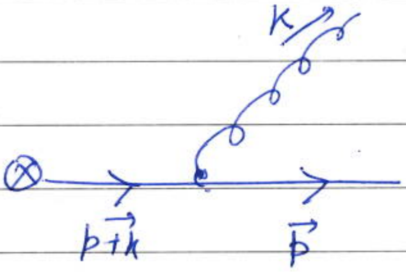
Why Do we see jets in QCD?

Consider single gluon radiation of a quark line

$$i\mathcal{M} = $$ 

$$\sim \frac{\epsilon^{*}\cdot p}{p\cdot k} \times g\, T^a$$

$$\left.\begin{aligned} p^\mu &= (E, 0, 0, E) \\ k^\mu &= (E', E'\sin\theta, 0, E'\cos\theta) \\ \epsilon^\mu &= (0, -\cos\theta, 0, \sin\theta) \end{aligned}\right\} \Rightarrow \begin{aligned} \epsilon\cdot p &= E\sin\theta \\ p\cdot k &= EE'(1-\cos\theta) \end{aligned}$$

free of any IR div after suitable virtual diagrams

$$|\mathcal{M}|^2 = C g^2 \frac{\sin^2\theta}{(1-\cos\theta)^2} \times \frac{1}{E'^2} \xrightarrow[\theta\to 0]{} C g^2 \frac{1}{E'^2}\frac{1}{\theta^2}$$

color factor

$$|\mathcal{M}|^2 \sim C g^2 \frac{1}{E'^2}\frac{1}{\theta^2}$$ ← enhanced probability for soft and collinear emissions

This differs from QED since $g^2 \sim \alpha_s \gg \alpha_{em}$, further there is color factor like C_F or C_A which gives further enhancement to radiation.

QCD loves lots of radiation. Lots of collinear radiation $\Rightarrow$ jets.

Usually when we calculate a crosssection we will do some integration over phase space (we don't observe individual gluon θ and E')

$$\underbrace{\sigma}_{\text{some observable}} \sim \underbrace{\int \frac{d^3k'}{E'}}_{\text{integrating over phase space}} |\mathcal{M}|^2 = \int dE' \cancel{d}E' \int d\theta \sin\theta |\mathcal{M}|^2 \xrightarrow[\theta \text{ limit}]{\text{small}} \int dE' E' \int d\theta\theta \; \alpha |\mathcal{M}|^2$$

~~setting~~

$$\sigma \sim C_F g^2 \int \frac{dE'}{E'} \int \frac{d\theta}{\theta}$$

$$\Big\downarrow \text{ setting some realistic integration limits}$$

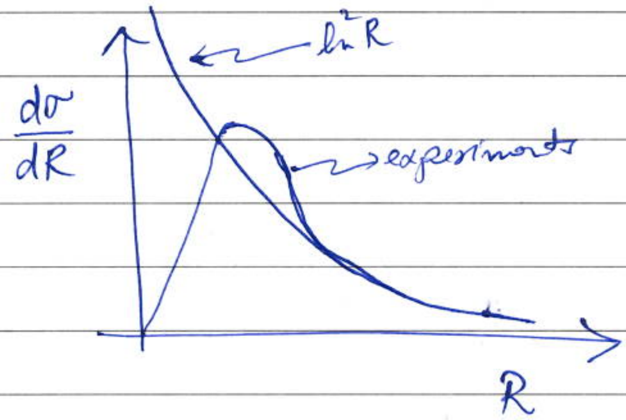
$$\sigma \sim C_F g^2 \int_{\Lambda_{QCD}}^{Q} \frac{dE'}{E'} \int_{\frac{\Lambda_{QCD}}{Q}}^{\sim 1} \frac{d\theta}{\theta} \sim \underbrace{C_F \alpha_s(Q) \ln^2 \frac{\Lambda_{QCD}}{Q}}_{\text{large double logarithm}}$$

large double logarithms are a consequence of soft and collinear gluon emissions.

Whenever an observable enforces jetty states, we are bound to get large logarithms.

For example, if we are observing cone-radius R then, typically lower bounds will be replaced by ~~$E'QR$ and θR~~ QR and R for E' and θ respectively.

