Large deviation principle and beyond in a single-electron box

Shilpi Singh, Joonas Peltonen, Ivan Khaymovich, Jonne Koski, Christian Flindt and Jukka Pekola

Aalto University, Finland

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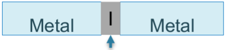
Single-electron box

Tunnel junctions

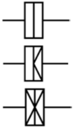


Examples:

NIN

NIS

SIS



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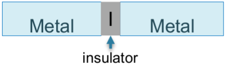

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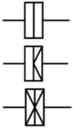
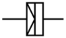


SEB



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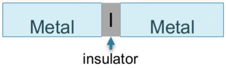

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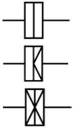

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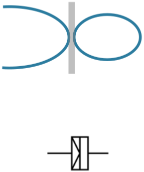
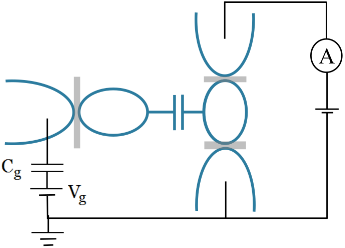
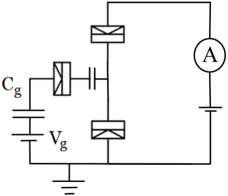


Detection scheme





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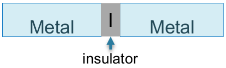

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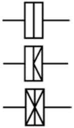

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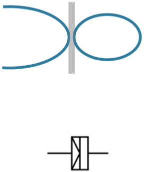

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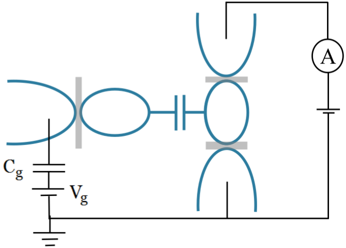

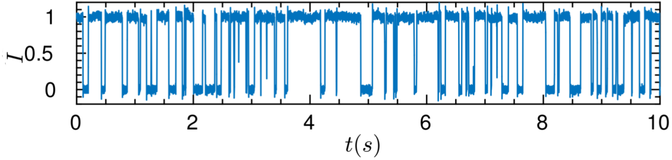

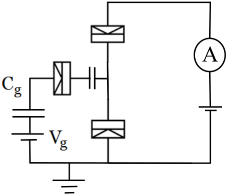

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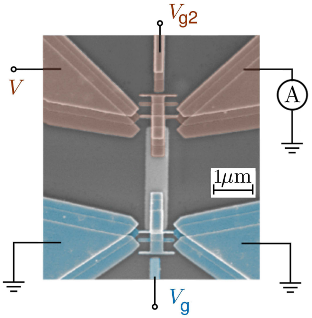
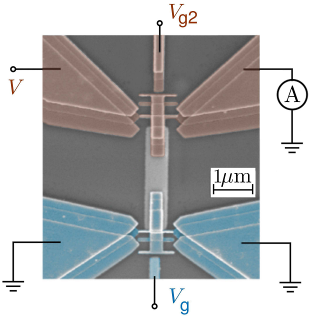
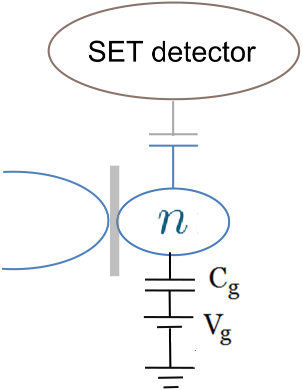
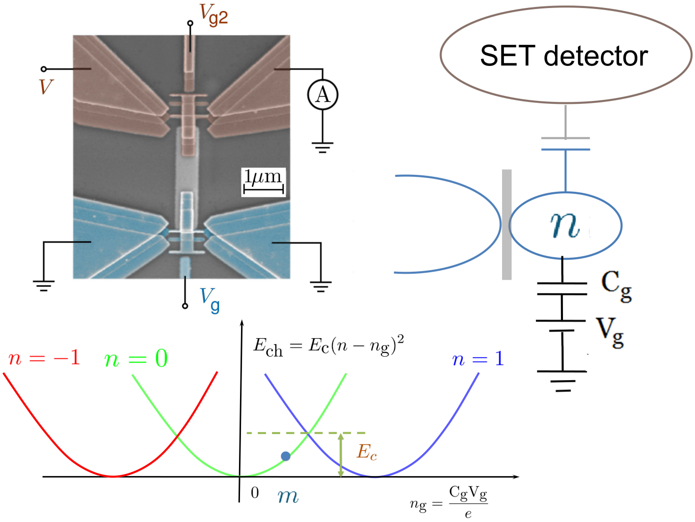


