Multiple Time scale phenomena on Complex Networks

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Complexity of real world systems

- Complex systems composed of Interacting sub units
- nonlinearity in the intrinsic dynamics of their sub systems
- complex interaction patterns among them

- variability and heterogeneity of the interacting sub systems can add another level of complexity

- heterogeneity arising from differing dynamical time scales of nonlinear systems
- with a heterogeneous pattern of interactions

- cooperative dynamics in interacting nonlinear systems of differing time scales using the frame work of complex networks

Multiple-timescale phenomena

- electrical activity of neurons- functional hierarchy is achieved among neurons
- chemical reactions
- Electro-optical processes
- turbulent flows
- population dynamics
- Climate- atmosphere& ocean

Collective behavior & phenomena

- Suppression and recovery of oscillations
- Synchronization- frequency synchronization
- Self organization
- Frequency locked clusters and travelling sequences

References

The study presented have appeared in the following publications

- Suppression of dynamics and frequency synchronization in coupled slow and fast dynamical systems- Kajari Gupta and G Ambika, , EP J B 89(6), 1-8 (2016)- DOI: 10.1140/epjb/e2016-70068-8

- Role of time scales and topology on the dynamics of complex networks- Kajari Gupt and G. Ambika- Chaos 29, 033119 (2019); doi: 10.1063/1.5063753

- Frequency locking and travelling burst sequences in community structured network of inhibitory neurons with differing time-scales- Kunal Mozumdar and G Ambika- Commun Nonlinear Sci Numer Simulat 69 320–328, (2019) - https://doi.org/10.1016/j.cnsns.2018.09.026

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2. Kunal Mozumdar, MS student, IISER Pune

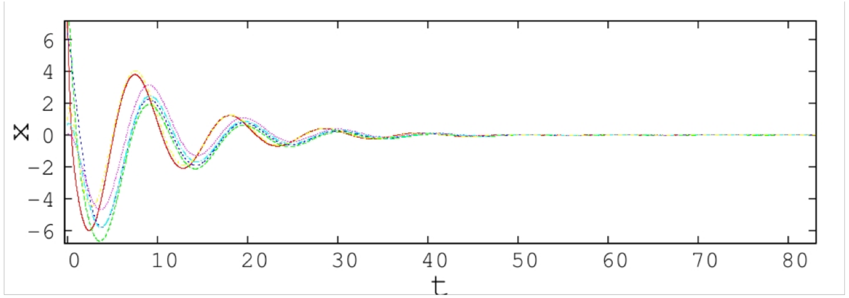
Rössler systems on Random networks

$$\dot{x}_i = \tau_i(-y_i - z_i) + \tau_i \epsilon \sum_{j=1}^{N} A_{ij}(x_j - x_i)$$

