Nuclear Astrophysics: Supernova Evolution and Explosive Nucleosynthesis

Gabriel Martínez Pinedo

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Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Outline



- Astrophysical reaction rates
 - Hydrostatic Burning Phases
 Hydrogen Burning

 - Advanced burning stages
- Core-collapse supernova
 Neutrinos and supernovae
- Nucleosynthesis heavy elements
 Neutrino-driven winds

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A new Era for Nuclear Astrophysics



Improved supernovae models



- New radioactive ion beam facilities (RIBF, SPIRAL 2, FAIR, FRIB) are being built or developed that will study many of the nuclei produced in explosive events. Hydrostatic burning phases studied in underground labs (LUNA)
- We need improved theoretical models to fully exploit the potential offered by these facilities.

Introduction Hydrostatic Burning Phases

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What is Nuclear Astrophysics?

- Nuclear astrophysics aims at understanding the nuclear processes that take place in the universe.
- These nuclear processes generate energy in stars and contribute to the nucleosynthesis of the elements and the evolution of the galaxy.







The solar abundance distribution

solar abundances

system at the time

K. Lodders, Astrophys. J. **591**, 1220-1247 (2003)

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Cosmic Cycle



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Nucleosynthesis processes

In 1957 Burbidge, Burbidge, Fowler and Hoyle and independently Cameron, suggested several nucleosynthesis processes to explain the origin of the elements.



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Composition of the Universe after Big Bang

Matter Composition





Stars are responsible of destroing Hydrogen and producing "metals".

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Star formation



Gaseous Pillars • M16 PRC95-44a • ST Scl OPO • November 2, 1995 J. Hester and P. Scowen (AZ State Univ.), NASA

- Stars are formed from the contraction of molecular clouds due to their own gravity.
- Contraction increases temperature and eventually nuclear fusion reactions begin. A star is born.
- Contraction time depends on mass: 10 millions years for a star with the mass of the Sun; 100,000 years for a star 11 times the mass of the Sun.

The evolution of a Star is governed by gravity

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What is a star?





- A star is a self-luminous gaseous sphere.
- Stars produce energy by nuclear fusion reactions. A star is a self-regulated nuclear reactor.
- Gravitational collapse is balanced by pressure gradient: hydrostatic equilibrium.

$$dF_{\text{grav}} = -G\frac{mdm}{r^2} = [P(r+dr) - P(r)]dA = dF_{\text{pres}}$$
$$dm = 4\pi r^2 \rho dr$$
$$-G\frac{m\rho}{r^2} = \frac{dP}{dr}$$

• Further equations needed to describe the transport of energy from the core to the surface, and the change of composition (nuclear reactions). Supplemented by an EoS: $P(\rho, T)$.

• Star evolution, lifetime and death depends on mass. Two groups

- Stars with masses less than 9 solar masses (white dwarfs)
- Stars with masses greater than 9 solar masses (supernova explosions)

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Core evolution

Hydrostatic equilibrium together with properties equation of state determines the evolution of the star core.



red: transition from relativistic to non-relativistic electrons.

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Where does the energy come from?

Energy comes from nuclear reactions in the core.

$$^{1}\text{H} \rightarrow ^{4}\text{He} + 2e^{+} + 2v_{e} + 26.7 \text{ MeV}$$





The Sun converts 600 million tons of hydrogen into 596 million tons of helium every second. The difference in mass is converted into energy. The Sun will continue burning hydrogen during 5 billions years. Energy released by H-burning:

6.45 × 10¹⁸ erg g⁻¹ = 6.7 MeV/nuc Solar Luminosity: 3.85×10^{33} erg s⁻¹

$$E = mc^2$$

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Nuclear Binding Energy

Liberated energy is due to the gain in nuclear binding energy.



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Type of processes

Transfer (strong interaction)

$$^{15}N(p, \alpha)^{12}C, \qquad \sigma \simeq 0.5 \text{ b at } E_p = 2.0 \text{ MeV}$$

Capture (electromagnetic interaction)

³He(
$$\alpha, \gamma$$
)⁷Be, $\sigma \simeq 10^{-6}$ b at $E_p = 2.0$ MeV

Weak (weak interaction)

$$p(p, e^+ v)d$$
, $\sigma \simeq 10^{-20}$ b at $E_p = 2.0$ MeV
b = 100 fm² = 10⁻²⁴ cm²

Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

Decay

$$5^{56}\text{Ni} \rightarrow 5^{56}\text{Co} + e^+ + \nu_e$$

$$1^{5}\text{O} + \gamma \rightarrow {}^{14}\text{N} + p$$

$$\frac{dn_a}{dt} = -\lambda_a n_a$$

In order to dissentangle changes in the density (hydrodynamics) from changes in the composition (nuclear dynamics), the abundance is introduced:

 $Y_a = \frac{n_a}{n}, \quad n \approx \frac{\rho}{m_u}$ = Number density of nucleons (constant)

$$\frac{dY_a}{dt} = -\lambda_a Y_a$$

Rate can depend on temperature and density

Introduction Astrop	hysical reaction rates	Hydrostatic Burning Phases	Core-collapse supernova	Nucleosynthesis heavy elements
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Types of reactions

