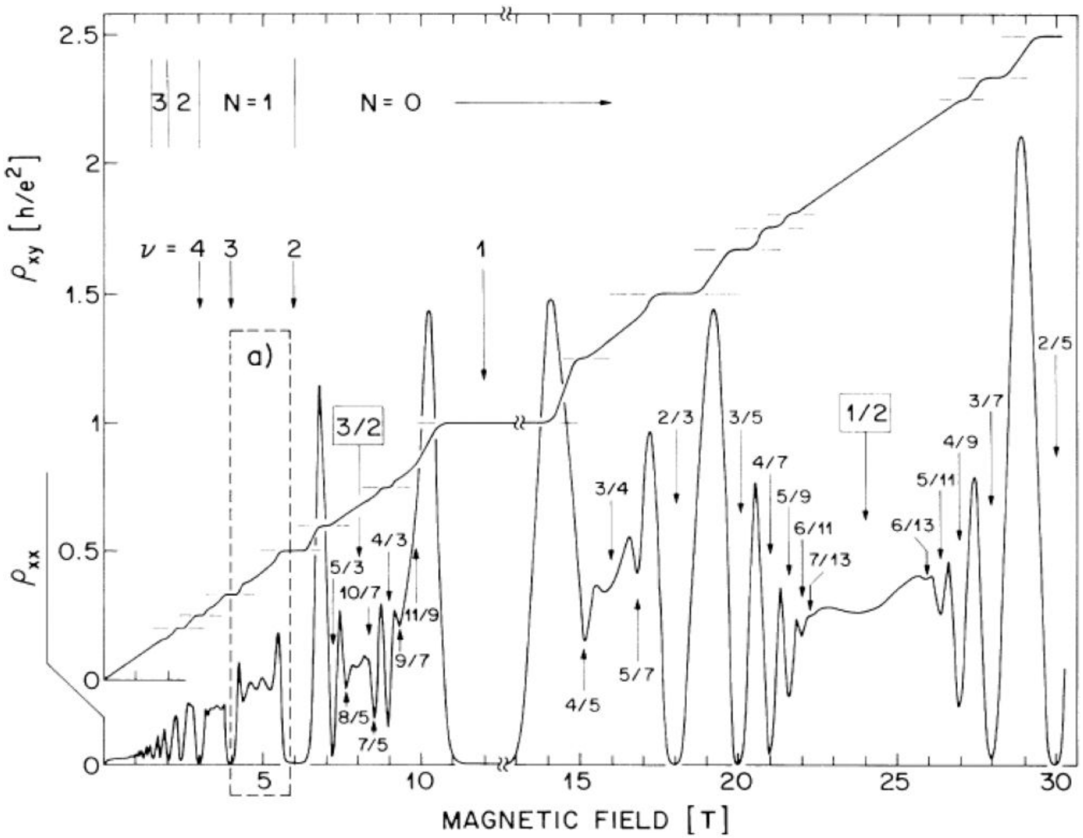
Mean field approximations to n body Hamiltonians in FQHE

Sreejith G J
IISER Pune

G J Sreejith, Y Zhang, J K Jain, Phys. Rev. B 96, 125149 (2017)
B Kusmierz, G J Sreejith (in preparation)

FQH States



FQHE in the lowest Landau level:

Can be described using a model of single Landau level of electrons interacting through a two-body Coulomb interaction.

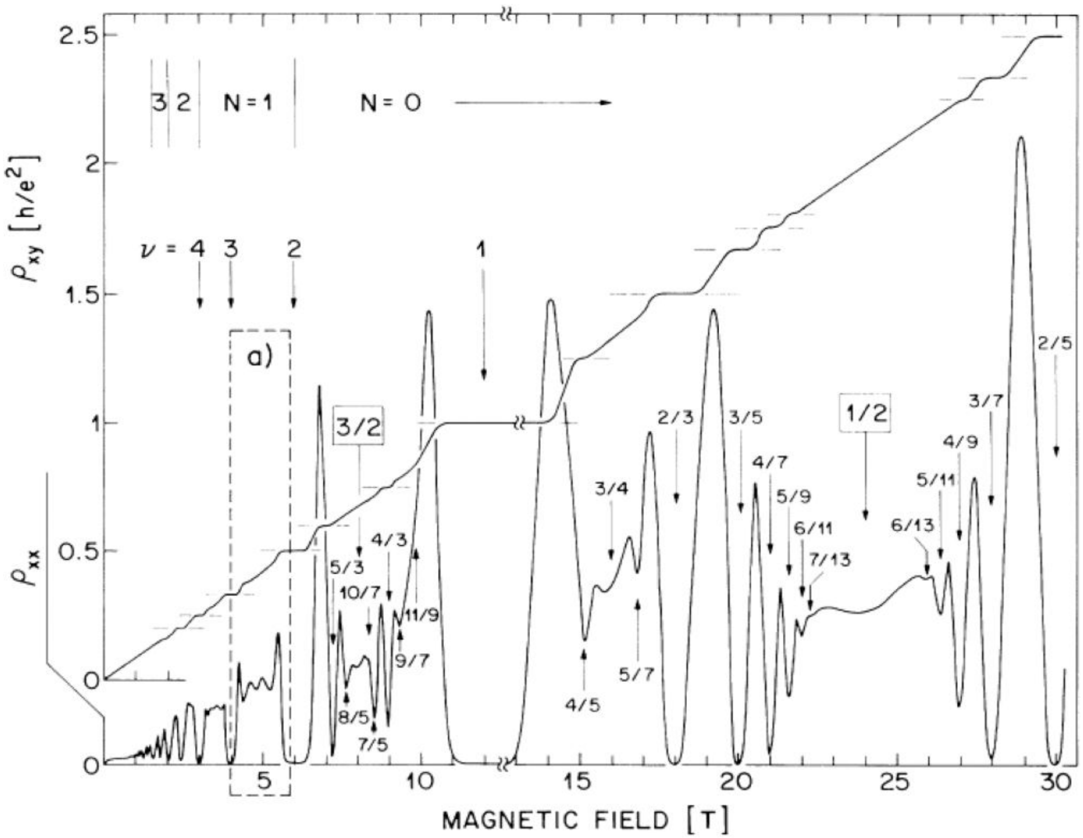
FQH occurs at odd-denominator filling fractions -- well-understood in terms of emergent particles called composite fermions

