# Phase Transition in an Ising Model without Detailed Balance

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# Phase Transitions in Out-of-Equilibrium Systems

Active systems consisting of self-propelled particles that can generate systematic motion from internal or ambient sources of energy.

These essentially out-of-equilibrium systems exhibit phase transitions, such as motility induced liquid-gas phase separation.

Nonequilibrium phase transition in an Ising-like system

# Two-dimensional ferromagnetic Ising model on a square lattice with nearest-neighbour interactions

Update rule: modified version of the Metropolis rule that does not satisfy detailed balance

**Metropolis rule**:

Proposed change $\sigma_i \to -\sigma_i$: change in energy $\Delta E$.

If $\Delta E \leq 0$, accept change with probability 1

If $\Delta E > 0$, accept change with probability $\exp[-\Delta E/T]$

# Modified update rule:

Define $\delta E \equiv \Delta E + E_0$.
If $\delta E \leq 0$, accept change with probability 1
If $\delta E > 0$, accept change with probability $\exp[-\delta E/T]$

$E_0 < 0$: spin flips are promoted ("active")
$E_0 > 0$: spin flips are made less probable ("persistent").

On a square lattice, $\Delta E = 0,\ \pm 4J,\ \pm 8J$.
If $E_0 \le -8J$, $\delta E \le 0$, i.e. all proposed spin flips are accepted. This corresponds to the Metropolis rule with effective temperature $T_{\text{eff}} \to \infty$.

If $E_0 \ge 8J$, $\delta E \ge 0$.
Proposed change $\sigma_i \to -\sigma_i$ is accepted with probability $\exp[-(\Delta E + E_0)/T$.
Proposed change $-\sigma_i \to \sigma_i$ is accepted with probability $\exp[-(-\Delta E + E_0)/T$.

$$\frac{P(\sigma_i \to -\sigma_i)}{P(-\sigma_i \to \sigma_i)} = \exp[-2\Delta E/T]$$

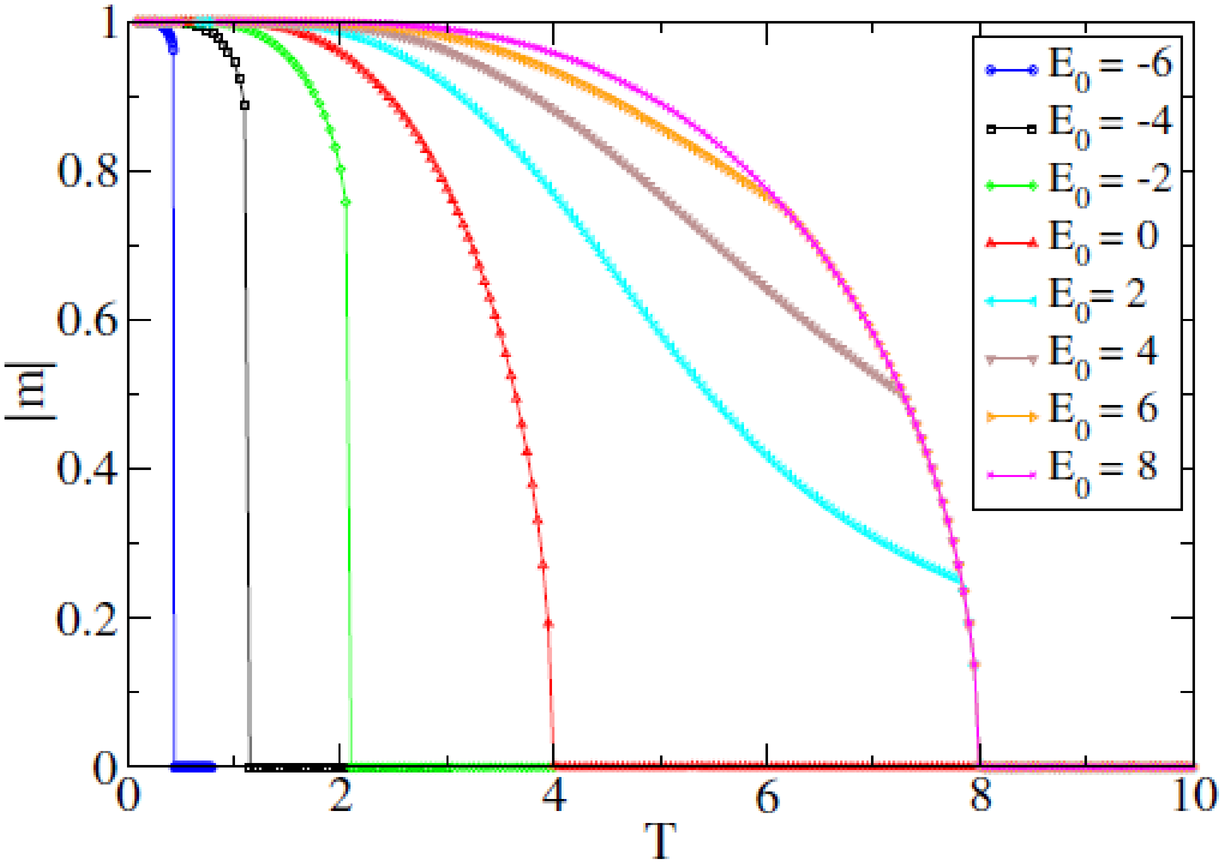
This corresponds to detailed balance at effective temperature $T_{\text{eff}} = T/2$.