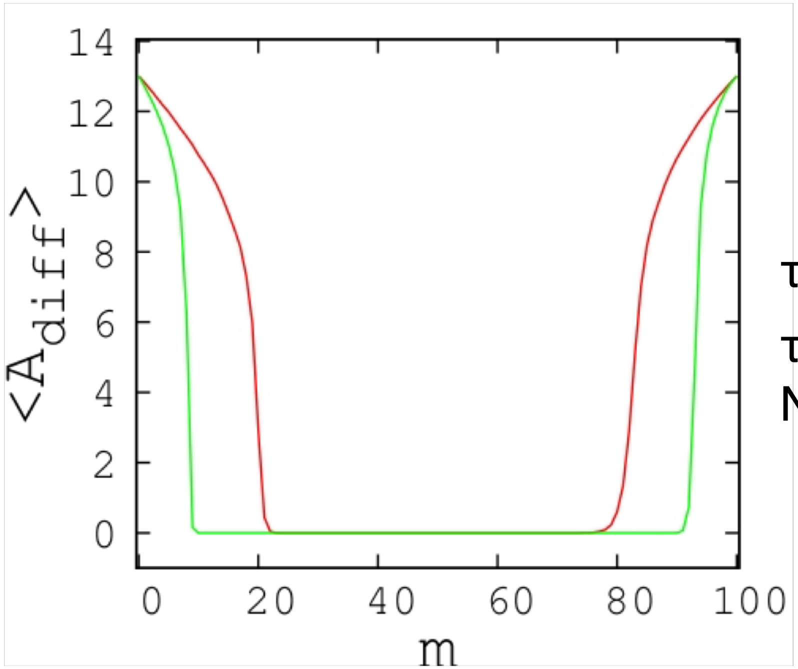
$$\dot{y}_i = \tau_i(x_i + a y_i)$$

$$\dot{z}_i = \tau_i(b + z_i(x_i - c))$$

Suppression and Recovery of oscillations



m=50, τ=0.35, ε = 0.05

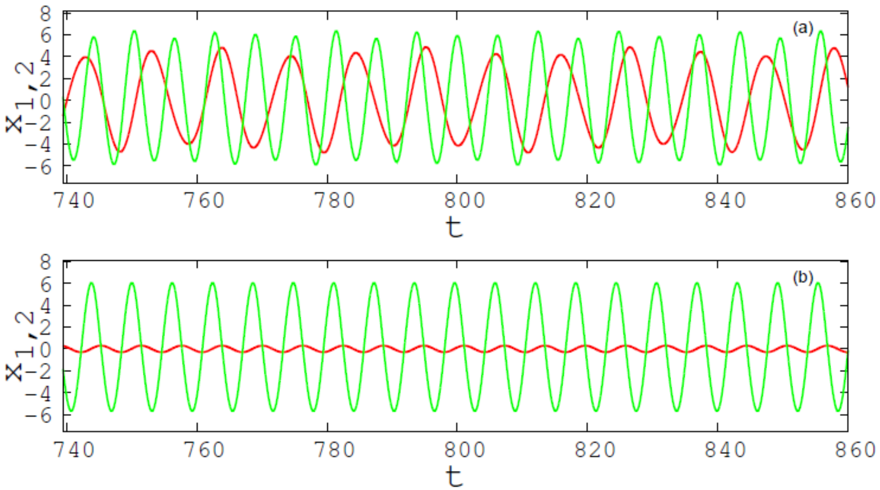


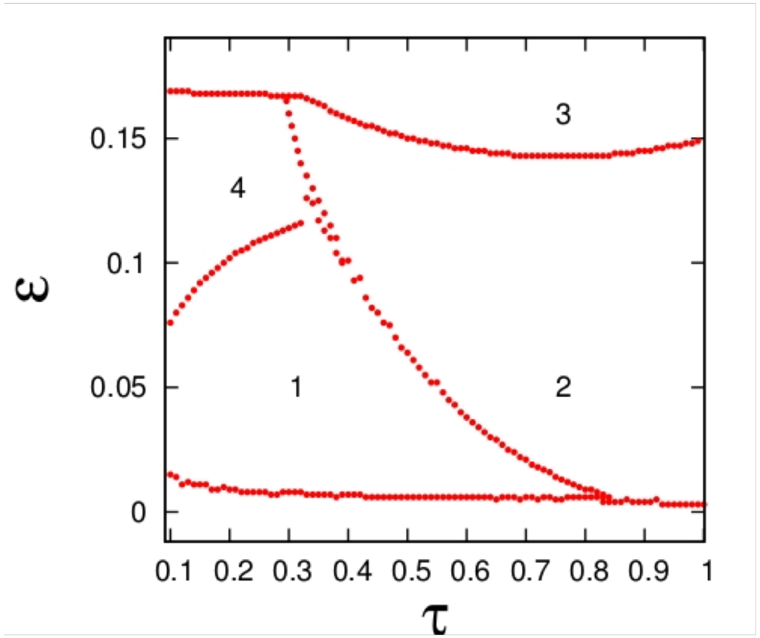
τ=0.35 and ε =0.01 (red curve)

τ=0.35 and ε =0.05 (green curve)

N=100, p=0.5.

Frequency synchronization

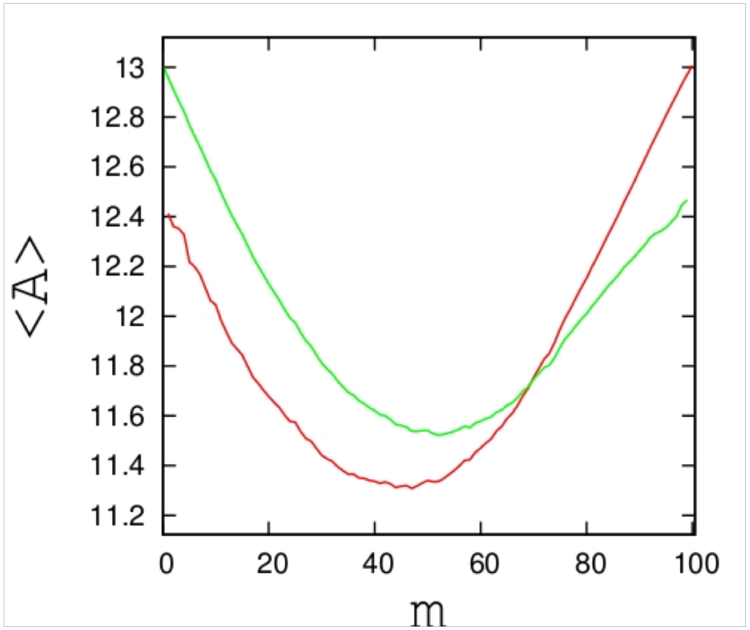




m=50,p=0.5,N=100

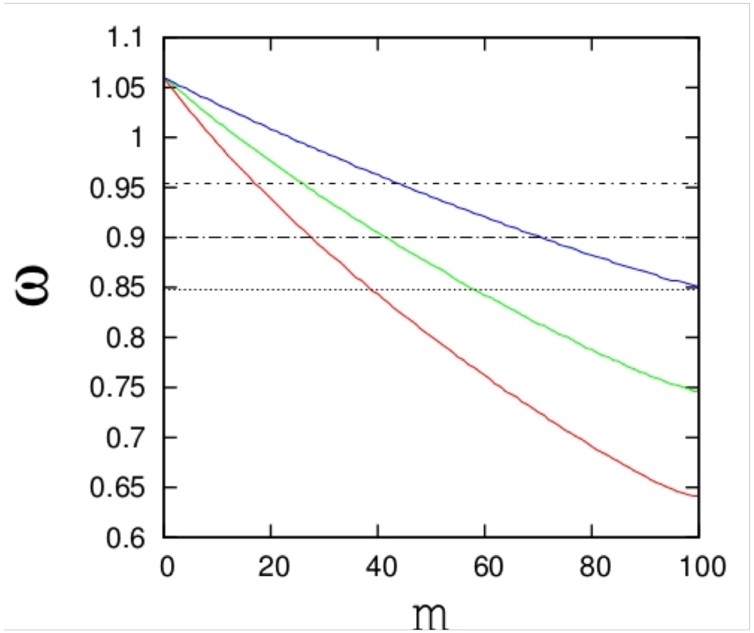
Crossover phenomena in the emergent dynamics