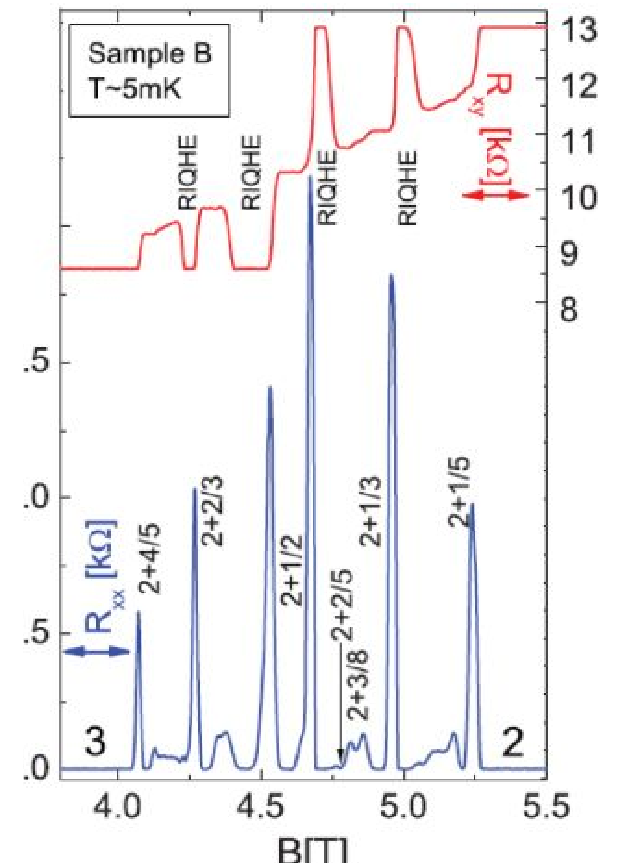
No FQHE at filling fraction ½

Willet et al PRL 1987, Willet, Prog in Physics 2013

Jain 1989,
Halperin, Lee Read 1993

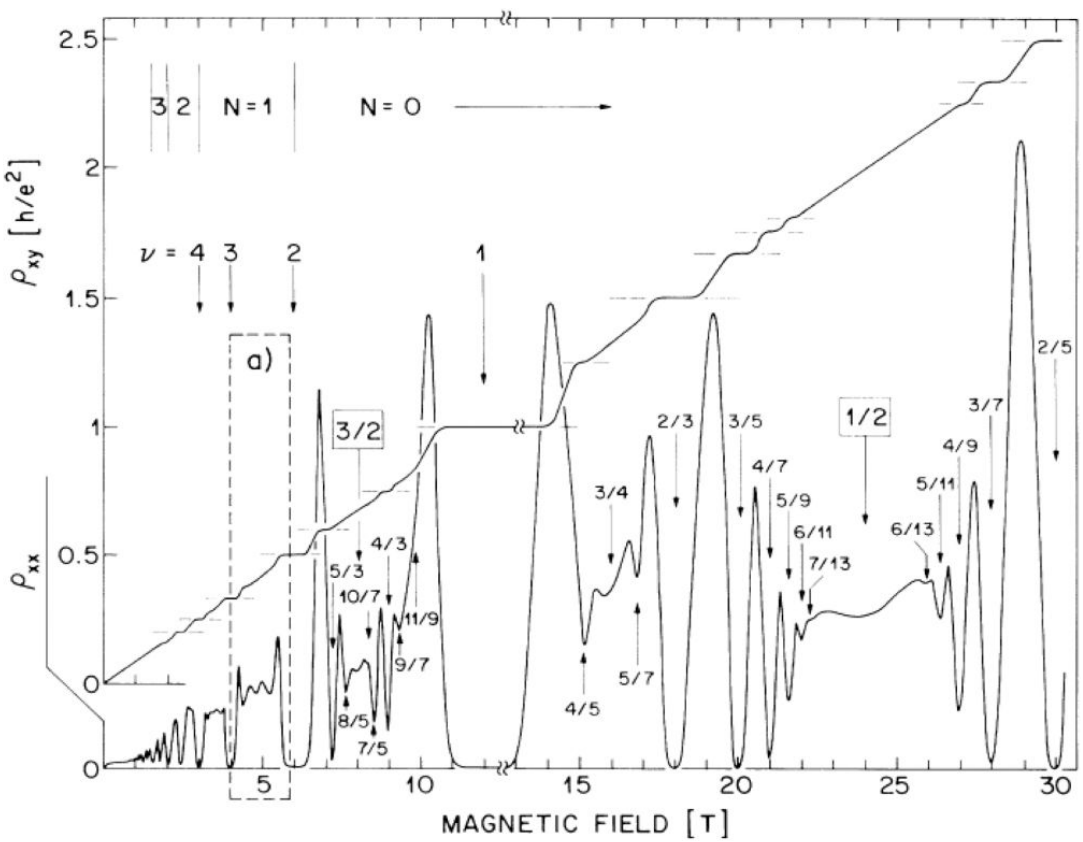
FQH States





Willet et al PRL 1987
Willet, Prog in Physics 2013

Even denominator FQHE



Willet et al PRL 1987
Willet, Prog in Physics 2013

FQHE in the lowest Landau level:

Can be described using a model of single Landau level of electrons interacting through a two-body Coulomb interaction.

