Transport and the breakdown of single-parameter scaling at the localization transition in quasiperiodic systems

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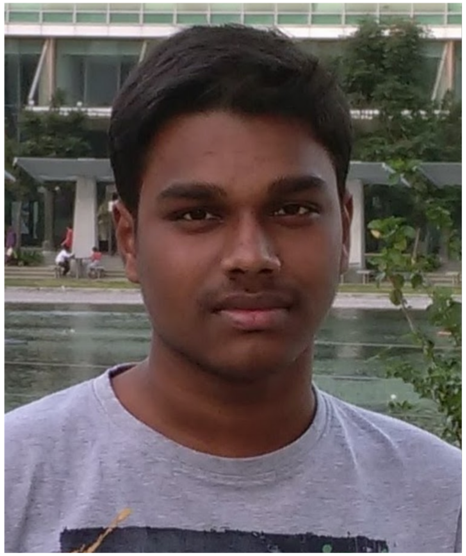


Jagannath Sutradhar (IISc)

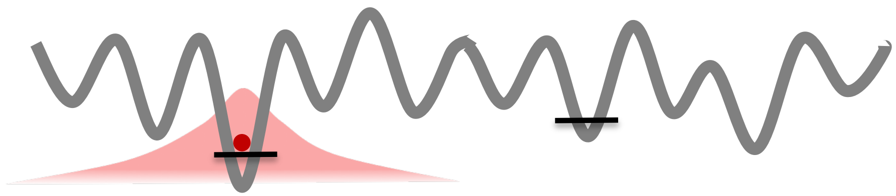
Subroto Mukerjee (IISC)

Rahul Pandit (IISc)

arXiv:1810.12931 (2018)

Anderson localization

Single-particle localization (Anderson 1958)

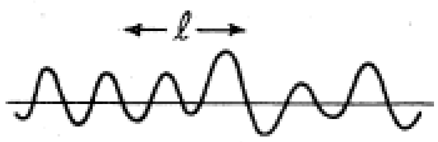


$$\mathcal{H} = -t \sum_{<ij>} (c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i$$

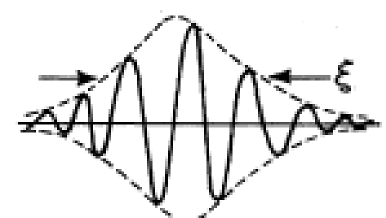
Random → $\epsilon_i \in [-W, W]$

Two distinct possibilities →

Extended states



Localized states



$|\psi(r)|^2 \sim \frac{1}{L^d}$

Critical point

$(d \geq 3)$

W_c, ϵ_c

$|\psi(r)|^2 \sim \frac{e^{-\frac{r}{\xi}}}{\xi^d}$

W, ϵ

→ Transport in finite system of linear dimension L

Conductance → $G(L) = \sigma L^{d-2}$ $\quad$ $G(L) \sim G_0 \exp(-L/\xi)$

Diffusive metal (Ohm's law) $\quad$ Insulator

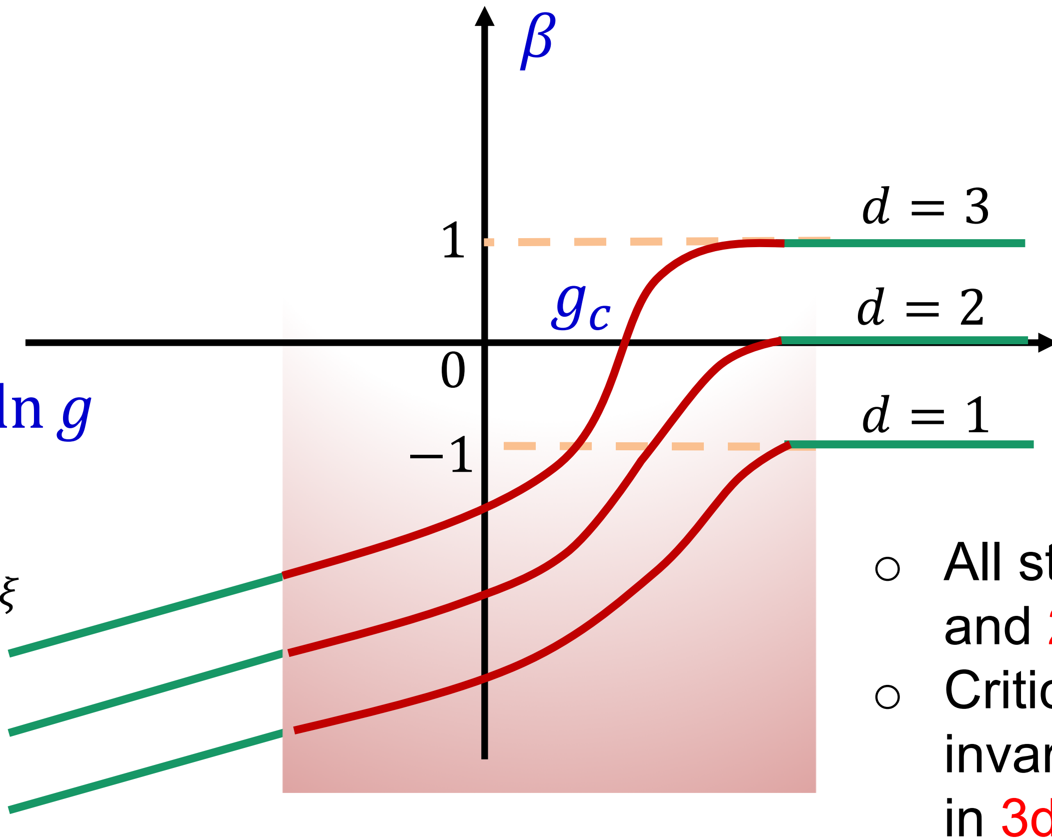
Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