Reversal of amplitudes of oscillations



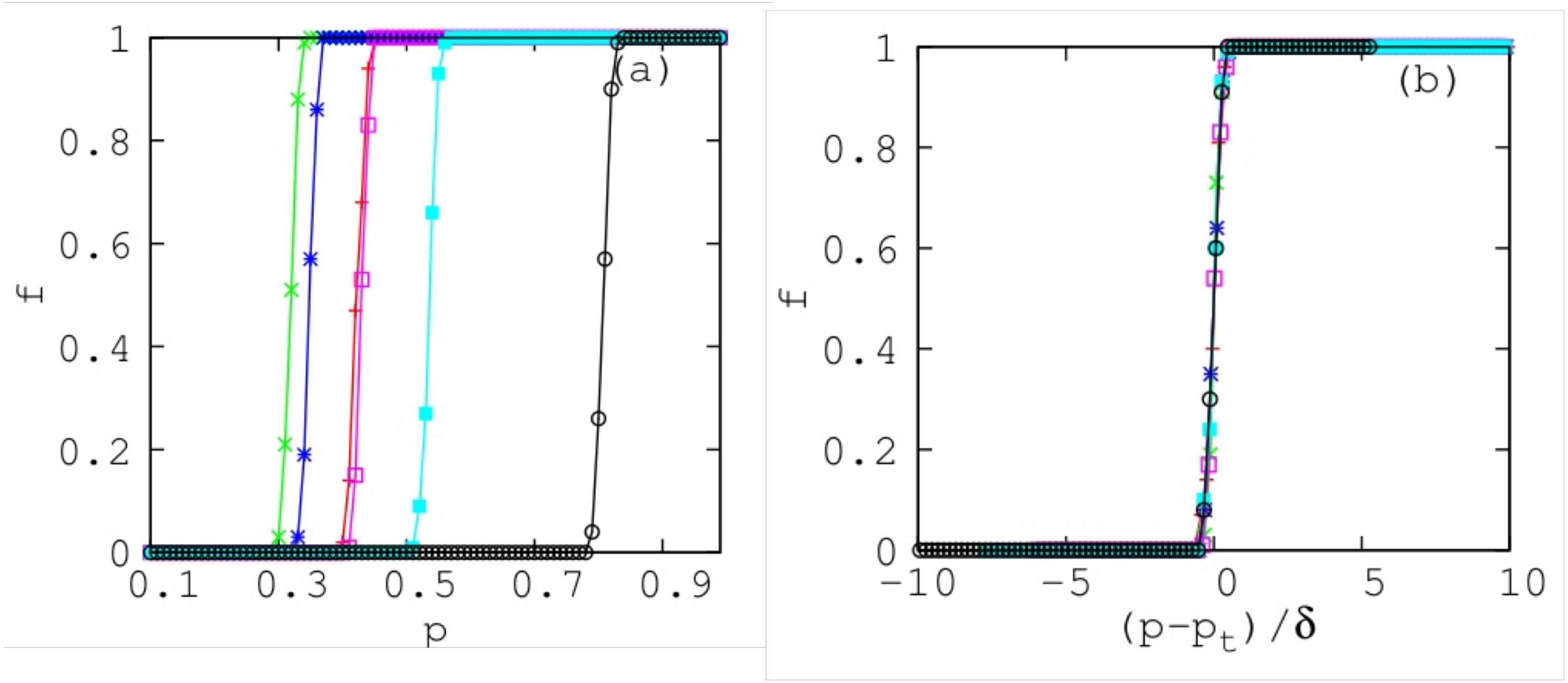
p=0.5, τ = 0.7, ε = 0.05

Crossover to frequency suppression



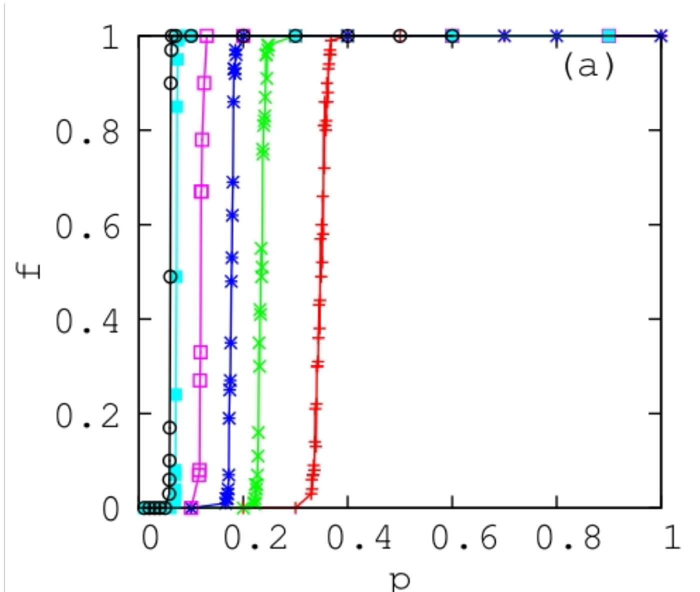
ε =0.05, p=0.5
τ=0.6(red),0.7(green),0.8(blue)

