KOREA INSTITUTE FOR ADVANCED STUDY

Carnot Efficiency in an Irreversible Process

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Issues

¶ Carnot efficiency versus Reversibility

- reversible engines $\Rightarrow$ Carnot (maxium) efficiency

  [Carnot(1824), General physics textbooks]

- irreversible engines $\underset{?}{\Rightarrow}$ Carnot efficiency

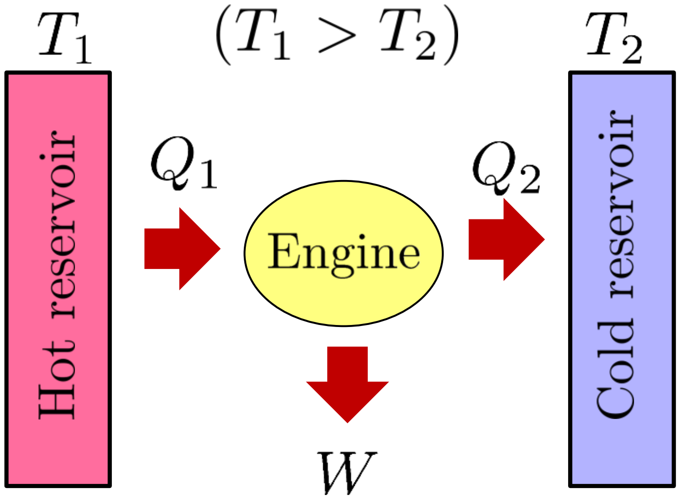
¶ Efficiency versus Irreversibility

- Irreversibility reduces the engine efficiency.

  (only for near reversible engines)

- Irreversibility can enhance the efficiency for highly irreversible engines ?

Heat engine



- energetics : $W = Q_1 - Q_2$ $\qquad (k_B = 1)$

★ thermodyn. : $\Delta S = -\dfrac{Q_1}{T_1} + \dfrac{Q_2}{T_2} \geq 0$

- efficiency : $\eta = \dfrac{W}{Q_1} \leq \eta_C = 1 - \dfrac{T_2}{T_1}$

$$\eta = \eta_C - \frac{T_2 \Delta S}{Q_1} = \eta_C - \eta \frac{T_2 \Delta S}{W}$$

(nonzero power)

† How to reach η_C ?

◇ $\Delta S = 0$ (reversible, quasi-static)

◇ $\Delta S > 0$ (irreversible, $Q_1 \to \infty$) $\qquad Q_2 \to \infty \quad \left(\frac{T_2}{T_1} Q_1 \leq Q_2 \leq Q_1\right)$

- $Q_2 = \alpha Q_1$ $\left(\frac{T_2}{T_1} \leq \alpha \leq 1\right)$ $\Rightarrow \dfrac{T_2 \Delta S}{Q_1} = \alpha - \dfrac{T_2}{T_1}$ $= 0$ at $\alpha = \dfrac{T_2}{T_1}$ $\Rightarrow \Delta S = 0$ $(\gamma < 0)$

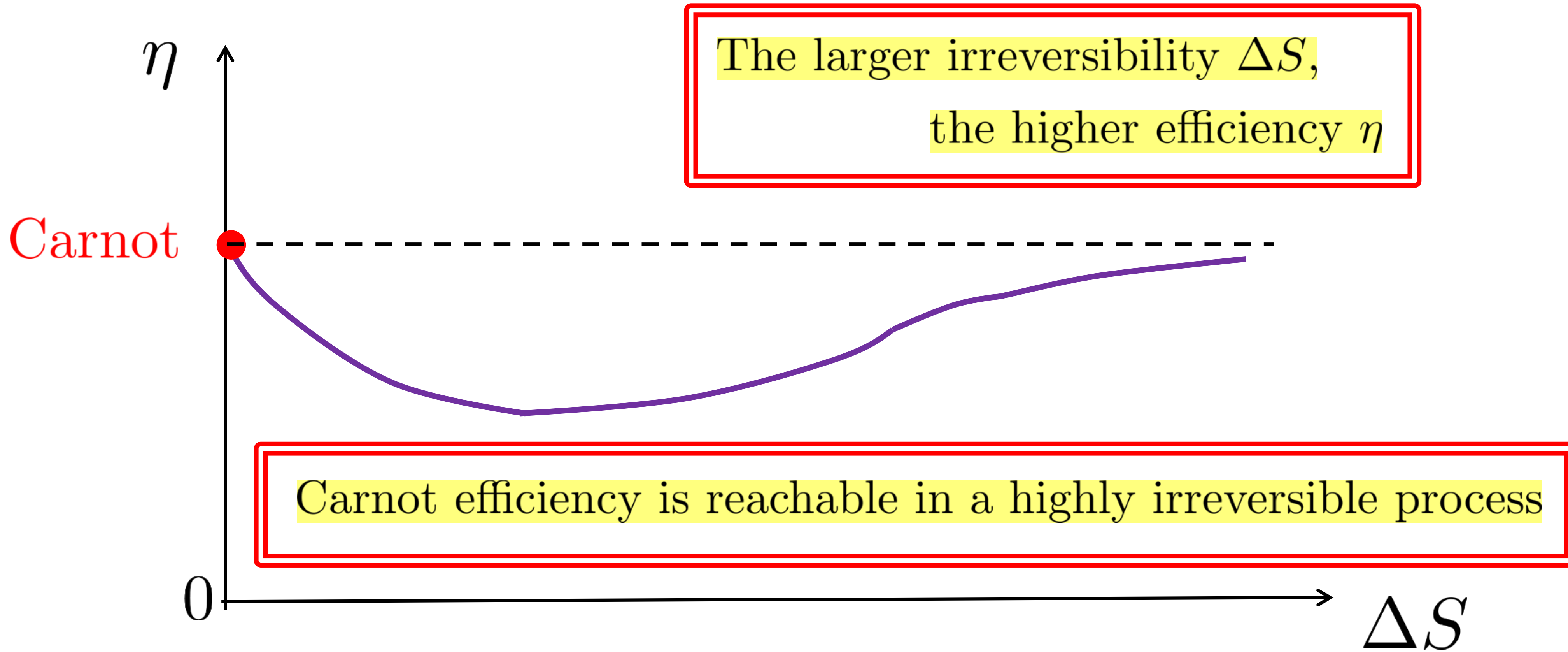
(otherwise, $\eta < \eta_C$) $\qquad$ (reversible)

- $Q_2 \simeq \dfrac{T_2}{T_1} Q_1 + \mathcal{O}(Q_1^\gamma)$ $(\gamma < 1)$ $\Rightarrow$ $\Delta S \sim Q_1^\gamma$ $\xrightarrow[Q_1 \to \infty]{}$ $\eta \to \eta_C$ $(0 \leq \gamma < 1)$

(highly irreversible with Carnot eff.)

