Singularity in the Dynamics of a Quantum Spin System

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Objective

★ To study non equilibrium dynamics of many body system : Dynamics of spin system after a quench has revealed many interesting features.

★ We consider an integrable spin system, namely transverse Ising chain and study the return probability starting from a generic initial state

★ The return probability shows non-analytic behaviour indicating "Dynamical Quantum Phase Transition"

Heyl, Rep. Prog. Phys. 81, 054001 (2018), arXiv:1709.07461
Heyl, arXiv:1811.02575
Zvyagin, arXiv:1701.08851

Scheme

Construct a generic initial state on a spin chain involving even and odd parity states

Study its time evolution under Transverse Ising Hamiltonian

$$\mathcal{H} = -\sum_{j=1}^{N} s_j^x s_{j+1}^x - \Gamma \sum_{j=1}^{N} s_j^z$$

Loschmidt Echo / Amplitude : $\mathcal{L} \equiv |\langle \Psi(0) | \Psi(t) \rangle|^2$
Measure of return probability, decoherence due to dynamics

Bhattacharyya, Dasgupta, EPJB (2017), in preparation (2019)

Eigenstates of Transverse Ising Model

$$\mathcal{H} = -\sum_{j=1}^{N} s_j^x s_{j+1}^x - \Gamma \sum_{j=1}^{N} s_j^z$$

s^x, s^z are Pauli spin matrices

Quantum Phase Transition at Γ=1
M_x M_z are non-analytic

Pfeuty (1970), Lieb, Schultz, Mattis (1961)

$\mathcal{H} = \sum_k \mathcal{H}_k$, $\mathcal{H}_k$-s commute $(0 \le k \le \pi)$. Eigenstates :

$$-\lambda_k(\Gamma), \quad |\mathrm{GS}_k\rangle = i\cos\theta_k |1_k 1_{-k}\rangle - \sin\theta_k |0_k 0_{-k}\rangle$$

$$+\lambda_k(\Gamma), \quad |\mathrm{ES}_k\rangle = i\sin\theta_k |1_k 1_{-k}\rangle + \cos\theta_k |0_k 0_{-k}\rangle$$

Even parity

$$0, \quad |0_k 1_{-k}\rangle$$

$$0, \quad |1_k 0_{-k}\rangle$$

Odd parity

$$\lambda_k = 2\sqrt{\Gamma^2 + 1 + 2\Gamma\cos k}, \quad \tan\theta_k = \frac{\sin k}{\Gamma + \cos k + \sqrt{\Gamma^2 + 1 + 2\Gamma\cos k}}$$

Hamiltonian evolution conserves parity.
The odd parity states |01>, |10> are often ignored.

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has been studied for quench of Hamiltonian of integrable systems
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Transverse Ising model : global quench of external field from Γ_1 to Γ_2

$$r(t) = (-1/N)\log \mathcal{L} = -\int_0^{\pi} dk \, \log\left[1 - \sin^2(\lambda_k t/2)\sin^2(\phi_k)\right]$$