For $-8J < E_0 < 8J$, it is not possible to find an unique effective temperature for which the transition probabilities for all possible values of $\Delta E$ satisfy detailed balance.

# Non-equilibrium Ising model

Does it exhibit a order-disorder transition?
If it does, then are the critical exponents the same as those of the equilibrium 2D Ising transition?

# Curie-Weiss Molecular Field Theory




Discontinuity in m for negative Eo.

Discontinuity in temperature derivative of m for positive values of Eo <8J

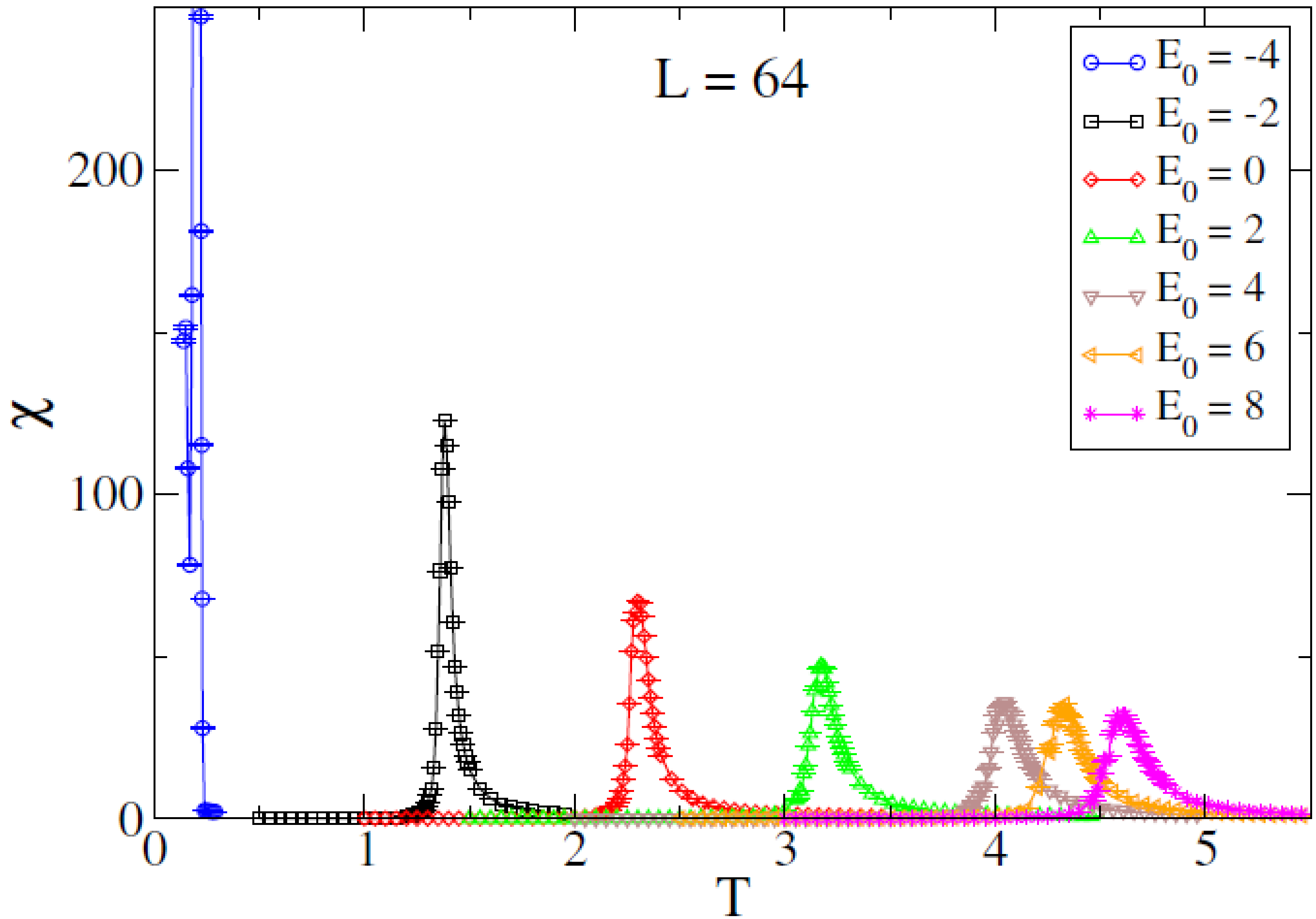
# Monte Carlo simulations with modified update rule

