Shock Propagation Following an Intense Explosion

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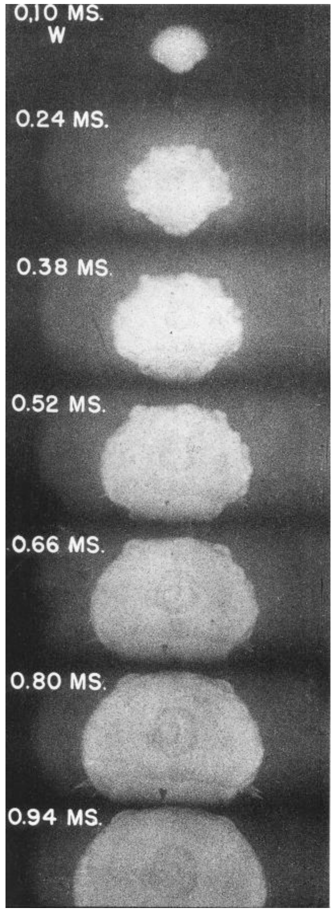
A Blast Wave



- **How does the radius increase with time?**
- **How do pressure, density, and temperature vary with distance?**

Growth of Radius

$$R(t) = f(E_0, t, \rho_a, \cancel{T_a})$$

$$[E_0] = ML^2T^{-2}$$

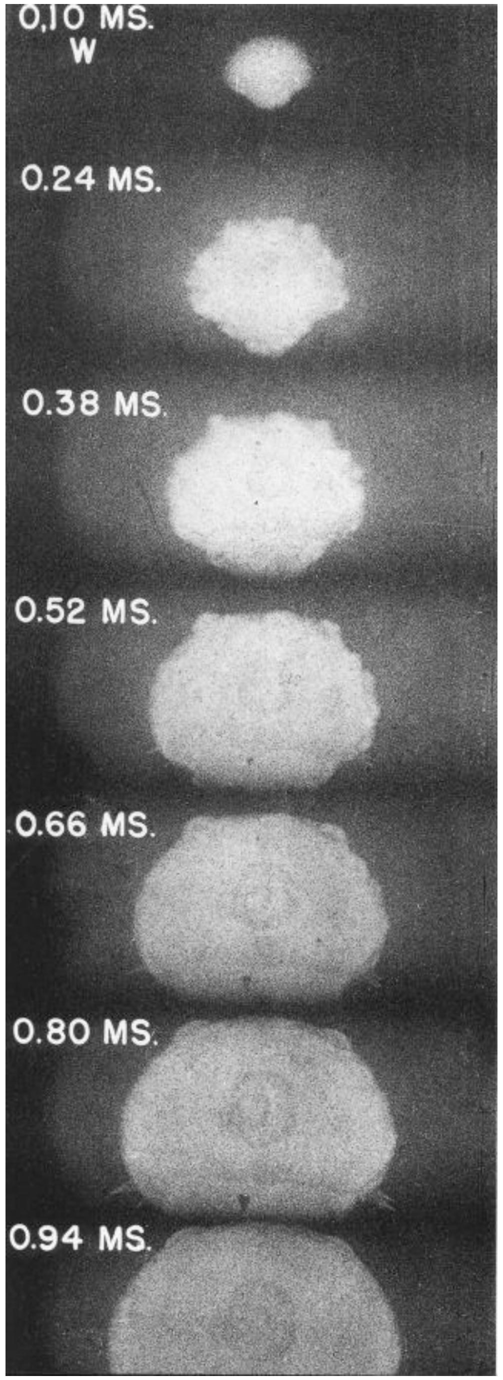
$$[\rho_0] = ML^{-d}$$

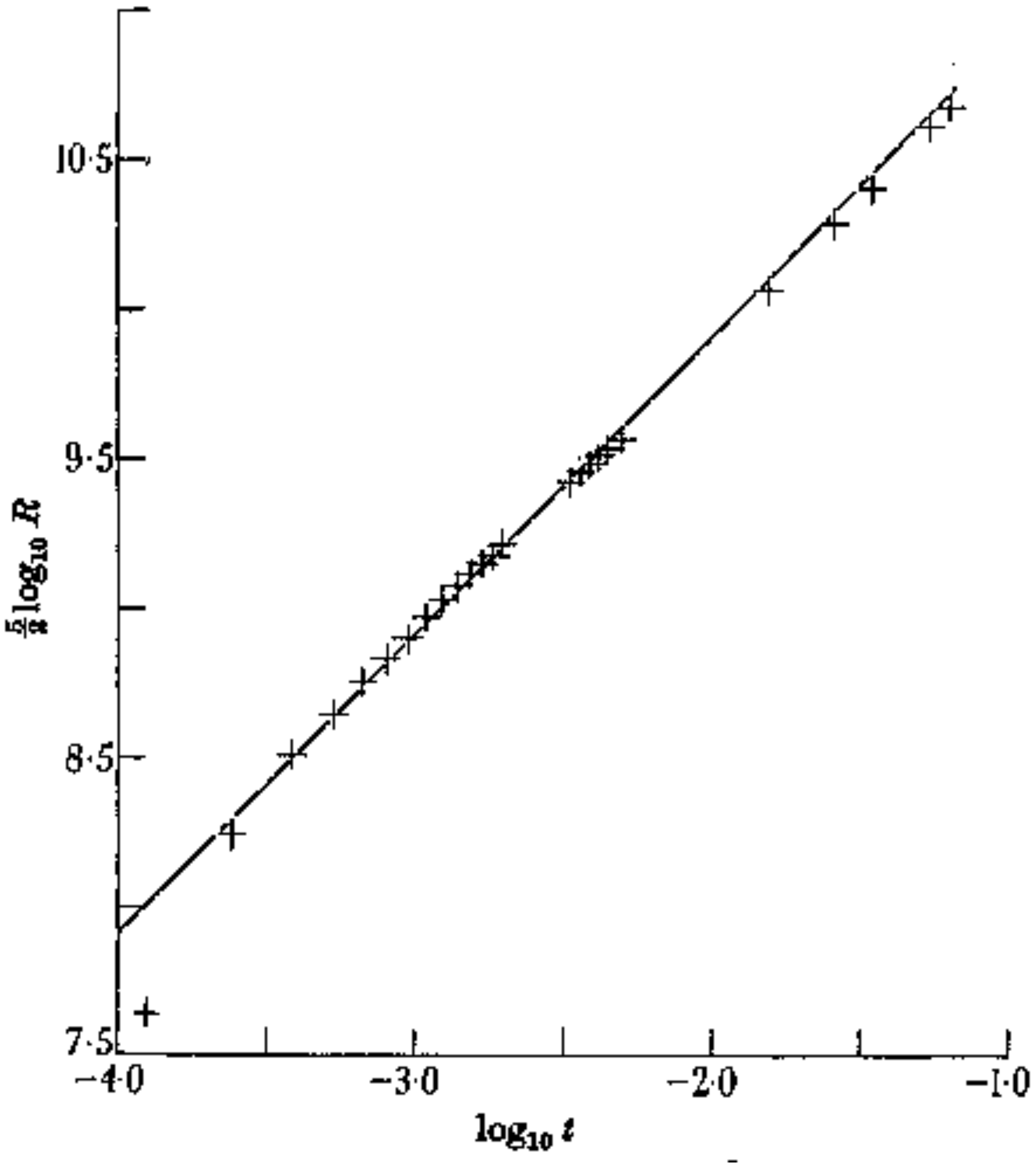
$$[t] = T$$

$$R(t) = c\left(\frac{E_0 t^2}{\rho_0}\right)^{\frac{1}{d+2}}$$

$$d = 3 \implies R(t) \propto t^{2/5}$$

Nuclear explosion