FQH occurs at odd-denominator filling fractions -- well-understood in terms of emergent particles called composite fermions

No FQHE at filling fraction ½

Second Landau level:

Strong FQH state at filling fraction ½.

General structure of the FQH states not very well understood

Pfaffian, Anti-pfaffian and Particle hole symmetry

$$\psi \sim \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right] \prod_{i<j=1}^{N} (z_i - z_j)^2$$

$$\text{Antisymmetrize}\left[\frac{1}{z_1 - z_2}\frac{1}{z_3 - z_4}\frac{1}{z_5 - z_6}\cdots\right]$$

P-wave superconductor of emergent particles called composite fermions

Pfaffian, Anti-pfaffian and Particle hole symmetry

$$\psi \sim \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right] \prod_{i<j=1}^{N} (z_i - z_j)^2$$

$$\text{Antisymmetrize}\left[\frac{1}{z_1 - z_2}\frac{1}{z_3 - z_4}\frac{1}{z_5 - z_6}\cdots\right]$$

Pfaffian which occurs at a filling fraction of 1/2 is not particle hole symmetric

'Anti Pfaffian' defined as the PH conjugate of the Pfaffian is a **different** topologically ordered state at the same filling fraction ½ (Levin, Halperin, Rosenow PRL 2007, Lee et al PRL 2007)

Two-body interaction within a Landau level is PH symmetric ⇒ System spontaneously breaks PH symmetry. Inter Landau level mixing could provide a breaking field.

Aside: Actual resolution of this might be slightly more exotic (Banerjee et al arXiv:1710.00492, Mross et al arXiv 1711.06278; Wang et al arXiv 1711.11557; Simon arXiv:1801.09687)

LL Mixing - ineffective particle hole symmetry breaking

$$H_{\text{eff}} = H_0 + \kappa \sum_{p \in \text{intermediate states}} \frac{V\,|p\rangle\,\langle p|\,V}{E_0 - E_p}$$

Leads to correction to the two-body interaction Hamiltonian (which do not break particle hole symmetry)
+
PH symmetry breaking three-body interactions between the electrons. (Bishara, Nayak PRB 2009, Simon, Rezayi PRL 2011, Sodeman, MacDonald 2013)

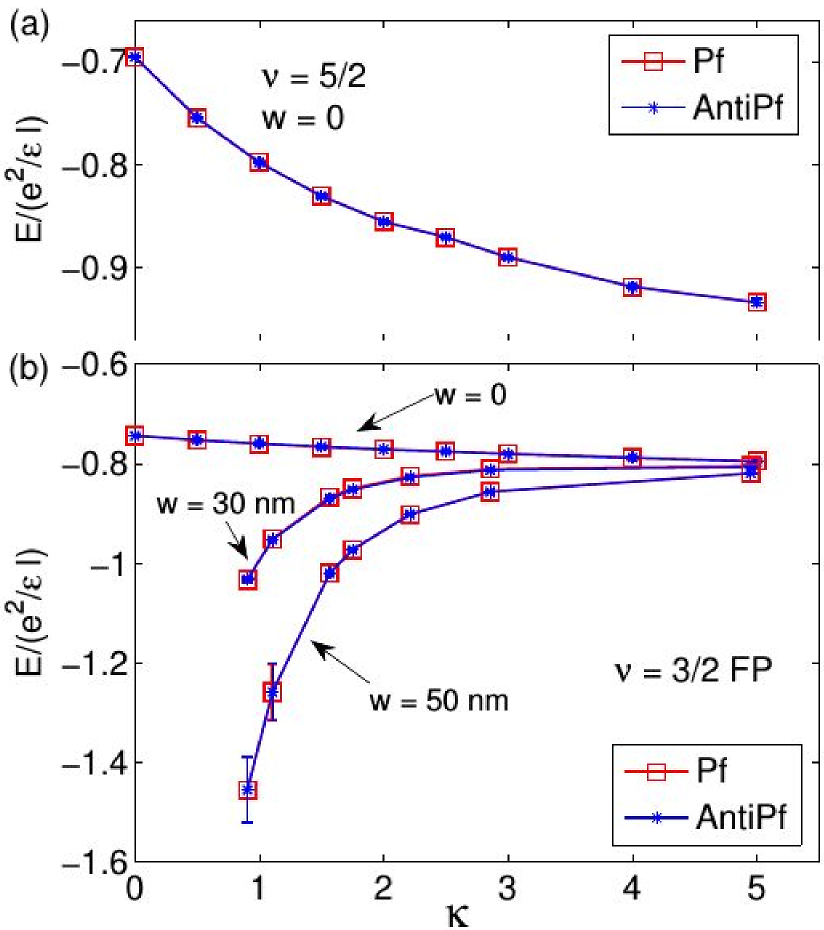
Some calculations point to a Pf ground state (Wojs et al PRL 2010, Pakrouski et al PRX 2015)

Others suggest a APf ground state (Rezayi, Simon PRL 2011, Zalatel et al 2015, Rezayi PRL 2017)

They all point to the fact that the effect of particle hole symmetry breaking is weak.