- For large Q_1, $Q_2 \simeq \frac{T_2}{T_1} Q_1 + a Q_1^{\gamma} \Rightarrow \Delta S \simeq \frac{a}{T_2} Q_1^{\gamma}$ $(0 \leq \gamma < 1, a > 0)$

$$\eta = \eta_C - \frac{T_2 \Delta S}{Q_1} = \eta_C - a \left(\frac{a}{T_2 \Delta S} \right)^{1/\gamma - 1} \xrightarrow[\Delta S \to \infty]{} \eta_C$$



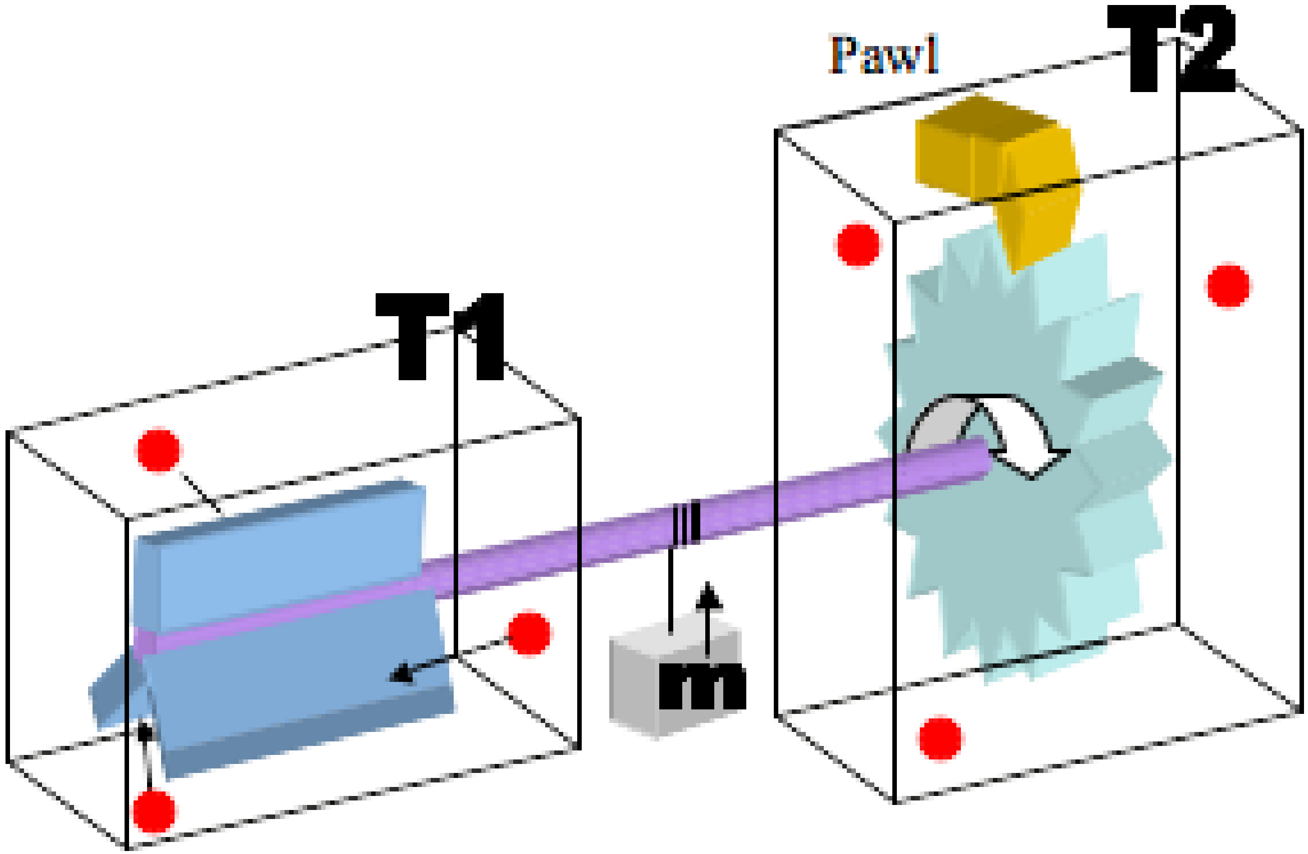
Any realistic system

showing the Carnot efficiency

in a highly irreversible process?

Feynman-Smoluchowski Ratchet (FSR)

[Feynman Lecture I (1983)]



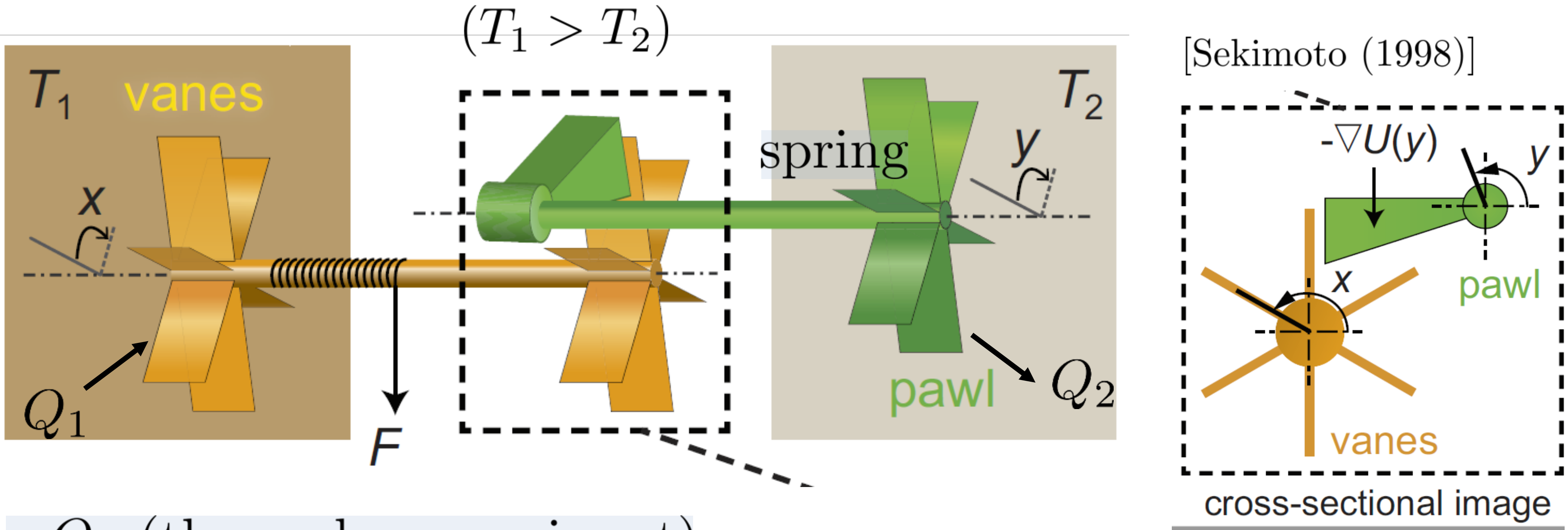
[Smoluchowski (1912)]

[Feynman (1962)]

When $T_1 > T_2$, " ratcheting". Can extract some work (pull the load up).