Single-electron box





Single-electron box

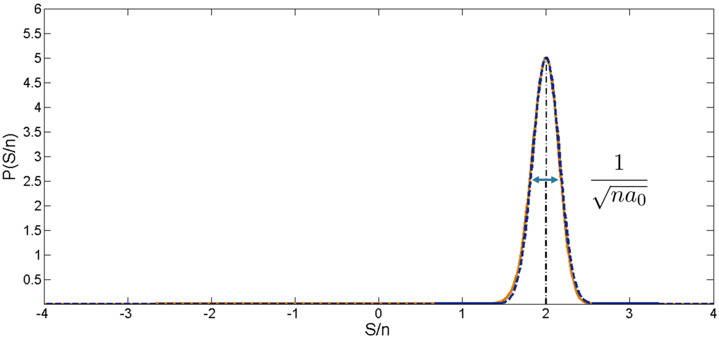


Central limit theorem

$S = \sum_{k=1}^{n} s_k ,\ n >> 1$

s_k : independently distributed random variables

$$P(S/n) \sim \exp\left[-na_0(S/n - s_0)^2\right]$$

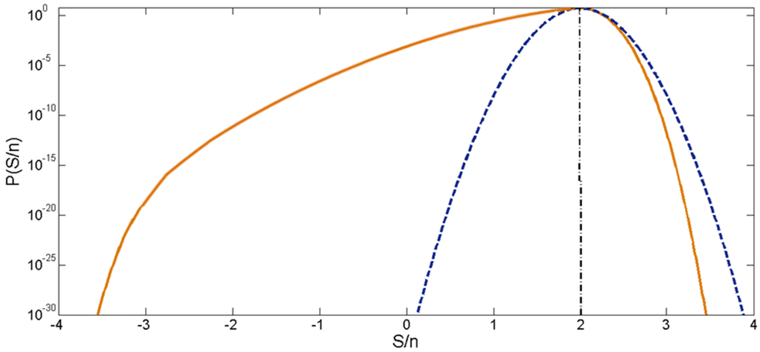


H. Touchette, Phys. Rep. 478, 1(2009)

Central limit theorem

$S = \sum_{k=1}^{n} s_k,\ n >> 1$ s_k : independently distributed random variables

$$\boxed{P(S/n)} \sim \exp\left[-n a_0 (S/n - s_0)^2\right]$$



H. Touchette, Phys. Rep. 478, 1(2009)

Large deviation principle

$$\lim_{n\to\infty} P(S/n) = \sqrt{\frac{na_0}{\pi}} \exp\left[-na_0(S/n - s_0)^2\right]$$

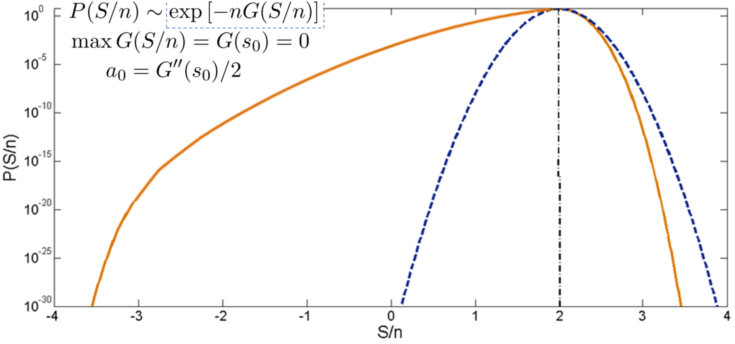
$$P(S/n) \sim \boxed{\exp\left[-na_0(S/n - s_0)^2\right]}$$

$$\lim_{n\to\infty} \frac{1}{n} \ln P(\sigma = S/n) = -G(\sigma)$$

$$P(S/n) \sim \boxed{\exp\left[-nG(S/n)\right]}$$

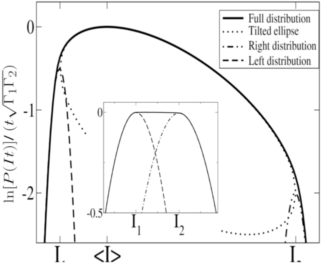
$$\max G(S/n) = G(s_0) = 0$$

$$a_0 = G''(s_0)/2$$



H. Touchette, Phys. Rep. 478, 1(2009)

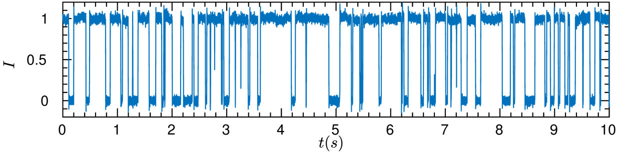
Bistable systems

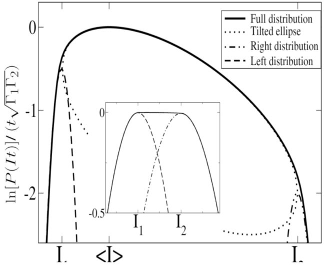


$$\frac{\ln P(It)}{t} \simeq -\frac{\left[\sqrt{\Gamma_1|I_2 - I|} - \sqrt{\Gamma_2|I - I_1|}\right]^2}{I_2 - I_1}$$

Jordan et. al. Phys. Rev. Lett. 93, 260604(2004)