LL Mixing - ineffective particle hole symmetry breaking

$$H_{\text{eff}} = H_0 + \kappa \sum_{p \in \text{intermediate states}} \frac{V\,|p\rangle\langle p|\,V}{E_0 - E_p}$$



Effect of LL mixing on particle hole symmetry breaking is weak

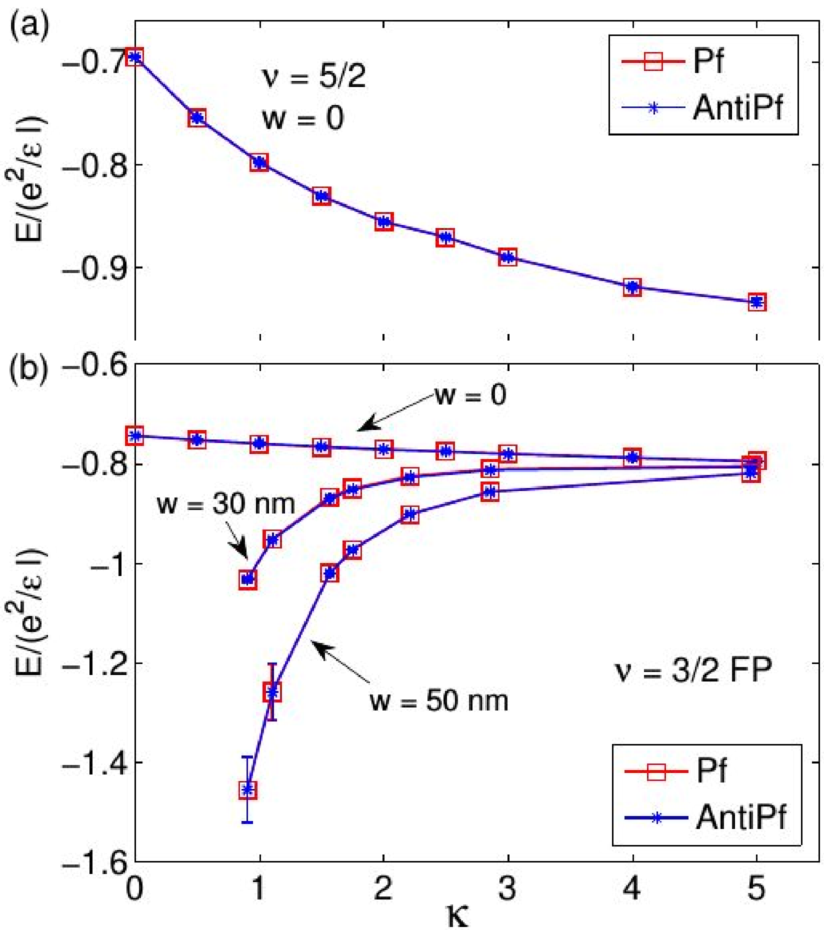
⇒ The generic three-body interactions induced by LL mixing should effectively act as a PH symmetric two-body interaction.

Generic three-body interaction Hamiltonian → Mean Field Construction → two-body interaction (Always PH symmetric)

GJS, Y Zhang, Jain PRB 2017

LL Mixing - ineffective particle hole symmetry breaking

$$H_{\text{eff}} = H_0 + \kappa \sum_{p \in \text{intermediate states}} \frac{V\,|p\rangle \langle p|\,V}{E_0 - E_p}$$



Effect of LL mixing on particle hole symmetry breaking is weak

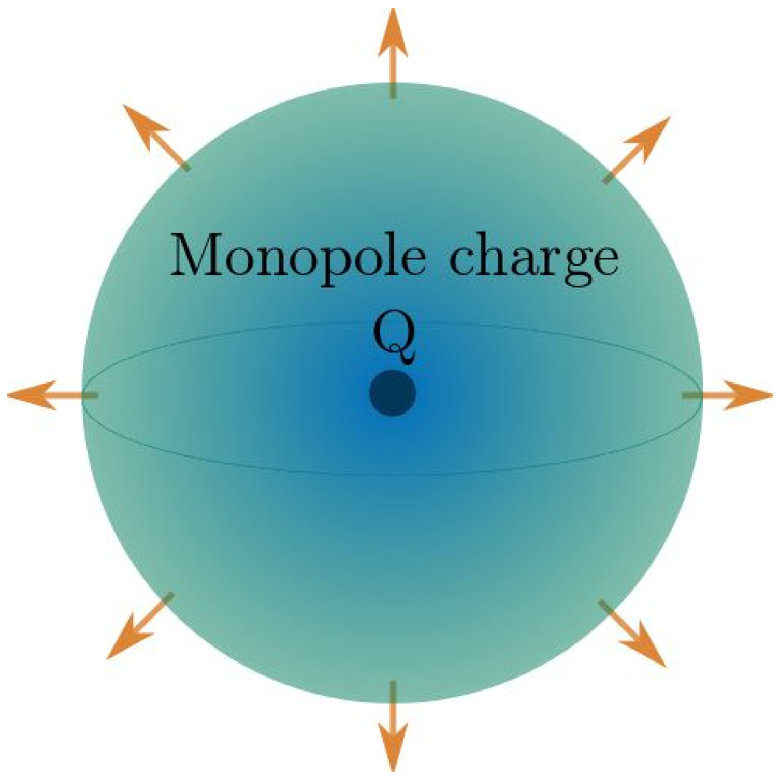
⇒ The generic three-body interactions induced by LL mixing should effectively act as a PH symmetric two-body interaction.

Generic three-body interaction Hamiltonian $\xrightarrow{\text{Mean Field Construction}}$ two-body interaction (Always PH symmetric)

three-body Haldane pseudopotentials $\xrightarrow{\text{Mean Field Construction}}$ two-body Haldane pseudopotentials

GJS, Y Zhang, Jain PRB 2017

Spherical geometry and Haldane pseudopotentials



Bulk physics of the system can be effectively studied in a spherical geometry

Translational, rotational symmetry of the bulk of the system manifests as complete rotational symmetry on the sphere.

Energy depends only on the L^2 quantum number.

