Analytical methods for extreme fluctuations in generalised exclusion processes

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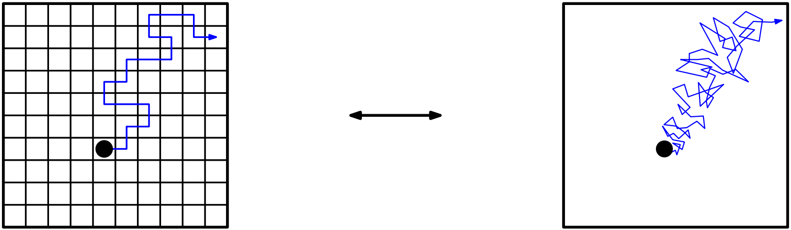
Layout

- **Introduction**: lattice gas models, large deviations
- **Integrable exclusion process**: definition, known results (dynamical phase transition)
- **Non-integrable exclusion process**: analytical methods for extreme deviations, examples, illustrations
- **Conclusion**

Statistical physics: from micro to macro



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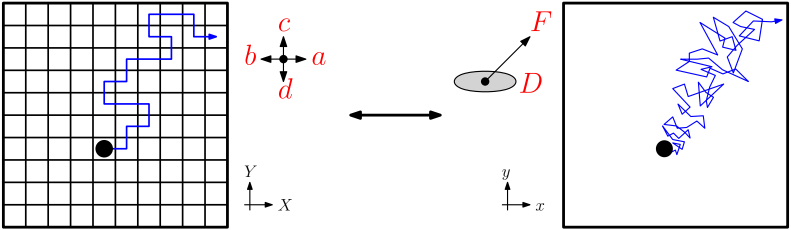

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}(X,Y) =$$

$$a\ \mathrm{P}(X-1,Y) + b\ \mathrm{P}(X+1,Y)$$
$$+\, c\ \mathrm{P}(X,Y-1) + d\ \mathrm{P}(X,Y+1)$$
$$-\,(a+b+c+d)\mathrm{P}(X,Y)$$

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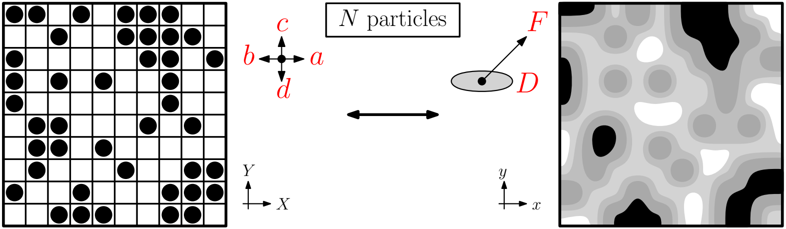

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}(X,Y) = a\ \mathrm{P}(X-1,Y) + b\ \mathrm{P}(X+1,Y) + c\ \mathrm{P}(X,Y-1) + d\ \mathrm{P}(X,Y+1) - (a+b+c+d)\mathrm{P}(X,Y)$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}(C) = \sum k_{C,C'}\mathrm{P}(C') - k_{C',C}\mathrm{P}(C)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P} = M\ \mathrm{P}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(x,y) = -\nabla\cdot\Big[\underbrace{\vec{F}\rho - D\vec{\nabla}\rho + \sqrt{2D\rho/N}\ \vec{\xi}}_{\vec{j}}\Big]$$

Large deviations

Large deviation functions :

- Example : N coin tosses, heads p, tails $1-p$

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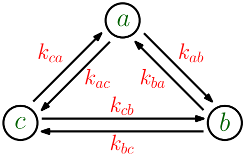

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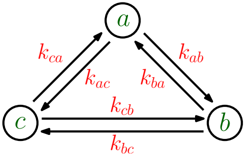

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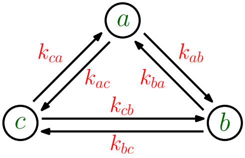

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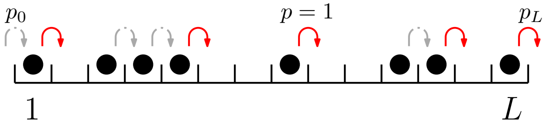
- In practice : $k_{ab} \to k_{ab}\ \mathrm{e}^{\mu}$
  $M \to M_\mu$ with a largest e.v. $E(\mu)$ such that

$$g_2(j_{ab}) = \mu j_{ab} - E(\mu) \quad ; \quad \frac{\mathrm{d}}{\mathrm{d}\mu}E(\mu) = j_{ab}$$

