Dynamical large deviations and quantum non-equilibrium

Juan P. Garrahan

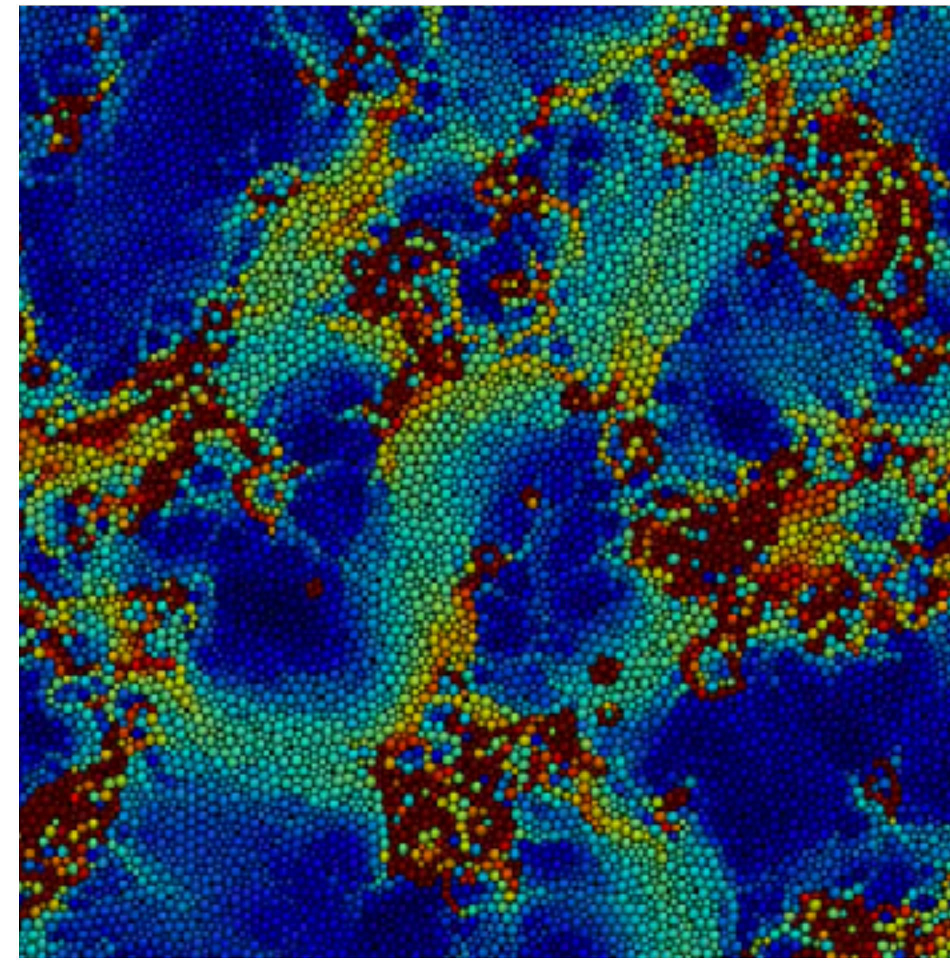
Physics & Astronomy, U. Nottingham

Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems

nottingham.ac.uk/science/schools-centres-and-institutes/cqne

Dynamics is more than statics

Canonical example → (classical) glasses



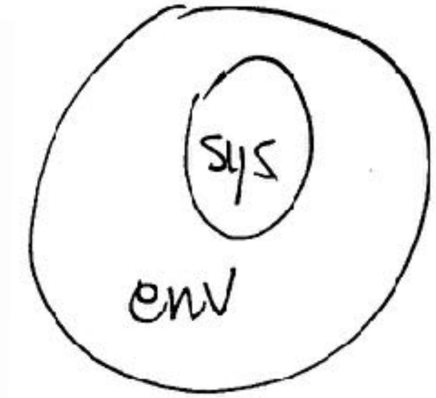
Requires "statistical mechanics of trajectories" via LDs

{Merolle-JPG-Chandler **PNAS 2005**, JPG-Jack-Lecomte-Pitard-van Wijland, **PRL 2007**
Hedges-Jack-JPG-Chandler, **Science 2009**}

Do this for open quantum systems via QLDs

JPG-Lesanovsky, *Thermodynamics of Quantum Jump Trajectories*, Phys. Rev. Lett. **16**, 160601 (2010)

Open quantum systems: $H_{tot} = \underset{\substack{\uparrow \\ \text{sys}}}{H} + H_{env} + H_{int}$



$$\partial_t |\psi_{tot}\rangle = -i H_{tot} |\psi_{tot}\rangle$$

trace over env: $|\psi_{tot}\rangle \longrightarrow \rho =$ mixed state for system (density matrix)

Approx (weak H_{int}, large/fast bath): Quantum Master Eq

$$\partial_t \rho = -i [H, \rho] + \sum_\mu J_\mu \rho J_\mu^\dagger - \tfrac{1}{2} \{J_\mu^\dagger J_\mu, \rho\} \equiv \mathcal{L}(\rho)$$

"escape rate"

super-op.

(Lindblad, Gorini et al)

$$\rho(t) = e^{t\mathcal{L}} \rho(0) \xrightarrow{t \to \infty} \rho_{stat.\,state}$$

QME $\partial_t \rho = -i\left[H, \rho\right] + \sum_\mu J_\mu \rho J_\mu^+ - \frac{1}{2}\left\{J_\mu^+ J_\mu, \rho\right\} \equiv \mathcal{L}(\rho)$

preserves properties of ρ: $\rho^+ = \rho$ $\quad \mathrm{Tr}\rho = 1$ $\quad \mathrm{Tr}\rho^2 \leq \mathrm{Tr}\rho$

Classical:

$\partial_t |P\rangle = \mathbb{W}|P\rangle$

$\langle - | \mathbb{W} = 0$ $\leftarrow$ prob. cons. $\longrightarrow$ $\mathcal{L}^+(\mathbb{1}) = 0$

$\mathbb{W}|\text{st.st.}\rangle = 0$

$\mathbb{W}$ generates stochastic traj between configs $|c\rangle$

NB: QMEq contains Classical M.Eq.

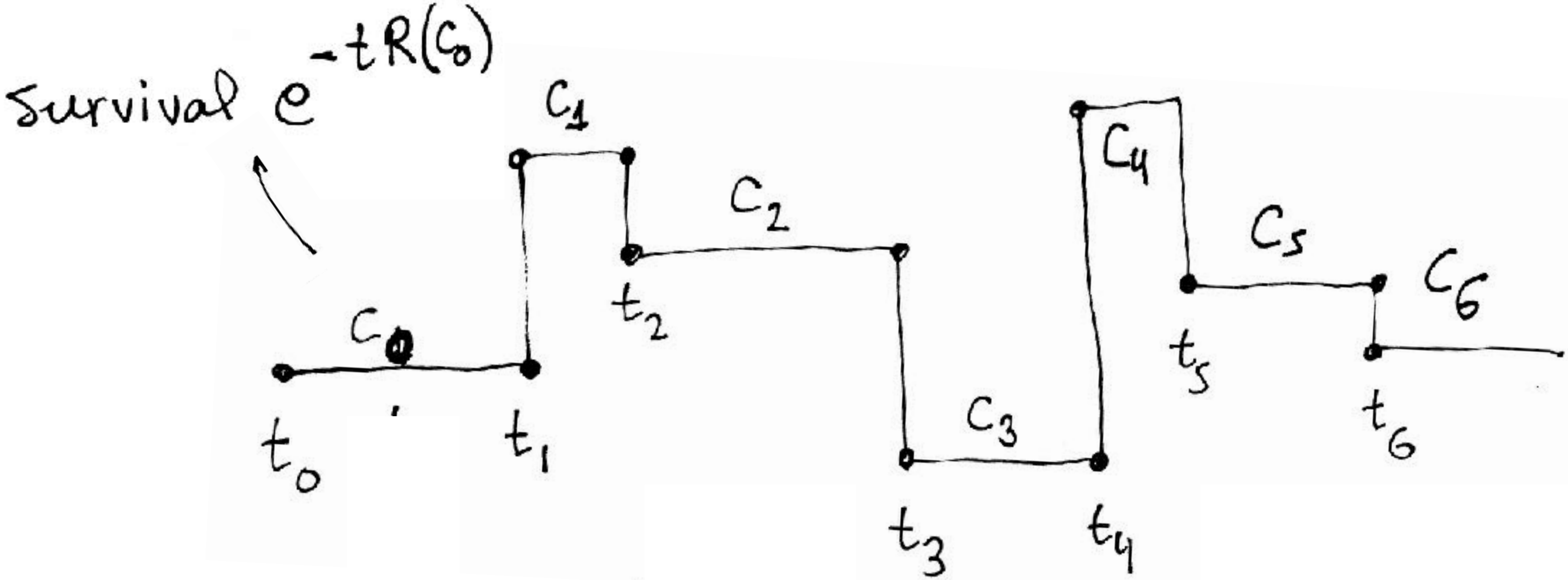
Open Quantum:

$\partial_t \rho = \mathcal{L}(\rho)$

$\mathcal{L}(\rho_{st.st.}) = 0$

Master super op. $\mathcal{L}$ generates Quantum Markov process between pure states $|\psi\rangle$

<u>Classical</u>: $\partial_t |P\rangle = \mathbb{W} |P\rangle$ $\mathbb{W}$ generates stochastic traj between configs $|c\rangle$



$P(c,t)$ = average over traj

Open Quantum: $\partial_t \rho = \mathcal{L}(\rho)$ Quantum Markov process

$$
\begin{tikzcd}
 & \lvert\psi_{t_1}\rangle \arrow[dd, squiggly] & & \lvert\psi_{t_2}\rangle \arrow[dd, squiggly] & \\
\lvert\psi_{t_0}\rangle \arrow[ur, "H_{eff}"] & & & & \cdots \\
 & J_{\mu_1}\lvert\psi_{t_1}\rangle \arrow[uurr] & & J_{\mu_2}\lvert\psi_{t_2}\rangle \arrow[ur] &
\end{tikzcd}
$$

$$H_{eff} = H - \frac{i}{2}\sum_\mu J_\mu^\dagger J_\mu$$

escape rate op.

$$P_{survival}(t) = \langle\psi| e^{itH_{eff}^\dagger} e^{-itH_{eff}} |\psi\rangle$$

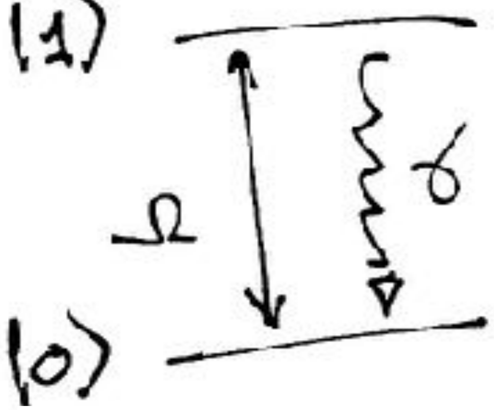
$$\rho(t) = \text{average over many } \lvert\psi_t\rangle$$

Each jump "observed" in environment → q. jump trajectory

eg. emitted photons

time

Example: 2-level system laser driven at $T=0$

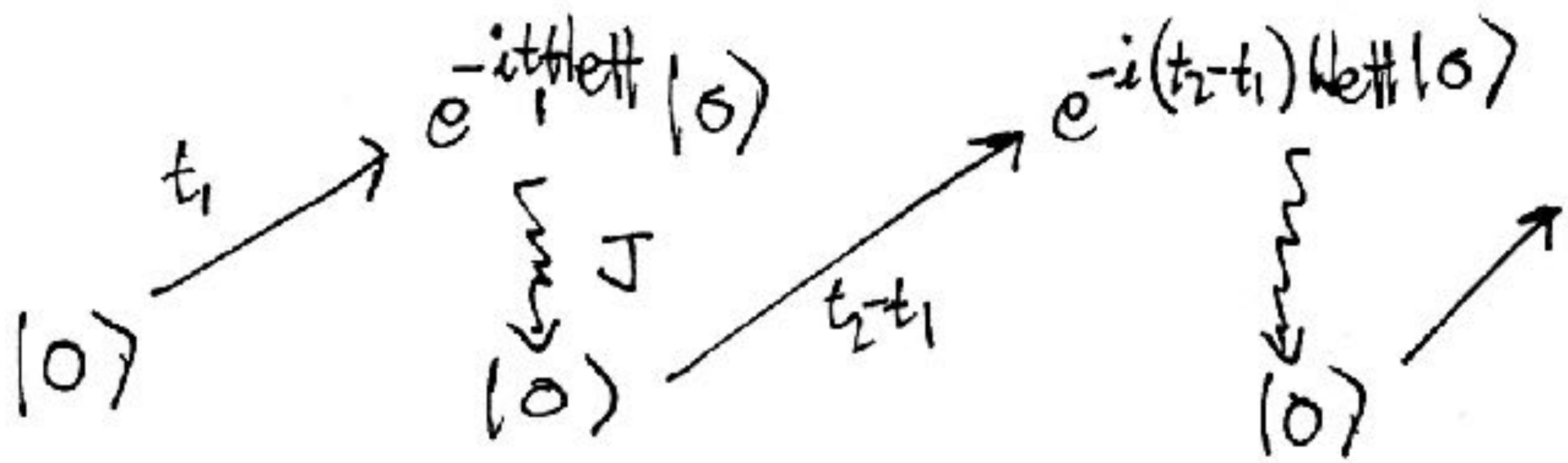


$$\left.\begin{aligned} H &= \Omega\,\sigma_x \\ J &= \sqrt{\gamma}\,\sigma_- \end{aligned}\right\} \to \partial_t \rho = -i\left[\Omega\sigma_x, \rho\right] + \gamma\,\sigma_-\rho\,\sigma_+ - \frac{\gamma}{2}\{n, \rho\}$$

$\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$, $\sigma_- = |0\rangle\langle 1|$, $n = |1\rangle\langle 1|$

$$\rho(t) \xrightarrow{t\to\infty} \rho_{\text{stat. state}} = \begin{pmatrix} 1/6 & i/3 \\ -i/3 & 5/6 \end{pmatrix}$$

(for $\gamma = 4\Omega$ for simpl.)



renewal process
$\neq$ Poisson
(survival $e^{-\gamma t}$)

$H_{eff} = \Omega H - \frac{i}{2}\gamma n$ $\qquad P_{|0\rangle}(t) = e^{-2\Omega t}\left(1 + \Omega t + (\Omega t)^2\right) = \langle 0| e^{it H_{eff}^\dagger} e^{-it H_{eff}}|0\rangle$