Bistable systems





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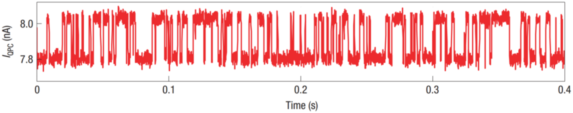
n	τ
s_k	$I(t)$
$S/n = \sum_n s_k/n$	$\mathcal{I}(\tau) = \frac{1}{\tau}\int_{t_0}^{t_0+\tau} dt I(t)$

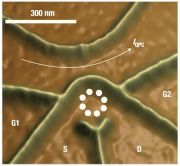
S. di Santo et. al., Phys. Rev. Lett. 116, 240601 (2016)

E. Dieterich et. al., Nat. Phys. 11, 971 (2015)

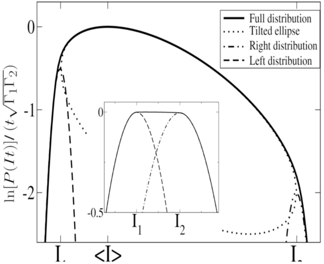
Jordan et. al. Phys. Rev. Lett. 93, 260604(2004)

Bistable systems





$n \to t$



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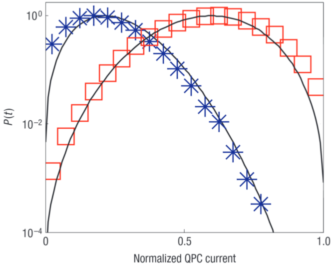


Jordan et. al. Phys. Rev. Lett. 93, 260604(2004)

Sukhorukov et. al. Nat. Phy. 3, 243(2007)

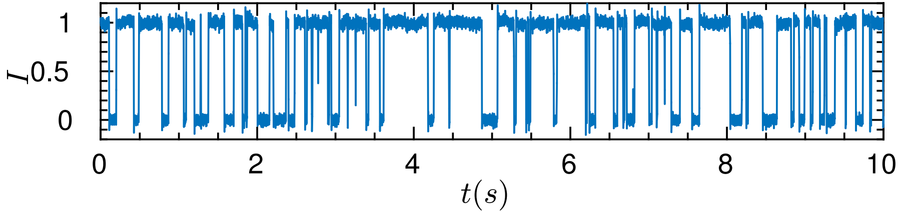
Aim

To investigate

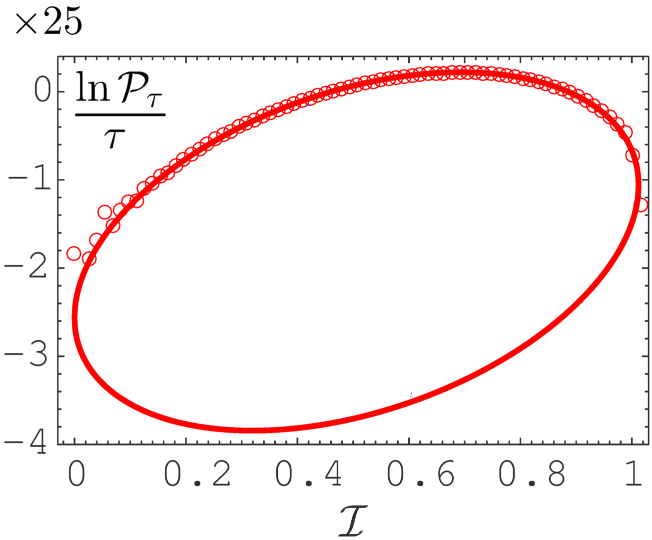
- the universal ellipse.
- the distribution crossover from short-time to long-time statistics.
- the symmetry properties of distribution.

Time averaged current

$$\mathcal{I}(\tau) = \frac{1}{\tau}\int_{t_0}^{t_0+\tau} dt I(t)$$



Universal ellipse



To investigate

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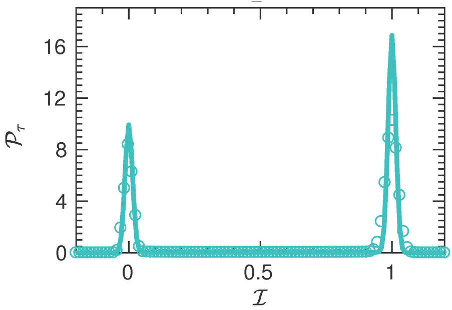
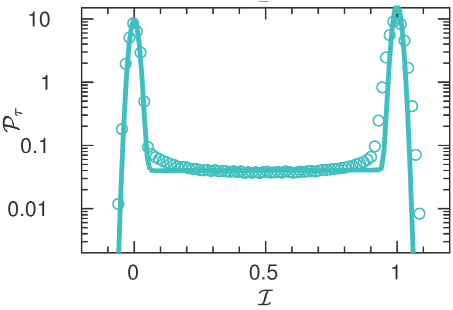
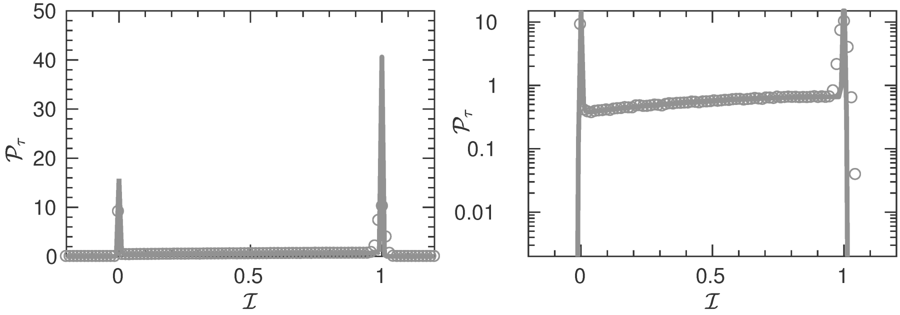
Time averaged current distribution: 2 ms