Taylor, Proc. Roy. Soc. A (1950)

Spatial variation

- **Mass**

$$\partial_t \rho + \partial_r(\rho v) + 2r^{-1}\rho v = 0$$

- **Momentum**

$$\partial_t v + v\partial_r v + \rho^{-1}\partial_r p = 0$$

- **Energy**

$$\partial_t(p\rho^{-\gamma}) + v\partial_r(p\rho^{-\gamma}) = 0$$

Assumptions

- **Local Equilibrium**
  - ⋆ **An equation of state**
  - ⋆ **Thermal energy in terms of local pressure and density**
  - ⋆ **Ideal gas law**
- **Heat flux term dropped in energy conservation**

Boundary conditions

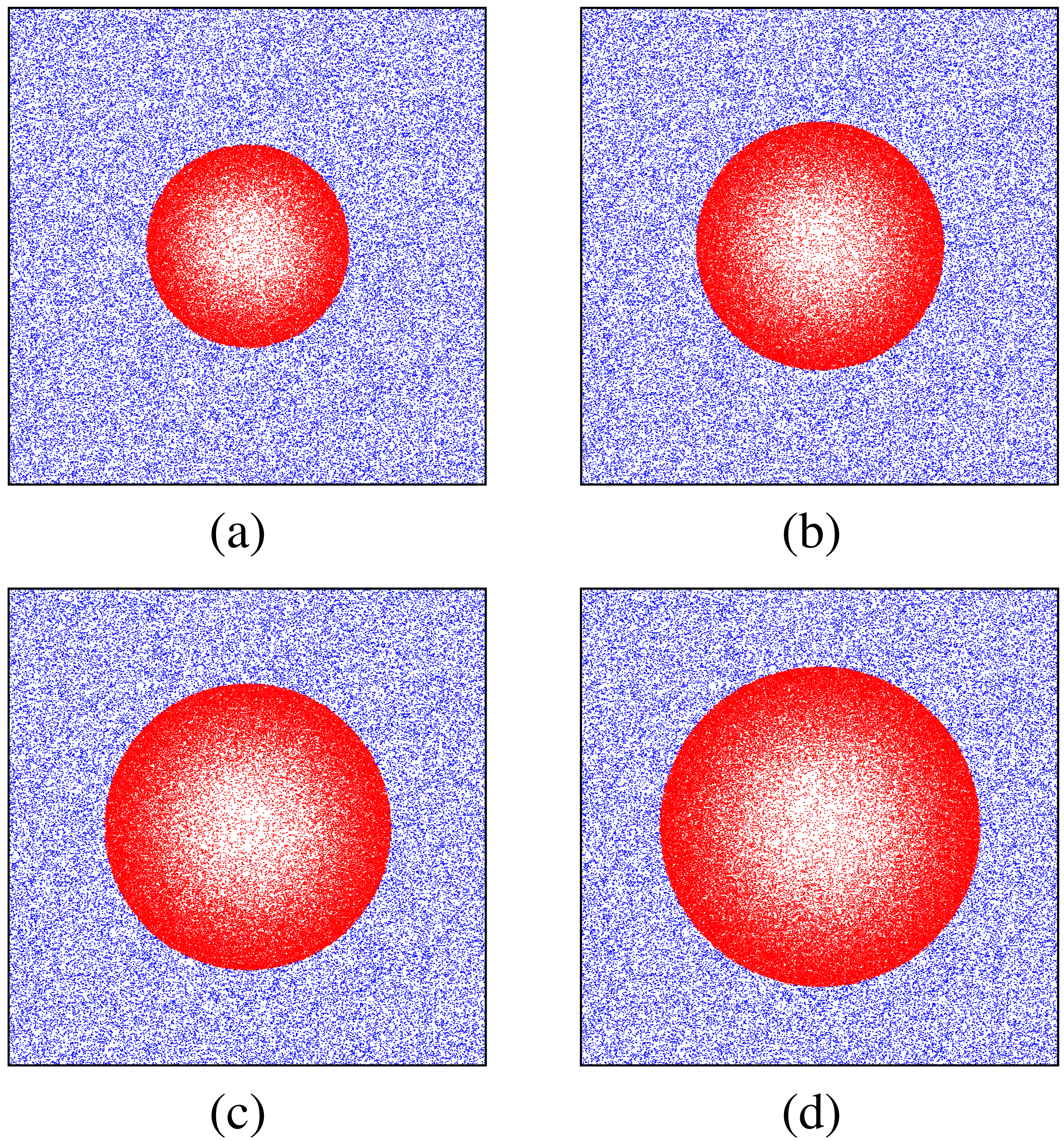
- **Discontinuities at shock front**
- **Rankine Hugoniot conditions**