A rotational symmetric Hamiltonian can be parametrized using a few ‘Haldane pseudopotentials’ instead of all matrix elements.

$$\sum_{i<j=1}^{N} \sum_{L} V_L P_L^{ij}$$

S H Simon, R Rezayi, N Cooper PRB 2006
Haldane 1983

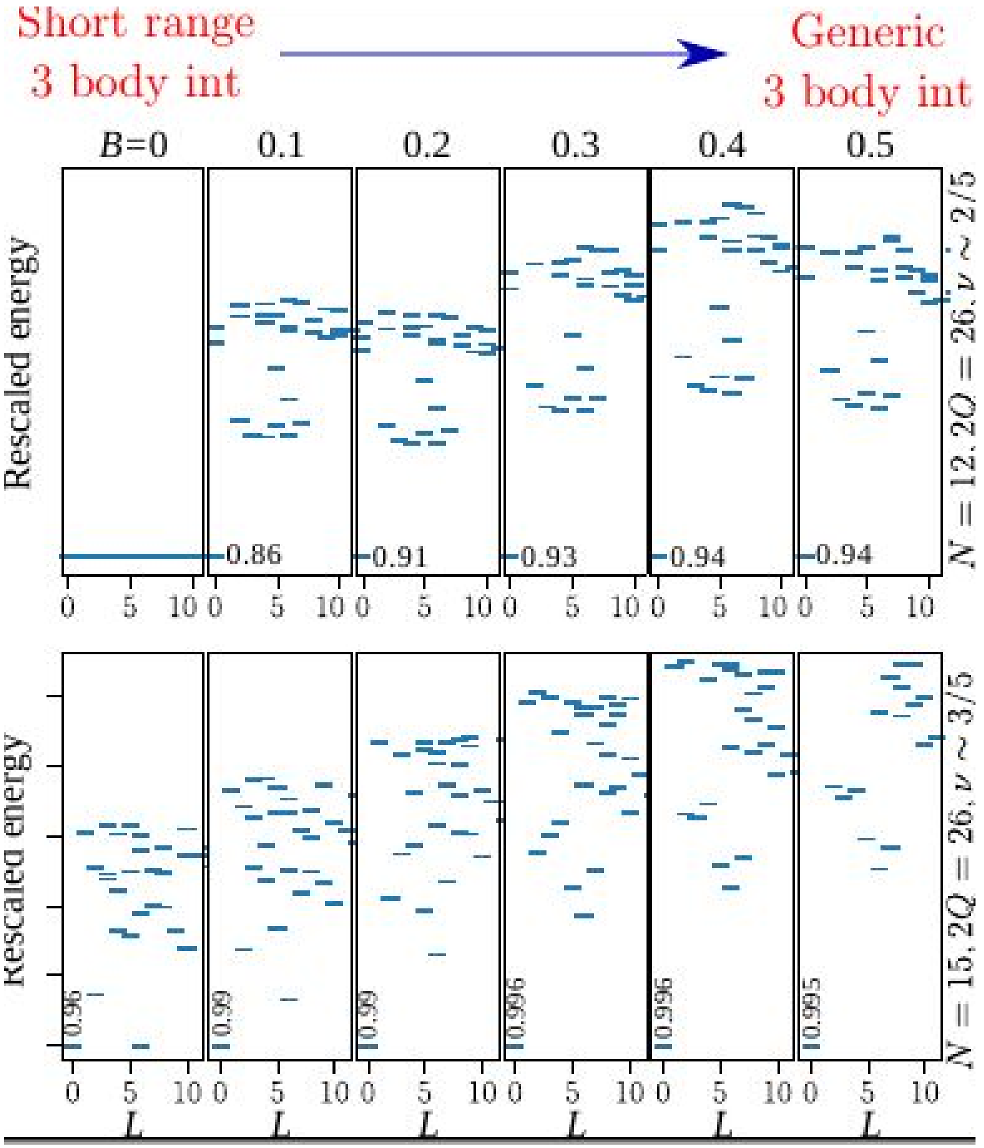
three-body interaction beyond the shortest range

Short range three-body interaction

$$H^{3BI-short} = \sum_{i<j<k=1}^{N} P^{ijk}_{L=3}$$

Generic three-body interaction

$$H^{3BI} = \sum_{i<j<k=1}^{N} \sum_{L} V_L P^{ijk}_{L}$$



GJS, Y Zhang, Jain PRB 2017

Mean field approximation to the 3-body interaction

Can the generic three-body interaction be approximated with a two-body interaction ?

$$H^{3BI} = \sum_{i<j<k=1}^{N} \sum_{L} V_L P_L^{ijk}$$

$$= \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c_{p_1}^\dagger c_{p_2}^\dagger c_{p_3}^\dagger c_{q_3} c_{q_2} c_{q_1}$$

Mean field approximation to the 3-body interaction

Can the generic three-body interaction be approximated with a two-body interaction ?

$$H^{3BI} = \sum_{i<j<k=1}^{N} \sum_{L} V_L P_L^{ijk}$$

$$= \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c_{p_1}^{\dagger} c_{p_2}^{\dagger} c_{p_3}^{\dagger} c_{q_3} c_{q_2} c_{q_1}$$

$$MF(H^{3BI}) = H^{(2)} = \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c_{p_1}^{\dagger} c_{p_2}^{\dagger} \left\langle c_{p_3}^{\dagger} c_{q_3} \right\rangle c_{q_2} c_{q_1}$$

For any gapped homogeneous FQH state

$$= \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c_{p_1}^{\dagger} c_{p_2}^{\dagger} \delta_{p_3 q_3} c_{q_2} c_{q_1}$$