'Gang of four', E. Abrahams et al. (1979)

β function →

$$\frac{d\ln(g(L))}{d\ln L} = \beta(g)$$

(continuous and monotonic) function of dimensionless conductance $g = G/(e^2/h)$ only



$g \gg 1$

$g \propto L^{d-2}$

$\beta = d - 2$

$g \ll 1$

$g = g_0 e^{-L/\xi}$

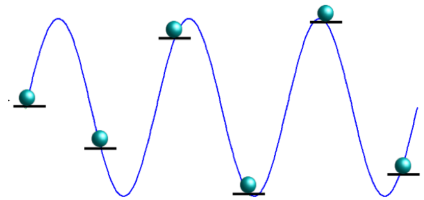
$\beta = \ln\left(\frac{g}{g_0}\right)$

- All states localized in 1d and 2d.
- Critical point with scale invariant conductance g_c in 3d.

Anderson localization is not limited to random systems

Quasiperiodic systems can have localized states and metal-insulator transitions, *even in 1d*

1d Aubry-Andre model



$$\mathcal{H} = -t \sum_{<ij>} (c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i$$

$$\epsilon_i = 2V \cos(2\pi b i + \phi)$$

b, irrational number,

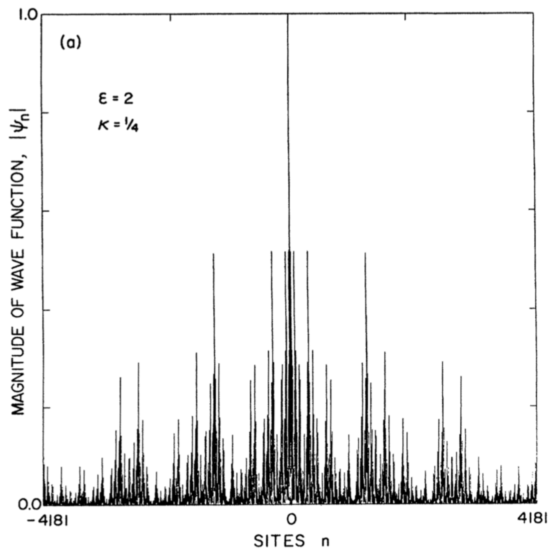
e.g. $b = \frac{\sqrt{5}-1}{2} = \lim_{n\to\infty} \frac{F_{n-1}}{F_n}$

Fibonacci numbers, $F_n = 1,2,3,5,\ldots$

- Deterministic but never repeating
- Disorder averaging ≡ Averaging over ensemble of ϕ