where λ_k is energy for field Γ_2 and ϕ_k is some function of Γ_1 and Γ_2

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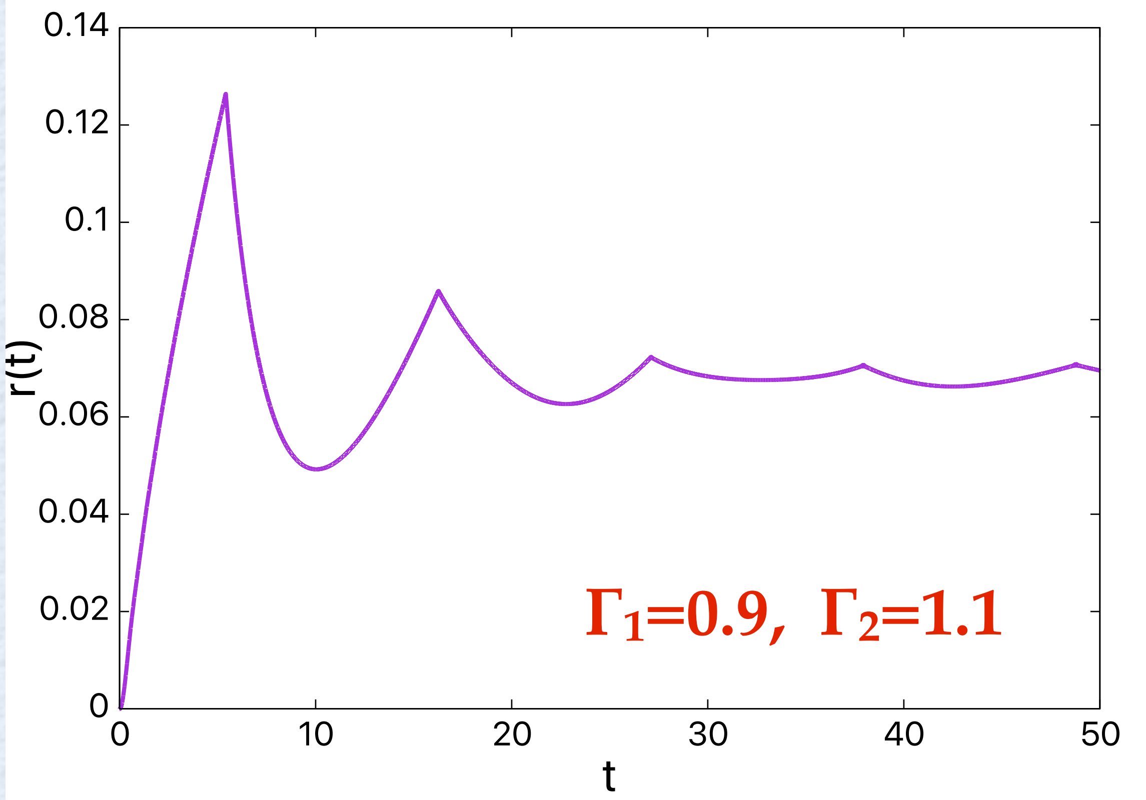

At some $k = k_c$, $\sin(\phi_k) = 1$.
At that k_c, for some $t = t_c$, $\sin(\lambda_k t/2) = 1$
$\Rightarrow$ Log term diverges, and singularity appears

Only when $\Gamma_1 < 1, \Gamma_2 > 1$ OR $\Gamma_1 > 1, \Gamma_2 < 1$

Singularity indicates DQPT
(Dynamical Quantum Phase Transition)
Some measurable quantities are non-analytic

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Normalisation needs $|\alpha_k|^2 + |\beta_k|^2 + 2|\gamma_k|^2 = 1$ for all k

Measure of even parity $\mathcal{E} = |\alpha_k|^2 + |\beta_k|^2$ and odd parity $\mathcal{O} = 2|\gamma_k|^2$

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Note : For ground state $\mathcal{O} = 0$, $\alpha_k = i\cos\theta_k$, $\beta_k = -\sin\theta_k$

We start with

$$|\Psi(t=0)\rangle = \otimes_k \left[\sqrt{\frac{\mathcal{E}}{2}}\,(|11\rangle_k + |00\rangle_k) + \sqrt{\frac{1-\mathcal{E}}{2}}\,(|01\rangle_k + |10\rangle_k)\right]$$

Calculate $|\Psi(t)\rangle = \exp\left(-i\mathcal{H}t\right)|\Psi(0)\rangle$, Return Prob : $\mathcal{L} \equiv |\langle\Psi(0)|\Psi(t)\rangle|^2$

Rate constant $r(t) = -(1/N)\log\mathcal{L} = -\int_0^\pi dk\, \log[1-\mathcal{E}+\mathcal{E}\cos\left(\lambda_k t\right)]$