Mean field approximation to the 3-body interaction

Can the generic three-body interaction be approximated with a two-body interaction ?

$$MF(H^{3BI}) = H^{(2)} = \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c^\dagger_{p_1} c^\dagger_{p_2} \left\langle c^\dagger_{p_3} c_{q_3} \right\rangle c_{q_2} c_{q_1}$$

For any incompressible homogeneous FQH state

$$= \sum_{\mathbf{p},\mathbf{q}} V_{p_1,p_2,p_3;q_1,q_2,q_3} c^\dagger_{p_1} c^\dagger_{p_2} \delta_{p_3 q_3} c_{q_2} c_{q_1}$$

$$MF(H^{3BI}) = H^{(2)} = \sum_{\mathbf{p},\mathbf{q}} \bar{V}_{p_1,p_2;q_1,q_2} c^\dagger_{p_1} c^\dagger_{p_2} c_{q_2} c_{q_1}$$

$$\bar{V}_{p_1,p_2;q_1,q_2} = \sum_{x} V_{p_1,p_2,x;q_1,q_2,x}$$

Mean field mapping: Properties

Linear $$MF(H_a) + MF(H_b) = MF(H_a + H_b)$$

Rotational symmetry:
If *H* is rotationally symmetric, the interaction *MF(H)* is rotationally symmetric.

$$MF(H) = \sum_{i<j=1}^{N} V_L^{\mathrm{2body}} P_L^{ij}$$

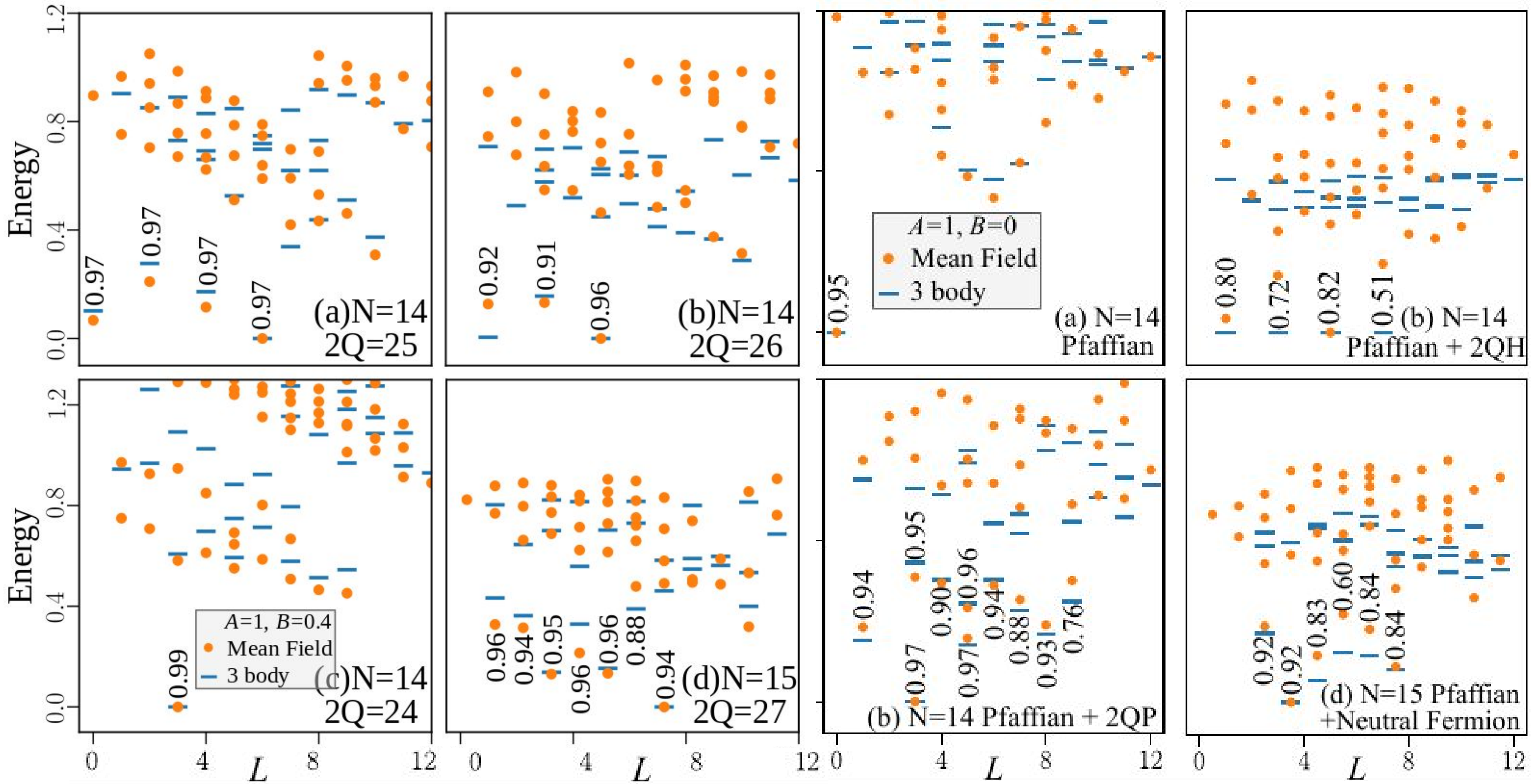
Thus *MF* is a well-defined linear mapping from three-body Haldane pseudopotentials to two-body Haldane pseudopotentials

PH symmetry: The mapping indeed gives a particle hole symmetric Hamiltonian by definition (it is a two-body interaction).
It indeed gives the particle hole symmetrization of the three-body interaction.