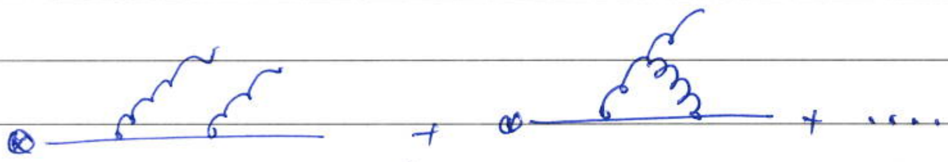
$$\frac{d\sigma}{dR} \sim C_F \alpha_s(Q) \ln^2 R$$



At higher orders :—

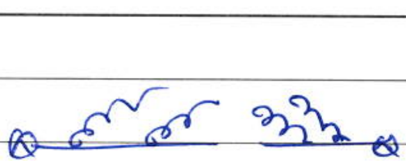
$$\frac{d\sigma}{dR} \sim \alpha_s(R)\left(a_0 + a_1 \ln R + a_2 \ln^2 R\right) + \alpha_s^2\left(b_0 + b_1 \ln R + b_2 \ln^2 R + b_3 \ln^3 R + b_4 \ln^4 R\right) + \ldots$$

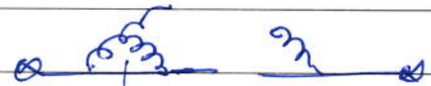
Situation gets worse owing to diagrams :—



$\alpha_s^2 \ln^4 R$ → mechanism same as one radiation

$\alpha_s^2 \ln^3 R$ → new mechanism & new color factor

 $\sim \alpha_s^2 \ln^4 R$

 $\sim \alpha_s^2 \ln^3 R$

additional virtual soft gluon loop gives an additional $\ln R$

Perturbation theory does not converge !!

However, Physics of $\alpha_s \ln^2$, $\alpha_s^2 \ln^4$, $\alpha_s^3 \ln^6$, --- terms is same thus we should be able to sum up these terms.

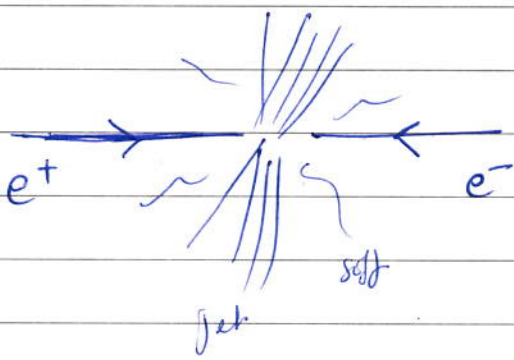
Reorganize Perturbation theory :—

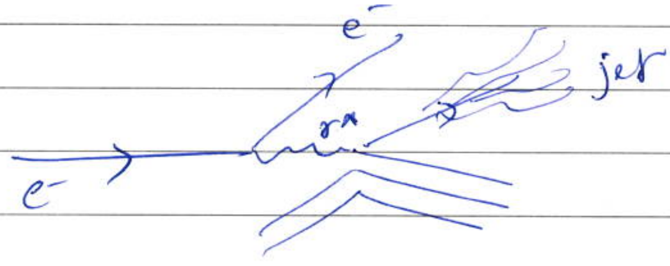
$$\frac{d\sigma}{dR} = \sigma_0 \Big\{ \alpha_s \big[C_{12} L^2 + C_{11} L + C_{10} \big] \rightarrow \text{LO}$$
$$+ \alpha_s^2 \big[C_{24} L^4 + C_{23} L^3 + C_{22} L^2 + C_{21} L + C_{20} \big] \rightarrow \text{NLO}$$
$$+ \alpha_s^3 \big[C_{36} L^6 + C_{35} L^5 + C_{34} L^2 + \cdots \big] \rightarrow \text{NNLO}$$
$$+ \cdots \Big\}$$

Leading Log   Next to Leading log

Calculate all terms in first tower, then the next tower and so on.

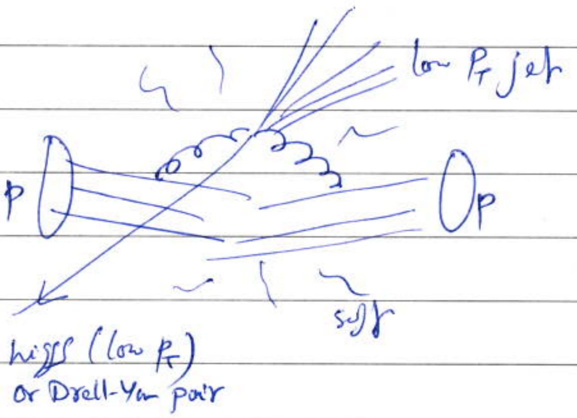
Before we continue about how to do that. Lets look at a few variety of observables

