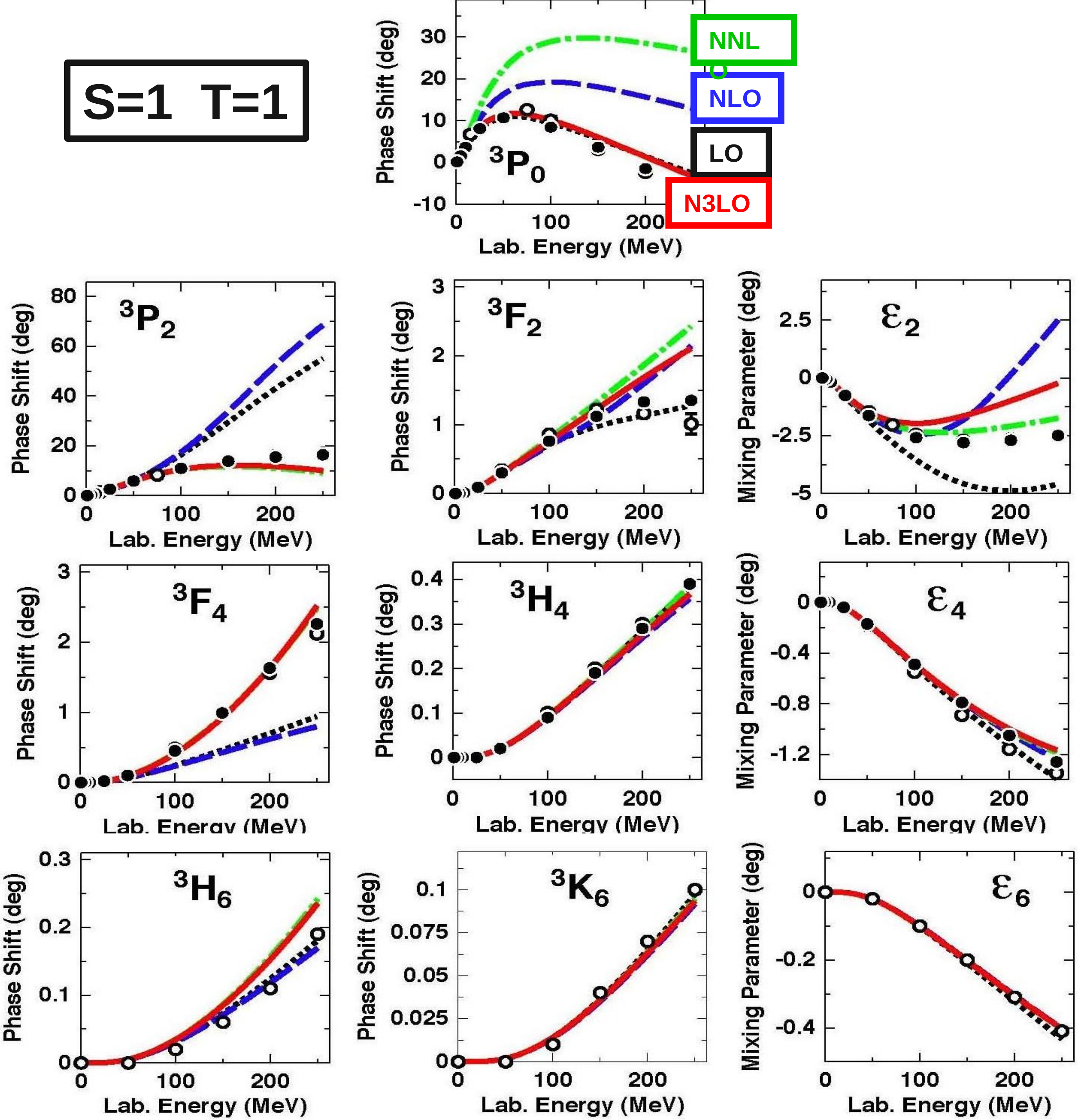
Option 1: Nonperturbative infinite-cutoff renormalization up to N3LO



S=0 T=1

Different partial waves are windows on different ranges of the force.