Nuclei in the astrophysical environment can suffer different reactions:

Capture processes

$$a + b \rightarrow c + d$$

$$\frac{dn_a}{dt} = -n_a n_b \langle \sigma v \rangle$$

$$\frac{dY_a}{dt} = -\frac{\rho}{m_u} Y_a Y_b \langle \sigma v \rangle$$

decay rate: $\lambda_a = \rho Y_b \langle \sigma v \rangle / m_u$

• 3-body reactions:

$$3^{4}\text{He} \rightarrow {}^{12}\text{C} + \gamma$$
$$\frac{dY_{\alpha}}{dt} = -\frac{\rho^{2}}{2m_{u}^{2}}Y_{\alpha}^{3}\langle\alpha\alpha\alpha\rangle$$

decay rate: $\lambda_{\alpha} = Y_{\alpha}^2 \rho^2 \langle \alpha \alpha \alpha \rangle / (2m_u^2)$

Hydrostatic Burning Phases

Reaction rates

Consider n_a and n_b particles per cubic centimeter of species a and b. The rate of nuclear reactions

$$a + b \rightarrow c + d$$

is given by:

$$r_{ab} = n_a n_b \sigma(v) v, \quad v = \text{relative velocity}$$

In stellar environment the velocity (energy) of particles follows a thermal distribution that depends of the type of particles.

Nuclei (Maxwell-Boltzmann)

$$N(v)dv = N4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) dv = N\phi(v)dv$$

• Electrons, Neutrinos (if thermal) (Fermi)

$$N(p)dp = \frac{g}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{(E(p)-\mu)/kT} + 1} dp$$

photons (Bose)

$$N(p)dp = \frac{2}{(2\pi\hbar)^3} \frac{4\pi p^2}{e^{pc/kT} - 1} dp$$

Introduction	Astrophysical reaction rates	Hydrostatic Burning Phases	Core-collapse supernova	Nucleosynthesis heavy e
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Stellar reaction rate

The product σv has to be averaged over the velocity distribution $\phi(v)$ (Maxwell-Boltzmann)

$$\langle \sigma v \rangle = \int_0^\infty \phi(v) \sigma(v) v dv$$

that gives:

$$\langle \sigma v \rangle = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 \sigma(v) \exp\left(-\frac{mv^2}{2kT}\right) dv, \quad m = \frac{m_1 m_2}{m_1 + m_2}$$

or using $E = mv^2/2$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

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Neutron capture (compound picture)



REACTION : 1+2 - C - 3+4 + Q (Q>0)

$$A + n \to B + \gamma$$

$$\sigma_n \approx \pi \lambda^2 |\langle B + \gamma | H_{II} | C \rangle \langle C | H_I | A + n \rangle|^2 \propto \lambda^2 T_{\gamma} (E_n + Q) T_n(E_n)$$

T transmision coefficient, E_n neutron energy, $Q = m_A + m_n - m_B = S_n(B)$, $Q \gg E_n$.

$$\sigma_n \propto \lambda_n^2 T_n(E_n), \quad T_n(E_n) \propto v_n P_l(E_n)$$

 $P_l(E_n)$, propability tunneling through the centrifugal barrier of momentum l. Normally s-wave dominates and $P_0(E_n) = 1$.

$$\sigma_n \propto \frac{1}{v_n^2} v_n = \frac{1}{v_n}, \quad \langle \sigma_n v \rangle = \text{constant}$$

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Charged-particle reactions

Stars' interior is a neutral plasma made of charged particles (nuclei and electrons). Nuclear reactions proceed by tunnel effect. For the p + p reaction the Coulomb barrier is 550 keV, but the typical proton energy in the Sun is only 1.35 keV.



Cross section given by:

$$\sigma(E) = \frac{1}{E} e^{-2\pi\eta} S(E), \quad \eta = \frac{Z_1 Z_2 e^2}{\hbar} \sqrt{\frac{m}{2E}} = \frac{b}{E^{1/2}}$$

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S factor

S factor makes possible accurate extrapolations to low energy.



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Gamow window

Using definition S factor:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE$$



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Gamow window

Assuming the S factor is constant over the gamow window and approximating the integrand by a Gaussian one gets:

$$\langle \sigma v \rangle = \left(\frac{2}{m}\right)^{1/2} \frac{\Delta}{(kT)^{3/2}} S(E_0) \exp\left(-\frac{3E_0}{kT}\right)$$

with

$$E_0 = \left(\frac{bkT}{2}\right)^{2/3} = 1.22(Z_1^2 Z_2^2 A T_6^2)^{1/3} \text{ keV}$$
$$\Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 kT} = 0.749 (Z_1^2 Z_2^2 A T_6^5)^{1/6} \text{ keV}$$