FSR can attain η_C at an appropriate value of m (detail balance: reversible).

cannot attain η_C due to inherent irreversible current. [Parrondo/Español (1996)]



- Q_1 (thermal energy input)
- $Q_2 = Q_{\rm col} + Q_{\rm hop}$ (nonzero $Q_{2\rm v} \Rightarrow \eta < \eta_C$) [Parrondo/Español (1996)]

⋆ collisions without a hop $\Rightarrow W = 0$, $Q_2 = Q_{\rm col}$: dissipation

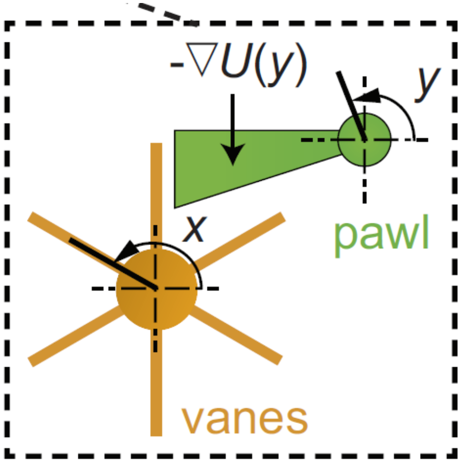
$\Rightarrow$ efficiency lowered ($\eta < \eta_C$)

⋆ successful hops $\Rightarrow W = \pm F x_0$, $Q_2 = Q_{\rm hop} \simeq U_{\rm threshold}$

¶ How to get rid of $Q_{\rm col}$? any controllable limit? Yes! later !

¶ high energy barrier (heat) limit $\Rightarrow$ vanes, pawl: almost EQ

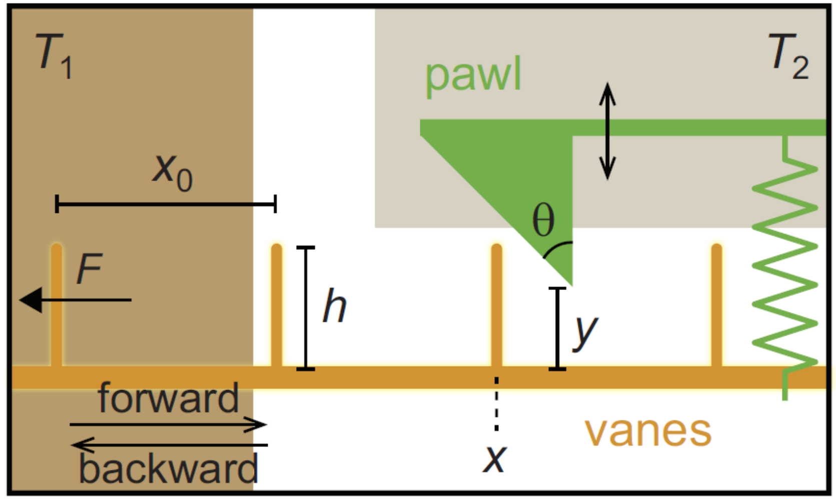
Model



cross-sectional image

$\Rightarrow$

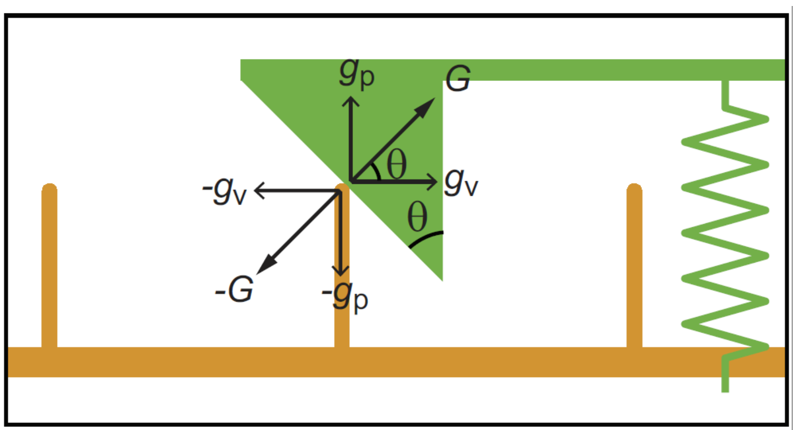
★ pawl: vertical motion only



$U(y) = \frac{1}{2}ky^2$

★ vanes: horizontal motion only

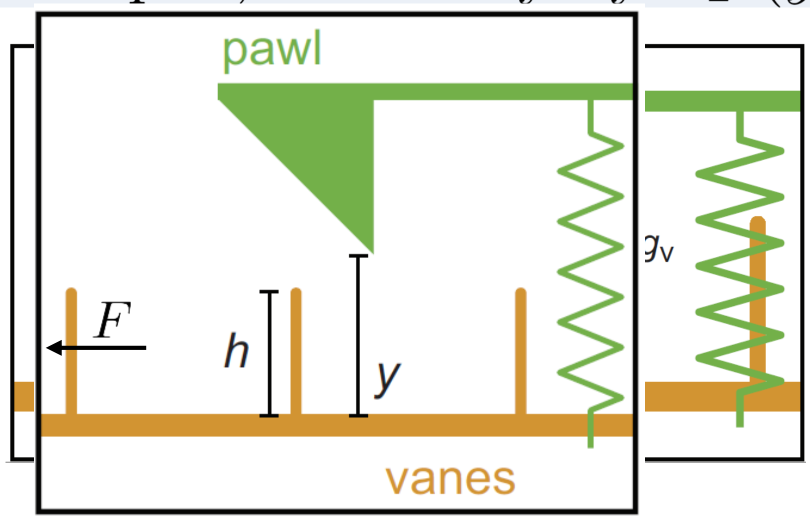
- pawl closed ($y < h$)