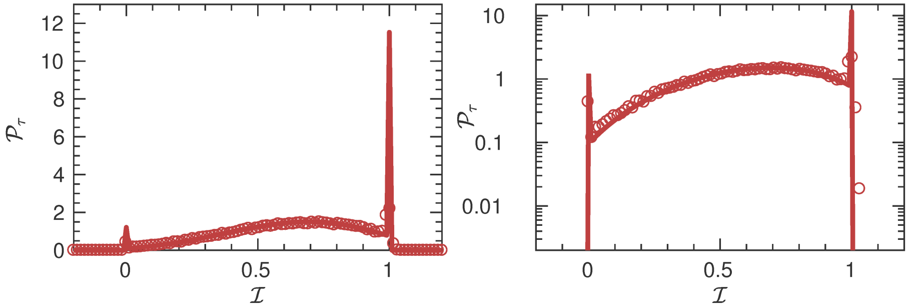




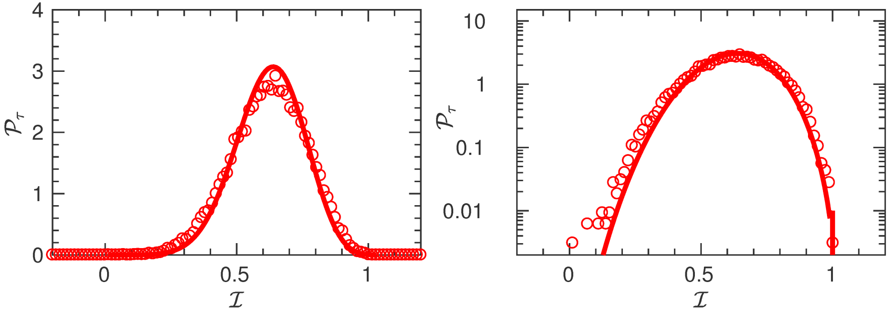
Time averaged current distribution: 80 ms



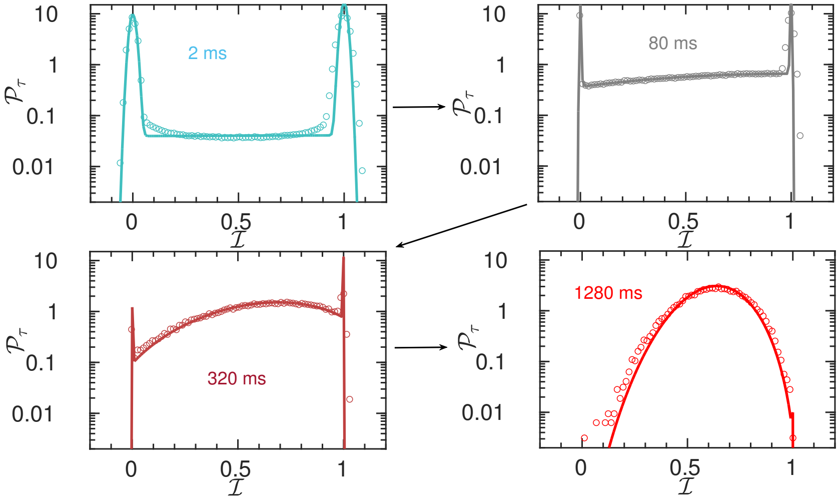
Time averaged current distribution: 320 ms



Time averaged current distribution: 1280 ms



Time averaged current



Model

- A classical 2-level system
- governed by rate equation

$$dp_1/dt = \Gamma_+ p_0 - \Gamma_- p_1$$

- probability

$$p_0 = 1 - p_1$$

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- governed by rate equation

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- probability

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- Using multi-jump approach, we obtain distribution

$$\mathcal{P}_\tau = \frac{e^{-[\Gamma_+(1-\mathcal{I})+\Gamma_-\mathcal{I}]\tau}}{\Gamma_+ + \Gamma_-}\Bigg\{\Gamma_-\delta\left(\mathcal{I}\right) + \Gamma_+\delta\left(1-\mathcal{I}\right) \\ + 2\tau\Gamma_+\Gamma_-\Pi(\mathcal{I})\left[I_0(x) + \tau[\Gamma_-(1-\mathcal{I}) + \Gamma_+\mathcal{I}]\frac{I_1(x)}{x}\right]\Bigg\}$$