- **In terms of scaled variables, PDEs reduce to ODEs**
- **Solved by Taylor, Sedov, Neumann**
- **A classic problem in gas dynamics**

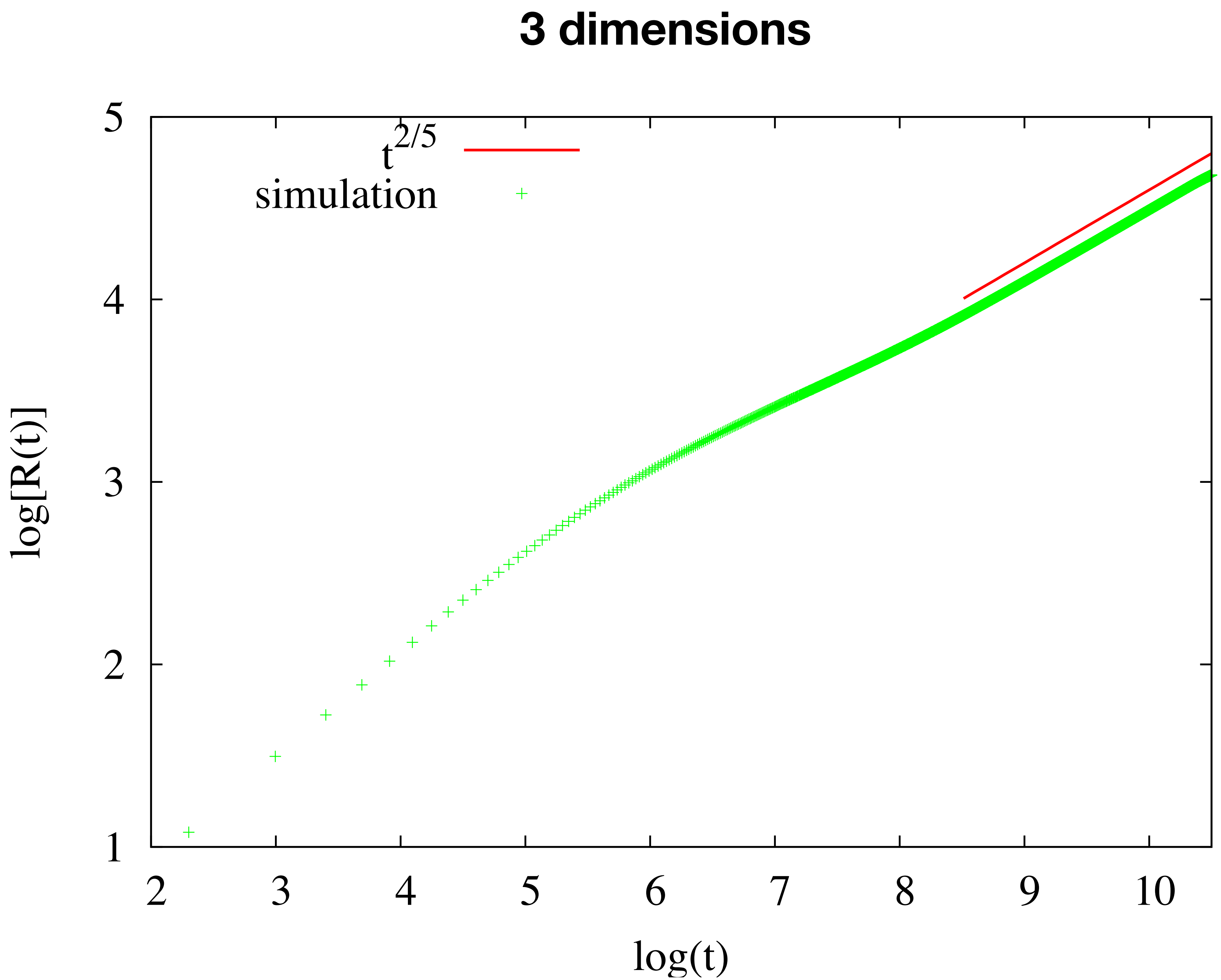
In this talk

- **Most studies focus on modifications of the PDEs to include different effects like radiation, conduction, instabilities, etc.**
- **Surprisingly, there are no detailed studies of microscopic models.**
- **How do the hydrodynamic results compare with large scale simulations of a particle based microscopic model?**
- **Are the assumptions valid?**