★ elastic collision of m and m_{p}

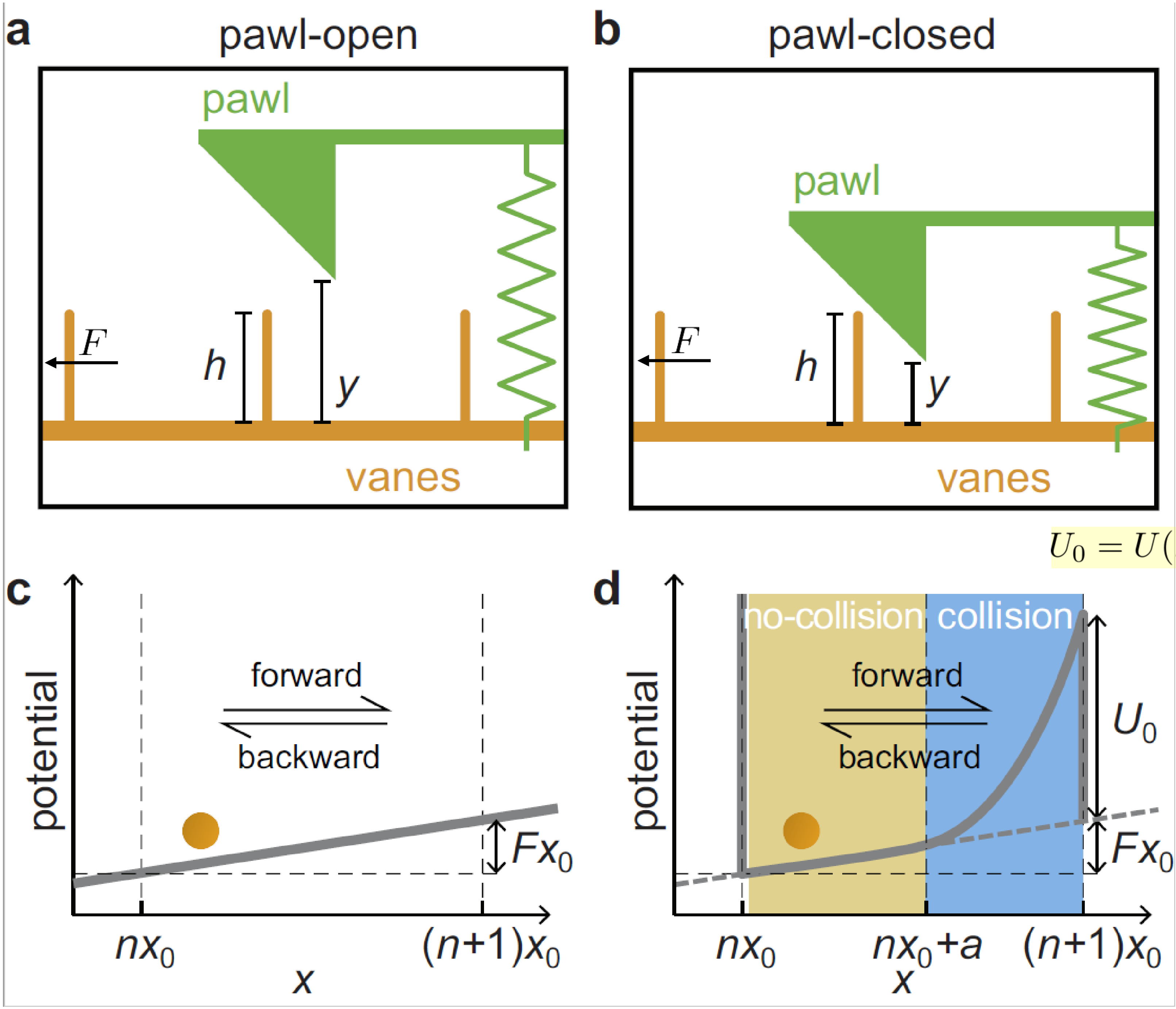
† forward hop possible ($W = Fx_0$)

- pawl open, thermally by T_2 ($y > h$)



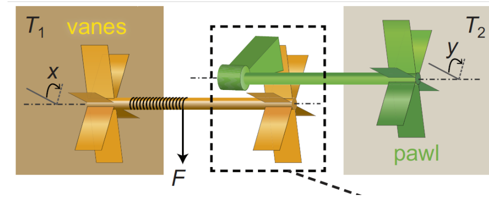
†★ perfect blocking of backward hop

backward hop mostly ($W = -Fx_0$)



$$U_0 = U(h) = \frac{1}{2}kh^2$$

¶ high energy barrier limit

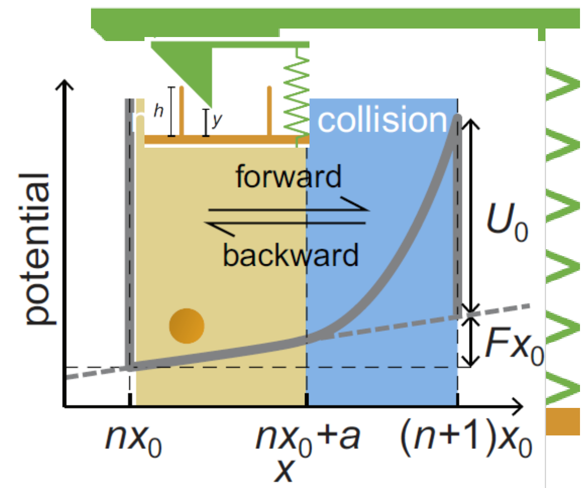


$T_2 < T_1 \ll U_0, Fx_0 \quad \Rightarrow$ almost always pawl closed for $U_0/T_2 \gg 1$.

⋆ pawl-open probability: $p_o \approx \int_h^\infty dy \sqrt{\frac{2k}{\pi T_2}} e^{-\frac{ky^2}{2T_2}} \approx \sqrt{\frac{T_2}{\pi U_0}} e^{-\frac{U_0}{T_2}}$ (equil.)