J. Bergli et. a., New J. Phys. 11, 025002 (2009)

Distribution

$$\mathcal{P}_\tau = A\left\{T_1 + T_2\right\}$$

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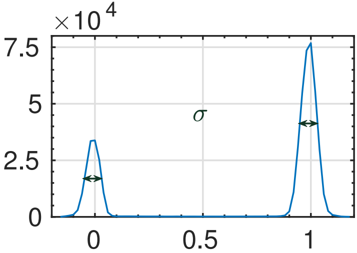
$$\Pi(\mathcal{I}) = \theta(\mathcal{I})\theta(1-\mathcal{I}) \qquad x = 2\tau\sqrt{\Gamma_+\Gamma_-\mathcal{I}(1-\mathcal{I})}$$

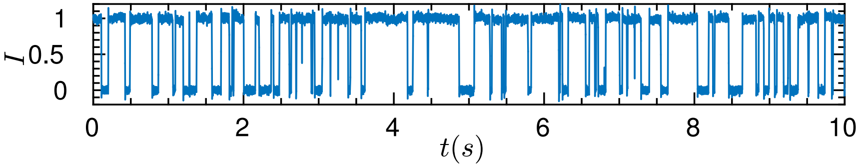
$I_l(x)$: Modified bessel function of the first kind.

Including noise

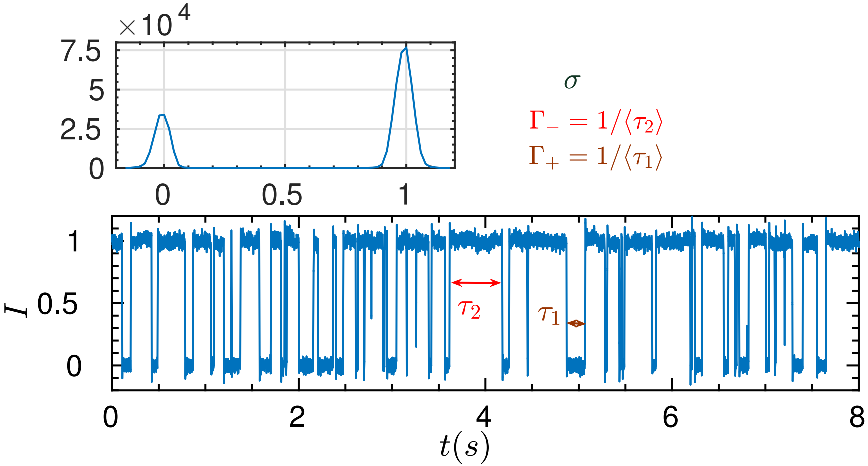
$$\widetilde{\mathcal{P}}(\widetilde{\mathcal{I}}) = \int_0^1 \mathcal{P}(\mathcal{I})\mathcal{P}_\xi(\widetilde{\mathcal{I}}-\mathcal{I})d\mathcal{I}$$

$$\mathcal{P}_\xi(\xi(\tau)) = \frac{1}{\sqrt{2\pi/K\sigma}} e^{-\xi(\tau)^2/2(\sigma^2/K)}$$