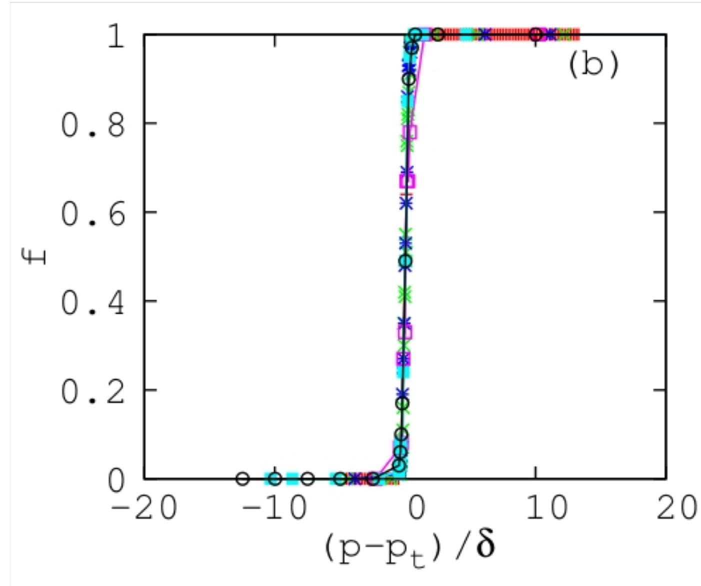
Transition to amplitude death and connectivity of the network

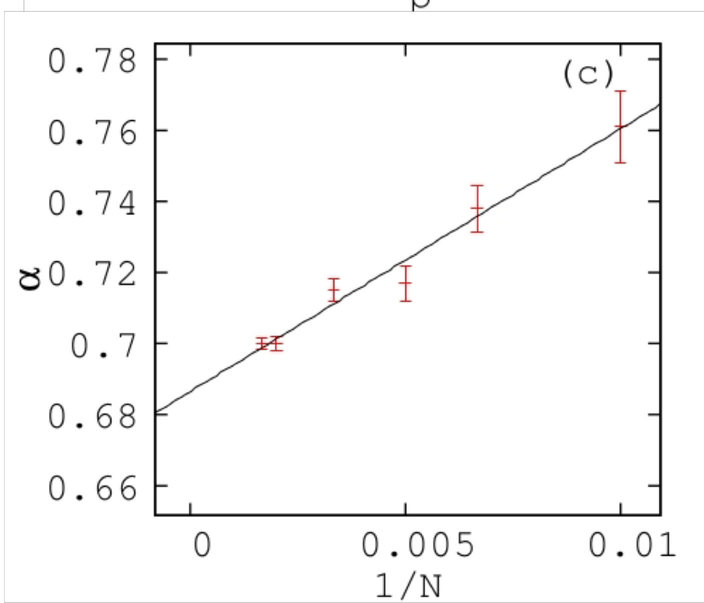


Fraction of realizations f for the transition to AD, plotted with the probability p of connections, for m=30(red), 40(green), 50(blue), 60(magenta), 70(cyan), 80(black)

Scaling with size of network







$\tau = 0.35, \ \varepsilon = 0.01$

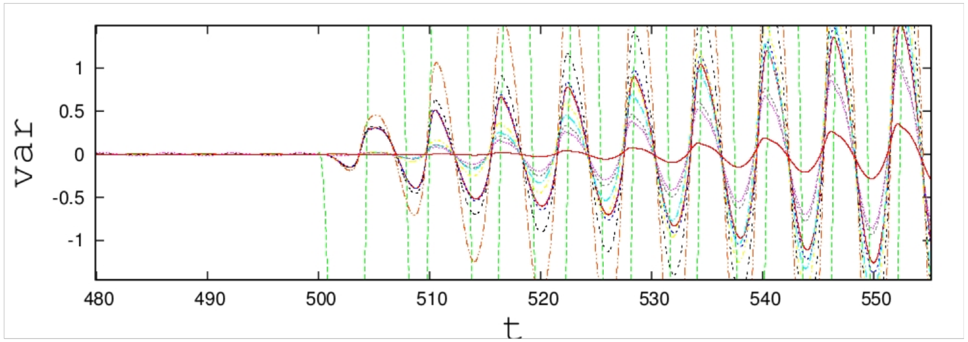
N=100(red), 150(green), 200(blue), 300(magenta), 500(cyan), 600(black),

$$f = (p - p_c)^{\alpha}.$$

$\alpha = 2/3$

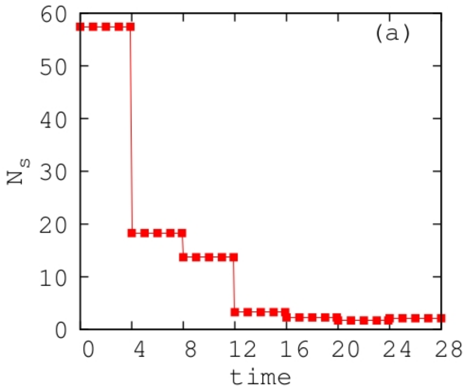
Scale free networks of slow and fast systems

