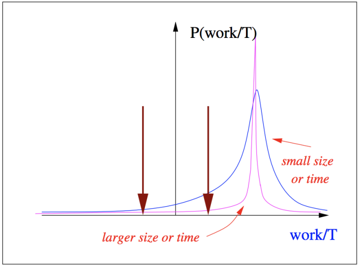
## Distribution of work done during a time t.

## obtained repeating the same protocol




$$\frac{P(\text{work})}{P(-\text{work})} \sim e^{t\ \text{work}/T}$$

**several different versions, depending on initial conditions and the nature of the thermal bath**

**in all but one case, the result holds almost without assumptions**

***what exacltly are experiments testing?***

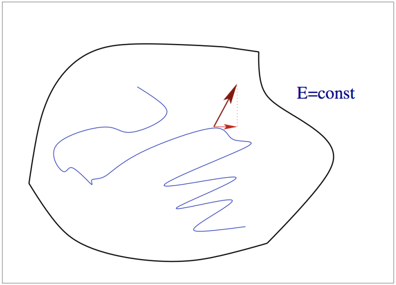
# Stochastic thermal bath: Langevin dynamics

$$\begin{cases} \dot{q}_i & = \frac{\partial \mathcal{H}}{\partial p_i} = \frac{p_i}{m} = v_i \\ \dot{p}_i & = -\frac{\partial \mathcal{H}}{\partial q_i} - \underbrace{f_i(\mathbf{q})}_{Forcing} + \underbrace{\eta_i(t) - \gamma p_i}_{Thermal\ Bath} \end{cases}$$

$$\langle \eta_i(t) \eta_j(t') \rangle = 2\gamma T \delta_{ij} \delta(t - t')$$

**no problem with ergodicity, the noise 'takes you everywhere'**

# Gaussian (Hoover) thermostat




The motion is restricted to the energy shell

However, under forcing, the distribution is not flat

## Gaussian thermostat: deterministic

$$\begin{cases} \dot{q}_i &= \frac{p_i}{m} \\ \dot{p}_i &= -\frac{\partial \mathcal{H}}{\partial q_i} + \underbrace{\gamma(t) p_i}_{thermostat} - \underbrace{f_i(\mathbf{q})}_{forcing} \end{cases}$$

**energy is conserved provided** $\gamma(t) = \frac{\mathbf{f} \bullet \mathbf{p}}{\mathbf{p}^2}$**.**

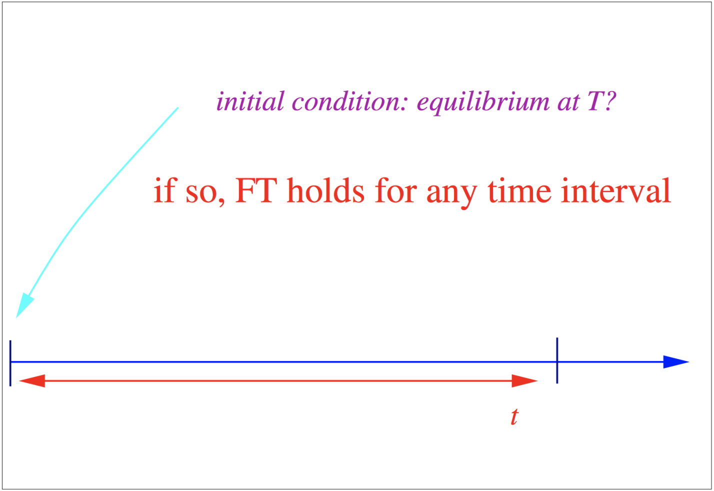
**ergodicity is not guaranteed!**

## Gaussian thermostat: noisy

$$\begin{cases} \dot{q}_i &= \frac{p_i}{m} \\ \dot{p}_i &= -\frac{\partial \mathcal{H}}{\partial q_i} + \underbrace{\gamma(t) p_i}_{thermostat} - \underbrace{f_i(\mathbf{q})}_{forcing} - \underbrace{\eta_i^{\parallel}}_{\substack{conservative \\ noise}} \end{cases}$$

- $\eta_i^{\parallel}$ is a noise tangential to the energy surface
- again, energy is conserved provided $\gamma(t) = \frac{\mathbf{f} \bullet \mathbf{p}}{\mathbf{p}^2}$.