$$m(T,L) = \frac{1}{N}\sum_i \sigma_i,$$

$$U_4(T,L) = 1 - \frac{\langle m^4\rangle}{3\langle m^2\rangle^2},$$

$$c(T,L) = \frac{1}{Nk_BT^2}\left(\langle E^2\rangle - \langle E\rangle^2\right)$$

$$\chi(T,L) = \frac{1}{Nk_BT}\left(\langle M^2\rangle - \langle M\rangle^2\right)$$




Transition temperature increases from 0 as Eo is increased from -8 to 8

# Finite-size scaling

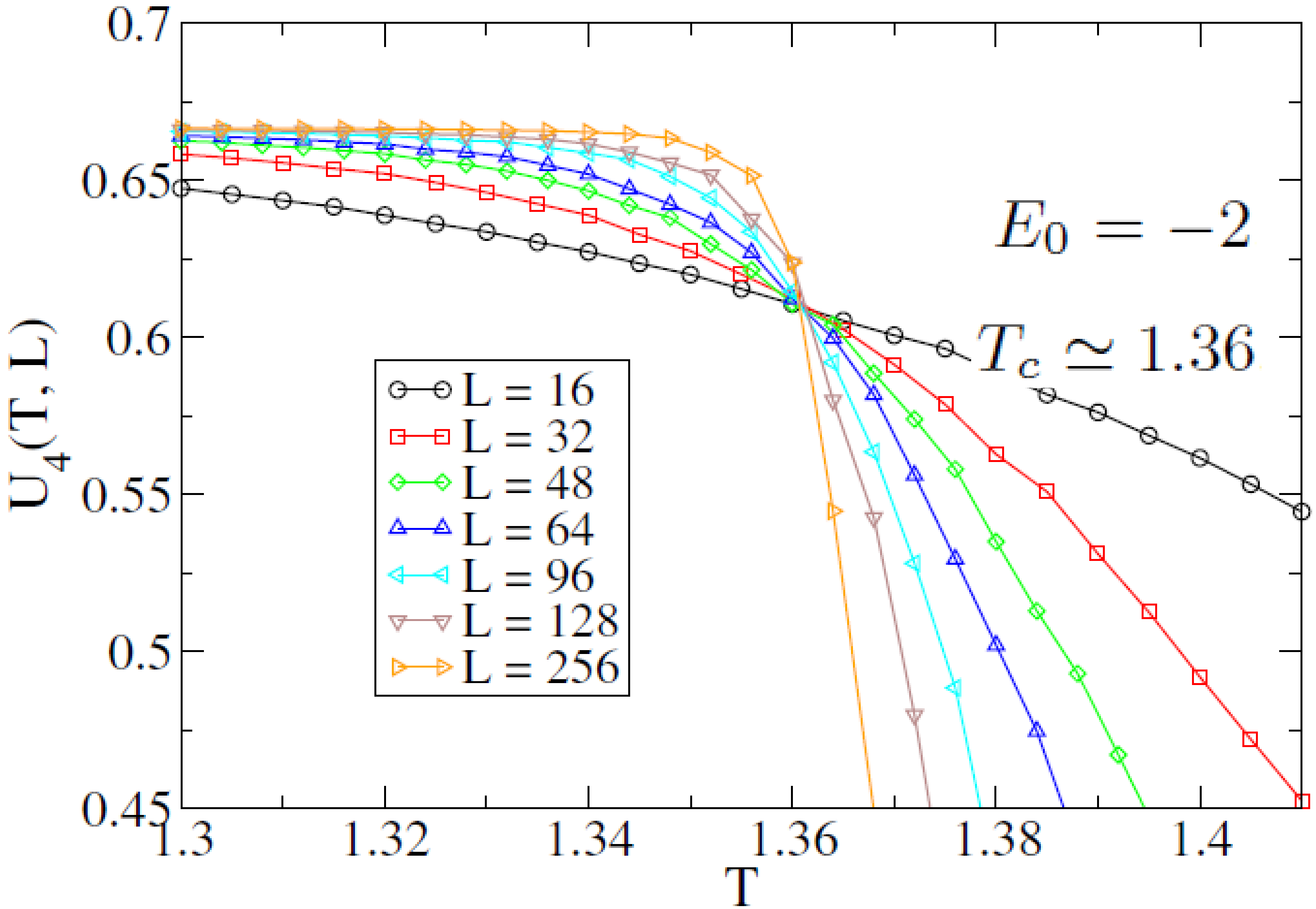
$$m(T,L) = L^{-\beta/\nu}\mathcal{M}\left((T-T_c)L^{1/\nu}\right),$$