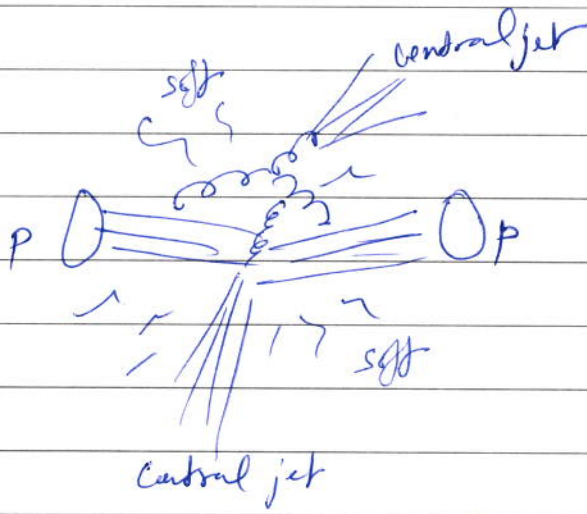




e+ e⁻ annihilation
(thrust, event shapes)

DIS / SIDIS
$\left(\frac{d\sigma}{dp_T dx}, \frac{d\sigma}{dx}\right)$



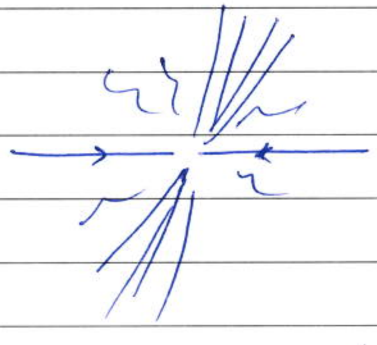


higgs p_T distribution at LHC

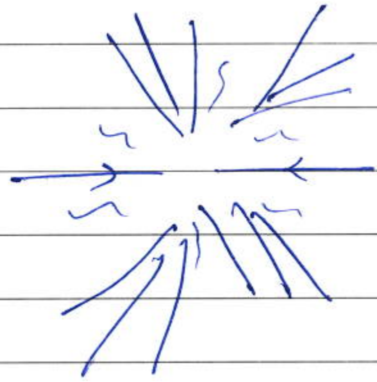
central jets (n-jettiness, exclusive jet cross-section)

Event Shapes → inclusive observable that give description of the entire event.

Event Shapes in e^+e^-



an event with two back to back jets



event with multijets

Such events can be differentiated by thrust

$$T = \frac{1}{Q} \max_{\hat{t}} \sum_{i \in X} |\hat{t} \cdot \vec{p}_i|$$

for each $\hat{t}$ calculate $\sum_{i \in X} \hat{t} \cdot p_i$ and then maximize this value.

$$\boxed{0.5 \leq T \leq 1}$$

maximizing $\hat{t}$ is called thrust axis and T is called thrust of the event. Thrust axis is also the direction of momentum flow.

Particularly interesting way to write thrust as:-

$$\tau = 1 - T \simeq \frac{1}{Q} \sum_{i \in X} |\vec{p}_{\perp i}| \, e^{-|\eta_i|}$$

also called thrust

massless hadron approx.

$$\boxed{0.5 \geq \tau \geq 0}$$

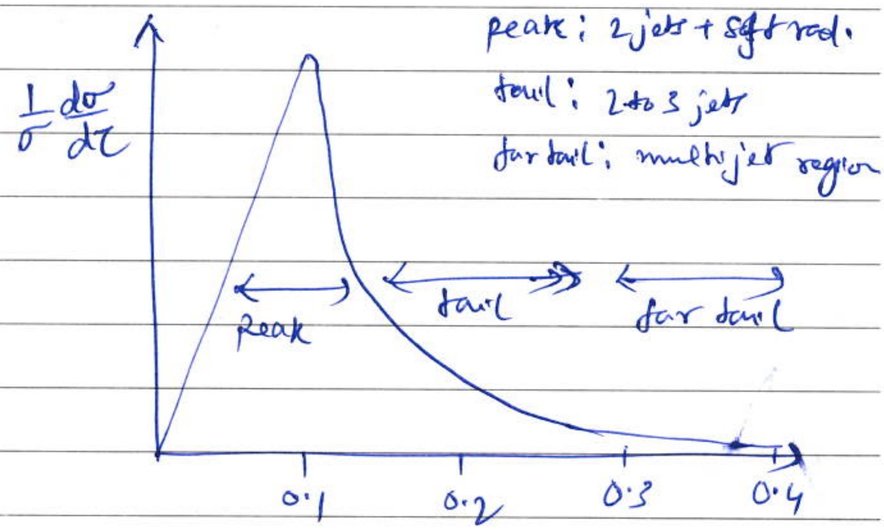
- Thrust axis provides a natural way of dividing the event into two hemispheres, one for each jet.