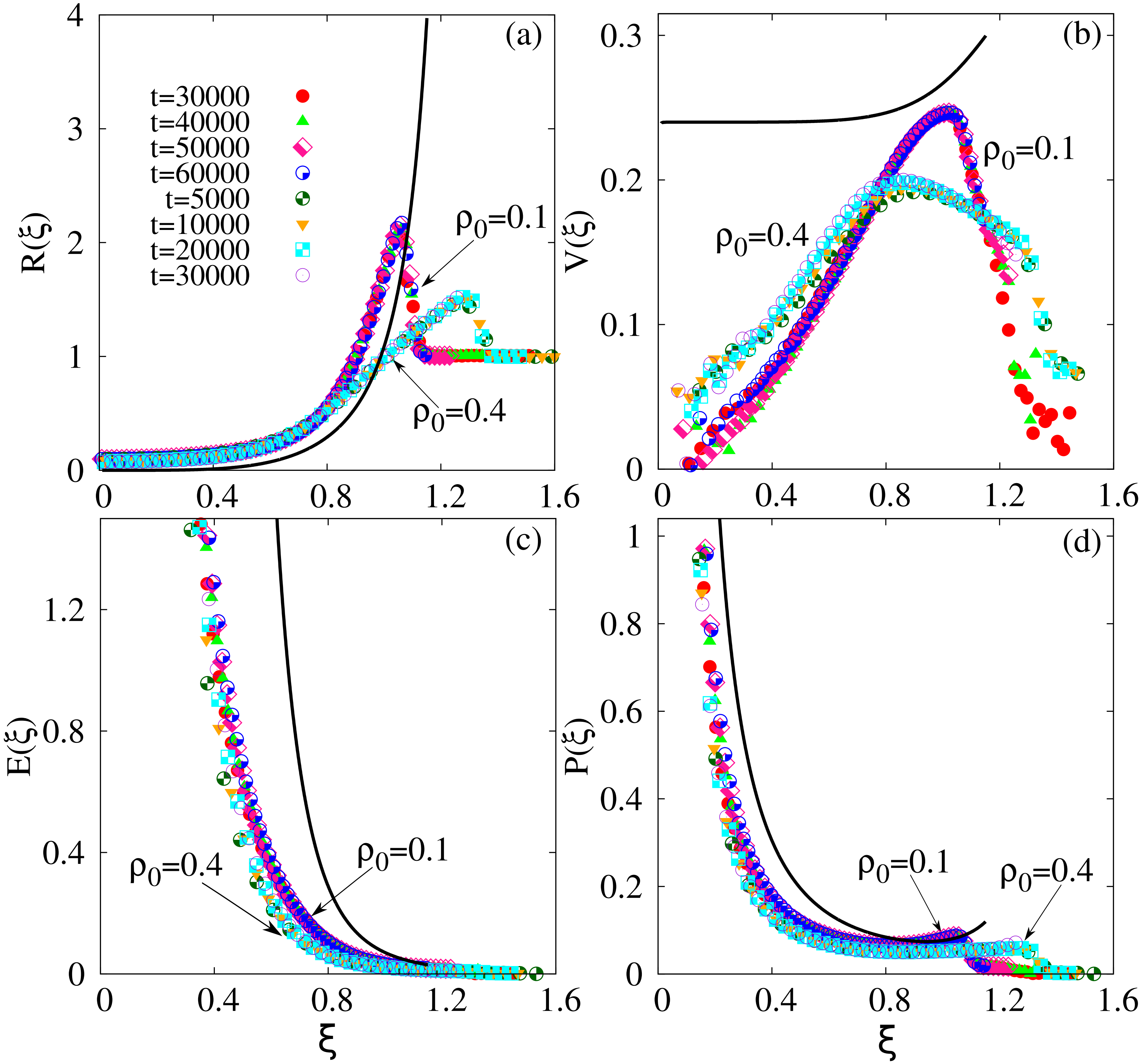
A microscopic model



Benchmarking



Comparison with TvNS



$$\xi = r\left(\frac{E_0 t^2}{\rho_0}\right)^{-1/5}$$

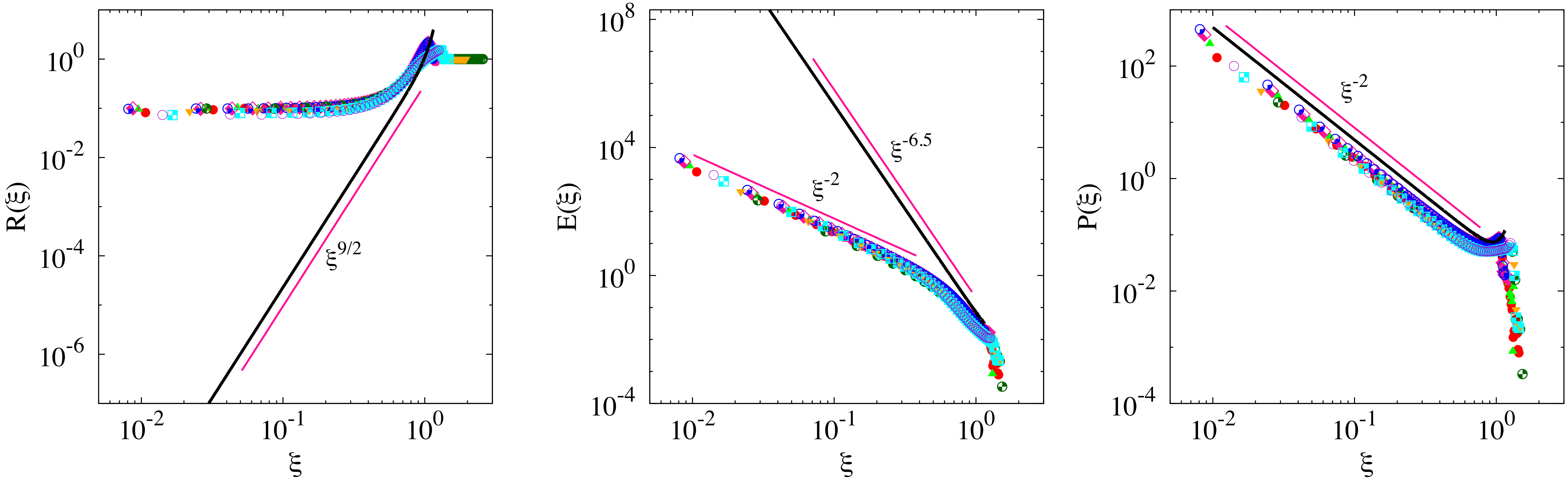
$$p = \frac{\rho_0 r^2}{t^2} P(\xi)$$

$$\rho = \rho_0 R(\xi)$$