1d Aubry-Andre model

Self-similar critical states



All states delocalized

Ballistic transport

$g(L) \propto L^{d-1}$

All states localized

$g(L) \sim \exp(-L/\xi)$

V

Self-dual Critical point $(d = 1)$

$V_c = 1$

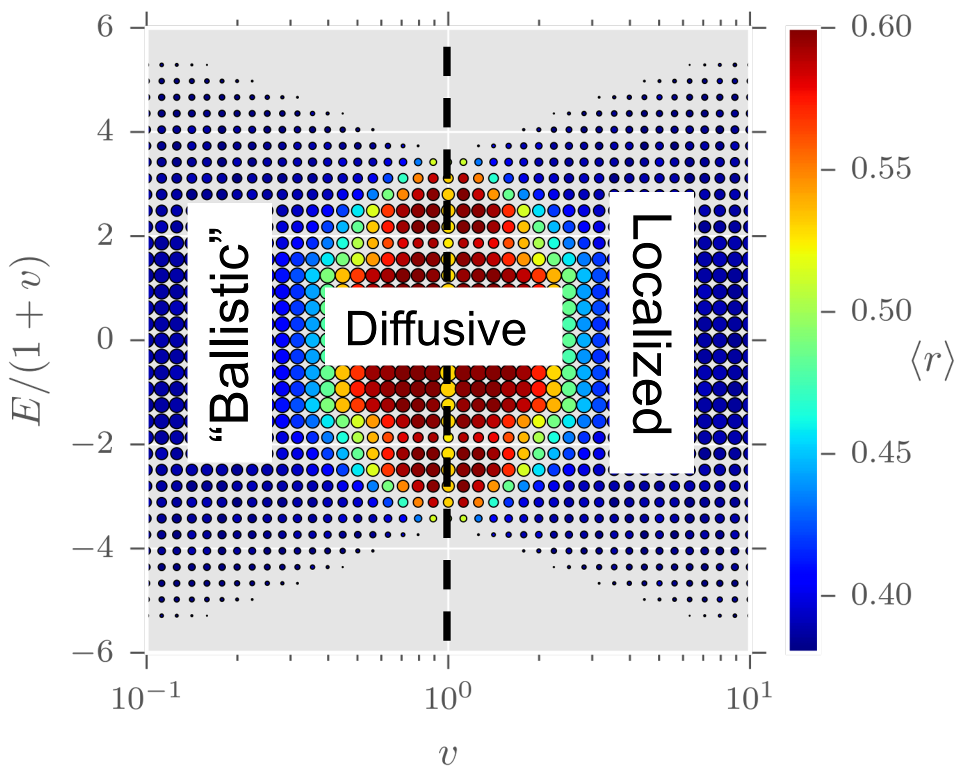
Momentum space ← **Duality** → Real space

Ostlund et al. (1983), Ostlund & Pandit (1984), Kohmoto (1983), ..

Self-dual generalizations of Aubry-Andre model to higher dimensions

Devakul & Huse, Phys. Rev. B 96 (2017) "Higher dimensional Aubry-Andre model"

Three dimensions



Diffusive to localized transition

$$\xi \sim (V - V_c)^{-\nu}$$

$$\nu \simeq 1.6$$

*Same as 3d Anderson transition

Could there be a single-parameter scaling description of metal-insulator transition in quasiperiodic systems?

- Is there a well-defined β-function *a la* 'Gang of Four'?
- Nature of transport at the critical point and its dependence on dimension?