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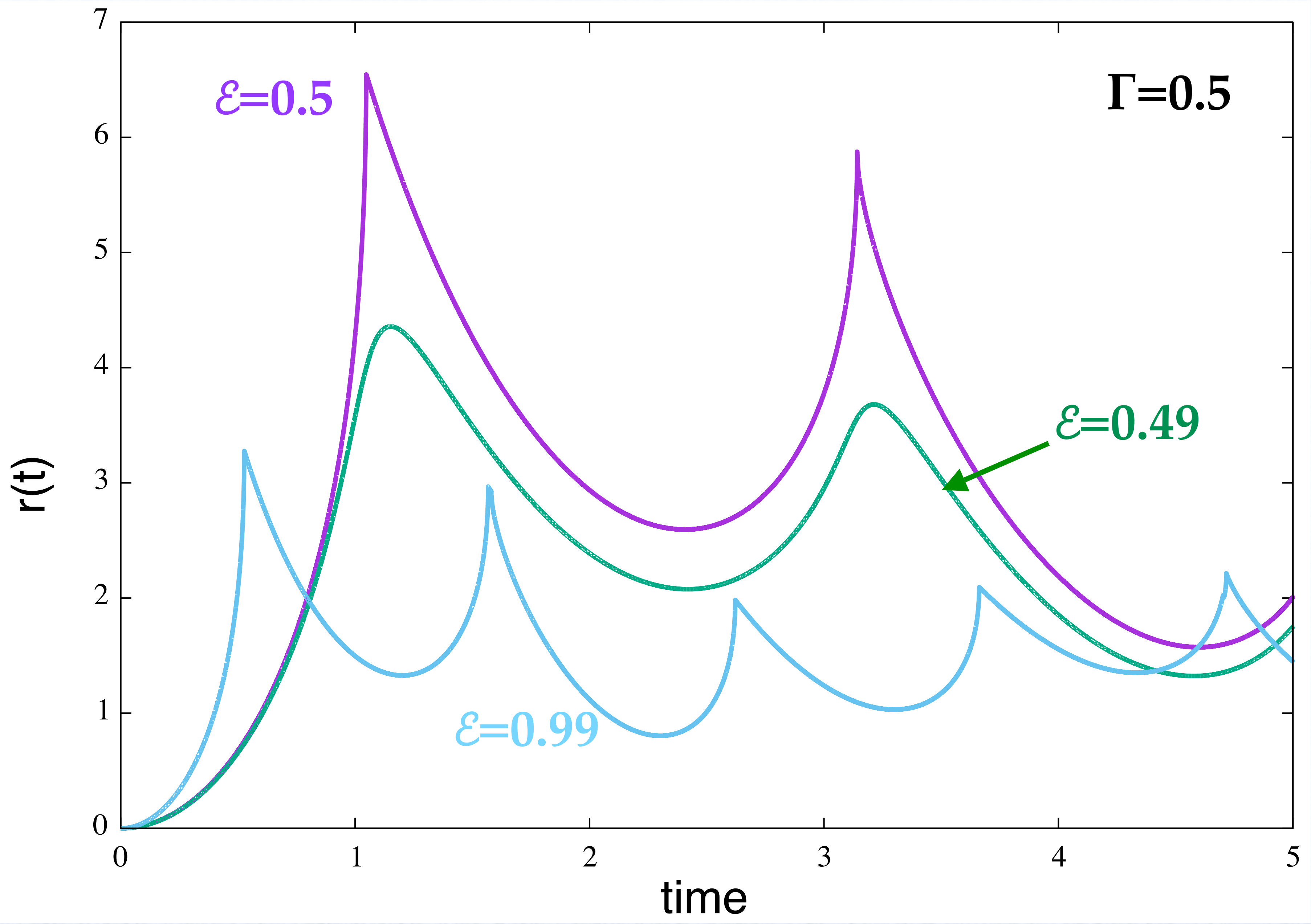
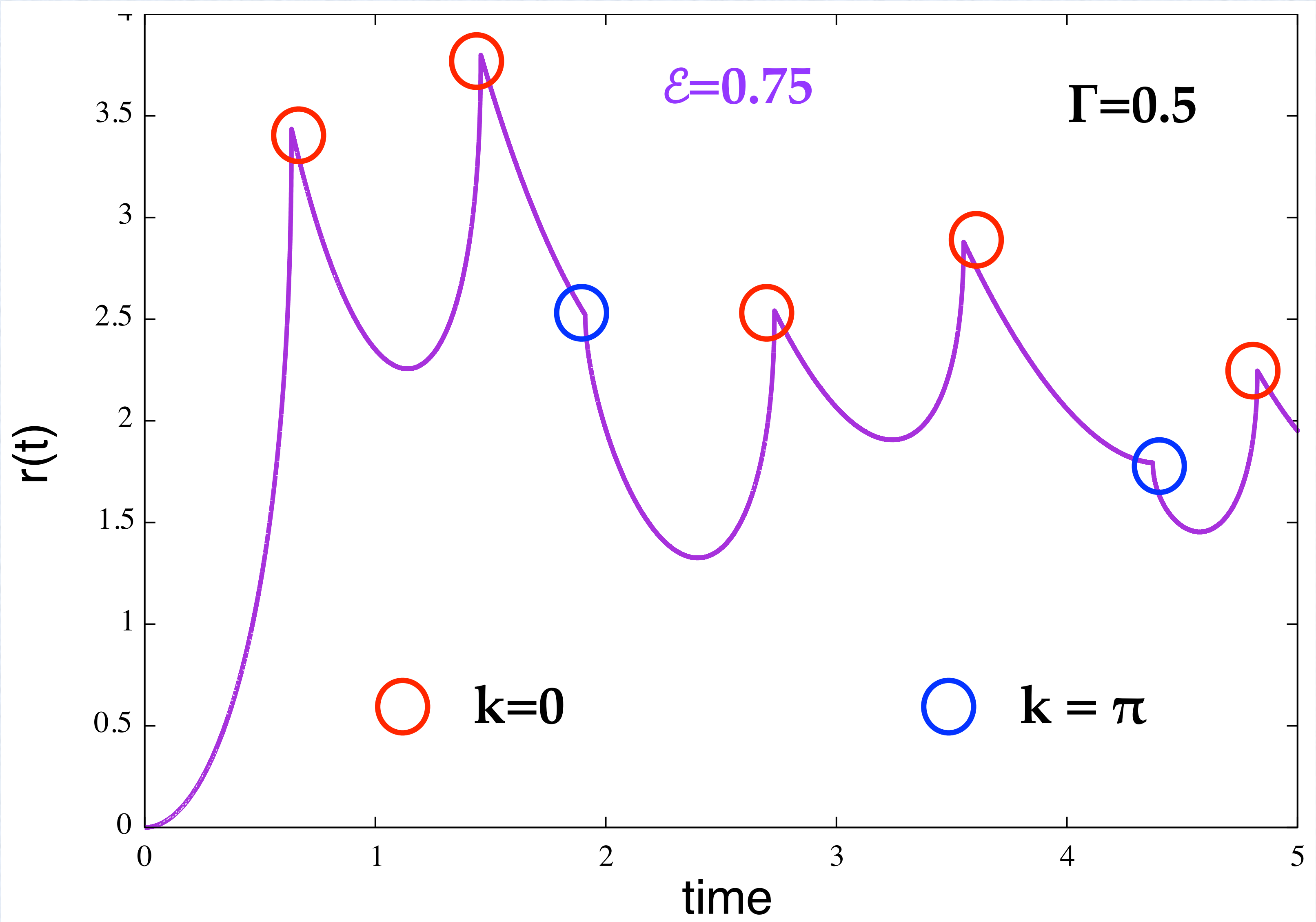