$$MF(H) = H + PHC(H)$$

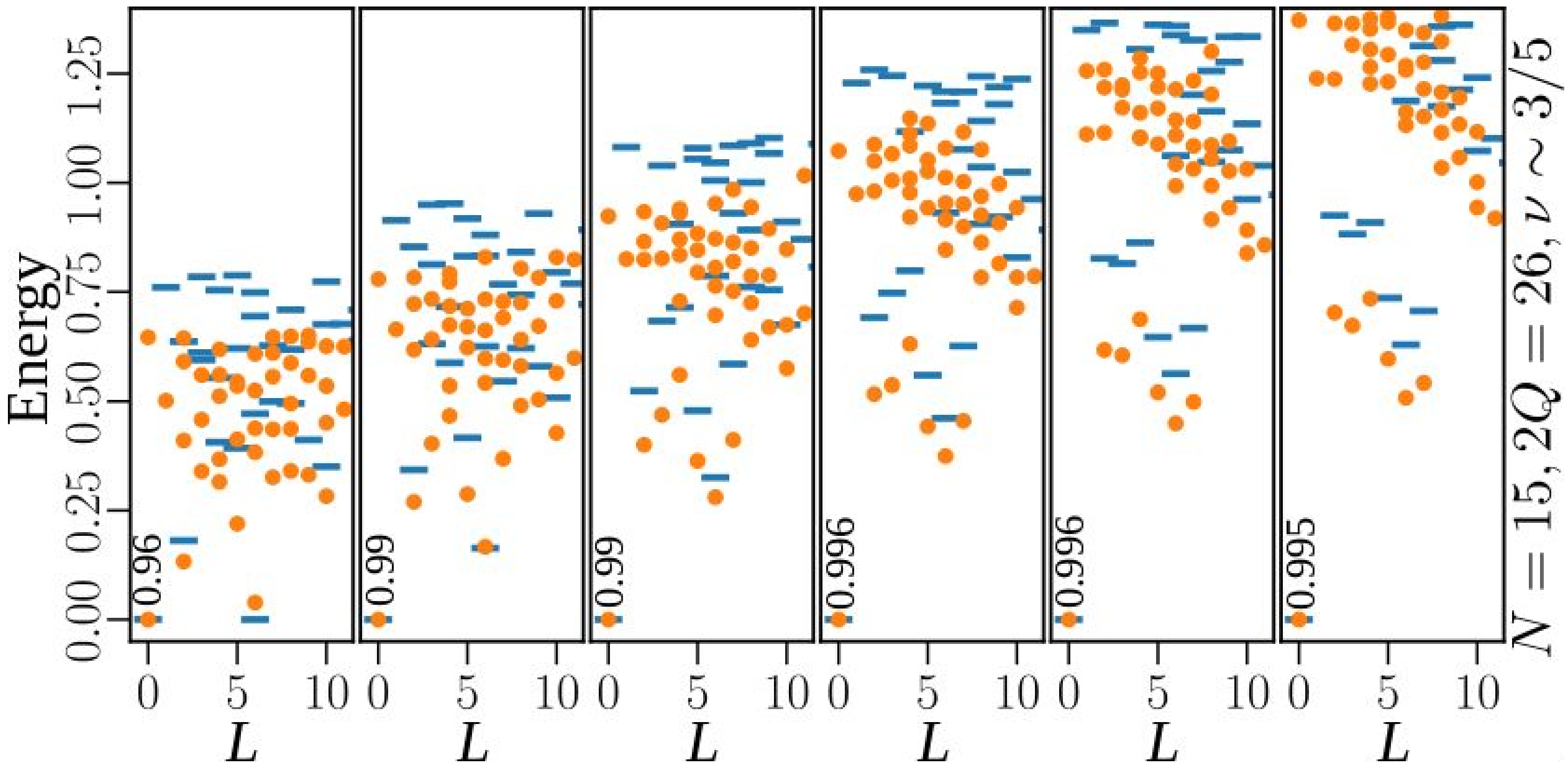
M Peterson, K Park, S D Sharma PRL 2008;
GJS, Y Zhang, Jain PRB 2017; Kusmierz, GJS (in prep);

Testing the mean field mapping



GJS,Y Zhang, Jain PRB 2017;

Testing the mean field mapping



Summary

Various short-range n-body terms in the Hamiltonian specify constraints on the many-particle correlations in the low-energy wavefunctions.

Generic n-body constraints on the many-particle wavefunctions imply and are implied by constraints on a smaller number of particles.

Fewer-body terms should reproduce the low-energy spectrum --> mean field approximation

Can be generalized to the $n>3$ body case. (Kusmierz, GJS (in preparation))