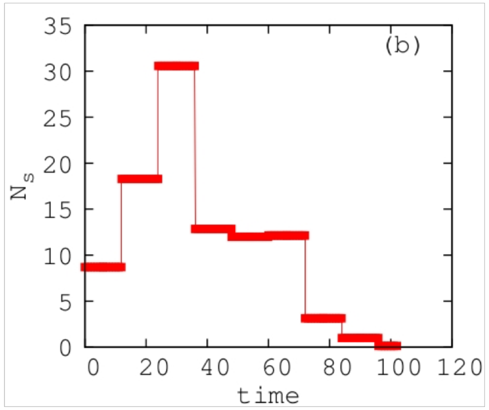
Onset of desynchronization



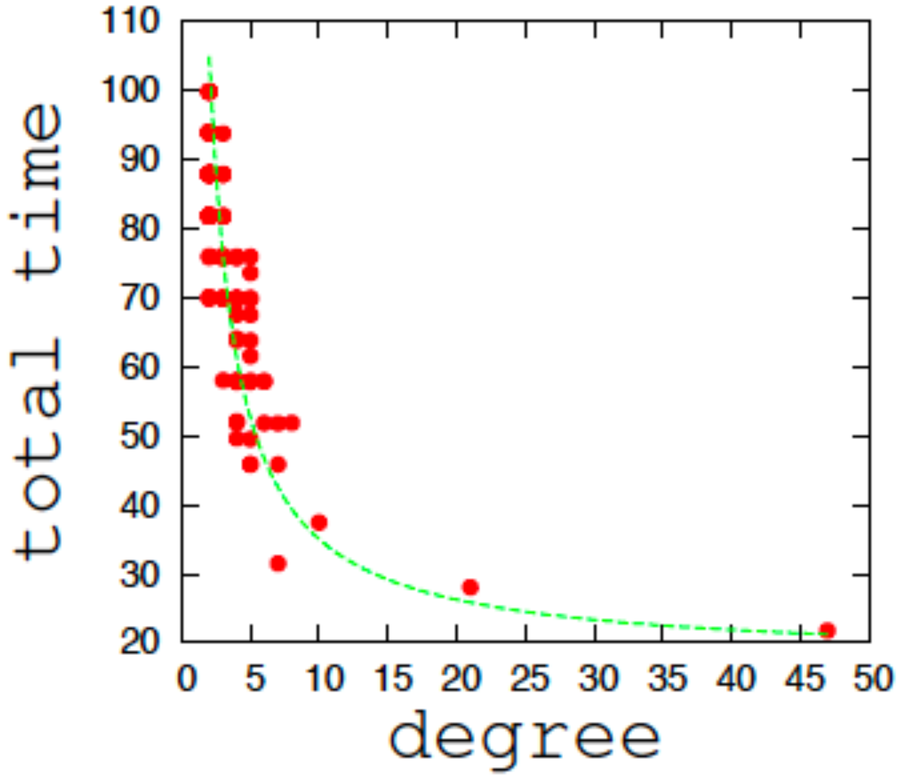
$\tau = 0.3, \varepsilon = 0.03$

Number of systems that move away from synchrony(Ns) in a range of time





Spreading of slowness- time for desynchronization

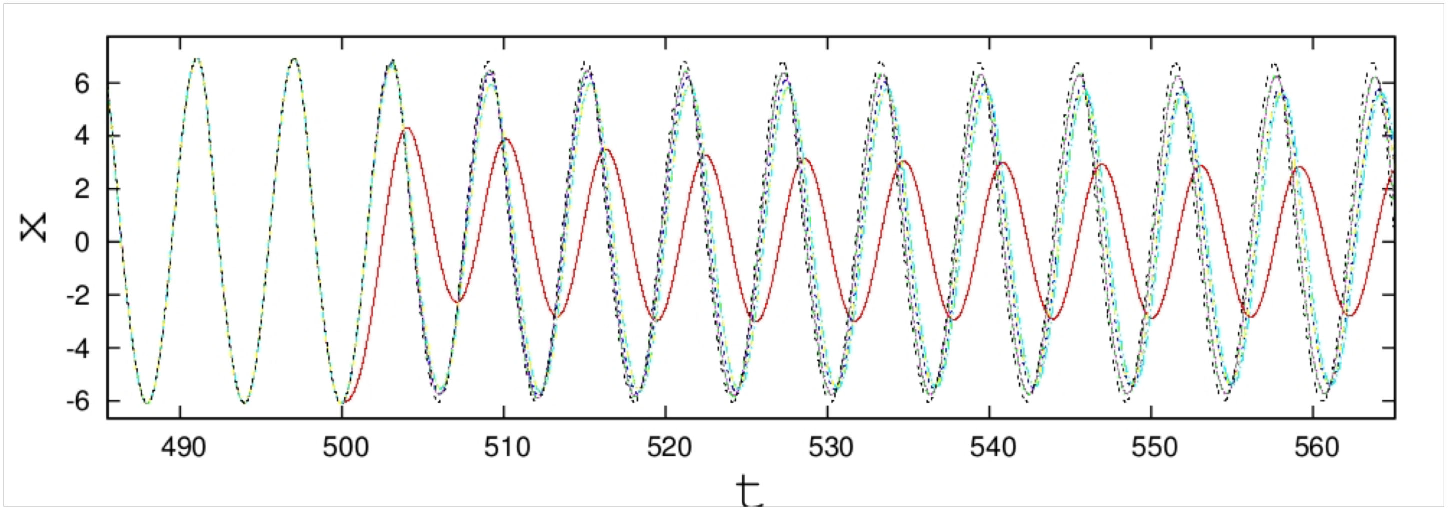


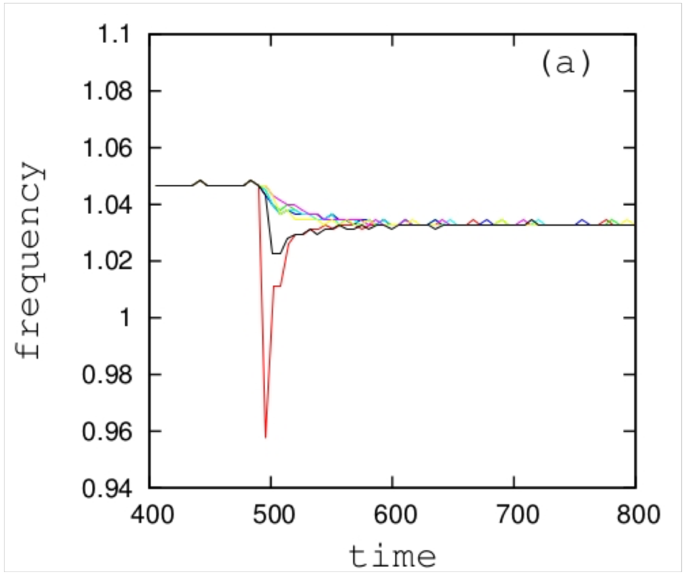
$$T(k) = (a/k) + b$$

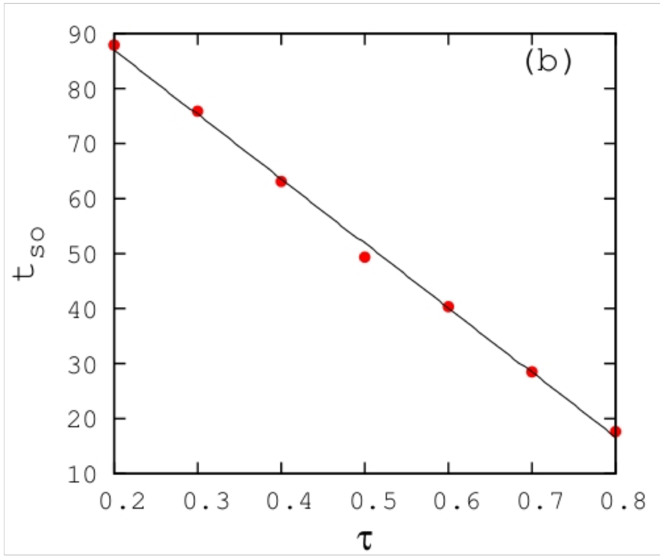
a = 180 and b = 18