⋆ pawl-closed probability: $p_c = 1 - p_0 \approx 1$

(Arrhenius)



- forward hopping rate: $r_{\rm f} \approx p_c N_c e^{-\frac{Fx_0+U_0}{T_1}}$
- backward hopping rate: $r_{\rm b} \approx p_o N_o$

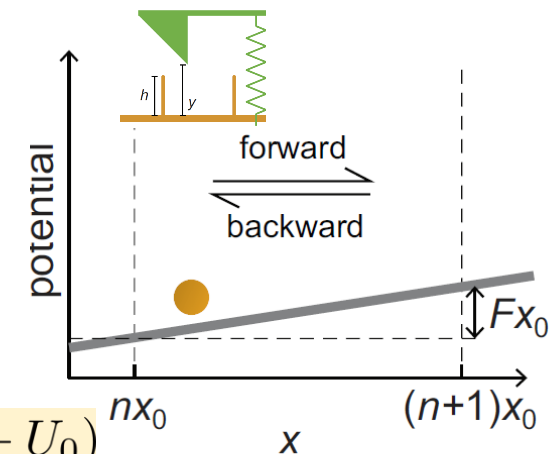
$$\dot{Q}_2 = r_{\rm f} U_0 \quad + \dot{Q}_{\rm col}$$

$$\dot{Q}_1 = r_{\rm f}(Fx_0 + U_0) - r_{\rm b} F x_0 \; + \dot{Q}_{\rm col}$$

$$\dot{W} = (r_{\rm f} - r_{\rm b}) F x_0$$

$$\eta = \frac{(r_{\rm f} - r_{\rm b})Fx_0}{(r_{\rm f} - r_{\rm b})Fx_0 + r_{\rm f} U_0} = \eta(F)$$

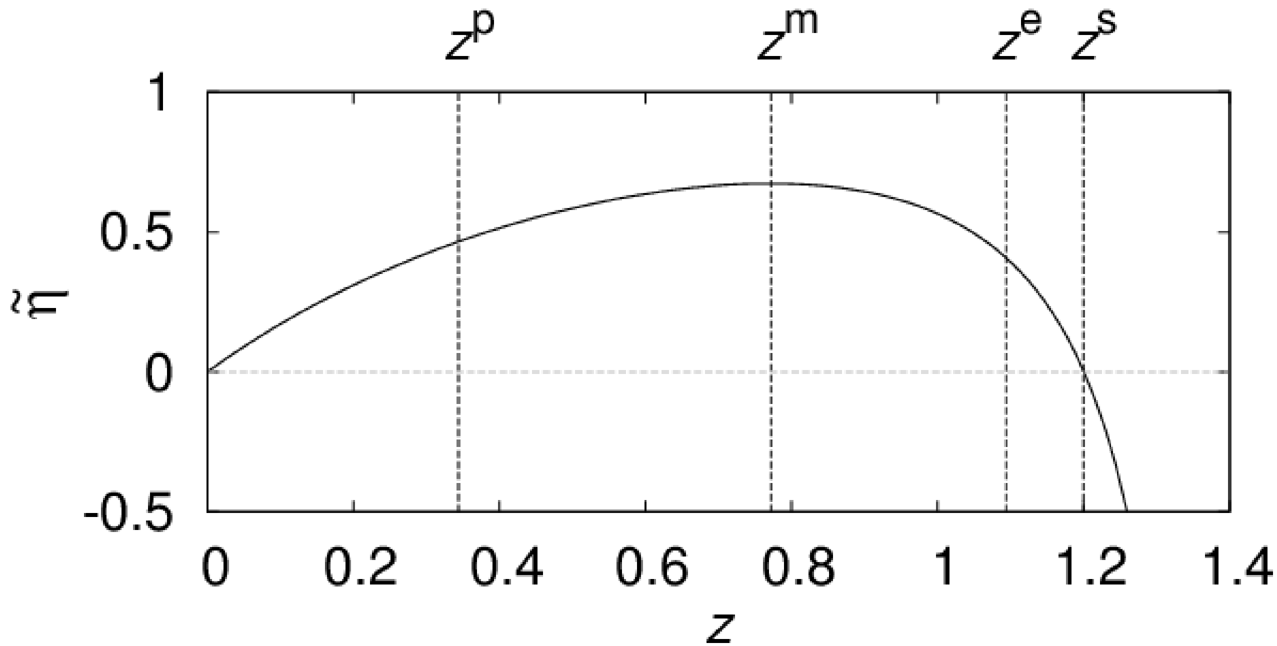
$$\dot{S} = -\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} = \dot{S}(F)$$



(Feynman)

$$\dot{Q}_2 = (r_{\rm f} - r_{\rm b}) U_0$$

$$\dot{Q}_1 = (r_{\rm f} - r_{\rm b})(Fx_0 + U_0)$$



$\tilde{\eta} = \eta/\eta_C$

$z = Fx_0/[(\eta_C U_0/T_2)T_1]$

$U_0 = 5, T_1 = 2, T_2 = 1, x_0 = 2$

$N_c = 0.045, N_o/N_c = 2.4$

$0 \leq Fx_0 \leq 7$

⋆ efficiency maximum at z^{m} $(U_0/T_2 \gg 1)$

$$z^{\mathrm{m}} \approx 1 - \frac{T_2}{2\eta_C U_0} \ln\left(\frac{\eta_C U_0}{T_2}\right) \Rightarrow 1$$

$$\eta^{\mathrm{m}} \approx \eta_C - \frac{(1-\eta_C)T_2}{2U_0} \ln\left(\frac{\eta_C U_0}{T_2}\right) \Rightarrow \eta_C$$ (Carnot eff.)

$$\dot{W}(z^{\mathrm{m}}) \approx r_{\mathrm{f}}(z^{\mathrm{m}})T_1 \frac{\eta_C U_0}{T_2} \Rightarrow 0 \quad (r_{\mathrm{f}}(z^{\mathrm{m}}) \sim N_c e^{-U_0/T_2})$$ (vanishing power)