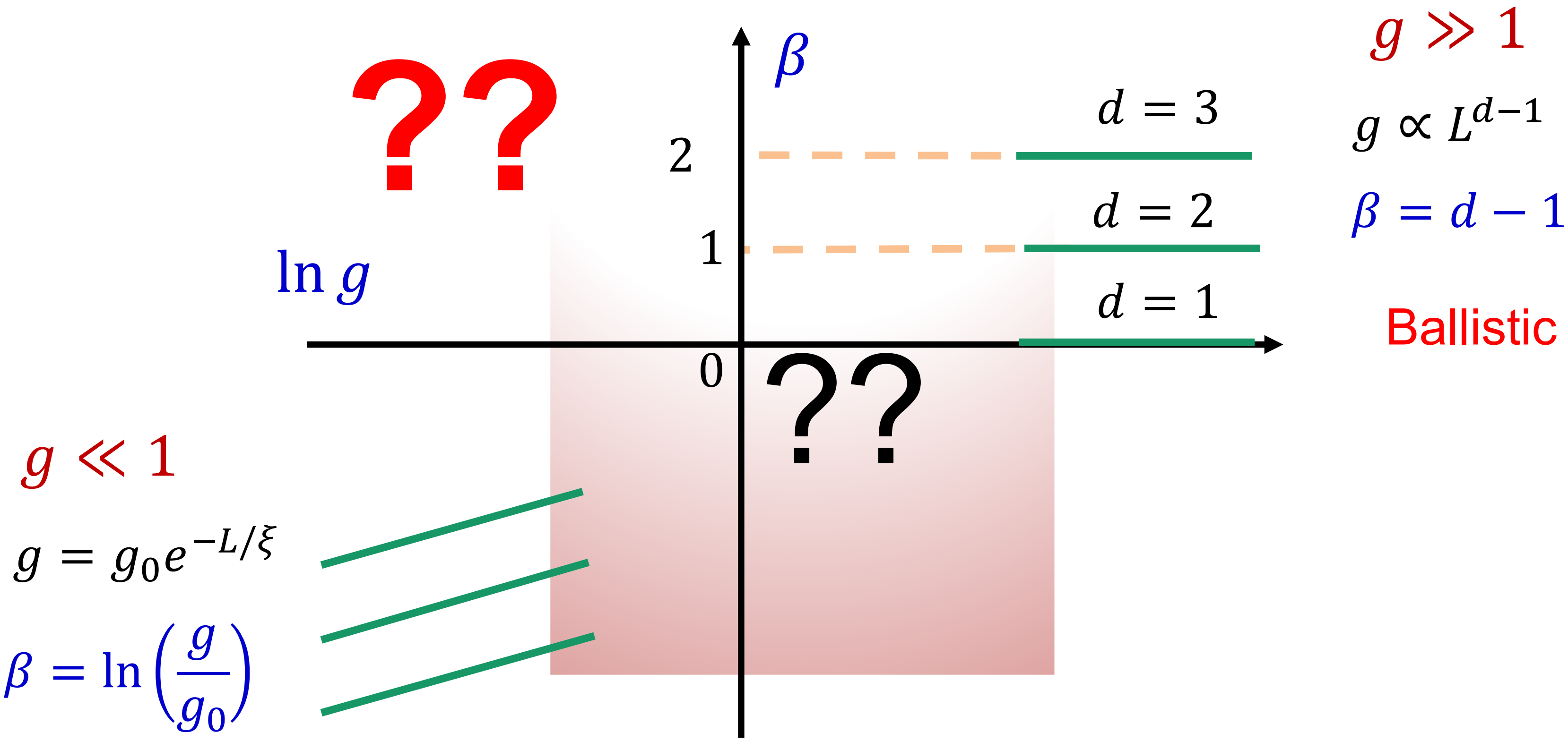
$$g(L) \propto L^{\alpha}$$

- Diffusive, $\alpha = d - 2$?
- Ballistic, $\alpha = d - 1$?
- Subdiffusive, $\alpha < d - 2$?
- Superdiffusive, $d - 2 < \alpha < d - 1$?

✓ 1d Aubry-Andre → Subdiffusive at 'high temperatures'

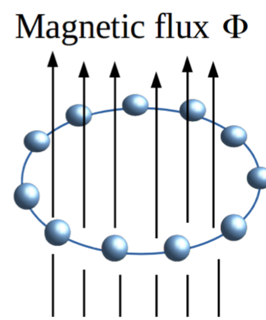
Purkayastha et al. (2017, 2018), Varma et al. (2017)

- What happens in higher dimensions?
- Does having same critical exponent ν as 3d Anderson imply single-parameter scaling for 3d Aubry-Andre?



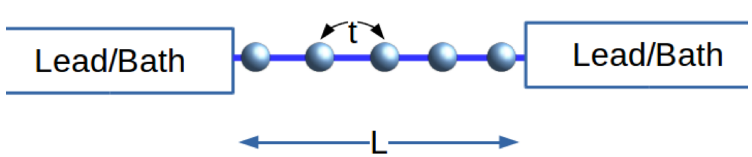
Zero-temperature conductance

- ‘Gang of Four’ → Thouless conductance
  (‘Closed system conductance’)



$$g_T(E) \sim \frac{\left(\frac{\partial^2 E}{\partial \Phi^2}\right)}{\delta_E}$$