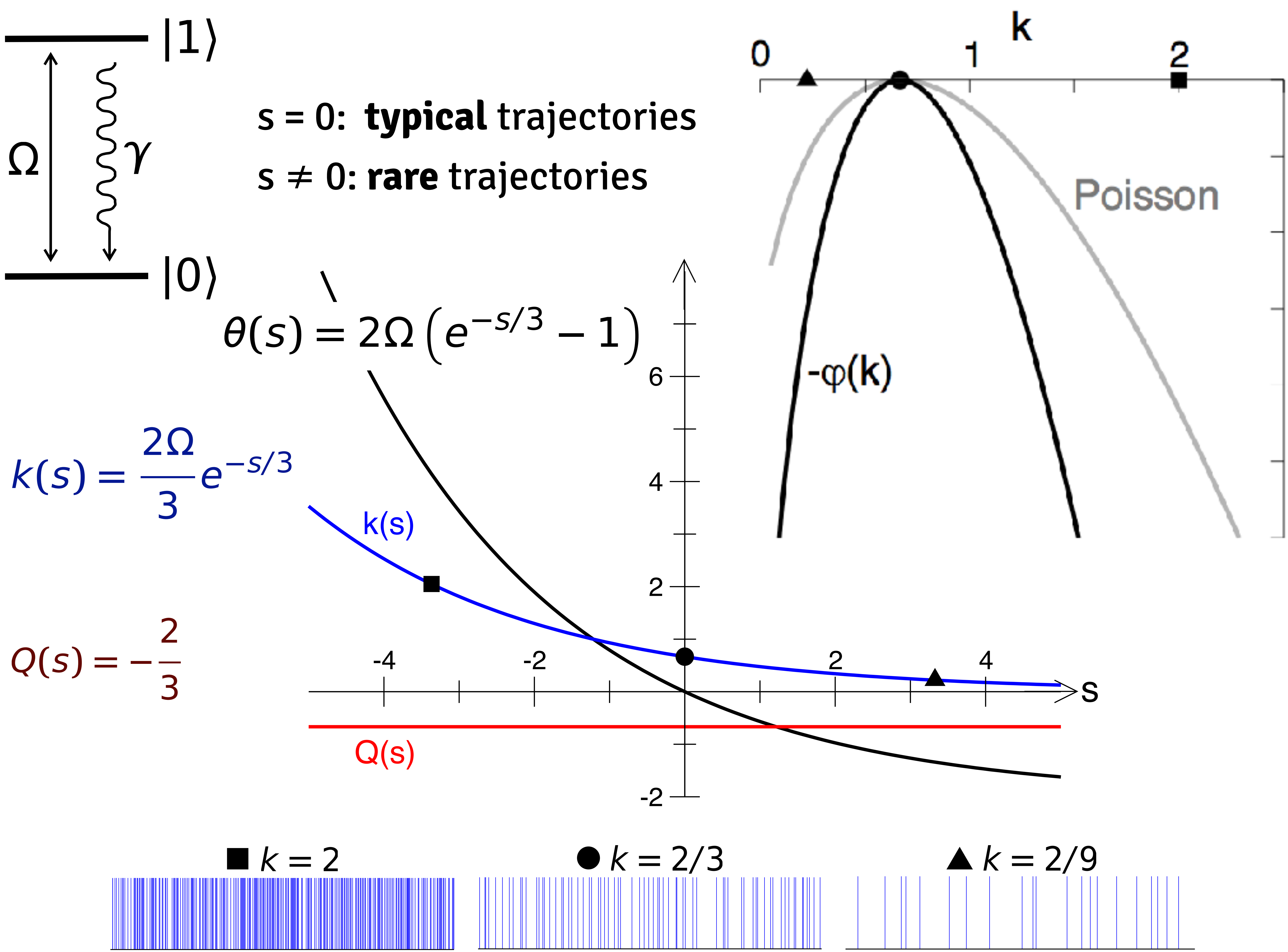
$P_t(K) = \text{Prob}(K \text{ jumps up to } t) \approx e^{-t\varphi(K/t)} \to$ L.D. rate funct.

$Z_t(s) = \sum_K e^{-sK} P_t(K) \approx e^{t\theta(s)}$ → related by legendre tr.

We can get $\theta(s)$ from tilted super-op.

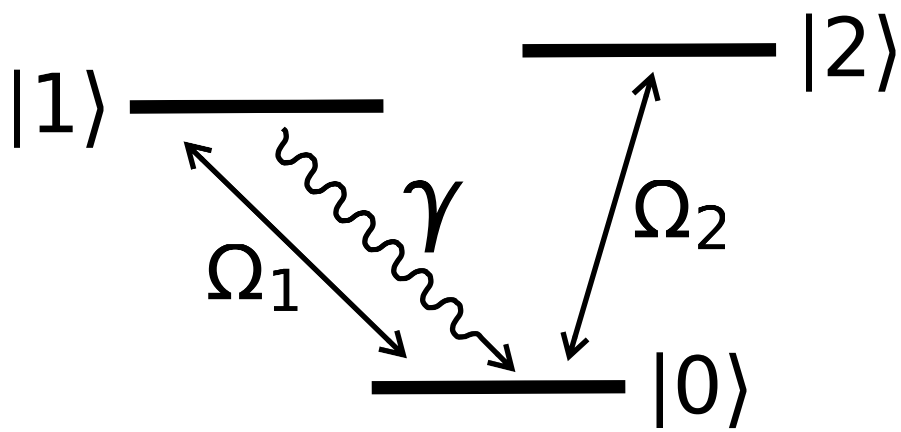
$\mathcal{L}_s \cdot = -i[H, \cdot] + e^{-s} \sum_\mu J_\mu \cdot J_\mu^+ - \frac{1}{2}\{J_\mu^+ J_\mu, \cdot\}$ $\qquad \theta(s) =$ largest eval.

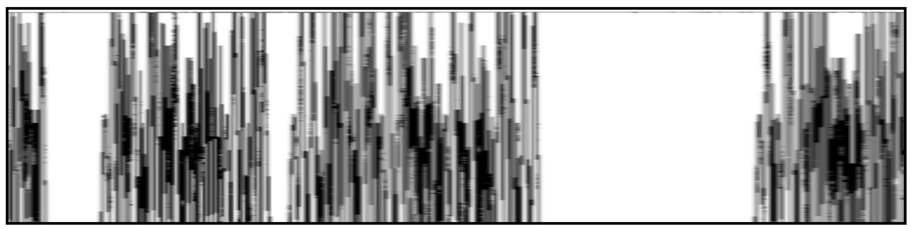
For 2-level $(\gamma = 4\Omega)$: $\mathcal{L}_s$ $(4\times4) \to \theta(s) = 2\Omega(e^{-s/3} - 1)$



|1⟩
|0⟩
Ω
γ
s = 0: **typical** trajectories
s ≠ 0: **rare** trajectories
$\theta(s) = 2\Omega\left(e^{-s/3} - 1\right)$
$k(s) = \frac{2\Omega}{3} e^{-s/3}$
$Q(s) = -\frac{2}{3}$
k(s)
Q(s)
s
k
Poisson
-φ(k)
$k = 2$
$k = 2/3$
$k = 2/9$