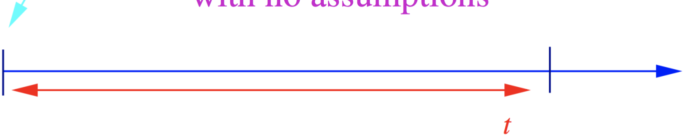
**ergodicity is restored, but what happens in the small noise limit?**


initial condition: equilibrium at T?
if so, FT holds for any time interval
t

*initial condition: equilibrium at T?*

if so, FT holds for any time interval

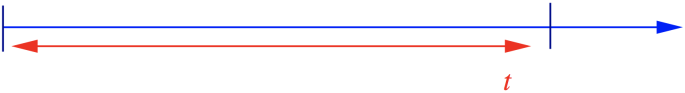
with no assumptions



*STATIONARY INITIAL CONDITION*

FT holds for large time intervals

WE NEED A THERMOSTAT

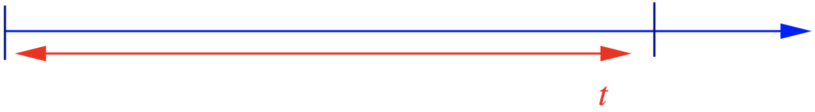


*STATIONARY INITIAL CONDITION*

FT holds for large time intervals

"Stochastic" THERMOSTAT

again, the FT is valid with no assumptions

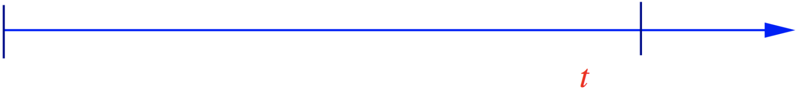


*STATIONARY INITIAL CONDITION*

FT holds for large time intervals

"Deterministic" THERMOSTAT

Gallavotti–Cohen: special ergodicity assumptions




WHAT IS THEIR MEANING?

## The fluctuation theorem is a mathematical truth if:

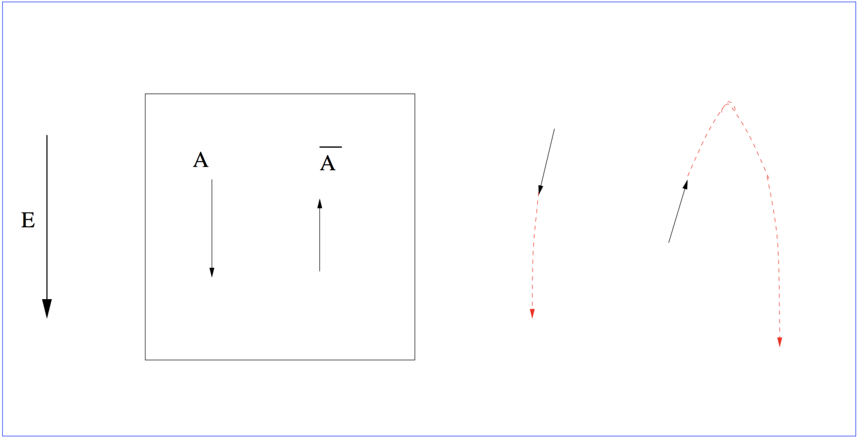
- sampling times are long compared to the longest timescale of the problem
- the thermostat either is stochastic, or can be substituted by one without changing the result.

## MAKING A VIRTUE OF NECESSITY:

Chaotic principle (Gallavotti): just as in equilibrium you assume ergodicity to go to thermodynamics

propose that the fluctuation theorem shows us the way for out of equilibrium: assume that a given system has the necessary properties to give the FT

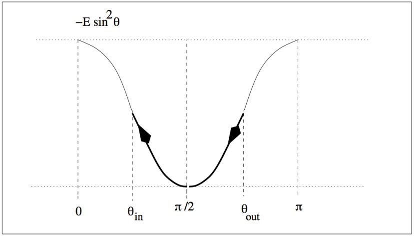
# A simple example Bonetto-Gallavotti



$p_x^2 + p_y^2 = 1$ at all times, and:

$$\begin{aligned} \dot{p}_x &= E - E p_x^2 \\ \dot{p}_y &= -E p_x p_y \\ \dot{\theta} &= -E \sin\theta = -\frac{d}{d\theta}[-E\cos\theta] \end{aligned}$$