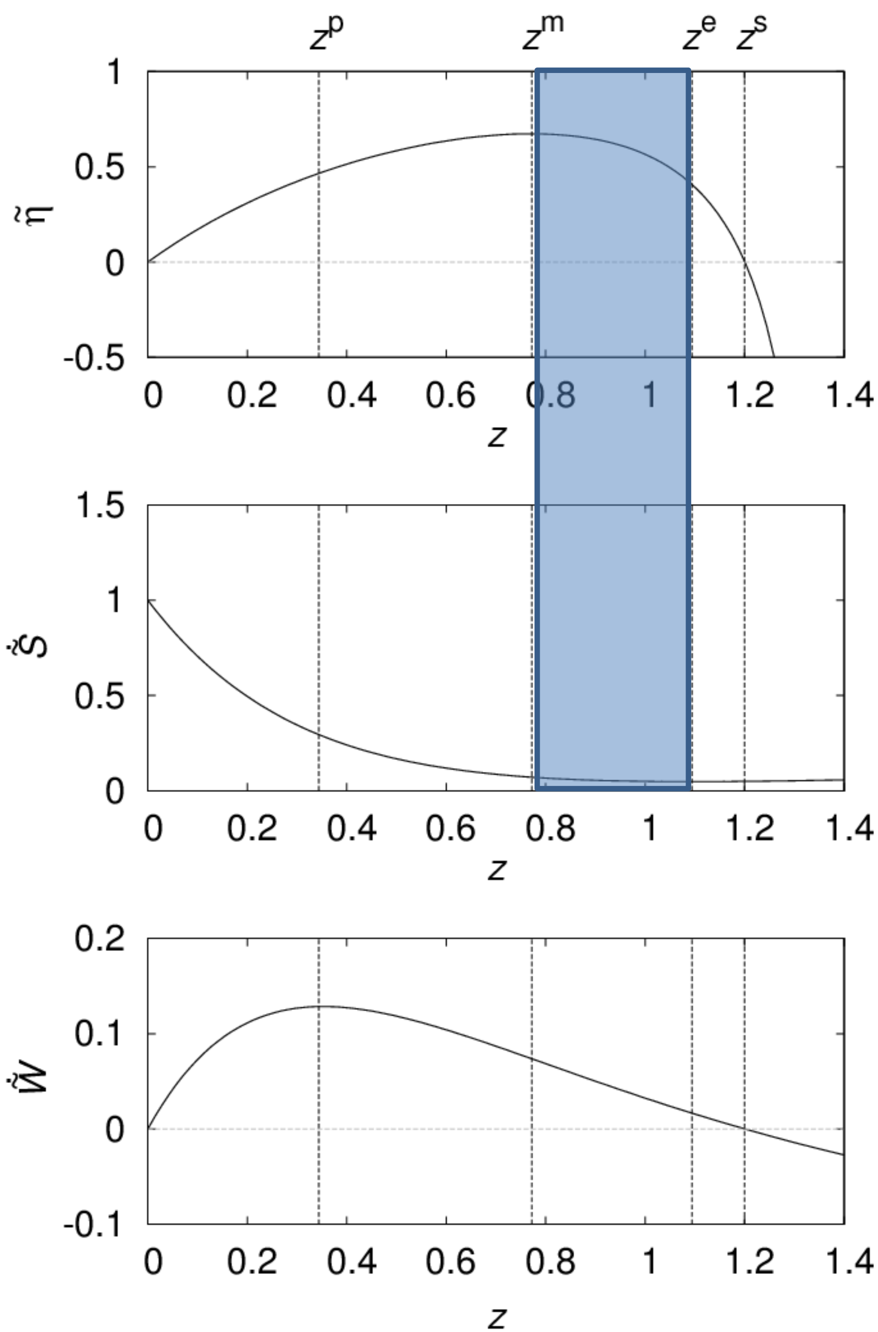
$$\dot{S}(z^{\mathrm{m}}) \approx r_{\mathrm{f}}(z^{\mathrm{m}}) \frac{1}{2} \ln\left(\frac{\eta_C U_0}{T_2}\right) \Rightarrow 0$$

$$\Delta S = \frac{1}{2} \ln\left(\frac{\eta_C U_0}{T_2}\right) \Rightarrow \infty$$ (highly irreversible)

⋆ stalling point $(r_{\mathrm{f}} = r_{\mathrm{b}})$

$$z^{\mathrm{s}} \approx 1 + \frac{T_2}{2\eta_C U_0} \ln\left(\frac{U_0}{T_2}\right) \Rightarrow 1$$

[Shiraishi/Saito/Tasaki (2016)]



$$\tilde{\eta} = \eta/\eta_C$$

$$\tilde{\dot{S}} = \dot{S}/[N_c e^{-U_0/T_1}(\eta_C U_0/T_2)]$$

$$\tilde{\dot{W}} = \dot{W}/[N_c e^{-U_0/T_1}(\eta_C U_0/T_2)T_1]$$

$$z = Fx_0/[(\eta_C U_0/T_2)T_1]$$

$U_0 = 5, T_1 = 2, T_2 = 1, x_0 = 2$

$N_c = 0.045, N_o/N_c = 2.4 \quad 0 \le Fx_0 \le 7$

⋆ efficiency maximum

$$z^{\mathrm{m}} \approx 1 - \frac{T_2}{2\eta_C U_0} \ln\left(\frac{\eta_C U_0}{T_2}\right)$$

⋆ EP rate mimimum

$$z^{\mathrm{e}} \approx 1 + \frac{T_2}{\eta_C U_0}$$

The larger irreversibility,
the higher efficiency.

⋆ Power maximum

$$z^{\mathrm{p}} \approx \frac{T_2}{\eta_C U_0} \qquad \eta^{\mathrm{p}} \approx \frac{T_2}{(1-\eta_C)U_0}$$

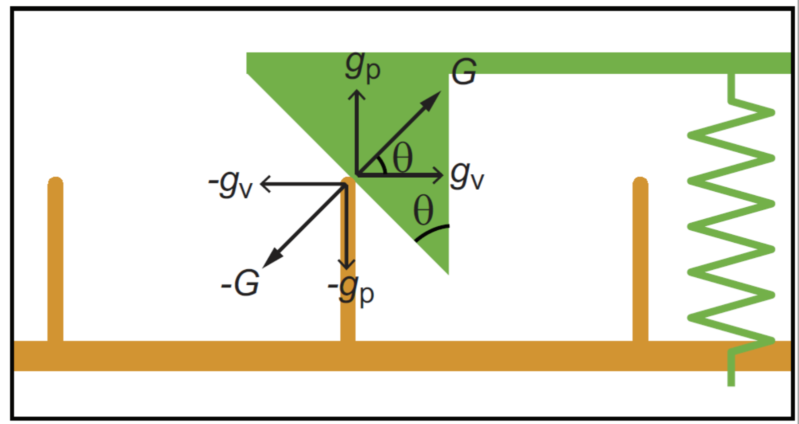
$\dot{Q}_{\rm col}$?

¶ small mass-ratio limit

$$\frac{m_{\rm p}}{m} \ll 1$$

m: mass of vanes

$m_{\rm p}$: mass of pawl



- each elastic collision transfers kinetic energy $\sim \frac{m_{\rm p}}{m}(T_1 - T_2)$

(thermal averaged)

- collision frequency should be very high per each hop

and diverges as $\frac{m_{\rm p}}{m} \to 0$ limit.