TASEP - definition

Totally asymmetric simple exclusion process (TASEP)



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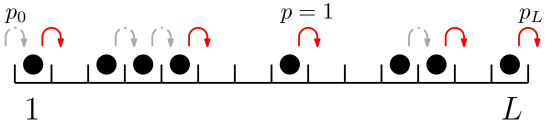

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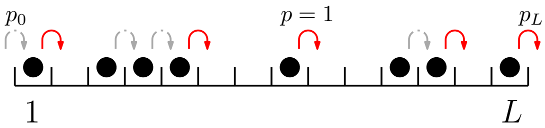

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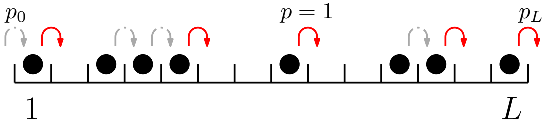

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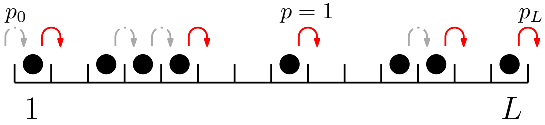

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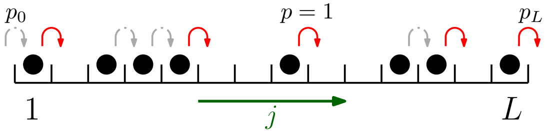

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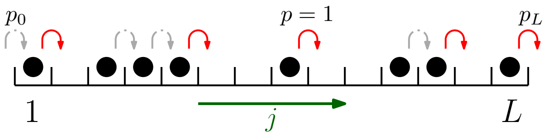

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$$M_\mu \to E(\mu) \to g(j)$$

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$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}\rho = -\nabla j \\ j = \rho(1-\rho) - \frac{1}{2L}\nabla\rho + \sqrt{\frac{\rho(1-\rho)}{L}}\xi \end{cases}$$

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Periodic: [Derrida, Lebowitz ; 1998], [Popkov, Schutz, Simon ; 2007]

Requires long-range correlations: $\langle n_k n_l \rangle_c \sim -(k-l)^{-2}$ for $j \gg \frac{1}{4}$.

Non-integrable generalisations: where to start

$$M_\mu = -M^d + M^+(\mu)$$

- M^d: diagonal, escape rates (sum of rates from each configuration)
- $M^+(\mu)$: off-diagonal, weighted jump rates, of the form

$$\mathrm{e}^{\mu_i}\; A(i,\mathcal{C}',\mathcal{C})\; S_i^- S_{i+1}^+$$

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High current: free fermions

$$M_\mu \sim \mathrm{e}^{\frac{\mu}{L+1}} M^+ \qquad \text{with} \qquad \mu \to \infty$$

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$$M^{+} = e^{-V} \left(p_0 S_1^{+} + \sum_{i=1}^{L-1} p_i S_i^{-} S_{i+1}^{+} + p_L S_L^{-} \right) e^{V}$$

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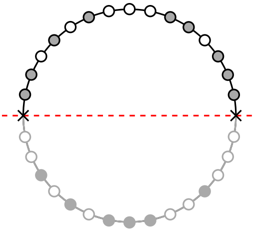

$$\mathrm{cst} \sim \frac{L}{\pi}$$

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Low current: resolvent method

$$M_\mu = -M^d + \varepsilon M^+ \qquad \text{with} \qquad \varepsilon = \mathrm{e}^{\frac{\mu}{L+1}},\ \mu \to -\infty$$

$\Rightarrow$ perturbations in highest eigenspace of $-M^d$, order $> L+1$

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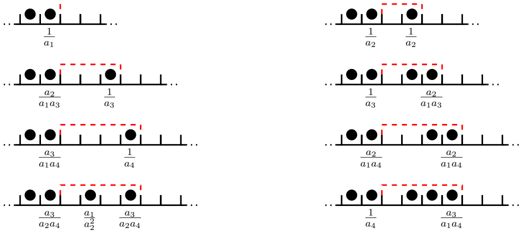
3 steps: • describe $\mathcal{S}$ • analyse cycles • estimate minors of M_{eff}

Low current: result



Width K bounded independently of L.