- Thrust does not care about how fat or narrow the jet is, it always divides an event into two hemispheres.

$\tau \ll 1$ $\Rightarrow$ two back to back pencil like jets

$\tau \gtrsim 0.2$ $\Rightarrow$ 3 jet like event

$\tau \gtrsim 0.3$ $\Rightarrow$ multi-jet scenario

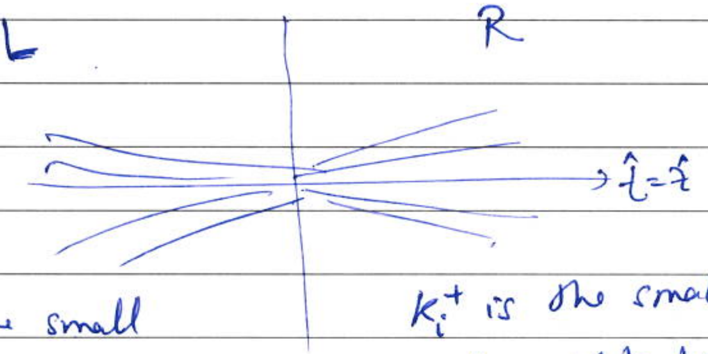


Another interesting way to write τ is :-

$$\tau = \frac{1}{Q}\Big[\underbrace{\sum_{i \in R}}_{\text{right hemisphere}} |k_i^+| + \underbrace{\sum_{i \in L}}_{\text{left hemisphere.}} |k_i^-|\Big] \qquad \left.\begin{matrix} k_i^+ \\ k_i^- \end{matrix}\right\} \text{light cone coord.}$$

In terms of light cone-coord

$$k_i^+ = k_i^0 - k_i^3 \qquad k^\mu = (k_i^+, k_i^-, k_{i\perp})$$
$$k_i^- = k_i^0 + k_i^3$$
$$k_{i\perp} = k_{i\perp}$$



k_i^- is the small component in the left hemisphere.

k_i^+ is the small component in the right hemisphere

Other event shapes in e^+e^-

all momenta measured with respect to the Thrust axis

$$B = \frac{1}{2Q} \sum_{i \in X} |\vec{p}_{\perp i}| \quad \leftarrow \text{Broadening}$$

$$\tau_a = \frac{1}{Q} \sum_{i \in X} |\vec{p}_{\perp i}| \, e^{-(1-a)|\eta_i|} \quad \leftarrow \text{angularity}$$

$$e = \frac{1}{Q} \sum_{i \in X} |\vec{p}_{\perp i}| \, f_e(\eta_i)$$

various choice of f_e defines various event shapes.

IRC safety is required for perturbative QCD calculatio

For angularities

$$\tau_a = \frac{1}{Q}\left[\sum_{i\in R}(k_i^+)^{1-a/2}(k_i^-)^{a/2} + \sum_{i\in L}(k_i^-)^{1-a/2}(k_i^+)^{a/2}\right]$$

$-\infty < a < 2$ IR safe
$a > 2$ IR unsafe.

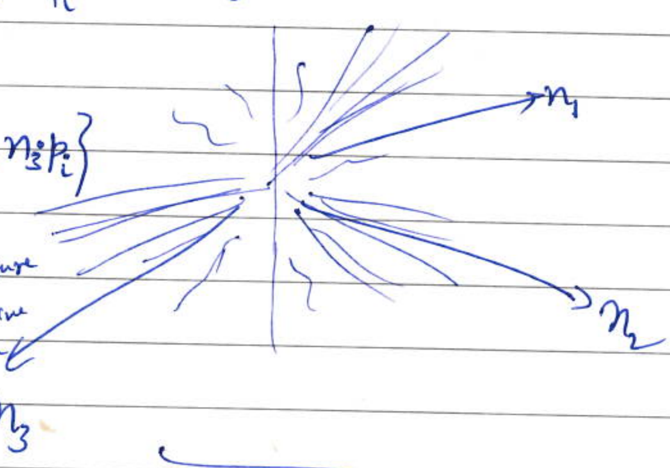
Angularities are IR safe for $a \leq 2$ thus thrust and broadening are IR safe.