Ex. **3-level system**





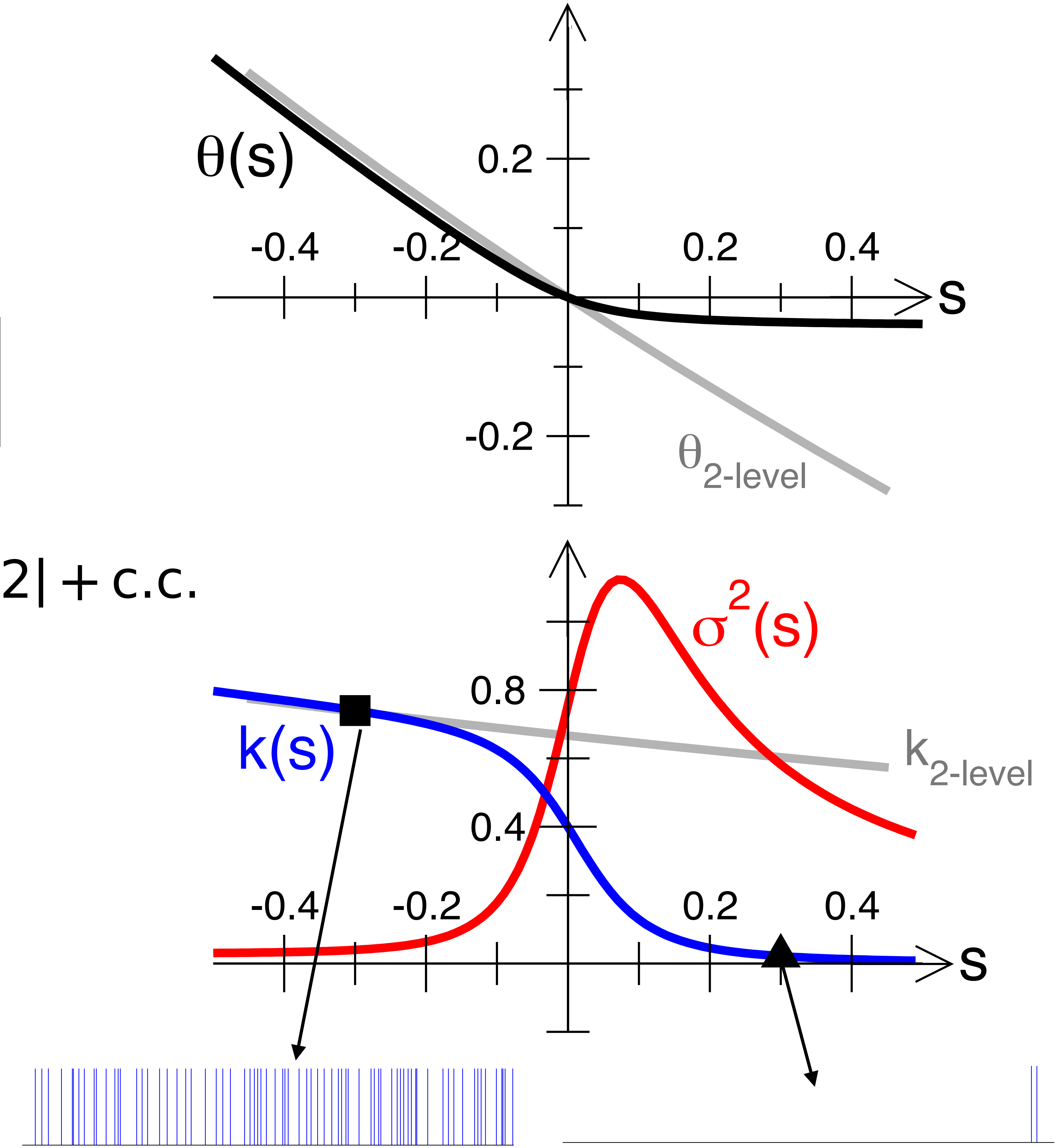
$$\Omega_2 \ll \Omega_1$$

$$H = \Omega_1|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \text{c.c.}$$

$$J = \sqrt{\gamma}|1\rangle\langle 0|$$

$$\mathcal{L} \to \mathcal{L}_s \to \theta(s)$$

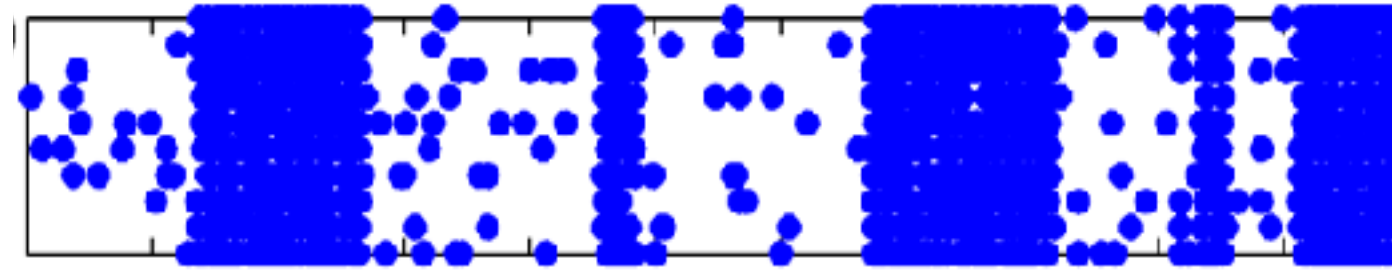
1st-order crossover between active and inactive phases



Example: **dissipative TF Ising model:**

$$H = \Omega \sum_i \sigma_i^x + V \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$$