$r(t)$ shows kinks

for $0.5 \leq \mathcal{E} < 1.0$

NOT for $\mathcal{E} < \mathcal{O}$

for ALL Γ



$$r(t) = -\int_0^{\pi} dk \, \log[1 - \mathcal{E} + \mathcal{E}\cos(\lambda_k t)]$$

At the kinks $1 - \mathcal{E} + \mathcal{E}\cos(\lambda_k t) = 0$ with $k = 0$ or $k = \pi$
The curve is vertical at the left or right of the kink respectively

Appearance of Dynamical Quantum Phase Transition (DQPT)

$$\text{Hamiltonian} \quad \mathcal{H}(\Gamma) = -\sum_{j=1}^{N} s_j^x s_{j+1}^x - \Gamma \sum_{j=1}^{N} s_j^z$$

Quench Protocol :

Start from the ground state of $\mathcal{H}(\Gamma_1)$ and evolve under $\mathcal{H}(\Gamma_2)$
Kinks (DQPT) appear when quenched through the critical point, that is, Γ_1 and Γ_2 are on two sides of the critical point (ferro-para or para-ferro quench). Only even parity states considered

This work :

Start from a generic state
Kinks (DQPT) appear when weightage of even parity is larger than the weightage of odd parity (driven by imbalance of parity)
For all values of Γ.