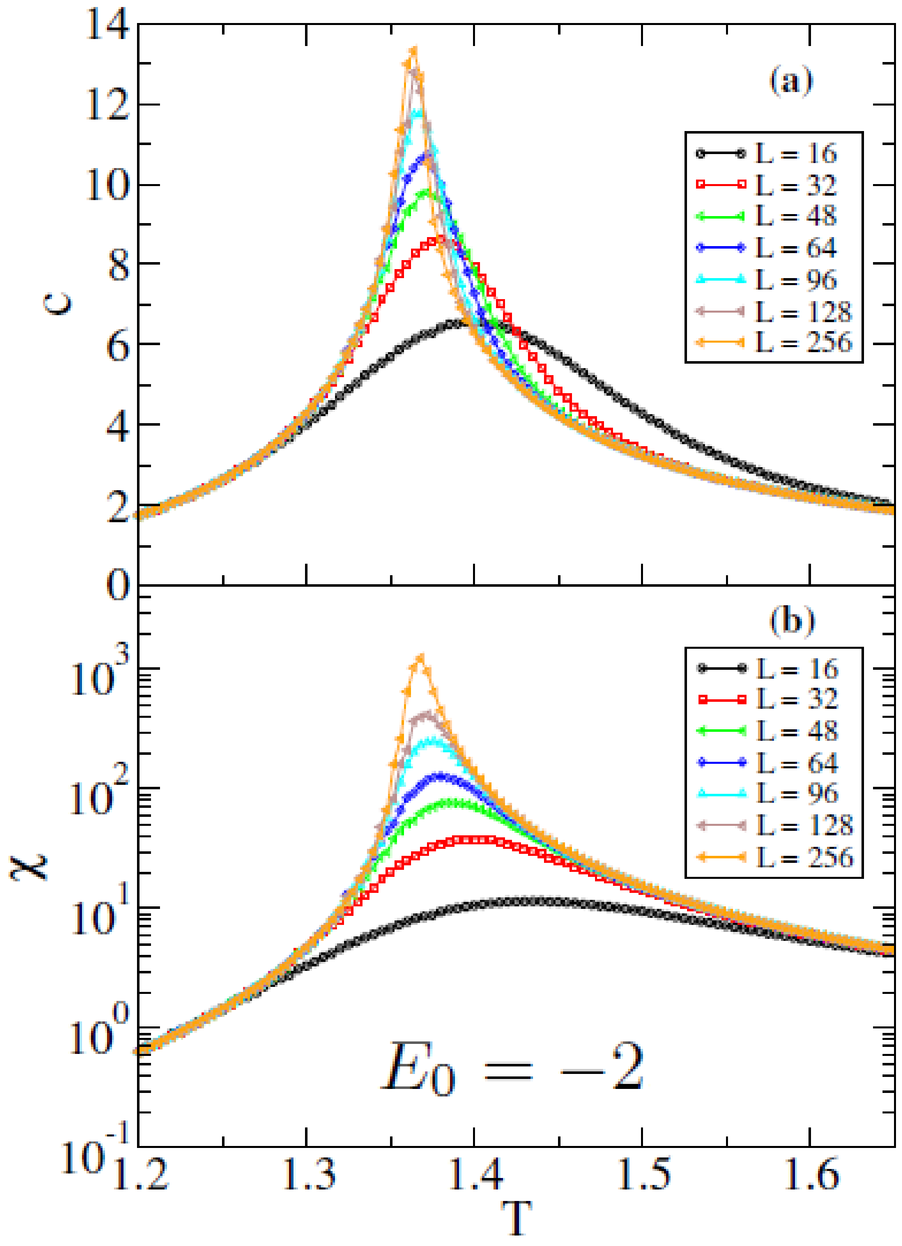
$$U_4(T,L) = \mathcal{U}\left((T-T_c)L^{1/\nu}\right),$$