 $(A = m/m_u \text{ and } T_6 = T/10^6 \text{ K})$ Examples for solar conditions $(T = 15 \times 10^6 \text{ K})$:

reaction	E_0 (keV)	$\Delta/2$ (keV)	$\exp(-3E_0/kT)$	T dependence
p + p	5.9	3.2	1.1×10^{-6}	$T^{3.6}$
${}^{14}N + p$	26.5	6.8	1.8×10^{-27}	T^{20}
$^{12}C + \alpha$	56.0	9.8	3.0×10^{-57}	T^{42}
$^{16}O + ^{16}O$	237.0	20.2	6.2×10^{-239}	T^{182}

Reaction rate depends very sensitively on temperature

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Rate Examples: (p, γ)



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Rate Examples: (α, γ)



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Rate examples: (n, γ)



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Hertzspung-Russell diagram



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Hydrogen burning: ppl-chain

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Step 1:
$$p + p \rightarrow {}^{2}$$
He (not possible)
 $p + p \rightarrow d + e^{+} + v_{e}$
Step 2: $d + p \rightarrow {}^{3}$ He
 $d + d \rightarrow {}^{4}$ He (*d* abundance too low)
Step 3: 3 He $+ p \rightarrow {}^{4}$ Li (4 Li is unbound)
 3 He $+ d \rightarrow {}^{4}$ He $+ n$ (d abundance too low)
 3 He $+ {}^{3}$ He $\rightarrow {}^{4}$ He $+ 2p$

d + d not going because Y_d is small and d + p leads to rapid destruction. ³He + ³He goes because Y_{3He} gets large as nothing destroys it.

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The relevant S-factors

 3 He(3 He, 2*p*) 4 He:

 $p(p, e^+v_e)d$: $p(d, \gamma)^3$ He: $S_{11}(0) = (4.00 \pm 0.05) \times 10^{25} \text{ MeV b}$ calculated $S_{12}(0) = 2.5 \times 10^{-7} \text{ MeV b}$ measured at LUNA $S_{33}(0) = 5.4 \text{ MeV b}$ measured at LUNA



Laboratory Underground for Nuclear Astrophysics (Gran Sasso).

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Reaction Network ppl-chain

$$\frac{dY_p}{dt} = -Y_p^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{pp} - Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd} + Y_3^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$
$$\frac{dY_d}{dt} = \frac{Y_p^2}{2} \frac{\rho}{m_u} \langle \sigma v \rangle_{pp} - Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd}$$
$$\frac{dY_3}{dt} = Y_d Y_p \frac{\rho}{m_u} \langle \sigma v \rangle_{pd} - Y_3^2 \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$
$$\frac{dY_4}{dt} = \frac{Y_3^2}{2} \frac{\rho}{m_u} \langle \sigma v \rangle_{33}$$

Stiff system of coupled differential equations.

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pp chains

Once ⁴He is produced can act as catalyst initializing the ppll and pplll chains.





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The other hydrogen burning: CNO cycle



requires presence of ¹²C as catalyst.

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Neutrino spectrum (Sun)

This is the predicted neutrino spectrum



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Energy generation: CNO cycle vs pp-chains



Introduction	Astrophysical	reaction	

Hydrostatic Burning Phases

Consequences

- Stars slightly heavier than the Sun burn hydrogen via CNO cycle.
- CNO cycle goes significantly faster. Such stars have much shorter lifetimes

Mass (M_{\odot})	lifetime (yr)
0.8	1.4×10^{10}
1.0	1×10^{10}
1.7	2.7×10^{9}
3.0	2.2×10^{8}
5.0	6×10^{7}
9.0	2×10^{7}
16.0	1×10^{7}
25.0	7×10^{6}
40.0	1×10^{6}



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Helium Burning

- Once hydrogen is exhausted the stellar core is made mainly of helium. Hydrogen burning continues in a shell surrounding the core.
- ${}^{4}\text{He} + p$ produces ${}^{5}\text{Li}$ that decays in 10^{-22} s.
- Helium survives in the core till the temperature become large enough ($T \approx 10^8$ K) to overcome the coulomb barrier for ${}^{4}\text{He} + {}^{4}\text{He}$. The produced ${}^{8}\text{Be}$ decays in 10^{-16} . However, the lifetime is large enough to allow the capture of another ${}^{4}\text{He}$:

$$3^{4}\text{He} \rightarrow {}^{12}\text{C} + \gamma$$

- Hoyle suggested that in order to account for the large abundance of Carbon and Oxygen, there should be a resonance in ¹²C that speeds up the production.
- ${}^{12}C$ can react with another ${}^{4}He$ producing ${}^{16}O$

$${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O} + \gamma$$

• These two reactions make up helium burning.

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Hoyle State and tripple α reaction

Red giant structure


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Influence ${}^{12}C(\alpha, \gamma)$ in nucleosynthesis



End of helium burning

Nucleosynthesis yields from stars may be divided into production by stars above and below $9\ M_{\odot}.$

stars with $M \leq 9 \text{ M}_{\odot}$ These stars eject their envelopes during helium shell burning producing planetary nebula and white dwarfs. Constitute the site for the s process.

stars with $M \gtrsim 9 \text{ M}_{\odot}$ These stars will ignite carbon burning under non-degenerate conditions. The subsequent evolution proceeds in most cases to core collapse. These stars make the bulk of newly processed matter that is returned to the interstellar medium.