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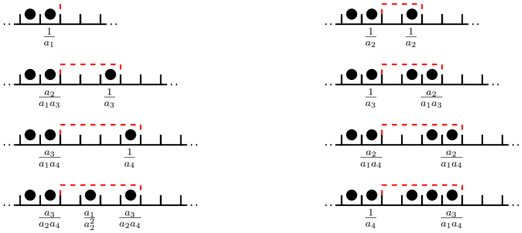


Width K bounded independently of L.

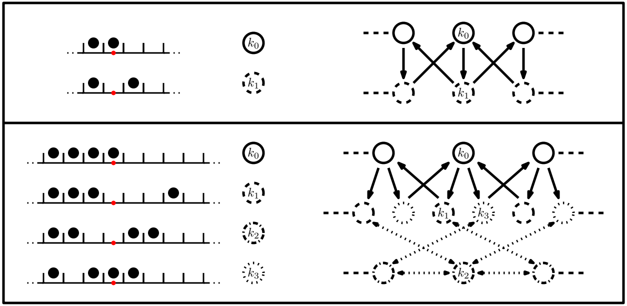
$$E(\mu) + z_0 \sim \mathrm{e}^{C\mu} \ll \mathrm{e}^{2^{-K-1}\mu}$$

$$\boxed{g(j) \sim z_0 + \mathrm{cst}\ j\log(j)}$$

with cst of order at most L^0.

Low current: example

$$K = 2$$



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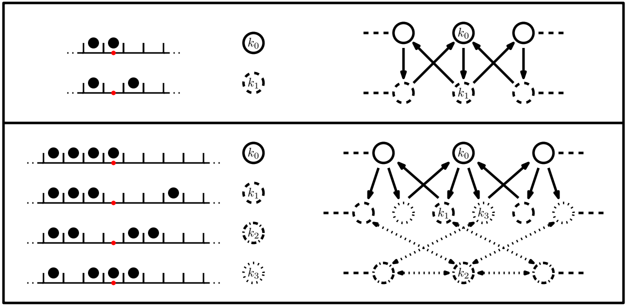

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Low current: locality is important

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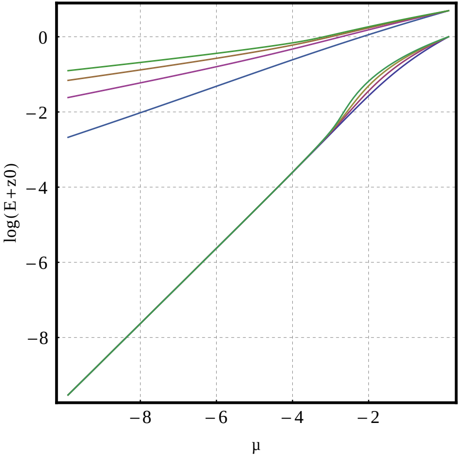
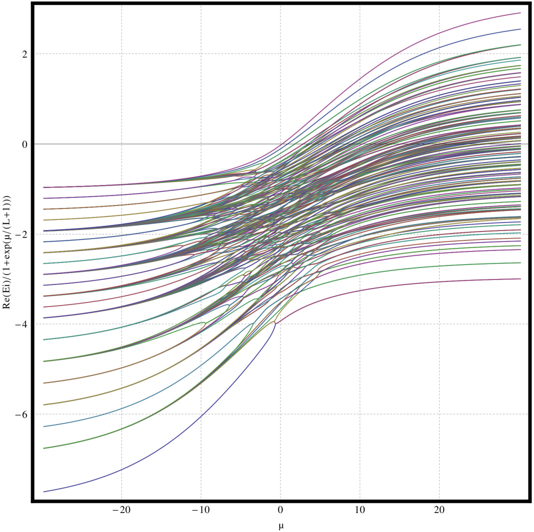


With V_0:
L=10, 8, 6, 4

Without V_0:
L=10, 8, 6, 4

TASEP - Numerics

TASEP with interactions:



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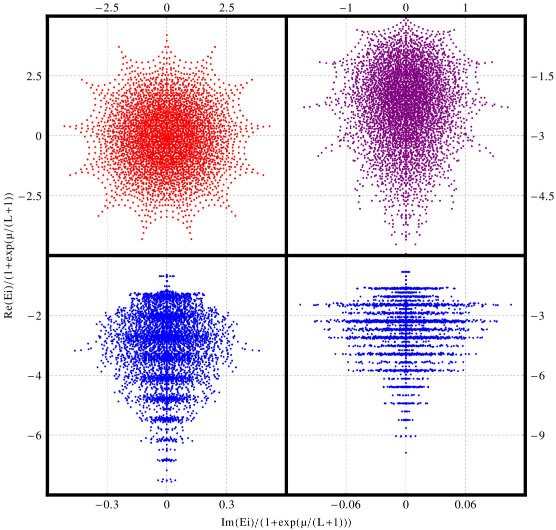

Dynamical phase transition: two scales

Proposed hydrodynamic equation: $j = \sigma(\rho) - \frac{D(\rho)}{2L}\nabla\rho + \sqrt{\frac{\chi(\rho)}{L}}\xi$

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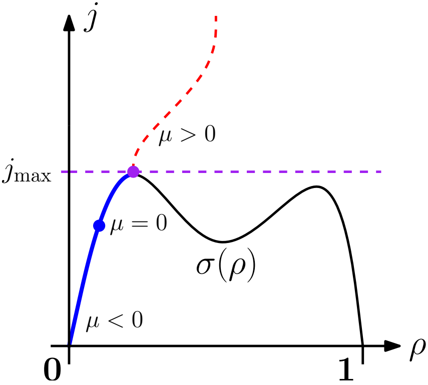
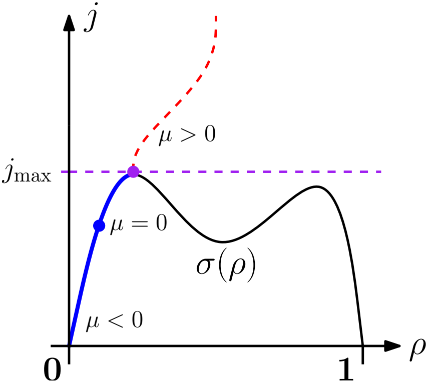


Dynamical phase transition: two scales

Proposed hydrodynamic equation: $j = \sigma(\rho) - \frac{D(\rho)}{2L}\nabla\rho + \sqrt{\frac{\chi(\rho)}{L}}\xi$



KMP, where $\sigma(\rho) = \rho^2$, has no transition [Baek, Kafri, Lecomte ; 2017].

Teaser: Large deviations of the activity

ASEP with low activity:

$$M_\mu = -M^d + \varepsilon \left(M^+ + M^- \right) \qquad \text{with} \qquad \varepsilon = \mathrm{e}^{\frac{\mu}{L+1}},\ \mu \to -\infty$$

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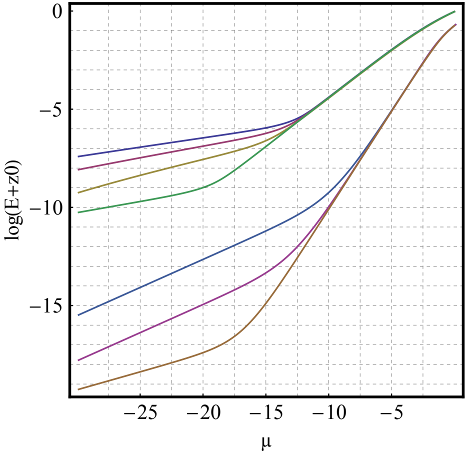

L, q, boundary rates:

10, 10^{-4}, 1
8, 10^{-4}, 1
6, 10^{-4}, 1
8, 10^{-6}, 1

6, 10^{-2}, 1/2
6, 10^{-4}, 1/2
10, 10^{-6}, 1/2

Conclusion

Ref: *Generic Dynamical Phase Transition in One-Dimensional Bulk-Driven Lattice Gases with Exclusion*, A.L.,

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Thank you!