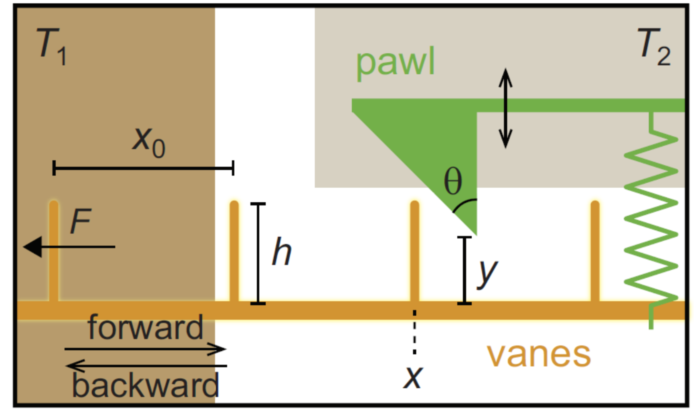
Guess $\quad \dot{Q}_{\rm col} \sim N_c p_c \left(\frac{m_{\rm p}}{m}\right)^{\omega} (T_1 - T_2) \qquad$ with $0 < \omega < 1$.

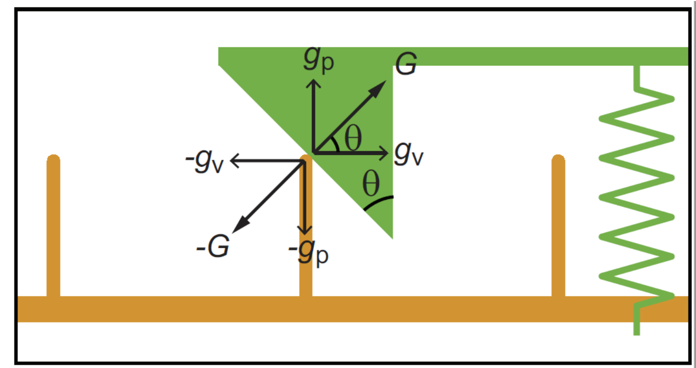
Numerical simulations

¶ Langevin equations with elastic collisions

$$\text{vane: } m\ddot{x} = -F - \gamma_1 \dot{x} + \xi_1 - g_{\mathrm{v}}(x,y)$$

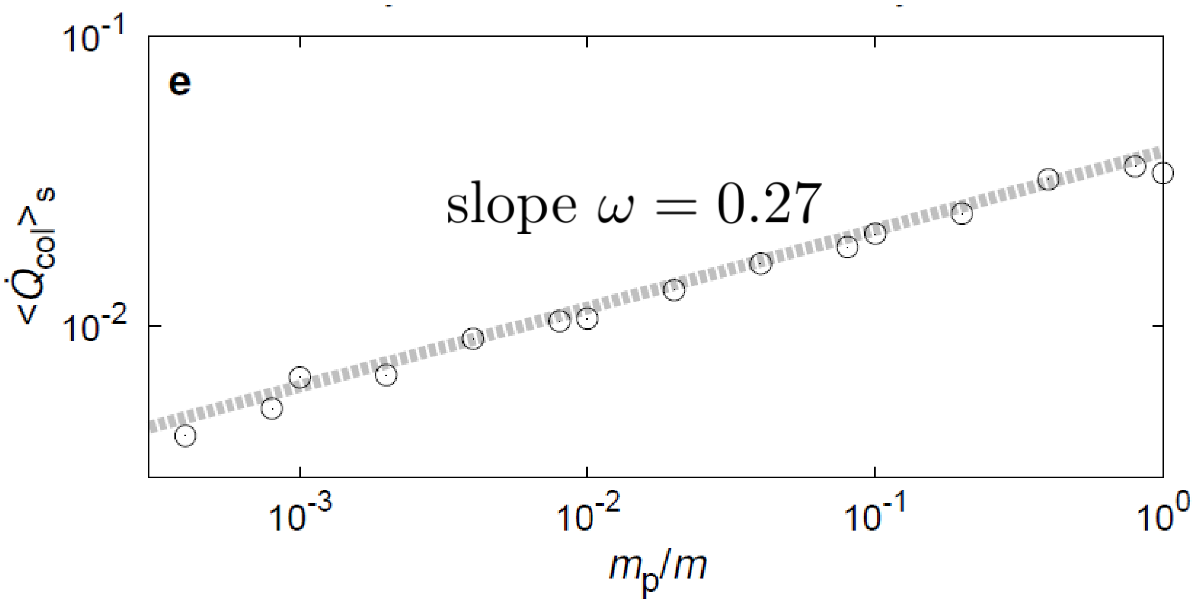
$$\text{pawl: } m\ddot{y} = -ky - \gamma_2 \dot{y} + \xi_2 + g_{\mathrm{p}}(x,y)$$





- numerical integrations

★ measure W, Q_1, Q_2 (Stratonovich)



$U_0 = 50, T_1 = 2, T_2 = 1, x_0 = 2, F = 1$

$$\dot{Q}_{\mathrm{col}} \sim N_c p_c \left(\frac{m_{\mathrm{p}}}{m}\right)^{\omega} (T_1 - T_2)$$

$$\Rightarrow 0 \text{ as } \frac{m_{\mathrm{p}}}{m} \to 0$$

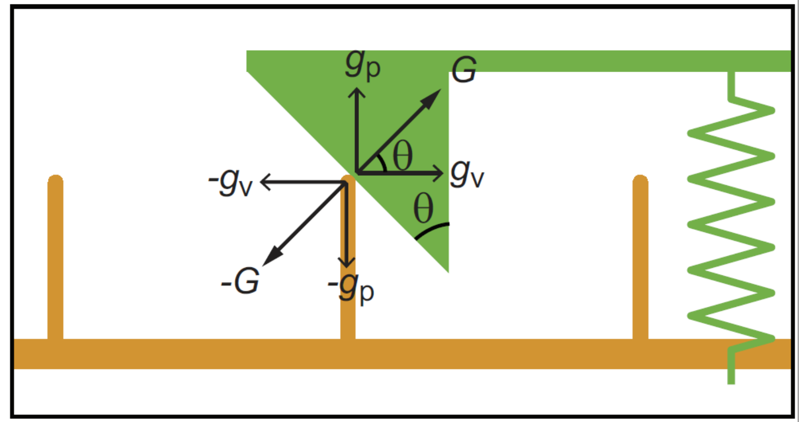
$\dot{Q}_{\rm col}$?