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Stellar Evolution



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Stellar life

Nuclear burning stages

(e.g., 20 solar mass star)

Fuel	Main Product	Secondary Product	T (10 ⁹ K)	Time (yr)	Main Reaction
н	He	¹⁴ N	0.02	10 ⁷	4 H → ^{CNO} 4He
He	0, C	¹⁸ O, ²² Ne s-process	0.2	10 ⁶	3 He ⁴ → ¹² C ¹² C(α,γ) ¹⁶ O
C	Ne, Mg	Na	0.8	10 ³	¹² C + ¹² C
Ne	O, Mg	AI, P	1.5	3	²⁰ Ne(γ,α) ¹⁶ O ²⁰ Ne(α,γ) ²⁴ Mg
O	Si, S	CI, Ar, K, Ca	2.0	0.8	¹⁶ O + ¹⁶ O
Si	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	²⁸ Si(γ,α)

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Supernova types

(a) Type la supernova

- The spectrum has no hydrogen or helium lines, but does have a strong absorption line of ionized silicon (Si II).
- Produced by runaway carbon fusion in a white dwarf in a close binary system (the ionized silicon is a by-product of carbon fusion).
- (b) Type Ib supernova
- The spectrum has no hydrogen lines, but does have a strong absorption line of un-ionized helium (He I).
- Produced by core collapse in a massive star that lost the hydrogen from its outer layers.



Wavelength (nm)

- (c) Type Ic supernova
- The spectrum has no hydrogen lines or helium lines.
- Produced by core collapse in a massive star that lost the hydrogen and the helium from its outer layers.

(d) Type II supernova

- The spectrum has prominent hydrogen lines such as ${\rm H}_{\alpha}.$
- Produced by core collapse in a massive star whose outer layers were largely intact.

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SN1987A

Type II supernova in LMC (~ 55 kpc)



- $E_{\rm grav} \approx 10^{53} \, {\rm erg}$
- $E_{\rm rad} \approx 8 \times 10^{49} \, {\rm erg}$
- $E_{\rm kin} \approx 10^{51} \, {\rm erg} = 1 \, {\rm Bethe}$



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Presupernova Star

- Star has an onion like structure.
- Iron is the final product of the different burning processes.
- As the mass of the iron core grows it becomes unstable and collapses when it reaches around $1.4~M_{\odot}$.



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Schematical Evolution



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Early iron core

- The core is made of heavy nuclei (iron-mass range A = 45-65) and electrons. Composition given by Nuclear Statistical Equilibrium. There are Y_e electrons per nucleon.
- The mass of the core M_c is determined by the nucleons.
- There is no nuclear energy generation which adds to the pressure. Thus, the pressure is mainly due to the degenerate electrons, with a small correction from the electrostatic interaction between electrons and nuclei.
- As long as $M_c < M_{\rm ch} = 1.44(2Y_e)^2 M_{\odot}$ (plus slight corrections for finite temperature), the core can be stabilized by the degeneracy pressure of the electrons.

Onset of collapse

There are two processes that make the situation unstable:

- Silicon burning is continuing in a shell around the iron core. This adds mass to the iron core increasing M_c.
- Ilectrons can be captured by protons (free or in nuclei):

$$e^- + A(Z, N) \to A(Z - 1, N + 1) + \nu_e.$$

This reduces the pressure and keep the core cold, as the neutrinos leave. The net effect is a reduction of Y_e and consequently of the Chandrasekhar mass (M_{ch})

Nuclear Statistical Equilibrium

At high temperatures compositium can be aproximated by Nuclear Statistical Equilibrium.

- Compositium is given by a minimum of the Free Energy: F = U - TS. Under the constrains of conservation of number of nucleons and charge neutrality.
- It is assumed that all nuclear reactions operate in a time scale much shorter than any other timescale in the system.

Nuclear Statistical Equilibrium favors free nucleons at high temperatures and iron group nuclei at low temperatures. Production of nuclei heavier than iron requires that nuclear statistical equilibrium breaks at some point.

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Nuclear Statistical Equilibrium

The minimum of the free energy is obtained when:

$$\mu(Z,A) = (A-Z)\mu_n + Z\mu_p$$

implies that there is an equilibrium between the processes responsible for the creation and destruction of nuclei:

$$A(Z,N) \rightleftarrows Zp + Nn + \gamma$$

Processes mediated by the strong and electromagnetic interactions proceed in a time scale much sorter than any other evolutionary time scale of the system.