n-jettiness in e^+e^-:

associate p_i with n_j if $p_i \cdot n_j$ is minimum

$$\tau_3(n_1,n_2,n_3) = \frac{1}{Q}\sum_{i\in X} \min\{n_1\cdot p_i,\ \underbrace{n_2\cdot p_i}_{\text{distance measure or metric relative to a direction}},\ n_3\cdot p_i\}$$

$n_j = (1, \hat{n}_j)$



$$\tau_3 = \min_{\hat{n}_1,\hat{n}_2,\hat{n}_3} \tau_3(n_1,n_2,n_3)$$

This divides the event into 3 regions; thus providing a way to find jets.

Alternatively one can define by maximization

$$T(\hat{n}_1,\hat{n}_2\hat{n}_3) = \frac{1}{Q}\sum_{i\in X}\max(\hat{n}_1\cdot p_i, \hat{n}_2\cdot p_i, \hat{n}_3\cdot p_i)$$

$T = \max T(\hat{n}_1,\hat{n}_2,\hat{n}_3)$

n-jettiness in p-p

In pp collision, we have two default jet axes n_a, n_b corresponding to beam direction. Maximization or minimization is done by including these two axes in the analysis, and a particle is associated with one of the beams or one of the signal directions thus dividing the event into $n+2$ regions.

n-subjettiness

A jet reconstructed by a jet algorithm or n-jettiness is then further divided into subjets by taking n-template directions in a jet and associating particles in the jet with one of these directions as it was done in n-jettiness.

By changing the distance measure or metric relative to the direction, several generalizations of n-jettiness are possible.

$\tau_n \ll 1 \implies$ event has n-jets or less

Example, To veto events with 4 or more jets choose $\tau_3 \ll 1$.

$\hookrightarrow$ central jets in pp

Thrust Cross-section

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{2\alpha_s}{3\pi}\left[-4\left[\frac{\ln\tau}{\tau}\right]_+ - \left[\frac{3}{\tau}\right]_+ \cdots\right] + \mathcal{O}(\alpha_s^2)$$

$$R(\tau) = \int_0^\tau d\tau' \frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} = 1 + \frac{2\alpha_s}{3\pi}\left[-2\ln^2\tau - 3\ln\tau\right] + \mathcal{O}(\alpha_s^2)$$

→ double logs.

- Resum the double logs by factorization and then ressummation via R.G. equations:—

Factorization is a statement of seperating physics at different scales.

To find out the scales involved let's look at relevant degrees of freedom or modes:—

* final state has only onshell particles
* each final state particle in right hemisphere should give $k_i^+ \sim \tau$ and each particle in left hemisphere should give $k_i^- \sim \tau$.

Thus Convenient to take, $\tau \sim \lambda^2$

$$\left.\begin{aligned} p_i^{coll} &\sim Q(1,\lambda^2,\lambda) \\ p_i^{anticoll} &\sim Q(\lambda^2,1,\lambda) \end{aligned}\right\} p^2 \sim \lambda^2$$

ultrasoft ⟵ $p_s \sim Q(\lambda^2,\lambda^2,\lambda^2) \Rightarrow p_s^2 \sim \lambda^4$

M T W T F S S --/--/----

Note: Additionally sometimes Glauber modes can be present $p^{\text{Glauber}} \sim (\lambda^2, \lambda^2, \lambda)$