$$J_i = \sqrt{\kappa}\, \sigma_i^-$$

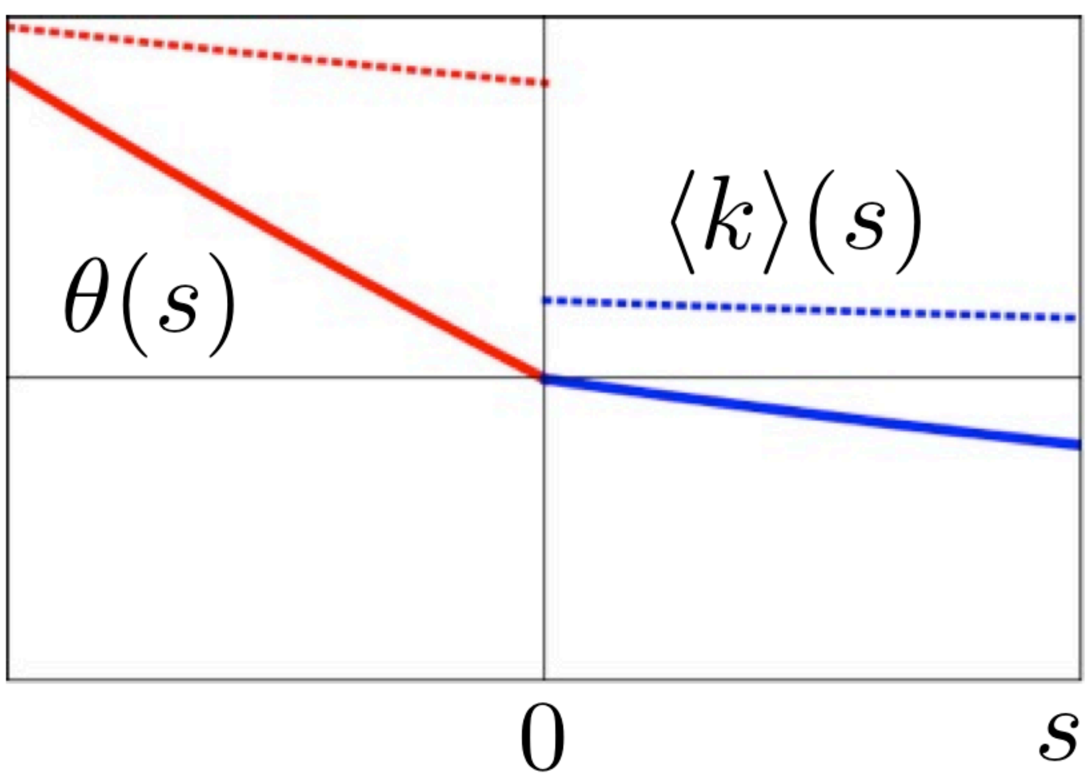


dynamics intermittent

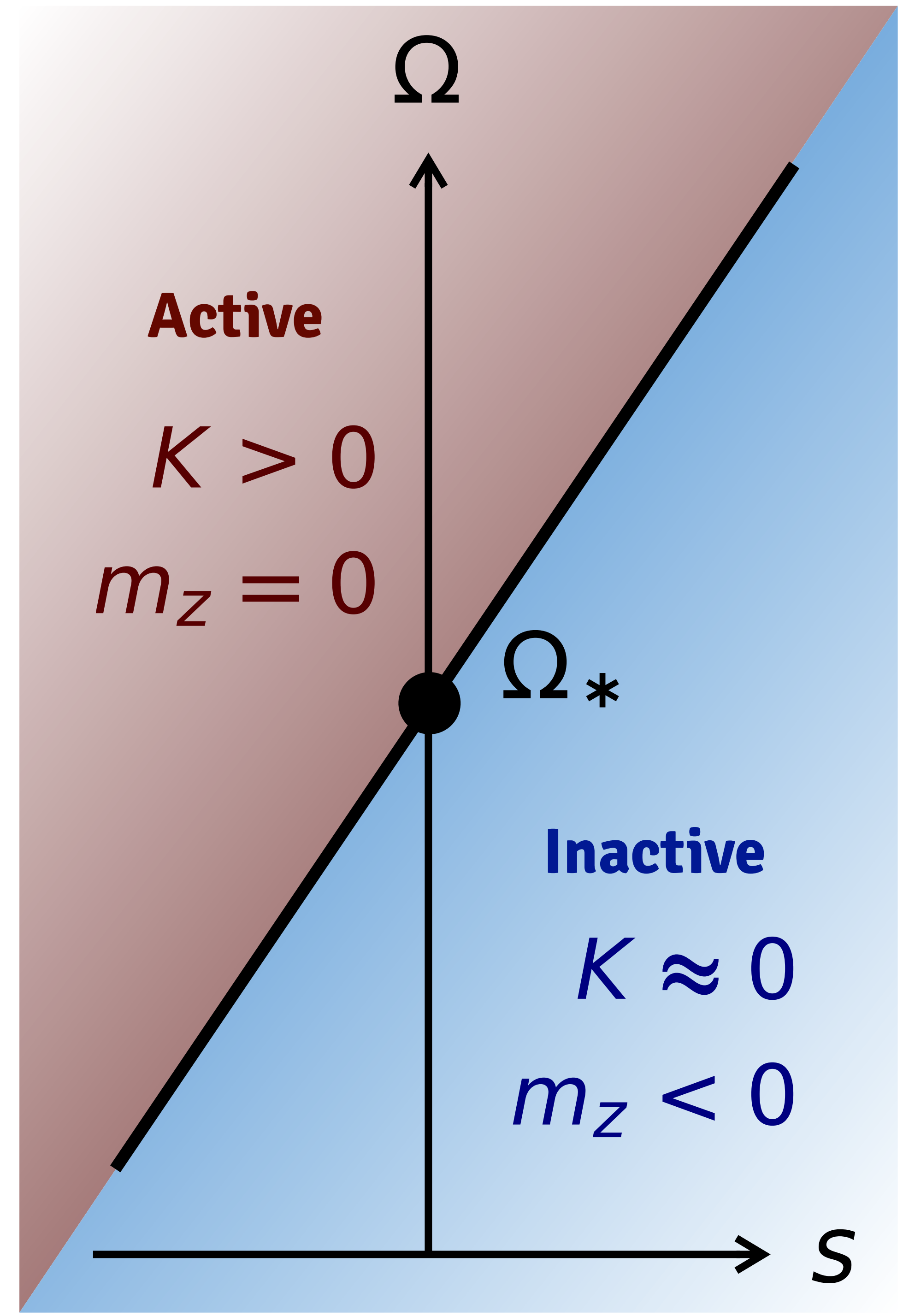
static: **ferro → para**

$$(\downarrow\downarrow \cdots \downarrow) \longrightarrow (\rightarrow\rightarrow \cdots \rightarrow)$$

$$\mathcal{L} \rightarrow \mathcal{L}_s \rightarrow \theta(s)$$



1st order dynamical transition (N→∞)

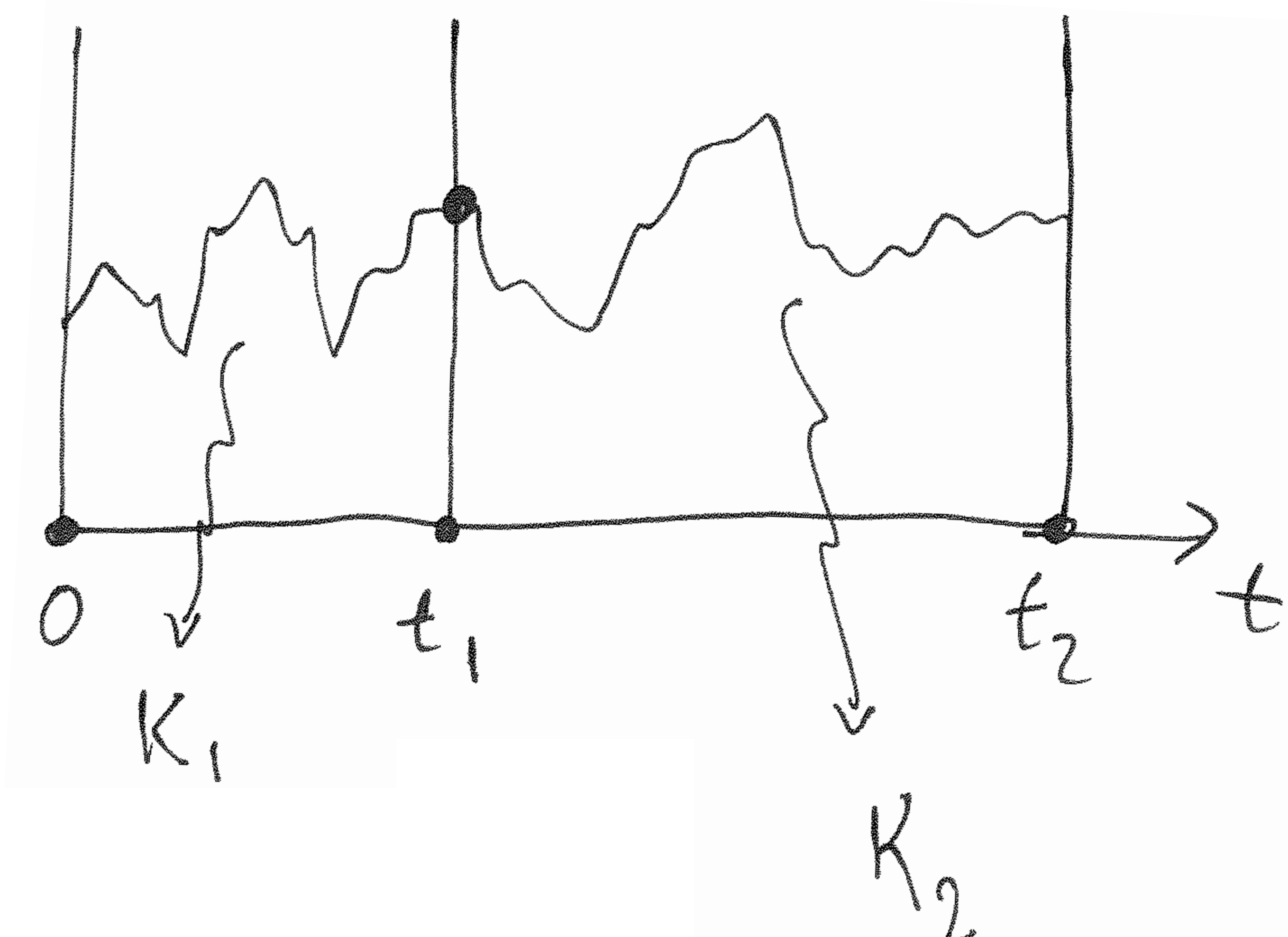


{Ates-Olmos-JPG-Lesanovsky **PRA 2012**, Lesanovsky-van Horssen-Guta-JPG **PRL 2013**, Rose-Macieszczak-Lesanovsky-JPG, **PRE 2016**}