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Nuclear abundances in NSE

Nuclei follow Boltzmann statistics:

$$\mu(Z,A) = m(Z,A)c^{2} + kT \ln\left[\frac{n(Z,A)}{G_{Z,A}(T)} \left(\frac{2\pi\hbar^{2}}{m(Z,A)kT}\right)^{3/2}\right]$$

with $G_{Z,A}(T)$ the partition function:

$$G_{Z,A}(T) = \sum_{i} (2J_i + 1)e^{-E_i(Z,A)/kT} \approx \frac{\pi}{6akT} \exp(akT) \left(a \sim A/9 \text{MeV}\right)$$

Results in Saha equation:

$$Y(Z,A) = \frac{G_{Z,A}(T)A^{3/2}}{2^A} \left(\frac{\rho}{m_u}\right)^{A-1} Y_p^Z Y_n^{A-Z} \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3(A-1)/2} e^{B(Z,A)/kT}$$

Composition depends on two parameters: Y_p , Y_n . They are determined from the conditions:

• $\sum_{i} Y_{i}A_{i} = 1$ (conservation number nucleons)

•
$$\sum_{i} Y_i Z_i = Y_e$$
 (charge neutrality)

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Composition



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Composition



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Initial conditions

The dominant contribution to the pressure comes from the electrons. They are degenerate and relativistic:

$$P \approx n_e \mu_e = n_e \varepsilon_F$$

 μ_e is the chemical potential, fermi energy, of the electrons:

$$\mu_e \approx 1.11 (\rho_7 Y_e)^{1/3} \text{ MeV}, \quad \frac{\rho Y_e}{m_u} = n_e$$

For $\rho_7 = 1$ ($\rho = 10^7$ g cm⁻³) the chemical potential is 1 MeV, reaching the nuclear energy scale. At this point is energetically favorable to capture electrons by nuclei.

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How to determine the evolution

- Composition determined by NSE, function of temperature, density and Y_{e} .
- Weak interactions are not in equilibrium $(\mu_e + \mu_p \neq \mu_n + \mu_v)$. Change of Y_e has to be computed explicitly:

$$\begin{split} Y_e &= \sum_i Y_i Z_i \\ \dot{Y}_e &= -\sum_i \lambda_{ec}^i Y_i + \sum_i \lambda_{\beta^-}^i Y_i \end{split}$$

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Presupernova evolution



- T = 0.1-0.8 MeV, $\rho = 10^7-10^{10}$ g cm⁻³. Composition of iron group nuclei.
- Important processes:
 - electron capture:
 - $e^- + (N,Z) \rightarrow (N+1,Z-1) + \nu_e$
 - β^- decay: $(N,Z) \rightarrow (N-1,Z+1) + e^- + \bar{\nu}_e$
- Dominated by allowed transitions (Fermi and Gamow-Teller)
- Evolution decreases number of electrons (Y_e) and Chandrasekar mass $(M_{\rm ch} \approx 1.4(2Y_e)^2 M_{\odot})$

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Laboratory vs. stellar electron capture



Capture of K-shell electrons to tail of GT strength distribution. Parent nucleus in the ground state Capture of electrons from the high energy tail of the FD distribution. Capture to states with large GT matrix elements (GT resonance). Thermal ensemble of initial states.

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Beta-decay



 GT_+ and GT_- sum rules related by Ikeda sum rule:

$$S_{-} - S_{+} = 3(N - Z)$$

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GT in charge exchange reactions

GT strength could be measured in Charge-Exchange reactions:

- GT₋ proved in (*p*, *n*), (³He, *t*).
- GT₊ proved in (*n*, *p*), (*t*, ³He), (*d*, ²He).

Mathematical relationship ($E_p \ge 100 \text{ MeV/nucleon}$):

$$\frac{d\sigma}{d\Omega dE}(0^\circ) \approx f(E_x)B(GT)$$

$$B(GT) = \frac{g_A^2}{2J_i + 1} |\langle f|| \sum_k \sigma^k t_{\pm}^k ||i\rangle|^2$$

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Independent Particle Model



- The IPM allows for a single transition $(f_{7/2} \rightarrow f_{5/2})$. It does not correctly reproduce the fragmentation of GT strength (correlations).
- To account for correlations, it is necessary to explicitly consider the "residual" interaction between nucleons.

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Gamow-Teller strength and rates





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KVI results using $(d, {}^{2}\text{He})$



GT strength in ⁴⁸Sc, ⁵⁰V, ⁵⁸Ni, ⁶⁴Ni also measured.

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Consequences weak rates

A. Heger et al., PRL 86, 1678 (2001)



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Collapse phase



Important processes:

cross sections ~ E_{ν}^2

- electron capture on protons: $e^- + p \rightleftharpoons n + v_e$
- electron capture on nuclei: $e^- + A(Z, N) \rightleftharpoons A(Z-1, N+1) + v_e$

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(Un)blocking electron capture at N=40





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Electron capture: nuclei vs protons



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Effects Realistic calculation

With Marek, Rampp, Janka & Buras (Approx. General Relativistic model) 15 M_{\odot} presupernova model from A. Heger & S. Woosley



Electron capture on nuclei dominates over capture on protons

 All models converge to a "norm" stellar core at the moment of shock formation.