Total time taken for all oscillators in the network to move away from synchrony is plotted against the degree of source node

Self organization of the network to frequency synchronized state

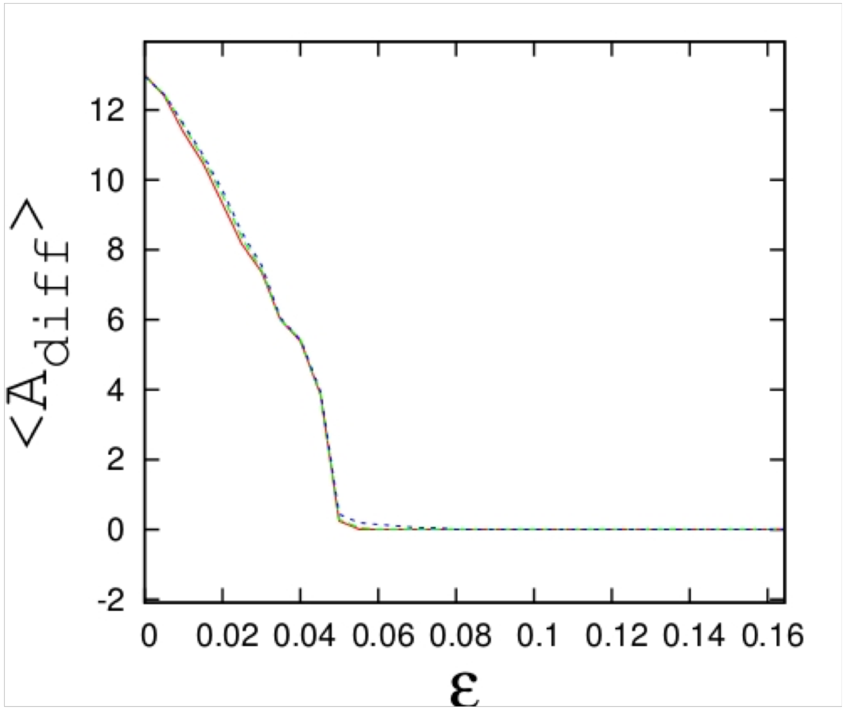






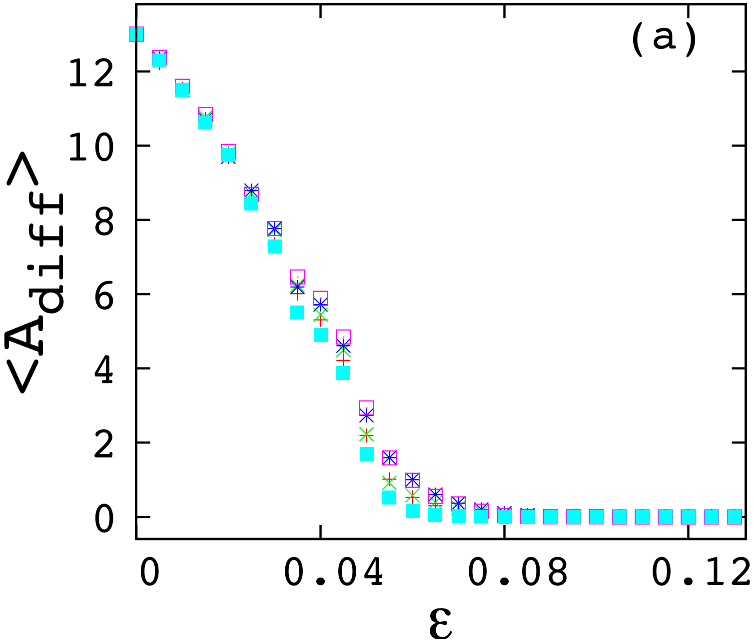
Scale free network with multiple time scales

$$\tau_i = 2/k_i$$

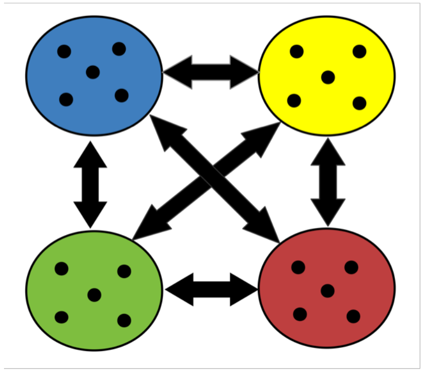


N=100(red), 500(green), 1000(blue)