¶ small mass-ratio limit

$$\frac{m_{\rm p}}{m} \ll 1$$

m: mass of vanes

$m_{\rm p}$: mass of pawl

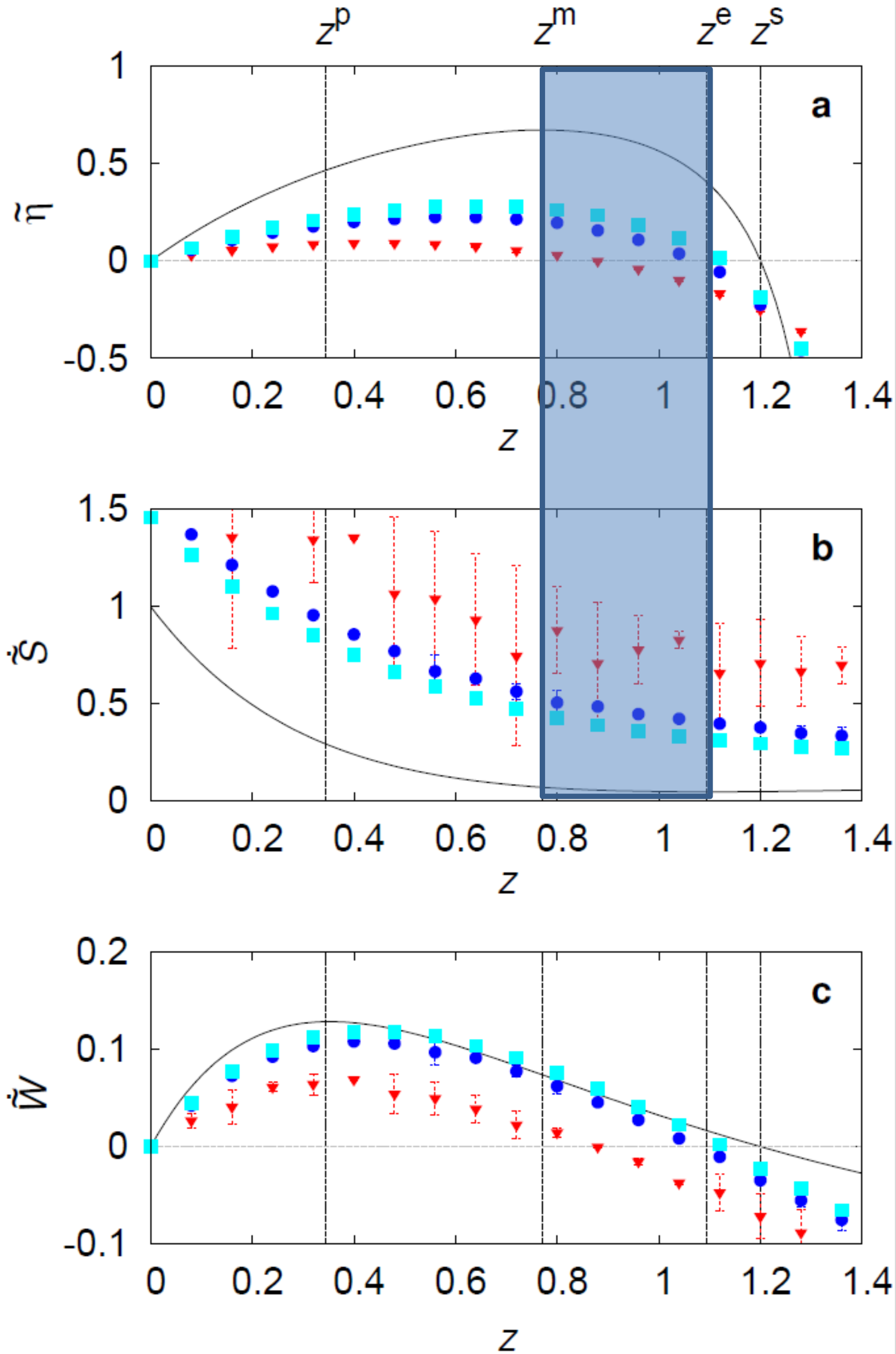


- each elastic collision transfers kinetic energy $\sim \frac{m_{\rm p}}{m}(T_1 - T_2)$ (thermal averaged)
- collision frequency should be very high per each hop and diverges as $\frac{m_{\rm p}}{m} \to 0$ limit.

Guess $\quad \dot{Q}_{\rm col} \sim N_c p_c \left(\frac{m_{\rm p}}{m}\right)^{\omega} (T_1 - T_2) \qquad$ with $0 < \omega < 1$.

[simulation: $\omega = 0.27(3)$]

⋆ $\dot{Q}_{\rm col} \ll \dot{Q}_{\rm hop} \quad \Rightarrow \quad \frac{m_{\rm p}}{m} \ll \left[e^{-(U_0+Fx_0)/T_1} U_0/(\eta_C T_1)\right]^{1/\omega}$



$$U_0 = 5, T_1 = 2, T_2 = 1, x_0 = 2, F = 1$$

$$\frac{m_\mathrm{p}}{m} = 10^{-1}, 10^{-2}, 10^{-3}$$

$$\frac{m_\mathrm{p}}{m} \ll \left[e^{-(U_0+Fx_0)/T_1} U_0/(\eta_C T_1)\right]^{1/\omega}$$

$$\sim 3.5 \times 10^{-6}$$

non-negligible $\dot{Q}_\mathrm{col}$

The larger irreversibility,
the higher efficiency.

$\star$ molecular motor (kinesin)

$$U_0/T_2 \approx 8, \;\; Fx_0/T_2 \approx 12$$

$$r_\mathrm{f} \approx 100 s^{-1}$$

Summary and discussion

¶ Carnot efficiency is reachable in an irreversible process (FSR).

$$\eta = \eta_C - \frac{T_2 \Delta S}{Q_1}$$

$Q_1 \simeq U_0 + F x_0$ (highly irreversible)

$\Delta S \simeq \frac{1}{2} \ln \left(\frac{\eta_C U_0}{T_2} \right)$ (vanishing power)

¶ We confirm that it is possible to have

"The larger the irreversibility $\dot{S}$, the higher the efficiency η".

(in realistic situations like kinesin)

$\Rightarrow$ opens a new way of designing a highly efficient engine

¶ Resolving the long-standing debate on FSR

Careful (Sekimoto) setup with kinetic collisions

high energy barrier and small mass-ratio limits

$\Rightarrow$ Carnot efficiency is possible in the FSR.