Theory parameters

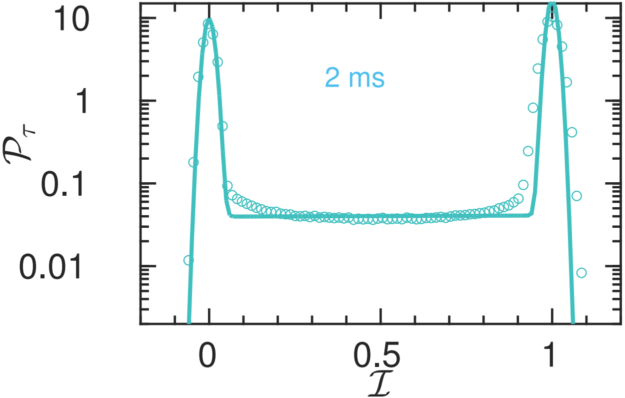


σ

$\Gamma_- = 1/\langle\tau_2\rangle$

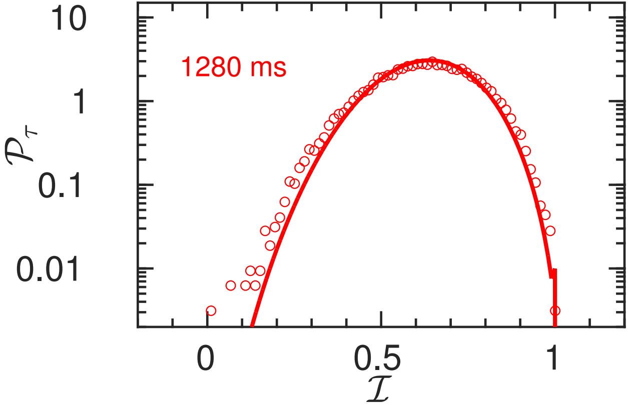
$\Gamma_+ = 1/\langle\tau_1\rangle$

Small τ



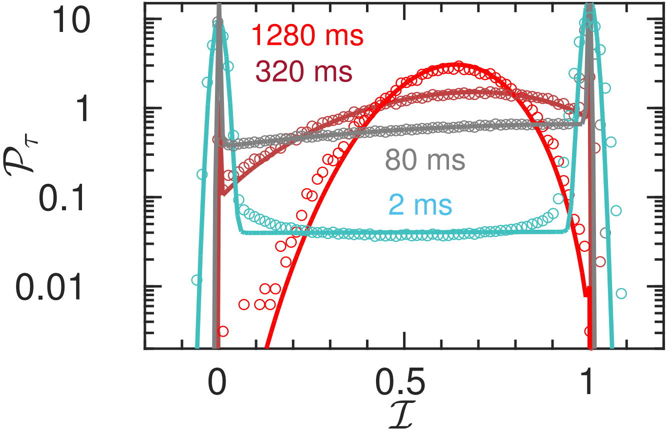
$$\mathcal{P}_\tau \approx \frac{e^{-[\Gamma_+(1-\mathcal{I})+\Gamma_-\mathcal{I}]\tau}}{\Gamma_+ + \Gamma_-} \left\{\Gamma_- \delta\left(\mathcal{I}\right) + \Gamma_+ \delta\left(1-\mathcal{I}\right)\right\}$$

Large τ

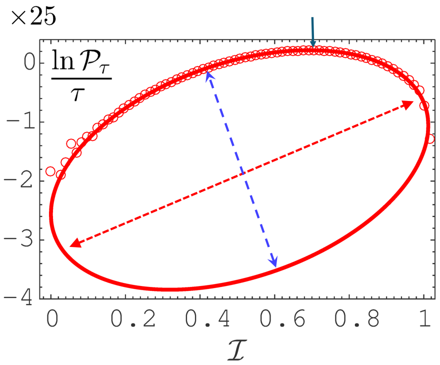


$$\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} \simeq -\left[\sqrt{\Gamma_+(1-\mathcal{I})} - \sqrt{\Gamma_-(\mathcal{I})}\right]^2$$

Comparing theory to experiment

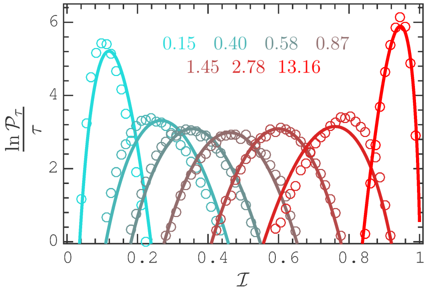


Characterising the ellipse



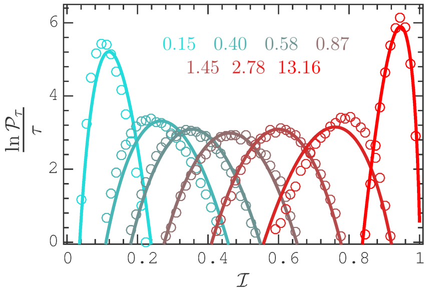
$$\left[\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau\sqrt{\Gamma_+\Gamma_-}} + \frac{\Gamma_+ + (\Gamma_- - \Gamma_+)\mathcal{I}}{\sqrt{\Gamma_+\Gamma_-}}\right]^2 + [2\mathcal{I} - 1]^2 \simeq 1$$

Varying transition rates



Universal Semicircle

$$\mathcal{G}(\mathcal{I}) \equiv \frac{1}{2\sqrt{\Gamma_+\Gamma_-}}\left\{\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} - \Gamma_+(1-\mathcal{I}) - \Gamma_-\mathcal{I}\right\}$$



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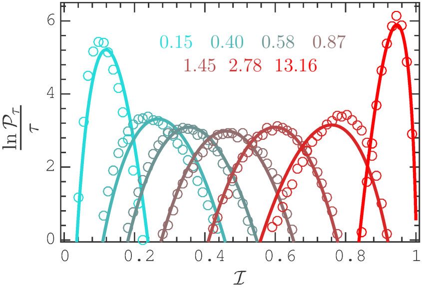

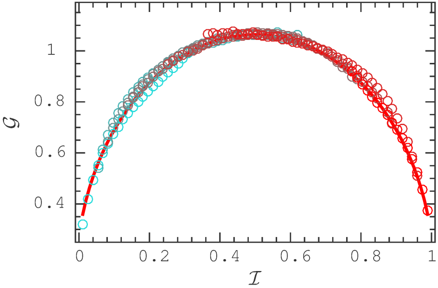

To investigate

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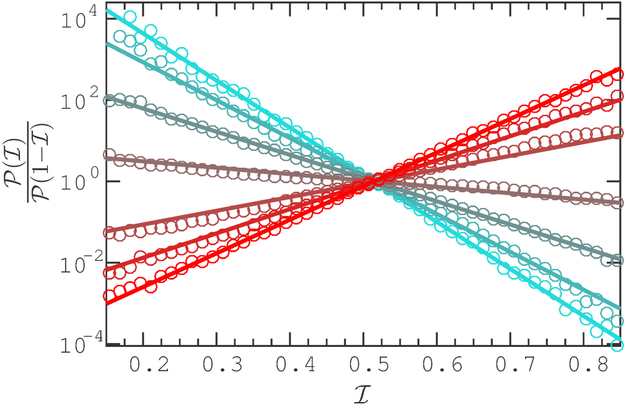
- the distribution crossover from short-time to long-time statistics.
- the symmetry properties of distribution.