$$c(T,L) = L^{\alpha/\nu}\mathcal{C}\left((T-T_c)L^{1/\nu}\right),$$

$$\chi(T,L) = L^{\gamma/\nu}\mathcal{X}\left((T-T_c)L^{1/\nu}\right),$$

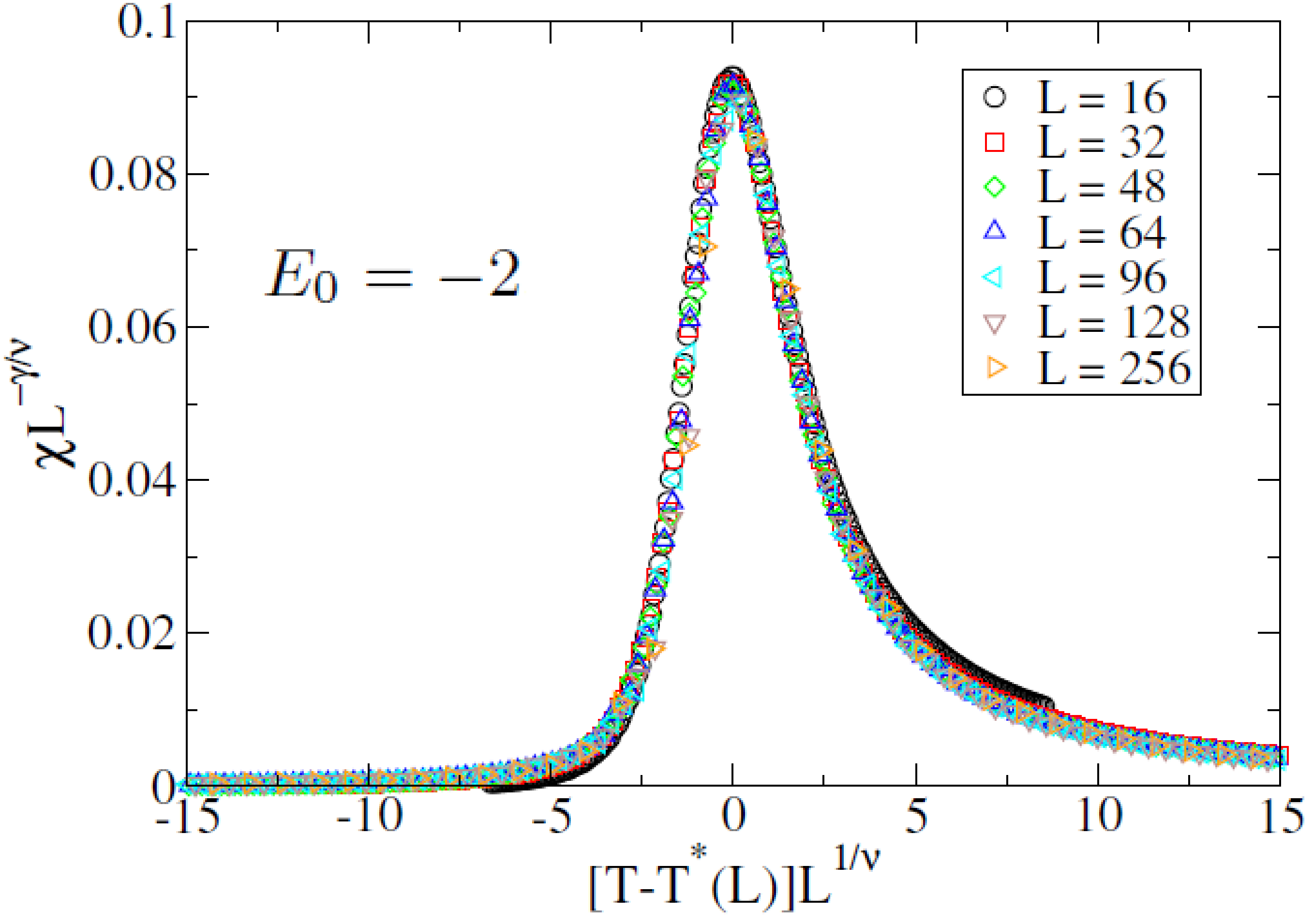