S=1 T=1



NNLO
NLO
LO
N3LO
$^3P_0$
$^3P_2$
$^3F_2$
ε_2
$^3F_4$
$^3H_4$
ε_4
$^3H_6$
$^3K_6$
ε_6
Phase Shift (deg)
Mixing Parameter (deg)
Lab. Energy (MeV)

Option 1: Nonperturbative infinite-cutoff renormalization up to N3LO

Observations and problems

- In lower partial waves (≅ short distances), in some cases convergence, in some not; data are not reproduced.
- In peripheral partial waves (≅ long distances), always good convergence and reproduction of the data.
- Thus, long-range interaction o.k., short-range not (should not be a surprise: the EFT is designed for $Q < \Lambda_\chi$).
- At all orders, either one (if pot. attractive) or no (if pot. repulsive) counterterm, per partial wave: What kind of power counting scheme is this?
- Where are the systematic order by order improvements?

Option 1: Nonperturbative infinite-cutoff renormalization up to N3LO

Observations and problems

No good!

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Option 2: Perturbative, using DWBA (Valderrama '09)

- Renormalize LO non-perturbatively using modified Weinberg counting.
- Use the distorted LO wave to calculate higher orders in perturbation theory.
- At NLO, 3 counterterms for 1S0 and 6 for 3S1: a power-counting scheme that allows for systematic improvements order by order emerges.
- Results for NN scattering o.k., so, in principal, this scheme works.

Option 2: Perturbative, using DWBA (Valderrama '09), cont'd

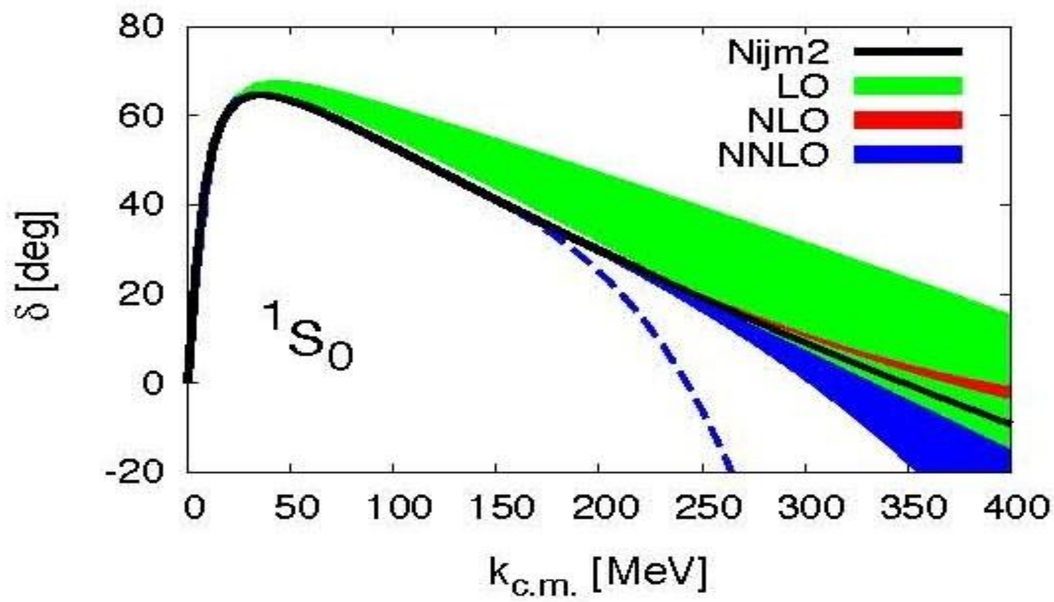

FIG. 1: Phase shifts for the $^1S_0$ channel with non-perturbative OPE and perturbative TPE. The non-perturbative OPE computation contains one counterterm which is determined by fixing the $^1S_0$ scattering length, $a_{0,s} = -23.74\,\mathrm{fm}$, while the perturbative TPE computation contains a correction to the LO counterterm plus two additional counterterms which are used to fit the Nijmegen II phase shifts [42] (equivalent to the Nijmegen PWA [43]) in the range $k = 0.2 - 0.8\,\mathrm{fm}^{-1}$. The error bands are generated varying the cut-off within the $0.6 - 0.9$ fm range. The dashed blue line represents the N^2LO results for $r_c = 0.1$ fm.