**a pendulum!**

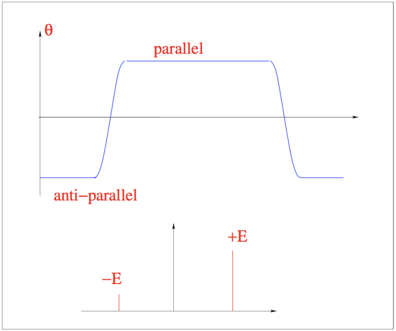


**The system rapidly converges to moving parallel to the field, and there is no reversal of power.**

**The stationary fluctuation relation does not work!**

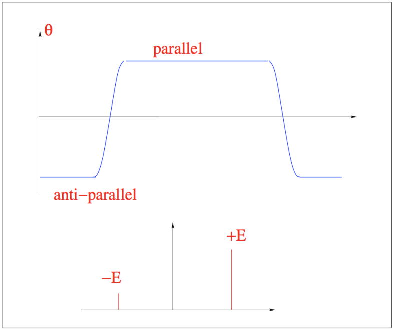
**However, if we add a little bit of noise...**

## The system visits attractor (parallel) and repellor (anti-parallel) in just the right proportion to satisfy the FT



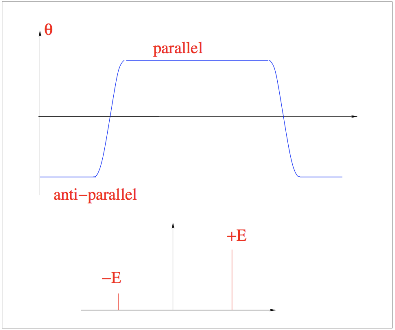

## the noise level only dictates the time to cross from one to the other

## The system visits attractor (parallel) and repellor (anti-parallel) in just the right proportion to satisfy the FT



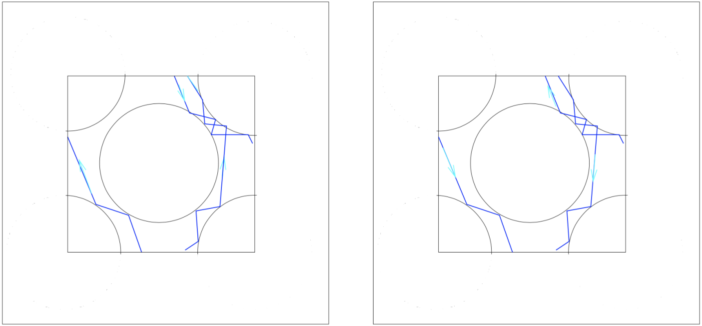

## the noise level only dictates the time to cross from one to the other

**The system visits attractor (parallel) and repellor (anti-parallel) in just the right proportion to satisfy the FT**

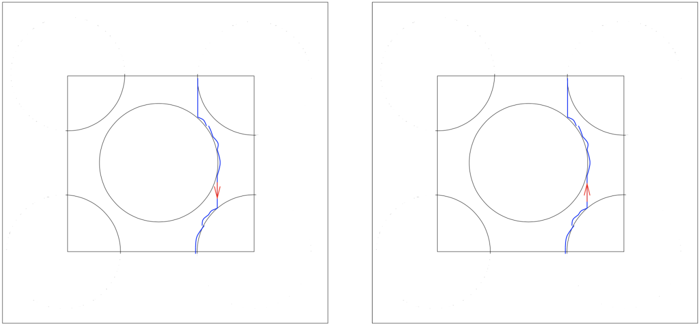



**the noise level only dictates the time to cross from one to the other**

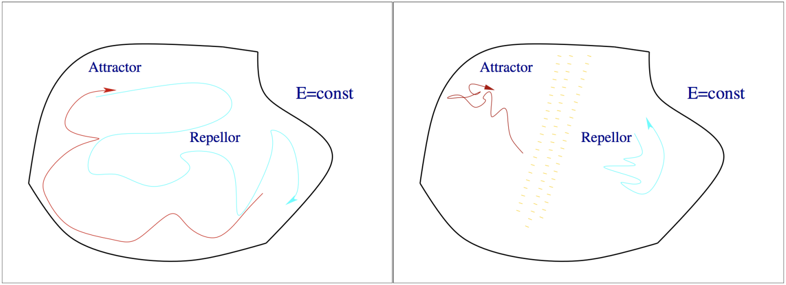
# A chaotic system without forcing: attractor and repellor are intertwined



# A chaotic system with forcing: attractor and repellor divorce



# ON THE ENERGY SHELL:




in a driven system you may need noise to 'push you across the desert'

## IN CONCLUSION:

The Fluctuation Theorem is not only a test of the ergodicty within the attractor

but also a check of the difficulty of passage between attractor and repellor

**The ’chaotic hypothesis’ is then the statement that nothing changes dramatically from small noise to no noise: i.e. stochastic stability**

**Because noise is unavoidable in an experimental system, it seems that failure of observing the fluctuation relation with the appropriate temperature $T$ can only be attributed to experimental timescales being shorter than the 'ergodic' timescale to visit attractor *and* repellor**