# Binder cumulant

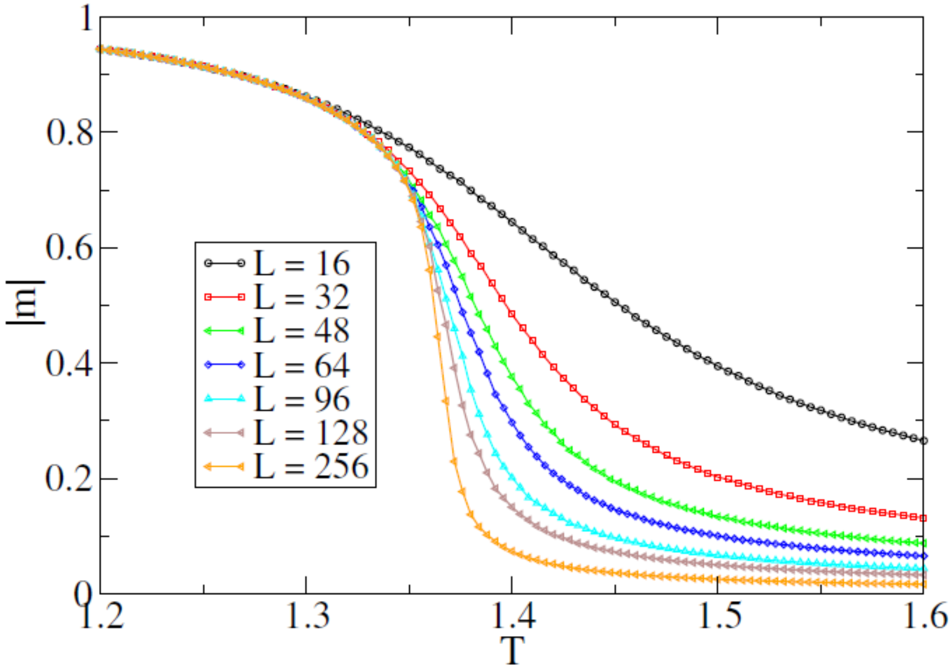





# Specific Heat and Susceptibility

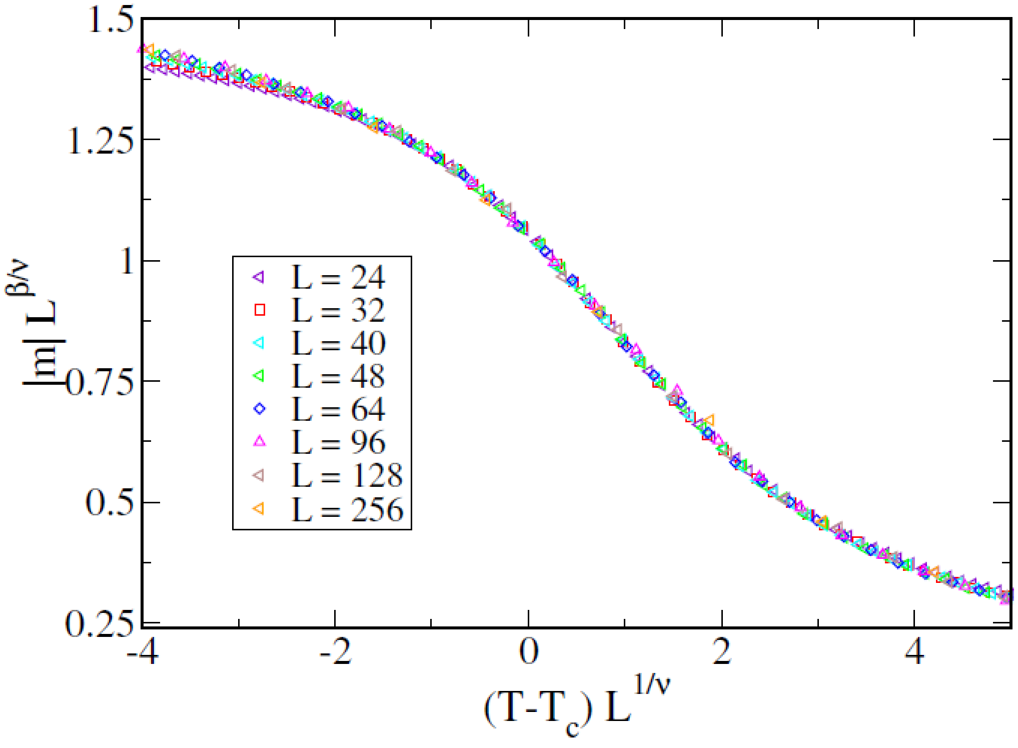
# Scaling collapse for the susceptibility