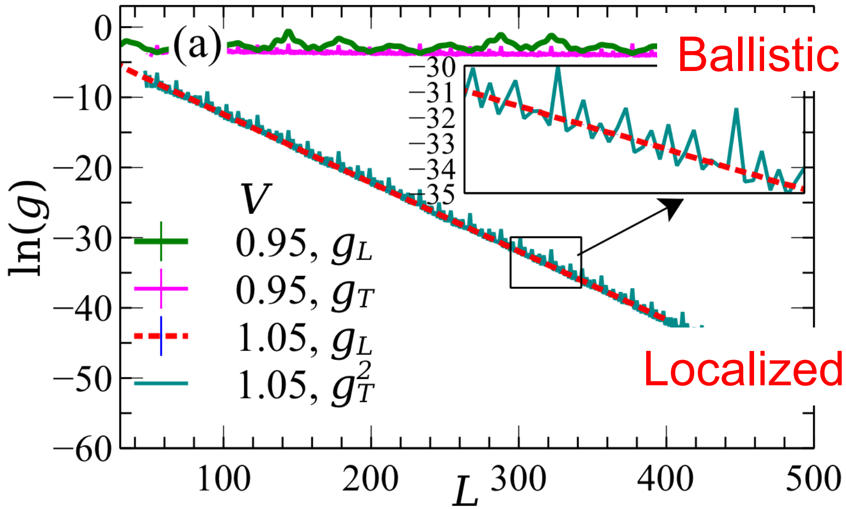
- Open-system (Landauer) conductance



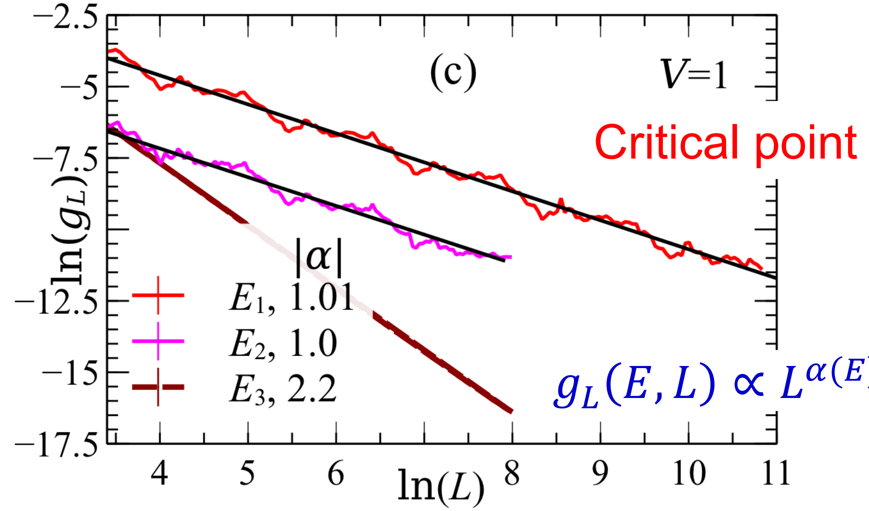
$g_L(E)$ ← Recursive Green’s function method