$$v = \frac{r}{t} V(\xi)$$

$$\varepsilon = \frac{k_B}{m_0} T(\xi)$$

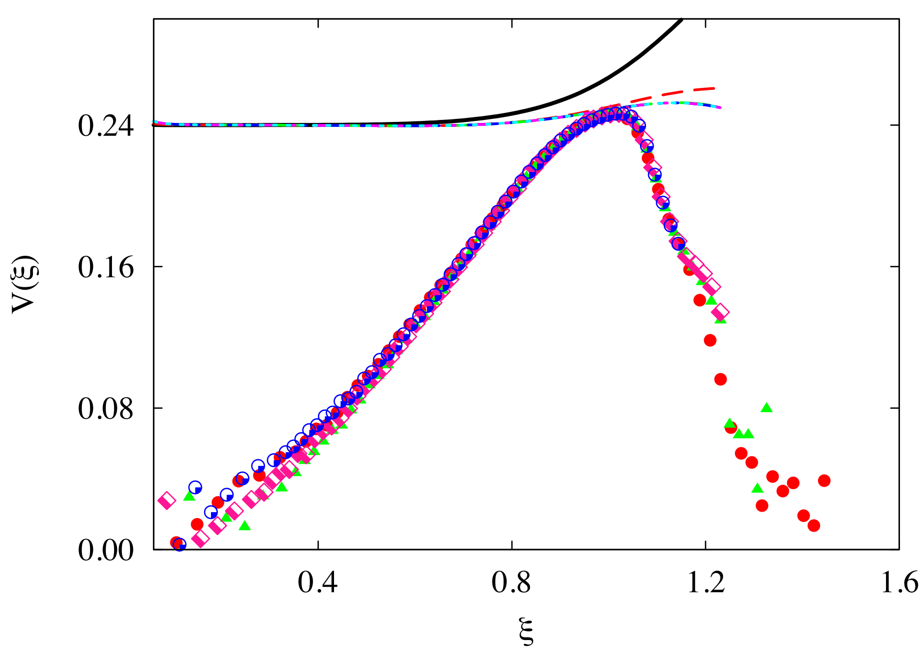
Comparison with TvNS

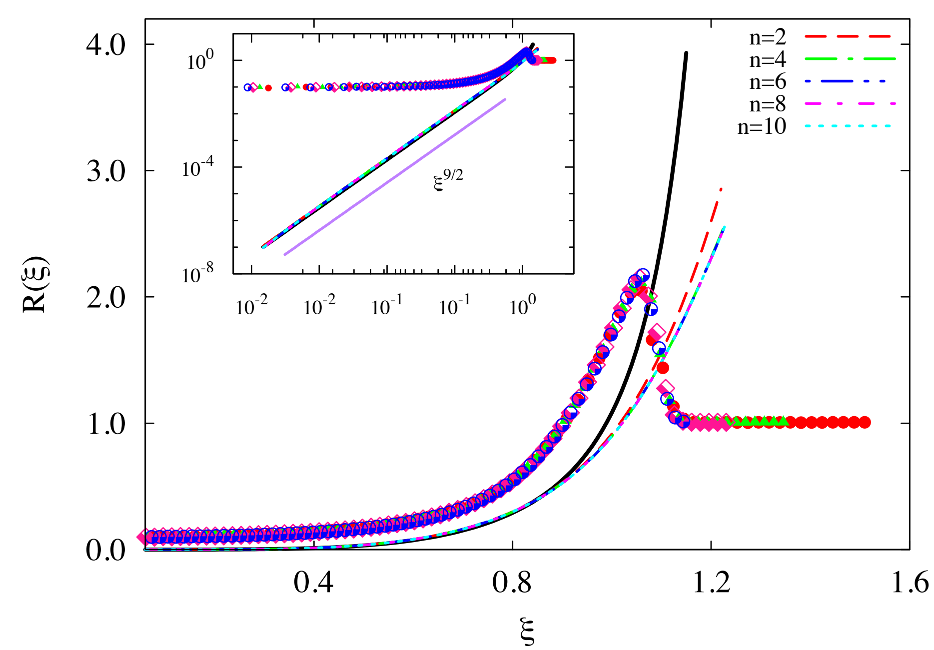


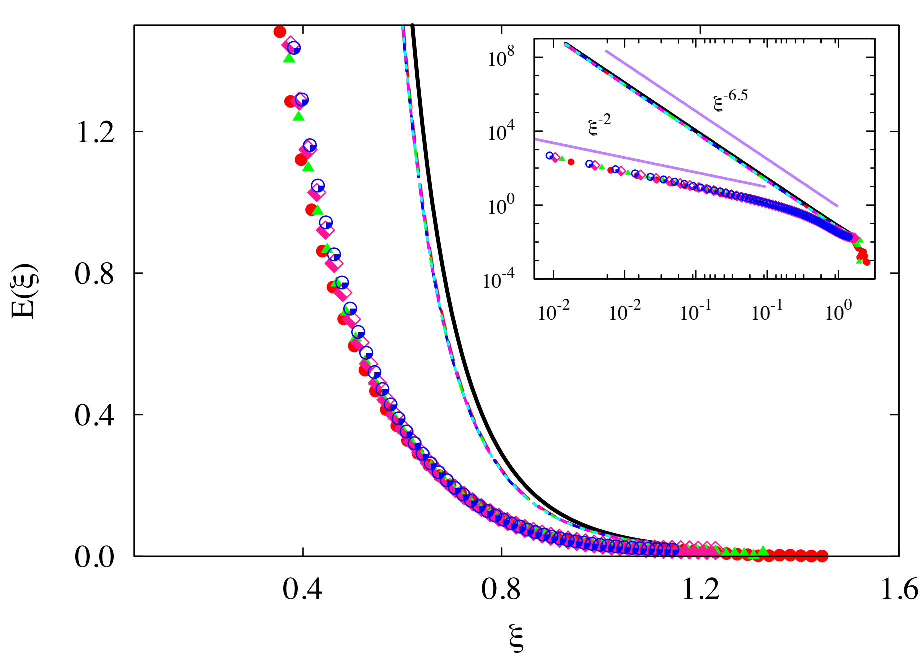
Where does it go wrong?