Hydrostatic Burning Phases

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Neutrino interactions in the collapse

30

20 ε_{νε} (MeV) 40

Bruenn and Haxton (1991) Based on results for ⁵⁶Fe 6 E-Neutrino Interaction Rates 5 $v_e + A \rightarrow v_e + A$ 4 3 $v_e + A \rightarrow e^- + A$ (this work) 2 Log[Rate (sec⁻¹)] + A' (FFN/B) 0 $\rho = 10^{10} \text{ g cm}^{-3}$ -1 T = 10¹⁰ K $Y_{a} = 0.45$ -2 s = 1.189 k this work A = 62.1-3 Z = 27.7-4

-5

0

10

- Elastic scattering: $v + A \rightleftharpoons v + A$ (trapping)
- Absorption: $v_e + (N, Z) \rightleftharpoons e^- + (N - 1, Z + 1)$
- *v-e* scattering:
 - $v + e^- \rightleftharpoons v + e^-$ (thermalization)
- Inelastic ν -nuclei scattering: $\nu + A \rightleftharpoons \nu + A^*$

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Neutrino trapping in supernovae

During the collapse of the core of a massive star the densities become so large that even neutrinos become dynamically trapped in the collapsing core at densities $\sim 10^{12}$ g cm⁻³. The neutrino mean free path ($\lambda_{\nu} = 1/n\sigma$) can be estimated from the expression for the cross section (assume matter composed of nuclei with A = 110 Z = 40).

$$\sigma(E_{\nu}) = \frac{G_F^2}{4\pi\hbar^4 c^4} E_{\nu}^2 \left[N - (1 - 4\sin^2\theta_W) Z \right]^2$$

$$1/\lambda_{\nu} = \frac{\rho G_F^2}{4\pi (\hbar c)^4 A m_u} E_{\nu}^2 N^2 \approx 2.5 \times 10^{-9} \rho_{12} E_{\nu}^2 N^2 / A$$
$$\lambda_{\nu} \approx 220 \text{ m} (E_{\nu} = 20 \text{ MeV})$$

The diffusion time for a distance of 30 km is:

$$t = \frac{3L^2}{c\lambda_{\nu}} \approx 41 \text{ ms}$$

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Importance trapping

• After trapping and thermalization, neutrinos becomes degenerate. They are described by a Fermi-Dirac distribution with chemical potential μ_{ν} given by the weak equilibrium condition:

$$\mu_{\nu} = \mu_e - (\mu_n - \mu_p)$$

- The presence of neutrinos stops electron capture processes and a sizable electron fraction survives the collapse.
- The inner core collapses as a homologous unit.

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Homologous collapse



- After thermalization an inner homologous core forms in which the local sound velocity is larger than the infall velocity.
- Matter in the outer core falls at supersonic velocities.

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Bounce and v_e burst



- Collapse continues until central density becomes around twice nuclear matter density.
- Sudden increase in nuclear pressure stops the collapse and a shock wave is launched at the sonic point. The energy of the shock depends on the Equation of State.
- The passage of the shock dissociates nuclei into free nucleons which costs ~ 8 MeV/nucleon. Additional energy is lost by neutrino emission produced by electron capture (v_e burst).
- Shock stalls at a distance of around 100 km.

Hydrostatic Burning Phases

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Spherical simulation



Hydrostatic Burning Phases

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Neutrino burst



- Burst is produced when shock wave reaches regions with densities low enough to be transparent to neutrinos
- Burst structure does not depend on the progenitor star.
- Future observation by a supernova neutrino detector. Standard neutrino candles.

Hydrostatic Burning Phases

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Delayed explosion mechanism: neutrino heating



Main processes:

```
 \begin{array}{l} \nu_e + n \rightleftarrows p + e^- \\ \bar{\nu}_e + p \rightleftarrows n + e^+ \end{array}
```

Concept of gain radius due to Bethe. Corresponds to the region where cooling (electron positron capture) and heating (neutrino antineutrino absorption) are equal.

Cooling:
$$143 \left(\frac{kT}{2 \text{ MeV}}\right)^6 \text{ MeV/s}$$

Heating:
$$110\left(\frac{L_{\overline{v}_e,52}\epsilon_{\overline{v}_e}^2}{r_7^2}Y_n + \frac{L_{\overline{v}_e,52}\epsilon_{\overline{v}_e}^2}{r_7^2}Y_p\right) \text{MeV/s}$$

Gravitational energy of a nucleon at 100 km: 14 MeV Energy transfer induces convection and requires multidimensional simulations.

Hydrostatic Burning Phases

Nucleosynthesis beyond iron



The stable nuclei beyond iron can be classified in three categories depending of their origin:

- s-process
- r-process
- p-process (γ-process)

Hydrostatic Burning Phases

Nucleosynthesis beyond iron

Three processes contribute to the nucleosynthesis beyond iron: s-process, r-process and p-process (γ -process).



- s-process: relatively low neutron densities, $n_n = 10^{10-12} \text{ cm}^{-3}$, $\tau_n > \tau_\beta$
- r-process: large neutron densities, $n_n > 10^{20} \text{ cm}^{-3}$, $\tau_n < \tau_{\beta}$.
- p-process: photodissociation of s-process material.

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The r-process

The r-process is responsible for the synthesis of half the nuclei with A > 60 including U, Th and maybe the super-heavies.