Earlier work for equal wightage of even and odd parity

$$|\Psi(t=0)\rangle = \otimes\left[\alpha_k|11\rangle_k + \beta_k|00\rangle_k + \gamma_k|10\rangle_k + \gamma_k|01\rangle_k\right]$$

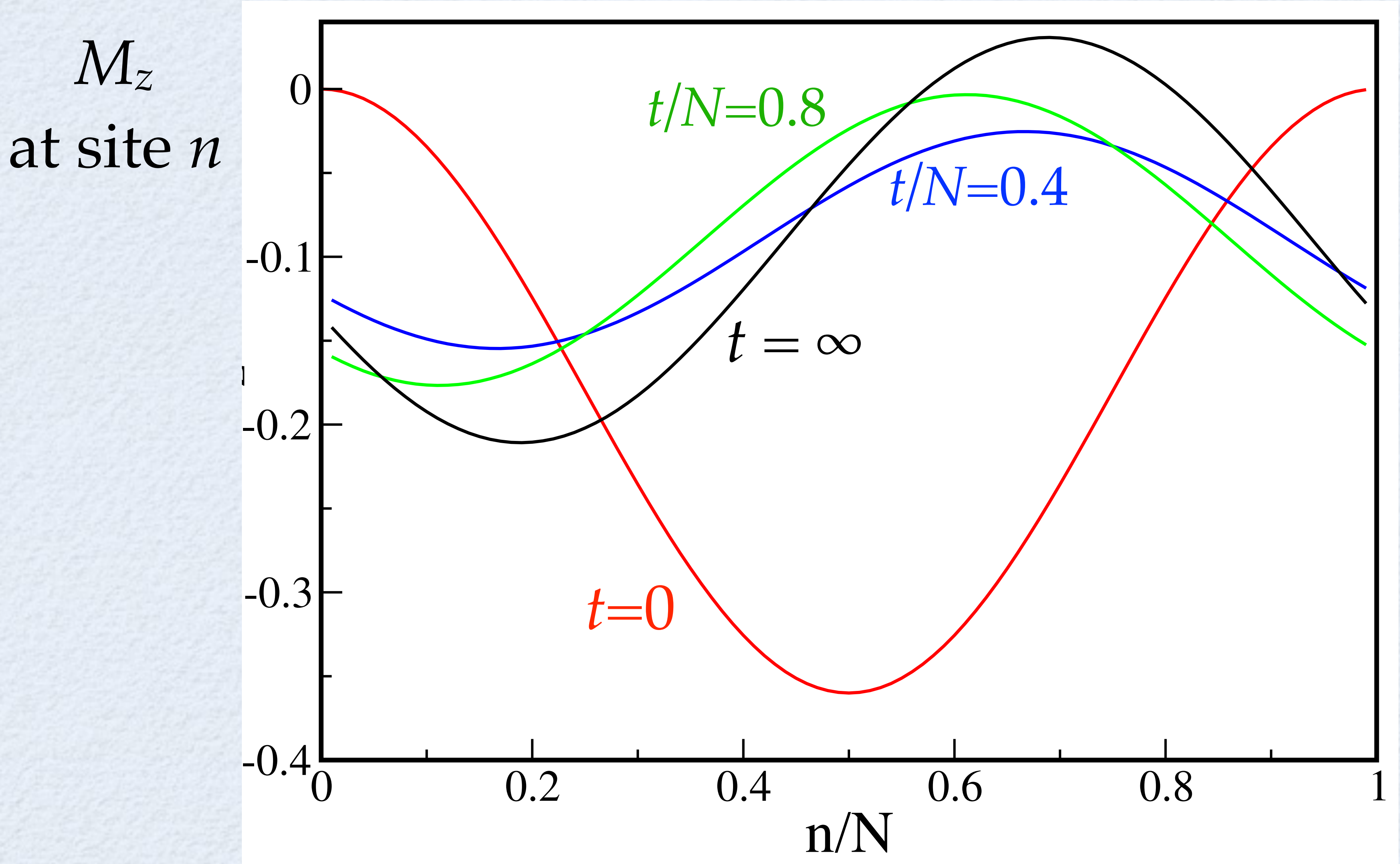
Weightage of even parity and odd parity are equal