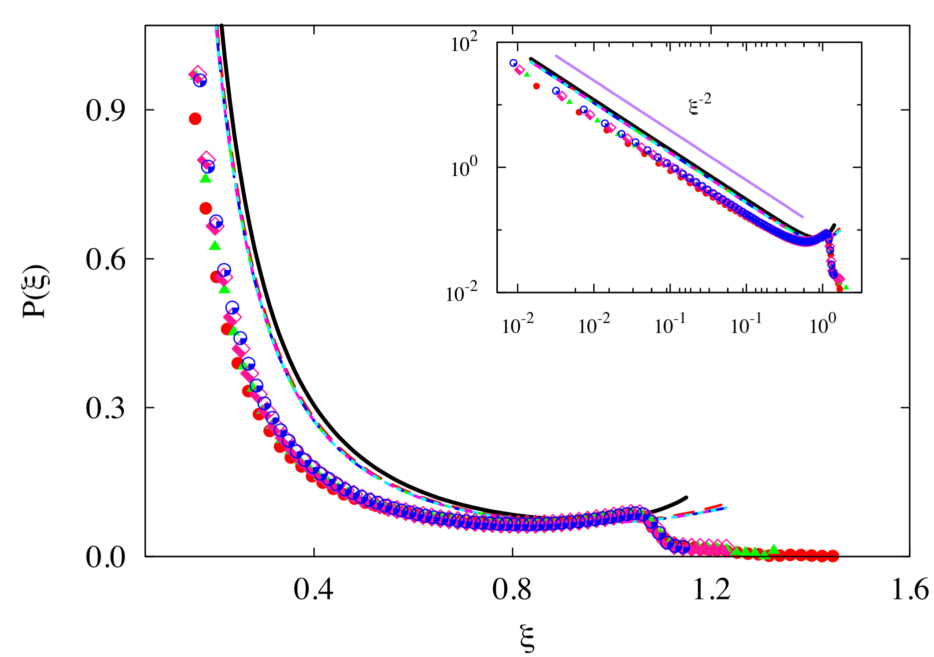
Excluded volume effects

- **Replace ideal gas law by viral expansion (10 terms)**