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

measure M (eg ± 1) ie perturbation



I observe K_1 $[0, t_1]$

You measure M (w/o telling me)

I observe K_2 $[t_1, t_2]$

Can I use info about "future" to get better estimate of "past" (retrodiction)

cf. Gammelmark-Julsgaard-Mølmer 2013
several others

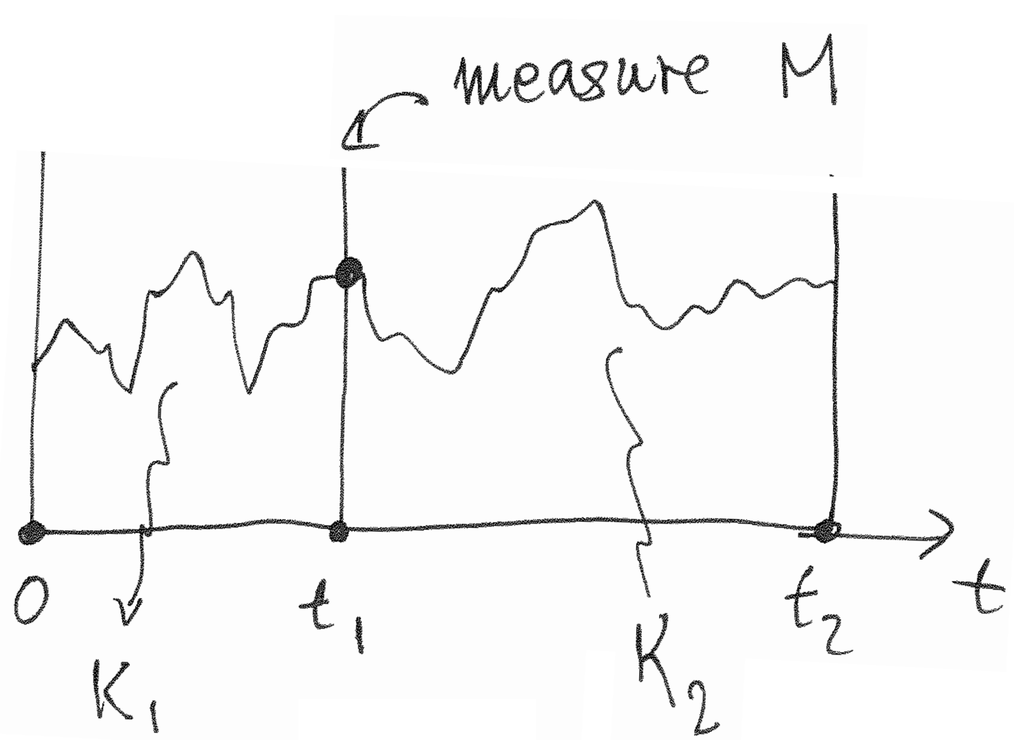
Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

M has $\{m\}$ outcomes $\rightarrow$ $p(m)$

if $m = \pm 1$ $\quad p \equiv p(m=+1)$ is all we need $\Rightarrow \langle m \rangle = 2p - 1$

$$\langle m \rangle = \langle - | \mathcal{M}_m | \rho_{ss} \rangle = \mathrm{Tr}\left[\mathbb{1} \cdot \mathcal{M}_m(\rho_{ss}) \right]$$



Ways to estimate p:

- no info

$$p = \frac{1}{2}\left[\langle - | \mathcal{M}_m | \rho_{ss} \rangle + 1 \right]$$

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

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$$\langle m \rangle = \langle - | \mathcal{M}_m | \rho_{ss} \rangle = \mathrm{Tr}\left[\mathbb{1} \cdot \mathcal{M}_m(\rho_{ss}) \right]$$

- info about past

$$p(m \mid \text{past}) = \frac{1}{2}\left[\langle - | \mathcal{M}_m | \rho_{K_1} \rangle - 1 \right]$$

state of system at t_1 given K_1 was observed

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

M has $\{m\}$ outcomes $\rightarrow$ $p(m)$

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$$\langle m \rangle = \langle - | \mathcal{M}_m | \rho_{ss} \rangle = \mathrm{Tr}\left[\mathbb{1} \cdot \mathcal{M}_m(\rho_{ss}) \right]$$

- info about past & future:

$$p(m | K_1, K_2) = \frac{1}{2}\left[\frac{\langle E_{K_2} | \mathcal{M}_m | \rho_{K_1} \rangle}{\langle E_{K_2} | \rho_{K_1} \rangle} - 1 \right]$$

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

M has $\{m\}$ outcomes $\rightarrow$ $p(m)$

if $m = \pm 1$ $\quad p \equiv p(m=+1)$ is all we need $\Rightarrow \langle m \rangle = 2p-1$

$$\langle m \rangle = \langle - | \mathcal{M}_m | \rho_{ss} \rangle = \mathrm{Tr}\left[\mathbb{1} \cdot \mathcal{M}_m(\rho_{ss}) \right]$$

$$\boxed{p(1|k_1 k_2) \geq p(1|k_1) \geq p(1)}$$

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

What are ρ_{K_1} and E_{K_2}?

$$\langle -| \overbrace{J_{t_2} \cdots\cdots J_{t_1+1}}^{\text{"future" } K_2} \; \mathcal{M} \; \underbrace{\overbrace{J_{t_1} \cdots J_2 \, J_1}^{\text{"past" } K_1}}_{\text{eg. Kraus ops}} |\rho_0\rangle$$

measurement

$$\langle E_{K_2}| \qquad\qquad\qquad\qquad |\rho_{K_1}\rangle$$

- Problem: conditioned state hard to calc.

Large deviations and prediction/retrodiction

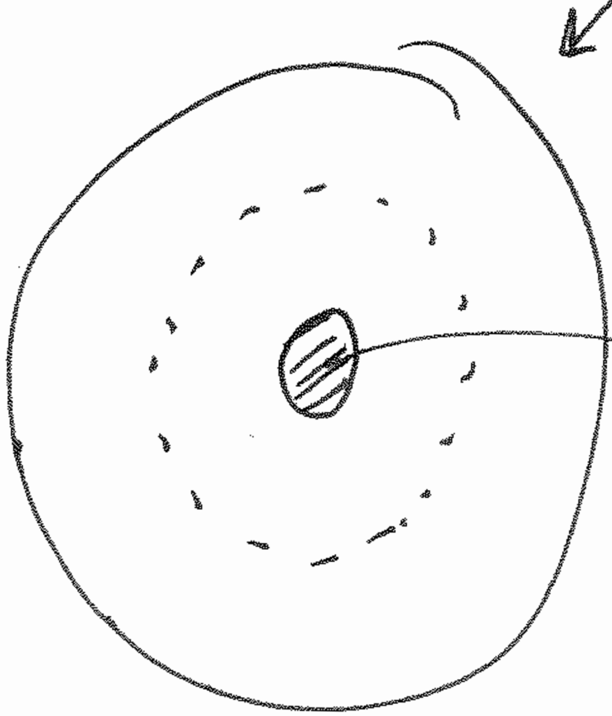
{Kiukas-Guta-Lesanovsky-JPG, to be published}

$$\dot{\rho} = \mathbb{W}(\rho) \rightarrow \rho(t) = e^{t\mathbb{W}}\rho(0)$$ generator

trajectories stochastic $|\psi(t)\rangle$

all traj dynamics (w/their prob)