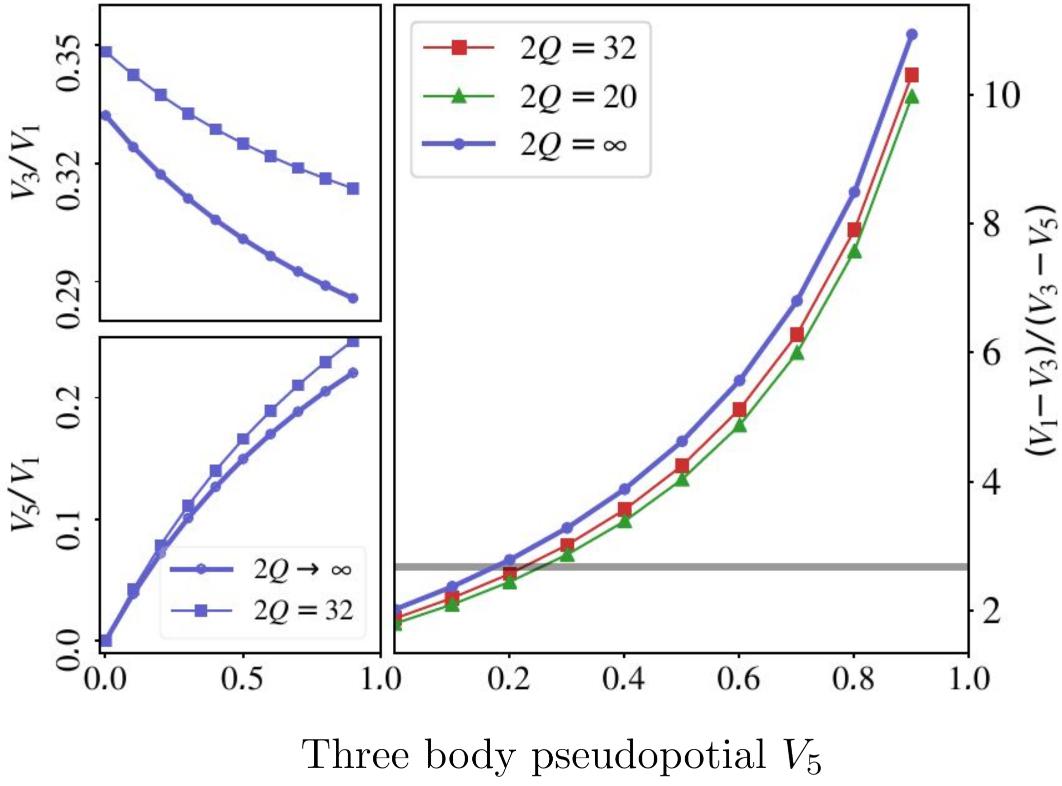
Mean field pseudopotentials

MF approximation to the short range three-body interaction gives the two-body Hamiltonian:

$$H = \sum_{ij} 3P^{ij}_{L=1} + 1P^{ij}_{L=3}$$

ie V_3=1 and V_1=3

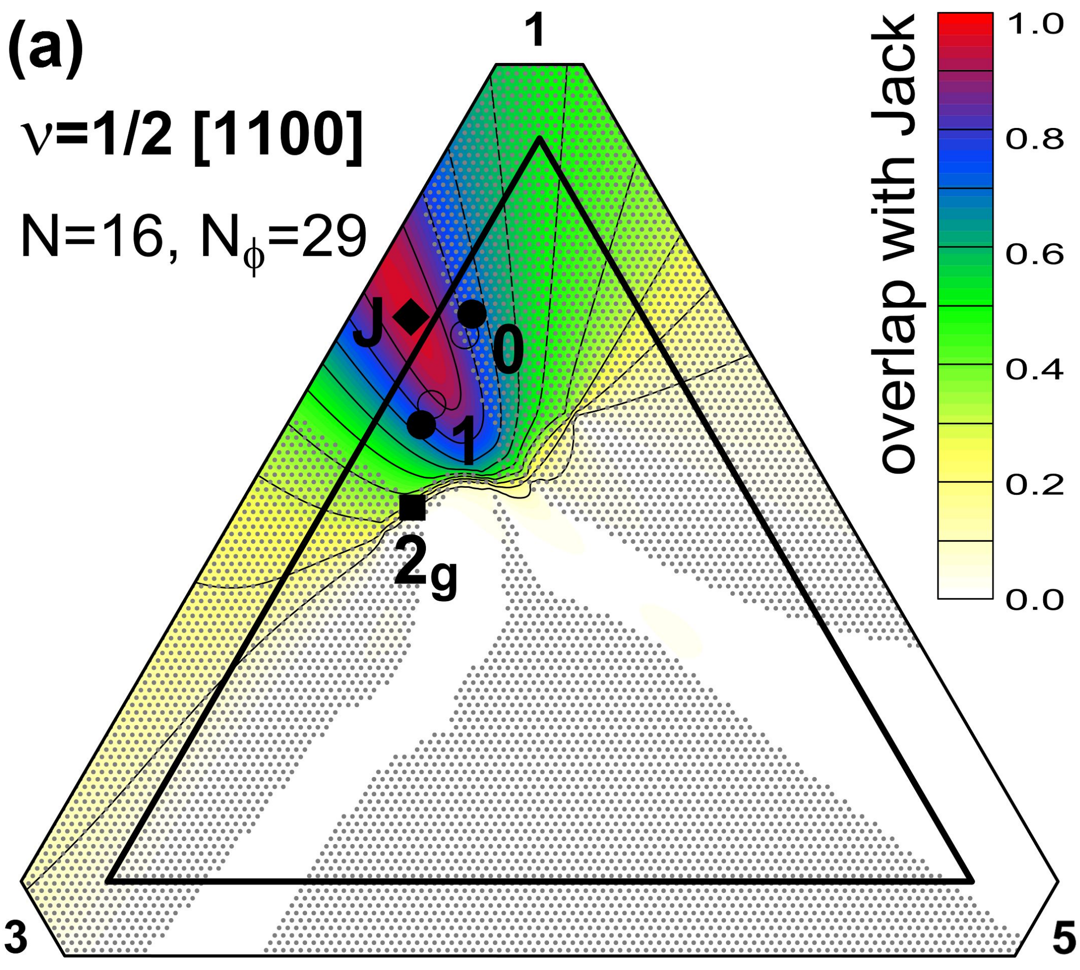
With increasing range, the mean field Hamiltonian contains one more pseudopotential.



GJS,Y Zhang, Jain PRB 2017;

Optimal two-body interaction for the Pfaffian state

Scan for an optimal two-body interaction that produces the Pfaffian ground state



Mean field approximation to three-body interaction gives the optimal parameters for two-body interaction that produces the Pfaffian state

A Wojs, B Kusmierz (unpublished)
B Kusmierz, G J Sreejith (in preparation)