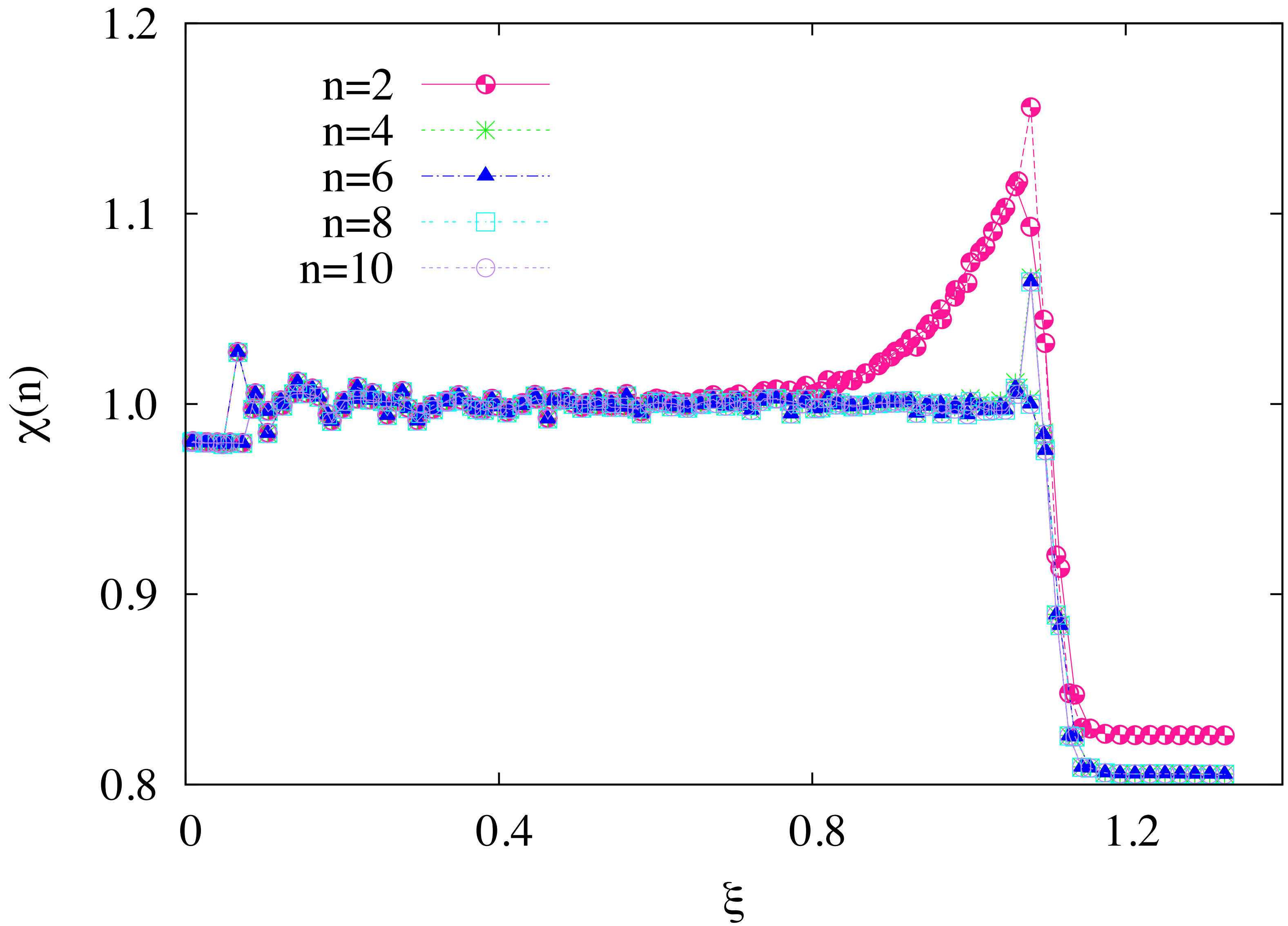




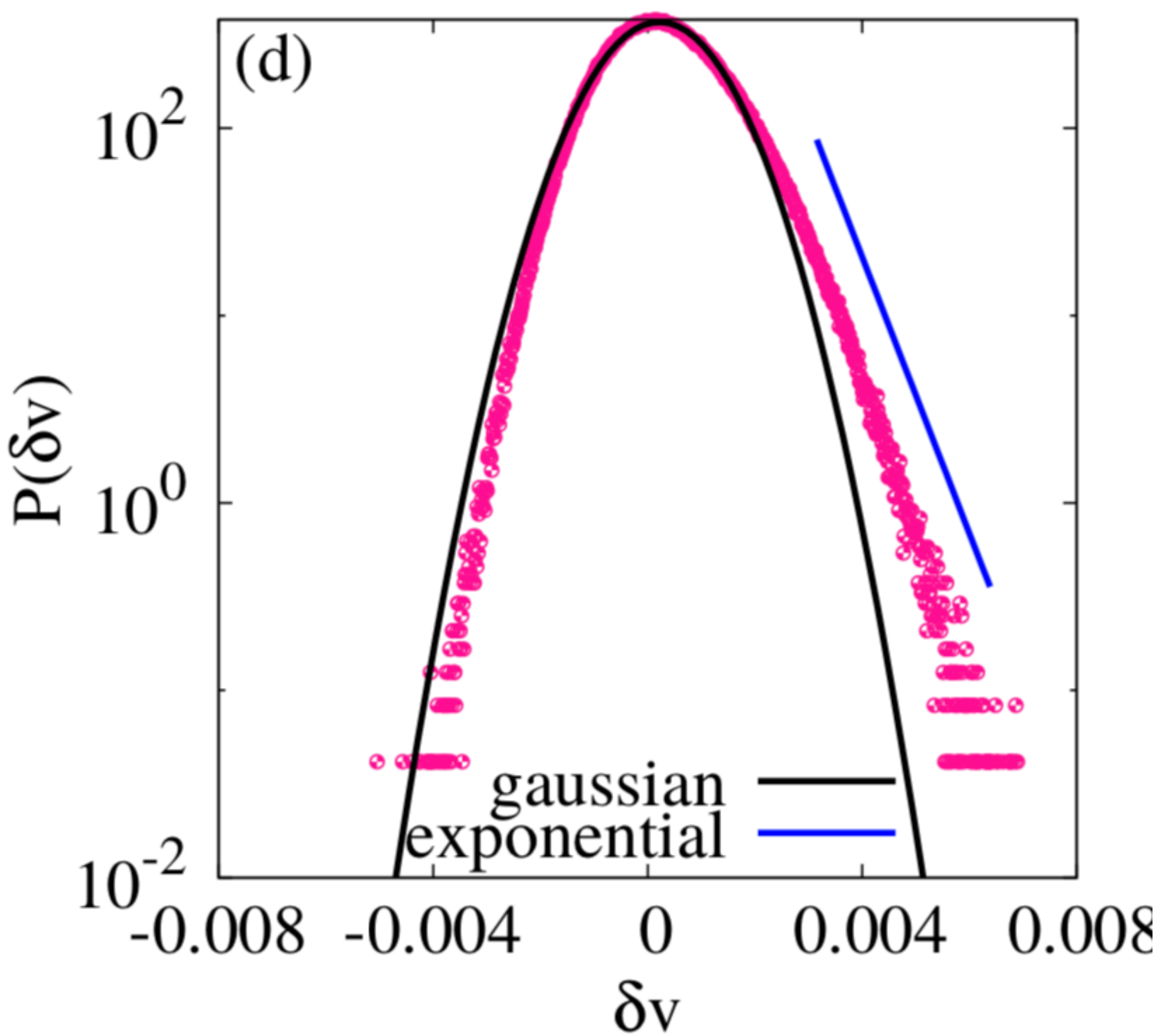


Assumption of equation of state?