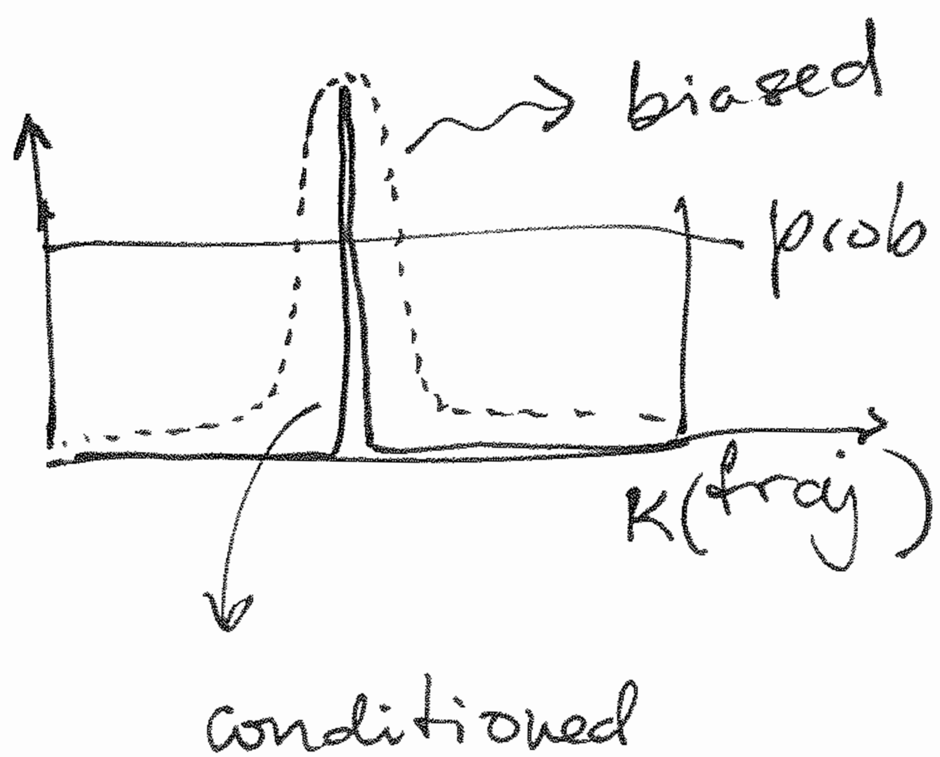
ensemble



traj with

$\hat{K}[x_t] = K$

conditioned ensemble



Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

use equivalence

conditioned ≡ biased

classical = Chetrite-Touchette

open quantum = Kiukas-Guta-Lesanovsky-JPG, **PRE 2015**

$$|\rho_{K_1}) \longrightarrow |\rho_s) \quad \text{from } \mathbb{W}_s : \quad \rho_s = e^{t_1 \mathbb{W}_s} \rho(t=0)$$

with $s: \langle \hat{K} \rangle = K_1$

$$\langle E_{K_2}| \longrightarrow \langle E_s| = e^{(t_2 - t_1)\mathbb{W}_s^{+}} (\mathbb{I}) \qquad s: \langle \hat{K} \rangle = K_2$$

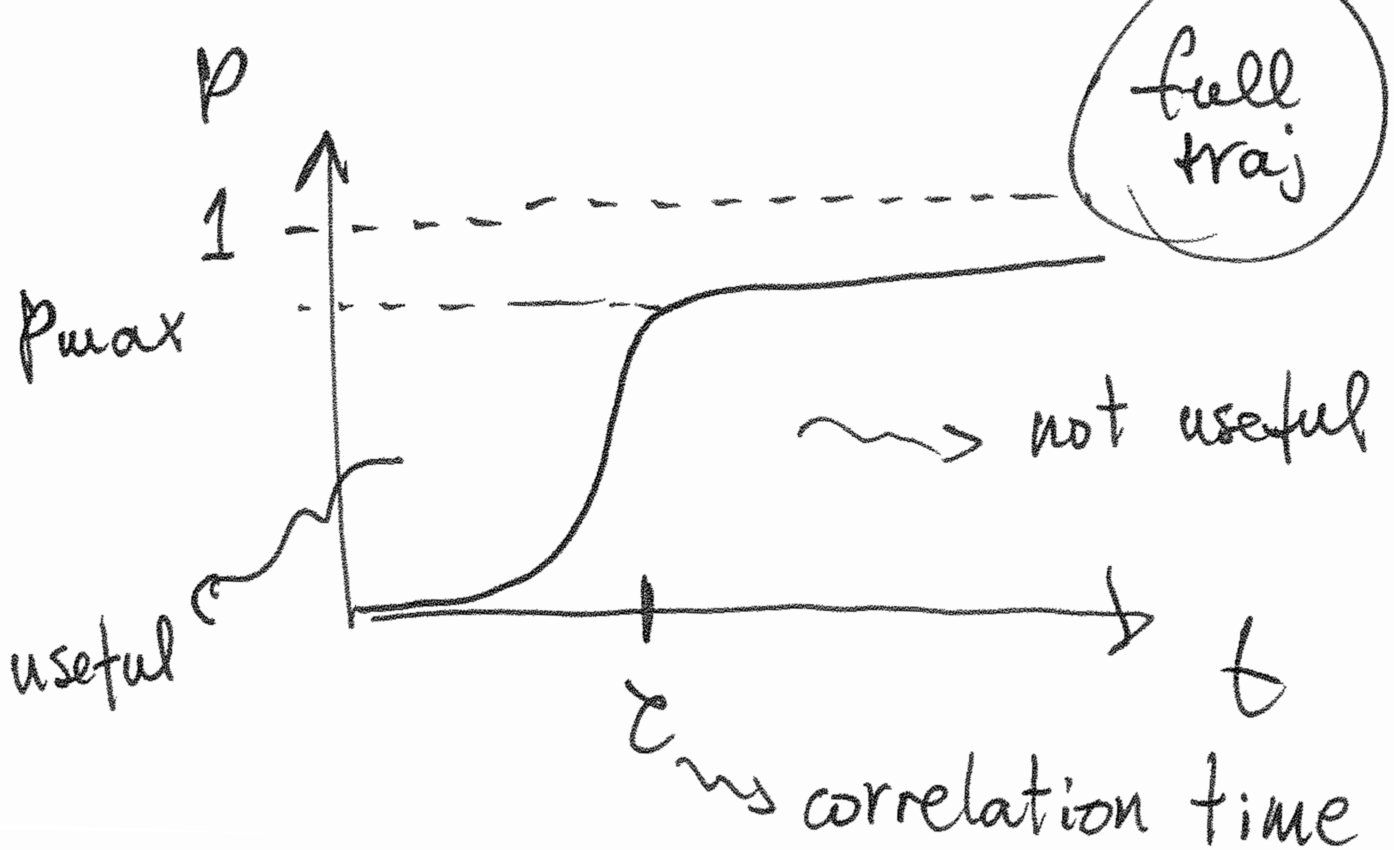
$$p(m | K_1 K_2) \approx \langle E_{s_2} | M | \rho_{s_1}) \Big/ \langle E_{s_2} | \rho_{s_1})$$