Fluctuation relation



$$\frac{1}{\tau}\ln\left[\frac{P_\tau(\mathcal{I}=\bar{I}+\mathcal{J})}{P_\tau(\mathcal{I}=\bar{I}-\mathcal{J})}\right]=\Omega\mathcal{J}$$

$$\Omega = 2(\Gamma_+ - \Gamma_-)/(\langle I_+\rangle - \langle I_-\rangle)$$

M. Esposito et. al., Rev. Mod. Phys. 81, 1665 (2009)
G. Gallavotti et. al., J. Stat. Phys. 80, 931 (1995)
G. Gallavotti et. al., Phys. Rev. Lett. 74, 2694 (1995)

Conclusions

We observed

- the universal ellipse in a single-electron box.
- the distribution crossover from short-time to long-time statistics.
- the Fluctuation relation in long time limit.

Phys. Rev. B 94, 241407 (2016)

Thank you for your attention!

$$\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} \simeq -\frac{\left[\sqrt{\Gamma_+(\langle I_+\rangle - \mathcal{I})} - \sqrt{\Gamma_-(\mathcal{I} - \langle I_-\rangle)}\right]^2}{\langle I_+\rangle - \langle I_-\rangle}$$

$$\Rightarrow \frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} \simeq -\left[\sqrt{\Gamma_+(1-\mathcal{I})} - \sqrt{\Gamma_-(\mathcal{I})}\right]^2$$

$$\Rightarrow \frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} \simeq -\left[\Gamma_+(1-\mathcal{I}) + \Gamma_-\mathcal{I} - 2\sqrt{\Gamma_+(1-\mathcal{I})\Gamma_-\mathcal{I}}\right]$$

$$\Rightarrow \left[\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} + \Gamma_+ + (\Gamma_- - \Gamma_+)\mathcal{I}\right]^2 \simeq 4\Gamma_+\Gamma_-\left[\mathcal{I} - \mathcal{I}^2 - \frac{1}{4}\right] + \Gamma_+\Gamma_-$$

$$\Rightarrow \left[\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} + \Gamma_+ + (\Gamma_- - \Gamma_+)\mathcal{I}\right]^2 + 4\Gamma_+\Gamma_-\left[\mathcal{I} - \frac{1}{2}\right]^2 \simeq \Gamma_+\Gamma_-$$

$$\Rightarrow \left[\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\sqrt{\Gamma_+\Gamma_-}\,\tau} + \frac{\Gamma_+ + (\Gamma_- - \Gamma_+)\mathcal{I}}{\sqrt{\Gamma_+\Gamma_-}}\right]^2 + [2\mathcal{I} - 1]^2 \simeq 1$$

$$\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau} \simeq -\left[\sqrt{\Gamma_+(1-\mathcal{I})} - \sqrt{\Gamma_-(\mathcal{I})}\right]^2 \Rightarrow \left[\frac{\ln \mathcal{P}_\tau(\mathcal{I})}{\tau\sqrt{\Gamma_+\Gamma_-}} + \frac{\Gamma_+ + (\Gamma_- - \Gamma_+)\mathcal{I}}{\sqrt{\Gamma_+\Gamma_-}}\right]^2 + [2\mathcal{I} - 1]^2 \simeq$$

Path integral representation of jumps

<u>Probability of trajectory with M jumps</u>

$$p_0(t=0)=1-p_1(t=0)=\frac{\Gamma_-}{\Gamma_++\Gamma_-}$$

- M = 0

$$P_0^+=p_0(t=0)e^{-\Gamma_+\tau}$$

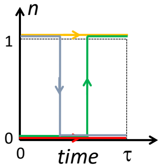
$$P_0^-=p_1(t=0)e^{-\Gamma_-\tau}$$

- M = 1

$$P_1^+(t_1)=p_0(t=0)e^{-\Gamma_+t_1}\Gamma_+(t_1)e^{-\Gamma_-(\tau-t_1)}$$

$$P_1^-(t_1)=p_1(t=0)e^{-\Gamma_-t_1}\Gamma_-(t_1)e^{-\Gamma_+(\tau-t_1)}$$

n(t), M=0, n_0=0
n(t), M=0, n_0=1
n(t), M=1, n_0=0
n(t), M=1, n_0=1



- M

$$P_M^+(t_1,\dots,t_M)=p_0(t=0)e^{-\Gamma_+t_1}\Gamma_+(t_1)e^{-\Gamma_-(t_2-t_1)}\dots\Gamma_{(-1)^{M-1}}(t_M)e^{-\Gamma_{(-1)^M}(\tau-t_M)}$$