$$\mathcal{E} = \mathcal{O}\,:\; |\alpha_k|^2 + |\beta_k|^2 = 2|\gamma_k|^2$$

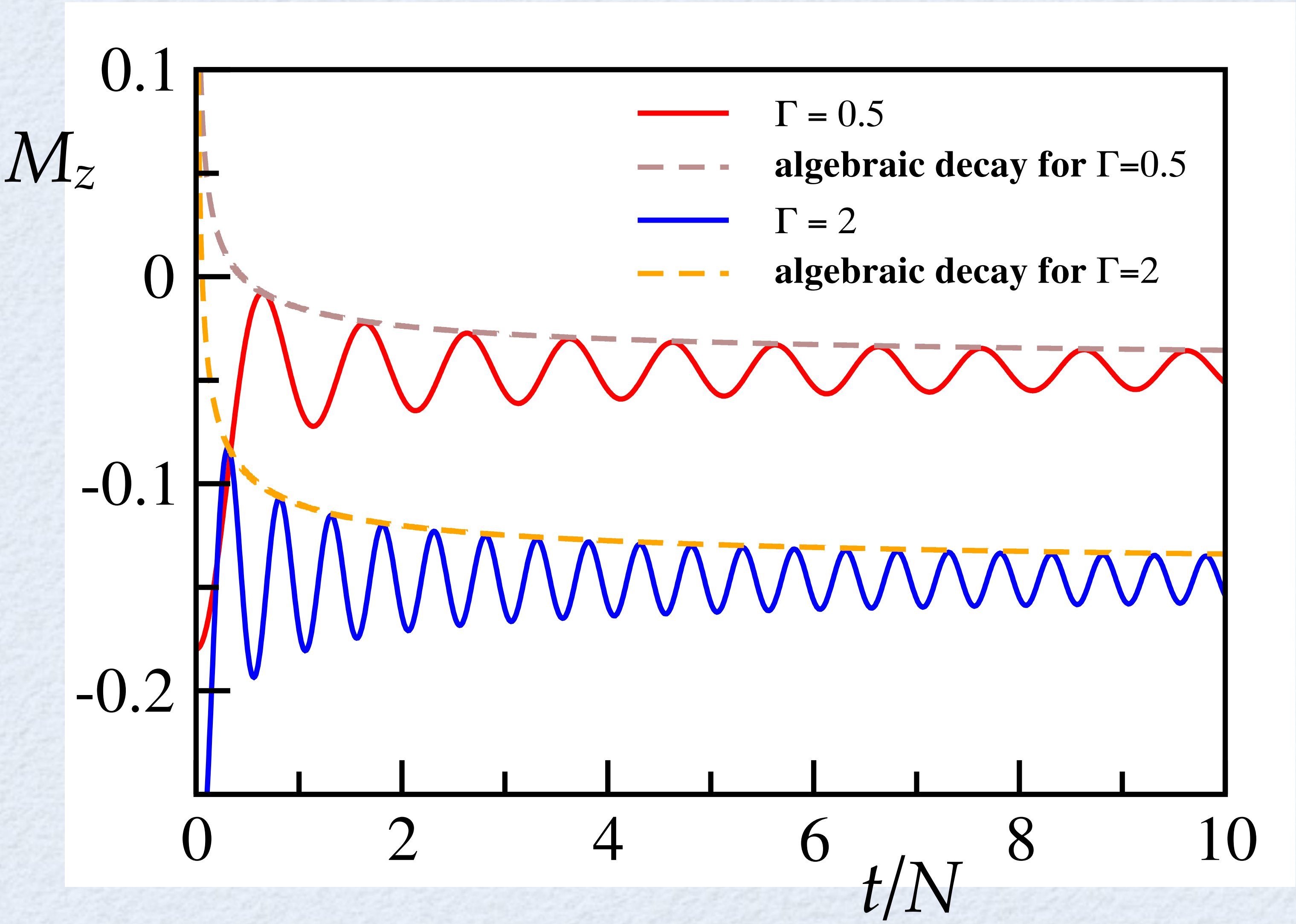
- Can be handled analytically, for calculation of transverse magnetisation
- The initial state has spatially inhomogeneous transverse magnetisation. (This happens for *nonzero* γ_k only.)