➢ No single-parameter scaling in 1d and 2d quasiperiodic model

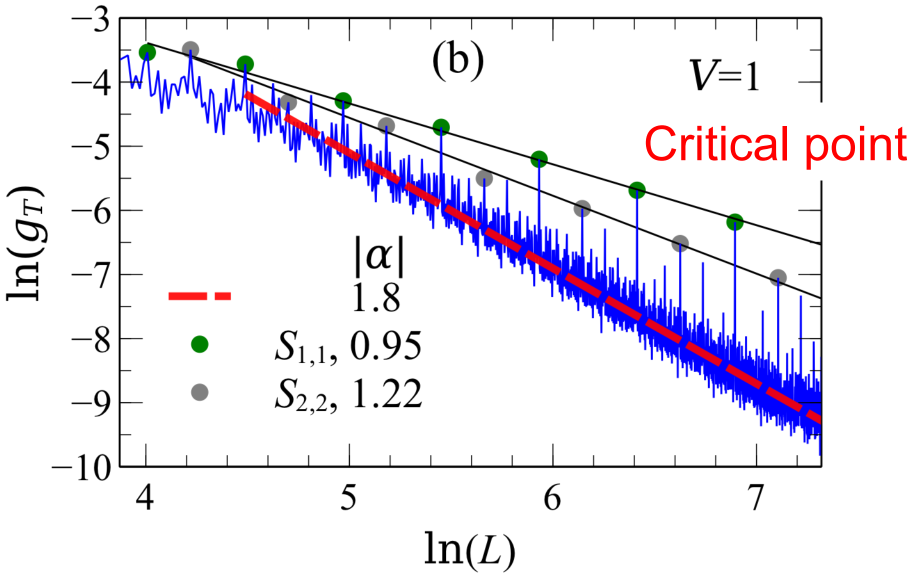
One dimension



Open system



Closed system

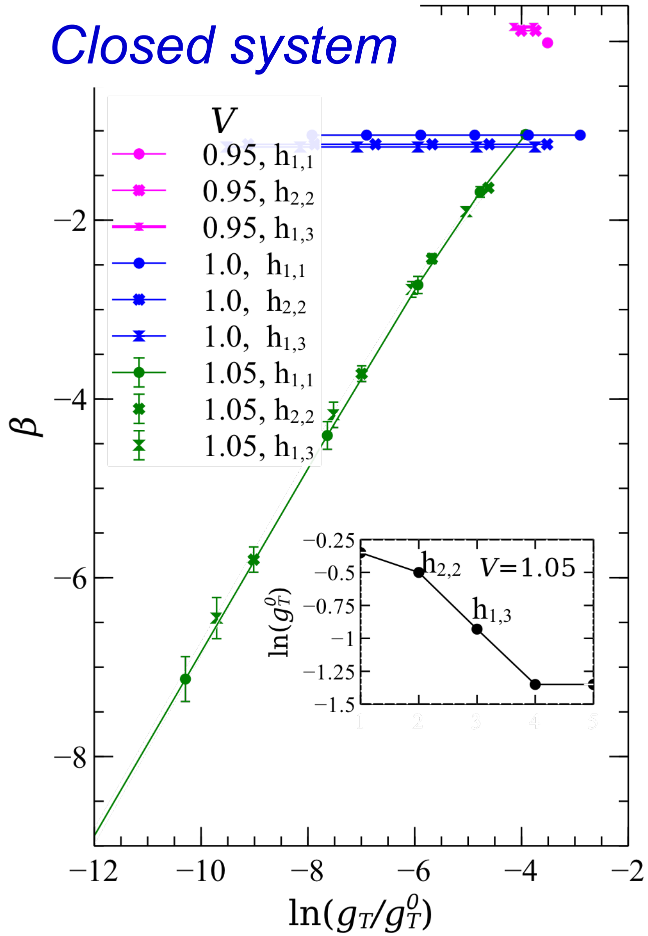


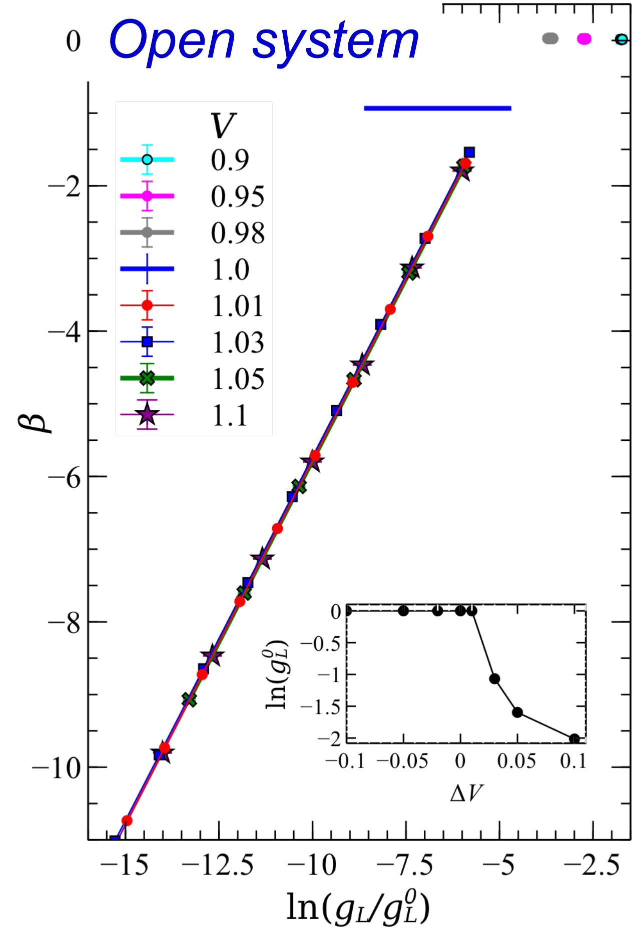
○ Even typical conductances are non-monotonic function of L