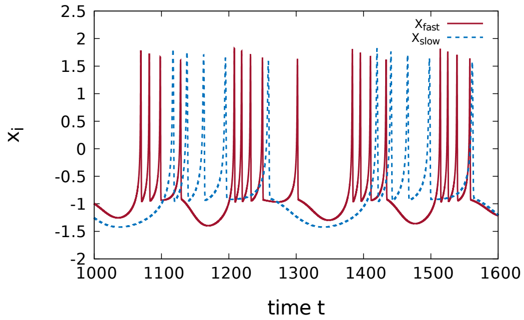
Normal distribution of time scales, mean=0.5 and std=0.15



Community structured network of HR neurons with differing time scales



N = 120 and M = 4



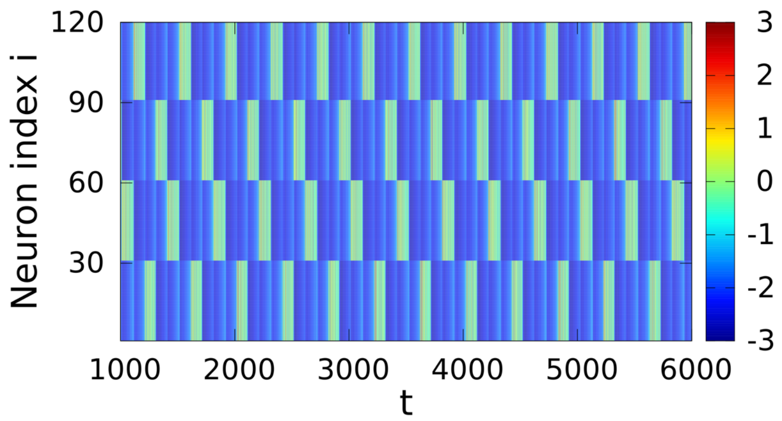
$$\dot{x}_i = \eta_i\Big(y_i - x_i^3 + 3x_i^2 - z_i + I_e - \beta(V - x_i)\sum_{j=1}^{N} a_{ij}\Big(\frac{1}{e^{-\lambda(x_j+K)}}\Big)\Big)$$

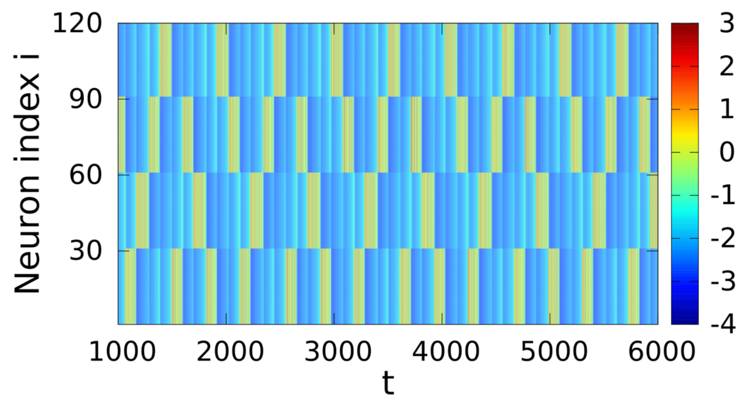
$$\dot{y}_i = \eta_i(1 - 5x_i^2 - y_i)$$

$$\dot{z}_i = \eta_i(\epsilon(4(x_i + x_r) - z_i))$$

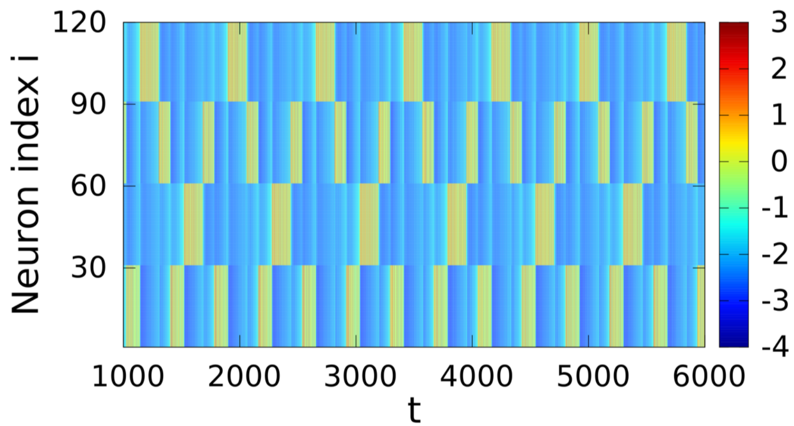
$$B_{lk} = \begin{cases} [0], \forall\, l = k \\ [1], \forall\, l \neq k \end{cases} \qquad A = \begin{bmatrix} [0] & [1] & [1] & [1] \\ [1] & [0] & [1] & [1] \\ [1] & [1] & [0] & [1] \\ [1] & [1] & [1] & [0] \end{bmatrix}$$