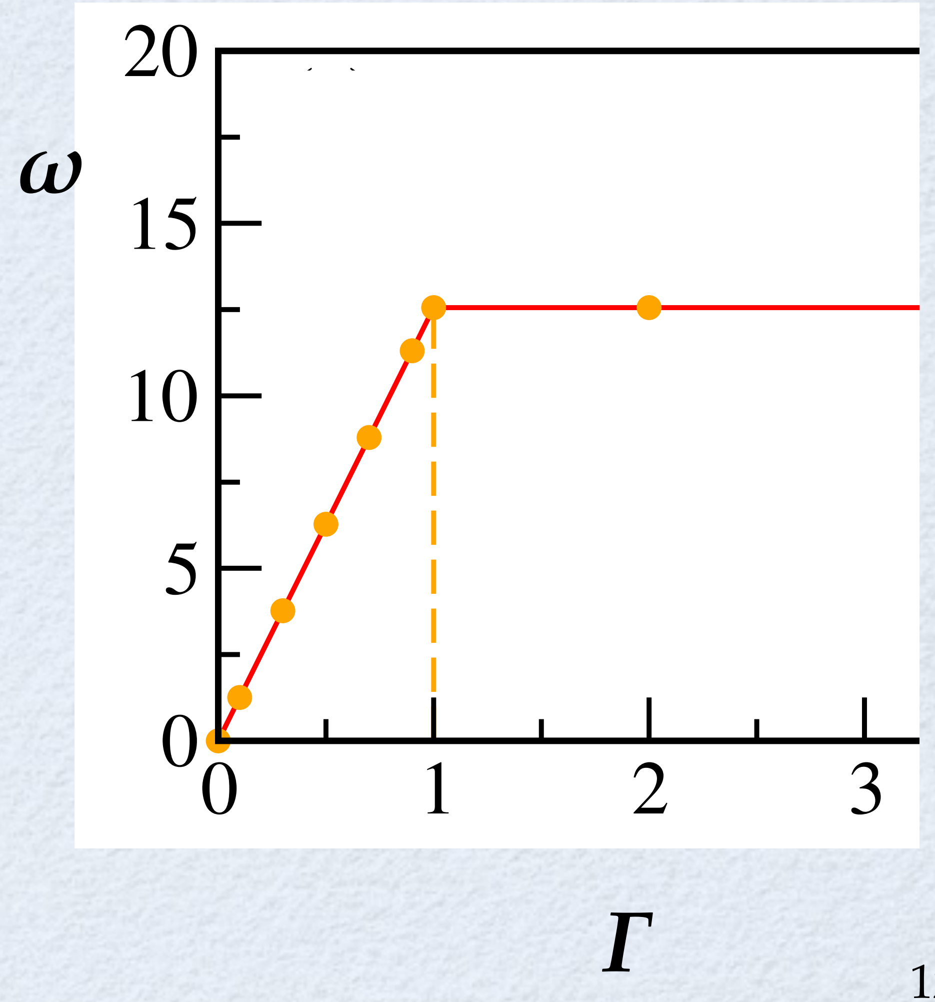
Main Results : (for $\mathcal{E}=0$, $|\alpha_k|^2 + |\beta_k|^2 = 2|\gamma_k|^2$)

- Whatever be the strength of the field, the magnetisation of the system does not become homogeneous even after infinite time