$$P_M^-(t_1,\dots,t_M)=p_1(t=0)e^{-\Gamma_-t_1}\Gamma_-(t_1)e^{-\Gamma_+(t_2-t_1)}\dots\Gamma_{(-1)^M}(t_M)e^{-\Gamma_{(-1)^{M+1}}(\tau-t_M)}$$

$$P(N_\tau)=\sum_{M;\eta=\pm}\int P_M^\eta(t_1,\dots,t_M)\delta(N_\tau-N(\eta;t_1,\dots,t_M))dt_1\dots dt_M$$

$$N(\eta;t_{1,\dots,M})=\tau^{-1}\sum_{j=1}^{M}(-1)^{j+\eta}t_j+\mathrm{odd}(M+\eta)$$

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Path integral representation of jumps

Simplifications

$$p_0(t=0)=1-p_1(t=0)=\frac{\Gamma_-}{\Gamma_+ + \Gamma_-}$$

$$P_M^+(t_1,\ldots,t_M)=p_0(t=0)e^{-\Gamma_+ t_1}\Gamma_+(t_1)e^{-\Gamma_-(t_2-t_1)}\ldots\Gamma_{(-1)^{M-1}}(t_M)e^{-\Gamma_{(-1)^M}(\tau-t_M)}$$

$$P_M^-(t_1,\ldots,t_M)=p_1(t=0)e^{-\Gamma_- t_1}\Gamma_-(t_1)e^{-\Gamma_+(t_2-t_1)}\ldots\Gamma_{(-1)^M}(t_M)e^{-\Gamma_{(-1)^{M+1}}(\tau-t_M)}$$

$$P(N_\tau)=\sum_{M;\eta=\pm}\int P_M^\eta(t_1,\ldots,t_M)\delta(N_\tau-N(\eta;t_1,\ldots,t_M))dt_1\ldots dt_M$$

$$N(\eta;t_{1,\ldots,M})=\tau^{-1}\sum_{j=1}^{M}(-1)^{j+\eta}t_j+\mathrm{odd}(M+\eta)$$

$$P_M^\pm(t_1,\ldots,t_M)\prod_{j=1}^{M}dt_j=p_{0,1}(t=0)\exp[\gamma_d N-\gamma_+](\gamma_+\gamma_-)^{\lfloor M/2\rfloor}(\gamma_\pm)^{\mathrm{odd}(M)}\prod_{j=1}^{M}\frac{dt_j}{\tau}$$

$$\boxed{P(N_\tau)=\frac{e^{\gamma_d N_\tau-\gamma_+}}{\gamma_+ + \gamma_-}\left\{\gamma_-\delta(N_\tau)+\gamma_+\delta(N_\tau-1)+2\gamma_+\gamma_-\left[I_0(x)+(\gamma_d N_\tau+\gamma_-)\frac{I_1(x)}{x}\right]\right\}}$$

$I_k(x)$ - modified Bessel function of k-th order

$$x=2\sqrt{\Gamma_+\Gamma N_\tau(1-N_\tau)},\quad \gamma_\pm=\Gamma_\pm\tau,\quad \gamma_d=(\Gamma_+-\Gamma_-)\tau$$

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Including noise : Alternative slide

- Assumptions for noise ζ_k
  - delta-correlated : ζ_k and ζ_l are independent for $k \neq l$
  - stationary : the distribution $P_0(\zeta_k)$ is the same for all k
- noise convolution

$$\widetilde{\mathcal{P}}(\widetilde{\mathcal{I}}) = \int_0^1 \mathcal{P}(\mathcal{I})\mathcal{P}_\xi(\widetilde{\mathcal{I}}-\mathcal{I})d\mathcal{I} = \frac{\gamma_- e^{-\gamma_+}\mathcal{P}_\xi(\widetilde{\mathcal{I}}) + \gamma_+ e^{-\gamma_-}\mathcal{P}_\xi(\widetilde{\mathcal{I}}-1)}{\gamma_{\rm tot}}$$
$$+ \frac{2\gamma_+\gamma_-}{\gamma_{\rm tot}} \int_0^1 e^{\gamma_d \mathcal{I} - \gamma_+} \left[I_0(x) + (\gamma_- + \gamma_d \mathcal{I}) \frac{I_1(x)}{x} \right] \mathcal{P}_\xi(\widetilde{\mathcal{I}} - \mathcal{I}) d\mathcal{I}$$

$$\mathcal{P}_\xi(\xi(\tau)) = \frac{K}{2\pi} \int e^{-iK(\xi(\tau)q - \sigma q^2)} dq = \frac{1}{\sqrt{2\pi/K}\sigma} e^{-\xi(\tau)^2/2(\sigma^2/K)}$$

which is also a Gaussian with the variance σ^2/K.