offshell modes cannot be present in Lagrangian

$$p_c^2 \gg p_{us}^2$$

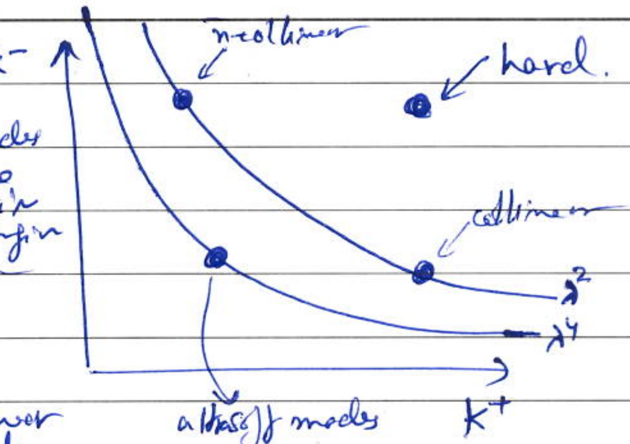
$$p_c^{coll} + p_{us} \sim p^{coll}$$

$p^{coll} + p^{anticoll} \sim (1, 1, \lambda) \sim$ hard mode

$p^{coll} + p_s^{soft} \sim (1, \lambda, \lambda) \sim$ hard collinear mode

not present in theory

$p.$



* A suitable effective theory to describe these modes is SCET

* EFT has advantage that modes are cleanly separated
  - Systematic expansion in power counting parameter (λ)
  - power corrections are well-separated providing theoretical control on errors
  - propagation to higher order perturbative corrections is much faster (each loop integral has reduced number of dimensionful parameters)

EFT in a nutshell

Full theory fields & operators are matched on to EFT fields and operators:

$$\mathcal{L}_{Full} = \underbrace{\mathcal{L}^{(0)}_{EFT}}_{\lambda^0} + \underbrace{\mathcal{L}^{(1)}_{EFT}}_{\lambda^1} + \underbrace{\mathcal{L}^{(2)}_{EFT}}_{\lambda^2} + \cdots$$

EFT is generated by integrating out offshell modes like hard gluons or quarks, hard-collinear gluons/quarks.

EFT fields are constructed by ~~large~~ removing large momentum component from the fields, i.e.

~~$\psi(x) = \sum e^{ip\cdot x}$~~ ~~$\bar{n} = (1,0,0,1)$~~ ~~$p^+ = \bar{n}\cdot p$~~

Define light cone direction n:-

$$n^2 = 0\ ,\ \bar{n}^2 = 0 \qquad n = (1,0,0,1)$$
$$n\cdot\bar{n} = 2 \qquad \bar{n} = (1,0,0,-1)$$

$\xi_n \to n$-collinear quark field
$\xi_{\bar{n}} \to \bar{n}$-collinear quark field.

ξ_n and $\xi_{\bar{n}}$ fields should be responsible for creating/annihilating quarks along n- and $\bar{n}$-directions.

<u>More formally</u>

$$\psi(x) = \sum_n \sum_{p_n^-,\, p_{n\perp}} e^{ip_n^- x^+ + ip_{n\perp}\cdot x_\perp}\, \xi_{n,p_{n\perp}}(x) \qquad p_n^- = \bar{n}\cdot p,\quad \bar{n} = (1,-\hat{n}),\quad n = (1,\hat{n})$$

$$p_n^- \sim 1 \qquad p_{n\perp} \sim \lambda$$

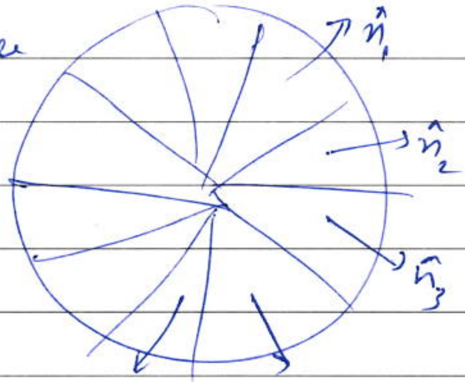
$$q = \underbrace{p}_{\text{label momenta}} + \underbrace{k}_{\text{residual momenta}}$$

label momenta is pulled out of the field.

Thus dynamic that remains in $\xi_{n,p_{n\perp}}(x)$ is only order $(\lambda^2, \lambda^2, \lambda^2)$ momenta.

Energy conservation, choice of direction etc. freezes values of $\bar{p}_n$, $p_{n\perp}$ etc.

Effectively we have divided phase space into several cones



Any interaction that leads to exchange of momenta between the cones is hard or offshell in nature and thus is integrated out; it does not appear in dynamics.

~~* SCET Lagrangian can be ...~~

* Similar decomposition is applied to gluon field.