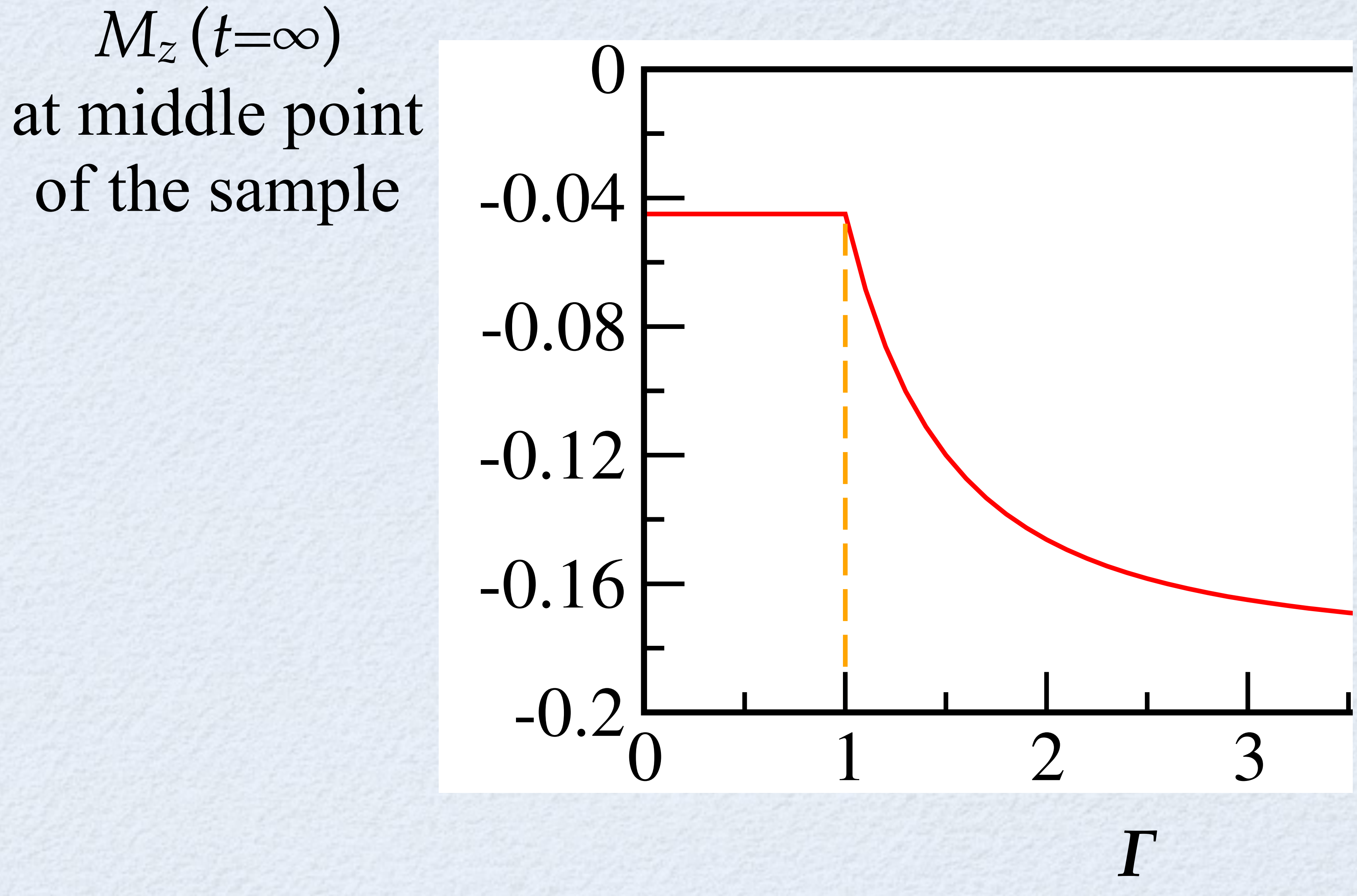


- At each site, the transverse magnetisation decays in the form of damped oscillation.
- The frequency of oscillation varies linearly with field for ferromagnetic phase and remains constant in paramagnetic state. The envelope of oscillation decays algebraically with exponent 0.5





- The local magnetisation after infinite time varies differently with field in ordered and disordered phases.



Summary and Open Questions

- We investisgate the singularities in the Loschmidt echo starting from a generic state. The singularities are found to be driven by the difference of weightage of even parity and odd parity states (for $\mathcal{E} \neq \mathcal{O}$)

- How does the non-local observables (say, the longitudinal magnetisation) behave? Do they bear a signature of dynamical transition?

- How can one realise experimentally a generic state, like the one considered here?

Thank you for your attention