Option 2: Perturbative, using DWBA (Valderrama '09)

- **Renormalize LO non-perturbatively with infinite cutoff using modified Weinberg counting.**
- **Use the distorted LO wave to calculate higher orders in perturbation theory.**
- **At NLO, 3 counterterms for 1S0 and 6 for 3S1: a power-counting scheme that allows for systematic improvements order by order emerges.**
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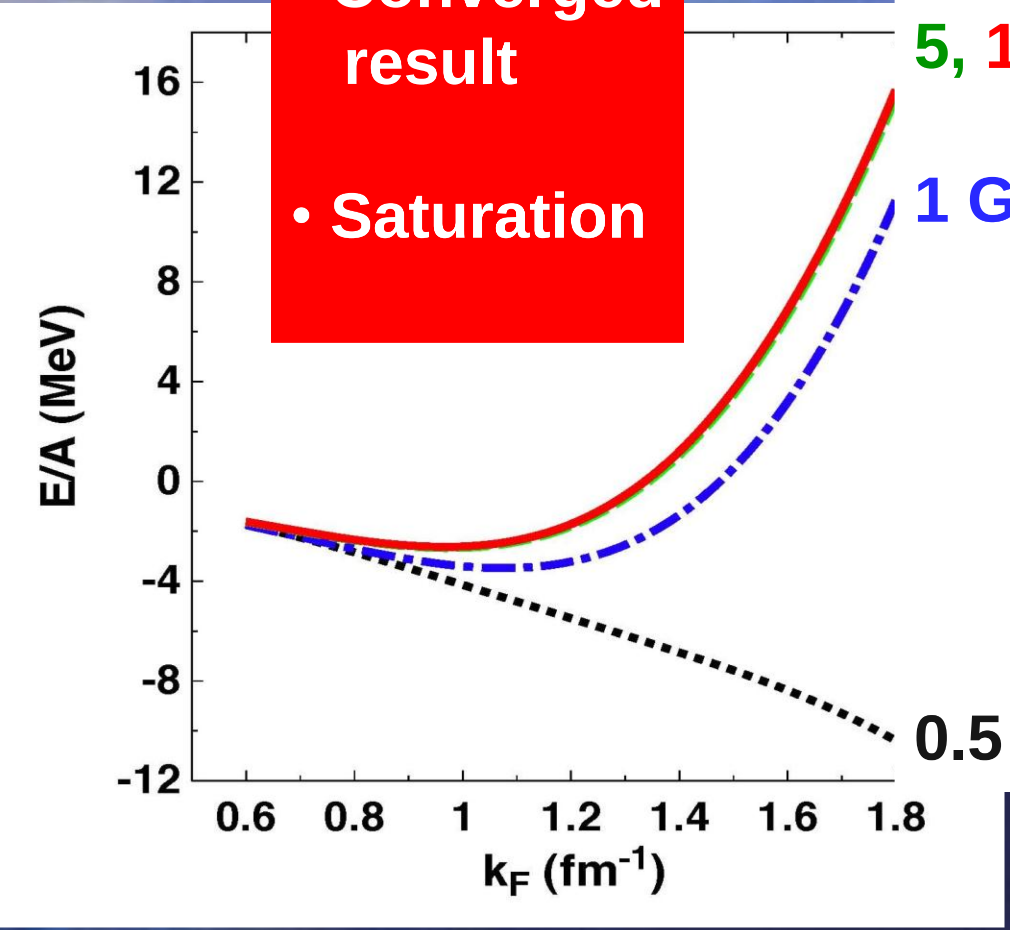
- **But how practical is this scheme for nuclear structure?**