<u>SCET components</u>:-

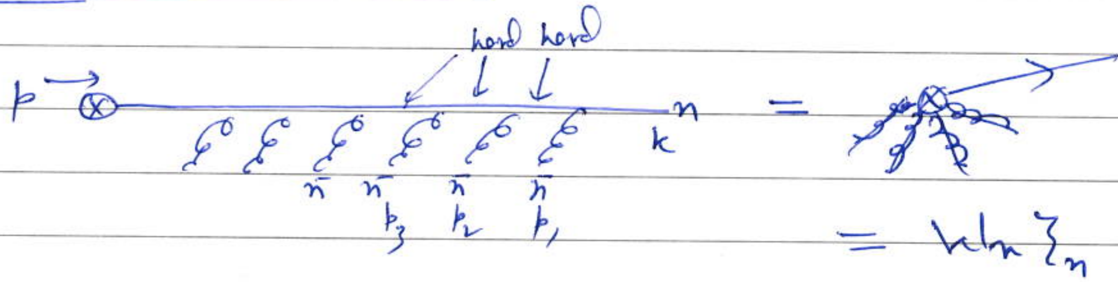
$$\xi_n, \ \xi_{\bar{n}}, \ \xi_s, \ \xi_{us}$$

$$A_n, A_{\bar{n}}, A_s, A_{us}$$

$$iD^\mu = \frac{n^\mu}{2}\bar{\mathcal{P}} + \mathcal{P}_\perp^\mu + \frac{\bar{n}^\mu}{2}\, in\cdot D$$

where $iD = i\partial^\mu + g A^\mu_{us}$

Collinear SCET Wilson lines:



$$= W_n \xi_n$$

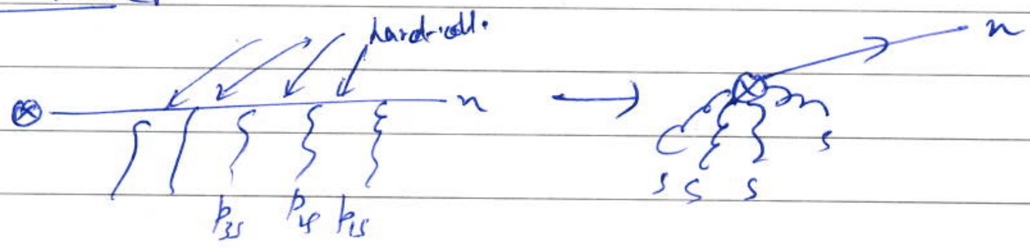
$$p = \underbrace{(p_1 + p_2 + \dots)}_{\bar{n}\text{-collinear}} + k \to n\text{-collinear}$$

$$W_n = \left[\sum_{perm} \exp\left(-g \frac{1}{\bar{n}\cdot \mathcal{P}} \bar{n}\cdot A_{n,q}\right) \right] \leftarrow \text{in mom. space}$$

$$W_n(x^+) = P \exp\left(ig \int_{-\infty}^{x^+} ds\, \bar{n}\cdot A_n(s\bar{n}) \right) \leftarrow \text{in x-space}$$

path ordered

Soft Wilson line:



$$= S_n \xi_n$$

ultrasoft Wilson line

$$Y(x) = P \exp\left(ig \int_{-\infty}^{x} ds\, n\cdot A^a_{us}(ns) T^a \right)$$

All Wilson lines are generated due to eikonalization of the offshell propagators

Current Matching:

$$\bar{\psi}\gamma^\mu\psi \longrightarrow C(P,\bar{P})\,\bar{\xi}_n W_n \underbrace{Y_n^\dagger \gamma^\mu Y_{\bar{n}}}_{} W_{\bar{n}} \xi_{\bar{n}}$$

u-soft gauge invariant

collinear gauge invariant

QCD cross-section;

$$\sigma = \sum_{X}^{res.} (2\pi)^4 \delta^{(4)}(q-P_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0|J_i^{\nu\dagger}|X\rangle\langle X|J_i^\mu|0\rangle$$

res = restrictions can also be written in terms of explicit measurement δ-functions.

res. $\Rightarrow$ we are measuring jetty states.

then $|X\rangle = |X_n\rangle|X_{\bar{n}}\rangle|X_s\rangle$

more robustly, we can make use of measurement operators and keep states without restriction. Fields and operators will automatically take care of restrictions.

$$\sigma = \sum_{X}^{\text{res.}} (2\pi)^4 \delta^{(4)}(q - p_{X_n} - p_{X_{\bar n}} - p_{X_s}) \, |C(Q,\mu)|^2$$ hard stuff

ultrasoft stuff only $\leftarrow \times \langle 0 | Y_{\bar n}^\dagger Y_n | X_s \rangle \langle X_s | Y_n^\dagger Y_n | 0 \rangle$

collinear stuff $\leftarrow \times \langle 0 | \gamma\cdot\bar n \, W_n^\dagger \xi_n | X_n \rangle \langle X_n | \bar\xi_n W_n | 0 \rangle$