Main parameter determining the nucleosynthesis is the neutron-to-seed ratio n_s .

$$A_f = A_i + n_s$$

Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

r-process and metal-poor stars





- Stars poor in heavy r-process elements but with large abundances of light r-process elements (Sr, Y, Zr)
- Production of light and heavy r-process elements seems to be decoupled.
- Possible astrophysical scenario: neutrino-driven wind in core-collapse supernova

- Stars rich in heavy r-process elements (Z > 56) and poor in iron (r-II stars, [Eu/Fe] > 1.0).
- Robust abundance patter for Z > 56, consistent with solar r-process abundance.
- These abundance seems the result of events that do not produce iron. [Qian & Wasserburg, Phys. Rept. **442**, 237 (2007)]
- Astrophysical scenario is unknown. It should involve ejection of very neutron-rich matter.



Honda et al, ApJ 643, 1180 (2006)

Core-collapse supernova Nucleosynthesis heavy elements

Evolution composition: assuming NSE



Core-collapse supernova Nucleosynthesis heavy elements

Evolution composition: assuming NSE



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Evolution composition: assuming NSE



Core-collapse supernova Nucleosynthesis heavy elements

Evolution composition: assuming NSE



Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Impact of light nuclei

• Nuclei with A = 5 and A = 8 are not stable.



• Nuclei heavier than A = 7 can only be produced by 3-body reactions:

 $2~^4{\rm He}+n\to ^9{\rm Be}+\gamma$ suppressed due to low densities (Probability $\sim\rho^2$) and high temperatures (destruction $\sim\rho T^3$).

 $3^{4}\text{He} \rightarrow {}^{12}\text{C} + \gamma$

α -rich freeze out

- Previous discussion assumes that nuclear reactions responsible for the build up of heavy nuclei proceed faster than the expansion timescale.
- It needs to be compared with the timescale for destruction of alpha particles by the 3α reaction:

$$\frac{1}{\tau_{\alpha}} = \left| \frac{1}{Y_{\alpha}} \frac{dY_{\alpha}}{dt} \right| = \frac{\rho^2}{2m_u^2} Y_{\alpha}^2 \langle \alpha \alpha \alpha \rangle$$
$$T = 6 \,\text{GK}, \quad \langle \alpha \alpha \alpha \rangle / m_u^2 = 7.6 \times 10^{-11} \,\text{cm}^6 \,\text{g}^{-2} \,\text{s}^{-1}, \quad \rho Y_{\alpha} = 2.5 \times 10^5 \,\text{g cm}^{-3}$$
$$\tau_{\alpha} = 0.4 \,\text{s}$$

• For faster expansions build up of heavy nuclei is suppressed leaving substantial amounts of free protons or neutrons.

Hydrostatic Burning Phases

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Sensitivity to entropy and Y_e

 $s_{\gamma} \sim 7$ photon-to-baryon ratio (B. S. Meyer, Phys. Rept. 227, 257 (1993))



Fig. 2. The timescale for the triple-alpha reaction to occur in a gas of pure ⁴He nuclei as a function of temperature for the indicated values of the photon-to-baryon ratio.



Fig. 3. The regions in the entropy per baryon versus Y_c plane where major freeze-out-from-NSE nucleosynthesis processes occur. This plot has the third axis giving the dynamical timescale suppressed. BBN stands for Big Bang nucleosynthesis.

Hydrostatic Burning Phases

Contours constant Q_{α}

 α separation energies determine the heaviest nuclei that are build before $\alpha\text{-rich}$ freeze out.



FIG. 1.—Contours of (a) constant proton and (b) a-particle separation energy. The contours are given by the first isotope of each element to have a separation energy less than the specified value. The proton separation energies are given only for nuclei with even Z in order to avoid complications from pairing.

From Woosley & Hoffman, Astrophys. J. 395, 202 (1992).

Core-collapse supernova Nucleosynthesis heavy elements

Evolution Abundances

Calculation assuming: s = 250 k, $Y_e = 0.4$, $\tau_{dyn} = 8$ ms, $T(t) = T_0 e^{-t/\tau_{dyn}}$



Core-collapse supernova Nucleosynthesis heavy elements

Evolution Abundances



Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Neutron to seed in adiabatic expansions



FIG. 9.—Y_n/Y_{sered} in a contour plot as a function of initial entropy S and Y_n for an expansion timescale of 0.05 s as expected from SNe II conditions

From Freiburghaus et al., Astrophys. J. 516, 381 (1999)

Hydrostatic Burning Phases

Neutron to seed in adiabatic expansions

$$n_s \sim s^3 / (Y_e^3 \tau_{\rm dyn}), \quad T(t) = T_0 e^{-t/\tau_{\rm dyn}}$$



Combinations Y_e , s, and τ_{dyn} necessary for producing the A = 195 peak. From Y.-Z. Qian, Prog. Part. Nucl. Phys. **50**, 153 (2003)

Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Nuclear physics needs



figure from H. Schatz

Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Classical r-process, $(n, \gamma) \rightleftharpoons (\gamma, n)$ equilibrium

- Need: mix of suitable heavy seed nuclei (A=56-90) and neutrons
 - sufficient large number density of neutrons (max at least ~1e24 cm⁻³)
 - sufficient large neutron/seed ratio (at least ~100)



Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

$(n, \gamma) \rightleftharpoons (\gamma, n)$ equilibrium

If the r-process occurs in $(n, \gamma) \rightleftharpoons (\gamma, n)$ equilibrium:

$$\mu(Z, A+1) = \mu(Z, A) + \mu_n$$

$$\frac{Y(Z, A+1)}{Y(Z, A)} = n_n \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2} \left(\frac{A+1}{A}\right)^{3/2} \frac{G(Z, A+1)}{2G(Z, A)} \exp\left[\frac{S_n(Z, A+1)}{kT}\right]$$

The maximum of the abundance defines the r-process path:

$$S_n^0(MeV) = \frac{T_9}{5.04} \left(34.075 - \log n_n + \frac{3}{2} \log T_9 \right)$$

For $n_n = 5 \times 10^{21}$ cm⁻³ and T = 1.3 GK corresponds at $S_n = 3.23$ MeV, $S_{2n} = 6.46$ MeV

Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Dynamical r-process calculations

Dynamical calculations show that the r-process can occur under two different regimes with quite different demands from nuclear physics. [A. Arcones, GMP, Phys. Rev. C **83**, 045809 (2011)]

- High temperature "hot" r-process [classical $(n, \gamma) \rightleftharpoons (\gamma, n)$ equilibrium]
- Low temperature "cold" r-process [competition between (n, γ) and β^- , Blake & Schramm, ApJ **209**, 846 (1976)]



Hydrostatic Burning Phases

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hot vs. cold r-process

Hot r-process



Cold r-process



Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Why such a large differences?

Currently available mass models show big differences in the predicted masses before and after the neutron shell closures, where one expects transitions from deformed to spherical nuclei.



Hydrostatic Burning Phases

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FAIR: a new era in our understanding of the r-process

the FAIR reach for nuclear masses.



Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Neutrino emission from the proto-neutron star



- Gravitational binding energy: $E_{\rm grav} \approx G M^2 / R \sim 10^{53}$ erg.
- Neutrino emission lasts around 10 s with energies $E_{\nu} \sim 10$ MeV.
- Enormous neutrino fluxes around the neutron star surface: $\Phi_{\nu} = 10^{43} \text{ cm}^{-2} \text{ s}^{-1}$ at 20 km. Gravitational binding energy nucleon ~ 100 MeV.
- With $E_{\nu} \sim 10$ MeV the typical neutrino-nucleon cross section is 10^{-41} cm⁻². This results in interaction times of 10 ms.

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Influence of neutrinos on nucleosynthesis

Main processes:

$$v_e + n \rightarrow p + e^-$$

 $\bar{v}_e + p \rightarrow n + e^+$

Neutrino interactions determine the proton to neutron ratio, the ejecta are proton rich if:



- Early times (up to 1-2 seconds): proton-rich ejecta (vp-process).
- Later times: neutron-rich ejecta (r-process)??



Core-collapse supernova Nucleosynthesis heavy elements

Long term evolution neutrino luminosities and average energies

Long-term simulations of the collapse and explosion of an 8.8 $\ensuremath{\text{M}_{\odot}}$ ONeMg core,





Hüdepohl et al., PRL 104, 251101 (2010)

Neutrino driven wind

- At T = 10 GK starts the formation of α -particles (⁴He).
- Between T = 8 GK and T = 3 GK, the formation of seeds occurs.
 Dominating reactions are:
 - $3\alpha \rightleftharpoons {}^{12}C + \gamma$ (proton-rich ejecta)
 - $2\alpha + n \rightleftharpoons {}^{9}\text{Be} + \gamma$ ${}^{9}\text{Be} + \alpha \rightarrow {}^{12}\text{C} + n$ (neutron-rich ejecta).
- At lower temperatures proton (*vp*-process) or neutron (r-process) captures take place.



GMP, Physik Journal 7, 51 (2008)
Hydrostatic Burning Phases

Core-collapse supernova Nucleosynthesis heavy elements

Impact of neutrino interactions on proton-rich ejecta

Once neutrino interactions are consistently included in the nucleosynthesis network, nuclei with A > 64 are produced.



Introduction Astrophysical reaction rate

Hydrostatic Burning Phases

The *vp*-process

- Without neutrino interactions proton-rich ejecta form N = Z iron-group nuclei with A < 64.
- However, nucleosynthesis occurs at the presence of substantial neutrino fluxes.



Introduction Astrophysical reaction rate

Hydrostatic Burning Phases

The *vp*-process

- Without neutrino interactions proton-rich ejecta form N = Z iron-group nuclei with A < 64.
- However, nucleosynthesis occurs at the presence of substantial neutrino fluxes.
- Antineutrino absorption and expansion time scales are similar (~ 1 s)

Neutrinos speed-up the matter flow

 $\overline{v}_e + p \rightarrow e^+ + n$ $n + {}^{64}\text{Ge} \rightarrow {}^{64}\text{Ga} + p$ ${}^{64}\text{Ga} + p \rightarrow {}^{65}\text{Ge} \quad \dots$

These reactions constitute the νp -process C. Fröhlich, *et al.*, PRL **96**, 142502 (2006)



Introduction Astrophysical reaction rate

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Trajectories from supernova simulation (Janka)