Nonperturbatively renormalized LO interaction and nuclear matter energy predictions



- Converged result
- Saturation

cutoff **Λ:**
5, 10 GeV
1 GeV
0.5 GeV

However, there is a However …

- Saturation at $k_f \approx 1.0\ fm^{-1}$ and $E/A = -2.6$ MeV.
- Empirical value : $E/A \approx -16$ MeV.
- Severe underbinding!
- Why?

The tensor force of the renorm. LO interaction is extraordinarily strong

	Renorm. LO	N3LO	CD-Bonn	AV18	Hamada-Johnston (1962)
Deuteron D-state probability	7.2%	4.51%	4.85%	5.76%	7.0%
Wound integral	40.5%	5.0%	5.8%	10.1%	21.1%

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- But how practical is this scheme in nuclear structure?
- LO interaction has huge tensor force, huge wound integral; bad convergence of the many-body problem. Impractical!

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For considerations of the NN amplitude o.k.
But impractical for nuclear structure applications.

What now?

Option 3: Rethink the problem from scratch

- **EFFECTIVE** field theory for $Q \leq \Lambda_\chi \approx 1$ GeV.
- So, you have to expect garbage above Λ_χ.
- The garbage may even converge, but that doesn’t convert the garbage into the good stuff (Epelbaum & Gegelia ‘09).
- So, stay away from territory that isn’t covered by the EFT.

Lepage 1997: take 3 steps

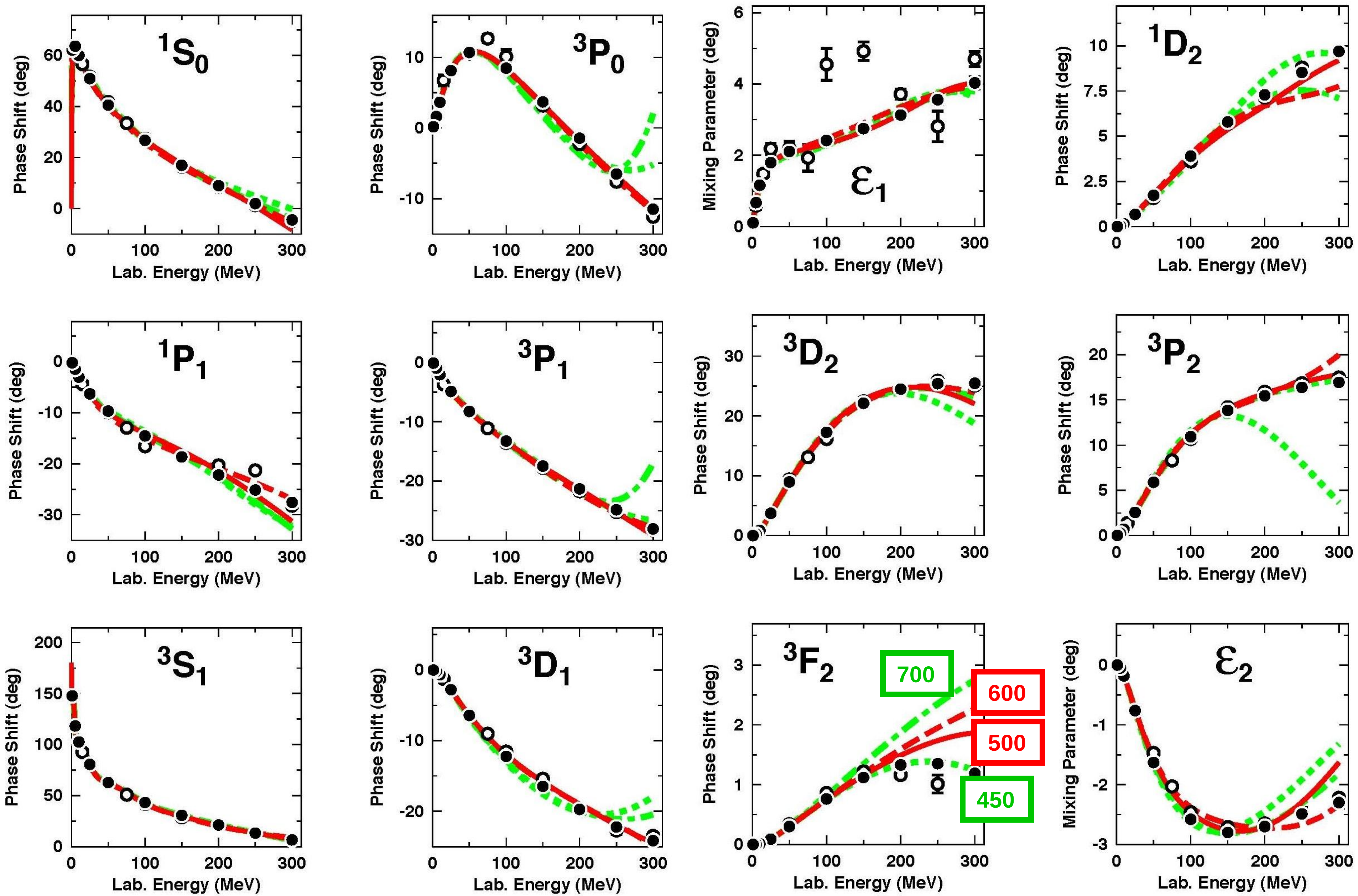
1. Incorporate the correct long-range behavior: The long-range behavior of the underlying theory must be known, and it must be built into the effective theory.