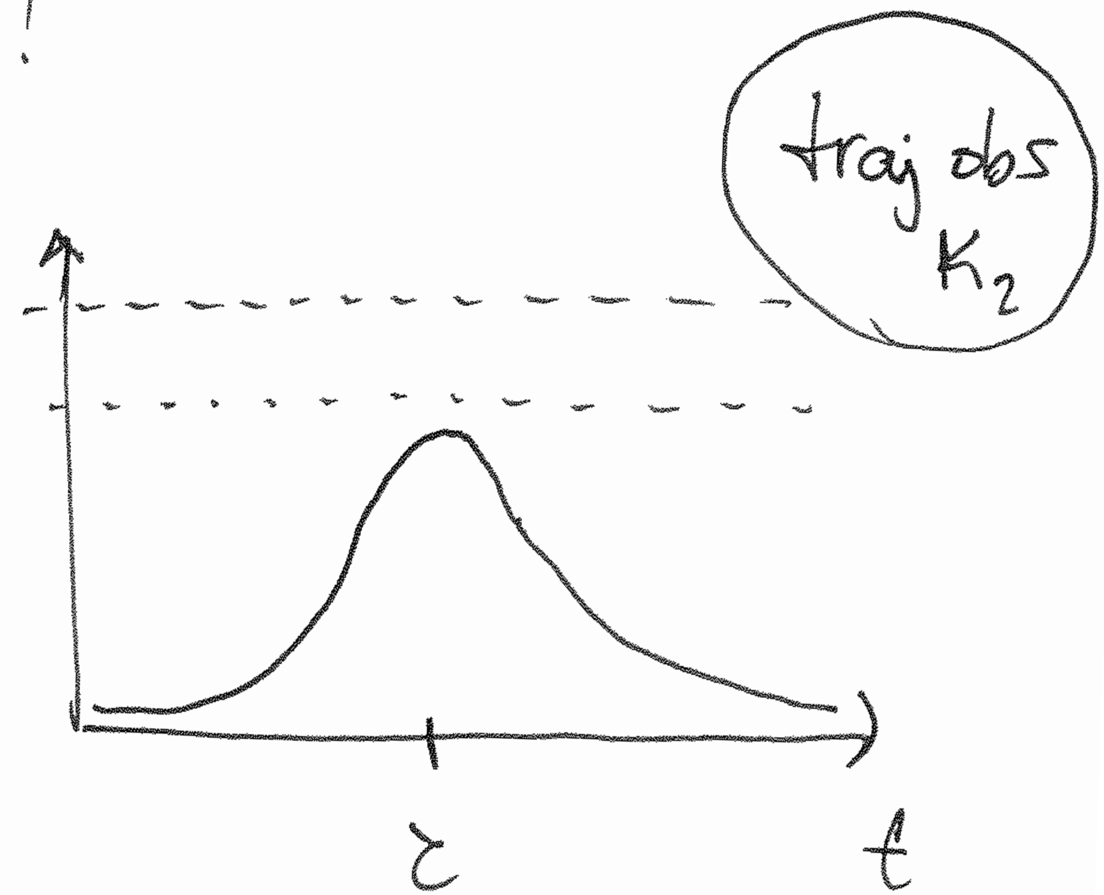
from s-ensemble

Large deviations and prediction/retrodiction

{Kiukas-Guta-Lesanovsky-JPG, to be published}

- How long to observe for afterwards?





Final thing: "entanglement" between past & future

(using MPS to describe pure state of sys + output)

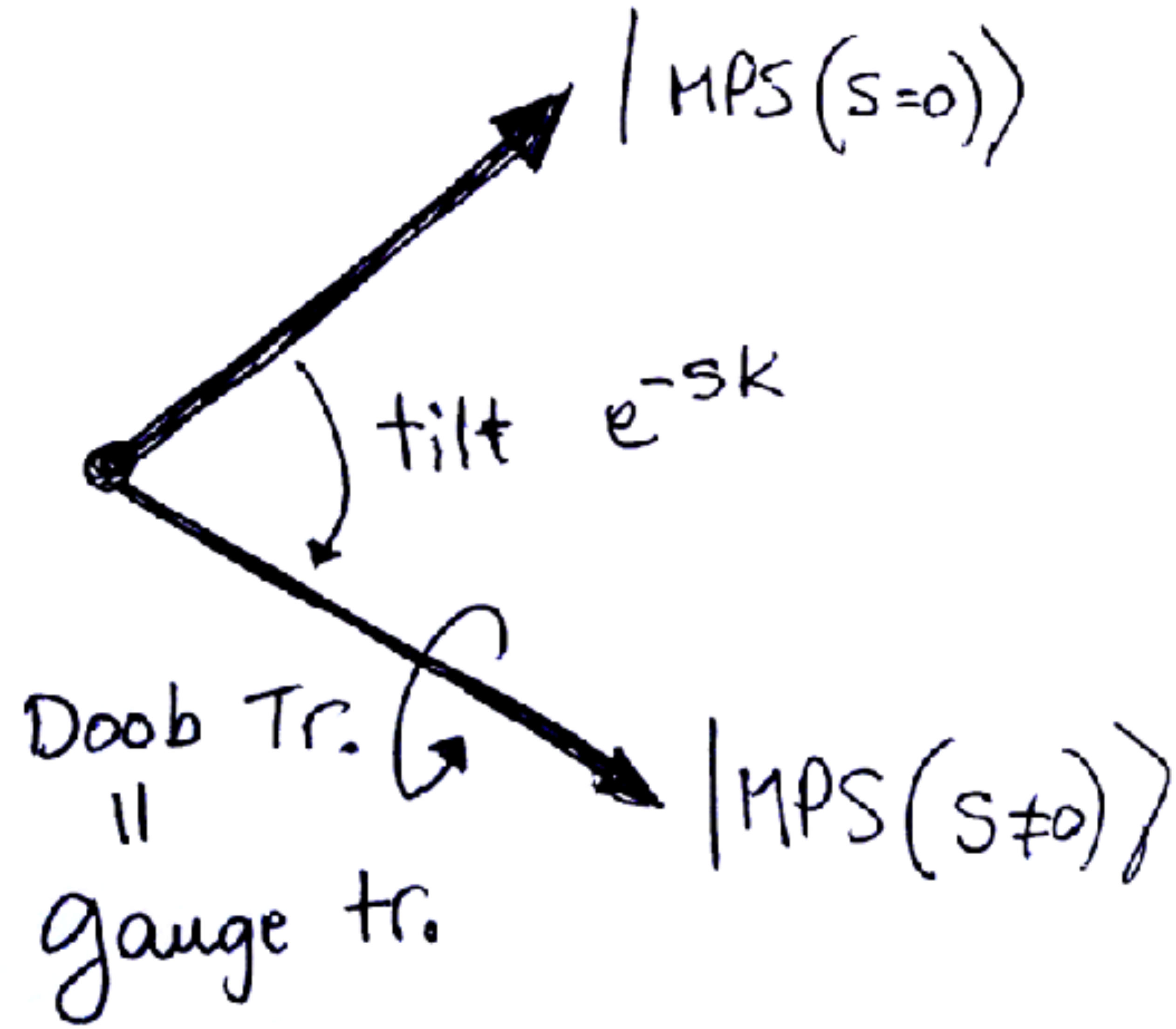
Ensembles of trajectories as Matrix Product States

{cf. Cirac-Verstraete-Osborne-Eisert, Lesanovsky-van Horssen-Guta-JPG **PRL 2013**}

$$|\mathrm{MPS}(s=0)\rangle = \sum_K \int_{0\leq t_1\cdots t_K\leq t} e^{-i(t-t_K)H_{\mathrm{eff}}} \cdots J_{k_2}\, e^{-i(t_2-t_1)H_{\mathrm{eff}}} J_{k_1}\, e^{-it_1 H_{\mathrm{eff}}} |\psi_0\rangle \otimes |t_K \cdots t_2, t_1\rangle$$

{Jack-Sollich
Chetrite-Touchette}

{JPG, **JSM 2016**}



Missing: quantum open Level 2·5

SUMMARY:

Dynamical LDs applied to open quantum dynamics

- Provides ensemble method for quantum jump trajectories
  → **think of quantum dynamics using thermodynamic concepts**

- Conditioned & biased ensembles in **prediction-retrodiction**

- Trajectory ensembles as Matrix Product States

I. Lesanovsky

M. Guta

J. Kiukas

M. van Horssen
D. Rose
K. Macieszczak

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