Travelling burst sequences





$$\boldsymbol{\eta}_1 = (1, \eta, 1, \eta)$$

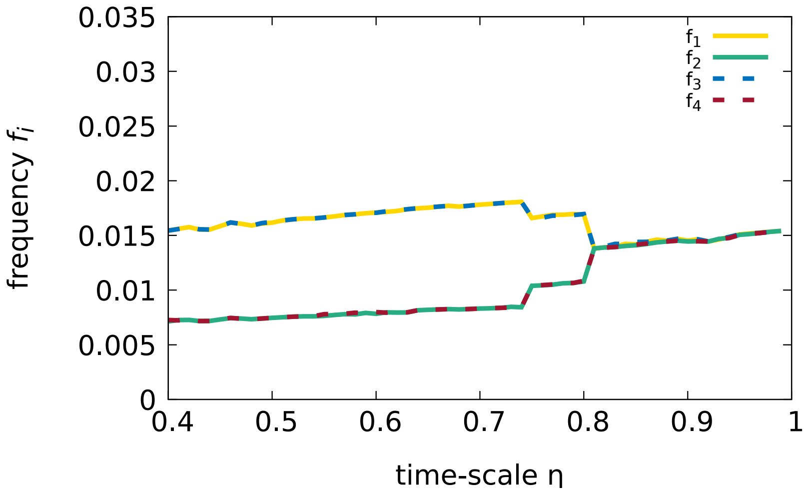


Spatio-temporal plots with time-scale mismatch at β= 0.1:
(a) η= 0.9, (b) η = 0.8 (c) = η 0.4

Burst sequences

Burst sequences in four modules			
Configuration ($\boldsymbol{\eta}_i$)	Sequnce ($P_j^{\eta_i}$)	(p:q)	Range of η
$\boldsymbol{\eta}_1 = (1, \eta, 1, \eta)$	$P_1^{\eta_1} = \overline{F_1F_2S_1S_2}$ $P_2^{\eta_1} = \overline{F_1S_1F_2S_2F_1F_2S_1F_1S_2F_2}$ $P_3^{\eta_1} = \overline{S_2F_2F_1S_1F_2F_1}$	(2:2) (4:6) (2:4)	(0.83,1) (0.76,08.82) (0.4,0.76)
$\boldsymbol{\eta}_2 = (\eta, \eta, \eta, 1)$	$P_1^{\eta_2} = \overline{S_3S_2S_1F}$ $P_2^{\eta_2} = \overline{FS_3S_2FS_1S_3FS_2S_1F}$ $P_3^{\eta_2} = \overline{S_3FS_2S_1F}$ $P_4^{\eta_2} = \overline{FS_3FS_2FS_1}$	(3:1) (6:4) (3:2) (3:3)	(0.83,1) (0.6,0.83) (0.54,0.6) (0.3,0.54)
$\boldsymbol{\eta}_3 = (1, 1, 1, \eta)$	$P_1^{\eta_3} = \overline{F_3F_2F_1S}$ $P_2^{\eta_3} = \overline{SF_3F_1F_2F_3SF_1F_2F_3F_1SF_2F_3F_1F_2}$ $P_3^{\eta_3} = \overline{SF_1F_3F_2F_1F_3SF_2F_1F_3F_3F_1SF_3F_2F_1F_3F_2}$ $P_4^{\eta_3} = \overline{SF_3F_2F_1F_3F_2F_1F_3F_2F_1}$ $P_5^{\eta_3} = \overline{F_3F_2F_1}$	(1:3) (3:12) (3:15) (1:9) (0:3)	(0.78,1) (0.72,0.78) (0.66,0.72) (0.63,0.66) (0.3,0.63)

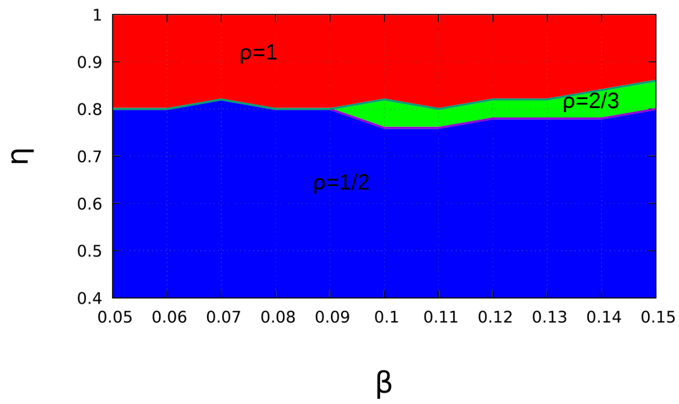
Frequency locked clusters



Frequency vs time-scale parameter at β = 0.1

two distinct clusters of slow and fast frequencies

$$f_i = \frac{2\pi}{K_i} \sum_{k=1}^{K_i} \frac{1}{{\tau_i}^{k+1} - {\tau_i}^{k}}$$



Synchronized frequency locked states
Red ρ = 1,
Green ρ = 2/3
Blue ρ = 1/2

Summary

- Suppression and recovery of of dynamics in coupled Rossler oscillators
- Collective phenomenon - frequency synchronization
- Cross over in amplitudes and frequency as number of slow systems increases
- Transition to AD in terms of the probability of connections p scales with network size, the index of scaling being 2/3

- Hubs can function as control nodes in the emergent dynamics
- Spread of slowness through the network due to one node being slow
- Self-organization of the whole network from completely synchronized state to one of frequency synchronization

- Pattern of travelling Burst sequences in HR neurons on a modular network with inhibitory couplings
- Sequential order in the burst patterns and the temporal order in frequency locking