$\bar n$-collinear stuff only $\leftarrow \times \langle 0 | \bar\xi_{\bar n} W_{\bar n} | X_{\bar n} \rangle \langle X_{\bar n} | \gamma\cdot n \, W_{\bar n}^\dagger \xi_{\bar n} | 0 \rangle$

<u>Factorization theorem for thrust</u>

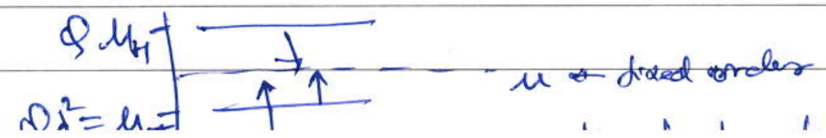
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \, dp_R^2 \; J(p_L^2,\mu) J(p_R^2,\mu) S(k,\mu) \times \delta\left(\tau - \frac{p_L^2 + p_R^2}{Q^2} - \frac{k}{Q}\right)$$

Each ~~operator~~ function has a matrix element defn. in terms of collinear and soft fields.

For jet fn. natural scale is $\mu \simeq Q\lambda^2$

" soft fn. " " " $\mu \simeq Q\lambda^4$

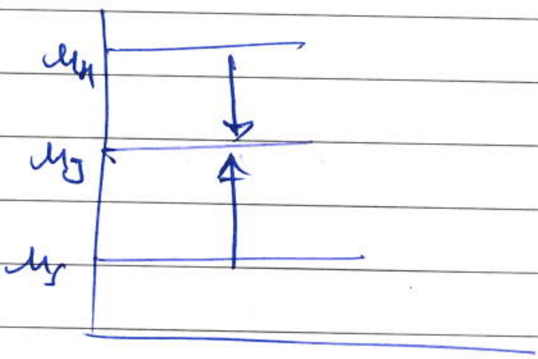
For hard fn. natural scale is $\mu \simeq Q$.



In practice it's easier to choose $\mu = \mu_J$ and run hard and soft fn to μ_J.

Thus,

$$\sigma = \sigma_0 \left[U_H(\mu_J, Q)\, H(Q) \right] J(\mu_J) \otimes J(\mu_J) \otimes \left[U_S(\mu_J, \mu_S)\, S(\mu_S) \right]$$



R.G. evolution in a toy model

$$S_{bare}(m,\mu) = 1 + \alpha_s(\mu)\frac{a_0}{\epsilon}\frac{\mu^{2\epsilon}}{m^{2\epsilon}}$$

$$= 1 + \alpha_s(\mu)\frac{a_0}{\epsilon} + 2\alpha_s(\mu)\, a_0 \log\left(\frac{\mu}{m}\right)$$

Renormalize,

$$S_{bare} = Z_s\left(\mu, 1/\epsilon\right) S_{ren}(\mu)$$

$$\Downarrow$$

$$Z_s = 1 + \alpha_s(\mu)\, a_0/\epsilon$$

$$S_{ren} = 1 + 2\alpha_s(\mu)\, a_0 \log(\mu/m)$$

R.G. eqn.,

$$\mu\frac{d}{d\mu} S_{bare} = 0 \Rightarrow Z\frac{dS_{ren}}{d\ln\mu} = -\frac{dZ}{d\ln\mu} S_{ren}(\mu)$$

$$\Rightarrow \frac{dS_{ren}}{d\ln\mu} = \gamma_s(\mu)\, S_{ren}(\mu)$$

$$\gamma_s(\mu) = \left(Z_s^{-1}\frac{dZ}{d\ln\mu}\right) = -2a_0\alpha_s(\mu)$$

Solution:

$$S_{ren}(m,\mu) = \exp\left[\int_{\mu_0}^{\mu}\gamma(\mu')\frac{d\mu'}{\mu'}\right] \cdot S_{ren}(m,\mu_0)$$

scale natural

$$S_{\gamma m}(m,\mu) = \left(\frac{\mu}{\mu_0}\right)^{-2a_0\alpha_s(\mu)} S_{\gamma m}(m,\mu_0)$$

Choosing $\mu_0 = m$ makes $S_{\gamma m}$ convergent. Series for $S_{\gamma m}$ will contain no large logs. All large logs appear in the exponential.

* Repeating same process for other functions leads to ressummation of the cross-section.