2. Introduce an ultraviolet cutoff to exclude high-momentum states, or, equivalently, to soften the short-distance behavior: The cutoff has two effects. First it excludes high-momentum states, which are sensitive to the unknown short-distance dynamics; only states that we understand are retained. Second, it makes all interactions regular at $r = 0$, thereby avoiding the infinities that plague the naive approach of the previous section.

3. Add local correction terms to the effective hamiltonian: These mimic the effects of the high-momentum states excluded by the cutoff in step 2. Each correction term consists of a theory-specific coupling constant, a number, multiplied by a theory-independent local operator. The correction terms systematically remove dependence on the cutoff. Their locality

Option 3, cont'd: finding a stable range of cutoffs below 1GeV

- A very systematic investigation up to N3LO does not (yet) exist.
- But there is ample circumstantial evidence on the market already (see next slide).



$^1S_0$
$^3P_0$
ε_1
$^1D_2$
$^1P_1$
$^3P_1$
$^3D_2$
$^3P_2$
$^3S_1$
$^3D_1$
$^3F_2$
ε_2
Phase Shift (deg)
Mixing Parameter (deg)
Lab. Energy (MeV)
700
600
500
450

Conclusions

- **Substantial advances in chiral nuclear forces during the past decade. The major milestone of the decade: “high precision” NN pots. at N3LO, good for nuclear structure.**
- **But there are still issues:**
- **Subleading 3NFs: additional and stronger 3NFs are needed; essentially technical and, in principal, straightforward.**
- **Renormalization: more subtle, more controversial, more interesting.**

Our views on reno

- Forget about non-perturbative infinite-cutoff reno: not convergent (in low partial waves ≅ short distances), should not be a surprise; no clear power counting scheme, no systematic improvements order by order.
- Perturbative beyond LO: may be o.k. for the NN amplitude; but impractical in nuclear structure applications for several different reasons as explained.
- Identify "Cutoff independence" within a range $\leq \Lambda_\chi \approx 1$ GeV. Most realistic approach (Lepage). Semi proven already.

And so,

Have we finally cracked the nuclear force problem?

Not quite,
but that's why we are here!