$$\chi(n) = \frac{P(\xi)}{E(\xi)R(\xi)\left[1 + \sum_{k=2}^{n} B_k \rho_0^{k-1} R(\xi)^{k-1}\right]}$$



Velocity Fluctuations

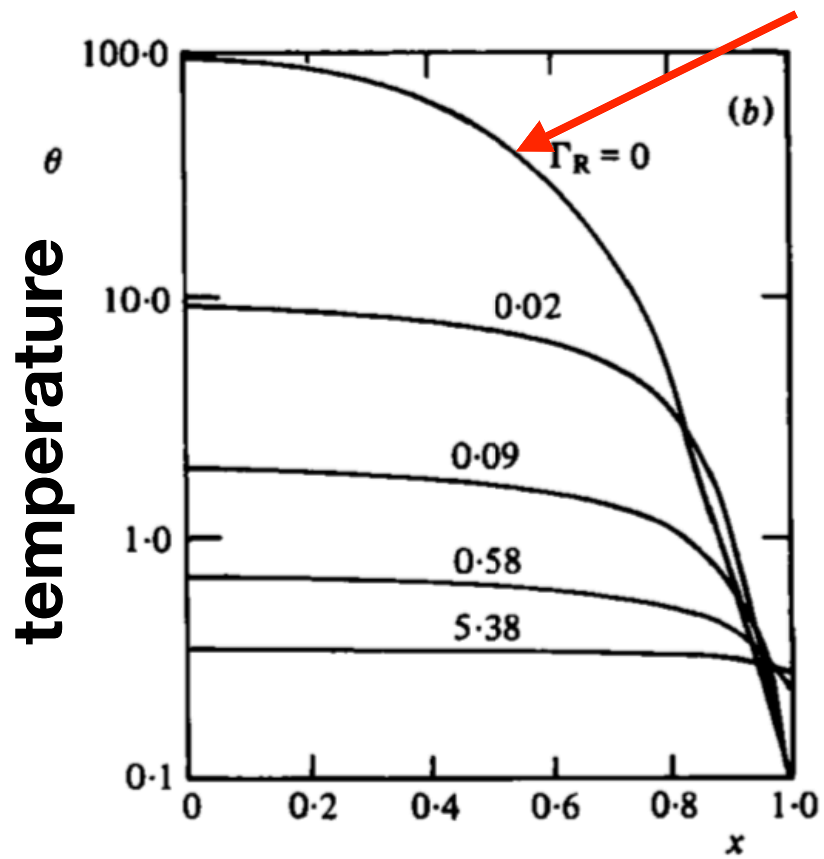


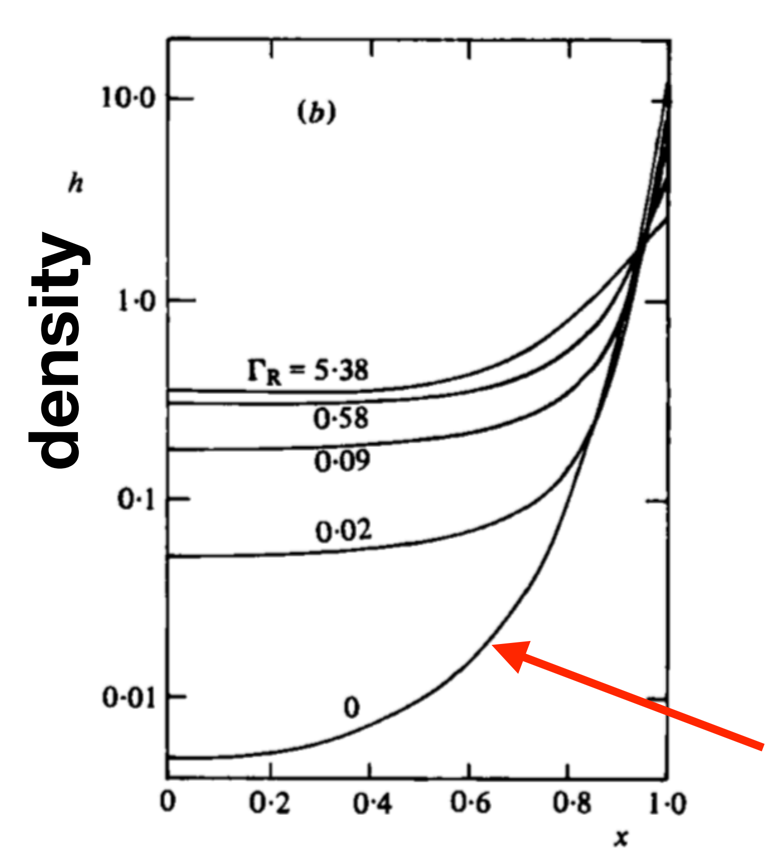
Summary

- Revisited problem of shock propagation following an intense explosion
- The hydrodynamic results do not match with simulations of a particle-based model
- Assumption of existence of local equation of state is consistent with simulation results
- But, velocity fluctuations are not Gaussian
  - ⋆ Whether these are responsible for the discrepancy can be checked by re-assigning velocities to ensure local equilibrium
- Heat conduction?

Heat Conduction

- With conduction, the boundary condition at centre is zero heat flux, or gradient in temperature in zero
- Heat conduction regularises diverging temperature at shock centre
- Within kinetic theory, heat conduction term not important. Have to assume conductivity proportional to $T^{1/6}$





Ghoniem et al, J. Fluid. Mech. (1982)

- However, the profile near the shock front should not be affected
- Also with heat conduction, is there a quantitative match?