→ β-function ill defined
strictly no single-parameter scaling

β-function from *overall scaling* →

β-function from overall scaling →





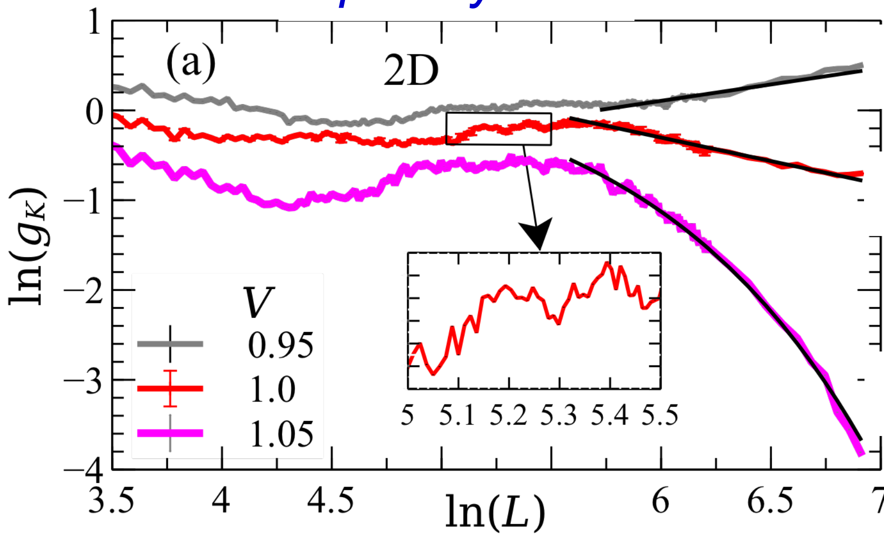
- β-function jumps discontinuously at the transition

- No one-parameter scaling collapse.
- Multifractal fluctuations of g_T as a function of L

arXiv:1810.12931 (2018)

➢ Similar story in 2d, no single parameter scaling

Open system



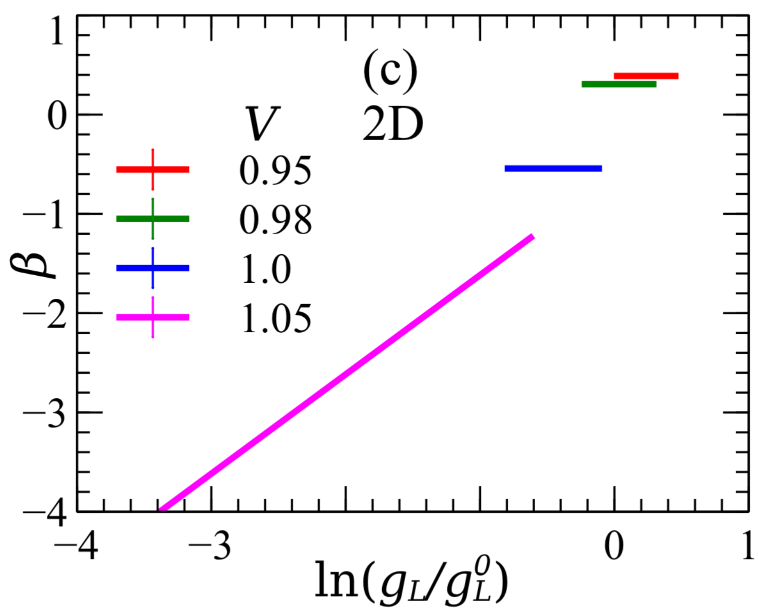
Superdiffusive

Subdiffusive critical state

Localized

Non-monotonic $g_L(L)$

→ ill-defined β-function



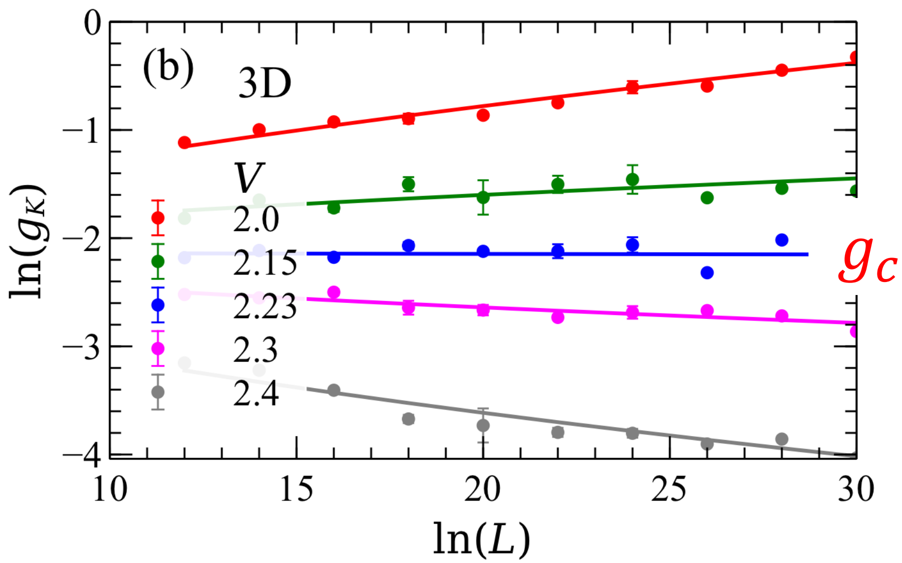
- Discontinuous jump of overall β-function across the transition

