Transport in a classical disordered systems

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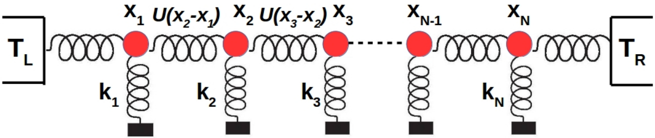
Brief Introduction

- Disordered systems without noninteracting particles exhibit Anderson localization (AL) whereby normal modes (NMs) of the system form spatially localized states, e.g, 1D systems found to be a thermal insulator.
- A natural question is to ask what happens upon introducing the interactions. Is the system show a transition to conducting state? If so, what is the critical strength of interaction?
- In quantum systems, these questions has been rigorously studied in Literature. But in classical systems, there is no complete understanding.
- We study the energy transport in a classical disordered nonlinear system where nonlinearity results in the destruction of AL because of nonintegrability of the system.

The Model



$$\mathcal{H} = \sum_{i=1,N} \left[\frac{p_i^2}{2m_i} + k_i \frac{x_i^2}{2} \right] + \sum_{i=1}^{N+1} \nu \frac{(x_i - x_{i-1})^4}{4}, \qquad (1)$$

$m_i = 1, k_i \in [1 - \Delta, 1 + \Delta]$, Δ: Disordered strength.

$$\begin{aligned} \ddot{x}_1 &= -k_1 x_1 - \nu[(x_1 - x_0)^3 + (x_1 - x_2)^3] - \gamma \dot{x}_1 + \eta_L, \\ \ddot{x}_i &= -k_i x_i - \nu[(x_i - x_{i-1})^3 + (x_i - x_{i+1})^3], \; i = 2, \cdots, N-1 \\ \ddot{x}_N &= -k_N x_N - \nu[(x_N - x_{N-1})^3 + (x_N - x_{N+1})^3] - \gamma \dot{x}_N + \eta_R. \end{aligned}$$

- The Gaussian white noise satisfy the fluctuation dissipation relation: $\langle \eta_{L,R}(t) \eta_{L,R}(t') \rangle = 2\gamma k_B T_{L,R} \delta(t - t')$ with $\langle \eta_{L,R} \rangle = 0$.

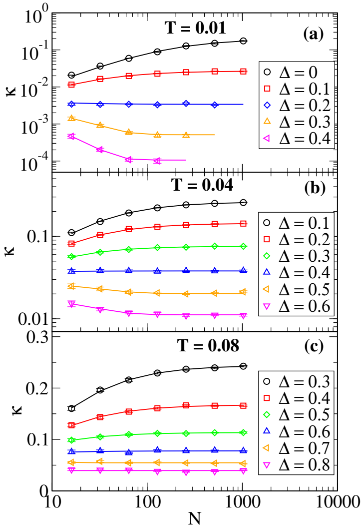
Numerical Details

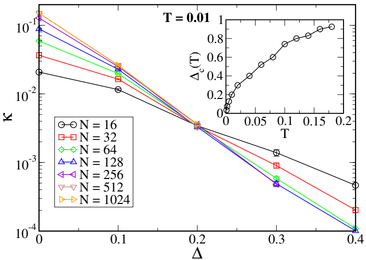
- Quantity of interest is to measure $\langle J_N \rangle = \sum_l \langle f_{l,l-1} \dot{x}_l \rangle / (N-1)$, $\kappa = \lim_{N\to\infty} JN/(T_L - T_R)$.

$$f_{l,l-1} = -\frac{\partial U(x_l - x_{l-1})}{\partial x_l} = \nu (x_{l-1} - x_l)^3. \qquad (2)$$

- Important point is to note that Eq. of motion is invariant under $T_{L,R} \to sT_{L,R}$, $\{x_l\} \to \{s^{1/2} x_l\}$ and $\nu \to \nu/s$, which implies $\kappa(sT_L, sT_R, \nu) = \kappa(T_L, T_R, s\nu)$.
- We set $\nu = 1$, $T = (T_L + T_R)/2$, and $(T_L - T_R) = T/2$ (i.e. $T_L = 1.25T$, $T_R = 0.75T$).
- Simulation is performed using velocity-Verlet algorithm. $t_{\rm eq} \sim 10^{10} - 10^{11}$, $dt = 0.005$. Numerical data is averaged over 50 disorder samples.

Simulation Results





- $\Delta < \Delta_c(T)$

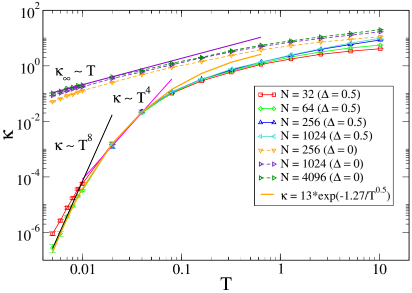
$$\kappa_N(\Delta, T) = \kappa_\infty(\Delta, T)\,\frac{1}{1 + \ell/N}$$

- $\Delta > \Delta_c(T)$

$$\kappa_N(\Delta, T) = A\, e^{-N/\xi} + \kappa_\infty(\Delta, T)$$

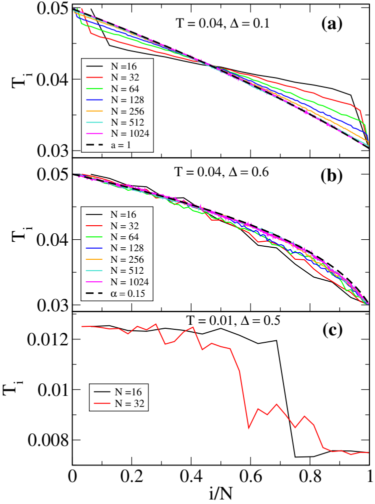
- Point symbols are the simulated values of κ whereas the solid lines are the best non-linear curve fits κ below and above $\Delta_c(T)$.

Conductivity as a function of temperature



- Without disorder ($\Delta = 0$), $\kappa \sim T$ and saturates at high T.
- With disorder, κ decays higher than any power-law.
- Going down to $T = 0.005$, $\kappa \sim T^8$, which crosses over T^4 at $T \gtrsim 0.01$.
- Below $T = 0.005$, we reach our computational limitation because of the huge impact of fluctuations (t_{eq} increases rapidly).
- A black solid line is the best fit: $\kappa(T) \sim \exp(-1.27/T^{1/2})$.

Temperature Profile



- $T_i = \langle p_i^2 \rangle$.
- The analytical fits to the T-profiles can be obtained by solving $-\kappa(T)\partial T(x)/\partial x = J$ with $\kappa \sim T$ for low Δ and $\kappa \sim e^{-B/T^{1/2}}$ for high Δ.
- Fig. (c) for a very small $T = 0.01$ and high $\Delta = 0.5$ shows a kind of step profile as an indication of MBL.

  (Roeck, Dhar, Huveneers, and Schütz, J. Stat. Phys, 167, 1143, 2017)

Conclusions

- We studied $\kappa(N, \Delta, T)$ in a classical disordered nonlinear system.
- We found $\Delta_c(T)$ below and above which the scaling of $\kappa_N(\Delta, T)$ is remarkably different.
- For ordered chain, $\kappa \sim T$.
- For disorder chain, $\kappa \sim \exp(-B/T^{1/2})$.
- The system show the signatures of MBL.