# Finite-size scaling collapse for the magnetization

$$E_0 = -2$$



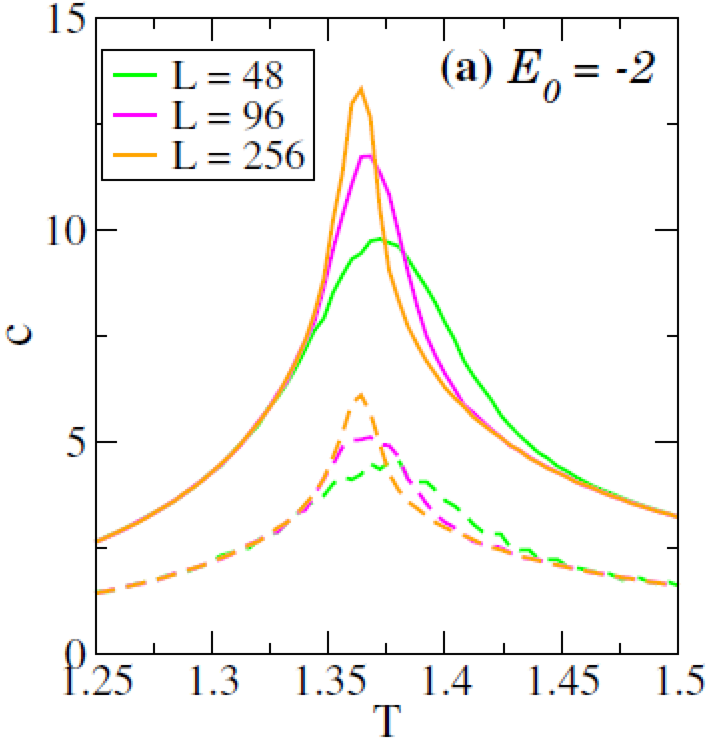
# Summary of results

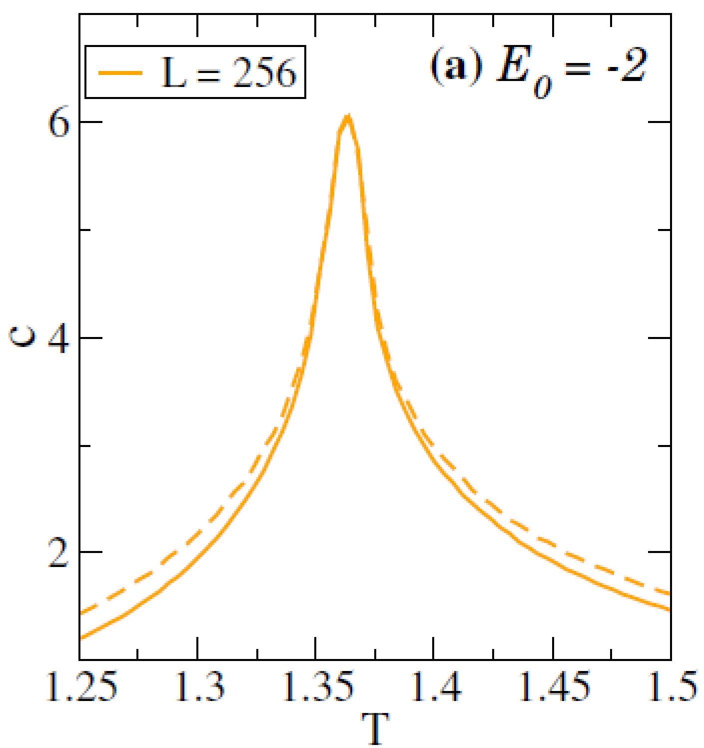
| | $T_c$ | $\alpha$ | $\beta$ | $\gamma$ | $\nu$ |
|---|---|---|---|---|---|
| $E_0 = 0$ (known results) | 2.269 | 0 | 0.125 | 1.75 | 1 |
| $E_0 = 2$ (This paper) | 3.1274(1) | 0 | 0.123(3) | 1.70(5) | 0.98(3) |
| $E_0 = -2$ (This paper) | 1.3608(5) | 0 | 0.124(5) | 1.71(4) | 0.99(2) |

The phase transition in the non-equilibrium model is in the same universality class as that of the equilibrium Ising model.

# Further test of equilibration (Boltzmann statistics)

For a system at equilibrium, $$\frac{dE}{dT} = \frac{1}{k_B T^2}\left(\langle E^2\rangle - \langle E\rangle^2\right)$$



# Conclusion:

The steady state of the model considered here is not described by Boltzmann statistics.

However, the phase transition in this model is in the same universality class as that of the equilibrium Ising model.

Lack of detailed balance is irrelevant in the RG sense??