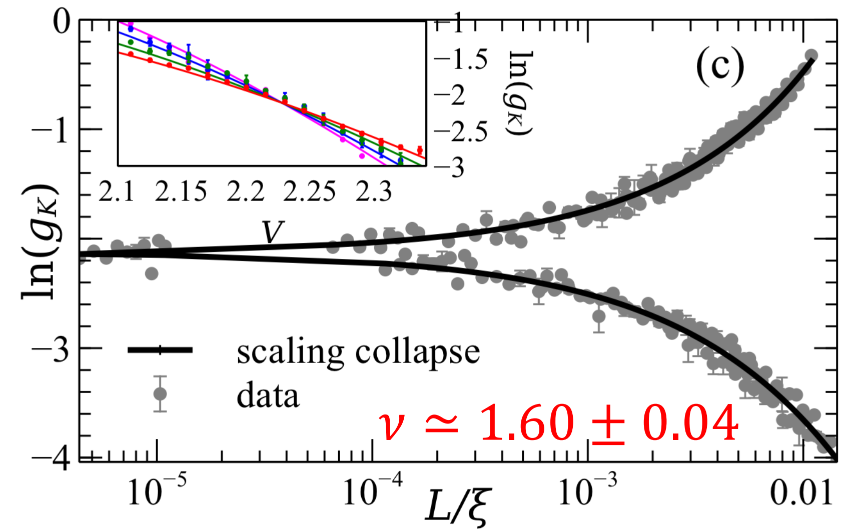
*System sizes up to 1000^2

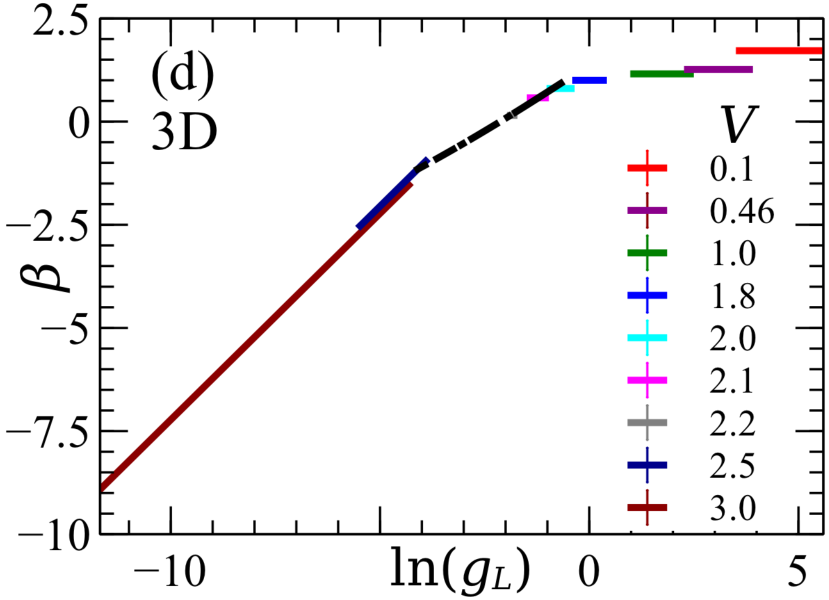
➢ *Approximate* single-parameter scaling in 3d quasiperiodic system

Open system

Single-parameter scaling →







○ Continuous and monotonic β-function across the diffusive-localized transition

○ No sharp diffusive-ballistic transition in open system,

Crossover →
Diffusive → Superdiffusive → Ballistic
→ Breakdown of duality

Summary and outlook

- Violation of single-parameter scaling in 1d and 2d Aubry-Andre and the way it fails.
- Approximate single-parameter scaling in 3d.
- Subdiffusive critical states in 1d, 2d and 3d.

Theory, e.g. RG, for higher-dimensional quasiperiodic systems?
Effect of interaction and many-body localization (MBL)?

Why care about localization in quasiperiodic systems?

- Experimental realizations of MBL in cold atomic systems ← 1d Aubry Andre + interaction
- Distinct universality classes for MBL transition in 1d random and quasiperiodic systems.
- MBL is more robust in quasiperiodic systems than in the random systems due to lack of rare regions → Possibility of MBL in higher dimensions